STRATEGIC CONSENSUS AND HETEROGENOUS VOTERS: PROVISION OF PUBLIC GOODS WITH RANDOM VOTER TURNOUT

Toke S. Aidt
University of Cambridge

Jayasri Dutta,
University of Birmingham

ABSTRACT

This paper argues that uncertain or random voter turnout plays a key role in mediating conflicts of interest between voters and politicians on the one hand and heterogenous groups of voters on the other. Random voter turnout creates an incentive for politicians to seek consensus because it is unclear ex ante who will hold the majority among those who turn out to vote. We argue that this leads to efficient provision of public goods if preference heterogeneity is not too large, and that it protects (large) minority groups against the tyranny of the majority. We also argue that compulsory voting may not be desirable because it reduces randomness in turnouts.

1. INTRODUCTION

Political processes are designed to resolve conflicts among groups of citizens with conflicting objectives and goals. The manner in which such conflicts are resolved is of interest not only because it informs normative discussions of fairness and legitimacy but also because it helps predict outcomes under different institutional arrangements. Voting is the cornerstone of the democratic political process and the properties of different types of voting systems have attracted a lot of attention from public choice scholars.[1] This includes the fundamental question of why voters vote.[2]

While there exists a substantial body of research on this question, the issue of voter turnout is largely ignored in applied work on the political economy of fiscal policy. The canonical models – the median voter model, the probabilistic voting model as well as the various agency models of elections – all *assume* that voters turn out to vote in each election.[3] Yet, the evidence suggests otherwise. Not only is average voter turnout low in many countries, it also fluctuates substantially over time and space. To illustrate this point, Table 1 records turnout rates in national elections for 24 countries for the period 1970 to 2000. Of particular importance to the argument of this paper is the fact that the coefficient of variation[4] – a direct measure of turnout uncertainty – is large in many countries and often exceeds 5% of the average.

Another important fact about electoral turnout is the large variation in the average turnout of voters with different demographic and socioeconomic characteristics. This is clearly illustrated by Table 2 which documents large and variable gaps in inter-group electoral participation in western democracies during the period 1996 to 1999.

[1]See, e.g., Mueller (2003).
[2]See Dhillon and Peralta (2002) or Aidt (2000) for a discussion of this literature.
[3]For a good introduction to these models, see, e.g., Persson and Tabellini (2000) or Hettich and Winer (1999, chapter 2).
[4]The coefficient of variation is defined as the standard deviation divided by the mean multiplied by 100.

Table 1: The average turnout rates and the coefficient of variation (Parliamentarian elections) in selected OECD countries 1970-99

Country	Average	Coefficient of Variation
Australia	83.5	1.6
Austria	83.3	7.2
Belgium	87.6	3.8
Canada	65.7	6.6
Denmark	84.9	2.9
Finland	77.3	8.5
France	63.9	7.5
Germany	78.9	7.3
Greece	84.6	2.1
Iceland	88.9	2.0
Italy	92.4	2.8
Japan	67.5	13.8
South Korea	74.3	7.9
Luxembourg	65.1	9.2
Mexico	54.8	18.2
Netherlands	80.8	7.7
Norway	79.7	4.0
Portugal	81.1	7.8
Spain	76.8	5.8
Sweden	85.1	4.0
Switzerland	40.3	9.9
United Kingdom	72.9	3.2
United States	45.1	17.4
New Zealand	82.4	4.8
All countries	74.6	19.5

Source: IDEA (2003).

Table 2: Turnout rates in general elections by gender for differerent social groups in 19 western democracies 1996-99

Social Group	Men	Women
	%	%
Young (18-30)	72.5	72.9
Old (65+)	86.9	83.2
Low income	82.1	76.9
High income	83.9	80.7
Primary	77.3	74.7
Secondary	79.4	79.4
Graduate	85.9	84.2
Full time employed	84.4	78.1
Unemployed	67.5	64.5
Students	71.7	74.7
Retired	84.9	81.1

Source: Norris (2001).

In this paper, we argue that *uncertain* voter turnout has important but largely overlooked implications for how particular political processes mediate conflicts of interest between voters and politicians on the one hand and heterogenous groups of voters on the other. A simple example can illustrate our reasoning. Consider a country in which decisions are made by simple majority rule. The country is divided into two regions, called North and South, and is populated by 100 voters distributed with 60 voters in the South and 40 voters in the North. So, if turnout were certain (and all voters showed up to vote), then politicians could win elections simply by pandering to the South.

Suppose, however, that turnout depends on weather conditions and that turnout is only half if the weather is foul. Moreover, suppose that the probability of "good weather" is 0.5, that the probability of "foul weather" is 0.5 and that weather conditions are independent across regions. Table 3 shows the number of voters who turn out to vote in each region as a function of the weather. We observe that region South holds the majority in three out of four cases (i.e., with probability 0.75) but that region North – the minority region – holds the majority among those who show up to vote with probability 0.25. So, ex ante, politicians might be wary pandering only to the majority of the South: if the weather turns out to be good in the North and foul in the South, the voters of region North will be casting the decisive vote.

Table 3: The number of voters who turn out to vote in the two regions as a function of the weather

		South	
		Good	Foul
North	Good	60 40	30 40
	Foul	60 20	30 20

Note: The first entry in each cell is for the South.

Building on this logic, we explore the consequences of uncertain voter turnout for electoral accountability and competition between heterogenous groups of voters. We do so within one of the canonical models of electoral politics, namely the so-called retrospective voting model. This model was suggested by Barro (1973) and further developed by Ferejohn (1986), and has been extensively used by Persson and Tabellini (2000, 2003) and many others in recent work on comparative public finance.[5] The model portrays elections as a vehicle through which voters

[5] See e.g., Persson et al. (1997) and Coate and Morris (1999).

(the principal) can dismiss or replace under-performing politicians (the agent) at election day. The main purpose of elections within this conception is to hold politicians accountable ex post for the choices they made while in office, i.e., the model highlights the accountability role of elections. In situations with heterogenous groups of voters, elections serve the additional purpose of aggregating conflicting preferences.[6] In this case, the model typically predicts that policy outcomes are biased in favor of the majority group at the expense of the minority. Moreover, competition between groups of voters allows politicians far too much leeway and renders electoral accountability ineffective (Ferejohn, 1986; Aidt and Magris, 2006). Turnout uncertainty changes all of this in fundamental ways.

Turnout uncertainty has two surprising implications. Firstly, politicians *always* implement policies that satisfy the demands of *all* voters including minority groups. Secondly, voters, in turn, make demands that politicians want to satisfy. We call this *strategic consensus*.[7] Strategic consensus insures politicians against turnout risk and voters against partisan choices that ignore the interests of minority groups. In contrast to an economy with certain electoral turnout, the interest of the minority *is* always included in the political calculus. Using this logic, we study the classical public finance choice between targeted transfers

[6] Generally, elections serve a number of different functions. They aggregate preferences and information, they select politicians and they allow voters to hold the selected politicians accountable. We focus on the first and last of these roles, but acknowledge that the other roles are also important in practice.

