

# Dynamics and Stability of Political Systems

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# Motivation

- Towards a general framework?
- One attempt: **Acemoglu, Egorov and Sonin:** *Dynamics and Stability of Constitutions, Coalitions and Clubs*.
- General approach motivated by political economy, though potentially applicable to organizational economics, club theory, and international relations as well.

# Toward General Insights

- Key trade-off between
  - **Payoffs:** different arrangements imply different payoffs and individuals care about payoffs.
  - **Power:** different arrangements reallocate decision-making (political) power and thus affect future evolution of payoffs.
- *Strategy:* Formulate a general dynamic framework to investigate the interplay of these two factors in a relatively “detail-free” manner.
  - Details useful to go beyond general insights.

## Simple Example

- Consider a simple extension of franchise story
- Three states: absolutism  $a$ , constitutional monarchy  $c$ , full democracy  $d$
- Two agents: elite  $E$ , middle class  $M$

$$w_E(d) < w_E(a) < w_E(c)$$

$$w_M(a) < w_M(c) < w_M(d)$$

- $E$  rules in  $a$ ,  $M$  rules in  $c$  and  $d$ .
- Myopic elite: starting from  $a$ , move to  $c$
- Farsighted elite: stay in  $a$ : move to  $c$  will lead to  $M$  moving to  $d$ .
- Same example to illustrate resistance against socially beneficial reform.

# Naïve and Dynamic Insights

- *Naïve insight*: a social arrangement will emerge and persist if a “sufficiently powerful group” prefers it to alternatives.
- Simple example illustrates: power to change towards a more preferred outcome is not enough to implement change
  - because of further dynamics
- Social arrangements might be stable even if there are powerful groups that prefer change in the short run.
- **Key**: social arrangements change the distribution of political power (decision-making capacity).
- **Dynamic decision-making**: future changes also matter (especially if discounting is limited)

# Applications

- Key motivation: changes in constitutions and political regimes.
- Extension of franchise (Acemoglu and Robinson 2000, 2006, Lizzeri and Persico 2004)
- Members of a club decide whether to admit additional members by majority voting (Roberts 1999)
- Society decides by voting, what degree of (super)majority is needed to start a reform (Barbera and Jackson 2005)
- EU members decide whether to admit new countries to the union (Alesina, Angeloni, and Etro 2005)
- Inhabitants of a jurisdiction determine migration policy (Jehiel and Scotchmer 2005)
- Participant of (civil) war decides whether to make concessions to another party (Fearon 1998, Schwarz and Sonin 2008)
- Dynamic political coalition formation: Junta (or Politburo) members decide whether to eliminate some of them politically or physically (Acemoglu, Egorov, and Sonin 2008)

# Voting in Clubs or Dynamic Franchise Extension

- Suppose that individuals  $\{1, \dots, M\}$  have a vote and they can extend the franchise and include any subset of individuals  $\{M + 1, \dots, N\}$ .
- Instantaneous payoff of individual  $i$  a function of the set of individuals with the vote (because this influences economic actions, redistribution, or other policies)
- Political protocol: majority voting.
- $\{1, \dots, M\}$  vote over alternative proposals.
- If next period the franchise is  $\{1, \dots, M'\}$ , then this new franchise votes (by majority rule) on the following period's franchise etc.
- Difficult dynamic game to analyze.
- But once we understand the common element between this game and a more general class of games, a tight and insightful characterization becomes possible.

# Model and Approach

- Model:
  - Finite number of individuals.
  - Finite number of states (characterized by economic relations and political regimes)
  - Payoff functions determine instantaneous utility of each individual as a function of state
  - Political rules determine the distribution of political power and protocols for decision-making within each state.
  - A dynamic game where “politically powerful groups” can induce a transition from one state to another at any date.
- Question: what is the **dynamically stable state** as a function of the initial state?



# Main Results of General Framework

- An axiomatic characterization of “outcome mappings” corresponding to dynamic game (based on a simple *stability* axiom incorporating the notion of forward-looking decisions).
- Equivalence between the MPE of the dynamic game (with high discount factor) and the axiomatic characterization
- Full characterization: *recursive* and *simple*
- Under slightly stronger conditions, the stable outcome (dynamically stable state) is unique given the initial state
  - but depends on the initial state
- Model general enough to nest specific examples in the literature.
- In particular, main theorems directly applicable to situations in which states can be ordered and static payoffs satisfy single crossing or single peakedness.

## Simple Implications

- A particular social arrangement is made stable by the instability of alternative arrangements that are preferred by sufficiently many members of the society.
  - stability of a constitution does not require absence of powerful groups opposing it, but the absence of an alternative stable constitution favored by powerful groups.
- Efficiency-enhancing changes are often resisted because of further social changes that they will engender.
  - Pareto inefficient social arrangements often emerge as stable outcomes.

