The Rise of Environmentalism, Pollution Taxes and Intra-industry Trade

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Appendix I

This appendix shows how production patterns, emission patterns, profits, consumer's surplus and tax revenue are affected by a change in t_A. From the firms' first order conditions, we get for the market in country A:

$$F_{AA}(x_{AA}, x_{BA}, t_A, t_B) = p'_A x_{AA} + p_A - c_A - t_A = 0 \tag{1}$$

$$F_{BA}(x_{AA}, x_{BA}, t_A, t_B) = p'_A x_{BA} + p_A - c_B - t_B = 0. \tag{2}$$

The Implicit Function Theorem states that $F_{AA} = 0$ and $F_{BA} = 0$ implicitly define continuous functions

$$x_{AA} = x_{AA}(t_A, t_B) \tag{3}$$

$$x_{BA} = x_{BA}(t_A, t_B) \tag{4}$$

and that these functions have continuous partial derivative if F_{AA} and F_{BA} have continuous partial derivatives and the following Jacobian determinant is non-zero:

$$\begin{aligned} \Delta_A &= |J_A| = \begin{vmatrix} p''_A x_{AA} + 2p'_A & p''_A x_{AA} + p'_A \\ p''_A x_{BA} + p'_A & p''_A x_{BA} + 2p'_A \end{vmatrix} \\ &= (p''_A x_{AA} + 2p'_A)(p''_A x_{BA} + 2p'_A) - (p''_A x_{AA} + p'_A)(p''_A x_{BA} + p'_A) \\ &= p'_A \left[(p''_A x_{BA} + p'_A) + (p''_A x_{AA} + 2p'_A) \right]. \end{aligned} \tag{5}$$

It follows from the second order conditions ($p''_A x_{AA} + 2p'_A < 0$) and the assumption that quantities are strategic substitutes ($p''_A x_{BA} + p'_A < 0$) that $\Delta_A > 0$.

1) Total differentiate equations (1) and (2) and rewrite in matrix notation:

$$J_A \begin{pmatrix} \frac{\partial x_{AA}}{\partial t_A} \\ \frac{\partial x_{BA}}{\partial t_A} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{6}$$

$$J_A \begin{pmatrix} \frac{\partial x_{AA}}{\partial t_B} \\ \frac{\partial x_{BA}}{\partial t_B} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{7}$$

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Using Cramer's rule, we find that

$$\frac{\partial x_{AA}}{\partial t_A} = \frac{(p''_A x_{BA} + 2p'_A)}{\Delta_A} < 0 \tag{8}$$

$$\frac{\partial x_{BA}}{\partial t_A} = -\frac{(p''_A x_{BA} + p'_A)}{\Delta_A} > 0 \tag{9}$$

Total supply to market A is $x_A = x_{AA} + x_{BA}$ and

$$\frac{\partial x_A}{\partial t_A} = \frac{-(p''_A x_{BA} + p'_A) + (p''_A x_{BA} + 2p'_A)}{\Delta_A} = \frac{p'_A}{\Delta_A} < 0 \tag{10}$$

Define $\Phi_{BA}(t_{A,}t_B) = p''_A x_{BA} + 2p'_A$ and $\overline{\Phi}_{BA}(t_{A,}t_B) = p''_A x_{BA} + p'_A$. Take the partial derivatives:

$$\frac{\partial \Phi_{BA}}{\partial t_A} = \frac{p'''_A x_{BA} p'_A}{\Delta_A} + \frac{p''_A}{\Delta_A}\left(-p''_A x_{BA} + p'_A\right) \tag{11}$$

$$\frac{\partial \overline{\Phi}_{BA}}{\partial t_A} = \frac{p'''_A x_{BA} p'_A}{\Delta_A} - \frac{p''_A}{\Delta_A}\left(p''_A x_{BA}\right) \tag{12}$$

Moreover, notice that

$$\frac{\partial \Delta_A}{\partial t_A} = p'''_A \frac{(p'_A)^2}{\Delta_A}(x_{AA} + x_{BA}) + p''_A \frac{\Delta_A + 4(p'_A)^2}{\Delta_A} \tag{13}$$