[7] See Aidt and Dutta (2004).

and universal public goods.[8] In an economy characterized by turnout uncertainty, we show that i) the minority is protected against the tyranny of the majority, ii) universal public goods are only provided if the Lindahl-Samuelson condition is satisfied, and iii) in economies with sufficiently high public sector productivity, the minority group is at least as well off as the majority group and often strictly better off. None of this is true if voter turnout is non-random. While the first result is robust to the introduction of preference heterogeneity, inter-group differences in the valuation of private goods may cause politicians to *over-provide* public goods. Moreover, minority groups who value private goods relatively little are unlikely to be better off than majority groups with a high valuation of private goods.

The rest of the paper is organized as follows. In section two, we present the model and discuss the main assumptions. In section three, we present the main analysis. In section four, we present the results. In section five, we discuss the broader implications of the analysis. The appendix contains some derivations and proofs.

---

[8]This question has received much attention in the recent literature on positive public finance (e.g., Persson and Tabellini, 2000; Lizzeri and Persico, 2001).

2. THE MODEL

Society consists of two groups of voters, $i = 1,2$; politicians are indicated by index 0. Voters and politicians have an infinite time horizon. Time is indexed by t. A group is defined as a subset of voters who are affected in the same way by public policy. Group affiliation may be determined by observable characteristics such as geographical location, age, gender, or religion. Per-period utility, u_{it}, is discounted with the common discount factor $\beta \in (0,1)$. There are n_1 voters in group 1 and n_2 voters in group 2. The size of the total (voter) population is $n = n_1 + n_2$. Assuming that $n_1 > n_2$, we refer to group 1 as the majority and group 2 as the minority, although either group may in actual fact hold the majority among those who turn out to vote (see below).

Each period, the politician collects taxes up to a maximum of T, spends some of this on universal public goods and/or on group-specific transfer payments, and keeps the rest for himself as a *political rent*.[9] Denoting the cost of providing utilities to the two groups of voters c_t, we can write the politician's per-period political rent as $u_{0t} = T - c_t$ if in office, and $u_{0t} = 0$ otherwise. Politicians apply the same discount factor as voters.

The politician, elected at t, cannot make binding promises on the level and pattern of public spending before

[9]This formulation of the conflict of interest between voters and politicians is due to Persson et al. (1997) and used extensively in Persson and Tabellini (2000). It should be understood as a metaphor for the more general phenomenon that politicians might divert their efforts towards activities that are not in the interests of their electorate.

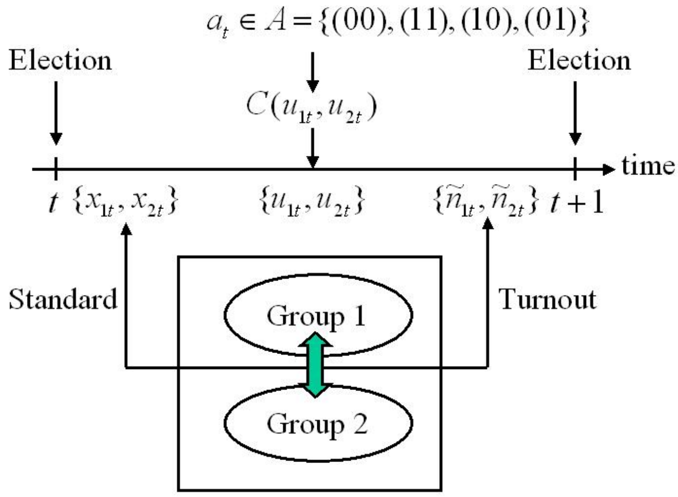
he enters office. Since his own payoff decreases with c_t, he would, in the absence of further incentives, choose $c_t = 0$ and provide no amenities to the electorate. Voters know this, and threaten to vote retrospectively against a politician who does not provide them with a minimum level of utility. At the beginning of each period, voters in each group announce a performance standard, denoted x_{1t} and x_{2t}. At the election at the end of the period, they then vote in favor of reelection of the incumbent politician if, and only if the policy implementation observed generates at least that level of utility, i.e., if, and only if $u_{it} \geq x_{it}$.

Importantly, neither group can guarantee to turn out in full force at elections. Suppose a politician delivers on the performance standard set by group 1, who holds the majority ex ante but fails to deliver on the standard set by group 2 ($u_{2t} < x_{2t}$). On the day of the election, $\tilde{n}_{it}$ voters from group i actually show up to vote, and the politician can lose his bid for reelection if $\tilde{n}_{2t} > \tilde{n}_{1t}$. The central assumption of our analysis is that electoral turnout is uncertain. Voters can commit to vote according to the announced performance standards *if they show up to vote*, but cannot commit to a particular turnout rate. This is captured by the following assumption.

Assumption 1 Electoral turnout, $\tilde{n}_t = (\tilde{n}_{1t}, \tilde{n}_{2t})$, is random. The ex ante probability that the turnout of group 1 is greater than that of group 2, $P(\tilde{n}_{1t} \geq \tilde{n}_{2t})$, is equal to p and constant over time. Moreover, $p \in (0,1)$.

Here, we specify the parameter p directly. It can be derived from more basic considerations, however. In this analysis, it is important that $0 < p < 1$, so that neither group can guarantee reelection. This is more likely to be the case when turnout shocks are correlated within groups. This is, for example, the case when differences in weather conditions affect the turnout of voters in different geographical locations or foul weather keeps certain types of voters, such as the poor or old, at home (Roemer, 1998), and when differences in group sizes are not too large (e.g., because election districts are designed to be of equal size). Typically p will be a function of group sizes and we expect that $p \rightarrow 1$ when $n_1 - n_2 \rightarrow \infty$. Thus, what we have in mind is a situation in which the two groups are of moderately different sizes.

Figure 1: The timeline of the model



The game between the incumbent politician and the two groups of voters unfolds over time as illustrated in Figure 1. At the beginning of each period, voters in each group announce the (utility) standard that the politician needs to satisfy to get their votes in the next election. The standards are chosen by the two groups non-cooperatively and at the same time. The politician observes the standards and determines whether to comply, and if so, how many standards to meet. We denote the set of actions available to the politician by $A = \{(00),(10),(01),(11)\}$ with elements $a_t = (00)$ (meet neither standard); $a_t = (10)$ (meet group 1's standard only); $a_t = (01)$ (meet group 2's standard only); and $a_t = (11)$ (meet both standards). At the end of the period, a new election is held and voters randomly turn up to vote. Those who turn up vote according to the announced performance standard. The politician either wins or loses. In the latter case, he is replaced by an identical challenger; in the former case, he gets (at least) another term in office. After the election, the game continues to the next period where a similar sequence of events takes place.

3. ANALYSIS

Ultimately, our goal is to understand the role of turnout uncertainty in shaping competition between heterogenous groups of voters. The first step towards this goal is to characterize the so-called *political cost function.* The second step is to use the political cost function to characterize equilibrium outcomes of the game described above. We restrict attention to history-independent subgame perfect Nash equilibria.[10]

[10]Formally, the model describes a dynamic common agency game with absorbing states and perfect information. The two groups of voters are principals, and the elected politician their common agent. Uncertainty in rewards arises from uncertainty about which of the two principals will have the "casting vote", or final say, in the only reward available:

3.1. THE POLITICAL COST FUNCTION

The political cost function defines the minimum expenditure the politician must incur to provide voters with a given utility level. Specifically, we define $C(x_{1t}, x_{2t})$ as the minimum cost of simultaneously providing utility levels $u_{1t} \geq x_{1t}$ and $u_{2t} \geq x_{2t}$ to voters in the two groups at time t. Likewise, we define $C_i(x_{it})$ as the minimum cost of *separately* providing the utility level $u_{it} \geq x_{it}$ to group i, $i = 1,2$.

