Interest-Rate Policy and Stability of Banking Systems*

Hans Gersbach

CER-ETH—Center of Economic Research
at ETH Zurich and CEPR
Zürichbergstrasse 18
8092 Zurich, Switzerland
hgersbach@ethz.ch

Jan Wenzelburger

Centre for Economic Research
Keele University
Keele, ST5 5BG, UK
j.wenzelburger@econ.keele.ac.uk

Keele Economic Research Papers No. 17

Abstract

We investigate a banking system subject to repeated macroeconomic productivity shocks. By lowering the interest rates, the central bank can increase the intermediation margin, which fosters the recapitalization of banks. We show that without lowering interest rates, the banking system faces the risk of collapsing. When interest policy is aimed only at avoiding a banking collapse, the economy converges with certainty to a consumption trap. In the consumption trap, the entire bank savings are needed to cover the banks' obligations and GDP is minimal. We present a positive analysis of conditions for interest-rate policies which resolve banking crises. Moreover, we provide an explanation of why banking crises such as the crisis in Japan may cause long-lasting economic downturns.

Keywords: Financial intermediation, macroeconomic risks, banking crises, deposit insurance, banking regulation.

JEL Classification: D41, E4, G2.

First version: Feb. 2007, this version: Jul. 2009.

*A previous version was entitled "The Dynamics of Deposit Insurance and the Consumption Trap" and has been published as Discussion Paper No. 343 at the University of Heidelberg and as CESifo Discussion Paper No. 509. We would like to thank Volker Böhm, Domenico Delli Gatti, Ulrich Erlenmaier, Jean-Charles Rochet, Gilles Saint-Paul, Oz Shy, Lars Siemers, Rune Stenbacka, Eva Terberger-Stoy, Jean Tirole, Anne Villamil, participants of seminars in Bielefeld, Heidelberg, Helsinki, Milan, Toulouse, at the CESifo conference in Munich 2001.

1 Introduction

Motivation

The frequency of severe banking crises has increased significantly over the last few decades. The current crisis will be the most costly ever. A banking crisis occurs when a large number of banks either fail to meet regulatory capital requirements or are illiquid, or even insolvent. There are at least four empirical facts concerning banking crises which are important from a macroeconomic perspective. Firstly, banking crises are most often caused by economic downturns. In differentiating between the sunspot view and the business-cycle view of banking crises, Gorton (1988) shows in a seminal empirical investigation that bank panics are systematically linked to business cycles. Subsequent work by González-Hermosillo et al. (1997), Kaminsky & Reinhart (1999), and Demirgüç-Kunt & Detragiache (1998) identify a number of factors causing financial fragilities which may ultimately lead to a systemic banking crisis. These results suggest that banking crises tend to occur when the macroeconomic environment is weak, particularly when output growth is low.

Secondly, the literature provides ample evidence that the costs of banking crises in terms of GDP losses may become very large, even if banks are allowed to continue to operate (e.g., see Caprio & Klingebiel (1997), Lindren, Garcia & Saal (1996), Caprio & Honohan (1999), or von Peter 1999). Further negative real output effects of banking crises beyond economic downturns have been identified in an interesting study by Dell'Ariccia, Detragiache & Rajan (2007).

Thirdly, many contributions (e.g., see Dekle & Kletzer (2003) and Hoshi & Kashyap 2004) suggest that the persistent weakness of the Japanese banking sector has been responsible for Japan's long-lasting recession after 1990. According to Dekle & Kletzer (2003), four important characteristics are held responsible for this dismal development: the predominance of commercial bank intermediation characterizing Japan as a bank-centered financial system, the prospect of deposit insurance implicitly guaranteed by the government, regulatory forbearance that allowed banks to operate without fulfilling regulatory requirements, and the low profitability of the Japanese banking sector for more than 10 years.

Finally, some countries have responded to banking crises and the associated economic downturns by lowering short-term interest rates. This has long been advocated by Hellman, Murdock & Stiglitz (2000), Krugman (1998) and many others. Indeed, the US Federal Reserve Bank, the Bank of England and the European Central Bank have responded to the recent subprime crisis by lowering short-term interest rates. The Bank of Japan lowered nominal interest rates in the nineties quite drastically which lead to gradual reduction of the real interest rate, cf. Hoshi & Kashyap (2004). The nominal short-term interest rate reached virtually zero in February 1999 and remained close to zero for many years except for a brief period. Real interest rates, which were high in the first half of the nineties, declined and fluctuated between zero and two percent in the second half of the nineties.

Model

The empirical findings motivate the present paper in which we develop a dynamic macroeconomic model with financial intermediation. This model allows us to investigate the interdependence between banking competition, financial stability and central bank's interest policy. It complements earlier business cycle models with financial intermediation with the seminal contributions of Boyd & Prescott (1986) and Williamson (1987) and the more recent approaches by Uhlig (1995) and Schreft & Smith (1997).

We consider an overlapping generations (OLG) model in which each generation consists of two types of agents, consumers and entrepreneurs, who live for two periods. In each period, the output of entrepreneurs is subject to an exogenous macroeconomic productivity shock. Entrepreneurs are financed by banks who offer deposit and loan contracts. Banks are delegated monitors in the sense of Diamond (1984), that is, they monitor borrowers and alleviate the moral hazard problems of entrepreneurs. According to Hellwig (1994), alleviating such agency problems constitutes a major market friction that necessitates financial intermediation, and this is a core activity of commercial banks which dominate in countries like Japan. In accordance with current practice, we assume that deposit contracts are implicitly insured by the government and, following Hellwig (1998), that deposit contracts cannot be conditioned on macroeconomic events. Furthermore, we assume that depositors believe that deposits are implicitly guaranteed by the government. Banks face double-sided Bertrand competition capturing a situation in which the profitability of a banking industry is low. There is regulatory forbearance that allows banks to operate without fulfilling capital requirements. The central bank may cut interest rates thereby lowering deposit rates which, in turn, increases the intermediation margin and fosters the recapitalization of banks.

Two key notions are of central importance for the model of this paper: banking crisis and banking collapse. A state of the economy will be called a *banking crisis* when the capital basis of the banking system is below some well-specified regulatory level. In such a state banks can continue to operate as their capital basis is still positive and they have enough funds to pay back depositors. A state of the economy in which banks are unable to pay back depositors is called *banking collapse*. We choose an OLG economy as a convenient setting in which deposit withdrawals to finance old-age consumption occur naturally and allowing the investigation of the effects of a banking crisis and interest rate policies on later generations.

Results and Literature

The present paper contains three main results. Firstly, without interest rate cutting, the banking system faces the risk of collapsing. This risk is caused by the asymmetric impact of unobservable macroeconomic shocks on bank capital and the inability of the banking system to accommodate credit losses. Adverse shocks may cause a decline in the repayment capacity of entrepreneurs and incur credit losses for banks. These may increase over time in such a way that a banking collapse becomes inevitable, as banks do not recapitalize sufficiently with positive macroeconomic shocks.

Secondly, we establish precise conditions under which a banking crisis can be resolved by means of interest rate cutting by the central bank. These lower the refinancing costs of banks and thus increase their capability to accumulate earnings for recapitalization. Although an interest-rate cutting can always avoid a banking collapse, it must create intermediation margins that are high enough to compensate banks for credit losses.

Thirdly, even with the most drastic interest-rate policy which is obtained by setting interest rates to zero, an economy with a banking crisis may experience a long-lasting recession. As a large share of new funds from the young generation must be used to fulfill old liabilities, aggregate investment and hence aggregate income in subsequent periods will be suppressed. The banking system can only gradually recover from the crisis, because its earnings are low. As a consequence, the economy may remain in a state of low aggregate income for many periods. We identify a critical level of bank capital below which the lowering of deposit interest rates by the central bank is necessary to ensure that no banking crisis occurs. This level of bank capital is determined by the fundamentals of the economy and might serve as a guideline for determining bank capital requirements.

Overall, our results may help to explain why banking systems with characteristics similar to those of the Japanese system might exhibit persistent weakness and to what extent interest-rate policies are capable of resolving banking crises. So far the issue of banking crises in the presence of repeated macroeconomic productivity shocks has received relatively little attention in the literature. One important contribution is

Blum & Hellwig (1995) who show that strict capital adequacy rules may reinforce macroeconomic fluctuations. Our analysis suggests that capital requirements can serve as an indicator of when to intervene with measures that promote recapitalization of banks.

There is a large microeconomic literature on bank runs building on the seminal contributions of Diamond & Dybvig (1984) and Allen & Gale (1998, 2004). It is therefore important to emphasize that banking crises in our model are not triggered by bank runs as, for example, in Ennis & Keister (2003). While bank runs may explain banking crises such as the one in Argentina in 2000, the banking crisis in Japan cannot be attributed to bank runs and for this reason requires a different explanation. In our model bank runs cannot occur, since informational externalities are absent and deposit contracts last for one period only so that there is no uncertainty about deposit withdrawals.

The paper is organized as follows. After the introduction of our model in Section 2, Section 3 treats the evolution of the banking system. We discuss measures to avoid a banking collapse in Section 4 and focus on the consumption trap in Section 5. Feasible policy measures that resolve and prevent banking crises are discussed in Section 6. Alternative forms of interventions are discussed in the concluding Section 7. All technical proofs are found in the appendix. Appendix C contains an example which can be solved analytically.

2 Model

2.1 Entrepreneurs and depositors

Consider an overlapping generations (OLG) model with one physical good that can be used for consumption or investment. The OLG structure can also be viewed as an economy in which agents with infinitely long lives optimize myopically.

Time is infinite in the forward direction and divided into discrete periods indexed by t. Each generation consists of a continuum of agents with two-period lives, indexed by $[0, 1]$. Each agent of each generation receives an endowment e of goods when young and none when old. The endowment may be thought of as being obtained from short-term production with inelastically supplied labor. Generations are divided into two classes. One fraction of agents, indexed by $[0, \eta]$, are potential entrepreneurs. The other fraction, indexed by $(\eta, 1]$, are consumers. Potential entrepreneurs and consumers differ in that only the former have access to investment technologies.

Consumers are endowed with intertemporal preferences over consumption with c_t^1, c_t^2 denoting youthful and old-age consumption of a typical consumer born in period t, respectively. For simplification, let $u(c_t^1, c_t^2) = \ln(c_t^1) + \delta \ln(c_t^2)$ be the intertemporal utility function of a consumer, where $\delta\,(0 < \delta < 1)$ is the discount factor. Each young consumer is a depositor at commercial banks. If a consumer faces a particular deposit rate and assumes that deposits including interest rates will be paid with certainty, the well-known solution of the consumer's intertemporal optimization problem yields inelastic savings: $s = \frac{\delta e}{1+\delta}$. Aggregate savings of the consumer sector is $S = (1-\eta)s$.

Potential entrepreneurs are assumed to be risk-neutral and consume only when old. Each entrepreneur has to decide whether to save her endowment or to invest in a production project that converts period-t goods into period-$t+1$ goods. The funds required for each investment project are fixed to $e + I$ so that an entrepreneur must borrow I additional units of the good from banks to undertake the investment project. Entrepreneurs are heterogeneous in the quality of their investment projects which depends on their index $i \in [0, \eta]$. The quality parameter of entrepreneur i is assumed to be private information. If entrepreneur i obtains additional resources I and decides to invest, her output y_i in the next period is determined by

$$y_i = q\,(1+i) f(e+I).$$

Here f denotes a standard atemporal neoclassical production function and the parameter $q \in \mathbb{R}_+$ describes the unobservable macroeconomic shock which affects the productivity of each entrepreneur and thus causes fluctuations of aggregate output.