Unless we put restrictions on p'''_A and p''_A, the sign of these derivatives is ambiguous. Therefore, it is not surprising that the effect of a change in t_A on marginal production is ambiguous as well. Assuming that $p''' = 0$, the two (relevant) second derivatives of $x_{AA}(t_A, t_B)$ and $x_{BA}(t_A, t_B)$ are:

$$\begin{aligned} \frac{\partial^2 x_{AA}}{\partial t_A^2} \quad | \quad {}_{p'''_A=0} &= \frac{\Delta_A \frac{\partial \Phi_{BA}}{\partial t_A} - \Phi_{BA} \frac{\partial \Delta_A}{\partial t_A}}{\Delta_A^2} \\ &= -\frac{p'_A p''_A}{\Delta_A^3}\left\{\left(1 + \frac{2p''_A}{p'_A}\right)\Delta_A + 4p'_A \Phi_{BA}\right\} \end{aligned} \tag{14}$$

$$\begin{aligned} \frac{\partial^2 x_{BA}}{\partial t_A^2} \quad | \quad {}_{p'''_A=0} &= -\frac{\Delta_A \frac{\partial \overline{\Phi}_{BA}}{\partial t_A} - \overline{\Phi}_{BA} \frac{\partial \Delta_A}{\partial t_A}}{\Delta_A^2} \\ &= -\frac{p''_A}{\Delta_A^3}\left\{p''_A x_{BA}(2\Delta_A + 4\left(p'_A\right)^2) + p'_A(\Delta_A + 4(p'_A)^2)\right\} \end{aligned} \tag{15}$$

If $p''_A = 0$, the two second derivatives are zero.

2) Discharge of emission from the firm in country A is $e_A = x_{AA} + x_{AB}$ with

$$\frac{\partial e_A}{\partial t_A} = \frac{(p''_A x_{BA} + 2p'_A)}{\Delta_A} + \frac{(p''_B x_{BB} + 2p'_B)}{\Delta_B} < 0, \tag{16}$$

where $\Delta_B = p'_B \left[(p''_B x_{AB} + p'_B) + (p''_B x_{BB} + 2p'_B) \right] > 0$. Discharge of emission from firm B is $e_B = x_{BB} + x_{BA}$ with

$$\frac{\partial e_B}{\partial t_A} = -\frac{(p''_B x_{BB} + p'_B)}{\Delta_B} - \frac{(p''_A x_{BA} + p'_A)}{\Delta_A} > 0. \tag{17}$$

Total emission is $E = e_A + e_B$ with

$$\frac{\partial E}{\partial t_A} = \frac{p'_A}{\Delta_A} + \frac{p'_B}{\Delta_B} < 0. \tag{18}$$

3) The (total) profit function of firm A is:

$$\pi^*_A(.) = p_A(.)x_{AA}(.) + p_B(.)x_{AB}(.) - (c_A + t_A)x_A(.). \tag{19}$$

From the Envelope Theorem, we get:

$$\begin{aligned} \frac{\partial \pi_A}{\partial t_A} &= p'_A(.)\, x_{AA}(.) \frac{\partial x_{BA}}{\partial t_A}(.) + \\ & p'_B(.)\, x_{AB}(.) \frac{\partial x_{BB}}{\partial t_A}(.) - \\ & (x_{AA}(.) + x_{AB}(.)) < 0 \end{aligned} \tag{20}$$

Use equation (20) to calculate:

$$\begin{aligned} \frac{\partial^2 \pi_A}{\partial t_A^2} &= p''_A \left[\frac{\partial x_{AA}}{\partial t_A} + \frac{\partial x_{BA}}{\partial t_A} \right] x_{AA} \frac{\partial x_{BA}}{\partial t_A} + \\ & p''_B \left[\frac{\partial x_{BB}}{\partial t_A} + \frac{\partial x_{AB}}{\partial t_A} \right] x_{AB} \frac{\partial x_{BB}}{\partial t_A} + \\ & p'_A(.) \left[\frac{\partial x_{AA}}{\partial t_A} \frac{\partial x_{BA}}{\partial t_A} + x_{AA} \frac{\partial^2 x_{BA}}{\partial t_A^2} \right] + \\ & p'_B(.) \left[\frac{\partial x_{AB}}{\partial t_A} \frac{\partial x_{BB}}{\partial t_A} + x_{AB} \frac{\partial^2 x_{BB}}{\partial t_A^2} \right] - \\ & \left[\frac{\partial x_{AA}}{\partial t_A} + \frac{\partial x_{AB}}{\partial t_A} \right]. \end{aligned} \tag{21}$$

The sign of $\frac{\partial^2 \pi_A}{\partial t_A^2}$ is, in general, ambiguous. However, if $p''_A = p''_B = 0$, then $\frac{\partial^2 \pi_A}{\partial t_A^2} = -\frac{7}{9}(\frac{1}{p'_A} + \frac{1}{p'_B}) > 0$.

