Bargaining and social structure

Edoardo Gallo*

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Abstract

We consider large, decentralized markets in which buyers and sellers obtain information about past deals through their social network and they use this information to make more accurate demands in bilateral bargaining rounds. We show that the equilibrium depends crucially on the peripheral (least connected) individuals in each group, who weaken the position of the whole group. Comparative statics shows that groups with high density and/or low variability in number of connections across individuals allow their members to obtain a better deal. An empirical analysis of the observed price differential between Asian and white buyers in New York's wholesale fish market is consistent with these predictions. An extension explores an alternative set-up in which buyers and sellers belong to the same social network: if the network is regular and the agents are homogeneous then the unique equilibrium division is 50-50.

JEL: C73, C78, D83. **Keywords:** network, communication, noncooperative bargaining, core-periphery networks, 50-50 division.

NOTE TO THE ORGANIZERS: This is an early version of the paper. I am currently working on a new version which will include further theoretical results and (conditional on securing funding) an experimental test of the model which will replace the empirical component in the current version (section 5).

*Address: University of Oxford, Nuffield College, New Road, Oxford OX1 1NF, UK. Email: *edoardo.gallo@economics.ox.ac.uk*. I am very grateful to Peyton Young for his invaluable help and guidance throughout this project, and to Kathy Graddy for making available the Fulton fish market dataset. Thanks to Carlos Alós-Ferrer, Larry Blume, Sam Bowles, Jean-Paul Carvalho, David Easley, Aytek Erdil, Andrea Galeotti, Sanjeev Goyal, Alan Kirman, Rocco Macchiavello, Matteo Marsili and Margaret Meyer for helpful comments and suggestions. In addition to participants at several conferences, I would like to thank seminar participants at the University of Oxford, Cambridge University, Université d'Aix-Marseille III (GREQAM), the University of Manchester, and the Oxford Internet Institute. Any remaining errors are the sole responsibility of the author.

The problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we make use never exists in concentrated or integrated form, but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess. The problem is thus in no way solved if one can show that all of the facts, if they were known in a single mind [...], would uniquely determine its solution: instead we must show how a solution is produced by the interactions of people each of whom possesses only partial knowledge.

F. A. von Hayek, "The use of knowledge in society." AER, XXXV (4), September 1945.

Individuals who belong to close-knit groups often enjoy an advantage in many market interactions. For instance, Greif [1993] describes how in the 11th century Maghribi merchants joined into tightly integrated communities to facilitate trading across the Mediterranean in an environment characterized by a high degree of uncertainty and incomplete information. Rauch [2001] reviews empirical evidence that the presence of ethnic immigrant communities significantly increases international trade volumes, especially for commodities whose price is variable or uncertain. The goal of this paper is to explore one type of advantage that these groups provide and to relate the internal structure of interactions of a group to the observed market outcomes.

The core idea is that *belonging to a group gives an informational advantage*: individuals who are part of a group use their interactions to gather information about past transactions which they employ in future bilateral negotiations. This set-up is relevant for markets with a large number of individuals that are characterized by incomplete information, uncertainty about the price of the goods, and private bilateral negotiations. In these markets an individual is unable to collect information on the current price of a good due to the size of the market and the unobservability of private transactions, and therefore she turns to other members of her group to gather information about recent transactions before starting a trade.

Specifically, this paper develops a bargaining model between agents belonging to different groups in which the equilibrium outcome depends on the structure of interactions within each group. In the long-term every agent in a group receives the same share of the good, but the share varies across groups depending on their internal structure. This share depends crucially on the least connected individuals in the group because these individuals are the most susceptible to respond to noise present in the information they receive from other group members they interact with. A consequence of this result is that the optimal internal structure for a group has minimal variability in the number of connections across individuals. Moreover, a testable comparative statics prediction is that groups with high density and low variability in number of connections across individuals allow their members to obtain a better deal.

The model is applicable to markets with a *large number of agents* where there is *incomplete information* on the price of a *homogeneous commodity*: there are no posted prices and the agents *communicate* with each other to *learn* about the current price. Similarly to classical bargaining models, each transaction is a *private, bilateral negotiation* between two agents and the outcome in equilibrium will depend on the risk profile of the agents in each group. However, the agents are boundedly rational: they base their bid on *information on past transactions* they have collected *from other agents* in their group, and they are unaware of the game they are playing or of the utility profile of their opponent.

There are many markets that share these characteristics. A prominent example is markets in developing countries where prices fluctuate due to exogenous factors affecting supply, and where the lack of strong institutions is an obstacle to the adoption of publicly displayed prices allowing the proliferation of decentralized bilateral transactions.[1] A second example is markets in illegal commodities: the need to perform secret transactions leads to incomplete information, private bilateral exchanges and fluctuations in price due to frequent disruptions of the supply chain.[2] A third example is some wholesale markets where transactions are private between one buyer and one seller and the prices are very sensitive to exogenous factors that affect the supply chain. For instance, many wholesale fish markets are characterized by private, bilateral transactions and prices fluctuate due to exogenous factors such as wind and wave height that affect the volume of the daily catch of fish.[3]

Section 5 analyzes a dataset of prices from the Fulton wholesale fish market in New York. A puzzling finding is that Asian buyers pay a significantly lower price than white buyers for a homogeneous product sold by a white seller. The theoretical predictions from the comparative statics analysis offer a potential explanation: the group of Asian buyers is denser than the group of white buyers, and therefore Asians obtain more information about the ongoing price of fish and they exploit this informational advantage to obtain a lower price from the seller. The empirical analysis presents corroborating evidence in support of this rationale: the price difference emerges only after a few hours, and the variability of prices within the Asian group decreases over time while it remains constant within the white group. These findings suggest that the price difference emerges only after enough information is circulating within each group and that the process of information sharing is faster within the group of Asians. A final piece of evidence comes from several studies in the sociology literature which argue that Asian immigrants form very dense

[1] See, for example, Aker [2008] for evidence from grain markets in Niger.

[2] See, for example, Levitt and Venkatesh [2007] for evidence from the Chicago street-level prostitution market.

[3] Kirman [2001] provides a detailed description of wholesale fish markets. Another example of a wholesale market that can be described by this model is the fruit and vegetable market investigated in Kirman et al. [2005].

social networks.

The rest of the paper is organized as follows. Section 1 gives a more detailed overview of the paper and surveys the related literature. Section 2 presents the model. Section 3 derives the bargaining solution and discusses its implications for the desirable network structure for the members of a group. Section 4 investigates how changes in the internal structure of a group affect the solution. Section 5 analyzes a dataset of prices in the Fulton fish market and it shows corroborating evidence that the predictions of the model explain the price differential between Asian and white buyers. Section 6 investigates a different set-up of the model where buyers and sellers belong to the same communication network. Section 7 concludes. Appendix A contains the proofs omitted in the main text, and appendix B extends the model to allow for indirect communication.

1 Overview of the model and related literature

Consider a population of agents consisting of two disjoint groups of n_B buyers and n_S sellers. At each time t a buyer b and a seller s are randomly drawn to play the Nash demand game: b demands a fraction x_t and s demands a fraction y_t. If $x_t + y_t \leq 1$ then b and s get their demands, otherwise they get nothing. Note that the role of buyers and sellers is completely interchangeable: the following description of the model will focus on buyers only for expository purposes.

Each buyer maximizes a well-behaved utility function. At each time t the buyer receives a sample of previous demands by sellers: she chooses an optimal reply to the cumulative distribution of demands with probability $1 - \epsilon$, and a non-optimal reply, i.e. a "mistake," with probability ϵ.[4] The amount of information that buyer b receives from another buyer b' is the realization of a Poisson process connecting b to b'. Thus, the total information sample that b receives before the bargaining round consists of all the information coming from the realization of the Poisson processes that connect b to the other buyers she communicates with. A network g^B is an abstract representation of the average communication flows in the group of buyers: the strength of a link $g_{bb'}$ is equal to the rate of the Poisson process connecting b to b'.

Theorem 1 proves that if the communication networks of buyers and sellers are connected and if they are not complete networks then the process without mistakes always converges to a convention. A convention means that each buyer always makes the same demand x and each seller always makes the same demand $1 - x$. The condition on the network structure guarantees that the information available to each agent on the history of demands is sufficiently incomplete to avoid the whole process getting stuck in a cycle.

[4] Throughout the paper, the buyer is female and the seller is male.

Theorem 2 proves that the process with mistakes converges to a unique stable division, which is the *asymmetric Nash bargaining solution (ANB)* with weights that depend on the network structure. Specifically, the weights are determined by the subset of peripheral agents in each group with the least number of and/or weakest communication links. A consequence of this result is that, given a budget of links to allocate, the desirable architectures for a group are *quasi-regular networks*, i.e. networks where all the agents are connected by strong links and have a very similar number of connections.

The solution in theorem 2 allows the exploration of how changes in the network structure affect the equilibrium division. The changes are modeled in terms of first and second order stochastic dominance shifts in the weighted degree distribution, i.e. variations in the relative frequencies of agents with different number of connections. Theorem 3 shows that individuals belonging to a group with *a high density of connections and/or a low variability of connections across individuals will fare better.*

Section 5 analyzes a dataset on transaction prices in the Fulton wholesale fish market in New York. A puzzling finding first highlighted by Graddy [1995] is that Asian buyers pay a significantly lower price than white buyers for the same product sold by a white seller. The predictions of theorem 3 provide the foundations for a story that explains the price difference. Graddy's field observations suggest that buyers communicate within their ethnic group to learn the daily price of fish. Moreover, sociological evidence shows that the group of Asians is very dense and therefore it is a better channel of information than the group of whites on the uncertain price of the product. Over time the Asians receive more information and they exploit this additional knowledge to make more accurate offers and obtain a lower price from the seller. An empirical analysis offers corroborating evidence for this story by showing that the price difference emerges only after a few hours, and the variability of prices within the Asian group decreases over time while it remains constant within the white group.

An extension explores how the theoretical predictions change if buyers and sellers belong to the same communication network, allowing in this way buyers to receive information from other buyers *and* sellers. The unique stable division is still the *ANB solution* in theorem 2. However, the effect of varying the network structure is now different: a denser communication network leaves the ANB unchanged, but *less variability of connections across individuals narrows down the difference* between the shares of the two groups. If the network is a regular network, then the solution is the *50-50 division.* The desirable architectures for the buyers are *core-periphery networks*: the buyers form a core network where they are connected by strong links and they have a very similar number of connections, while the sellers are at the periphery where each one of them is connected by one link to a buyer.

In previous contributions there are at least two complementary explanations of why belonging to a group leads to a competitive advantage in a market with incomplete information. The first one was originally advanced by Greif [1993]: an individual trader in a group can rely on the other members of the group to inflict a costly punishment to a cheater by cutting all future trade between any member of the group and the cheater. He illustrates this with a simple model in a repeated game framework, and he draws on historical records to discuss its relevance for trading in the 11th century.

The second explanation is the core idea behind the model presented here: an individual in a group has access to information from other group members and this leads to a competitive advantage in a market where information is incomplete. Rauch and Casella [2003] proposed a model where information-sharing within ethnic groups influences resource allocation in international trade markets affected by incomplete information. Rauch and Trindade [2002] show that the information-sharing story fits observed international trade flows better than the collective punishment one. One of the key differences between this paper and these previous contributions is that it explicitly models the role of the network structure of interactions within a group.

In the economics of networks literature a number of papers investigate how a network that constrains agents' interactions affects the outcome of a bargaining process. Selected contributions include Calvó-Armengol [2001], Calvó-Armengol [2003], Corominas-Bosch [2004], Polanski [2007] and Manea [2008]. The framework adopted here is conceptually different. In all the references listed above, the network is a constraint on the *interactions* that agents are allowed to have. On the other hand, in this paper the network is a constraint on the *information* about past bargains that agents have as they enter a bargaining round. Moreover, the focus of this paper is also different. Previous work in the literature investigates how the position of *one* agent in a network affects her *individual* payoffs. Here the aim of the paper is to understand how the *overall* structural properties of the network determine the payoff that every individual in the *whole group* receives, independently on their position in the network.

Methodologically, this paper is based on the evolutionary bargaining framework first formulated by Young [1993a]. The bargaining procedure and the behavior of agents is the same as in Young's model: individuals from two groups of bargainers are randomly matched to play the Nash demand game and they make demands by choosing best replies based on an incomplete knowledge of precedents. The novel element introduced here is the modeling of the process by which agents receive information to play the game: information travels through a communication network that connects the agents in each group.

The introduction of the network demands the construction of a different Markov pro-

cess to describe the evolution of the system, which requires a novel equilibrium analysis. The equilibrium outcome depends on the underlying network and therefore this allows the comparative statics analysis in section 4, which would not be possible in the model without the network. Another key advantage of introducing the network is that it opens up the possibility of testing the comparative statics predictions of the model on real market data, as the empirical exercise carried out in section 5 illustrates. Finally, section 6 analyzes the case when buyers and sellers belong to the same communication network: the equilibrium outcome is unchanged, but the comparative statics predictions are different, and this type of analysis is only feasible in the model with the network.

The contribution of this paper is threefold. First, it constructs a model to investigate the role of a group's communication structure in markets with a large number of agents where there is incomplete information. This provides a theoretical underpinning to previous empirical studies that emphasized the informational role of social structure in determining market outcomes. Second, it derives predictions on the effects of changes in the communication structure on the equilibrium outcome. Third, it illustrates the relevance of these predictions in the context of the New York wholesale fish market.

2 The Model

This section presents the main elements of the model: the network concepts used, the adaptive play bargaining process, and the Markov process which describes the evolution of the system.

Networks. A *weighted, undirected network* g is represented by a symmetric matrix $[g_{ij}]^{n\times n}$, where $g_{ij} \in \mathbb{R}_+$. The entry g_{ij} indicates the *strength* of the *communication link* between i and j. If $g_{ij} > 0$ then agents i and j are connected and they communicate directly with each other. If $g_{ij} = 0$ then i and j are not connected in the communication network. Throughout this paper let $g_{ii} \equiv \overline{g}$, i.e. an agent is connected with herself and the strength of this self-connection is the same for all agents.