The depositors of the economy in an arbitrary period t consist of all consumers and those entrepreneurs who save; its borrowers consist of entrepreneurs who invest. In an OLG economy deposit withdrawals of the old generation occur naturally to finance old-age consumption and banks may use new funds from a young generation of depositors to balance old liabilities. Banks in an OLG economy may thus continue to operate even when their equity is negative.

2.2 Banking sector

As depositors cannot observe the quality parameters of entrepreneurs and cannot verify whether or not an entrepreneur invests, our economy faces market frictions that according to Hellwig (1994) necessitate financial intermediation. To alleviate these

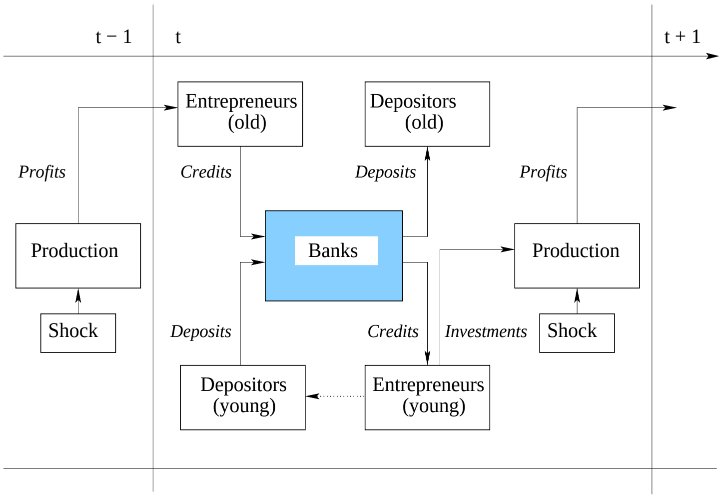

Figure 1: Sectors of the economy.

market frictions, we assume that there are $n > 1$ (commercial) banks, indexed by $j = 1, \ldots, n$. Banks finance investment projects as delegated monitors in the sense of Diamond (1984) as they ensure that entrepreneurs invest and as they secure repayments which in case of a default is the liquidation value of an investment project. For simplicity, monitoring is assumed to be costless.[1] Banks maximize profits accruing to current shareholders which are the old entrepreneurs. Bank ownership is transferred to the next generation of entrepreneurs through bequests.[2] The interaction between depositors, entrepreneurs, and banks within a typical period t is depicted in Fig. 1.

In each period, a (financial) intermediation game takes place in the form of a *double-sided Bertrand competition* between banks who offer deposit and loan contracts. Each bank j can sign deposit contracts $D(r_j^d)$, where $1 + r_j^d$ is the repayment offered for 1 unit of resources. Loan contracts of bank j are denoted by $C(r_j^c)$, where $1 + r_j^c$ is the repayment required from entrepreneurs for 1 unit of funds. All deposits and loan contracts last for one period. The balance sheet of a bank at the beginning of a typical period consists of liabilities in the form of deposits, assets in the form of loan repayments, and bank capital.

[1]Monitoring may be thought of as inspecting entrepreneurs' cash flow when customers pay, or efforts to collateralize assets created in the process of investing. The simplest way to model such a technology explicitly is to assume that banks, at the monitoring costs of $m \geq 0$ per loan, can secure a repayment of γI, $0 < \gamma < 1$ from entrepreneurs who received a loan of size I and shirk. For sufficiently high γ, entrepreneurs who borrow will invest. In equilibrium, the interest rate spread will then cover the monitoring costs. To simplify the exposition, we assume that monitoring outlays per credit contract are negligible.

[2]This feature will be further discussed at the end of Section 2.4.

Entrepreneurs are contract takers operating under limited liability. Given a loan interest rate r^c, the expected profit of an entrepreneur i investing is

$$\Pi(i, r^c) := \int_{\mathbb{R}_+} \max\Big\{q\,(1+i)f(e+I) - I(1+r^c), 0\Big\}\, h(q)\mathrm{d}q,$$

where $h\,(q)$ denotes the probability density function describing the distribution of the unobservable macroeconomic shocks. Note that $\Pi(i, r^c)$ is non-decreasing in quality levels i and non-increasing in loan rates r^c. Risk-neutral entrepreneurs will choose the most profitable contract offered. Given loan and deposit contracts of banks, an entrepreneur i will apply for a loan at bank j_0, if

$$r^c_{j_0} = \min_{1\le j\le n} \big\{r^c_j\big\} \quad \text{and} \quad \Pi(i, r^c_{j_0}) \geq e \max_{1\le j\le n}\{1 + r^d_j\}. \tag{1}$$

Otherwise she will choose the bank with the best deposit contract and save her endowments. Since banks are able to secure repayments, they do not have to worry about low-quality entrepreneurs applying for loans. Low-quality entrepreneurs are always better off with saving endowments.[3] Depositors and entrepreneurs choose randomly between banks which offer the same contract. We assume that aggregate uncertainty in these choices are canceled out such that banks which offer the same contract obtain the same market share of loans or deposits, respectively.[4]

Throughout the remainder of the paper we presume two institutional features which are in place in most countries. Firstly, deposits are implicitly guaranteed by the government and depositors trust the guarantees. Secondly, deposit contracts are not conditioned on productivity shocks such that the macroeconomic risk remains on the balance sheets of banks. These institutional features have been discussed extensively in the literature, e.g., see Hellwig (1998). We take them as given.

2.3 Intermediation equilibria

The Bertrand competition comprises two possible intermediation games. In the first scenario, banks set deposit and loan interest rates. In the second scenario, the central bank affects the intermediation equilibrium by lowering interest rates.

The modelling of central bank policy deserves comments. We built a real model in order to highlight how banking crises can occur and may be alleviated by interest rate

[3]The case in which banks are allowed to screen applicants and to price loan risk contingent on default risk is left for future research.

[4]The exact construction of individual randomness so that this statement holds can be found in Al-Najjar (1995) and Alós-Ferrer (1999).

policies. In line with the New-Keynsian models, we assume that the central bank can vary real interest rates as prices only adjust gradually when nominal interest rates are set. Changes in real interest rates by the central banks translate into changes of financing costs of banks and thus into changes of the deposit rates. We therefore associate changes in the real deposit rate to changes in the central bank interest rate policy. We call r^d_{CB} the deposit rate induced or set by the central bank.[5]

In view of Fig. 1, the time-line of actions within a typical period t is as follows:

1. Old entrepreneurs pay back with limited liability.

2. In the first scenario, banks set interest rates on deposits and loans. In the second scenario, banks' realized profits are too low or they have made losses. Then the central bank lowers interest rates. Banks will set interest rates on loans. In both scenarios, banks offer deposit contracts to consumers and deposit and credit contracts to entrepreneurs.

3. Consumers and entrepreneurs decide which contracts to accept. Resources are exchanged, and banks pay back depositors. The current level of bank capital is determined. Excess bank capital is distributed among shareholders.

4. Young entrepreneurs produce subject to an unobservable macroeconomic productivity shock.

The solution of the intermediation game in an arbitrary period t will be referred to as a (temporary) intermediation equilibrium. The economically meaningful and interesting situation is when aggregate savings of consumers are never sufficient to fund all entrepreneurs. Since the interest rate elasticity of aggregate savings of consumers is zero, this condition takes the form

$$S := (1 - \eta)\, s < \eta\, I.$$

Let d denote the current capital level of the banking system. Assume, for a moment, that there exists a critical entrepreneur i_E such that all entrepreneurs $i \in [0, i_E)$ save

[5]The interest rate set by the central bank and the deposit rates could be identical. For instance, if banks can refinance themselves unboundedly at the central bank, it would not be profitable for banks to offer a higher deposit rate than the interest rate set by the central bank. Moreover, competition among banks will induce that they do not offer a lower deposit rate than the one set by the central bank.

their endowments whereas all entrepreneurs $i \in [i_E, \eta]$ invest. The fundamental balance between total bank savings and the loan volume then is

$$S + i_E\, e + d = \big[\eta - i_E\big]\, I. \tag{2}$$

Total bank savings are composed of aggregate savings of consumers S, aggregate savings of entrepreneurs who do not invest $i_E\, e$, and the capital of the banking system d. These three sources of funds are used to finance investing entrepreneurs. Aggregate investments in the economy consists of the loan volume and the equity of investing entrepreneurs $[\eta - i_E]e$.[6]

The balance equation (2) imposes two boundary values for d. We use $\overline{d} := \eta I - S > 0$ to denote the value of bank capital that would allow all entrepreneurs $i \in [0, \eta]$ to invest, since $S + \overline{d} = \eta I$. If $d > \overline{d}$, then banks have more capital than is needed to finance all entrepreneurs, and excess resources are available independent of any interest rates. Similarly, let $\underline{d} := -[S + \eta e]$ denote the lowest level of (negative) bank capital that still allows for a balance of liabilities in a particular period. If $d = \underline{d}$, then all entrepreneurs $i \in [0, \eta]$ are required to save in order to pay back obligations to the previous generation. This prevents any financing of new investment projects. $d > \underline{d}$ ensures that there are enough funds to meet the liabilities of the previous generation and to finance new investment projects. If $d < \underline{d}$, then the banking system can no longer fulfill its obligations and is declared bankrupt. This situation will be referred to as a *banking collapse*.

An intermediation problem arises when $d \in [\underline{d}, \overline{d}]$. For each $d \in [\underline{d}, \overline{d}]$ there exists a unique $i_E \in [0, \eta]$, referred to as the *critical entrepreneur* and given by

$$i_E = i_E(d) := \frac{\overline{d} - d}{e + I} = \frac{\eta I - S - d}{e + I}, \tag{3}$$

such that loans are balanced by total bank savings as expressed in equation (2). In Fact 1, Appendix A, we establish existence and uniqueness of two types of *intermediation equilibria*. In an intermediation equilibrium *without interest rate cutting*, all banks set the same loan and deposit interest rate $r_* = r_*(d) > 0$ depending solely on the capital level d such that for each bank $j = 1, \dots, n$,

$$r_j^{d*} = r_j^{c*} = r_*. \tag{4}$$

In an intermediation equilibrium *with interest rate cutting*, the central bank lowers deposit interest rates $r_{CB}^d < r_*$ when the banking system violates certain capital requirements, which will be made explicit below. Fact 1 in Appendix A states that in

[6]Note that aggregate savings in the economy are composed of the sum of total bank savings and the equity of investing entrepreneurs.

this case all banks set the same loan interest rate $r_*^c = r_*^c(d, r_{CB}^d)$ which depends on the capital level d as well as on the interest rate r_{CB}^d, so that for each bank $j = 1, \dots, n$,

$$r_j^{d*} = r_{CB}^d < r_* \quad \text{and} \quad r_j^{c*} = r_*^c > r_*. \tag{5}$$

Fact 1 states that in both intermediation games total bank savings and loan volume are balanced, such that all entrepreneurs with quality levels $i < i_E(d)$ save, while all those with quality levels $i \geq i_E(d)$ invest.[7]

For the purpose of this paper we will use the properties of equilibrium interest rates summarized in the following Corollary:

Corollary 1
Equilibrium interest rates of the intermediation game satisfy the following:

(i) $r_*(\cdot)$ *is decreasing in capital levels* $d \in [\underline{d}, \overline{d}]$*;*

(ii) $r_*^c(\cdot, r_{CB}^d)$ *is decreasing in capital levels* d *for each* $r_{CB}^d \in [0, r_*(d)]$*;*

(iii) $r_*^c(d, \cdot)$ *is decreasing in* $r_{CB}^d \in [0, r_*(d)]$ *for each* $d \in [\underline{d}, \overline{d}]$*;*

(iv) for each $d \in [\underline{d}, \overline{d}]$, $r_*^c\big(d, r_*(d)\big) = r_*(d)$ *and* $r_*^c(d, r_{CB}^d) > r_*(d)$ *for all* $r_{CB}^d \in \big[0, r_*(d)\big)$.