4) Consumer's surplus for consumers in country A can be written as $u\,(x_{AA}(.) + x_{BA}(.)) - p_A\,(x_{AA}(.) + x_{BA}(.))\,[x_{AA}(.) + x_{BA}(.)]$. Using the fact that $p_A = \frac{\partial u}{\partial C_A}$, we get:

$$\frac{\partial CS_A}{\partial t_A} = -\left[x_{AA}(.) + x_{BA}(.) \right] p'_A(.) \left[\frac{\partial x_{AA}}{\partial t_A} + \frac{\partial x_{BA}}{\partial t_A} \right] < 0. \tag{22}$$

The second derivative is:

$$\begin{aligned}\frac{\partial^2 CS_A}{\partial t_A^2} &= -\left[[x_{AA}(.) + x_{BA}(.)]\, p_A^{''}(.) + p_A^{'}(.)\right]\left[\frac{\partial x_{AA}}{\partial t_A} + \frac{\partial x_{BA}}{\partial t_A}\right]^2 \\ &\quad + [x_{AA}(.) + x_{BA}(.)]\, p_A^{'}(.)\left[\frac{\partial^2 x_{AA}}{\partial t_A^2} + \frac{\partial^2 x_{BA}}{\partial t_A^2}\right]. \qquad (23)\end{aligned}$$

The sign of $\frac{\partial^2 CS_A}{\partial t_A^2}$ is, in general, ambiguous. However, if $p''_A = 0$, then $\frac{\partial^2 CS_A}{\partial t_A^2} = \frac{-1}{9p'_A} > 0$

5) The tax revenue collected in country A is $R_A = t_A e_A$ and

$$\frac{\partial R_A}{\partial t_A} = t_A \frac{\partial e_A}{\partial t_A} + e_A \qquad (24)$$

The second derivative is:

$$\frac{\partial^2 R_A}{\partial t_A^2} = t_A \frac{\partial^2 e_A}{\partial t_A^2} + 2\frac{\partial e_A}{\partial t_A} \qquad (25)$$

If $p''_A = 0$, then $\frac{\partial^2 R_A}{\partial t_A^2} = \frac{4}{3}(\frac{1}{p'_A} + \frac{1}{p'_B}) < 0$.

Appendix II

In this appendix, we analyze the second order condition associated with

$$\theta_A \frac{\partial W_A}{\partial t_A} + \frac{\gamma_A^P}{\overline{\gamma}_A^P}\frac{\partial \pi_A}{\partial t_A} - \gamma_A^E(\beta_A E_A \frac{\partial E_A}{\partial t_A} + \delta_A E_B \frac{\partial E_B}{\partial t_A}) = 0, \qquad (26)$$

where

$$\frac{\partial W_A}{\partial t_A}(.) = \theta_A(\frac{\partial CS_A}{\partial t_A} + \frac{\partial R_A}{\partial t_A} + \frac{\partial \pi_A}{\partial t_A} - \beta_A E_A \frac{\partial E_A}{\partial t_A} - \overline{\gamma}_A^E \delta_A E_B \frac{\partial E_B}{\partial t_A}). \qquad (27)$$

Calculate:

$$\begin{aligned}\frac{\partial \Lambda_A(.)}{\partial t_A} &= \theta_A \frac{\partial^2 R_A(.)}{\partial t_A^2} + \theta_A \frac{\partial^2 CS_A(.)}{\partial t_A^2} + (\theta_A + \gamma_A^P)\frac{\partial^2 \pi_A(.)}{\partial t_A^2} \qquad (28) \\ &\quad -(\theta_A + \gamma_A^E)\left(\beta_A \left(\frac{\partial E_A}{\partial t_A}\right)^2 + \beta_A E_A \frac{\partial^2 E_A}{\partial t_A^2}\right) \qquad (29) \\ &\quad - \left(\theta_A \overline{\gamma}_A^E + \gamma_A^E\right)\left(\delta_A \left(\frac{\partial E_B}{\partial t_A}\right)^2 + \delta_A E_B \frac{\partial^2 E_B}{\partial t_A^2}\right)\end{aligned}$$

The sign of the derivative is, in general, ambiguous but $\frac{\partial \Lambda_A(.)}{\partial t_A} < 0$ if β_A and δ_A are sufficiently large. If $p''_A = p''_B = 0$ and $p'_A = p'_B = p'$, then

$$\begin{aligned}\frac{\partial \Lambda_A(.)}{\partial t_A} &= \frac{1}{p'}(\theta_A - \gamma_A^P \frac{14}{9}) \\ &-(\theta_A + \gamma_A^E)\left(\beta_A \left(\frac{2}{3}\frac{1}{p'}(2\alpha_{AA} - \alpha_{BA})\right)^2\right) \\ &- \left(\theta_A \overline{\gamma}_A^E + \gamma_A^E\right)\left(\delta_A \left(\frac{2}{3}\frac{1}{p'}(\alpha_{BB} - 2\alpha_{AB})\right)^2\right)\end{aligned} \tag{30}$$

which is surely negative if $(\theta_A > \frac{14}{9})$.