The *neighborhood* of i in g is $L_i(g) = \{j \in N | g_{ij} > 0\}$.[5] $d_i(g) \equiv |L_i(g)|$ denotes the size of i's neighborhood, or the *degree* of i, in g. $z_i(g) \equiv \sum_{j\in L_i(g)} g_{ij}$ is the *weighted degree* of i in g. Let $Z(g) = \max_{i\in N} z_i(g)$ be the maximum weighted degree of any agent in the network g. A *complete network* is a network that belongs to the class of networks $g^C = \{g | g_{ij} > 0, \forall i, j \in N\}$ where every pair of agents is connected. A *regular network* $g_{d,a}$ of degree d and link strength a is a network that belongs to the class of

[5]Note that this definition is slightly different than the standard one adopted in the literature because it allows for i's neighborhood to include i as well. This is because in our framework agent i's own degree g_{ii} is allowed to be positive. This difference affects the ensuing definitions as well.

networks $\overline{g}_{d,a} = \{g|g_{ij} = \{0,a\}; d_i(g) \equiv d;\ \forall i,j \in N; a \in \mathbb{R}_+\}$. A *regular weighted network* g_k of weighted degree k is a network that belongs to the class of networks $\overline{g}_k = \{g|z_i(g) = k; \forall i \in N; k \in \mathbb{R}_+\}$.

The *weighted degree distribution* of a network is a description of the relative frequencies of agents that have different degrees. Let $p(z)$ denote the weighted degree distributions of network g, i.e. the fraction of nodes that have weighted degree z in network g, and let $\mu[p(z)]$ denote the mean of the distribution. The comparative statics analysis in this paper will investigate changes in the communication structure that are captured by stochastic dominance shifts in this degree distribution. The following are more formal definitions of these notions.

Definition 1. A distribution p' first order stochastic dominates (FOSD) another distribution p if $\rho'(x) \leq \rho(x)$ for any $x \in [0, Z]$, where $\rho(x) = \sum_{z=0}^{x} p(z)$ is the cumulative distribution of $p(z)$. The FOSD shift is variance-preserving if $Var[p(z)] = Var[p'(z)]$.

Definition 2. A distribution p'' strictly second order stochastic dominates (SOSD) another distribution p if $\sum_{z=0}^{x} \rho''(z) \leq \sum_{z=0}^{x} \rho(z)$ for any $x \in [0, Z]$. The SOSD shift is mean-preserving if $\mu[p(z)] = \mu[p''(z)]$.

If $p'(z)$ FOSD $p(z)$ then a network g' is *denser* than a network g. Note that in the context of weighted networks denser means that agents in g' have on average a higher number and/or stronger links than agents in g. If $p''(z)$ SOSD $p(z)$ then a network g'' is *more homogeneous* than a network g. Similarly, more homogeneous means that there is less variability across agents in g'' in terms of the number and/or strength of their connections than across agents in g.

Adaptive play bargaining process. Consider two finite, non-empty and disjoint groups of individuals $B = \{1, ..., n_B\}$ and $S = \{1, ..., n_S\}$: the buyers and sellers. In each period t one buyer and one seller drawn at random meet to divide a pie of size normalized to one. They play the Nash demand game: b demands a fraction x_t and s demands a fraction y_t, if $x_t + y_t \leq 1$ then b and s get their demands, otherwise they get nothing. Assume that the set of possible divisions is discrete and finite, and let δ be the smallest possible division. The sequence $h = \{(x_1, y_1), ..., (x_t, y_t)\}$ is the complete *global history* up to and including period t. Each agent remembers the last m rounds of the bargaining game that she has played, where m stands for the memory of the agent and $m > \max\{Z(g^B), Z(g^S)\}$.

Agents receive information to play the game as follows. Suppose agent $b \in B$ is picked to play the game at $t + 1$: in the $\Delta t = 1$ time period she receives information from some of the other buyers in B about past bargaining rounds. Information arrival is modeled as a Poisson process. Specifically, in the $\Delta t = 1$ time interval, the probability

$P(s_{bj}(\Delta t = 1) = k)$ that b receives a sample $s_{bj}(\Delta t = 1)$ of k past bargains from agent j is equal to:

$$P(s_{bj}(\Delta t = 1) = k) = \frac{e^{-g_{bj}} g_{bj}^k}{k!}$$

where g_{bj} is the rate of arrival of information to b from j. By standard properties of Poisson processes, the expected amount of information b receives from j before each bargaining round is $E[P(s_{bj})] = g_{bj}$. Thus, we have that $\sum_{j \in L_b(g)} E[P(s_{bj})] = E[P(s_b)] = \sum_{j \in L_b(g)} g_{bj} = z_b(g)$, and therefore before each bargaining round the expected size of the sample of past demands is $\lceil z_b(g) \rceil$, where $\lceil . \rceil$ is the ceiling function to round it up to the nearest integer. Clearly, at each point in time the realization of the Poisson process that determines how much information b receives from j may be higher or lower than g_{bj}, but over a long period of time the average amount of information per time period that b receives from j will be equal to g_{bj}. Thus, the network g captures the average information flows between each pair of agents in the group over a long period of time.[6]

The variability of an agents' information sample over time poses significant challenges to an analytical investigation of the model. In order to overcome this, throughout the paper we impose the *mean-field assumption* that the *total* amount of information an agent b receives is the same across bargaining rounds. More formally, assume that the size of the information sample of the buyer b is constant and equal to the amount of information b receives in expectation given the Poisson processes involving b, i.e. $s_b(t) \equiv \lceil \sum_{j \in B} g_{bj} \rceil = \lceil z_b(g) \rceil$. Note that this assumption still allows for the realization of each individual Poisson process to vary, and it fixes only the sum of the realizations of all the Poisson processes. Thus, in some bargaining rounds agent b may receive most of the information from her neighbor b', while in other rounds b' may not provide any information. However, the total size of the information sample b receives before playing each bargaining round is always the same.

Agents are boundedly rational as they are not aware of the game they are embedded in and they base their decision exclusively on the information they receive. Specifically, agents do not have prior knowledge or beliefs about the utility function of the other side, and they do not know the distribution of utility functions in the general population. Agent b chooses an optimal reply to the cumulative probability distribution $G(y)$ of the demands y_j made by sellers in the sample that she receives, where $G(y) = \frac{h}{s_b(t)}$ if and only if there are exactly h demands y_l in the sample $s_b(t)$ such that $y_l \leq y$.

Agent b has a concave and strictly increasing von Neumann-Morgenstern utility function $u(x)$. Assume that $u(x)$ is defined for all $x \in [0, 1]$ and that it is normalized so that $u(0) = 0$. Buyer b's expected payoff from demanding x is then equal to $Eu(x) \equiv$

[6]Note also that there is no need to assume that the Poisson process is truncated given that the memory m can be arbitrarily large.

$u(x)G(1-x)$. Thus, b chooses x_{t+1} so as to maximize $Eu(x)$, and if there are several values of x to choose from then each one of them is chosen with positive probability.

The set-up for seller s is analogous, and the utility function of the sellers will be denoted by $v(y)$.

Markov process. Let $\mathbf{S}$ be the state space, whose elements are sets of vectors $\mathbf{s} = \{\mathbf{v}_1, ..., \mathbf{v}_n\}$, where $\mathbf{v}_i$ stands for agent i's memory, which is a vector of size m, and $n \equiv n_B + n_S$. If $i \in B$ then $\mathbf{v}_i = \{y^i_{k-m+1}, ..., y^i_k\}$, i.e. the entries of $\mathbf{v}_i$ are the m last demands made by sellers in bargaining rounds involving i. Similarly, if $i \in S$ then $\mathbf{v}_i = \{x^i_{k-m+1}, ..., x^i_k\}$. Let $q_b(x \mid \mathbf{s})$ be agent b's best-reply distribution, i.e. $q_b(x \mid \mathbf{s}) > 0$ if and only if demanding x is b's best-reply to a sample received when the system is in state $\mathbf{s}$. Analogously, $q_s(y \mid \mathbf{s})$ is seller s's best-reply distribution.

Assume that the process starts at an arbitrary time $t_0 > n \cdot m$, and denote the initial state by $\mathbf{s}^0$. At each $t > t^0$, one pair of agents $(b, s) \in B \times S$ is drawn at random with probability $\pi(b, s)$, where $\pi(b, s) > 0$, $\forall (b, s) \in B \times S$. At time t, consider a state $\mathbf{s} = \{\mathbf{v}_b, \mathbf{v}_s, \mathbf{v}_{-b}, \mathbf{v}_{-s}\}$, where $\mathbf{v}_b = \{y^b_{k-m+1}, ..., y^b_k\}$ and $\mathbf{v}_s = \{x^s_{k-m+1}, ..., x^s_k\}$. Define $\mathbf{s}'$ to be a successor of $\mathbf{s}$ if it has the form $\mathbf{s}' = \{\mathbf{v}'_b, \mathbf{v}'_s, \mathbf{v}_{-b}, \mathbf{v}_{-s}\}$, where $\mathbf{v}'_b = \{y^b_{k-m+2}, ..., y^b_{k+1}\}$ and $\mathbf{v}'_s = \{x^s_{k-m+2}, ..., x^s_{k+1}\}$. The transition probability $P_{\mathbf{ss}'}$ of moving from state $\mathbf{s}$ to state $\mathbf{s}'$ is then equal to:

$$P_{\mathbf{ss}'} = \sum_{b \in B} \sum_{s \in S} \pi(b, s) q_b(x_{t+1} \mid \mathbf{s}) q_s(y_{t+1} \mid \mathbf{s}) \tag{1}$$

Mistakes. In the process described so far agents always give a best reply to the sample they happen to pick. In reality, people make mistakes for a variety of reasons: human beings are poor at computing probabilities and they might miscalculate the expected utility from an offer, they are prone to get distracted, they experiment, or sometimes they are outright irrational. The following is a more formal definition of a mistake.

Definition 3. Let $\mathbf{s} = \{\mathbf{v}_b, \mathbf{v}_s, \mathbf{v}_{-b}, \mathbf{v}_{-s}\}$ and let $\mathbf{s}' = \{\mathbf{v}'_b, \mathbf{v}'_s, \mathbf{v}_{-b}, \mathbf{v}_{-s}\}$ be a successor of $\mathbf{s}$, where $\mathbf{v}_b = \{y^b_{k-m+1}, ..., y^b_k\}$, $\mathbf{v}_s = \{x^s_{k-m+1}, ..., x^s_k\}$, $\mathbf{v}'_b = \{y^b_{k-m+2}, ..., y^b_{k+1}\}$ and $\mathbf{v}'_s = \{x^s_{k-m+2}, ..., x^s_{k+1}\}$. The demand x^s_{k+1} is a mistake by b if it is not a best response to any sample b could have received given that the system is in state $\mathbf{s}$. A mistake y^s_{k+1} by s is defined similarly.

Another concept that will be useful in the analysis of the perturbed process is the *resistance* in moving from one state $\mathbf{s}$ to another state $\mathbf{s}'$.

Definition 4. Let $\mathbf{s}$ and $\mathbf{s}'$ be two states of the system. The *resistance* $r(\mathbf{s}, \mathbf{s}')$ is the least number of mistakes required for the system to go from state $\mathbf{s}$ to $\mathbf{s}'$.

Note that if $\mathbf{s}'$ is a successor of $\mathbf{s}$ then $r(\mathbf{s}, \mathbf{s}') \in \{0, 1, 2\}$ given that the maximum number of mistakes in any one-time transition is two, i.e. both the buyer and seller involved in that bargaining round make a mistake.

Now let ϵ be the absolute probability that agents in the model make mistakes, and let λ_b, λ_s be the relative probabilities that buyers and sellers do so respectively. Thus, $\epsilon\lambda_b$ and $\epsilon\lambda_s$ are the probabilities that buyers and sellers make a mistake. Denote by $w_b(x \mid \mathbf{s})$ the buyer's conditional probability of choosing x given that the current state is $\mathbf{s}$ and that she is not giving a best-response offer to the sample picked, and define $w_s(y \mid \mathbf{s})$ analogously. Assume $\lambda_b, \lambda_s, \epsilon > 0$ and that $w_b(x \mid \mathbf{s})$, $w_s(y \mid \mathbf{s})$ have full support.

This process also yields a stationary Markov chain on $\mathbf{S}$ that can be described by the probability of moving from a state $\mathbf{s}$ to a successor state $\mathbf{s}'$, similarly to equation (1) above. Assume that the process starts at an arbitrary time $t_0 > n \cdot m$, and denote the initial state by $\mathbf{s}^0$. At each $t > t^0$ one pair of agents $(b, s) \in B \times S$ is drawn at random with probability $\pi(b, s)$, where $\pi(b, s) > 0$, $\forall (b, s) \in B \times S$. Let $\mathbf{s}$ be the state at time t, and let $\mathbf{s}'$ be a successor of $\mathbf{s}$, where $\mathbf{s}$ and $\mathbf{s}'$ are defined above. The transition probability $P^{\epsilon}_{\mathbf{ss}'}$ of moving from state $\mathbf{s}$ to state $\mathbf{s}'$ is then equal to:

$$P^{\epsilon}_{\mathbf{ss}'} = \sum_{b \in B} \sum_{s \in S} \pi(b, s)[(1 - \epsilon\lambda_b)(1 - \epsilon\lambda_s) q_b(x_{t+1} \mid \mathbf{s}) q_s(y_{t+1} \mid \mathbf{s}) + \epsilon^2 \lambda_b \lambda_s w_b(x_{t+1} \mid \mathbf{s}) w_s(y_{t+1} \mid \mathbf{s})] +$$
$$+ \epsilon\lambda_b(1 - \epsilon\lambda_s) w_b(x_{t+1} \mid \mathbf{s}) q_s(y_{t+1} \mid \mathbf{s}) + \epsilon\lambda_s(1 - \epsilon\lambda_b) w_s(x_{t+1} \mid \mathbf{s}) q_b(y_{t+1} \mid \mathbf{s}) \tag{2}$$

The limit of the perturbed process is clearly the unperturbed one: $\lim_{\epsilon \to 0} P^{\epsilon}_{\mathbf{ss}'} = P_{\mathbf{ss}'}$. Note that this Markov process is more complex than it needs to be to generate the results presented in this paper because it allows for the relative probabilities that buyers and sellers make mistakes to vary due to the λ_b and λ_s factors. This heterogeneity does not affect the results because the analysis is asymptotic in the limit as mistakes go to zero.