The politician can please voters by providing a universal public good, g_t, or targeted, lump sum transfers, $\tau_{it} \geq 0$, $i = 1,2$, or a combination of the two.[11] The public good is produced by a linear technology

$$g_t = Ak_t, \tag{1}$$

where k_t denotes the tax revenues devoted to the production of the public good. $A > 0$ is a productivity parameter that captures the efficiency of the public sector. The public budget constraint requires that

$$n_1\tau_{1t} + n_2\tau_{2t} + k_t \leq T \tag{2}$$

for each t. We assume that all voters value the universal public good in the same way, and that utility is linear in public and private goods:

$$u_{it} = g_t + \theta_i\tau_{it}; \quad \theta_i > 0 \;\; i = 1,2. \tag{3}$$

---

re-election. There is no aggregate uncertainty, as one of the principals will have the casting vote for sure.

[11] We do not allow transfers to be targeted specifically to individual voters, but only to groups of voters in different geographical locations, age-groups, professions, etc.

We note that all voters like public goods and dislike political rents, but disagree on who should have the transfers. Moreover, we allow for heterogenous taste for the private good, i.e., θ_1 may be different from θ_2. We, therefore, note that voters are heterogenous along three dimensions: they differ in the characteristics that define group membership; they disagree about who should have transfers; and, finally, they may value the private good differently.

A utilitarian social planner would provide the public good (by spending $k = T$) if, and only if the Lindahl-Samuelson condition – saying that the sum of the marginal benefit of the public good (n) exceeds the marginal cost of producing the good ($\max\{\theta_1, \theta_2\}/A$) – holds, or if:

$$A \geq \frac{\max\{\theta_1, \theta_2\}}{n}. \quad (4)$$

Moreover, given that the marginal utility of private goods varies across groups, the social planner will, if the Lindahl-Samuelson condition fails redistribute income from the group with the lowest valuation to the group with the highest valuation. If $\theta_1 = \theta_2$, the planner has not any particular reason to redistribute income.

The self-interested politician will choose the level and composition of public spending to maximize his rent, but needs to take into account that if more is spent on targeted transfers, less is available for public goods and political rents. The politician also keeps in mind that transfers must be provided to all voters in the relevant group, implying that the cost of targeted redistribution is sensitive to group sizes. In contrast, public goods allow the politician to satisfy all voter demands simultaneously irrespective of group sizes. This distinction turns out to be important.

We derive the political cost function in the Appendix and summarize the key features below. The cost-efficient method of meeting the performance standards depends on A, the productivity level in the public sector, relative to the size of the two groups and to their taste for private goods, as follows:

1. Let $A < \left(\frac{n_1}{\theta_1} + \frac{n_2}{\theta_2} \right)^{-1}$. The political cost function is:

$$C(x_{1t}, x_{2t}) = \frac{n_1}{\theta_1} x_{1t} + \frac{n_2}{\theta_2} x_{2t}; \quad (5)$$

$$C_1(x_{1t}) = \frac{n_1}{\theta_1} x_{1t}; \quad (6)$$

$$C_2(x_{2t}) = \frac{n_2}{\theta_2} x_{2t}. \quad (7)$$

Here, the productivity of the public sector, A, is so low that the politician will not provide any public goods at all. Transfers are the cheapest way to buy voter approval. Consequently, no public goods are provided $k_t = g_t = 0$ and if voters in group i ask for the utility level x_{it}, the cost to the politician of providing the utility level is $\frac{n_i}{\theta_i} x_{it}$.

2. Let $A \in [\left(\frac{n_1}{\theta_1} + \frac{n_2}{\theta_2} \right)^{-1}, \min\{\frac{\theta_1}{n_1}, \frac{\theta_2}{n_2}\})$. The political cost function is

$$C(x_{1t}, x_{2t}) = \frac{\min[x_{1t}, x_{2t}]}{A} + \frac{n_1}{\theta_1}(x_{1t} - \min[x_{1t}, x_{2t}]) + \frac{n_2}{\theta_2}(x_{2t} - \min[x_{1t}, x_{2t}]); \quad (8)$$

$$C_1(x_{1t}) = \frac{n_1}{\theta_1} x_{1t}; \tag{9}$$

$$C_2(x_{2t}) = \frac{n_2}{\theta_2} x_{2t}. \tag{10}$$

Here, the politician provides public goods only if he wishes to satisfy both standards. In particular, he satisfy the demands of the least demanding group (which could be either of the two groups) with public goods and top up with a targeted transfer to the more demanding group. On the other hand, if he only wants to satisfy the standard of one group, the cheapest way to do so is to provide targeted transfers to that group only.

3. Let $A \in [\min\{\frac{\theta_1}{n_1}, \frac{\theta_2}{n_2}\}, \max\{\frac{\theta_1}{n_1}, \frac{\theta_2}{n_2}\})$. For $\frac{\theta_1}{n_1} < \frac{\theta_2}{n_2}$, the political cost function is

$$C(x_{1t}, x_{2t}) = \frac{x_{1t}}{A} + \frac{n_2}{\theta_2} \max(x_{2t} - x_{1t}, 0); \tag{11}$$

$$C_1(x_{1t}) = \frac{x_{1t}}{A}; \tag{12}$$

$$C_2(x_{2t}) = \frac{n_2}{\theta_2} x_{2t}. \tag{13}$$

For $\frac{\theta_1}{n_1} \geq \frac{\theta_2}{n_2}$, the political cost function is

$$C(x_{1t}, x_{2t}) = \frac{x_{2t}}{A} + \frac{n_1}{\theta_1} \max(x_{1t} - x_{2t}, 0); \tag{14}$$

$$C_1(x_{1t}) = \frac{n_1}{\theta_1} x_{1t}; \tag{15}$$

$$C_2(x_{2t}) = \frac{x_{2t}}{A}. \tag{16}$$

Here, the political costs are minimized by satisfying the demands from the group with the lowest valuation of the private good relative to group size (the lowest θ_i / n_i) with public goods and meeting further demands from the other group with transfers. We notice that if $\theta_1 = \theta_2$, then group 1 – the majority – is pleased with public goods, while further demands from the minority are meet with transfers.

4. Let $A \geq \max\{\frac{\theta_1}{n_1}, \frac{\theta_2}{n_2}\}$. The political cost function is

$$C(x_{1t}, x_{2t}) = \frac{\max[x_{1t}, x_{2t}]}{A}; \tag{17}$$

$$C_1(x_{1t}) = \frac{x_{1t}}{A}; \tag{18}$$

$$C_2(X_{2t}) = \frac{x_{2t}}{A}. \tag{19}$$

Here, the productivity of the public sector is so high that all demands are met by public goods rather than transfers. A politician who wants to meet the standard of one group will automatically provide (some) utility to the other.