By Corollary 1, interest-rate cutting with $r_{CB}^d < r_*$ will lead to positive intermediation margins $r_*^c - r_{CB}^d > 0$ for all banks. By lowering deposit interest rates, the central bank will thus increase banks' profits.

The critical feature of the model is that without interest rate cutting, banks receive no premium for the macroeconomic risk of their loans, so that $r_j^{c*} - r_j^{d*} = 0$ for all banks j. Without interest rate cutting, (4) shows that the intermediation game yields the competitive outcome in which total bank savings and loans are balanced at a common interest rate for loans and deposits. The reason for this effect is double-sided Bertrand competition which allows entrepreneurs to switch market sides and save their endowments when offered unfavorable loan contracts. The assumption of Bertrand competition thus captures a benchmark situation of a banking industry with low profitability as, for example, has been the case in Japan or in the recent worldwide banking crises 2008 when many banks in Europe or in the US had low er even negative profits. Gersbach & Wenzelburger (2008) show that the initial result of this paper,

[7] The intermediation game will be further discussed in Appendix A.

namely that interest rate cutting may become necessary to avoid a banking collapse, does not depend on the fact that banks' intermediation margins are zero. It carries over to the case when intermediation margins reflect positive but too low premia on macroeconomic risks. In order to investigate how central bank's interest policy may help to avoid and resolve banking crises, we use the current, simpler framework.

Observe that an interest-rate cutting with $r^d_{CB} = 0$ is equivalent to allowing the banking industry to form a cartel. In such a cartel, aggregate profits are maximized, which is equivalent to setting $r^d_* = 0$ and r^c_* such that total bank savings and loans are balanced. We focus directly on controlling short-term real interest rates. During the Japanese banking crisis, the short-term nominal interest rates have been set at zero. During deflation the real interest rate in Japan was of course higher than zero, but declined considerably in the second half of the nineties. During the recent worldwide banking crises, the major central banks have substantially lowered short-term nominal interest rates, thereby lowering short-term real interest rates as inflation expectations remained low.

2.4 Profits and capital of banks

As an immediate consequence of Fact 1, all banks are identical and adopt the same strategies in an intermediation equilibrium. We can therefore focus directly on the banking system and proceed by calculating its profits and the amount of capital at the end of a period. We present the key economic equations for the case in which the central bank sets r^d_{CB}. The case without interest rate cutting is then obtained as a special case by setting $r^d_{CB} = r_*(d)$, which in turn implies $r^c_* = r_*(d)$.

Let $d \in [\underline{d}, \overline{d}]$ be the current capital level at the beginning of an arbitrary period and $r^d_{CB} \leq r_*(d)$ and $r^c_*(d, r^d_{CB})$ be the corresponding equilibrium interest rates. Since loans are balanced by total bank savings, the banking system raises funds $S + e\, i_E(d)$ that have to be paid back with interest at the end of the subsequent period. The loan volume of the banking system is given by $[\eta - i_E(d)]I$.

At the end of the period, banks will receive repayments from all entrepreneurs who have invested. These will be denoted by $P = P(d, q, r^d_{reg})$ and are given by

$$P\big(d, q, r^d_{CB}\big) = \int_{i_E(d)}^{\eta} \min\Big\{q\,(1+i)f(e+I),\; I\big[1 + r^c_*(d, r^d_{CB})\big]\Big\}\mathrm{d}i. \tag{6}$$

The repayments P depend significantly on the macroeconomic productivity shock q. Given a macroeconomic shock q, the *capital function* G of the banking system is defined by

$$G(d,q,r^d_{CB}) = P(d,q,r^d_{CB}) - [S + e\, i_E(d)]\,(1 + r^d_{CB}). \tag{7}$$

$G(d,q,r^d_{CB})$ is the level of bank capital (or equity) of the banking system at the end of the period *before* dividends are paid out to current shareholders. The end-of-period profits of the banking system are given by the change in capital that the banking system experiences in the period under consideration and hence are $G(d,q,r^d_{CB}) - d$. To focus on the evolution of bank capital, it is convenient to work with the capital function rather than with the profit function.

Aggregate losses of the banking system are determined by the difference between full repayments of all entrepreneurs and actual repayments P and are given by

$$L(d,q,r^d_{CB}) = [\eta - i_E(d)]I\left[1 + r^c_*(d)\right] \; - \; P(d,q,r^d_{CB}). \tag{8}$$

Using (2), the capital function (7) now takes the convenient form

$$G(d,q,r^d_{CB}) = d + dr^d_{CB} + [\eta - i_E(d)]I\left[r^c_*\big(d,r^d_{CB}\big) - r^d_{CB}\right] - L(d,q,r^d_{CB}). \tag{9}$$

Thus capital of the banking system at the end of a period consists of its initial capital d and its profits which are represented by the last three terms in (9). The profits are composed of interest rate earnings on d, the intermediation margin times the loan volume $[\eta - i_E(d)]I$, and losses L. Observe that in the case without interest rate cutting, the third term in (9) vanishes due to zero intermediation margins.

Finally, we specify how banks distribute positive capital among old entrepreneurs who are the shareholders of the banks. If $d \leq \overline{d}$, we assume that no capital is distributed. If $d > \overline{d}$, then excess capital $d - \overline{d}$ is distributed as dividends. This assumption is made in order to give the banking system the best chance of accumulating reserves against adverse macroeconomic shocks. The assumption could be further justified in two ways. First, when current owners are altruistic regarding their children in the sense of a warm glow model, they may refrain from withdrawing their capital. Second, the bank regulator might impose a zero payout ratio in order to enhance the recapitalization of the banking system. Since excess capital is distributed among old entrepreneurs, dividend prospects alter neither their savings nor their investment decisions and hence do not affect our intermediation results.

2.5 Interest-Rate Cutting

We assume that the banking system is subject to a capital adequacy rule.[8] Here, a capital-adequacy rule is defined as a threshold for the ratio between the capital of a bank and its loan volume. Since the banks in our model behave symmetrically, this capital requirement will be formulated for the whole banking system. A *(prospective) capital-adequacy rule* is the requirement that the banking system fulfills

$$\frac{d}{[\eta - i_E(d)]I} \geq \alpha, \tag{10}$$

where $0 \leq \alpha \leq 1$. The capital requirement (10) defines a threshold for the capital level of the banking system as a percentage α of the current loan volume $[\eta - i_E(d)]I$. In the first Basel Accord α was set at 0.08. The critical capital level d_{reg} below which (10) is violated is given by

$$d_{\text{reg}} := \frac{-\alpha I \underline{d}}{e + I - \alpha I} \geq 0. \tag{11}$$

We distinguish between two forms of banking crises: *critical states*, when capital in the banking sector is below regulatory capital d_{reg} but still positive so that $d \in (0, d_{\text{reg}})$, and *bad states*, when $d \leq 0$ and the banking system is insolvent.

We assume that a bank regulator suspends strict enforcement of the capital-adequacy rule and the central bank lowers the deposit interest rate as soon as $d < d_{\text{reg}}$.[9] Such a rule is referred to as an *interest-rate cutting.* We note that strict enforcement of capital-adequacy rules would force all banks to reduce loans and hence could trigger a credit crunch.[10] Formally, an *interest-rate policy rule* is defined by a function

$$\psi : [\underline{d}, \overline{d}] \longrightarrow \mathbb{R}_+, \quad r^d_{CB} = \psi(d), \tag{12}$$

such that $\psi(d) = r^d_{CB} \leq r_*(d)$ in the case with interest rate cutting when $d \in [\underline{d}, d_{\text{reg}})$ and $\psi(d) = r_*(d)$ in the case without interest rate cutting when $d \in [d_{\text{reg}}, \overline{d}]$. The latter corresponds to an intermediation equilibrium without interest rate cutting. Hence, the policy rule ψ is designed in such a way that the central bank reduces interest rates if and only if $d < d_{\text{reg}}$.

[8] The pros and cons of capital adequacy rules in the presence of bank moral hazard have been hotly debated in the literature, e.g. Dewatripont & Tirole (1994), Hellwig (1995), Gehrig (1996), and Holmström & Tirole (1997). The welfare costs associated with capital requirements may be quite high, see the recent important contribution by Van den Heuvel (2007).

[9] The recent worldwide banking crises has highlighted that bank regulators will soften capital requirements in such crises, and central banks react by lowering interest rates.

[10] Further comments on this regulatory option are found in the conclusions.

3 Evolution of the Banking System

We are now ready to describe the evolution of capital of the banking system. We allow the banking system to start with an arbitrary capital level $d_0 \in (\underline{d}, \overline{d}]$. The case of an initially positive level $d_0 > 0$ may be interpreted as follows: In previous periods, either capital requirements have forced the banking system to accumulate d_0 or the banking system has operated in an oligopolistic or monopolistic manner. In period $t = 0$ a liberalization shock occurs, and the banking system encounters tough price competition.

3.1 Bank capital

The stochastic difference equation which drives the evolution of bank capital is now set up as follows. Let $\underline{d} < d_t \leq \overline{d}$ denote the level of bank capital at the beginning of period t. Then banks raise funds $S + i_E(d_t)\, e$ that have to be paid back with interest in the subsequent period $t+1$. Given the capital function G as defined in (7) and a policy rule ψ as defined in (12), the new level of bank capital d_{t+1} is thus determined by

$$d_{t+1} = \min\left\{\overline{d},\, G\big(d_t, q_t, \psi(d_t)\big)\right\}, \qquad (13)$$

where q_t is the macroeconomic shock in period t. Equation (13) is a stochastic difference equation, where, for simplicity, we assume from now on that the sequence of shocks $\{q_t\}_{t\in\mathbb{N}}$ follows an i.i.d. process.[11]

As long as the capital at the end of period t is below $\overline{d}$, no dividends are paid out to shareholders, and the level of bank capital d_{t+1} at the beginning of the subsequent period $t+1$ is equal to the level of capital at the end of period t. If, on the contrary, capital at the end of period t is above $\overline{d}$, then $d_{t+1} = \overline{d}$ and excess capital is payed out to old shareholders. If $\underline{d} \leq d_{t+1} < 0$, then banks have incurred losses, and d_{t+1} is the amount of liabilities that could not be covered by loan repayments from entrepreneurs. In this case, banks in period $t+1$ must raise enough funds to reimburse the amount $-d_{t+1}$ to the depositors born in period t who withdraw their deposits for old-age consumption. For $d_t = \underline{d}$ we have $i_E(d_t) = \eta$ such that all funds are needed to meet previous obligations. The banking system is bankrupt and collapses for $d_{t+1} < \underline{d}$, since previous obligations can no longer be met and (2) is violated.

[11]This implies that the sequence of bank capital levels $\{d_t\}_{t\in\mathbb{N}}$ generated by (13) is a Markov process, see Lasota & Mackey (1994).

We shall see that the return on loans of the banking system, which is given by

$$\frac{P(d,q,r^d_{\text{reg}})}{[\eta - i_E(d)]I} - 1, \quad d > \underline{d}, \tag{14}$$

is the crucial quantity governing the evolution of bank capital. For our later analysis, three observations are of central importance. In view of (9), the first one is that the capital base of the banking system can only increase if the intermediation margin on the loan volume $[\eta - i_E(d)]I$ and the earnings on capital d exceed losses L. According to Lemma 1, which is formalized next, this is equivalent to saying that the capital base increases only if the return on loans exceeds the deposit interest rate times the deposit-loan ratio.