# Illustration

- Voting in clubs.
- Dynamic taxation with endogenous franchise.
- Stability of constitutions.
- Political eliminations.
- From **Acemoglu, Egorov and Sonin**: *Dynamics of Political Selection*
  - a small amount of incumbency advantage can lead to the emergence and persistence of very incompetent/inefficient governments (without asymmetric information)
  - a greater degree of democracy does not necessarily ensure better governments
  - **but**, a greater degree of democracy leads to greater **flexibility** and to better governments in the long run in stochastic environments.

## Related Literature

- Papers mentioned above as applications or specific instances of the general results here.
- Dynamic coalition formation—Ray (2008).
- Dynamic political reform—Lagunoff (2006).
- Farsighted coalitional stability—Chwe (1994).
- Dynamic economic interactions with transferable utility—Gomes and Jehiel (2005).
- Dynamic inefficiencies with citizen candidates—Besley and Coate (1999).

## Model: Basics

- Finite set of individuals  $\mathcal{I}$  ( $|\mathcal{I}|$  total)
  - Set of coalitions  $\mathcal{C}$  (non-empty subsets  $X \subset \mathcal{I}$ )
- Each individual maximizes discounted sum of payoffs with discount factor  $\beta \in [0, 1)$ .
- Finite set of states  $\mathcal{S}$  ( $|\mathcal{S}|$  total)
- Discrete time  $t \geq 1$
- State  $s_t$  is determined in period  $t$ ;  $s_0$  is given
- Each state  $s \in \mathcal{S}$  is characterized by
  - Payoff  $w_i(s)$  of individual  $i \in \mathcal{I}$  (normalize  $w_i(s) > 0$ )
  - Set of winning coalitions  $\mathcal{W}_s \subset \mathcal{C}$  capable of implementing a change
  - Protocol  $\pi_s(k)$ ,  $1 \leq k \leq K_s$ : sequence of agenda-setters or proposals ( $\pi_s(k) \in \mathcal{I} \cup \mathcal{S}$ )

# Winning Coalitions

## Assumption

**(Winning Coalitions)** For any state  $s \in \mathcal{S}$ ,  $\mathcal{W}_s \subset \mathcal{C}$  satisfies two properties:

- (a) If  $X, Y \in \mathcal{C}$ ,  $X \subset Y$ , and  $X \in \mathcal{W}_s$  then  $Y \in \mathcal{W}_s$ .  
(b) If  $X, Y \in \mathcal{W}_s$ , then  $X \cap Y \neq \emptyset$ .

- (a) says that a superset of a winning coalition is winning in each state
- (b) says that there are no two disjoint winning coalitions in any state
- $\mathcal{W}_s = \emptyset$  is allowed (exogenously stable state)
- Example:
  - Three players 1, 2, 3
  - $\mathcal{W}_s = \{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$  is valid (1 is dictator)
  - $\mathcal{W}_s = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$  is valid (majority voting)
  - $\mathcal{W}_s = \{\{1\}, \{2, 3\}\}$  is not valid (both properties are violated)

# Dynamic Game

- 1 Period  $t$  begins with state  $s_{t-1}$  from the previous period.
- 2 For  $k = 1, \dots, K_{s_{t-1}}$ , the  $k$ th proposal  $P_{k,t}$  is determined as follows. If  $\pi_{s_{t-1}}(k) \in \mathcal{S}$ , then  $P_{k,t} = \pi_{s_{t-1}}(k)$ . If  $\pi_{s_{t-1}}(k) \in \mathcal{I}$ , then player  $\pi_{s_{t-1}}(k)$  chooses  $P_{k,t} \in \mathcal{S}$ .
- 3 If  $P_{k,t} \neq s_{t-1}$ , each player votes (sequentially) yes (for  $P_{k,t}$ ) or no (for  $s_{t-1}$ ). Let  $Y_{k,t}$  denote the set of players who voted yes. If  $Y_{k,t} \in \mathcal{W}_{t-1}$ , then  $P_{k,t}$  is accepted, otherwise it is rejected.
- 4 If  $P_{k,t}$  is accepted, then  $s_t = P_{k,t}$ . If  $P_{k,t}$  is rejected, then the game moves to step 2 with  $k \mapsto k + 1$  if  $k < K_{s_{t-1}}$ . If  $k = K_{s_{t-1}}$ ,  $s_t = s_{t-1}$ .
- 5 At the end of each period (once  $s_t$  is determined), each player receives instantaneous utility  $u_i(t)$ :

$$u_i(t) = \begin{cases} w_i(s) & \text{if } s_t = s_{t-1} = s \\ 0 & \text{if } s_t \neq s_{t-1} \end{cases}$$

