

Estimating Euler Equations with Noisy Data: Two Exact GMM Estimators.¹

Sule Alan²

Faculty of Economics
University of Cambridge
UK

Department of Economics
York University
Canada

Orazio Attanasio

Department fo Economics
University College London
UK

Martin Browning

Department of Economics
University of Oxford
UK

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²Corresponding author: sa479@cam.ac.uk, phone: +44 1223 335 283.

Abstract

In this paper we exploit the specific structure of the Euler equation and develop two alternative GMM estimators that deal explicitly with measurement error. The first estimator assumes that the measurement error is lognormally distributed. The second estimator drops the distributional assumption at the cost of less precision. Our Monte Carlo results suggest that both proposed estimators perform much better than conventional alternatives based on the exact Euler equation or its log-linear approximation, especially with short panels. An empirical application to the PSID yields plausible and precise estimates of the coefficient of relative risk aversion and the discount rate.

Keywords: Nonlinear models; Measurement error; Euler equation

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1 Introduction

Since Hall's (1978) seminal paper, the Euler equation for consumption derived from the intertemporal problem faced by a generic consumer has played an important role in applied consumption research ¹. The reasons are well known and have been discussed at length in many places. First, we can allow for differences in important, unobserved and slowly evolving variables such as lifetime wealth, retirement expectations, future fertility plans, bequest motives etc. Second, the Euler equation approach allows us to take into account wage shocks, employment shocks, health shocks and other shocks even if we only observe these imperfectly (or not at all) in the data. Finally, within this framework it is trivial (in the theory sense) to allow coherently for non-separabilities between market goods and other factors (such as demographics, housing, labour supply and that impact on the marginal utility of expenditures); see, for example, Attanasio and Browning (1995). All of this makes the Euler equation approach superbly fitted for the task of estimating some of the structural parameters of fully parametric structural models. Unfortunately, the relationships that are derived from all plausible preferences are highly non-linear. This creates a number of econometric problems, ranging from the performance of non-linear GMM estimators in small samples to the effect of measurement error (see Alan and Browning (2003) and Attanasio and Low (2004)). The latter problem cannot be credibly ignored when working with microeconomic data, (see, for example, Shapiro (1984), Altonji and Siow (1987) and Runkle (1991)).

Standard non-linear GMM estimation yields inconsistent estimates in the presence of (neglected) measurement error (see, for instance, Amemiya (1985)). One solution that has been followed in the literature has been to work with a log linearized version of the Euler equation, a solution first used by Hansen and Singleton (1982). Such a procedure has several advantages, ranging from the robustness to the presence of classical measurement error, to the fact that it

¹See Browning and Lusardi (1996) and Attanasio (1999).

yields a specification linear in parameters and is therefore amenable to estimation using pseudo panel methods such as those proposed by Browning, Deaton and Irish (1985). These advantages are, however, outweighed by several problems. First and foremost, there are many situations in which log linearizing the Euler equation does not give a specification linear in parameters and therefore one is again subject to the same problems mentioned above. Situations of this type include a variety of interesting models including, for instance, models with habit formation. Second, the ‘constant’ of a log-linearized Euler equation includes conditional higher moments of consumption growth and interest rate, so that one loses identification of the discount factor; an intertemporal allocation parameter that is of great importance especially in the macro-finance literature. Moreover, if such conditional moments are not constant, the error term will include innovations to these moments that might be correlated with the instruments that one typically use in estimating the Euler equation. Attanasio and Low (2004) show that with long panels we can consistently estimate the elasticity of intertemporal substitution using the log-linearized Euler equation, even when the income process is heteroskedastic but long panels (Attanasio and Low suggest about 40 periods) are rare. Consequently one would like to work with an estimator that is robust to these potential problems.²

In the wider measurement error literature, resolutions of the problem for nonlinear estimators have only been possible in particular circumstances (see Hausman et al (1991), Hausman et al (1995), Schennach (2004), Hausman (2001), Wansbeek (2001), Hong and Tamer (2003)). In this paper we exploit the specific structure of the Euler equation to propose two alternative GMM estimators that deal explicitly with measurement error. Both assume that the measurement error is ‘classical’ in the sense of being stationary and multiplicative (but not necessarily with a unit mean). For the first estimator we additionally assume that the measurement error is

²Alan and Browning (2003) propose an alternative non Euler equation procedure which avoids the need to model any driving processes. This requires parameterising and simulating the martingale that expectations errors follow. Attanasio and Browning (1995) propose a different approach to the problem which consists in starting with a flexible but log-linear in parameters specification for the marginal utility of consumption. One then has to integrate such a specification to obtain the corresponding utility function.