Lemma 1
For each $d \in (\underline{d}, \overline{d}]$, each $0 \le r^d_{CB} \le r_*(d)$, and each $q \in \mathbb{R}_+$, the following holds:

$$G(d,q,r^d_{\text{reg}}) \gtreqless d \iff \frac{P(d,q,r^d_{\text{reg}})}{[\eta - i_E(d)]I} - 1 \gtreqless \left(\frac{S + i_E(d)\, e}{[\eta - i_E(d)]I}\right) r^d_{CB}.$$

Lemma 1 follows directly from (7) and (8). The second observation is that banks' capital at the end of a period is non-decreasing in macroeconomic shocks, since their repayments from entrepreneurs P are likewise non-decreasing in macroeconomic shocks. The last observation, formally stated in Lemma 5 of Appendix B, is that bank capital can be increased if the central bank lowers deposit interest rates, thereby reducing repayment obligations to depositors.

In the next section we focus on the repayment behavior of entrepreneurs.

3.2 Repayments of entrepreneurs and aggregate income

Let $\underline{d} \le d \le \overline{d}$ be the current level of bank capital at the beginning of an arbitrary period. Assume that the central bank has set the deposit interest rate at some value $r^d_{CB} \le r_*(d)$ and that entrepreneurs have encountered the shock q. An entrepreneur with quality level i enters bankruptcy if she is unable to fully pay back her credit, that is, if

$$I\left[1 + r^c_*(d, r^d_{CB})\right] > q(1+i)f(e+I). \tag{15}$$

Clearly, if the lowest-quality entrepreneur $i_E(d)$ is able to fully repay her credit, then all higher quality entrepreneurs will also be able to do so. The critical shock above which no entrepreneur enters bankruptcy is thus given by

$$q_{\text{NB}}(d, r^d_{CB}) := \frac{I\left[1 + r^c_*(d, r^d_{CB})\right]}{\big(1 + i_E(d)\big)f(e+I)}. \tag{16}$$

In other words, no entrepreneur enters bankruptcy if shocks are sufficiently positive, i.e., $q \geq q_{\mathrm{NB}}(d, r^d_{CB})$, while for shocks below $q_{\mathrm{NB}}(d, r^d_{CB})$ entrepreneurs with insufficient quality levels enter bankruptcy. We infer from (6) that repayments of entrepreneurs are maximal and aggregate losses (8) of the banking system vanish for sufficiently positive shocks, i.e.,

$$L(d, q, r^d_{CB}) = 0 \quad \text{for all} \;\; q \geq q_{\mathrm{NB}}(d, r^d_{CB}). \tag{17}$$

By Corollary 1 (iv), the loan interest rate r^c_* increases when r^d_{CB} is lowered. Given a capital level d, this has two consequences for banks. First, entrepreneurs' repayments P defined in (6) increase, as they increase with r^c_*. This is illustrated in Fig. 2, in which P corresponds to the gray shaded area. Second, the critical shock (16) will be raised, thus forcing more entrepreneurs into bankruptcy. As a consequence, by lowering $r^d_{CB} \geq 0$ the central bank can only increase banks' repayments at the expense of an increasing number of bankruptcies.[12]

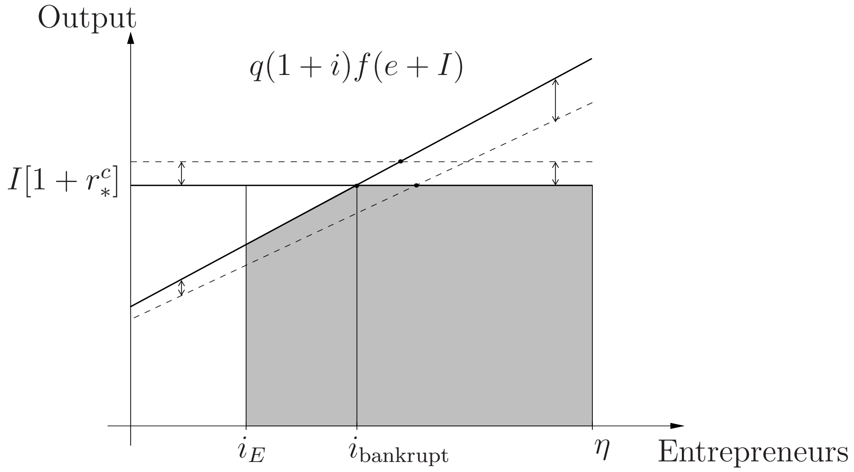

Figure 2: Repayments of entrepreneurs.

The return on loans, as introduced in equation (14), has the important property that it is non-increasing in bank capital. The reason is that higher levels of d allow banks to finance a larger portion of entrepreneurs, thus causing the average quality level of investments to decline. This important observation is expressed in the following Lemma 2. The formal proof is given in Appendix B.

[12]The fact that higher loan rates simultaneously increase repayments to banks and the fraction of bankrupt entrepreneurs is a property of many intermediation models. The subtle point here is that this effect is caused by deposit interest rate control.

Lemma 2
Let $r^d_{CB} \geq 0$ be an arbitrary deposit interest rate. Then for each $q \in \mathbb{R}_+$, the return on loans (14) is non-increasing in d.

Finally, we introduce aggregate income of the economy which is given by

$$Y(d,q) = e + \int_{i_E(d)}^{\eta} q\,(1+i) f(e+I)\,\mathrm{d}i. \tag{18}$$

Since $i_E(d)$ is decreasing in d, aggregate income Y is increasing in d and q. High capital levels of the banking system allow for a high loan volume that enables many entrepreneurs to invest.

A critical state of our economy is $\underline{d} = -[S + \eta e]$ in which all available funds are needed to meet previous obligations. Equilibrium interest rates are such that even the highest-quality entrepreneur $i_E(\underline{d}) = \eta$ saves her endowment. Profitable but risky investments can no longer be financed, aggregate income Y is minimal and equal to e. We therefore refer to $\underline{d}$ as the *consumption trap* of the economy.

4 Banking Collapse and Prevention

In this section we discuss scenarios in which the banking system collapses, and introduce *feasible policy rules* that will prevent such a banking collapse. We start with the observation that interest rate cutting may be necessary to prevent a banking collapse.

4.1 Banking Collapse without Interest Rate Cutting

Consider the case without interest rate cutting in which loan and deposit interest rates coincide. In this case, the capital function of the banking system (9) takes the form

$$G\big(d,q,r_*(d)\big) = d\big[1 + r_*(d)\big] - L\big(d,q,r_*(d)\big). \tag{19}$$

The most favorable situation is when all entrepreneurs meet their obligations. This occurs for all shocks $q \geq q_{\mathrm{NB}}\big(d, r_*(d)\big)$. Then aggregate losses L as given in (17) are zero and repayments to banks (6) are maximal and equal to

$$P\big(d,q,r_*(d)\big) = [\eta - i_E(d)]\, I\big[1 + r_*(d)\big].$$

In view of (13), the evolution of bank capital is now given by the following lemma:

Lemma 3
Let $d_\tau \in (\underline{d}, \overline{d}]$ be the capital base in some period τ, and suppose that no bankruptcies occur, i.e., $q_\tau \geq q_{\mathrm{NB}}\big(d_\tau, r_*(d_\tau)\big)$. Then $d_{\tau+1} = \min\Big\{d_\tau\big[1 + r_*(d_\tau)\big], \overline{d}\Big\}$.

Lemma 3 shows that if aggregate productivity shocks are sufficiently positive, bank capital grows according to the interest rate that banks earn on capital invested in the last period. Whether or not the banking system can survive will now depend on the evolution of bank capital in bad times, when adverse shocks cause aggregate losses.

Suppose for a moment, that negative macroeconomic shocks have caused $d_{T_0} < 0$ in some period T_0. To cover these losses, banks need new deposits on which they have to pay interest. Since intermediation margins are zero, it follows from (19) that bank capital in the subsequent period $T_0{+}1$ is at most equal to $d_{T_0}\big[1{+}r_*(d_{T_0})\big]$, provided that no bankruptcies occur. Since by Fact 1, Appendix A real interest rates are positive, $d_{T_0}\big[1 + r_*(d_{T_0})\big] < d_{T_0}$ and bank capital will decline.

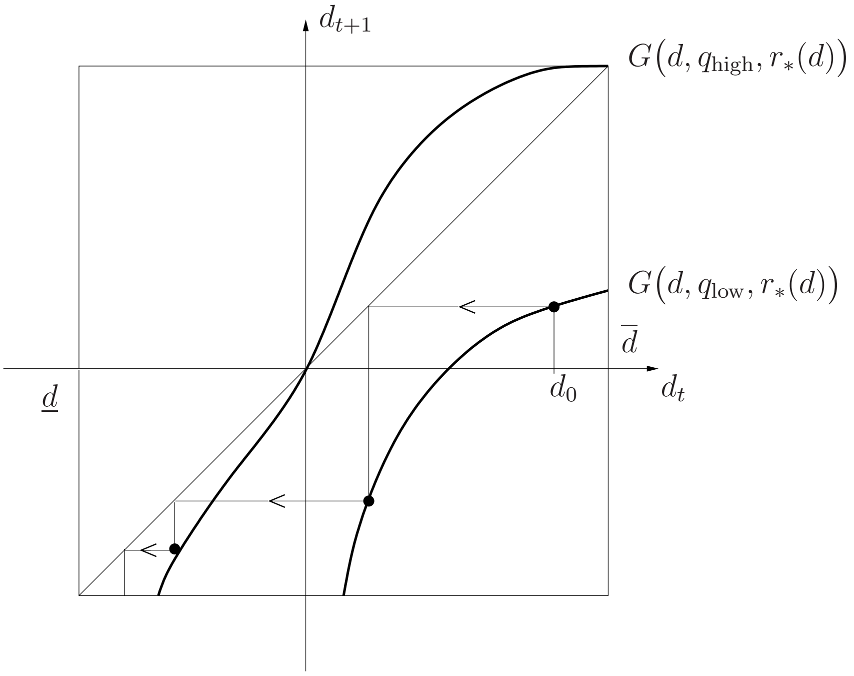

Figure 3: Collapse of the banking system.

This argument shows also that the future bank capital can at most be zero, once it has been depleted in period T_0 so that $d_{T_0} = 0$, and that it becomes negative as soon as losses occur. The statistical property of ergodicity of the i.i.d. sequence of macroeconomic shocks implies that losses occur in finite time with probability one.[13]

[13]This is a consequence of the Birkhoff Ergodic Theorem, e.g., see Lasota & Mackey (1994).

Hence, once $d_{T_0} \leq 0$, then bank capital decreases until the banking system collapses. This result is stated in the following Lemma 4.

Lemma 4
Suppose that the central bank does not lower interest rates. Then the banking system collapses with probability one, if one of the following conditions holds:

(i) The banking system has accumulated negative bank capital $d_{T_0} < 0$ in some period T_0.

(ii) Bank capital has been depleted such that $d_{T_0} = 0$ in some period T_0, and bankruptcies of entrepreneurs occur with positive probability, that is, $\text{Prob}\big(q < q_{\text{NB}}(0, r_*(0))\big) > 0$.