# Key Notation and Concepts

- Define binary relations:

- states  $x$  and  $y$  are payoff-equivalent

$$x \sim y \iff \forall i \in \mathcal{I} : w_x(i) = w_y(i)$$

- $y$  is weakly preferred to  $x$  in  $z$

$$y \succeq_z x \iff \{i \in \mathcal{I} : w_y(i) \geq w_x(i)\} \in \mathcal{W}_z$$

- $y$  is strictly preferred to  $x$  in  $z$

$$y \succ_z x \iff \{i \in \mathcal{I} : w_y(i) > w_x(i)\} \in \mathcal{W}_z$$

- Notice that these binary relations are **not** simply preference relations
  - *they encode information about preferences and political power.*



# Preferences and Acyclicity

## Assumption

**(Payoffs)** Payoff functions  $\{w_i(\cdot)\}_{i \in \mathcal{I}}$  satisfy:

(a) For any sequence of states  $s_1, \dots, s_k$  in  $\mathcal{S}$ ,

$$s_{j+1} \succ_{s_j} s_j \text{ for all } 1 \leq j \leq k-1 \implies s_1 \not\succeq_{s_k} s_k.$$

(b) For any sequence of states  $s, s_1, \dots, s_k$  in  $\mathcal{S}$  with  $s_j \succ_s s$  (for all  $1 \leq j \leq k$ )

$$s_{j+1} \succ_s s_j \text{ for all } 1 \leq j \leq k-1 \implies s_1 \not\succeq_s s_k.$$

- (a) rules out cycles of the form  $y \succ_z z, x \succ_y y, z \succ_x x$
- (b) rules out cycles of the form  $y \succ_s z, x \succ_s y, z \succ_s x$
- Weaker than transitivity of  $\succ_s$ .
- These assumptions cannot be dispensed with in the context of a general treatment because otherwise Condorcet-type cycles emerge.

## Preferences and Acyclicity (continued)

- We will also strengthen our results under:

### Assumption

**(Comparability)** For  $x, y, z \in \mathcal{S}$  such that  $x \succ_z z$ ,  $y \succ_z z$ , and  $x \approx y$ , either  $y \succ_z x$  or  $x \succ_z y$ .

- This condition sufficient (and “necessary”) for uniqueness.

## Approach and Motivation

- **Key economic insight:** *with sufficiently forward-looking behavior, an individual should not wish to transition to a state that will ultimately lead to another lower utility state.*
- Characterize the set of allocations that are consistent with this insight—without specifying the details of the dynamic game.
  - Introduce three simple and intuitive axioms.
  - Characterize set of mappings  $\Phi$  such that for any  $\phi \in \Phi$ ,  $\phi : \mathcal{S} \rightarrow \mathcal{S}$  satisfies these axioms and assigns an **axiomatically stable state**  $s^\infty \in \mathcal{S}$  to each initial state  $s_0 \in \mathcal{S}$  (i.e.,  $\phi(s) = s^\infty \in \mathcal{S}$  loosely corresponding to  $s_t = s^\infty$  for all  $t \geq T$  for some  $T$ ).
- Interesting in its own right, but the main utility of this axiomatic approach is as an input into the characterization of the (two-strategy) MPE of the dynamic game.

# Axiom 1

**(Desirability)** If  $x, y \in \mathcal{S}$  are such that  $y = \phi(x)$ , then either  $y = x$  or  $y \succ_x x$ .

- A winning coalition can always stay in  $x$  (even a blocking coalition can)
- A winning coalition can move to  $y$
- If there is a transition to  $y$ , a winning coalition must have voted for that

## Axiom 2

**(Stability)** If  $x, y \in \mathcal{S}$  are such that  $y = \phi(x)$ , then  $y = \phi(y)$ .

- Holds “by definition” of  $\phi(\cdot)$ :  $\exists T : s_t = \phi(s)$  for all  $t \geq T$ ; when  $\phi(s)$  is reached, there are no more transitions
- If  $y$  were unstable ( $y \neq \phi(y)$ ), then why not move to  $\phi(y)$  instead of  $y$

## Axiom 3

**(Rationality)** If  $x, y, z \in \mathcal{S}$  are such that  $z \succ_x x$ ,  $z = \phi(z)$ , and  $z \succ_x y$ , then  $y \neq \phi(x)$ .