What is important for what follows is that the political cost function is *sub-additive,* i.e., $C(x_{1t}, x_{2t}) \leq C(x_{1t}) + C(x_{2t})$. In plain words this means that it is at least as cheap for the politician to satisfy the performance standards of the two groups jointly as it is doing it separately. Sub-additivity arises from the fundamental role of public goods. Imagine that a politician wants to provide utility to one group of voters only. He can do this by making transfers to this group. If he wants to provide utility to both groups, it may be cheaper to provide universal public goods. The fact that public goods can be used to provide utility to everybody allows the cost function to be sub-additive. When, as in case 1, it is inefficient to provide public goods and the politician uses transfers to satisfy all demands, the political cost function becomes additive, i.e., $C(x_{1t}, x_{2t}) = C(x_{1t}) + C(x_{2t})$.

3.2. EQUILIBRIUM

Our model of the political economy of fiscal choices under turnout uncertainty is a special case of a more general model studied in Aidt and Dutta (2004). In that paper, we show that all equilibrium paths of the game described above have a property called *strategic consensus:* the politician prefers to meet all performance standards at all times, all groups of voters vote for the incumbent, and the incumbent is reelected with certainty, irrespective of turnout shocks. This is a surprising result as intuition would lead one to think that it must at least sometimes – e.g., when it is very unlikely that a group is ever going to hold the majority among those who turn out to vote – be optimal for a politician to be partisan and focus on one group only. This intuition is, however, wrong. To see why, consider the special case where the only policy instrument is group-specific transfers. To please voters, the politician must either be partisan and give transfers to one group only or seek consensus and give to both. The two groups of voters set their standards simultaneously. Suppose that group 1 has announced a standard that is so high that the politician prefers to take his chances and offer transfers only to group 2. This cannot be an equilibrium because group 1 gets nothing and it would do better by reducing its standard to a level such that it is in the best interest of the politician to offer it a positive transfer. In other words, whenever the politician is willing to implement a "partisan" outcome, the disfavored group has an incentive to lower its standard to induce the politician to make a "partisan" choice in its favor. This logic continues until the standards are such that the politician is just willing to implement a policy that satisfies both groups. The result is strategic consensus. Importantly, it does *not* follow from this logic that the two groups will "under-bid" each other until the politician captures the entire rent. This would only happen if the two groups were "perfect substitutes" in the sense that each could guarantee reelection for sure. In our model, however, the two groups of voters avoid Bertrand-style competition precisely because they are not "perfect substitutes" from the point of view of the politician: the consent of both is needed to secure the reelection reward with certainty. As a consequence, voters retain some control power, even when political costs are additive.

Although all equilibrium paths display strategic consensus, the distribution of payoffs depends critically on the properties of the political cost function. In an economy with sub-additive political costs, the following characterization result holds.[12]

Proposition 1 (Sub-additive Costs) If the political cost function is sub-additive, then the distribution of payoffs is determined by the following conditions:

$(\mathbf{SC_1^+})$ $C(x_{1t}, x_{2t}) = \beta T$;

$(\mathbf{SC_2^+})$ $C_1(x_{1t}) \geq \beta p T$;

$(\mathbf{SC_3^+})$ $C_2(x_{2t}) \geq \beta(1-p)T$.

Moreover, $(\mathbf{SC_2^+})$ and $(\mathbf{SC_3^+})$ hold with equality for an additive political cost function. Along all equilibrium paths, the politician receives payoffs $(1-\beta)T$ per period.

The proposition tells us how the payoff is divided between voters and the politician (i.e., how large the political rent is) and how the remainder is divided between the two groups of voters. The politician *always* gets the

[12]We state proposition 1 without proof. For a proof, see Aidt and Dutta (2004; theorem 1 and proposition 1).

political rent $(1-\beta)T$ per period. The remaining share of tax revenues, βT, is devoted to the task of generating utilities to the voters. Importantly, this distribution of payoffs is unaffected by turnout uncertainty. Thus, strategic consensus provides the politician with "full insurance" against random voter turnout and voters with insurance against "partisan" choices by the politician. When the political cost function is additive, the allocation of utility between the two groups of voters is uniquely determined by p. In contrast, economies with *strictly* sub-additive costs exhibit multiple equilibria in performance standards at each t and the division of payoffs between the two groups of voters cannot be pinned down uniquely.

To gain intuition into why the payoffs are distributed in this way, begin by recalling that the politician can always keep the entire tax revenue T and accept to be voted out of office at the next election. This defines his outside option. To avoid this outcome, which is a disaster for voters, the two groups must allow the politician to collect the rent u_0 each period. In particular, this rent must be such that its present value, $u_0/(1-\beta)$, is at least equal to T. This, in turn, implies that $u_0 \geq (1-\beta)T$. On the other hand, at Nash equilibrium it cannot be the case that voters leave more than that on the table for the politician to collect. To see why, suppose, for simplicity, that the politician can only use transfers to please the two groups of voters. This makes the political cost function additive and the equilibrium allocation of utilities is uniquely given by $u_0 = (1-\beta)T$, $u_1 = \theta_1 \beta p T$ and $u_2 = \theta_2 \beta (1-p) T$. Now, suppose that, say, group 1 asks for less than $\theta_1 \beta p T$ while group 2 asks for $\theta_2 \beta (1-p) T$. This implies that the politician is allowed to collect strictly more rent than $(1-\beta)T$ each period. This is not an equilibrium. The reason is that group 1 can increase its utility by asking for a bit more: the politician will still be happy to comply as long as it is better to do so than collecting the entire rent or to please group 2 only. The outcome of this equilibrating process is that the politician always gets $(1-\beta)T$, no more, no less.

4. THE RESULTS

A number of interesting results about the composition of public spending and the surprising role of turnout uncertainty in mediating inter-group conflict flow directly from this analysis. The results are valid for any $p \in (0,1)$ and so do not depend on the precise distribution of the turnout shocks. To prove the results, we combine proposition 1 with the specific cost function we derived above. This is done in the Appendix. Here, we focus on the general insights and the intuition behind them. We begin by focussing on the inter-group conflict that arises from the disagreement over who should get the transfer. That is, we assume that all citizens have the same taste for private goods and set $\theta_1 = \theta_2 = \theta$. We notice that in this case, $(n_1/\theta_1 + n_2/\theta_2)^{-1} = \theta/n$ and that $\theta/n_1 < \theta/n_2$ because group 1 is the majority.

Proposition 2 (Public goods) Assume that $\theta_1 = \theta_2 = \theta$. Along any equilibrium path, public goods are provided ($g_t > 0$) if, and only if

$$A > \frac{\theta}{n}. \tag{20}$$

Proof. The statement follows directly from the political cost function $C(x_{1t}, x_{2t})$ and the fact that all equilibrium paths exhibits strategic consensus

Proposition 3 (Transfers) Assume that $\theta_1 = \theta_2 = \theta$. Along any equilibrium path, transfers are used only if

$$0 < A < \frac{\theta}{n_2}. \qquad (21)$$

Further, only the minority gets transfers if $A \in (\theta / n_1, \theta / n_2)$.

Proof. The first statement follows from the political cost function $C(x_{1t}, x_{2t})$. The second statement follows from the fact that $A \in (\theta / n_1, \theta / n_2)$ implies that $\tau_1 = 0$ minimizes costs for any attainable x_{1t} and x_{2t}.