3 Equilibrium analysis

This section presents the results of the equilibrium analysis. Section 3.1 shows that the process without mistakes converges to a convention as long as the network is not complete. Section 3.2 derives the stochastically stable division. Section 3.3 characterizes the desirable communication network structure for the members of a group and discusses the relevance of this result to a long-standing debate in the sociology literature.

3.1 Convergence

First, consider the unperturbed process P. The first step in the analysis is to define an appropriate concept of stability for this system, and to show that in the long-term the process will reach it. Intuitively, the system will be in a stable state if after a certain time t any buyer will always make the same demand x because the sellers have always demanded $1-x$, and vice versa for the sellers. The following definition states this more formally.

Definition 5. A state $\mathbf{s}$ is a convention if any $\mathbf{v}_i \in \mathbf{s}$ with $i \in B$ is such that $\mathbf{v}_i = (1-x, ..., 1-x)$, and any $\mathbf{v}_j \in \mathbf{s}$ with $j \in S$ is such that $\mathbf{v}_j = (x, ..., x)$, where $0 < x < 1$. Hereafter, denote this convention by $\mathbf{x}$.

It is straightforward to see that the convention $\mathbf{x}$ is an absorbing state of P. If a buyer receives a sample in which all sellers' demands are equal to $1-x$ then she will demand x. Similarly, if a seller receives a sample in which all buyers' demands are equal to x then he will demand $1-x$. Clearly, this will go on forever so $\mathbf{x}$ is an absorbing state of P.

Lemma 1. *Every convention $\boldsymbol{x}$ is an absorbing state of the Markov process P in (1).*

The following theorem shows that if information about the history of play is sufficiently incomplete then the process P converges to a convention. The incompleteness of information is delivered by the network structure: if the network is not complete then some agents do not receive information on past demands in rounds played by individuals that do not belong to their neighborhoods.

Theorem 1. *Assume both g^B and g^S are connected and they are not complete networks. The bargaining process converges almost surely to a convention.*

The example networks in figure 1 help understanding the intuition behind the proof. The goal is to show that from any initial state $\mathbf{s}$ there is a positive probability p independent of t of reaching a convention within a finite number of steps. The assumption that g^B is not a complete network implies that there are at least two agents b' and b'' such that $g_{b'b''} = 0$. Moreover, given that g^B is connected, there are at least two agents like b' and b'' such that the intersection of their neighborhoods includes at least one agent b. The same applies to the sellers' network, where agents s and s'' are the equivalent of agents b' and b'' respectively.

Now, consider the following path which happens with positive probability from any state $\mathbf{s}$ at time t. First, b and s are picked to play the game for m periods, they draw samples σ and σ' respectively, they demand best-replies x and y respectively, and therefore we have a run $\xi = \{(x, y), ..., (x, y)\}$ such that $\mathbf{v}_b = (y, ..., y)$ and $\mathbf{v}_s = (x, ..., x)$. Second,

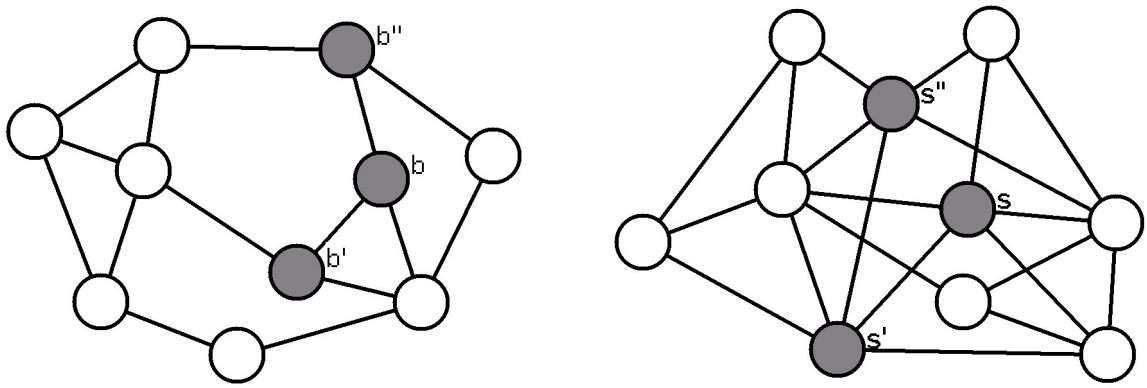

Figure 1: Example networks of buyers (left) and sellers (right). Gray-colored nodes are mentioned in the text to give intuition about the proof.

b' and s' are picked to play the game for m periods, they draw samples from $\mathbf{v}_b$ and $\mathbf{v}_s$ each time, they demand best-replies $1-y$ and $1-x$ respectively, and therefore we have a run $\xi' = \{(1-y, 1-x), ..., (1-y, 1-x)\}$ such that $\mathbf{v}_{b'} = (1-x, ..., 1-x)$ and $\mathbf{v}_{s'} = (1-y, ..., 1-y)$. Third, b'' and s'' are picked to play the game for m periods, they draw samples from $\mathbf{v}_b$ and $\mathbf{v}_{s'}$ each time, they demand best-replies $1-y$ and y respectively, and therefore we have a run $\xi'' = \{(1-y, y), ..., (1-y, y)\}$ such that $\mathbf{v}_{b''} = (y, ..., y)$ and $\mathbf{v}_{s''} = (1-y, ..., 1-y)$. Hereafter it is clear that there is a positive probability of reaching a convention $\mathbf{x} = (1-y, y)$.

Theorem 1 in Young [1993b] proves adaptive play converges almost surely to a convention in any weakly acyclic game with n agents as long as information is sufficiently incomplete. In Young [1993b]'s the incompleteness of the information is given by bounds on the size of the sample the agents can draw to base their play on. Here the incompleteness of information is given by the network structure: if the network is not complete there will be agents who cannot sample some past rounds because they were played by agents in their group with whom they do not communicate.

Appendix B extends the model to a setting where indirect communication is allowed so that agents receive information from friends of their friends up to a social distance r. The statement of the theorem extends naturally to this setting: if there are at least two agents at a distance higher than r then there is convergence to a convention. The rationale is the same: there is incompleteness of information because at least two agents are not able to sample the whole history of past demands. Clearly, theorem 1 above is the special case for $r = 1$.

Second, consider the perturbed process P^ϵ. Given that the distributions w_b and w_s have full support, P^ϵ is irreducible. Thus, P^ϵ has a unique stationary distribution. Moreover, P^ϵ is strongly ergodic, i.e. $\forall \mathbf{s} \in S$, μ_s^ϵ is with probability one the relative frequency with which state $\mathbf{s}$ will be observed in the first t periods as $t \to \infty$. The stability concept for this kind of perturbed process is a *stochastically stable convention*

by Foster and Young [1990].

Definition 6. A convention $\mathbf{s}$ is stochastically stable if $\lim_{\epsilon \to 0} \mu^{\epsilon}_{\mathbf{s}} > 0$. A convention $\mathbf{s}$ is strongly stable if $\lim_{\epsilon \to 0} \mu^{\epsilon}_{\mathbf{s}} = 1$.

Intuitively, in the long-run stochastically stable conventions will be observed much more frequently than unstable conventions when the probability ϵ of mistakes is small. A strongly stable convention will be observed almost all the time. The technique to compute the stochastically stable conventions is standard and it will not be explained in detail below, see Young [1998] for an excellent introduction.

Construct a weighted, directed network $[r_{\mathbf{s}^i \mathbf{s}^j}]^{k \times k}$, where the nodes are the states $\mathbf{s} \in \mathbf{S}$, the links are the resistances $r_{\mathbf{s}^i \mathbf{s}^j}$ connecting $\mathbf{s}^i$ to $\mathbf{s}^j$, and k is the total number of states in $\mathbf{S}$. Define an *$\boldsymbol{x}$-tree* $t_{\mathbf{x}} \in T_{\mathbf{x}}$ to be a collection of links in $[r_{\mathbf{s}^i \mathbf{s}^j}]^{k \times k}$ such that, from every node $\mathbf{x}' \neq \mathbf{x}$, there is a unique directed path to $\mathbf{x}$ and there are no cycles. This construction leads to the definition of the concept of *stochastic potential* of a convention $\mathbf{x}$.

Definition 7. The stochastic potential $\gamma(\mathbf{x})$ of a convention $\mathbf{x}$ is the least resistance among all $t_{\mathbf{x}} \in T_{\mathbf{x}}$. Mathematically:

$$\gamma(\mathbf{x}) = \min_{t_{\mathbf{x}} \in T_{\mathbf{x}}} \sum_{(\mathbf{x}', \mathbf{x}'') \in T_{\mathbf{x}}} r(\mathbf{x}', \mathbf{x}'') \tag{3}$$

Theorem 4 in Young [1993b] explains how to compute the stochastically stables states. The following is a special case of that result.

Theorem. ***[Young [1993b]]*** *Let μ^0 be a stationary distribution of the unperturbed process P. Then $\lim_{\epsilon \to 0} \mu^{\epsilon}_{\boldsymbol{s}} = \mu^0$. Moreover, $\mu^0 > 0$, i.e. $\boldsymbol{s}$ is stochastically stable, if and only if $\boldsymbol{s} = \boldsymbol{x}$ is a convention and $\gamma(\boldsymbol{x})$ has minimum stochastic potential among all conventions.*

3.2 Asymmetric Nash bargaining solution

Let us apply the methodology outlined above to find the division which the process will converge to. Define $B_{min} = \{j \in B \mid \lceil z_j(g^B) \rceil \leq \lceil z_b(g^B) \rceil, \forall b \in B\}$ to be the subset of buyers with the least weighted degree. Let $z_b^{min}(g^B) = \lceil z_j(g^B) \rceil$ for $j \in B_{min}$. Equivalent definitions apply to the sellers. The first step is to compute the minimum resistance to moving from the convention $\mathbf{x}$ to the basin of a different convention $\mathbf{x}'$. This is done in the following lemma.

Lemma 2. *The minimum resistance to moving from* $\boldsymbol{x}$ *to a state in some other basin is* $\lceil R(x) \rceil$*, where:*

$$R(x) = \min \left\{ z_b^{min}(g) \left(1 - \frac{u(x-\delta)}{u(x)} \right), z_s^{min}(g) \frac{v(1-x)}{v(1-\delta)}, z_s^{min}(g) \left(1 - \frac{v(1-x-\delta)}{v(1-x)} \right) \right\} \tag{4}$$

The intuition is as follows. Some agents have to make mistakes in order for the system to move from one convention to a state in the basin of another convention. The agents who will switch with the least number of mistakes in their sample are the ones who receive the smallest samples. This explains the factors $z_b^{min}(g)$ and $z_s^{min}(g)$ in equation (4). Now, consider the case when some sellers make a mistake. The smallest mistake they can make is to demand a quantity δ more than the conventional demand $1-x$. If they do this, buyers will attempt to resist up to the point when the utility from getting the conventional amount x some of the time, i.e. when sellers do not make a mistake, is equal to the utility from getting the lower amount $x-\delta$ all the time. This gives the first term in equation (4). The third term is the equivalent of the first one, but this time the buyers make a mistake and demand δ more than the conventional amount x.

Another possibility is that some buyers make a mistake, but this time they demand less than the conventional amount x. The "worst" mistake, from the buyers' point of view, would be to demand the minimum amount δ. If they do this, sellers will only switch at the point when the utility from getting the higher amount $1-\delta$ some of the time, i.e. when buyers make a mistake, is higher than the utility from getting the conventional amount x all the time. This gives the second term in (4). There should also be a fourth term, i.e. the equivalent of the second one with the roles of buyers and sellers reversed, but it is not included in (4) because it is never strictly smaller than the last term.

The expression for $R(x)$ in (4) is the minimum of three monotone functions: the first two are strictly decreasing in x, while the last one is strictly increasing in x. Thus, $R(x)$ is first strictly increasing and then strictly decreasing as x increases, so it achieves its maximum at a unique value x^*.[7] Using this fact, the following theorem shows that there is a unique stable division, which is the asymmetric Nash bargaining solution with weights that depend on the agents in each group with the least weighted degrees.

Theorem 2. *There exists a unique stable division* $(x^*, 1-x^*)$*. It is the one that maximizes the following product:*

$$u^{z_b^{min}}(x) v^{z_s^{min}}(1-x) \tag{5}$$

In other words, it is the asymmetric Nash bargaining solution with weights $z_b^{min}(g^B)$ *and*

[7] Technically, $R(x)$ can achieve its maximum at one value x^* or at two values x^* and $x^*+\delta$. As $\delta \to 0$ these two values clearly converge to a unique maximum x^*.

$z_s^{min}(g^S)$.

If the precision δ is sufficiently small then over time the two groups will settle on a conventional division, which is the asymmetric Nash bargaining solution. This solution crucially depends on the communication networks that buyers and sellers use to learn about past bargaining rounds to determine what to demand once they are picked to play. More precisely, *ceteris paribus* (i.e. agents' risk-aversion in the two groups is the same), the division depends on the agents in the group with the least number and/or weakest communication links. The intuition is that these agents will be the least informed when it comes to play the game, and therefore they will be the most susceptible to respond to mistakes from the other side. Over time, this susceptibility weakens the bargaining position of the whole group.

The proof of the theorem follows from two lemmas from Young [1993a]. The first lemma shows that a division $(x, 1-x)$ is generically stable if and only if x maximizes the function $R(x)$ in equation (4). The second lemma shows that the maxima of $R(x)$ converge to the asymmetric Nash bargaining solution which maximizes the product in (5). This solution is clearly analogous to the one in theorem 3 in Young [1993a]. The key difference is that the solution in theorem 2 above depends explicitly on the internal communication structure of the group of buyers/sellers. This allows the derivation of the comparative statics results in section 4 and the empirical analysis of the Fulton fish market.