lognormally distributed. This leads to a GMM in which the conditional expectation of the discounted change in the marginal utility is equal to a constant which is not equal to unity but which depends on the preference parameters and the variance of the measurement error. Colera (1994) follows the same parametric approach and assumes a lognormally distributed serially independent measurement error process. While her GMM estimator identifies the coefficient of relative risk aversion, the discount factor and the measurement error variance are not separately identified. Our estimator does identify all of the parameters of interest. Our second estimator drops the distributional assumption and solves out for the unknown, but constant, conditional mean. Chioda (2004) follows a somewhat similar route; however, the GMM estimator she proposes does not identify the discount factor. The trade-off between our two estimators is the conventional one: the first estimator is more efficient if the errors are indeed lognormally distributed but the second estimator is more robust. We present the small sample behavior of our two GMM estimators as compared with the usual exact Euler equation GMM estimator and conventional log-linearized GMM.

On the theoretical side, the novelty of our estimators stems from the fact that they exploit the information on the relationship between two-period apart consumption as well as standard one-period apart consumption information. It is this particular feature of our estimators which makes it possible to identify the discount factor. It is important to note that although both Ventura(1994) and Chioda (2004) (two papers most relevant to our work) also use the specific structure of the consumption Euler equation neither of their estimators identify the discount factor. There is now considerable agreement that individual discount factors are a key determinant of intertemporal decisions such as asset accumulation (Venti and Wise ,1998 and Samwick 2006), labor supply decision (MaCurdy 1981) and job search (Dellavigna and Paserman 2005).

Our Monte Carlo results suggest that both proposed estimators perform much better than conventional alternatives based on the exact Euler equation or its log-linear approximation, especially with short panels. As the sampling period increases, the bias in the coefficient of

relative risk aversion parameter becomes smaller for both exact and log-linear Euler equation estimation whereas the bias in the discount factor does not respond to the panel length in the case of the exact Euler equation. The novel feature of our estimators in relation to two conventional estimators used in the literature is that they outperform them in estimating both the coefficient of relative risk aversion and the discount factor especially when the available panel length is short. Both proposed estimators capture the true values of the coefficient of relative risk aversion and the discount factor remarkably well even in small samples and the resulting sampling distributions have reasonable dispersions.

In the empirical section we present an application using the US Panel Study of Income Dynamics (PSID). Although it contains only food expenditure, the survey suits our purposes since its panel length is reasonably long and it gives us the opportunity to place our results in the literature³. The formal justification for using food expenditures is that in the within period utility function, food quantities are additively separable for other quantities. For many policy implications the relevant parameter is not the elasticity of intertemporal substitution for food but that for total consumption expenditure. The former, however, is not an uninteresting parameter. Browning and Crossley (2000), for instance, show that the elasticity of intertemporal substitution for a necessity (such as food) is lower than the elasticity for total consumption. Thus our estimates for the former provide a lower bound for the latter. Moreover, if the subutility functions are discounted over time at the same rate⁴ then we can recover estimates of the discount factor from observing only food expenditures. Our empirical study using the two proposed estimators yields statistically significant estimates of the coefficient of relative risk aversion of about 1.5. This value lies between the estimate obtained by the log-linearized GMM (2.25) and that obtained by the exact GMM (1.15). Furthermore, our first estimator yields a high and precisely estimated discount factor estimate (0.99) while the second estimator provides

³There is a substantial body of literature that explores consumer behavior by estimating Euler equations using the PSID food expenditures and assuming that utility is separable in food and other expenditures. Examples include Hall and Mishkin(1982), Shapiro (1984), Zeldes (1989), Runkle (1991) and Dynan (2000).

⁴A universal assumption since otherwise we lose the exponential discounting form.

a lower but less precise estimate of this parameter.