- A winning coalition can move to  $y$  and to  $z$
- A winning coalition can stay in  $x$
- When will a transition to  $y$  be blocked?
  - If there is another  $z$  preferred by some winning coalition
  - If this  $z$  is also preferred to  $x$  by some winning coalition (so blocking  $y$  will lead to  $z$ , not to  $x$ )
  - If transition to  $z$  is credible in the sense that this will not lead to some other state in perpetuity

# Stable States

- State  $s \in \mathcal{S}$  is  $\phi$ -stable if  $\phi(s) = s$  for  $\phi \in \Phi$
- Set of  $\phi$ -stable states:  $\mathcal{D}_\phi = \{s \in \mathcal{S} : \phi(s) = s \text{ for } \phi \in \Phi\}$
- We will show that if  $\phi_1$  and  $\phi_2$  satisfy the Axioms, then
$$\mathcal{D}_{\phi_1} = \mathcal{D}_{\phi_2} = \mathcal{D}$$
  - Even if  $\phi$  is non-unique, notion of stable state is well-defined
  - But  $\phi_1(s)$  and  $\phi_2(s)$  may be different elements of  $\mathcal{D}$

# Axiomatic Characterization of Stable States

## Theorem

*Suppose Assumptions on Winning Coalitions and Payoffs hold. Then:*

- 1 *There exists mapping  $\phi$  satisfying Axioms 1–3.*
- 2 *This mapping  $\phi$  may be obtained through a recursive procedure (next slide)*
- 3 *For any two mappings  $\phi_1$  and  $\phi_2$  that satisfy Axioms 1–3 the the sets of stable states of these mappings coincide (i.e.,  $\mathcal{D}_{\phi_1} = \mathcal{D}_{\phi_2} = \mathcal{D}$ ).*
- 4 *If, in addition, the Comparability Assumption holds, then the mapping that satisfies Axioms 1–3 is “payoff-unique” in the sense that for any two mappings  $\phi_1$  and  $\phi_2$  that satisfy Axioms 1–3 and for any  $s \in \mathcal{S}$ ,  $\phi_1(s) \sim \phi_2(s)$ .*



## Recursive Procedure

### Theorem (continued)

Any  $\phi$  that satisfies Axioms 1–3 can be recursively computed as follows.

Construct the sequence of states  $\{\mu_1, \dots, \mu_{|\mathcal{S}|}\}$  with the property that if for any  $l \in (j, |\mathcal{S}|]$ ,  $\mu_l \not\succ_{\mu_j} \mu_j$ . Let  $\mu_1 \in \mathcal{S}$  be such that  $\phi(\mu_1) = \mu_1$ . For  $k = 2, \dots, |\mathcal{S}|$ , let

$$\mathcal{M}_k = \{s \in \{\mu_1, \dots, \mu_{k-1}\} : s \succ_{\mu_k} \mu_k \text{ and } \phi(s) = s\}.$$

Define, for  $k = 2, \dots, |\mathcal{S}|$ ,

$$\phi(\mu_k) = \begin{cases} \mu_k & \text{if } \mathcal{M}_k = \emptyset \\ z \in \mathcal{M}_k : \nexists x \in \mathcal{M}_k \text{ with } x \succ_{\mu_k} z & \text{if } \mathcal{M}_k \neq \emptyset \end{cases}.$$

(If there exist more than one  $s \in \mathcal{M}_k$ :  $\nexists z \in \mathcal{M}_k$  with  $z \succ_{\mu_k} s$ , we pick any of these; this corresponds to multiple  $\phi$  functions).

## Extension of Franchise Example

- Get back to the simple extension of franchise story
- Three states: absolutism  $a$ , constitutional monarchy  $c$ , full democracy  $d$
- Two agents: elite  $E$ , middle class  $M$

$$w_E(d) < w_E(a) < w_E(c)$$

$$w_M(a) < w_M(c) < w_M(d)$$

- $\mathcal{W}_a = \{\{E\}, \{E, M\}\}$ ,  $\mathcal{W}_c = \{\{M\}, \{E, M\}\}$ ,  
 $\mathcal{W}_d = \{\{M\}, \{E, M\}\}$
- Then:  $\phi(d) = d$ ,  $\phi(c) = d$ , therefore,  $\phi(a) = a$ 
  - Indeed,  $c$  is unstable, and among  $a$  and  $d$  player  $E$ , who is part of any winning coalition, prefers  $a$
  - Intuitively, if limited franchise immediately leads to full democracy, elite will not undertake it

## Example (continued)

- Assume  $\mathcal{W}_c = \{\{E, M\}\}$  instead of  $\mathcal{W}_c = \{\{M\}, \{E, M\}\}$
- Then:  $\phi(d) = d$ ,  $\phi(c) = c$ , and,  $\phi(a) = c$
- $a$  became unstable because  $c$  became stable