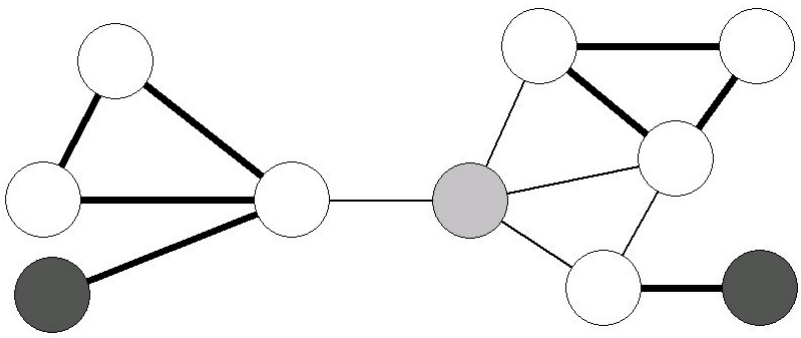

Figure 2: A weighted network with 10 agents: strong links (in bold) have weight 2 and weak links have weight 0.5. Color-coded nodes are the least connected agents.

Figure 2 is an example of a weighted network formed by 10 agents connected by two types of links: strong links with weight 2 and weak links with weight 0.5. The subset B_{min} of agents with the least information has three individuals, who are color-coded in the figure. Note that there are at least two typologies of agents who can belong to this subset. The first one is represented by the two agents color-coded in dark gray: they rely on a single source for information on past bargaining rounds, and in both cases the

source belongs to their own sub-community. They are strongly linked to their source, but they are very susceptible to potential mistakes in the information coming from her. They are *peripheral* agents in the network, who rely excessively on information from their *own community*. On the other hand, the second typology is represented by the agent color-coded in light gray: she relies on a high number of sources from both sub-communities, but they are only weakly connected with her. She is connected to different parts of the network making her very exposed to any kind of information circulating in the network, including potential mistakes. She is an agent with *weak links* who is very susceptible to information flowing in the network because she connects *across communities*. Section 3.3 below will include further discussion on this.

Obviously, as in standard bargaining models, the solution also depends on the utilities of the agents. *Ceteris paribus* (i.e. the least connected agents in each group have the same weighted degree), a group with less risk-averse agents will have a stronger bargaining position because these agents are more likely to take chances, and therefore they are more demanding.

Finally, it is possible to derive an equilibrium division that depends on the network structure in a richer way than the solution in theorem 2. Appendix B extends the model to a setting where indirect communication is allowed, so that agents receive information about past demands not only from their friends but also from friends of their friends up to a social distance r. In the extended model the statement of theorem 2 is unchanged except for the weights that are now determined by the agents with the smallest *decay r-centrality*, a metric that captures the number and/or strength of connections in their extended neighborhood up to a distance r.

3.3 The weakness of weak ties

What is the desirable communication structure for the members of a group of individuals that engage in this bargaining process with another group? In order to answer this question, let us define a class of *quasi-regular networks*, which are generated by a given regular network.

Definition 8. Consider the set G of undirected networks with n nodes and at most L links. Let $g_{d,a}$ be a regular network with degree $d = \left\lfloor \frac{2L}{n} \right\rfloor$ and link strength a, i.e. it belongs to $\overline{g}_{d,a}$ which is the class of largest regular networks in G. The network $g \in G$ is a quasi-regular network generated by $g_{d,a}$ if it can be obtained by randomly adding k links of any strength to $g_{d,a}$, where $k \in [0, L - \frac{n}{2})$.

A quasi-regular network is a network that is similar to a regular network in the sense that the links are distributed evenly among the nodes and there is minimal degree

variation. Note that if $L/n \in \mathbb{N}$, i.e. the links can be exactly divided among the nodes, then the set of quasi-regular network coincides with the class of regular networks $\overline{g}_{d,a}$. If $L/n \notin \mathbb{N}$ then each node has at least as many links as in the generating regular network, and the remaining links are randomly assigned. The following corollary shows that the desirable communication structure for a group is a quasi-regular network.

Corollary 1. *Fix a communication network g^S for the sellers. Consider the set G of all possible communication structures g^B among the n_B buyers such that the total number of links is $L < \frac{n_B}{2}(n_B - 1)$ and the strength of each link is in the $[\underline{s}, \overline{s}]$ range, where $\underline{s}, \overline{s} \in \mathbb{R}_+$. The subset of networks $G_B \subset G$ that gives the highest share to buyers are the quasi-regular networks generated by regular networks in $\overline{g}_{d,\overline{s}}$, where $d = \left\lfloor \frac{2L}{n_B} \right\rfloor$. The same statement holds reversing the roles of buyers and sellers.*

For illustrative purposes it is easier to give the intuition for the case where $L/n_B \in \mathbb{N}$. First, the desirable network must have communication links of maximum strength because they carry more information about past rounds, decreasing in this way buyers' susceptibility to sellers' mistakes. Second, a regular network is desirable because it is the network where the buyers with the lowest degree have the highest possible degree given the constraint L. Informally, (quasi)-regular networks are very steady: they have no weak points that could be more susceptible to sellers' mistakes.

There is a long-standing debate in the sociological literature on what constitutes a desirable network for a group of individuals. A seminal paper by Granovetter [1973] introduced the idea that weak ties play an important role in networks because they connect individuals with few characteristics in common and that have non-overlapping neighborhoods, allowing them to access non-redundant information. For instance, Granovetter [1995] shows that individuals with many weak ties are better at finding employment through their social networks. A rough summary of this view is that networks with a significant fraction of *weak ties* and *high degree variability* are desirable because they facilitate the flow of information.

On the other hand, Coleman [1988] argues that close-knit, homogeneous networks formed by strong bonds are desirable. The rationale is that these strong connections and their even distribution across all group members make it easier to establish an informal, decentralized monitoring of the flow of information. Moreover, there are no peripheral individuals who could be potential defectors. He gives the example of the network of wholesale diamond traders in New York: strong family, religious and community ties ensure that information about any cheating will be quickly available to all the members leading to the exclusion of the cheater from the community. A rough summary of this view is that networks composed exclusively by *strong ties* and *minimal degree variability*

without peripheral individuals are desirable because they facilitate monitoring of what is going on in the network.

In the context described by this model, corollary 1 shows that Coleman-type, quasi-regular networks exclusively formed by strong ties are desirable because they allow the effective sharing of information about past demands. However, it is important to understand that this is not an absolute statement about the two views, which are, in fact, complementary. There are two key aspects of this model which determine the desirability of a Coleman-type network in this context. First, the new information that circulates in the network is negative: mistakes made by the other side that individuals in the group should not respond to. Second, the final outcome is the establishment of a norm for the whole group, so the important factor is how structural properties of the group as a whole, not the structural position of single agents, influence the outcome. A regular network with strong ties ensures that each agent has a lot of information about the state of the system so that new negative information has a very low probability of affecting the group. Moreover, the regularity of the networks ensures that there are no weak points where negative information has a higher probability of "entering" the group. On the other hand, in a model where new information is positive and valuable (e.g. innovation, job opportunities) then the desirable network would probably be closer to the Granovetter's type because it would facilitate the effective circulation of positive information.

4 Comparative statics

The standard tool to analyze the effects of changes in the network structure is to look at first order stochastic dominance (FOSD) and second order stochastic dominance (SOSD) shifts in the degree distribution, as defined in section 2.[8] In the context of weighted networks, a FOSD shift leads to a network with "more" and/or "stronger" links, and a SOSD shift leads to a network with a more equal distribution of the number and/or strength of links.

The following theorem shows how the asymmetric Nash bargaining solution (ANB) in theorem 2 varies with changes in the degree distributions of the buyers and sellers' networks.

Theorem 3. *Let $(x^*, 1 - x^*)$ be the ANB for sets of agents B and S that communicate through networks g^B and g^S with weighted degree distributions $p_b(z)$ and $p_s(z)$. Consider the weighted degree distributions $p'_b(z)$ and $p''_b(z)$ of networks g'^B and g''^B respectively, and let $p'_b(z)$ FOSD $p_b(z)$ and $p''_b(z)$ SOSD $p_b(z)$.*

[8] See Goyal [2007] and Jackson [2008] for textbook treatments of this methodology.

(i) Let $(x^{'*}, 1 - x^{'*})$ be the ANB for sets of agents B and S with degree distributions $p'_b(z)$ and $p_s(z)$. Then $x^{'*} \geq x^*$.

(ii) Let $(x^{''*}, 1 - x^{''*})$ be the ANB for sets of agents B and S with degree distributions $p''_b(z)$ and $p_s(z)$. Then $x^{''*} \geq x^*$.

The same statement holds reversing the roles of buyers and sellers.

The theorem states that individuals who belong to a *denser* social group, i.e. with more numerous and/or stronger connections, will fare better. Similarly, individuals who belong to a *more homogeneous* social group, i.e. with more equally distributed connections in terms of the number and/or strength of links, will also be better off. The intuition is that agents in these groups will have access to more information about past deals experienced by other members in their group. Thanks to this informational advantage, they are less likely to respond to mistakes by the other side, and they are therefore able to maintain an advantageous bargaining position.

A more general comparative statics result than theorem 3 also holds: adding a link to the communication network of a group weakly increases the share individuals in that group obtain in equilibrium. There are two reasons behind the choice to formulate the comparative statics analysis in terms of shifts in the degree distribution. First, the statement in theorem 3 is more suitable to empirical verification. Second, the same approach applies to the extension in section 6 where buyers and sellers belong to the same network, while the more general statement does not hold there.

It is not straightforward to carry out an empirical test of the statement in theorem 3. It seems rather challenging to artificially engineer a shift in the degree distribution of a network, or to isolate exogenous shocks to a social structure that would result in these stochastic dominance shifts. However, from cross-sectional studies we know that homophily is a powerful determinant of social structure and that networks composed of different *types* of individuals often have internally different structures. Moreover, studies that compare cross-sections of different networks are much easier to undertake than tracking the evolution of a single network.

Only mild assumptions are required to extend the model to a context with several groups of buyers. Assume that there is one group of sellers and that there are k separate groups of buyers such that buyers communicate within their group but not across groups.[9] The main assumption that is required is that each seller knows which group a buyer belongs to and he only receives information from other sellers on previous transactions with buyers from that group. Moreover, when a seller determines which offer to make to a buyer from a certain group, he does not use information from transactions with

[9] An equivalent set-up is to assume that there is one group of buyers B connected by a network g^B which is composed of k *components*.

buyers in other groups. Mathematically, the whole system can be represented by k different processes that run "in parallel," and the dynamics/outcomes of one process are completely independent from the ones of the other processes. Clearly, the results in this paper apply to each one of these processes. The following corollary presents this set-up more formally and it states its implications.

Corollary 2. *Consider one group of sellers S who communicate through g^S, and k groups of buyers $B_1, ..., B_k$ who communicate through separate networks $g^1, \ldots, g^k$ with weighted degree distributions $p_1(z), \ldots, p_k(z)$ respectively. Assume $B_i \cap B_j = \emptyset$ and sellers know which group a buyer b belongs to. Then sellers will reach different conventions with different groups of buyers on the share x_i^* that buyers in B_i get. Moreover:*

(i) If $p_1(z)$ FOSD $p_2(z)$ FOSD ... FOSD $p_k(z)$, then $x_1^* \geq x_2^* \geq \ldots \geq x_k^*$

(ii) If $p_1(z)$ SOSD $p_2(z)$ SOSD ... SOSD $p_k(z)$, then $x_1^* \geq x_2^* \geq \ldots \geq x_k^*$

This corollary states a clear and testable prediction of the model: in a market with different groups of buyers where communication only occurs within groups, buyers that belong to denser and/or more homogeneous groups will fare better. The next section explores how these predictions shed light on the observed pricing patterns in the Fulton wholesale fish market.

5 An Application: The Fulton wholesale fish market

Wholesale fish markets have historically attracted the attention of economists because they offer fertile ground for econometric tests of a competitive market.[10] They also have several characteristics that make them an ideal setting to test this model: *(i)* all transactions are private between one buyer and one seller; *(ii)* bargaining is minimal and usually consists of take-it-or-leave-it offers; *(iii)* they are perfectly competitive with a large number of buyers/sellers, low entry costs, and no search costs; *(iv)* products are very homogeneous; *(v)* prices very considerably from day to day and they depend on exogenous factors largely unknown to the market participants; *(vi)* there are no inventories so separate days can be considered independently.

Here we will analyze a dataset on transaction prices in the Fulton wholesale fish market (FFM) in New York collected by Kathy Graddy in 1992.[11] The goal is to show that the predictions of the model provide an explanation for a puzzling finding found by Graddy [1995] in the pricing patterns observed in this market: Asian buyers pay a significantly lower price than white buyers for the same product sold by a white seller.

[10] See Kirman [2001] for a review.

[11] I am very grateful to Kathy Graddy for allowing me to use her dataset. See Graddy [2006] for a more detailed description of the dataset and of the FFM.

The FFM in lower Manhattan is the largest fish market in the US with 100-200 million pounds of fish sold per year.[12] Transactions start at 3am and end at 9am; there are about 35 sellers and several hundred buyers. The FFM dataset contains all 620 sales of whiting made by one white seller from April 13th till May 8th 1992. For each transaction the dataset contains the following: time of sale (month, day, hour and minutes); price per pound; quantity sold; customer information (unique identifier, ethnicity); size (small, medium or large), type (king or normal) and quality (1-5 scale) of whiting; mode of transaction (cash or credit; in person or by phone); geographical location (Manhattan, Brooklyn, other) and type of establishment (store or fry shop) owned by the buyer; and total quantity that the seller received and sold on that day.

Only a subset of 132 observations has been used for the main analysis conducted below. Duplicate entries and sales with missing data were omitted. The few transactions carried out on the phone were also excluded. Following Graddy [1995], only sales involving medium sized normal whiting of medium quality were included in order to focus on a homogeneous product. A common characteristic of fish markets is the presence of large fluctuations in prices in the last 1-2 hours of the market depending on whether there is excess demand or supply on a given day. In order to avoid this effect the data was restricted to transactions carried out between 3am and 7am, which correspond to the busiest hours of the market.[13]

Graddy [1995] found a puzzling result: Asian buyers pay a significantly lower price than white buyers for the same product sold by a white seller. She investigates a number of potential determinants of this result, but in her concluding remarks she writes that "[...] price discrimination is present. The reason behind price discrimination is less clear" (p. 87). It is difficult to explain why the price difference is not arbitraged away in a market with a healthy competition, no obvious entry barriers, low search costs and a homogeneous product.[14]

The model in this paper puts forward an alternative rationale for the observed price difference. There are *distinct groups of buyers* in the FFM depending on their ethnicity,

[12] In November 2005, the FFM moved to the Bronx, see Jacobs [2005] for a short account of the move.

