Part I Paper 3 Quantitative Methods in Economics

Paper Co-ordinator: Prof. Alexei Onatskiy (ao319@cam.ac.uk)

Paper Content

This paper has two components (Mathematics and Statistics) with the unifying principle of developing an understanding of and practical fluency in basic analytical techniques widely used in studying empirical and theoretical problems in Economics. The paper outline sets out the lecture courses for the two components of the paper separately. Candidates are required to cover both components of the paper: they will not be able to pass by concentrating exclusively either on Mathematics or on Statistics. The three-hour written examination for the paper will contain separate sections on Mathematics and Statistics, each carrying 50% of the mark for the whole course: candidates will be required to answer questions from all sections of the paper.

Mathematics - Aims

Mathematical techniques are an indispensable tool of economics. Using mathematics, an economist can formalise and solve problems that cannot be addressed in other ways. The aim of this component of the paper is to cover the key areas of mathematics needed to allow candidates to tackle the compulsory papers of the Economics Tripos successfully. The range of techniques and tools is comparable to A-Level Further Maths. However, some of the material, and almost all the applications to economics, are new for the majority of students. Moreover, in order to equip the students with a more holistic understanding of the material covered, the paper will explore the logic underlying the mathematical machinery, and give a flavour of building and verifying mathematical arguments.

Mathematics - Objectives

By the end of the paper, students should have a good understanding of key mathematical concepts and techniques and be able to apply these to economic problems.

Mathematics - Content

The mathematics teaching for the paper will assume that candidates are familiar with the material set out below (which is basically the content of the Core Mathematics modules C1 – C4 of a standard A-Level Mathematics course):

Sets, notation and basic facts; Definition of integers, rational and real numbers; Indices; Pairs of simultaneous linear equations; Quadratic equations; Graphs of linear and quadratic equations, and simple coordinate geometry; Definition of function, domain, range and inverse function, composition of functions; An intuitive notion of limits and continuity; Differentiation and its geometric interpretation, rate of change; Natural logarithm and exponential function; Differentiation of natural log and exponential functions; Product, quotient and chain rules for differentiation; Differentiation of polynomial functions; Multi-variable functions and partial derivatives; Integrals as anti-derivatives, definite integrals; Sequences, series, sum of geometric progression; Unconstrained optimisation of a function of one variable; Integration of 1/x and exponential function; Integration by substitution and by parts; Vectors (addition, subtraction and scalar product)

Candidates who took mathematics qualifications other than A-level (for example, IB or European qualifications) should check that they have covered all of these topics: If they have not, they should contact their Director of Studies for further information and advice on reading.

The specific mathematical concepts and techniques covered in the course are:

Calculus and optimisation

Functions from reals to reals, limits, continuity, and differentiation. Polynomials, exponential and logarithm functions. Convexity and concavity. Unconstrained optimisation. Sequences and series.

Taylor series approximations. Integration as anti-derivative, definite integrals, connection with probability distributions.

Multivariable functions, partial derivatives, total differentials. Unconstrained optimisation of functions of more than one variable. Homogenous functions and Euler's theorem. Concave and convex functions of more than one variable. Lagrangian techniques in optimisation, economic applications and interpretation (such as the Lagrange multiplier as a shadow price).

Linear algebra

Vector spaces, linear transformations, and matrices. Solving systems of linear equations: matrix inversion. Elementary properties of determinants. Positive and negative definite matrices, eigenvalues: applications in unconstrained optimisation problems. Simple applications of linear algebra in Economics.

Difference and differential equations

Simple difference and differential equations, models of price and quantity adjustment.

Lecture courses

There are 20 lectures across the first two terms.

Mathematics for Economists: Intro to Calculus, Partial Differentiation, Constrained Optimisation (Prof A Onatskiy, 12 hours, Weeks 1-4 Michaelmas Term M. 11-12, Tu. 11-12. Weeks 5-8 M.11-12) and 8 hours, Weeks 1-8 Lent Term T. 12-1)
Introduction to Probability and Statistics (Dr D Robertson, 12 hours, weeks 1-4,: Th. 11-12, M. 12-1, weeks5-8: M.12-1)

Introduction to Statistical Inference (Dr M Weeks, 8 hours, Mon. 12 weeks 1-8)

Reading (* denotes primary text)

* Sydsaeter, K and P Hammond, *Essential Mathematics for Economic Analysis* (5th edition), Prentice Hall. Covers the course syllabus, at an appropriate level for the majority of students.
- Pemberton, M & N Rau, *Mathematics for Economists* (4rd edition), Manchester University Press. A good text for those who have done Further Maths modules at A Level, or who plan to take the optional Mathematics paper in Part IIA. Some material (roughly, Chapters 24-32) goes beyond the course syllabus.

Examination

The Mathematics component of the 3-hour examination for this paper has two sections, labelled A and B. Section A questions are short answer questions, testing mathematical techniques, while section B questions are typically framed by in the context of an economic application that may require multiple techniques and tools. Candidates must answer all four questions from section A, and one question (out of two) from section B.

The examination questions will focus on testing the understanding of and familiarity with the mathematical techniques covered in the lectures, their application to economic problems, and the interpretation of results. They will not require complex numerical calculations. Although candidates are permitted to use approved electronic calculators in the examination, examiners will not set numerical questions (for example, questions requiring the numerical inversion of matrices) which can be answered purely by using 'built-in' features of the calculator.

For details of the examination structure, please refer to the Form and Conduct Notice pages on Moodle.