- Now assume  $\mathcal{W}_a = \mathcal{W}_c = \mathcal{W}_d = \{\{E, M\}\}$  and

$$w_E(a) < w_E(d) < w_E(c)$$

$$w_M(a) < w_M(c) < w_M(d)$$

- $a$  is disliked by everyone, but otherwise preferences differ
- Then:  $\phi(d) = d$ ,  $\phi(c) = c$ , and  $\phi(a)$  may be  $c$  or  $d$
- In any case,  $\mathcal{D} = \{c, d\}$  is the same

## Back to Dynamic Game

### Assumption

**(Agenda-Setting and Proposals)** For every state  $s \in \mathcal{S}$ , one (or both) of the following two conditions is satisfied:

(a) For any state  $q \in \mathcal{S} \setminus \{s\}$ , there is an element  $k : 1 \leq k \leq K_s$  of sequence  $\pi_s$  such that  $\pi_s(k) = q$ .

(b) For any player  $i \in \mathcal{I}$  there is an element  $k : 1 \leq k \leq K_s$  of sequence  $\pi_s$  such that  $\pi_s(k) = i$ .

- Exogenous agenda, sequence of agenda-setters, or mixture.
- This assumption ensures that all proposals will be considered (or all agenda-setters will have a chance to propose)

### Definition

**(Dynamically Stable States)** State  $s^\infty \in \mathcal{S}$  is a **dynamically stable state** if there exist a protocol  $\{\pi_s\}_{s \in \mathcal{S}}$ , a MPE strategy profile  $\sigma$  (for a game starting with initial state  $s_0$ ) and  $T < \infty$ , such that in MPE  $s_t = s^\infty$

## Slightly Stronger Acyclicity Assumption

**Assumption (Stronger Acyclicity)** For any sequence of states  $s, s_1, \dots, s_k$  in  $\mathcal{S}$  such that  $s_j \approx s_l$  (for any  $1 \leq j < l \leq k$ ) and  $s_j \succ_s s$  (for any  $1 \leq j \leq k$ )

$$s_{j+1} \succ_s s_j \text{ for all } 1 \leq j < k - 1 \implies s_1 \not\prec_s s_k.$$

Moreover, if for  $x, y, s$  in  $\mathcal{S}$ , we have  $x \succ_s s$  and  $y \not\prec_s s$ , then  $y \not\prec_s x$ .

- Stronger version of part (b) of Payoffs Assumption.
- First part:  $\succeq$ -acyclicity as opposed  $\succ$ -acyclicity
- Second part: slightly stronger than acyclicity
  - but weaker than transitivity within states, i.e.,  $x \succ_s s, y \not\prec_s s$ , then  $y \not\prec_s x$ , whereas transitivity would require  $x \succ_s s, s \succ_s y$ , then  $x \succ_s y$ , which implies our condition, but is much stronger.
- Alternative (with equivalent results): voting  $y$ es has a small cost.

# Noncooperative Characterization

## Theorem

**(Noncooperative Characterization)** *Suppose Assumptions on Winning Coalitions and Payoffs hold. Then there exists  $\beta_0 \in [0, 1)$  such that for all  $\beta \geq \beta_0$ , the following results hold.*

- 1 For any mapping  $\phi$  satisfying Axioms 1–3 there is a protocol  $\{\pi_s\}_{s \in \mathcal{S}}$  and a MPE  $\sigma$  of the game such that  $s_t = \phi(s_0)$  for any  $t \geq 1$ ; that is, the game reaches  $\phi(s_0)$  after one period and stays in this state thereafter. Therefore,  $s = \phi(s_0)$  is a dynamically stable state.

# Noncooperative Characterization (continued)

## Theorem

... Moreover, suppose that Stronger Acyclicity Assumption holds. Then:

2. For any protocol  $\{\pi_s\}_{s \in \mathcal{S}}$  there exists a MPE in pure strategies. Any such MPE  $\sigma$  has the property that for any initial state  $s_0 \in \mathcal{S}$ , it reaches some state,  $s^\infty$  by  $t = 1$  and thus for  $t \geq 1$ ,  $s_t = s^\infty$ . Moreover, there exists mapping  $\phi : \mathcal{S} \rightarrow \mathcal{S}$  that satisfies Axioms 1–3 such that  $s^\infty = \phi(s_0)$ . Therefore, all dynamically stable states are axiomatically stable.
3. If, in addition, Assumption (Comparability) holds, then the MPE is essentially unique in the sense that for any protocol  $\{\pi_s\}_{s \in \mathcal{S}}$ , any MPE strategy profile in pure strategies  $\sigma$  induces  $s_t \sim \phi(s_0)$  for all  $t \geq 1$ , where  $\phi$  satisfies Axioms 1–3.