A formal proof of Lemma 4 is given in Appendix B. The next crucial issue is whether an initial level of bank capital can be sufficient to prevent a banking collapse. Suppose to this end that the capital level of the banking system is $d_0 > 0$ at some given point in time, say $t = 0$. Suppose further that there exists a critical shock $q_{\text{crit}} > \underline{q}$ that reduces entrepreneurs' repayments in such a way that the capital base will decrease for shocks below q_{crit}, i.e.,

$$G\big(d, q, r_*(d)\big) < d \quad \text{for all} \quad d \in [0, \overline{d}], \; q \leq q_{\text{crit}}. \tag{20}$$

Then a series of sufficiently many shocks $q_0, \ldots, q_t$ below q_{crit} will lead to a series of decreasing capital bases

$$d_1 = G\big(d_0, q_0, r_*(d_0)\big) > \cdots > d_{t+1} = G\big(d_t, q_t, r_*(d_t)\big)$$

that will finally take on values below zero. Let T_0 denote the first time when negative bank capital occurs, i.e. $d_{T_0} < 0$. It follows from the ergodicity of the shock process that this event will occur within finite time T_0 with probability one, if shocks below q_{crit} occur with positive probability, i.e., if $\text{Prob}(q \leq q_{\text{crit}}) > 0$. As a consequence of Lemma 4, the banking system will then collapse with probability one. The intuition for this result is qualitatively illustrated in Figure 3, showing that the banking system starting with initial bank capital d_0 collapses after encountering two successive adverse shocks $q_{\text{low}} \leq q_{\text{crit}}$.[14] In view of Lemma 1, Condition (20) can be rephrased in terms of a productivity condition to yield the following Theorem:

[14] For uniformly distributed shocks, it will be shown in Appendix C that G is non-increasing in d.

Theorem 1
Suppose that there exists a critical shock q_{crit} such that the return on loans satisfies

$$\frac{P\big(d, q, r_*(d)\big)}{[\eta - i_E(d)]I} - 1 < \left(\frac{S + i_E(d)\, e}{[\eta - i_E(d)]I}\right) r_*(d) \quad \textit{for all } d \in [0, \overline{d}],\ q \leq q_{\text{crit}}.$$

If $\text{Prob}(q \leq q_{\text{crit}}) > 0$*, then the banking system collapses with probability one for each initial capital level* $d_0 \in [0, \overline{d}]$.

Theorem 1 states that, independently of the initial level of bank capital, a banking collapse cannot be prevented if the return on loans is lower than the deposit interest rate times the deposit-loan ratio with positive probability. Whether or not this condition holds depends essentially on the interplay between the productivity of entrepreneurs, the distribution of macroeconomic shocks, and the interest rate levels set by the banking system.

4.2 Preventing a banking collapse

Under the hypotheses of Theorem 1 the banking system will collapse with certainty. In this section we show that a collapse of the banking system in which the capital base has fallen below $\underline{d}$ may be prevented by capping deposit interest rates appropriately. By capping deposit interest rates $r^d_{CB} = \psi(d) < r_*(d)$, the central bank will enhance recapitalization of the banking system as banks are able to ask for higher loan interest rates $r^c_*(d, r^d_{CB}) > r_*(d)$.

The ultimate goal of any central bank policy must be to keep bank capital above $\underline{d}$ with certainty. Since as argued above bank capital is increasing in shocks (see Lemma 5, Appendix B), it suffices to implement a policy rule ψ which guarantees

$$G\big(d, \underline{q}, \psi(d)\big) \geq \underline{d} \quad \text{for all } d \in [\underline{d}, \overline{d}], \tag{21}$$

where $\underline{q}$ denotes the lowest macroeconomic shock. In view of (19), the monotonicity of the capital function G with respect to shocks implies that a critical capital level $d_{\text{crit}} \in [\underline{d}, \overline{d}]$ exists for which

$$G\big(d_{\text{crit}}, \underline{q}, r_*(d_{\text{crit}})\big) = \underline{d}. \tag{22}$$

This implies that for any capital base $d \leq d_{\text{crit}}$, the probability that the banking system will collapse in the next period may be positive unless the central bank lowers interest rates. Without restriction, let $d_{\text{crit}} \in [\underline{d}, \overline{d}]$ be the highest level satisfying (22).

To obtain a cap for the deposit interest rates which prevents a banking collapse, consider the extreme case in which $\underline{q} = 0$ and repayments to banks are zero, i.e., $P = 0$. Recalling that $\underline{d} = -(S + e\eta)$, it then follows from the capital function G as given in (7) that Condition (21) holds if banks' liabilities are below the total amount of resources in the economy, i.e., if

$$\big(S + e\, i_E(d)\big)\big[1 + \psi(d)\big] \leq S + e\,\eta \quad \text{for} \;\; d \in [\underline{d}, d_{\text{crit}}).$$

We thus obtain a cap for the policy function $r^d_{CB} = \psi(d)$, which is given by

$$r^d_{CB} = \psi(d) \leq \frac{S + e\,\eta}{S + e\, i_E(d)} - 1 \quad \text{for all} \;\; d \in [\underline{d}, d_{\text{crit}}). \tag{23}$$

Summarizing, we obtain the following:

Theorem 2
Let $d_{\text{crit}} \leq d_{\text{reg}}$ with d_{crit} as defined by (22). Then any policy rule ψ with $\psi(d) = r_*(d)$ for all $d \in [d_{\text{crit}}, \overline{d}]$ which satisfies (23) prevents a banking collapse with probability one.

Theorem 2 shows that the central bank may always fulfill (23) and thus prevent a banking collapse by directly setting $r^d_{CB} = 0$ once the capital base is below the critical level d_{crit}. Observe that the critical capital level d_{crit} is determined by the fundamentals of the economy as is expressed by equation (22). Therefore, only if regulatory capital is above the critical capital level, $d_{\text{crit}} \leq d_{\text{reg}}$, the central bank will be able to reduce interest rates early enough as to avoid the risk of a banking collapse.

5 The Consumption Trap

Let us take a closer look at interest-rate cutting rules that just avoid a banking collapse. Such a scenario could occur when the young generation determines the interest rate policy of the central bank. Suppose that the only interest of a young generation is to secure its own investments. Acting solely on their behalf, the central bank will not cut interest rates as long as bank capital is above d_{crit} with d_{crit} denoting the largest level satisfying (22). For capital levels below d_{crit}, the central bank will set the highest possible deposit interest rate, denoted by $r_{\text{crit}}(d)$, that avoids the risk of a banking collapse. Assume for a moment, that this critical interest rate $r_{\text{crit}}(d)$ exists such that $\underline{d} = G\big(d, \underline{q}, r_{\text{crit}}(d)\big)$ for $d \in [\underline{d}, d_{\text{crit}}]$. An interest rate cutting rule that merely avoids a banking collapse then takes the form

$$\psi_{\text{crit}}(d) = \begin{cases} r_{\text{crit}}(d) & \text{if } d \in [\underline{d}, d_{\text{crit}}), \\ r_*(d) & \text{if } d \in [d_{\text{crit}}, \overline{d}]. \end{cases} \tag{24}$$

Less reduction of interest rates would run the risk of a banking collapse, more reduction of interest rates would lower interest rates on savings and raise loan rates, thus decreasing the utility of consumers and entrepreneurs of the young generation.

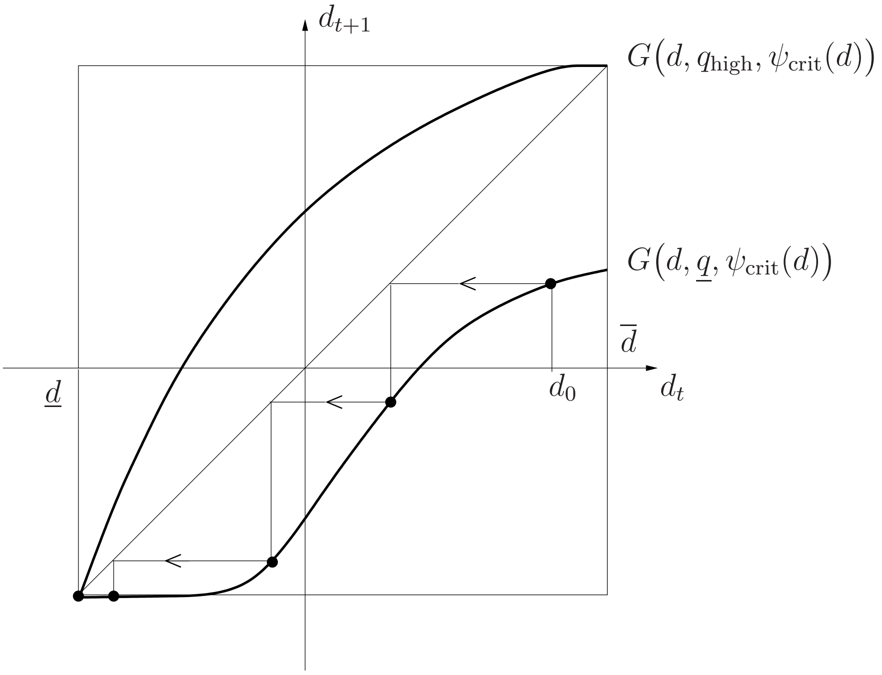

Figure 4: Convergence to the consumption trap.

The following Theorem 3 states conditions under which the interest rate cutting rule (24) will lead the economy into the consumption trap $\underline{d}$. Analogously to the reasoning leading to Lemma 4, a series of sufficiently adverse shocks $q_0, \ldots, q_t$ below a critical level q_{crit} will deplete the capital base of the banking system, so that it finally ends up with a capital base below d_{crit}. By the same reasoning as for Lemma 4, the ergodicity of the shock process then implies that the consumption trap is reached with probability one. This argument is illustrated in Fig. 4, showing that an economy may converge to the consumption trap for a sequence of four successive shocks $\underline{q}$. In this case the interest rate policy fails to recapitalize the banking system sufficiently and the consumption trap is reached. Using Lemma 1, we obtain the following:

Theorem 3

Suppose that the policy rule ψ_{crit} defined by (24) is applied. Assume, in addition, that *there exists a critical shock* $q_{\text{crit}} \geq \underline{q}$ *such that*

$$\frac{P\big(d, q, \psi_{\text{crit}}(d)\big)}{[\eta - i_E(d)]I} - 1 < \left(\frac{S + i_E(d)\, e}{[\eta - i_E(d)]I}\right) \psi_{\text{crit}}(d) \quad \textit{for all } d \in [d_{\text{crit}}, \overline{d}],\ q \leq q_{\text{crit}}.$$

If $\text{Prob}(q = \underline{q}) > 0$, *then for any initial level* $d_0 \in [\underline{d}, \overline{d}]$, *the economy converges to the consumption trap with probability one.*

The formal proof of Theorem 3 is given in Appendix B. Theorem 3 shows that an interest rate policy may not recapitalize a banking system sufficiently as to prevent a consumption trap in which aggregate income is minimal. In order to avoid the trap, the central bank must be able not only to keep bank capital strictly above $\underline{d}$ with certainty but also to prevent bank capital from converging to $\underline{d}$. Thus, a policy rule ψ is needed that allows the banking system to recapitalize in such a way that

$$G\big(d, q, \psi(d)\big) > d \tag{25}$$

with positive probability at least for levels d close to $\underline{d}$. Setting deposit interest rates to zero, it follows from Lemma 1 that a sufficient condition for (25) is that the return on loans is positive, i.e.,

$$\frac{P\big(d, q, \psi(d)\big)}{I[\eta - i_E(d)]} - 1 > 0 \tag{26}$$

with positive probability for capital levels d close to $\underline{d}$. As will be shown in the proof of the following Theorem 4, a sufficient condition for (26) is that the entrepreneur with the highest quality level is sufficiently productive.

Theorem 4
Let $r_*(0) > 0$, $d_{\text{crit}} \leq d_{\text{reg}}$, *and suppose there is positive probability that the entrepreneur with the highest quality level fully pays back her loan at zero loan interest rates, i.e.,* $q^\star(1+\eta)f(e+I) > I$ *for some* $q^\star \in \mathbb{R}_+$ *with* $\text{Prob}(q \geq q^\star) > 0$. *Then there exists a policy rule* ψ *that prevents the consumption trap with probability one.*

A formal proof of Theorem 4 is given in Appendix B. It shows that setting sufficiently low deposit interest rates $r^d_{CB} = \psi(d)$ creates high enough intermediation margins to prevent the economy from converging to the consumption trap, provided that the productivity of the economy is high enough.