Proposition 2 demonstrates that the politician only provides public goods if the Lindahl-Samuelson condition is satisfied. In this sense, strategic consensus implies efficient provision of public goods. Turnout uncertainty is critical for this result. To see this, suppose that the majority in group 1 always turn out in full strength at election day. In this case, the politician will cater only for this group. Suppose the group asks for the utility level x_{1t}. The politician will provide public goods to satisfy this demand only if $A \geq \theta / n_1$. This is because he only internalizes the benefit of public goods to the majority group – he is not concerned with the incidental fact that also the minority benefits. As a consequence, for any $A \in [\theta / n, \theta / n_1)$, the politician will *not* provide public goods (despite the Lindahl-Samuelson condition being satisfied) unless there is turnout uncertainty.

It is clear, however, that the politician supplies less public goods than the social planner, who spends all tax revenues on the purpose ($g_t^* = AT$ for $A \geq \theta / n$). Under-provision in this sense arises, as in Persson et al. (1997), because voters must allow their politicians to divert some funds, which could otherwise have been spent on public goods (or transfers), in order to discipline them not to expropriate everything. More surprisingly, for $A \in [\theta / n, \theta / n_1]$, all attainable equilibrium paths *over-provide* public goods relative to the wishes of the majority who for this ranges of productivity levels prefers transfers to public goods (and would get just that in the absence of turnout uncertainty). This happens because the consensus seeking politician finds it cheaper to satisfy the demands of one of the groups with public goods.

A comparison of propositions 2 and 3 shows how the politician makes use of the two policy instruments in economies with different productivity levels and group sizes. Begin by considering an economy with an inefficient public sector ($A < \theta / n$). In this economy, the politician prefers to use targeted transfers to get reelected. This makes the political cost function additive, and voters in each group receive actuarially fair insurance against partisan choices. In particular, the voters' share of total revenue (βT) is divided between the two groups according to their probability of being pivotal, i.e., each member of group 1 gets the transfer $p\beta T / n_1$, while each member of group 2 gets the transfer $(1-p)\beta T / n_2$. An implication, then, is that the majority is unable to (fully) expropriate (with the help of the politician) the wealth of the minority. Turnout uncertainty plays a critical role in generating this outcome. To see this, suppose, as in Ferejohn (1986), that voters *always* turn out to vote. Since reelection requires the support of the majority only, the wealth of the minority is expropriated completely by the politician who redistributes some to the majority and keeps the rest for himself.[13] Turnout uncertainty protects the minority against this because there is a chance that it is, in fact, the minority that holds the majority among those who show up to vote. We shall return to the practical implications of this result in section 5, but notice that the "protection" offered to minority groups by politicians is proportional to their chance of being pivotal. Thus, very small minority groups, whose chance of ever holding the majority amongst those who turn out to vote is tiny, get very limited "protection" against the tyranny of the majority.

Contrast this to an economy with high public sector productivity ($A \geq \theta / n_2$). In this economy, the politician prefers to satisfy all demands from voters with public goods and the political cost function becomes (strictly) sub-additive. An immediate implication is that all voters are treated equally and turnout uncertainty is no longer necessary to protect the minority from expropriation. To see why, return to the situation where turnout is certain and the politician can win the election by pleasing only the majority group. Since the cheapest way to do so is to provide public goods, everybody – including voters in the minority group – get the same benefits, even if the politician were to attempt to implement a "partisan" outcome.[14]

---

[13]If there are more than two groups, then outcomes are even worse. The politician will be looking for a minimum winning coalition. Competition to get included in this coalition provides a strong incentive for groups to offer their votes at a discount and sparks a process of underbidding. This leads to the result that none of the groups get any transfers at equilibrium.

[14]Persson and Tabellini (2000, chapter 9) and Aidt and Magris (2006) make a similar point.

Only in an economy with an intermediate productivity level ($A \in (\theta/n, \theta/n_2)$), the politician prefers to use a combination of public goods and transfers (to at most one group) to please voters. While for $A \in (\theta/n, \theta/n_1)$, the direction of the transfer depends on the particular equilibrium path attained, only the minority receives transfers when $A \in [\theta/n_1, \theta/n_2)$. This observation has a somewhat surprising implication.

Proposition 4 (Minority welfare) Assume that $\theta_1 = \theta_2 = \theta$. Along any equilibrium path, the minority is at least as well off as the majority if

$$A \geq \frac{\theta}{n_1}. \qquad (22)$$

Proof. To establish this, we note that $\theta/n_2 > A \geq \theta/n_1$ implies

$$C(x_{1t}, x_{2t}) = \frac{x_{1t}}{A} + \frac{n_2}{\theta}\max(x_{2t} - x_{1t}, 0) = \beta T. \qquad (23)$$

Hence, $g_t = x_{1t}$, $\tau_{1t} = 0$ and $\tau_{2t} \geq 0$. This implies that

$$u_{2t} \geq g_t = u_{1t} = x_{1t}. \qquad (24)$$

If $A \geq \theta/n_2$, we have $g_t = \max(x_{1t}, x_{2t})$, and $\tau_{1t} = \tau_{2t} = 0$ all t implying that $u_{1t} = u_{2t}$.

The result derives from the fact that it is often too expensive for politicians to satisfy the demands of the majority with transfers: the group is simply too large. Conversely, it is too expensive to satisfy additional demands by the minority with (more) public goods when $A < \theta/n_2$. Hence, for $\theta/n_2 > A \geq \theta/n_1$, the politician provides public goods to please the majority. The minority, of course, also benefits from this, and, in addition, in some, but not all equilibria, gets a transfer.

The case without inter-group taste differences yields interesting insights into how turnout uncertainty mediates the redistributional conflict between the two groups. Taste heterogeneity, however, also plays a role and introduces a new dimension of inefficiency. In particular, we can state the following result.

Proposition 5 (Public Goods and Preference Heterogeneity) Assume that $\theta_1 \neq \theta_2$. Along any equilibrium path, public goods are provided ($g_t > 0$) if, and only if

$$A \geq \left(\frac{n_1}{\theta_1} + \frac{n_2}{\theta_2} \right)^{-1}. \qquad (25)$$

For $A \in [\left(n_1/\theta_1 + n_2/\theta_2\right)^{-1}, \max\{\theta_1, \theta_2\}/n)$, it is inefficient to provide public goods.

Proof. The first part follows from inspection of the political cost function and the fact that all equilibria exhibit strategic consensus. The second part follows by recalling that the Lindahl-Samuelson condition for efficient provision of public goods requires

$$A \geq \frac{\max\{\theta_1, \theta_2\}}{n}. \qquad (26)$$

Simply manipulations show that

$$\left(\frac{n_1}{\theta_1} + \frac{n_2}{\theta_2} \right)^{-1} < \frac{\theta_1}{n} \text{ for } \theta_1 > \theta_2; \qquad (27)$$

$$\left(\frac{n_1}{\theta_1} + \frac{n_2}{\theta_2} \right)^{-1} < \frac{\theta_2}{n} \text{ for } \theta_1 < \theta_2. \qquad (28)$$

For $A \in [\left(n_1/\theta_1 + n_2/\theta_2\right)^{-1}, \max\{\theta_1, \theta_2\}/n)$, the politician provides public goods despite the fact that the Lindahl-Samuelson condition fails.