# Dynamic vs. Myopic Stability

## Definition

State  $s^m \in \mathcal{S}$  is *myopically stable* if there does not exist  $s \in \mathcal{S}$  with  $s \succ_{s^m} s^m$ .

## Corollary

- ① State  $s^\infty \in \mathcal{S}$  is a (dynamically and axiomatically) stable state only if for any  $s' \in \mathcal{S}$  with  $s' \succ_{s^\infty} s^\infty$ , and any  $\phi$  satisfying Axioms 1–3,  $s' \neq \phi(s')$ .
  - ② A myopically stable state  $s^m$  is a stable state.
  - ③ A stable state  $s^\infty$  is not necessarily myopically stable.
- E.g., state  $a$  in extension of franchise story



# Inefficiency

## Definition

**(Inefficiency)** State  $s \in S$  is *(strictly) Pareto inefficient* if there exists  $s' \in S$  such that  $w_i(s') > w_i(s)$  for all  $i \in \mathcal{I}$ .

State  $s \in S$  is *(strictly) winning coalition inefficient* if there exists a winning coalition  $\mathcal{W}_s \subset \mathcal{I}$  in  $s$  and  $s' \in S$  such that  $w_i(s') > w_i(s)$  for all  $i \in \mathcal{W}_s$ .

- Clearly, if a state  $s$  is Pareto inefficient, it is winning coalition inefficient, but not vice versa.

## Corollary

- 1 A stable state  $s^\infty \in S$  can be *(strictly) winning coalition inefficient and Pareto inefficient*.
- 2 Whenever  $s^\infty$  is not myopically stable, it is *winning coalition inefficient*.

# Applying the Theorems in Ordered Spaces

- The characterization theorems provided so far are easily applicable in a wide variety of settings.
- In particular, if the set of states is ordered and static preferences satisfy single crossing or single peakedness, all the results provided so far can be applied directly.
- Here, for simplicity, suppose that  $\mathcal{I} \subset \mathbb{R}$  and  $\mathcal{S} \subset \mathbb{R}$  (more generally, other orders on the set of individuals and the set of states would work as well)

# Single Crossing and Single Peakedness

## Definition

Take set of individuals  $\mathcal{I} \subset \mathbb{R}$ , set of states  $\mathcal{S} \subset \mathbb{R}$ , and payoff functions  $w_i(\cdot)$ . Then, single crossing condition holds if whenever for any  $i, j \in \mathcal{I}$  and  $x, y \in \mathcal{S}$  such that  $i < j$  and  $x < y$ ,  $w_i(y) > w_i(x)$  implies  $w_j(y) > w_j(x)$  and  $w_j(y) < w_j(x)$  implies  $w_i(y) < w_i(x)$ .

## Definition

Take set of individuals  $\mathcal{I} \subset \mathbb{R}$ , set of states  $\mathcal{S} \subset \mathbb{R}$ , and payoff functions  $w_i(\cdot)$ . Then, single-peaked preferences assumption holds if for any  $i \in \mathcal{I}$  there exists state  $x$  such that for any  $y, z \in \mathcal{S}$ , if  $y < z \leq x$  or  $x \geq z > y$ , then  $w_i(y) \leq w_i(z)$ .

# Generalizations of Majority Rule and Median Voter

## Definition

Take set of individuals  $\mathcal{I} \subset \mathbb{R}$ , state  $s \in \mathcal{S}$ , and set of winning coalitions  $\mathcal{W}_s$  that satisfies Assumption on Winning Coalitions. Player  $i \in \mathcal{I}$  is called quasi-median voter (in state  $s$ ) if  $i \in X$  for any  $X \in \mathcal{W}_s$  such that  $X = \{j \in \mathcal{I} : a \leq j \leq b\}$  for some  $a, b \in \mathbb{R}$ .

- That is, quasi-median voter is a player who belongs to any “connected” winning coalition.
- Denote the set of quasi-median voters in state  $s$  by  $M_s$  (it will be nonempty)

## Definition

Take set of individuals  $\mathcal{I} \subset \mathbb{R}$ , set of states  $\mathcal{S} \subset \mathbb{R}$ . The sets of winning coalitions  $\{\mathcal{W}_s\}_{s \in \mathcal{S}}$  has monotonic quasi-median voter property if for each  $x, y \in \mathcal{S}$  satisfying  $x < y$  there exist  $i \in M_x, j \in M_y$  such that  $i \leq j$ .

## A Weak Genericity Assumption

- Let us say that preferences  $w.(\cdot)$ , given the set of winning coalitions  $\{\mathcal{W}_s\}_{s \in \mathcal{S}}$ , are *generic* if for all  $x, y, z \in \mathcal{S}$ ,  $x \succeq_z y$  implies  $x \succ_z y$  or  $x \sim y$ .
- This is (much) weaker than the comparability assumption used for uniqueness above.
  - In particular, it holds generically.