Theorem 3 and Theorem 4 have important implications for the design of interest rate cutting rules. Even if each young generation lowers interest rates sufficiently to avoid a banking collapse of the banking system, the economy still faces the risk of running into a consumption trap. Although depositors are fully protected, this outcome may be highly inefficient because in the consumption trap aggregate income is minimal.

Since aggregate output Y is monotonically increasing in d, no matter what shock is encountered, later generations will always benefit from their predecessors' interest rate cutting to prevent the decline of d. An old generation, however, is indifferent between interest rate cutting rules as long as a banking collapse is avoided. Therefore, incentives to enforce interest rate cutting rules which prevent the consumption trap must be borne by concerns for future generations. To avoid the consumption trap, the discretion of a generation to determine central bank policy must be limited. This could be implemented through constitutional political arrangements, for example by restricting the government's freedom to alter previously adapted fiscal policies (e.g., see Azariadis & Galasso 1998). Such an analysis necessitates a welfare analysis which is beyond the scope of this paper. However, before such an analysis can be carried out, the nature of feasible interest rate cutting rules that resolve and prevent banking crises must first be understood. This will be done next.

6 Resolving and Preventing Banking Crises

In this section we discuss *feasible interest rate cutting rules* that resolve and prevent a banking crisis. To prevent a banking crisis, interest rate cutting must preserve *good states* of the banking system in which its capital levels lie within the interval $[d_{\text{reg}}, \overline{d}]$. To resolve a banking crisis, interest rate policy must reverse bad or critical states of the banking system to good states. *Critical states* are those in which the capital requirements are violated but in which the banking system still has positive capital that lies within $(0, d_{\text{reg}})$. In *bad states*, the banking system has accumulated negative capital whose levels lie within the interval $[\underline{d}, 0]$.

Preserving good states requires the banking system to sustain bank capital above regulatory capital d_{reg}. To reverse bad and critical states, the central bank must be capable of inducing a recapitalization of the banking system in such a way that the economy eventually returns to a good state.

6.1 Resolving a banking crisis

Suppose that $d_{T_0} < d_{\text{reg}}$ has been realized at time T_0 so that the central bank takes action. The central bank's task must be to compensate potential losses due to bad shocks by creating positive intermediation margins. In order to resolve the banking crisis, the central bank must set lower interest rates to promote a sufficient recapitalization of the banking system. To this end we seek a policy rule ψ such that

$$G\big(d, q, \psi(d)\big) > d, \quad d \in (\underline{d}, d_{\text{reg}}] \tag{27}$$

with positive probability. Suppose that there exists a critical shock q_{crit} such that the capital base will increase in such a way that (27) holds for shocks above q_{crit}. Then at some point in time τ, a series of sufficiently many shocks $q_\tau, \ldots, q_{\tau+t}$ above q_{crit} will lead to a series of increasing capital bases

$$d_{\tau+1} < \cdots < d_{\tau+t+1}$$

that will finally take on a value above d_{reg}. If $\text{Prob}(q \geq q_{\text{crit}}) > 0$, then the ergodicity of the shock process guarantees the existence of the required series of positive shocks such that bad stages are reversed. The intuition for this argument is provided in Fig. 5, in which a series of three successive positive shocks q_{high} leads to a capital level above regulatory capital d_{reg} after a series of three negative shocks q_{low} has caused a serious decline of the initial capital base d_0.

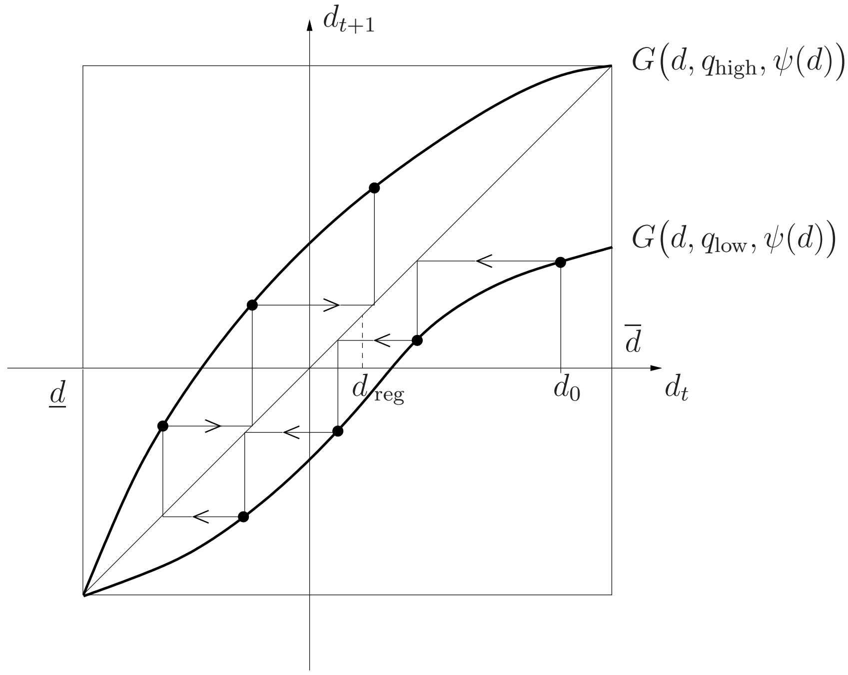

Figure 5: Resolving a banking crisis.

In view of Lemma 1, Condition (27) can be rephrased in terms of a productivity condition guaranteeing a recapitalization of the banking system so that it satisfies regulatory capital requirements within finite time. The following Theorem stating conditions under which a banking crisis can be resolved is now straightforward:

Theorem 5
Let the assumptions of Theorem 4 be satisfied such that a policy rule ψ exists which prevents a banking collapse. Suppose that there exists a critical chock q_{crit} such that the return on loans satisfies

$$\frac{P\big(d,q,\psi(d)\big)}{[\eta - i_E(d)]I} - 1 > \left(\frac{S + i_E(d)\,e}{[\eta - i_E(d)]I}\right)\psi(d) \quad \textit{for all } d \in (\underline{d}, d_{\text{reg}}],\ q \geq q_{\text{crit}}. \tag{28}$$

If $\text{Prob}(q \geq q_{\text{crit}}) > 0$, *then for any initial capital level* $d_0 \in (\underline{d}, \overline{d}]$, ψ *will resolve a banking crises as bad and critical states will be reversed with probability one.*

Since by Lemma 2 the return on loans is non-increasing in d, a sufficient condition for (28) is that these are positive at the regulatory capital level, i.e.,

$$\frac{P\big(d_{\text{reg}},q,0\big)}{[\eta - i_E(d_{\text{reg}})]I} - 1 > 0 \quad \text{for all} \quad q \geq q_{\text{crit}}.$$

Setting deposit interest rates to zero, $\psi(d) = 0$, for capital levels d below regulatory capital d_{reg} then suffices to resolve a banking crisis as Condition (28) is satisfied.

6.2 Preventing a banking crisis

We complete our analysis by stating necessary conditions that allow a banking system to sustain bank capital d above regulatory capital d_{reg} without interest rate cutting, thus preserving good states. Formally this requirement reads

$$G\big(d,q,r_*(d)\big) \geq d_{\text{reg}}, \quad \text{for all} \quad d \in [d_{\text{reg}}, \overline{d}],\ q \geq \underline{q}. \tag{29}$$

Replacing the capital function G by (9), Condition (29) is equivalent to

$$d\big(1 + r_*(d)\big) - d_{\text{reg}} \geq L\big(d,\underline{q},r_*(d)\big), \quad d \in [d_{\text{reg}}, \overline{d}].$$

This shows that in the worst case when the lowest macroeconomic shock $\underline{q}$ occurs the return on loans must not exceed banks' earnings on capital minus regulatory capital. An argument similar to the one leading to Lemma 1 shows that (29) is equivalent to

$$\frac{P\big(d,\underline{q},r_*(d)\big)}{[\eta - i_E(d)]I} - 1 \geq \left(\frac{S + i_E(d)\,e}{[\eta - i_E(d)]I}\right) r_*(d) - \frac{d - d_{\text{reg}}}{[\eta - i_E(d)]I}, \quad d \in [d_{\text{reg}}, \overline{d}]. \tag{30}$$

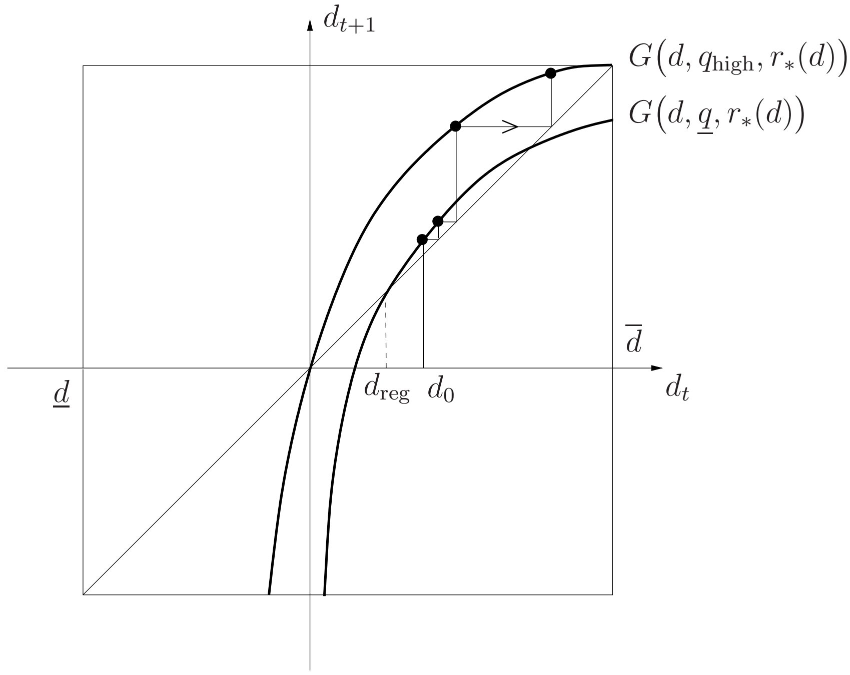

Figure 6: Preventing a banking crisis.

This demonstrates that in the worst case scenario, the return on loans must be greater than the deposit-loan ratio times the interest rate minus the ratio between 'excess capital' $d - d_{\text{reg}}$ and loan volume. Note, moreover, that the lowest shock $\underline{q}$ must be sufficiently positive in order to guarantee Condition (30). Summarizing, we obtain the following result.

Corollary 2
Suppose that equilibrium interest rates $r_*(d)$ satisfy (30). Then the banking system preserves good states with certainty and thus is capable of preventing a banking crisis without interest rate cutting by the central bank.

Corollary 2 complements Theorem 1. A situation in which the productivity of entrepreneurs is high enough for bank capital to never fall below regulatory capital d_{reg} is illustrated in Fig. 6. If, on the contrary, repayments from entrepreneurs P and thus the return on loans may become too small, then a banking system may not be able to preserve good states on its own. Apart from the regulatory policy and the level of regulatory capital, we see again here that the capability of entrepreneurs to cope with adverse shocks in relation to the interest rate levels is a crucial factor determining whether or not an economy can prevent a banking crisis without interest rate cutting.

In Appendix C, we will illustrate some of the key properties of the banking system when the shock process is i.i.d. and uniformly distributed. This parameterization allows for an explicit solution.

7 Conclusions

We investigated a model in which a series of adverse macroeconomic shocks may trigger a downward spiral of bank capital. This downward spiral will end up in a banking collapse in which the banking system has insufficient capital to carry out the intermediation services for future generations. If the return on loans of banks is too low, this banking collapse occurs with certainty unless interest rate cutting takes place.