A comparison of this proposition and proposition 2 shows that heterogeneity in the taste for private goods introduces inefficiency in the provision of public goods. In particular, the elected politician may *over-provide* public goods in the sense that he will provide even when the Lindahl-Samuelson condition fails. To see the intuition for this, we can rewrite condition (25) as:

$$n \geq \frac{n}{A}\left(\frac{n_1}{\theta_1}+\frac{n_2}{\theta_2}\right)^{-1}. \qquad (29)$$

The left hand side is the marginal benefit of providing one unit of the public good: for the politician seeking a consensus outcome (as well as for the social planner), the one unit of public good provides one unit of benefit to all n citizens. The right hand side is the marginal cost. For $\theta_1 \neq \theta_2$, this is smaller than the marginal cost facing the social planner ($\max\{\theta_1, \theta_2\}/A$). For the social planner, the opportunity cost of spending one unit of revenue on public goods is that less private goods can be enjoyed by the group with the *highest* valuation of such goods. For the politician, the opportunity cost is different. A unit of revenue spend on public goods cannot be spent on transferring income to *each* of the two groups. Since $\theta_1 \neq \theta_2$, this cost is lower than the opportunity cost faced by the planner. This result is interesting, not the least because it provides an example why politicians may choose an inefficient policy instrument (a public good) despite the fact that a more efficient instrument (transfers) is available.

Preference heterogeneity also has implications for how well the minority group fares. In a society without significant differences in the valuation of private goods, proposition 4 shows that the minority is often as well off as the majority. The reason is that everybody benefit from public goods *and* that it is expensive to offer transfers to the majority. A consequence of the later is that the transfers then often go to the minority. If there are significant differences in the taste for private goods, this logic may, however, break down. In particular, if the majority values the private good much more than the minority, it becomes cheaper for the politician to provide transfers to this group than to the minority group. As a consequence, turnout uncertainty no longer guarantees that the minority group is at least as well off as the majority group, except, of course, in the case where it is optimal for the politician to supply public goods only.

5. DISCUSSION

The analysis shows that turnout uncertainty has important implications for the fiscal choices made by self-interested politicians. In the absence of such uncertainty, politicians are tempted to be "partisan" in the sense of pleasing only those groups which are strictly required to secure reelection. Turnout uncertainty, on the other hand, induces politicians to seek consensus outcomes. The reason is simply that politicians cannot be sure ex ante who will hold the majority among those who turn out to vote. This significantly changes the dynamics of inter-group competition for political favors and implies that the political process aggregates the preferences of heterogenous groups of voters in such a way that all interests are given some weight.

Turnout uncertainty is one example of what we might call governance uncertainty (Aidt and Dutta, 2004). Governance uncertainty can arise from many other sources than random election turnout. It may, for example, reflect fluctuations in inter-group power relations with one group becoming more powerful and therefore more likely to be pivotal than another due to unpredictable events. In particular, the lobbying power of social groups may well fluctuate in this way. Under this interpretation the probability of being pivotal, p, can be seen as a manifestation of randomness in the cost of political mobilization. Combined with the insights from Olson (1965), a minority could be as likely as a majority group to be pivotal, not because it may in fact hold the majority among those who turn out to vote, but because it is better at organizing an effective lobby group.[15] The new twist highlighted by our theory to the Olsonian logic is that uncertainty about the cost of mobilization will play an important role in mediating competition between special interests.

Our model has some interesting similarities to the probabilistic voting model, in particular as applied to redistributive politics by Lindbeck and Weibull (1987), Dixit and Londregan (1996), and Hettich and Winer (1988, 1999). This body of research studies the incentives of competing political parties to targeted monetary transfers (or tax concessions) at specific groups of voters in order to "buy" votes. A key result is that transfers are targeted at swing voters, i.e., groups of voters whose voting probabilities are particularly sensitive to additional benefits because they are not ideologically committed. In our model, competition is between groups of voters rather than between political parties. The equilibrium payoff of a group is increasing in its likelihood of casting the decisive vote in the election. This is much in the spirit of probabilistic voting where groups of voters are rewarded according to how sensitive their vote decisions are at the margin. From a theoretical point of view, it is also interesting to notice that

---

[15]We are grateful to an anonymous referee for pointing out the link to Olson's work.

under turnout uncertainty, an equilibrium exists in our model under mild conditions on the political cost function. Existence of equilibria in the probabilistic voting model requires that voters' utility functions are sufficiently concave (see, e.g., Lindbeck and Weibull (1987, Theorem 2) or Lin et al. (1999)). While the utility functions in our application satisfy this condition, existence of equilibrium does not require this in our model. Another difference between the probabilistic voting model and our model concerns the ability of politicians to commit to platforms. The probabilistic voting model assumes that such commitment is possible; our model, building on the work of Barro (1973) and Ferejohn (1986), assumes that no such commitment is possible. It would be interesting in future work to try and bridge the two approaches. This would provide a more active role for challengers in our model and for a more realistic representation of political competition.

An important insight of the model is that turnout uncertainty limits redistribution and protects the minority. Standard political economy explanations for why the majority (typically the poor with income below the average) does not expropriate the wealth of the minority (typically the rich with income above the average) are based on two main ideas. Firstly, taxation is distortionary. This makes it too costly for the majority to demand complete equalization of after-tax income (Richard and Meltzer, 1981; Winer and Rutherford, 1993). Secondly, the rich can organize pressure groups and protect themselves that way against high taxes (Becker, 1983; Aidt, 2003). As an alternative to this, Roemer (1995) demonstrates that two-party competition can limit redistribution. This happens if the policy space has two dimensions and voters care sufficiently about a non-economic issue such as religion or race. The idea is intuitive: the party representing the poor, which in the absence of the non-economic issue would propose a tax rate of one, can enhance the welfare of its constituency by attracting votes from amongst those rich who care sufficiently about its position on the non-economic issue. It does so by proposing a more lenient tax policy. Finally, Corneo and Gruner (2000) provide a sociological explanation. They argue that social status is positively correlated with income, and appeal to the idea that fear of losing social status as a result of less income inequality might induce middle class voters to block redistributive policies that expropriate the wealth of the rich. Our model demonstrates that turnout uncertainty can provide an alternative answer to the puzzle of why redistribution is limited in a democracy.

With the possible exception of Becker (1983), none of the above explanations apply to the case with very small minorities, such as the Basarwa ethnic group in Botswana, First nations people in Canada, Blacks in the USA, and the Romas in Romania. Instead, they provide alternative explanations why the 51% majority does not expropriate the 49% minority. Our theory is no exception to this. It has little to say about why very small groups get protection in a democracy: it is simply so unlikely that these groups ever hold the majority among those who turn out to vote that politicians rationally offer them very limited protection.

Randomness in turnouts plays a positive role in our model and might be socially desirable. This has an interesting implication for the design of voting systems. In some countries, including Australia, voting is compulsory. The aim of this policy is to increase average turnout and to ensure that all citizens are participating in the political process. However, it has the downside, which is very clear from Table 1, of reducing turnout uncertainty. This makes "partisan outcomes" rather than "consensus outcomes" more likely. Thus, our analysis suggest a new trade off:

compulsory voting guarantees high average turnout, but eliminates turnout uncertainty.