# Theorem on Single Crossing and Single Peakedness

## Theorem

*Suppose the Assumption on Winning Coalitions holds.*

- 1 If preferences satisfy single crossing and the monotonic quasi-median voter property holds, then Assumptions on Payoffs above are satisfied and the axiomatic characterization (Theorem 1) applies.*
  - 2 If preferences are single peaked and all winning coalitions intersect (i.e.,  $X \in \mathcal{W}_x$  and  $Y \in \mathcal{W}_y$  imply  $X \cap Y \neq \emptyset$ ), then Assumptions on Payoffs are satisfied and Theorem 1 applies.*
  - 3 If, in addition, in part 1 or 2, preferences are generic, then the Stronger Acyclicity Assumption is satisfied and the noncooperative characterization (Theorem 2) applies.*
- 
- Note monotonic median voter property is weaker than the assumption that  $X \in \mathcal{W}_x \wedge Y \in \mathcal{W}_y \implies X \cap Y \neq \emptyset$ .

# Voting in Clubs

- $N$  individuals,  $\mathcal{I} = \{1, \dots, N\}$
- $N$  states (clubs),  $s_k = \{1, \dots, k\}$
- Assume single-crossing condition

for all  $l > k$  and  $j > i$ ,  $w_j(s_l) - w_j(s_k) > w_i(s_l) - w_i(s_k)$

- Assume “genericity”:

for all  $l > k$ ,  $w_j(s_l) \neq w_j(s_k)$

- Then, the theorem for ordered spaces applies and shows existence of MPE in pure strategies for any majority or supermajority rule.
- It also provides a full characterization of these equilibria.

# Voting in Clubs

- If in addition only odd-sized clubs are allowed, unique dynamically stable state.
- Equilibria can easily be Pareto inefficient.
- If “genericity” is relaxed, so that  $w_j(s_l) = w_j(s_k)$ , then the theorem for ordered spaces no longer applies, but both the axiomatic characterization and the noncooperative theorems can still be applied from first principles.
- Comparison to Roberts (1999): much simpler analysis under weaker conditions, and more general results (existence of pure-strategy equilibrium, results for supermajority rules etc.)
- Also can be extended to more general structure of clubs
  - e.g., clubs on the form  $\{k - n, \dots, k, \dots, k + n\} \cap \mathcal{I}$  for a fixed  $n$  (and different values of  $k$ ).



## An Example of Elite Clubs

- Specific example: suppose that preferences are such that

$$w_j(s_n) > w_j(s_{n'}) > w_j(s_{k'}) = w_j(s_{k''})$$

for all  $n' > n \geq j$  and  $k', k'' < j$

- individuals always prefer to be part of the club
- individuals always prefer smaller clubs.
- Winning coalitions need to have a strict majority (e.g., two out of three, three out of four etc.).
- Then,
  - $\{1\}$  is a stable club (no wish to expand)
  - $\{1, 2\}$  is a stable club (no wish to expand and no majority to contract)
  - $\{1, 2, 3\}$  is not a stable club (3 can be eliminated)
  - $\{1, 2, 3, 4\}$  is a stable club
- More generally, clubs of size  $2^k$  for  $k = 0, 1, \dots$  are stable.
- Starting with the club of size  $n$ , the equilibrium involves the largest club of size  $2^k \leq n$ .

## Example: Taxation

- Suppose there are  $k$  individuals  $1, 2, \dots, k$ , and  $k$  states  $s_1, s_2, \dots, s_k$ , where  $s_j = \{1, 2, \dots, j\}$ .
- Suppose winning coalition is a simple majority rule of players who are enfranchised:

$$\mathcal{W}_{s_j} = \{X \in \mathcal{C} : \#(X \cap s_j) > j/2\}.$$

- Suppose player  $i$ 's payoff is

$$w_i(s_j) = (1 - \tau_{s_j}) A_i + G_{s_j}$$

where  $A_i$  is player  $i$ 's productivity;  $G_{s_j}$  and  $\tau_{s_j}$  are the public good and the tax rate voting franchise is  $s_j$ .

- Assume  $A_i > A_j$  for  $i < j$ , so the first players are the most productive ones

## Example: Taxation (continued)

- $\tau_{s_j}$  is the tax rate determined by the median voter in the club  $s_j$  (or by one of the two median voters with equal probability in case of even-sized club)
- The technology for the production of the public good is

$$G_{s_j} = H \left( \sum_{i=1}^k \tau_{s_j} A_i \right),$$

where  $H$  is strictly increasing and concave.