[13] The goal of this data selection was to follow Graddy [1995] as closely as possible, but it is likely that there are very minor discrepancies between the two procedures. To be included in the subset of data analyzed here, transactions had to have the following characteristics: (i) time of trade and ethnicity of buyer are not missing; (ii) trade happened before 7am; (iii) trade was made in person; the fish was (iv) normal whiting of (v) medium quality (3 on a 1-5 scale) and of (vi) medium size. Moreover, one duplicate transaction was excluded. Steps (i)-(vi) reduced the dataset to $n = 132$ observations, while Graddy's selection procedure reduced it to $n = 131$.

[14] In a recent contribution, Graddy and Hall [2009] construct a structural model to explain the pricing data in the FFM: their simulations match the observed prices very well. The key assumption they make to reproduce the price differential between Asians and whites is that Asians have a higher price elasticity of demand than whites. The approach they use is different from this paper, and the stories proposed in Graddy and Hall [2009] and in this paper to explain the price differential complement each other.

and buyers communicate with other members of their group to learn the current price of fish. The *group of Asians is denser* and it is therefore a better channel of information on the uncertain price of the product. As the market unfolds the density of the communication network in their group gives the Asians an informational advantage: they *learn the ongoing price more accurately* and they exploit this additional knowledge to obtain a lower price from the sellers.

The argument will consist of four steps. First, we will document the price differential between Asians and whites by replicating the analysis in Graddy [1995]. Second, we will show that the price differential is not present in the first two hours of the market, and it emerges only afterwards. We interpret this as evidence of learning: the price differential emerges only in the course of the market as the buyers learn the daily price of fish. Third, we will provide further evidence of learning by showing that the variability of prices within the Asians decreases over time while the same does not hold for the whites. Finally, we will draw on a number of studies to argue that the key competitive advantage of Asian buyers is that they belong to a denser social group than whites, in agreement with the predictions of the model.

The objective of the first step is to reproduce the main finding in Graddy [1995]: this ensures consistency with the original study and it confirms that the analysis carried out here correctly picks out the price differential between Asians and whites. The first column in table 1 reproduces the regression analysis in Graddy [1995]. The price of each of the $n = 132$ trades is regressed on the following independent variables: an *ASIAN* dummy equal to 1 if the buyer is Asian; a *BLACK* dummy equal to 1 if the buyer is black; a *CASH* dummy equal to 1 if the purchase was paid in cash; a *MLOC* dummy equal to 1 if the buyer is from Manhattan or Brooklyn; a *STORE* dummy equal to 1 if the buyer's establishment is a store; and time and date dummies not shown in the table.[15]

The coefficients in column 1 support Graddy [1995]'s conclusions. The Asian dummy is negatively correlated with price and significant at the $p = 0.01$ level: Asian buyers pay a price that is approximately 5% lower than white buyers. All the other controls are not significant, apart from the *STORE* and the date dummies that are strongly significant due to the strong dependence of the price of fish on daily conditions.[16] Note that the

[15]The time dummies *TIM1, TIM2* and *TIM3* are equal to 1 if the purchase was made before 5am, in the 5am-6am and in the 6am-7am time periods respectively; the date dummy *DATE X* is equal to 1 if the purchase was made on day X. Two variables present in Graddy's regression have not been included: *AVQUAN* (i.e. average quantity purchased by the customer during the time period) and *REG* (i.e. the number of times the customer purchased during the time period). The results of the regressions presented here would not change if these variables were to be included.

[16]A difference between these regressions and Graddy's is that here the coefficient on the *STORE* dummy is positive and statistically significant in all regressions: store owners pay higher prices than non-store owners that buy fish for fry-shops. In order to ensure that this has no effect on the Asian dummy coefficients, all regressions were repeated further restricting the sample to store owners only, who

variables in the regression explain essentially all the variation in prices observed in the dataset.

TABLE 1 - Determinants of the Price of Whiting

Standard errors in brackets.

The coefficients on the date and time dummies are not reported.

*** Significant at the 0.01 level; ** 0.05 level; * 0.1 level.

Variables	**(1) All times**	**(2) Before 5am**	**(3) 5am-7am**
ASIAN	-.0488*** (.0150)	.0085 (.0231)	-.0455** (.0202)
BLACK	.0115 (.0195)	.0658 (.0290)	.0138 (.0249)
CASH	.0249 (.0147)	-.0078 (.0175)	.0194 (.0203)
MLOC	.0061 (.0147)	-.0214 (.0204)	.0069 (.0175)
STORE	.0465*** (.0128)	.0514*** (.0187)	.0537** (.0165)
R^2	.990	.998	.986
N. of observations	132	38	86
N. of (Asian; white)	(70;49)	(22;20)	(48;29)

However, the results in column 1 do not necessarily support the hypothesis that Asian buyers gain a competitive advantage through learning as opposed to some other mechanism. A stronger piece of evidence for a learning story would show that *the price differential emerges over the course of the trading day as learning takes place.* Column 2 in the table shows the coefficients for the same regression as column 1, but considering only early trades that happened before 5am.[17] The coefficient of the Asian dummy is now positive and insignificant: Asian buyers pay a price that is slightly higher than white buyers, and *statistically there is no difference* between the two.

The price differential in favour of Asian buyers emerges later in the trading day. Column 3 in the table shows the same regression as column 1, but considering only trades that happened between 5am and 7am.[18] The coefficient of the Asian dummy is negative and statistically significant at the $p = 0.05$ level: after the first two hours of the market, the price differential emerges and the Asian buyers trade at significantly lower prices compared to white buyers. This is strong evidence in favour of the hypothesis that different rates of social learning within the two groups drive the emergence of the price

constitute the large majority of buyers. The results do not change.

[17]The coefficients of column 2 do not change substantially if "early" trading is defined as transactions that happened between 4am and 5am, excluding in this way the first hour of the market.

[18]As in Graddy [1995], trades after 7am are excluded from the analysis because of large fluctuations in prices in the last 1-2 hours of the market due to excess demand or supply.

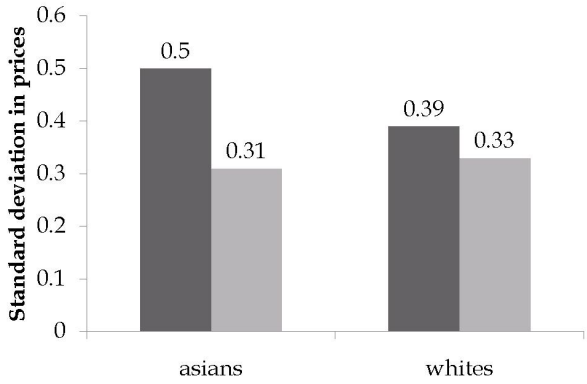

Figure 3: Standard deviation in prices paid by Asian and white buyers for the 4-5am (dark gray bar) and 6-7am (light gray bar) time periods.

differential: any alternative story based on individuals' and/or groups' characteristics would face the difficult task of explaining why these characteristics emerge only in the course of the trading day.

If buyers within the same group are learning the daily price of fish then the variability of prices paid by members of the same group should decrease over time. At the beginning of the market only a few transactions have taken place and therefore the scarce information on previous sellers' demands is of little guide to buyers: the prices that buyers pay should therefore vary a lot across buyers reflecting the lower accuracy of the small samples of information at their disposal. On the other hand, after some time buyers receive abundant information about sellers' past demands: the variation in prices paid by buyers should decrease reflecting the higher accuracy of buyers' demands which are now based on the large samples of information at their disposal. The hypothesis is therefore that *the variability of prices decreases faster for Asians than for whites*, given that social learning is faster within the Asian group,

Figure 3 compares the standard deviation in prices for transactions between 4am and 5am (in dark gray) to the standard deviation for transactions between 6am and 7am (in light gray) for Asians and whites. They both decrease over time, but the decrease for Asian buyers is visibly larger. A two-sample variance comparison test rejects at the 99% confidence level the null hypothesis that for Asian buyers the two standard deviations are the same. The same test for whites cannot reject the same null hypothesis at the 90% confidence level. Moreover, the result does not hinge on the difference in initial standard deviations: the test cannot reject at the 90% confidence level the null hypothesis that the standard deviation of prices at 4-5am is the same for Asian and white buyers.[19]

[19]The same results hold if we consider transactions *before 5am* instead of in the range 4-5am (this adds 6 additional observations). The results are the same if we use the robust equal variance test, which does not assume that the underlying distribution of prices is normal.

The final step is to show that social learning within the Asian buyers is faster than within the white buyers because the Asian buyers' group has a higher density of social connections. Unfortunately, there is no data available on the interactions among different buyers in the FFM, and therefore we will have to resort to findings from other studies to provide *corroborating evidence* in support of this hypothesis. First of all, Graddy's personal field observations of the FFM support the assumption that buyers are split in different groups along ethnic dimensions. She remarks that "[v]ery little social contact appears to take place between groups of Asian buyers and groups of white buyers" (p. 84) and "[b]uyers do not realize they are receiving better or worse prices than other buyers" (p. 83-84). This is in agreement with a large amount of empirical evidence reviewed in McPherson et al. [2001] that shows that "[h]omophily in race and ethnicity creates the strongest divides in our personal environments" (p. 415).

Social connections play an especially important role in business transactions in the overseas Asian community. Redding [1995]'s extensive study of overseas Chinese networks stresses that "co-operativeness [...] converts an otherwise disparate group of entrepreneurs into a significant economy" (p. 62) and "[p]ersonalism does in Asia what law does in the West [...] [w]ithout [what is termed *guanxi* or connections] nothing can be made to happen [...] the instinct of the Overseas Chinese to trust friends but no-one else is very deep-rooted" (p. 63). Redding [1995] explicitly states that the main purpose of social connections is to "seek the opportunities by trading rare information [and] share that information to build allegiances" (p. 65).

The importance of social connections for Asians in the US is not limited to the Chinese community. Xie and Goyette [2004] stress that "[m]ost Asian Americans are recent immigrants and as such maintain a strong identity with their home culture [...] Ethnic communities offer many practical resources to immigrants, including [...] information in native languages, and entrepreneurial opportunities" (p. 66). Rauch [2001] reviews further evidence of the high density of Asian social networks. The Asian buyers in the FFM seem to be no exception: in her fieldwork Kathy Graddy observed that "Asian buyers certainly spoke to one another and congregated much more frequently than white buyers."[20] Even though all these studies do not provide conclusive empirical evidence, they at least give broad support to the hypothesis that the group of Asian buyers is part of a denser social network than the group of white buyers.

Summing up, this analysis provides corroborating evidence in support of a differential social learning story as a driver of the price differential between Asian and white buyers. In the first hours of the market, there is scarce information on the daily price of fish because very few transactions have taken place and therefore buyers cannot rely on their

[20]This quote is taken from a personal communication with Kathy Graddy.

contacts to learn what the daily price is. After a good number of transactions have taken place, information on prices asked by sellers circulates among buyers who start learning the daily price of fish. The learning process occurs faster among Asian buyers, who are tightly connected with one another, and over time they cumulate an informational advantage that is reflected in the lower prices of their trades compared to white buyers.

It is not easy to find an alternative story that fits this analysis as well. For instance, an alternative rationale for the price differential could be that Asian buyers have better bargaining skills. Besides the fact that haggling is non-existant or minimal in this market, this story would have to explain why Asian buyers only employ their bargaining skills after 5am. Any other story based on individuals' and/or groups' characteristics that the analysis may not control for would have to explain why these characteristics become effective only after 5am.

Finally, there are several other contexts where the predictions of this model are relevant. A prominent example is international trade markets. James Rauch and others have shown that the density of ethnic immigrants' networks is an important determinant of international trade patterns.[21] Moreover, Rauch and Trindade [2002] explore two potential mechanisms driving this competitive advantage. The first one, proposed by Greif [1993], is a contract enforceability story: if a member of the social network has been cheated by a trader then every member of the network punishes the cheater by stopping future business with that trader. The second one, proposed by this paper, is that the social network gives an informational advantage: being part of the social network gives access to more accurate information about the price of a product.

Rauch and Trindade [2002] are able to distinguish between the two mechanisms by comparing trade volumes in products with "reference prices," whose price is well-known, to trade in products without "reference prices," whose price is uncertain. They find that ethnic Chinese social networks have a much larger effect on trade of products with uncertain price and they conclude that the main function of social networks is to provide an informational advantage. Similar research by Kumagai [2007] confirms that the same effect is present for Japanese ethnic networks. Furthermore, Kumagai [2007] shows that the effect is increasing with the density of the network, in agreement with the results presented in section 4.

6 A unique network of buyers and sellers

The previous example had the feature that buyers and sellers belonged to separate groups. However, there are many contexts in which this is not the case and both buyers and sellers

[21] See Rauch [2001] for a comprehensive review.

are part of the same community. Would the results of the model continue to hold in these contexts? This section investigates how the results in sections 3 and 4 change if buyers and sellers belong to the same communication network. Section 6.1 illustrates the changes to the model and derives the bargaining solution, section 6.2 discusses the implications for the desirable communication structure for the members of a group, and section 6.3 carries out the comparative statics analysis.

6.1 Set-up and bargaining solution

There are two main changes to the set-up of the model that are required to describe a context where buyers and sellers are part of the same communication network. First, consider the information arrival process. Assume agent $b \in B$ is picked to play the game at time $t+1$: in the $\Delta t = 1$ time period she receives information from other buyers in B *and* other sellers in S about past bargaining rounds. As before, the expected total amount of information b receives before each bargaining round is equal to $\sum_{j \in L_b(g)} E[P(s_{bj})] = E[P(s_b)] = \sum_{j \in L_b(g)} g_{bj} = z_b(g)$. The only difference is that here $z_b(g) = \sum_{j \in B,S} g_{bj}$: b's sample comes from both buyers and sellers. The expected realizations of the Poisson processes define a *weighted, undirected network* of buyers *and* sellers, which is represented by a symmetric matrix $[g_{ij}]^{n \times n}$.