REFERENCES

Aidt, Toke S. (2000). "Economic Voting and Information". *Electoral Studies* 19: 349-262.

Aidt, Toke S. (2003). "Redistribution and Deadweight Costs: The Role of Political Competition". *European Journal of Political Economy* 19(2): 205-226.

Aidt, Toke S. and Jayasri Dutta (2004). "Strategic Consensus". *Journal of Mathematical Economics* 40: 227-245.

Aidt, Toke S. and Francesco Magris, (2006). "Capital Taxation and Electoral Accountability". *European Journal of Political Economy* 22: 277-291.

Barro, Robert J. (1973). "The Control of Politicians: An Economic Model". *Public Choice* 14: 19-42.

Becker, Gerry S. (1983). "A Theory of Competition Among Pressure Groups for Political Influence". *Quarterly Journal of Economics* 98(3): 371-400.

Coate, Stephen and Stephen Morris (1999). "Policy Persistence". *American Economic Review* 89, 1327-1336.

Corneo, Giacomo and Hans-Peter Gruner (2000). "Social Limits to Redistribution". *American Economic Review* 90: 1491-1507.

Dhillon, Amrita and Susana Peralta (2002). "Economic Theories of Voter Turnout". *Economic Journal* 112: F332-F352.

Dixit, Avinash and John Londregan (1996). "The Determinants of Success of Special Interests in Redistributive Politics". *Journal of Politics* 58: 1132-1155.

Ferejohn, John (1986). "Incumbent Performance and Electoral Control". *Public Choice* 50(1-3): 5-25.

Hettich, Walter and Stanley L. Winer (1988). "Economic and Political Foundations of Tax Structure". *American Economic Review* 78: 701-12.

Hettich, Walter and Stanley L. Winer (1999). *Democratic Choice and Taxation: A Theoretical and Empirical Analysis*. Cambridge University Press.

IDEA (International Institute for Democracy and Electoral Assistance) (2001). http://www.idea.int/voter_turnout.

Lin, Tse-min, James M. Enelow, and Han Dorussen (1999). "Equilibrium in Multi-candidate Probabilistic Spatial Voting". *Public Choice* 98: 59-82.

Lindbeck, Assar and Jörgen W. Weibull (1987). "Balanced Budget Redistribution as the Outcome of Political Competition". *Public Choice* 52: 272-297.

Lizzeri, Alessandro and Nicola Persico (2001). "The Provision of Public Goods under Alternative Electoral Incentives". *American Economic Review* 91: 225-239.

Meltzer, Allan H. and Scott F. Richard (1981). "A Rational Theory of the Size of Government". *Journal of Political Economy* 89: 914-927.

Mueller, Dennis C. (2003). *Public Choice III*. Cambridge University Press.

Norris, Peter. (2001). "Women's Power at the Ballot Box". In IDEA (International Institute for Democracy and Electoral Assistance) *Voter Turnout – a Global Survey*. http://www.idea.int/voter_turnout.

Olson, Mancur (1965). *The Logic of Collective Action*. Harvard University Press.

Persson, Torsten, Gerard Roland and Guido Tabellini (1997). "Separation of Powers and Political Accountability". *Quarterly Journal of Economics* 112: 1163-1202.

Persson, Torsten and Guido Tabellini (2000). *Political Economics: Explaining Economic Policy*. The MIT Press.

Persson, Torsten and Guido Tabellini (2003). *The Economic Effects of Constitutions.* The MIT Press.

Roemer, John E. (1998). "Why the Poor do not Expropriate the Rich: An Old Argument in New Garb". *Journal of Public Economics* 70: 399-424.

Winer, Stanley L. and Thomas Rutherford (1993). "Coercive Redistribution and the Franchise. A Preliminary Investigation Using Computable General Equilibrium Modelling". Albert Breton, Gianluigi Galeotti, Poerre Salmon, Ronald Wintrobe (editors). *Preferences and Democracy*. Kluwer, p. 351-375.

ACKNOWLEDGEMENTS

Aidt: Faculty of Economics, University of Cambridge, U.K. (tsa23@econ.cam.ac.uk)

Dutta: Department of Economics, University of Birmingham, U.K. (j.dutta@bham.ac.uk)

We thank Robert Evans, Paola Profeta, Dalibor Eterovic, Stanley Winer, Vania Sena and the participants in seminars at University of Birmingham, ETH Zurich, CEPR, and PET 07 for useful comments and suggestions. We are also grateful for the constructive feedback provided by two anonymous reviewers.

APPENDIX

The political cost functions

To simplify notation, we omit all time subscripts. Write the cost to the politician if k is invested in the public good, τ_1 is transferred to group 1 and τ_2 is transferred to group 2 as

$$c(k,\tau_1,\tau_2) = k + n_1\tau_1 + n_2\tau_2.$$

Let $x = \{x_1, x_2\}$ be the utility standards announced by voters. The least cost of satisfying both standards is the solution to the following problem:

$$C(x_1,x_2) = \min_{k\geq 0;\tau_1\geq 0;\tau_2\geq 0} c(k,\tau_1,\tau_2)$$

subject to $Ak + \theta_1\tau_1 \geq x_1$, $Ak + \theta_2\tau_2 \geq x_2$ and the public budget constraint. Similarly, we can define the least cost of providing utility levels satisfying one of the standards only as

$$C_i(x_i) = \min_{k\geq 0,\tau_i\geq 0} c(k,\tau_i,0)$$

subject to $Ak + \theta_i\tau_i \geq x_i$ and the public budget constraint. It is clear that feasibility requires that $C(x_1,x_2) \leq T$ and

$C_i(x_i) \leq T$. The solutions to these problems depend on the size of A relative to n_1, n_2, n, θ_1 and θ_2. Consider first the derivation of $C(x_1, x_2)$. Logically there are five ways in which the politician can provide utility to the two groups. The case with $g > 0$, $\tau_1 > 0$ and $\tau_2 > 0$ can, however, be ruled out immediately because of the linear production technology. If the politician wants to target transfers at both groups, it most be cheaper to do so than providing any public goods at all. If, on the other hand, the politician wants to provide public goods, it must be cheaper to satisfy the demands of at least one group completely with public goods. This leaves us with four cases to consider:

1. $g = 0$, $\tau_1 > 0$ and $\tau_2 > 0$ with costs

$$C(1) = \frac{n_1}{\theta_1} x_1 + \frac{n_2}{\theta_2} x_2 .$$

2. $g > 0$, $\tau_1 = 0$ and $\tau_2 \geq 0$ with costs

$$C(2) = \frac{x_1}{A} + \frac{n_2}{\theta_2}(x_2 - x_1) \text{ for } x_2 \geq x_1 .$$

3. $g > 0$, $\tau_1 \geq 0$ and $\tau_2 = 0$ with costs

$$C(3) = \frac{x_2}{A} + \frac{n_1}{\theta_1}(x_1 - x_2) \text{ for } x_1 \geq x_2 .$$