The paper's main conclusion is that given a certain level of productivity of entrepreneurs, interest-rate cutting can always avoid a banking collapse. However, the central bank must induce large enough interest-rate margins in order to prevent an economy from running into long periods with low aggregate income, that is, a consumption trap.

In future research, alternative intervention policies that foster recapitalization of a banking system under financial distress should be considered. There are at least two policies that could be incorporated into our framework. The first one is a strict enforcement of capital requirements. This would force banks to reduce their loan volume in critical times thereby lowering aggregate production. Indeed, the example in Appendix C indicates that the reduction of the loan size required to restore loan/capital ratios when capital has fallen below the threshold level will cause a credit crunch and lowers aggregate production. The second type of intervention would be a direct transfer of resources to a troubled banking system. These funds could be obtained by taxing consumers and entrepreneurs. While such an intervention again decreases aggregate investment and hence production, it would, unlike interest-rate policy, not enhance banks' intermediation margins. The examination of the economic costs of these alternative intervention policies offers important avenues for further research.

The banking system in our model plays a dominant role in economies with emerging markets or in commercial bank-based industrial countries. In future research one should allow for financial markets that exist parallel to banks. In this case, banks may recapitalize themselves by issuing new equity. On the one hand, this could lower the likelihood of banking crises. On the other hand, interest-rate policy may become less powerful, as depositors may withdraw their bank deposits when deposit interest rates are capped. Investigation of these countervailing effects is a matter for future research.

In view of the new regulatory framework Basel II, commercial banks will invest more in screening of entrepreneurs. Gehrig & Stenbacka (2004) show that uncoordinated screening behavior of competing financial intermediaries creates a financial multiplier and may be an independent source of fluctuations. Screening activities are low or zero once the economy is hit by a sufficiently adverse macroeconomic shock. These negative consequences for investment and GDP are likely to reinforce the vulnerabilities of the banking system in our model, and may entail stronger interest rate cuttings. This is again left for future research.

A Bertrand Competition Between Banks

This appendix elaborates on the intermediation game introduced in Section 2.2. In accordance with current regulation policies in most countries, we assume that banks cannot ration deposit contracts and that interest rates on deposits are guaranteed by the government. Entrepreneurs face a binary decision problem as they are required to invest all their equity e in order to receive a loan contract of size I. Entrepreneurs are contract-takers.[15]

Formally, an intermediation equilibrium is a *subgame-perfect equilibrium* of the intermediation game, given by a tuple $\left\{\left\{r_j^{d*}\right\}_{j=1}^{n}, \left\{r_j^{c*}\right\}_{j=1}^{n}\right\}$, such that entrepreneurs take optimal credit application[16] and saving decisions, and no bank has an incentive to offer different deposit or loan interest rates.

Fact 1
Let $d \in [\underline{d}, \overline{d}]$ be arbitrary, and assume that $\Pi(0,0) > e$ such that the entrepreneur with the lowest quality level $i = 0$ will invest for zero loan interest rates.

A. *Suppose that banks set deposit and loan interest rates. Then there exists a unique subgame-perfect equilibrium of the intermediation game with*

(i) $r_* = r_j^{c*} = r_j^{d*}, \quad j = 1, \ldots, n;$

(ii) $r_* = r_*(d) > 0$ *is determined by* $\Pi\big(i_E(d), r_*(d)\big) = e\left(1 + r_*(d)\right)$.

B. *Suppose that the central bank sets the interest rate $r_{CB}^d \leq r_*(d)$. Then there exists a unique equilibrium of the intermediation game with*

(i) $r_*^c = r_j^{c*}, \quad j = 1, \ldots, n;$

(ii) $r_*^c = r_*^c(d, r_{CB}^d) > 0$ *is determined by* $\Pi\big(i_E(d), r_*^c(d, r_{CB}^d)\big) = e\left(1 + r_{CB}^d\right)$.

In both scenarios, all entrepreneurs $i \in [0, i_E(d))$ save, while all $i \in [i_E(d), \eta]$ invest.

Proof of Fact 1.
We start with Part **B**. In this case, the central bank fixes the refinancing costs of

[15] Hence, entrepreneurs apply for loans under the assumption that they will not be rationed by banks. If entrepreneurs were rejected nevertheless, they will turn to a bank with the highest deposit interest rate and save. Other rationing schemes are discussed in Gersbach (1998).

[16] Recall that monitoring of banks is assumed to be sufficiently efficient, so that entrepreneurs who obtained a loan will choose to invest.

banks.[17] At r_*^c, banks can exactly provide the amount of loans demanded because total bank savings and loans are balanced. By setting $r_j^c < r_*^c$, bank j attracts all borrowers. Since deposit interest rates are fixed, the only means of increasing savings is by rejecting borrowers. These will then switch banks in order to save. The associated increase in the amount of loans for bank j will not outweigh the decrease in profits per loan. Thus, the deviation is not profitable. The case $r_j^c > r_*^c$ is also not profitable. Since all borrowers would choose the bank offering r_*^c, no borrowers would apply, and bank j would simply face an excess of resources. The logic and intuition for Part **A** is similar, but much more complicated. We have to show that no bank deviates by offering higher deposit interest rates, or by offering higher deposit and loan rates, see Gersbach (1998).[18]

■

B Technical Appendix

We preface the formal proofs of the Lemmas and Theorems of the main text by a straightforward technical Lemma.

Lemma 5
For each $d \in [\underline{d}, \overline{d}]$ and each $0 \leq r_{CB}^d \leq r_*(d)$, we have

(i) $G(d, \cdot, r_{\text{reg}}^d)$ is non-decreasing in realizations of shocks $q \in \mathbb{R}_+$ and for each $q \in \mathbb{R}_+$,

$$\underline{d}(1 + r_{\text{reg}}^d) \leq G(d, q, r_{\text{reg}}^d) \leq [\eta - i_E(d)] I \left[r_*^c\big(d, r_{CB}^d\big) - r_{CB}^d \right] + d(1 + r_{\text{reg}}^d);$$

(ii) $G(d, q, \cdot)$ is decreasing in r^d and for each $q \in \mathbb{R}_+$,

$$G\big(d, q, r_*(d)\big) \leq G(d, q, r_{\text{reg}}^d) \leq G(d, q, 0).$$

Proof of Lemma 2.
Let $d_0 < d_1 \in (\underline{d}, \overline{d}]$ and $r_{CB}^d \geq 0$, $q > 0$ be arbitrary but fixed. By the mean value

[17] The interest rate set by the central bank and the deposit rates could be identical. For instance, if banks can refinance themselves unboundedly at the central bank, it would not be profitable for banks to offer a higher deposit rate than the interest rate set by the central bank. Moreover, competition among banks will induce that they do not offer a lower deposit rate than the one set by the central bank.

[18] The tedious and lengthy calculations are available upon request.

theorem for integrals there exists $i_0 \in [i_E(d_0), \eta]$ and $i_1 \in [i_E(d_1), \eta]$ with

$$\frac{P\big(d_0, q, r^d_{CB}\big)}{[\eta - i_E(d_0)]} = \min\big\{q\,(1+i_0)f(e+I),\ I\big[1 + r^c_*\big(d_0, r^d_{CB}\big)\big]\big\},$$

and

$$\frac{P\big(d_1, q, r^d_{CB}\big)}{[\eta - i_E(d_1)]} = \min\big\{q\,(1+i_1)f(e+I),\ I\big[1 + r^c_*\big(d_1, r^d_{CB}\big)\big]\big\},$$

respectively. Since $i_E(d)$ is decreasing, $i_1 \leq i_0$. The assertion then follows from Corollary 1 (ii).

■

Proof of Lemma 4.

(i) It follows from (19) and the monotonicity of $r_*(d)$ stated in Corollary 1 (i) that

$$G\big(d, q, r_*(d)\big) \leq d\big(1 + r_*(d)\big) \leq d\big(1 + r_*(0)\big) < d, \quad d \in [\underline{d}, 0),\ q \in \mathbb{R}_+.$$

This implies that $d_{t+1} = G\big(d_t, q_t, r_*(d_t)\big) < d_t$ for all subsequent periods $t \geq T_0$. Therefore, if $d_{T_0} < 0$ in period T_0, then the bank capital will be below $\underline{d}$ after a sufficient number of periods, and the banking system collapses with probability one.

(ii) It follows from (17) that losses are positive $L\big(0, q, r_*(0)\big) > 0$ for $q < q_{\mathrm{NB}}\big(0, r_*(0)\big)$, which occurs with positive probability. By virtue of the Birkhoff ergodic theorem (see e.g. Lasota & Mackey 1994), the ergodicity of the shock process implies that this event will occur in some finite time $T_1 > T_0$ with probability one. Then (19) implies $d_{T_1} < 0$ and, as shown in (i), the banking system collapses with probability one.

■

Proof of Theorem 3.

For simplicity, let $\underline{q} = 0$. The proof of the case $\underline{q} > 0$ is more tedious but follows the same lines. We first have to show that (24) is well defined. Since $r_*(d)$ is decreasing by Corollary 1 (i), it follows from Lemma 5 and the definition of d_{crit} in (22) that for each $d \in [\underline{d}, d_{\mathrm{crit}}]$,

$$-\big[S + i_E(d)e\big]\big[1 + r_*(d)\big] = G\big(d, 0, r_*(d)\big) \leq \underline{d} < G(d, 0, 0) = -\big[S + i_E(d)e\big].$$

Since G is continuous with respect to r^d_{CB}, there exists a critical interest rate $r_{\mathrm{crit}}(d)$ satisfying $\underline{d} = G\big(d, 0, r_{\mathrm{crit}}(d)\big)$ for $d \in [\underline{d}, d_{\mathrm{crit}}]$. The rest of the proof is analogous to that of Lemma 4: since $\text{Prob}(q = \underline{q}) > 0$, the ergodicity the shocks implies that $\underline{d}$ will be reached with probability one.

■

Proof of Theorem 4.
We have to show that there exists $d^\star > \underline{d}$ such that (26) holds for all $d \in (\underline{d}, d^\star]$ and all $q \geq q^\star$. By the mean value theorem for integrals, there exists $i_0(d) \in [i_E(d), \eta]$ with

$$\frac{P\big(d, q, \psi(d)\big)}{[\eta - i_E(d)]} = \min\big\{q[1 + i_0(d)]f(e + I),\ I\big[1 + r_*^c\big(d, \psi(d)\big)\big]\big\}. \tag{31}$$

By Corollary 1, $r_*^c\big(d, \psi(d)\big) > r_*(d) > 0$. Therefore, Condition (26) is equivalent to $q\,[1+i_0(d)]f(e+I) > I$. By assumption there exists $q^\star$ such that $q^\star[1+\eta]f(e+I) > I$. By continuity of (31) there exists $d^\star$ sufficiently close to $\underline{d}$ such that (26) can be guaranteed for $q > q^\star$. The ergodicity of the shock process then implies that the event $q_\tau > q^\star$ for some finite time τ will occur with probability one. Hence, $d_t > \underline{d}$ for all times t with probability one and bank capital d_t will never converge to $\underline{d}$.