Second, consider the elements $\mathbf{s}$ of the state space $\mathbf{S}$ of the Markov process. Here, $\mathbf{s} = \{\mathbf{v}_1, ..., \mathbf{v}_{2n}\}$, i.e. for each agent i there are two vectors $\mathbf{v}_i$ and $\mathbf{v}_{2i}$ of size m. If $i = 1, ..., n$ then $\mathbf{v}_i = \{y^i_{k-m+1}, ..., y^i_k\}$, i.e. if $i \in B$ then the entries of $\mathbf{v}_i$ are the last m demands made by sellers in bargaining rounds involving i, and if $i \in S$ then the entries of $\mathbf{v}_i$ are i's last m demands. Similarly, if $i = n+1, ..., 2n$ then $\mathbf{v}_i = \{x^i_{k-m+1}, ..., x^i_k\}$. Assume that when a buyer $b \in B$ is picked to play the game, she receives a sample of information from her neighborhood about past demands made by *sellers*, i.e. the demands in the sample come from $\mathbf{v}_1, ..., \mathbf{v}_n$. Similarly, when a seller $s \in S$ is picked to play the game, the demands in the sample come from $\mathbf{v}_{n+1}, ..., \mathbf{v}_{2n}$.

The unique stable division is unchanged from the case of separate communication networks of buyers and sellers.

Theorem 4. *There exists a unique stable division* $(x^*, 1-x^*)$. *It is the one that maximizes the following product:*

$$u^{z_b^{min}}(x) v^{z_s^{min}}(1-x)$$

In other words, it is the asymmetric Nash bargaining solution with weights $z_b^{min}(g)$ *and* $z_s^{min}(g)$.

Lemma 2 is unchanged and therefore the proof of theorem 4 follows the same argument as the proof of theorem 2, and it is therefore omitted. The size of the information sample

of the least informed member(s) of a group is the key determinant of the deal the group obtains in equilibrium. Whether this information comes from members of the same group or of the other group is inconsequential for the split of the pie. As in theorem 2, the buyers with the minimum weighted degree will be the least informed and therefore they will be more susceptible to respond to mistakes from the sellers. Over time, this susceptibility weakens the bargaining position of the whole group of buyers, leading to the establishment of the conventional split that maximizes the product in (5).

6.2 Core-periphery networks

The introduction of communication across groups does not change the ANB solution. However, the desirable architecture for the group of buyers in this setting is not the same as in section 3.3 because here the sellers are part of the network. The corollary below shows that the desirable communication structures for the buyers are *core-periphery networks* where the buyers are at the core and the sellers at the periphery. For expository purposes this section restricts the analysis to unweighted networks. However, the proof of the statement of corollary 3 in the appendix is for the general case of weighted networks.

First of all, we need to give a formal definition of core-periphery networks.

Definition 9. A *semi-bipartite network* $g(H)$ is a network with a subset of agents $H \subset N$, with $|H| \leq |N|/2$, such that $d_i(g) = 1$ for all $i \in H$ and if $i, j \in H$ then $g_{ij} = 0$.

Definition 10. Consider the set G of undirected, unweighted networks with n nodes. A *core-periphery* network is a connected and semi-bipartite network $g(H)$. The agents in $N \backslash H$ form the *core*, which is a quasi-regular network $g_{d,s}$, and the agents in H form the *periphery*.

The key characteristic of core-periphery networks is that they divide a society into two classes of individuals: on the one hand an elite of core individuals who are well-connected with each other, and on the other hand a group of peripheral individuals that are dependent on the elite and poorly connected with each other. It is intuitively clear why it would be desirable for the buyers to be at the core, the following corollary formalizes this intuition.

Corollary 3. *Consider the set G of all possible communication structures for a group of n agents comprising n_B buyers and $n_S < n_B$ sellers, and where the total number of links is $L \leq \frac{n_B}{2}(n_B - 1)$. Let $d = \left\lfloor \frac{2L+n_s-1}{n_B} \right\rfloor$ and let $s \in \mathbb{R}_+$ be the strength of each link. The networks which maximize the share buyers obtain in equilibrium are core-periphery networks where buyers form a quasi-regular network $g_{d,s}$ at the core and sellers are at the periphery. The same statement holds with the roles of buyers and sellers reversed.*

At the periphery sellers have the lowest number of links needed for the network to be connected and at the same time they take the least number of links away from the buyers. The sellers' information sample is as small as possible: it is equal to the strength s of one link. On the other hand, at the core buyers maximize the number of links of the least connected buyer(s) given the available budget L. By forming a quasi-regular network at the core, the buyers' information sample is as large as possible, as shown in corollary 1.

The proof in appendix A is more general than the statement above and it characterizes the subset of weighted networks that maximize the share of a group. There are three main steps in the proof. First, for the sellers to get the smallest possible share there must be at least one seller s_0 with only one weak link. Second, the sellers $s \in S \backslash s_0$ should have at least the lowest number of links needed for the network to be connected while at the same time take the least number of links away from the buyers. Thus each seller, apart from s_0, is connected by one strong link to a buyer. Third, following the argument of corollary 1, the buyers should form a regular network with strong links to maximize the smallest weighted degree among all the buyers. The remaining links can be assigned at random (as long as none of them links to s_0) so the core is a quasi-regular network of buyers and each seller has only one or a few links.

The key for a group to obtain a high share is to form a close-knit elite and leave the individuals of the other group at the periphery. Gellner [1983] argues that this is the prevailing social structure in agrarian societies.[22] A small minority of the population forms the literate elite, which is composed of members of specialized professions such as warriors, priests and administrators. These specialized elites form very close-knit communities that are bound together by shared norms within their profession, and that maintain minimal contacts with the rest of the population. At the other end of the spectrum the large majority of the population consists of "[s]mall peasant communities [that] generally live inward-turned lives, tied to the locality by economic need if not by political prescription" (p. 10). These small communities of peasants do not communicate with each other and they are linked to the elite only through weak connections necessary for the extraction of rent.

A further clue that underlines the crucial role of communication in carving out the social structure of agrarian societies comes from looking at the languages that are usually spoken in each group. The elite is mainly composed by literate individuals and "the tendency of liturgical languages to become distinct from the vernacular is very strong: it is as if literacy alone did not create enough of a barrier [...] as if the chasm between

[22]Thanks to Sam Bowles for pointing me to Gellner's work and for suggesting the interpretation that follows.

them had to be deepened, by making the language not merely recorded in an inaccessible script, but also incomprehensible when articulated" (p. 11). Everyone in the elite shares a common oral and written language, which individuals outside of the elite are not able to comprehend. On the other hand, small peasant communities suffer "a kind of culture drift [that] soon engenders dialectal and other differences. No-one [...] has an interest in promoting cultural homogeneity at this social level. The state is interested in extracting taxes [...] and has no interest in promoting lateral communication between its subject communities" (p. 10). In agreement with the predictions of the model, a close-knit elite is able to share information effectively and extract a high rent from the peripheral, isolated communities of peasants who are unable to communicate with each other.

6.3 Comparative statics and the 50-50 split

When buyers and sellers share the same communication network, any change in the social network structure affects both buyers *and* sellers, and therefore the comparative statics will differ from the case of separate networks. The following is the equivalent statement to theorem 4 in the modified set-up where buyers and sellers belong to the same communication network.

Theorem 5. *Let $(x^*, 1 - x^*)$ be the ANB for sets of agents B and S that communicate through a network g with degree distribution $p(z)$.*

(i) If $p'(z)$ is a variance-preserving FOSD shift of $p(z)$ then $x'^* = x^*$.

(ii) Assume that the least weighted degree for the sellers is (weakly) larger than the mean degree, i.e. $z_s^{min}(g) \geq \mu[p(z)]$. If $p''(z)$ is a mean-preserving SOSD shift of $p(z)$ then $x''^* \geq x^*$.

The same statement holds reversing the roles of buyers and sellers.

A shift to a *denser* communication network, without any changes in the variance of the distribution, leaves the equilibrium *ANB* unchanged. This is because the weighted degrees of the least connected buyers and sellers will change in absolute value, but not in relative value to each other. On the other hand, a shift to a *more homogeneous* communication network, holding constant the mean of the degree distribution, changes the equilibrium because it affects the relative values of the least connected buyers and sellers. Specifically, as the network becomes more homogeneous the difference between the shares of the two groups narrows down.

Theorem 5 further highlights how the introduction of a network to model information flows leads to new insights that are not accessible in a model without the network. As the statements of theorems 2 and 4 make clear, the fact that buyers and sellers belong to separate or the same communication network has no impact on the long-term equilibrium

division making these two cases indistinguishable. However, the introduction of the network allows a comparative statics analysis that highlights how changes in the network affect the equilibrium division. The comparative statics clearly differs if buyers and sellers belong to the same network, and this leads to the insight of theorem 5 that a shift in the distribution of connections that decreases the variability in number of connections across agents narrows down the difference in the shares that buyers and sellers obtain.

The model predicts that societies with more homogeneous social groups would have more equitable divisions. The limit network after a sufficient number of SOSD shifts is a regular weighted network: if all the agents have the same utility function, then the equilibrium division in a regular weighted network is the *50-50* split.

Corollary 4. *Let g be a regular weighted network and let all agents have the same utility function, then 50-50 is the unique stable division.*

In the extreme case of a regular weighted communication network the equilibrium division is 50-50, which suggests that this well-observed phenomenon may be more prevalent in societies with a very flat and non-hierarchical social structure. The mechanism that leads to the emergence of the 50-50 division in this model differs from other mechanisms previously advanced in the literature. Schelling [1960] advanced the idea that 50-50 is a prominent focal point, whose salience is exploited by two bargainers to coordinate on an efficient division. In Young [1993a]'s framework the 50-50 division emerges in societies where there are some individuals that exchange roles and, at different times, can be both buyers and sellers. On the other hand, in this model the driving force leading to the emergence of the 50-50 division is the homogeneity of the social structure of the society that buyers and sellers are embedded in.

7 Conclusion

This paper has investigated the informational advantage an individual derives from being part of a group in a large, decentralized market where there is incomplete information about past transactions. The communication patterns within the group determine the information the individual has before a private bilateral transaction, and the outcome of the bargaining hinges on the accuracy of this information. In the long-run equilibrium every member of the group obtains the same share of the good in each transaction, and the group communication network critically determines the market outcome. More specifically, the equilibrium division depends on the number and the strength of the connections of the least connected individuals in each group. An immediate consequence of this result is that individuals belonging to a group with a high density and a low

variability of connections across individuals fare better. Empirical evidence shows that this prediction is consistent with the price differential between Asian and white buyers in the New York fish market. Finally, a modified setting analyzes the case where buyers and sellers are embedded in the same communication network: the peripheral individuals are again pivotal, and the more equally distributed the connections are across agents the more similar are the shares of buyers and sellers.

At the empirical level the analysis of the Fulton fish market is just a preliminary step in the testing of the predictions of this model because it provides only illustrative evidence that the model sheds light on the observed market outcome. The identification of network effects in markets is a notoriously difficult task, and methodological advances in the econometric literature are necessary to tackle it properly. The option of conducting lab experiments to overcome the identification issue inherent in field data is usually not viable for network models, where it is difficult to reproduce social relations in the lab. However, the model in this paper may be quite suitable to an experimental investigation because the network here is simply a communication channel. It is relatively easy to construct protocols to constrain communication among subjects in the lab, and therefore it should be feasible to create an artificial market where there are groups of traders with different internal communication structures.

Network theorists have only recently started to examine models that investigate the role of network structure in determining market outcomes in markets with a large number of agents. In these models the mechanisms through which network structure affects market outcomes vary widely, reflecting the multiplicity of possible types of social interactions. This paper focused on the role of network structure as a carrier of market information. Hopefully this model may serve as a starting point for future work both theoretically and empirically in order to identify the role of network structure using real market data and in laboratory settings.

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A Appendix: Proofs

This appendix contains all the proofs omitted in the main body of the paper. Hereafter let $\delta = 10^{-p}$ ($p \in \mathbb{Z}_+$) be the precision of the demands, and assume $x_t, y_t \in D$, where D is the set of all p-place decimal fractions that are feasible demands.

Proof of Lemma 1. Suppose the process is in state $\mathbf{x}$ at time t, and pick any two agents $b \in B$ and $s \in S$ to play the Nash demand game at time $t+1$. For any sample b receives from her neighborhood, the cumulative distribution $G(y)$ of previous demands by sellers is a probability mass function with value 1 at $1-x$. Thus, b's best reply is always to demand x. Following a similar argument, the seller s' best reply is always $1-x$. It follows that the state of the system at $t+1$ is the same as it was at t, and therefore $\mathbf{x}$ is an absorbing state of P. □

Proof of Theorem 1. The goal is to show that from any initial state $\mathbf{s}$ there is a positive probability p independent of t of reaching a convention within a finite number of steps. Select individuals b, b', b_0 such that $b \in L_{b'} \cap L_{b_0}$ and $g_{b'b_0} = 0$. Similarly, select individuals s, s', s_0 such that $s' \in L_s \cap L_{s_0}$ and $g_{ss_0} = 0$. Note that such individuals must exist because by assumption the networks are connected and they are not complete networks. Figure 1 in section 3 illustrates two networks of buyers and sellers with individuals b, b', b_0 and s, s', s_0. Note that in figure 1 agents b_0 and s_0 are labeled b'' and s'' respectively. Consider the following steps from t onwards.

(i) $[t, t+m]$: There is a positive probability that b and s (or agents like them[23]) will play the game in every period $t \in [t, t+m]$. Also, there is a positive probability that b and s will draw samples σ and σ' respectively. Let x and y be the best replies of b and s to these samples respectively. Then there is a positive probability of obtaining a run of (x, y) for m periods in succession such that $\mathbf{v}_b = (y, ..., y)$ and $\mathbf{v}_s = (x, ..., x)$.[24]

(ii) $[t+m+1, t+2m]$: There is a positive probability that b' and s' (or agents like them[25]) will play the game in every period $t \in [t+m+1, t+2m]$. There is a positive

[23] An agent $b_i \in B$ that is "like" b is such that $b_i \in L_{b'} \cap L_{b_0}$. This condition allows b_i to potentially collect the same sample of information σ as b. Similarly, an agent s_i that is "like" s is such that $s_i \in L_{s' \setminus s_0}$, where $L_{s' \setminus s_0} = \{j \in N \mid j \in L_{s'}, j \notin L_{s_0}\}$.