Statistics - Aims

The statistical analysis of data is essential for the study of economic and social problems, and the discussion of issues of public policy. The aim of this component of the paper is to cover a range of basic statistical techniques which are both useful in their own right, and important in providing a foundation for the compulsory paper in Econometrics in Part IIA of the Tripos.

Statistics - Objectives

By the end of the course, students should be in possession of a good grasp of the elementary tools of descriptive statistics; should understand elementary principles of probability and statistical theory; should be competent in applying basic methods of statistical inference; and should be familiar with the use of spreadsheets to undertake graphical and statistical analysis of economic data.

Statistics - Content

The statistics teaching for the paper will assume that candidates are familiar with the basic material set out below (which is covered in the GCSE Mathematics paper):

Graphical techniques for representing data
Histograms, scatter diagrams, time series plots
Measures of central tendency for a dataset
Mean, median and mode

Candidates who took other mathematics qualifications should check that they have covered all of these topics: if they have not, they should contact their Director of Studies for further information and advice on reading.

The specific statistical concepts and techniques covered in the course are:

Descriptive statistics

The use of tables, graphs, diagrams and frequency distributions in summarizing and organizing statistical data; summary measures of central tendency, dispersion and skewness; simple measures of association.

Probability and distribution theory

Probability - events, outcomes and sample space; Venn diagrams; unions, intersections and complements; simple combinatorial formulae for sampling with and without replacement; conditional probability and Bayes' Theorem; the concept of a random variable.

Probability distributions – univariate discrete and continuous distributions; probability mass functions; cumulative distribution functions and probability density functions; expectations, variances and higher moments; expectation and variance of sums of independent random variables; Bernoulli trials and the Binomial distribution; simple discrete and continuous probability distributions, particularly Uniform and Normal distributions; Chi-squared, t and F distributions.

Sampling distributions - the use of sample statistics: the concept of an estimator; unbiasedness and efficiency; sampling distributions (large samples) - Law of Large Numbers and Central Limit Theorem (proofs not required); sample mean, sample variance, difference between sample means, difference between sample proportions; sampling distributions (small samples from parent normal populations) - sample mean.

Estimation and inference

Estimation and hypothesis testing - a simple treatment of point and confidence interval estimation and hypothesis testing (in each case the sample statistics used are those enumerated above under 'sampling distributions'); null and alternative hypotheses; critical regions; one-tailed and two-tailed tests; Type I and Type II errors; power functions.

Bivariate distributions and bivariate regression - bivariate probability distributions; the bivariate Normal distribution; conditional and marginal probability distributions; conditional expectation; statistical estimation of bivariate models where errors are independently and normally distributed with common variance; sampling distributions of regression coefficients under these assumptions; testing of simple hypotheses about regression coefficients; distribution of correlation coefficient under the null of zero correlation, and associated tests for significance.

Multiple regression - interpretation of multiple regression coefficients; dummy variables; significance tests for individual regression coefficients; graphical analysis of regression residuals.

Computational statistics - the use of spreadsheet packages to store and organise economic data, to generate simple graphs, and to compute the statistics outlined above.

Many of the concepts and techniques set out above are covered in Modules S1-S4 of the A-level courses in Mathematics and Further Mathematics. Students may find it helpful to bring their A-Level (or equivalent) notes, and any textbooks, with them to Cambridge. A detailed syllabus, which relates the material set out above to the content of Modules S1-S4, is available on the course website.

Lecture courses

There are 20 lectures across the first two terms.

Introduction to Probability and Statistics – Probability and Distributions, Hypothesis Testing (Dr D. Robertson, 12 lectures, weeks 1-8, Michaelmas Term)
Introduction to Statistical Inference – Correlation and Regression (Dr M. Weeks, 8 lectures, weeks 1-8, Lent Term)

Reading (* denotes primary text)

The following texts are recommended for the Statistics component of this paper: since they all cover broadly the same material you should choose one text which you feel is at the appropriate level for you.

* Ross, S M, *Introductory Statistics* (3rd edition), Academic Press.
- Larsen, R J and M L Marx, *An Introduction to Mathematical Statistics and its Applications* (5th edition), Pearson. This text adopts a more formal mathematical approach, and is therefore more suitable for those with Further Maths A level.
- Lind, D, W Marchal and R Mason, *Statistical Techniques in Business and Economics* (11th edition), McGraw-Hill. This text adopts a more practical approach to statistics, and provides relatively little mathematical detail.
- Mann, P S, *Introductory Statistics* (7th edition), Wiley.
* Goldberger, A, *Introductory Econometrics* Harvard University Press. This is the recommended text for the section of the course which covers correlation and regression analysis.
Pindyck, R and D Rubinfeld, *Econometric Models and Economic Forecasts* (4th edition), McGraw-Hill.

Examination

The Statistics component of the 3-hour examination for this paper has two sections, labelled C and D. Section C questions are short answer questions, while section D questions are longer and require more

detailed answers. Candidates must answer all four questions from section C, and one question (out of two) from section D.
Past examination papers for this paper are available on the Faculty website. Note that for examinations up to and including 2008 candidates were offered a choice of questions in Section C. In 2009 and thereafter only four questions were set, and candidates were required to answer all of these.

The examination questions will focus on testing the understanding of and familiarity with the statistical techniques covered in the lectures, their application to economic data, and the interpretation of results. They will not require the entry of large datasets, or long and complex numerical calculations. Although candidates are permitted to use approved electronic calculators in the examination, examiners will not set numerical questions (for example, the calculation of correlation or regression coefficients from raw data) which can be answered purely by using 'built-in' features of the calculator.

For details of the examination structure, please refer to the Form and Conduct Notice pages on Moodle.