■

C An Example

Consider an example in which the shock process $\{q_t\}_{t \in \mathbb{N}}$ is i.i.d. and uniformly distributed on the compact interval $[\underline{q}, \overline{q}] \subset \mathbb{R}_+$ with $\underline{q} > 0$. Set $\underline{r}(i) = (1+i)\underline{q}f(e+I)/I - 1$ and $\overline{r}(i) = (1+i)\overline{q}f(e+I)/I - 1$. Then the expected profit $\Pi(i, r)$ of entrepreneur i is

$$\Pi(i, r) = \begin{cases} (1 + i)f(e + I)\frac{\overline{q}+\underline{q}}{2} - I(1 + r) & \text{if } r \leq \underline{r}(i), \\ \frac{(1+i)f(e+I)}{2(\overline{q}-\underline{q})}\left[\overline{q} - \frac{I(1+r)}{(1+i)f(e+I)}\right]^2 & \text{if } \underline{r}(i) < r < \overline{r}(i), \\ 0 & \text{if } \overline{r} \leq r(i). \end{cases} \tag{32}$$

In view of Fact 1, solving the equation $\Pi(i_E, r_*) = e(1+r_*)$ for the equilibrium interest rate $r_* = r_*(d)$ gives

$$1 + r_* = \begin{cases} \frac{(1+i_E)f(e+I)}{e+I}\frac{\overline{q}+\underline{q}}{2} & \text{if } \frac{I}{e} \leq \frac{2\underline{q}}{\overline{q}-\underline{q}}, \\ \frac{(1+i_E)f(e+I)\overline{q}}{I}\left[1 + \frac{\overline{q}-\underline{q}}{\overline{q}}\frac{e}{I} - \sqrt{\left(1 + \frac{\overline{q}-\underline{q}}{\overline{q}}\frac{e}{I}\right)^2 - 1}\right] & \text{otherwise,} \end{cases} \tag{33}$$

where $i_E = i_E(d)$ denotes the critical entrepreneur as before. It is straightforward to see that $r_* \leq \underline{r}(i_E)$ if $I/e \leq 2\underline{q}/(\overline{q}-\underline{q})$ and it follows from (32) that in this case bankruptcies of entrepreneurs are ruled out. This implies in particular that good times can be preserved by reducing the credit size I, provided that $\underline{q} > 0$.

Therefore, interest rate cutting is only meaningful if $I/e > 2\underline{q}/(\overline{q}-\underline{q})$ and entrepreneurs can default with positive probability. When the central bank sets $r^d_{CB} \leq r_*(d)$, the loan interest rates $r^c_* = r^c_*(d, r^d_{CB})$ is given as

$$1 + r^c_* = \frac{\big(1 + i_E(d)\big) f(e+I)\, q_{\rm NB}(d, r^d_{CB})}{I}, \tag{34}$$

where the critical shock $q_{\rm NB}$ above which no bankruptcies occur takes the form

$$q_{\rm NB} = q_{\rm NB}(d, r^d_{CB}) = \begin{cases} \overline{q}\left[1 + \frac{\overline{q}-\underline{q}}{\overline{q}}\frac{e}{I} - \sqrt{\left(1 + \frac{\overline{q}-\underline{q}}{\overline{q}}\frac{e}{I}\right)^2 - 1}\right] & \text{if } r^d_{CB} = r_*(d), \\ \overline{q}\left[1 - \sqrt{\frac{2(\overline{q}-\underline{q})e}{\big(1+i_E(d)\big) f(e+I)\overline{q}^2}(1 + r^d_{CB})}\right] & \text{if } r^d_{CB} < r_*(d). \end{cases}$$

Notice that $q_{\rm NB} < \overline{q}$ and that $q_{\rm NB}$ is independent of the capital level in the case without interest rate cutting. Using (34), the bankruptcy condition (15) yields an entrepreneur with the lowest quality level who is not bankrupt after encountering a shock q. This entrepreneur is given by

$$i_B = i_B(q, d, r^d_{CB}) := \begin{cases} \eta & \text{if } q \leq q_{\rm TB}, \\ \frac{q_{\rm NB}(d, r^d_{CB})}{q}\big[1 + i_E(d)\big] - 1 & \text{if } q_{\rm TB} \leq q < q_{\rm NB}, \\ i_E(d) & \text{if } q \geq q_{\rm NB}, \end{cases} \tag{35}$$

where

$$q_{\rm TB} := q_{\rm NB}(d, r^d_{CB}) \left(\tfrac{1+i_E(d)}{1+\eta}\right).$$

If shocks are sufficiently positive $q \geq q_{\rm NB}$, then no entrepreneur defaults and aggregate losses of banks are zero. For shocks $q_{\rm TB} \leq q < q_{\rm NB}$, all investing entrepreneurs with quality levels $i_E < i < i_B$ default, whereas entrepreneurs with quality levels $i \geq i_B(q)$ fully pay back their loans. All entrepreneurs default if $q < q_{\rm TB}$ and losses are maximal. This event will occur with positive probability whenever $\underline{q} < q_{\rm TB}(d, r^d_{CB})$.

Using (35), repayments to banks $P = P(d, q, r^c_*)$ as given by (6) take the form

$$P = \begin{cases} q_{\rm NB}(1 + i_E) f(e+I)[\eta - i_E] & \text{if } q_{\rm NB} \leq q, \\ q_{\rm NB}(1 + i_E) f(e+I)\left[1 + \eta - \frac{1+i_E}{2}\left(\frac{q_{\rm NB}}{q} + \frac{q}{q_{\rm NB}}\right)\right] & \text{if } q_{\rm TB} < q < q_{\rm NB}, \\ q f(e+I)\frac{1}{2}[(1+\eta)^2 - (1+i_E)^2] & \text{if } q \leq q_{\rm TB}. \end{cases} \tag{36}$$

It is straightforward to verify that (36) is non-increasing in i_E for $i_E \in [0, \eta]$ in the case without interest rate cutting. Since banks' liabilities

$$[S + i_E e] \frac{(1 + i_E) f(e + I) q_{\mathrm{NB}}}{I}$$

are increasing in i_E, we see that future capital G is non-increasing in $i_E \in [0, \eta]$. Since $i_E(d)$ is decreasing in $d \in [\underline{d}, \overline{d}]$, it follows that without interest rate cutting, $G\big(d, q, r_*(d)\big)$ as given in (19) is non-decreasing in capital levels d.

Notice, finally, that an interest-rate policy $r_{CB}^d = \psi(d)$ could be defined by setting

$$1 + \psi(d) := g(d)\,\big[1 + r_*(d)\big], \quad d \in [\underline{d}, \overline{d}]$$

with an non-decreasing function g which satisfies $g(\underline{d}) = 0$ and $g(d) = 1$ for $d \geq d_{\mathrm{reg}}$. It can be verified that g may be chosen such that the resulting capital function G is non-decreasing in d.

References

Al-Najjar, N. I. (1995): "Decomposition and Characterization of Risk with a Continuum of Random Variables", *Econometrica*, 63, 1195–1224.

Allen, F. & D. Gale (1998): "Optimal Financial Crises", *The Journal of Finance*, 53(4), 1245–1284.

Alós-Ferrer, C. (1999): "Individual Randomness in Economic Models with a Continuum of Agents", Working paper no. 9807, University of Vienna.

Azariadis, C. & V. Galasso (1998): "Constitutional Rules and Intergenerational Fiscal Policy", *Constitutional Political Economy*, 9(1), 64–74.

Blum, J. & M. F. Hellwig (1995): "The Macroeconomic Implications of Capital Adequacy Requirements", *European Economic Review*, 39, 733–749.

Boyd, J. H. & E. C. Prescott (1986): "Financial Intermediary-Coalitions", *Journal of Economic Theory*, 38, 211–232.

Caprio, G. & P. Honohan (1999): "Restoring Banking Stability: Beyond Supervised Capital Requirements", *Journal of Economic Perspectives*, 13(4), 43–64.

Caprio, G. & D. Klingebiel (1997): "Bank Insolvency: Bad Luck, Bad Policy or Bad Banking?", in *World Bank Annual Conference on Development Economics, 1996*, ed. by M. Bruno & B. Pleskovic.

Dekle, R. & K. Kletzer (2003): "The Japanese Banking Crisis and Economic Growth: Theoretical and Empirical Implications of Deposit Guarantees and Weak Financial Regulation", Mimeo.

Dell'Ariccia, G., E. Detragiache & R. Rajan (2007): "The Real Effects of Banking Crises", *Journal of Financial Intermediation,* forthcoming.

Demirgüç-Kunt, A. & E. Detragiache (1998): "The Determinants of Banking Crises in Developing and Developed Countries", *IMF Staff Papers*, 45(1), 81–109.

Dewatripont, M. & J. Tirole (1994): *The Prudential Regulation of Banks.* MIT Press.

Diamond, D. (1984): "Financial Intermediation and Delegated Monitoring", *Review of Economic Studies*, 51, 393–414.

Diamond, D. & P. Dybvig (1984): "Bank Runs, Deposit Insurance, and Liquidity", *Journal of Political Economy*, 3, 401–419.

Ennis, H. M. & T. Keister (2003): "Economic Growth, Liquidity, and Bank Runs", *Journal of Economic Theory*, 109, 220–245.

Gehrig, T. (1996): "Market Structure, Monitoring and Capital Adequacy", *Schweizerische Zeitschrift für Volkswirtschaft und Statistik*, 132(4/2), 685–702.

Gehrig, T. & R. Stenbacka (2004): "Screening Cycles", Working paper, University of Freiburg.

Gersbach, H. (1998): "Financial Intermediation, Regulation and the Creation of Macroeconomic Risks", Working paper, University of Heidelberg.

Gersbach, H.& J. Wenzelburger (2008): "Do Risk Premia Protect from Banking Crises?", *Macroeconomic Dynamics*, 12.

González-Hermosillo, B., C. Pazarbaşioğlu & R. Billings (1997): "Determinants of Banking System Fragility: A Case Study of Mexico", *IMF Staff Papers*, 44(3), 295–314.

Gorton, G. (1988): "Banking Panics and Business Cycles", *Oxford Economic Papers*, 40, 751–778.

Hellman, T., K. Murdock & J. Stiglitz (2000): "Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Capital Requirements Enough?", *American Economic Review*, 90(1).

Hellwig, M. F. (1994): "Banking and Finance at the End of the Twentieth Century", WWZ-Discussion paper no. 9426, University of Basel.

——— (1995): "Systemic Aspects of Risk Management in Banking and Finance", *Schweizerische Zeitschrift für Volkswirtschaft und Statistik*, 131, 723–737.

——— (1998): "Banks, Markets, and the Allocation of Risks", *Journal of Institutional and Theoretical Economics*, 154, 328–345.

Holmström, B. & J. Tirole (1997): "Financial Intermediation, Loanable Funds, and the Real Sector", *The Quarterly Journal of Economics*, 112(3), 663–691.

Hoshi, T. & A. Kashyap (2004): "Japan's Economic and Financial Crisis: An Overview", *The Journal of Economic Perspectives*, forthcoming.

Kaminsky, G. L. & C. M. Reinhart (1999): "The Twin Crises: The Causes of Banking and Balance-of-Payments Problems", *American Economic Review*, 89(3), 473–500.

Krugman (1998): "It's Baaack! Japan's Slump and the Return of the Liquidity Trap", *Brooking Papers on Economic Activity*, 2, 137–187.

Lasota, A. & M. Mackey (1994): *Chaos, Fractals and Noise: Stochastic Aspects of Dynamics*, Applied Mathematical Sciences 97. Springer, New York a.o.

Lindren, C.-J., G. Garcia & M. Saal (1996): "Bank Soundness and Macroeconomic Policy", Discussion paper, International Monetary Fund, Washington, D.C.

Peter, von G. (1999): "The Banking System as a Wealth Redistribution Device", Working paper, Columbia University.

Schreft, S. L. & B. D. Smith (1997): "Money, Banking, and Capital Formation", *Journal of Economic Theory*, 73(1), 157–182.

Uhlig, H. (1995): "Transition and Financial Collapse", Discussion paper no. 66, CentER.

Van den Heuvel, S. (2007): "The Welfare Cost of Bank Capital Requirements", *Journal of Monetary Economics*, forthcoming.

Williamson, S. D. (1987): "Financial Intermediation, Business Failures, and Real Business Cycles", *Journal of Political Economy*, 95, 1197–1216.