[24] The argument here has been simplified on a number of dimensions for expository purposes: 1) it is not necessary that the same pair of agents plays in each of the m rounds, it is sufficient that they are "like" b or s (see footnote above); 2) it is not necessary that the m rounds are consecutive, as long as there is a finite time between them and they are still in the state $\mathbf{s}$ at the end of the third step below; 3) if different agents are involved in these rounds, then the state $\mathbf{s}$ of the system at the end of this step will not be such that there are two vectors $\mathbf{v}_b = (y, ..., y)$ and $\mathbf{v}_s = (x, ..., x)$, but such that there are m entries of vectors $\mathbf{v}_i \in \mathbf{s}$ equal to y and m entries of vectors $\mathbf{v}_j \in \mathbf{s}$ equal to x, with $i \in B$ and $j \in S$. The same observations apply to the second step below.

[25] an agent $b_i \in B$ that is "like" b' is such that $b_i \in L_{b \setminus b_0}$. This condition allows b_i to potentially collect the same sample of information ρ as b'. Similarly, an agent s_i that is "like" s' is such that $s_i \in L_s \cap L_{s_0}$.

probability that they will sample from $\mathbf{v}_b = (y, ..., y)$ and $\mathbf{v}_s = (x, ..., x)$ respectively. Thus, there is a positive probability of obtaining a run of $(1-y, 1-x)$ for m periods in succession such that $\mathbf{v}_{b'} = (1-x, ..., 1-x)$ and $\mathbf{v}_{s'} = (1-y, ..., 1-y)$.

(iii) $[t+2m+1, t+3m]$: There is a positive probability that b_0 and s_0 will play the game in every period $t \in [t+2m+1, t+3m]$. There is a positive probability that b_0 will sample from $\mathbf{v}_b = (y, ..., y)$ and that s_0 will sample from $\mathbf{v}_{s'} = (1-y, ..., 1-y)$. Their best reply will then be $(1-y, y)$, so there is a positive probability of obtaining a run of $(1-y, y)$ for m periods in succession such that $\mathbf{v}_{b_0} = (y, ..., y)$ and $\mathbf{v}_{s_0} = (1-y, ..., 1-y)$.

(iv) $[t+3m+1, t+4m]$: There is a positive probability that agents $b_1 \in L_{b_0}$ and $s_1 \in L_{s_0}$ play the game for the next m periods. There is a positive probability that their samples come from $\mathbf{v}_{b_0}$ and $\mathbf{v}_{s_0}$ respectively. Their best reply will then be $(1-y, y)$, so there is a positive probability of obtaining a run of $(1-y, y)$ for m periods in succession such that $\mathbf{v}_{b_1} = (y, ..., y)$ and $\mathbf{v}_{s_1} = (1-y, ..., 1-y)$.

(v) $[t+4m+1, t+5m]$: There is a positive probability that agents $b_2 \in \bigcup_{k=0}^{k=1} L_{b_k}$ and $s_2 \in \bigcup_{k=0}^{k=1} L_{s_k}$, with $b_2 \neq b_0, b_1$ and $s_2 \neq s_0, s_1$ play the game for the next m periods. There is a positive probability that their samples come from $(\mathbf{v}_{b_0}, \mathbf{v}_{b_1})$ and $(\mathbf{v}_{s_0}, \mathbf{v}_{s_1})$ respectively. Their best reply will then be $(1-y, y)$, so there is a positive probability of obtaining a run of $(1-y, y)$ for m periods in succession such that $\mathbf{v}_{b_2} = (y, ..., y)$ and $\mathbf{v}_{s_2} = (1-y, ..., 1-y)$.

(vi) Now iterate the following step for $p = 3, ..., n_{max}-1$, where $n_{max} = \max\{n_B, n_S\}$. $[t+(p+2)m+1, t+(p+3)m]$: There is a positive probability that agents $b_p \in \bigcup_{k=0}^{k=p-1} L_{b_k}$ and $s_p \in \bigcup_{k=0}^{k=p-1} L_{s_k}$, with $b_p \neq b_0, ..., b_{p-1}$ and $s_p \neq s_0, ..., s_{p-1}$ play the game for the next m periods. There is a positive probability that their samples come from $(\mathbf{v}_{b_0}, ..., \mathbf{v}_{b_{p-1}})$ and $(\mathbf{v}_{s_0}, ..., \mathbf{v}_{s_{p-1}})$ respectively. Their best reply will then be $(1-y, y)$, so there is a positive probability of obtaining a run of $(1-y, y)$ for m periods in succession such that $\mathbf{v}_{b_p} = (y, ..., y)$ and $\mathbf{v}_{s_p} = (1-y, ..., 1-y)$.

At time $t+(n_{max}+2)m$ the state of the system is such that $\mathbf{v}_i = (y, ..., y)$ $\forall i \in B$ and $\mathbf{v}_j = (1-y, ..., 1-y)$ $\forall j \in S$, i.e. the system has reached a convention. Thus, from any initial state $\mathbf{s}$ there is a positive probability of reaching a convention within $[n_{max}+2]m$ periods. Given that the number of states is finite, there is a positive probability p of reaching a convention within $[n_{max}+2]m$ periods, which concludes the proof. □

Proof of Lemma 2. Suppose that the process is at the convention $\mathbf{x} = (x, 1-x)$, where $x \in D^0 = \{x \in D : \delta \leq x \leq 1-\delta\}$. Obviously, to move from $\mathbf{x}$ to another convention $\mathbf{x}' = (x', 1-x')$ the agents need to make mistakes. Without loss of generality, assume that the sellers make the mistakes. Let π be a path of least resistance from $\mathbf{x}$ to $\mathbf{x}'$, and let $\mathbf{s}$ be the first state on this path. In order to get to $\mathbf{s}$, a buyer b_0 must have received a sample σ where by mistake some sellers have demanded a quantity that differs from

$1-x$, such that b_0's best reply to σ is to demand a quantity $x' \neq x$. The buyers who require the minimum number of mistakes to switch best reply are the ones receiving the smallest sample. Recall that $B_{min} = \{j \in B \mid \lceil z_j \rceil \leq \lceil z_b \rceil, \forall b \in B\}$ is the subset of buyers with the least weighted degree. Let $z_b^{min} \equiv z_j$ for $j \in B_{min}$ and let $b_0 \in B_{min}$. Denote by p the number of mistakes by sellers in σ.

Consider the sample σ and construct a different sample σ' such that every entry of σ that differs from $1-x$ is replaced by $1-x'$, and every entry of σ equal to $1-x$ stays the same. Note that if b_0's best reply to σ was x', then her best reply to σ' must also be x'. By the mean-field assumption, σ' is composed by a total of z_b^{min} demands: p demands are equal to $1-x'$ and $z_b^{min} - p$ are equal to $1-x$.

Now, let us construct an alternative path π' from $\mathbf{x}$ to $\mathbf{x}'$ such that π' is also a path of least resistance with p mistakes. Start with the system at the convention $\mathbf{x}$ at time t. Consider the time t_1 when the md_{b_0} bargaining rounds played by buyers $b \in L_{b_0}$ happened after t. Let p of these md_{b_0} bargaining rounds be such that the seller involved made a mistake and demanded $1-x'$. There is a positive probability that b_0 plays with seller $s_0 \in S$ at time t_1 and receives a sample σ', and therefore she plays the best-reply demand x. Moreover, there is a positive probability that in the next $m-1$ rounds that b_0 and s_0 are picked to play, they again play with each other. Moreover, there is a positive probability that in each of these rounds b_0 receives the sample σ', which could still be available, and plays the best-reply demand x'. Thus, at some time $t_2 > t_1$, $\mathbf{v}_{s_0} = \{x', ..., x'\}$.

There is a positive probability that at time $t_3 > t_2$ agents b_0 and $s_1 \in L_{s_0}$ are picked to play, and that b_0 receives the sample σ' and s_1 receives his sample exclusively from $\mathbf{v}_{s_0}$.[26] Thus, b_0 will play the best-reply demand x' and s_1 will play the best-reply demand $1-x'$. Moreover, there is a positive probability that in the next $m-1$ rounds that b_0 and s_1 are picked to play, they again play with each other. Moreover, there is a positive probability that in each of these rounds they receive the same samples they got at t_3, which could still be available, and they play the best-reply demands x' and $1-x'$ respectively. Thus, at some time $t_4 > t_3$, $\mathbf{v}_{s_1} = \{x', ..., x'\}$ and $\mathbf{v}_{b_0} = \{1-x', ..., 1-x'\}$.

Following the same argument as the proof of theorem 1 above, it is clear that the process can now converge to the new convention $\mathbf{x}'$ without any further mistakes. Clearly, the same argument can be used to construct an alternative least-resistant path which starts with the buyers making q mistakes. In order to determine which least-resistant path requires the lowest number of mistakes, one has to compute these two numbers and choose the smallest. This leads us to consider four possible cases: two depending on

[26]Note that s_1 can receive his sample exclusively from $\mathbf{v}_{s_0}$ only if the size m of this vector is larger than z_{s_0}. This is guaranteed by the assumption made in section 3.2 that the individual memory $m \geq \max\{z_b, z_s\}$, where $b \in B$ and $s \in S$. Note that a lower bound would also be sufficient, what is necessary is that m is large enough.

whether the buyers or sellers make mistakes, and two depending on whether they ask a quantity higher or lower than what they get under the convention $\mathbf{x}$.

(i) Sellers make a mistaken demand $1 - x' < 1 - x$

Suppose sellers make p mistaken demands. Clearly, $p \leq z_b^{min}$, which is the sample size for the buyers with the smallest sample. As above, let $b_0 \in B_{min}$. Buyer b_0 therefore receives a sample of p mistaken demands $1 - x'$ and $z_b^{min} - p$ conventional demands $1 - x$. If b_0 demands $x' > x$ then she expects to obtain utility $u(x')$ with probability (p/z_b^{min}). On the other hand, b_0 demands $x < x'$ then she expects to obtain utility $u(x)$ for sure (because if the seller makes a mistake and demands $1 - x'$ then $1 - x' + x < 1$ and each agent gets their demand). Thus, b_0 switches to x' if $p \geq z_b^{min} \frac{u(x)}{u(x')}$. The minimum p occurs with the largest possible $u(x')$, i.e. with $x' = 1 - \delta$, which is the largest possible mistake the sellers can make, so:

$$p = z_b^{min} \frac{u(x)}{u(1-\delta)} \tag{A.1}$$

(ii) Sellers make a mistaken demand $1 - x' > 1 - x$

Now suppose sellers make p mistaken demands, but they demand more than the conventional demand. Now, if b_0 demands $x' < x$ then she expects to obtain utility $u(x')$ for sure. On the other hand, if b_0 demands $x > x'$ then she expects to obtain utility $u(x)$ with probability $(z_b^{min} - p)/z_b^{min}$. Thus, b_0 switches to x' if $p \geq z_b^{min} \left(1 - \frac{u(x')}{u(x)}\right)$. The minimum p occurs with the largest possible $u(x')$, i.e. with $x' = x - \delta$, which is the largest possible mistake $x' < x$ the sellers can make, so:

$$p = z_b^{min} \left(1 - \frac{u(x-\delta)}{u(x)}\right) \tag{A.2}$$

(iii) Buyers make a mistaken demand $x' < x$

Following an argument similar to case *(i)*, the minimum number q of mistaken demands by buyers needed for the seller with the smallest sample to switch is equal to:

$$q = z_s^{min} \frac{v(1-x)}{v(1-\delta)} \tag{A.3}$$

(iv) Buyers make a mistaken demand $x' > x$

Following an argument similar to case *(ii)*, the minimum number q of mistaken demands by buyers needed for the seller with the smallest sample to switch is equal to:

$$q = z_s^{min} \left(1 - \frac{v(1-x-\delta)}{v(1-x)}\right) \tag{A.4}$$

Combining equations (A.1), (A.2), (A.3), and (A.4) it follows that the least number of

mistakes necessary to move out of the convention $\mathbf{x}$ is $\lceil R(x) \rceil$, where $R(x)$ is equal to:

$$R(x) = \min \left\{ z_b^{min} \frac{u(x)}{u(1-\delta)}, z_b^{min} \left(1 - \frac{u(x-\delta)}{u(x)}\right), z_s^{min} \frac{v(1-x)}{v(1-\delta)}, z_s^{min} \left(1 - \frac{v(1-x-\delta)}{v(1-x)}\right) \right\}$$

It is straightforward to show that the first term is at least as large as the last one for all $x \in D^0$, so it can be ignored. Thus, the minimum resistance to move out of the $\mathbf{x}$ convention is $\lceil R(x) \rceil$, where $R(x)$ is given by (4). □

Proof of Theorem 2. Lemma 2 in Young [1993a] shows that a division $(x, 1-x)$ is generically stable if and only if x maximizes the function $R(x)$ in (4). Lemma 3 in Young [1993a] shows that as $\delta \to 0$, the maxima of the function $R(x)$ converge to the asymmetric Nash bargaining solution in (5). The proofs of the equivalent statements to lemmas 2 and 3 for this model are essentially the same as in Young [1993a], and they are therefore omitted here. □

Proof of Corollary 1. Denote by G_Q the quasi-regular networks generated by regular networks in $\overline{g}_{d,a}$. The proof is by contradiction. Suppose there exists a network $g \in G$ such that $g \in G_B$ and $g \notin G_Q$. There are two possible cases:

(i) $g \in G_B$ and $G_Q \cap G_B = \emptyset$: If this is the case then $\min_{b \in B} z_b(g) > \min_{b \in B} z_b(\overline{g}_{d,a}) = \overline{s}d$, i.e. $\min_{b \in B} z_b(g) \geq \overline{s}d + \epsilon$. Given that the maximum link strength is $\overline{s}$, this implies that $\min_{b \in B} d_b(g) = \lfloor \frac{2L}{n_B} \rfloor + 1$ and the degree of all other buyers must be at least equal to this. But then the total minimum number of links is $\frac{n_B}{2} \min_{b \in B} d_b(g) > L$, which is a contradiction.