4. $g > 0$, $\tau_1 = 0$ and $\tau_2 = 0$ with costs

$$C(4) = \frac{\max\{x_1, x_2\}}{A} .$$

Note that $C(2) = C(3) = C(4)$ if $x_1 = x_2$. Suppose that $x_1 \geq x_2$. Then, we get

$$(A1) \quad C(1) \leq (<) C(3) \Leftrightarrow A \leq (<) \left(\frac{n_1}{\theta_1} + \frac{n_2}{\theta_2} \right)^{-1} ,$$

$$(A2) \quad C(3) \le (<) C(4) \Leftrightarrow A \le (<) \frac{\theta_1}{n_1}.$$

Suppose that $x_2 \ge x_1$. Then, we get

$$(A3) \quad C(1) \le (<) C(2) \Leftrightarrow A \le (<) \left(\frac{n_1}{\theta_1} + \frac{n_2}{\theta_2} \right)^{-1},$$

$$(A4) \quad C(2) \le (<) C(4) \Leftrightarrow A \le (<) \frac{\theta_2}{n_2}.$$

Note that $\left(\frac{n_1}{\theta_1} + \frac{n_2}{\theta_2} \right)^{-1} < \min\{\frac{\theta_1}{n_1}, \frac{\theta_2}{n_2}\}$. We can now derive the cost function for the five cases stated in the text.

1. Let $A < \left(\frac{n_1}{\theta_1} + \frac{n_2}{\theta_2} \right)^{-1}$. It follows from $(A1)-(A4)$ that

$$C(1) < C(3) < C(4) \text{ for } x_1 \ge x_2,$$

$$C(1) < C(2) < C(4) \text{ for } x_2 > x_1.$$

Hence,

$$C(x_1, x_2) = \frac{n_1}{\theta_1} x_1 + \frac{n_2}{\theta_2} x_2.$$

2. Let $A \in [\left(\frac{n_1}{\theta_1} + \frac{n_2}{\theta_2} \right)^{-1}, \min\{\frac{\theta_1}{n_1}, \frac{\theta_2}{n_2}\})$. It follows from $(A1)-(A4)$ that

$$C(3) \le C(1) \text{ and } C(3) < C(4) \text{ for } x_1 \ge x_2,$$
$$C(2) \le C(1) \text{ and } C(2) < C(4) \text{ for } x_2 > x_1.$$

Hence, defining $x^{\min} = \min\{x_1, x_2\}$, we can write

$$C(x_1, x_2) = \frac{x^{\min}}{A} + \frac{n_2}{\theta_2}(x_2 - x^{\min}) + \frac{n_1}{\theta_1}(x_1 - x^{\min}).$$

3. Let $A \in [\min\{\frac{\theta_1}{n_1}, \frac{\theta_2}{n_2}\}, \max\{\frac{\theta_1}{n_1}, \frac{\theta_2}{n_2}\})$. We need to consider two sub-cases. Suppose that $\frac{\theta_1}{n_1} < \frac{\theta_2}{n_2}$. It follows from $(A1)-(A4)$ that

$$C(4) \leq C(3) < C(1) \text{ for } x_1 \geq x_2,$$
$$C(2) < C(1) \text{ and } C(2) < C(4) \text{ for } x_2 > x_1.$$

Hence, we get

$$C(x_1, x_2) = \frac{x_1}{A} + \frac{n_2}{\theta_2}(\max\{x_2 - x_1, 0\}).$$

Suppose that $\frac{\theta_1}{n_1} \geq \frac{\theta_2}{n_2}$. It follows from $(A1)-(A4)$ that

$$C(4) \leq C(2) < C(1) \text{ for } x_2 \geq x_1,$$
$$C(3) < C(1) \text{ and } C(2) < C(4) \text{ for } x_1 > x_2.$$

Hence, we get

$$C(x_1, x_2) = \frac{x_2}{A} + \frac{n_1}{\theta_1}(\max\{x_1 - x_2, 0\}).$$

4. Let $A \geq \max\{\frac{\theta_1}{n_1}, \frac{\theta_2}{n_2}\}$. It follows from $(A1)-(A4)$ that

$$C(4) < C(3) < C(1) \text{ for } x_1 \geq x_2,$$

$$C(4) \leq C(2) < C(1) \text{ for } x_2 > x_1.$$

Hence, we get

$$C(x_1, x_2) = \frac{\max\{x_1, x_2\}}{A}.$$

To derive $C(x_i)$, we note that the relevant cases are

1. $g = 0$ and $\tau_i > 0$ with costs $C(1) = \frac{n_i}{\theta_i} x_i$.

2. $g > 0$ and $\tau_t = 0$ with costs $C(2) = \frac{x_i}{A}$.

It follows that $C(x_i)=\frac{n_i}{\theta_i}x_i$ for $A<\frac{\theta_i}{n_i}$ and that $C(x_i)=\frac{x_i}{A}$ for $A\geq\frac{\theta_i}{n_i}$.

Equilibria

Combining proposition 1 with the political cost function derived above, we can show the following.

1. Let $A<\left(\frac{n_1}{\theta_1}+\frac{n_2}{\theta_2}\right)^{-1}$. The political cost function is additive. It follows from proposition 1 that the equilibrium allocation is unique and given by $u_{1t}=\theta_1\tau_{1t}=\frac{\theta_1 p\beta T}{n_1}$, $u_{2t}=\theta_2\tau_{2t}=\frac{\theta_2(1-p)\beta T}{n_2}$ and $u_{0t}=(1-\beta)T$ for all t.

2. Let $A\in[\left(\frac{n_1}{\theta_1}+\frac{n_2}{\theta_2}\right)^{-1},\min\{\frac{\theta_1}{n_1},\frac{\theta_2}{n_2}\})$. The political cost function is strictly sub-additive. Proposition 1 then implies that there exist many possible equilibrium paths, but along all of these the politician wants to satisfy the standard of the least demanding group (which can be either group depending on which equilibrium is played) by public goods and then top up the utility of the other with transfers. This implies that some public goods are always provided and that at most one group receives transfers. The politician always gets $u_{0t}=(1-\beta)T$ for all t.

3. Let $A\in[\min\{\frac{\theta_1}{n_1},\frac{\theta_2}{n_2}\},\max\{\frac{\theta_1}{n_1},\frac{\theta_2}{n_2}\})$. The political cost function is strictly sub-additive and provision levels and the size of the transfer vary across equilibria. The direction of the transfer, if any, is, however, uniquely determined: it

goes towards the group with the lowest $\frac{\theta_i}{n_i}$. Notice in the absence of taste heterogeneity ($\theta_1 = \theta_2$), the transfer, if any, always goes to the minority. The politician always gets $u_{0t} = (1-\beta)T$ for all t.

4. Let $A \geq \max\{\frac{\theta_1}{n_1}, \frac{\theta_2}{n_2}\}$. Since the productivity of the public sector is high enough that all demands are met by public goods rather than by transfers. Although the cost function is (strictly) sub-additive, all equilibrium paths generate the same utility allocation, namely $g_t = u_{1t} = u_{2t} = A\beta T$ and $u_{0t} = (1-\beta)T$ for all t.[16] A politician who wants to meet the standard of one group will automatically provide (some) utility to the other. An implication, then, is that the utility allocation is independent of p.

[16]This outcome can be supported by many different performance standards.