## Example: Taxation (continued)

- In light of the previous theorem, to apply our results, it suffices to show that if  $i, j \in s_k, s_{k+1}$ , then

$$w_j(s_{k+1}) - w_j(s_k) > w_i(s_{k+1}) - w_i(s_k)$$

whenever  $i < j$ .

- This is equivalent to

$$(1 - \tau_{s_{k+1}}) A_j - (1 - \tau_{s_k}) A_j \geq (1 - \tau_{s_{k+1}}) A_i - (1 - \tau_{s_k}) A_i,$$

- Since  $A_j < A_i$ , this in turn is equivalent to

$$\tau_{s_{k+1}} \geq \tau_{s_k}.$$

- This can be verified easily, so the theorem for order spaces can be applied.

# Stable Constitutions

- $N$  individuals,  $\mathcal{I} = \{1, \dots, N\}$
- In period 2, they decide whether to implement a reform ( $a$  votes are needed)
- $a$  is determined in period 1
- Two cases:
  - Voting rule  $a$ : stable if in period 1 no other rule is supported by  $a$  voters
  - Constitution  $(a, b)$ : stable if in period 1 no other constitution is supported by  $b$  voters
- Preferences over reforms translate into preferences over  $a$ 
  - Barbera and Jackson assume a structure where these preferences are single-crossing and single-peaked
  - Motivated by this, let us assume that they are strictly single-crossing
- Stable voting rules correspond to myopically (and dynamically) stable states
- Stable constitutions correspond to dynamically stable states

# Political Eliminations

- The characterization results apply even when states do not form an ordered set.
- Set of states  $\mathcal{S}$  coincides with set of coalitions  $\mathcal{C}$
- Each agent  $i \in \mathcal{I}$  is endowed with political influence  $\gamma_i$
- Payoffs are given by proportional rule

$$w_i(X) = \begin{cases} \gamma_i/\gamma_X & \text{if } i \in X \\ 0 & \text{if } i \notin X \end{cases} \quad \text{where } \gamma_X = \sum_{j \in X} \gamma_j$$

and  $X$  is the “*ruling coalition*”.

- this payoff function can be generalized to any function where payoffs are increasing in relative power of the individual in the ruling coalition

## Political Eliminations (continued)

- Winning coalitions are determined by weighted (super)majority rule  $\alpha \in [1/2, 1)$

$$\mathcal{W}_X = \left\{ Y : \sum_{j \in Y \cap X} \gamma_j > \alpha \sum_{j \in X} \gamma_j \right\}$$

- Genericity:  $\gamma_X = \gamma_Y$  only if  $X = Y$
- Assumption on Payoffs is satisfied and the axiomatic characterization applies exactly.
- If players who are not part of the ruling coalition have a slight preference for larger ruling coalitions, then Stronger Acyclicity Assumption is also satisfied.

# Other Examples

- Inefficient inertia
- The role of the middle class in democratization
- Coalition formation in democratic systems
- Commitment, (civil or international) conflict and peace



# Political Selection

- A related problem: how does a society select its government (rulers, officials, bureaucrats)
  - different levels of competence
  - rents from being in office
  - some degree of incumbency advantage
- How do political institutions, affecting the degree of incumbency advantage, impact on the “efficiency” of governments?
- What types of political institutions enable greater flexibility, allowing the society to adopt to changes in environments by changing the government?

# Summary of Main Results

- More democratic regimes not necessarily better in deterministic environments.
- More democratic regimes are more resistant to shocks
  - because they are more *flexible*
  - they can absorb larger shocks
  - an ideal democracy will fully adjust to any shock
- Even negative political shocks may increase the competence of government
  - at the cost of less flexibility in the future
- Consequently, democratic regimes potentially preferable because of their flexibility advantage.

## Concluding Remarks

- A class of dynamic games potentially representing choice of constitutions, dynamic voting, club formation, dynamic coalition formation, organizational choice, dynamic legislative bargaining, international or civil conflict.
- Common themes in disparate situations.
- A framework for general analysis and tight characterization results.
- *Simple implications: social arrangements* are unstable **not** when some winning coalition (e.g., majority) prefers another social arrangement, but when it prefers another **stable** social arrangement
- We show that this gives rise to inefficiencies: a Pareto dominated state may be stable, even if discount factor is close to 1

# Persistence and Change in Institutions

- Using this framework in order to analyze issues of persistence and change systematically.
- Missing:
  - Stochastic shocks and more generally stochastic power switches.
  - Dynamics with intermediate discount factors.
  - Good mapping between the shoes of the first lecture and the general model.
  - And of course, strategy for empirical work
- Much to do as we go forward.