(ii) $g \in G_B$ and $G_Q \subset G_B$: If this is the case then either $\min_{b \in B} z_b(g) > \min_{b \in B} z_b(\overline{g}_{d,a})$ or $\min_{b \in B} z_b(g) = \min_{b \in B} z_b(\overline{g}_{d,a})$. The argument above shows that the former leads to a contradiction, so suppose that $\min_{b \in B} z_b(g) = \min_{b \in B} z_b(\overline{g}_{d,a}) = \overline{s}d$. Thus, $\min_{b \in B} d_b(g) = d$ and the degree of all other buyers must be at least equal to this. The minimum total number of links for this to hold is $d \cdot n_B/2$, which leaves a maximum of $L - d \cdot n_B/2 = L - \lfloor L \rfloor$ links to assign. But this means that g is a quasi-regular network, no matter how the remaining links are assigned and we have a contradiction. □

Proof of Theorem 3. Let us look at *(i)* and *(ii)* separately.

(i) First, consider the case $i = B$. The goal is to compare the $(x^*, 1-x^*)$ ANB solution for agents that communicate through networks g^B and g^S, and the $(x'^*, 1-x'^*)$ ANB solution for agents that communicate through networks g'^B and g^S, where $p'_b(z)$

FOSD $p_b(z)$. The claim is that $x'^* \geq x^*$. From equation (4) we have:

$$R(x) = \min\left\{z_b^{min}(g^B)\left(1-\frac{u(x-\delta)}{u(x)}\right), z_s^{min}(g^S)\frac{v(1-x)}{v(1-\delta)}, z_s^{min}(g^S)\left(1-\frac{v(1-x-\delta)}{v(1-x)}\right)\right\} \leq$$
$$\leq \min\left\{z_b^{min}(g'^B)\left(1-\frac{u(x-\delta)}{u(x)}\right), z_s^{min}(g^S)\frac{v(1-x)}{v(1-\delta)}, z_s^{min}(g^S)\left(1-\frac{v(1-x-\delta)}{v(1-x)}\right)\right\} = R'(x)$$

because, by definition of FOSD, $z_b^{min}(g^B) \leq z_b^{min}(g'^B)$. Thus, the unique division $(x'^*, 1-x'^*)$ that maximizes $R'(x)$ is such that $x'^* \geq x^*$, where $(x^*, 1-x^*)$ is the unique division that maximizes $R(x)$. The case $i = S$ is similar, and it is therefore omitted.

(ii) Note that by definition of SOSD, $z_b^{min}(g^B) \leq z_b^{min}(g''^B)$. Replacing $z_b^{min}(g'^B)$ by $z_b^{min}(g''^B)$, the proof of this statement is the same as the proof of *(i)* above. □

Proof of Corollary 3. Let us prove a more general statement by characterizing the subset of networks $G_B \subset G$ that maximize buyers' share, where G is the set of all possible networks g such that the total number of links is L and the strength of each link is in the $[\underline{s}, \overline{s}]$ range.

Assume that $n_s \leq n_b$. First, in order to minimize sellers' share, there must be a seller s_0 such that $d_{s_0} = 1$ and $g_{s_0 b_0} = \underline{s}$, i.e. s_0 has only one weak link with one buyer b_0. Second, for the network to be connected each seller $s \in S \backslash s_0$ must have one link g_{si}, and, to maximize the number of links of buyers, let $i \in B$, $g_{si} = \overline{s}$ and assign the links so that there is no buyer who is connected to more than one seller. Third, by corollary 1, the networks that maximize the buyers' share are quasi-regular networks generated by $\overline{g}_{d,\overline{s}}$, where $d = \left\lfloor \frac{2L+n_S-1}{2} \right\rfloor$. Here, the addition of the $n_S - 1$ term takes into account the strong links buyers have with the sellers $s \in S \backslash s_0$. The only restriction on the construction of the quasi-regular network is that the links assigned at random are first assigned so that b_0 and each buyer who is not linked to any seller is assigned one link, and then the remaining links are assigned at random as long as none of them links with s_0.

A similar argument to the proof of corollary 1 shows that the existence of a network g which gives a weakly higher share to buyers and which is not in G_B would lead to a contradiction. Clearly the unweighted, core-periphery networks in the statement of corollary 3 belong to G_B. □

Proof of Theorem 5. Let us look at *(i)* and *(ii)* separately.

(i) The goal is to compare the $(x^*, 1-x^*)$ ANB solution for agents that communicate through network g, and the $(x'^*, 1-x'^*)$ ANB solution for agents that communicate through network g', where $p'(z)$ FOSD $p(z)$ and $Var[p(z)] = Var[p'(z)]$. The claim is that $x'^* = x^*$. By definition of a variance-preserving FOSD shift, we have that $z_i(g) = \varsigma z_i(g')$ for each $i \in N$, where $\varsigma > 1$ and $\varsigma \in \mathbb{R}_+$. The variance-preserving FOSD shift is therefore

only a rescaling of $R(x)$ by a ς factor. Thus, the unique division $(x^*, 1-x^*)$ that maximizes $R(x)$ is also the unique division that maximizes $R'(x) = \varsigma R(x)$, i.e. $x^* = x'^*$.

(ii) First, assume that $z_s^{min}(g) > \mu[p(z)]$. By definition of a mean-preserving SOSD shift we have that $z_b^{min}(g^{''}) > z_b^{min}(g)$. Moreover, $z_s^{min}(g^{''}) < z_s^{min}(g)$ because of the definition of SOSD shift and the assumption that $z_s^{min}(g) > \mu[p(z)]$. Substituting these inequalities into the expression (4) for $R(x)$ it is straightforward to see that the unique division $(x''^*, 1-x''^*)$ that maximizes $R''(x)$ must be such that $x''^* \geq x^*$, where $(x^*, 1-x^*)$ is the unique division that maximizes $R(x)$. The case $z_b^{min}(g) > \mu[p(z)]$ is similar and it is therefore omitted. □

Proof of Corollary 4. Let all agents have the same utility $u(.)$. If g is a regular weighted network then $\beta \equiv z_b^{min}(g) = z_s^{min}(g) \equiv \sigma$. Substituting this into (5) one obtains that the unique stable division $(x^*, 1-x^*)$ is the one that maximizes $u(x)u(1-x)$, which is clearly $x^* = 0.5$. □

B Indirect communication

This appendix presents an extension of the basic model which allows for indirect communication, i.e. information traveling more than one step in the network. It shows that all the results are robust to the introduction of indirect communication and that they generalize in a straightforward way by replacing degree with the concept of *decay r-centrality*. The proofs are omitted given that only minor changes are required to adapt the proofs in appendix A.

It is necessary to first introduce some new notation. A *path* $p(i,j;g)$ between i and j in a graph g is a sequence of links $p(i,j;g) = \{g_{ii_1}, g_{i_1i_2}, ..., g_{i_pj}\}$ such that $g_{kl} > 0$ for all $g_{kl} \in p(i,j;g)$. The *length* of a path is $|p(i,j;g)|$, and if there is no path between i and j then the length is infinite. The *geodesic distance* $D(i,j;g)$ between i and j in g is the minimum number of links that need to be used along some network path to connect i and j. If there is no such path, then $D(i,j;g) = \infty$.

Define by $g^r_{ij} = \min\{g_{kl} \mid g_{kl} \in p(i,j;g), |p(i,j;g)| = D(i,j;g) = r\}$ the *information bottleneck* between i and j, and note that this is equal to g_{ij} if i and j are directly connected. The *r-neighborhood* of i in g is $L^r_i(g) = \{j \in N | D(i,j;g) \leq r\}$, where $r \in \mathbb{N}_+$ and clearly the case of $r = 1$ is simply the neighborhood. Let $\delta \in (0,1)$ be a discount factor that captures how much information decays as it travels through the network. Now an important definition:

Definition B.1. The *decay r-centrality* of i in g is $C_i(r,g) \equiv \sum_{j \in L^r_i(g)} g^r_{ij} \delta^{D(i,j;g)-1}$

This centrality metric captures how much information an agent i receives from other agents who are at a distance less than or equal to r in the network.

The definition of the *decay r-centrality distribution* is similar to the definition of the weighted degree distribution: it captures the relative frequencies of agents that have different extended neighborhood sizes from which they draw information. Let $p(C)$ denote the decay r-centrality distribution in network g. A difference with the basic framework is in the process of information arrival because now agents receive information from their extended neighborhood up to a distance r. Formally, in the $\Delta t = 1$ time interval, the probability $P(s_{bj}(\Delta t = 1) = k)$ that b receives a sample $s_{bj}(\Delta t = 1)$ of k past bargains from agent j is equal to:

$$P(s_{bj}(\Delta t = 1) = k) = \frac{e^{-g^r_{bj}\delta^{D(i,j;g)-1}} (g^r_{bj}\delta^{D(i,j;g)-1})^k}{k!}$$

where $g^r_{bj}\delta^{D(i,j;g)-1}$ is the rate of arrival of information to b from j. By standard properties of Poisson processes, the expected amount of information b receives from j before each bargaining round is $E[P(s_{bj})] = g^r_{bj}\delta^{D(i,j;g)-1}$. It is straightforward to see that the

expected total amount of information b receives before each bargaining round is equal to b's decay r-centrality:

$$\sum_{j \in L_b^r(g)} E[P(s_{bj})] = E[P(s_b)] = \sum_{j \in L_b^r(g)} g_{bj}^r \delta^{D(i,j;g)-1} = C_b(r, g)$$

The definition of the Markov process and the other components of the set-up of the model are unchanged.

Now consider the unperturbed process, as in section 3.1. The following theorem is the equivalent version of theorem 1 in the more general set-up: it shows that if information about the history of play is sufficiently incomplete then the unperturbed process converges to a convention, i.e. a $(x, 1-x)$ split as defined in Definition 5.

Theorem B.1. *Let r be the maximum distance at which information travels in a group. Assume both g^B and g^S are connected and there is at least one pair of agents $\{i, j\}$ in each network such that $D_{ij}(g^B) > r$ and $D_{ij}(g^S) > r$. The bargaining process converges almost surely to a convention.*

First of all, note that if $r = 1$ then the statement above reduces to the statement of theorem 1 as expected. Second, the intuition for the statement of the theorem is very similar to the special case of $r = 1$. As already mentioned, theorem 1 in Young [1993b] proves that adaptive play converges almost surely to a convention in any weakly acyclic game with n agents as long as information is sufficiently incomplete. Here the incompleteness of information is given by the network structure: if the network is such that there are at least two agents who are at a distance larger than r then there will be agents who cannot sample some past rounds because they were played by agents in their group with whom they do not communicate.

Third, note that the subset of networks on which the process converges shrinks as r increases. If $r = 1$ then there will be convergence in any network that is not the complete network because it is sufficient that two agents are not connected to ensure that they have incomplete information about the past history. At the other extreme if $r \to \infty$ then the statement of the theorem has almost no bite because information about past plays is available to anyone in the network as long as the network is connected. This illustrates the importance of the network structure to deliver the incompleteness of information that is crucial to prove convergence to a convention.

As in the case of $r = 1$ analyzed in section 3, it is necessary to introduce perturbations in the system in order to obtain sharper equilibrium predictions that select one out of the large number of possible conventions. Define $B_{min}^r = \{j \in B \mid \lceil C_j(r, g^B) \rceil \leq \lceil C_b(r, g^B) \rceil, \forall b \in B\}$ to be the subset of buyers with the least decay r-centrality. Let

$C_b^{min}(r, g^B) = \lceil C_j(r, g^B) \rceil$ for $j \in B_{min}^r$. Equivalent definitions apply to the sellers. The following theorem is the general version of theorem 2 for the case of an arbitrary r.

Theorem B.2. *There exists a unique stable division $(x^*, 1 - x^*)$. It is the one that maximizes the following product:*

$$u^{C_b^{min}}(x) v^{C_s^{min}}(1 - x) \tag{B.1}$$

In other words, it is the asymmetric Nash bargaining solution with weights $C_b^{min}(r, g^B)$ and $C_s^{min}(r, g^S)$.

Note that if $r = 1$ then decay r-centrality is the same as degree and therefore the statement of the theorem reduces to the one presented in section 3.2. The intuition is also very similar to the case of $r = 1$. The share a group obtains in equilibrium crucially depends on the communication network connecting the members of the group. Specifically, it hinges on the agents in the group with the smallest extended neighborhoods, in terms of the number and/or strength of the links connecting them to other agents up to the information radius r. The agents with the smallest extended neighborhoods will be the least informed when they have to bargain, and therefore they will be the most susceptible to respond to mistakes from the other side. In the long-run this susceptibility weakens the bargaining position of the whole group.

The extension of the comparative results in theorem 4 is also rather intuitive: it suffices to replace the degree distribution with the decay r-centrality distribution to generalize the theorem as the following statement shows.

Theorem B.3. *Let $(x^*, 1 - x^*)$ be the ANB for sets of agents B and S that communicate through networks g^B and g^S with decay r-centrality distributions $p_b(C)$ and $p_s(C)$. Consider the decay r-centrality distributions $p_b'(C)$ and $p_b''(C)$ of networks g'^B and g''^B respectively, and let $p_b'(C)$ FOSD $p_b(C)$ and $p_b''(C)$ SOSD $p_b(C)$.*

(i) Let $(x'^*, 1 - x'^*)$ be the ANB for sets of agents B and S with decay r-centrality distributions $p_b'(C)$ and $p_s(C)$. Then $x'^* \geq x^*$.

ii) Let $(x''^*, 1 - x''^*)$ be the ANB for sets of agents B and S with decay r-centrality distributions $p_b''(C)$ and $p_s(C)$. Then $x''^* \geq x^*$.

The same statement holds reversing the roles of buyers and sellers.

The interpretation of first and second order shifts of the decay r-centrality distribution are similar to the correspondent shifts of the degree distribution, which is the special case of $r = 1$. A first order shifts of the decay r-centrality distribution is approximately an increase in the density of the network, i.e. an increase in the number and/or strength

of the connections among individuals in the group. A second order shift of the decay r-centrality distribution is approximately a decrease in the variability of the number and/or strength of connections across agents.

The theorem therefore states that an increase in the density of connections (and/or a decrease in their variability across individuals) within a group leads to members of that group obtaining a higher share of the pie in equilibrium. The intuition is that members of this group will have access to better information about the history of past deals experienced by other members of their group. This informational advantage makes them less likely to respond to mistakes by the other side, and they are therefore able to maintain an advantageous bargaining position.

It is also possible to extend the results of the version of the model in section 6, where buyers and sellers belong to the same network, to allow for indirect communication among agents. The generalization of the results follows along the same lines as the generalization of the theorems for the version of the model with separate networks, and it is therefore left to the reader to explore it.