2 The Model and Proposed Estimators.

In this section we discuss the estimation by GMM of a non-linear Euler equation for consumption where the consumption data are contaminated by measurement error. We will assume that we have enough time series observations and variation in intertemporal prices to be able to estimate the parameters of interest, which include the elasticity of intertemporal substitution and the discount factor. For expositional purposes, we consider the simplest version of a standard life cycle model with uncertainty. We could easily consider more complicated settings. A consumer maximizes expected utility. Utility is intertemporally additive and the instantaneous utility function is of the iso-elastic form. Future utility is discounted at a rate $\beta < 1$. The individual consumer is not subject to liquidity constraints and has rational expectations. She can move resources over time by saving and borrowing in N assets with variable and stochastic returns. Her problem will then imply the following Euler equation for a generic asset:

$$E_t \left[\left(\frac{c_{t+1}^*}{c_t^*} \right)^{-\gamma} (1 + r_{t+1})\beta \right] = 1 \quad (1)$$

where c_t^* is consumption at time t , r_{t+1} the real interest rate between periods t and $t + 1$, β is the discount factor and γ the coefficient of relative risk aversion. Such an equation can be used to estimate the preference parameters β and γ , using nonlinear GMM. In order to implement GMM, we need a vector of instruments assumed to be included in the time- t information set of the consumer. In this particular example, in the absence of measurement error a constant and a single instrument (such as the lagged real rate or lagged consumption growth) would deliver identification. For our discussion it is useful to express 1 as follows:

$$\left(\frac{c_{t+1}^*}{c_t^*} \right)^{-\gamma} (1 + r_{t+1})\beta = \varepsilon_{t+1} \quad (2)$$

where ε_{t+1} is an expectational error uncorrelated with the information available at time t and, by definition, has unit conditional mean: $E_t(\varepsilon_{t+1}) = 1$.

Now suppose that observed consumption in time t , c_t^o , is observed subject to a multiplicative error: $c_t^o = c_t^* \eta_t$. The multiplicative assumption is a critical assumption for identification. We also assume the following.

Assumption 1: *For any household, the measurement error is stationary and independent of ‘everything’ (including lagged values of the measurement error, expectations errors, consumption levels and interest rates).*

Although the independence assumption is standard in the measurement error literature (for example, in the Euler equation literature, see Altonji and Siow 1987, Runkle 1991 and Dynan 2000), it is questionable. For example, it may be that poorer households have a clearer idea of their usual food expenditures than wealthy ones; in that case the measurement error will depend on the level of consumption. The stationarity assumption could also be problematic. For example, respondents may report food expenditures with more accuracy the longer they have been in the sample. Another possible source of non-stationarity is that accuracy may depend on time varying factors such as family size. Nonetheless, our assumptions are weaker than those usually employed. For example, we do not impose that the distribution has a unit mean nor that it is the same across households. For one of our estimators we need to strengthen this assumption to:

Assumption 2: *For any household, the measurement error is log Normal with the same variance across households and independent of ‘everything’.*

This is, of course, considerably restrictive as compared to 2 but note that it still allows for household specific means.⁵

If we write equation 2 in terms of observed consumption we have:

⁵As a referee pointed out to us, we could weaken this to allow for functional dependence of the measurement error variance on observables such as family size, education and age.

$$\left(\frac{c_{t+1}^o}{c_t^o}\right)^{-\gamma} (1+r_{t+1})\beta = \varepsilon_{t+1} \left(\frac{\eta_{t+1}}{\eta_t}\right)^{-\gamma} \quad (3)$$

Assumption 1 gives the following result for conditional expectations:

$$E_t \left(\varepsilon_{t+1} \left(\frac{\eta_{t+1}}{\eta_t} \right)^{-\gamma} \right) = E_t(\varepsilon_{t+1}) E_t \left(\left(\frac{\eta_{t+1}}{\eta_t} \right)^{-\gamma} \right) = E_t \left(\left(\frac{\eta_{t+1}}{\eta_t} \right)^{-\gamma} \right) = E \left(\left(\frac{\eta_{t+1}}{\eta_t} \right)^{-\gamma} \right) = \kappa \quad (4)$$

where the expectation is taken conditional on the time t information available to the econometrician. The assumption that measurement error is independent of the expectational error can be used to justify the first equality in equation 4. The stationarity of the measurement error distribution gives that the conditional expectation of the measurement error does not depend on t . Equation (4) implies that applying GMM to equation (3) ignoring measurement error would yield inconsistent estimates of the discount factor⁶. If the measurement error is constant ($\eta_t = \eta, \forall t$) then $\kappa = 1$. The exact value of κ depends on the distribution of the measurement error and the (unknown) value of γ . If, for instance, we assume that the measurement error is log-normal, that is $\ln \eta_t \sim N(\mu, \nu)$, then we have:

$$\kappa = E \left(\left(\frac{\eta_{t+1}}{\eta_t} \right)^{-\gamma} \right) = \exp\{\gamma^2 \nu\} \quad (5)$$

Notice that the mean of $\ln(\eta)$ does not enter this expression so that for this and subsequent results we do *not* have to assume that the measurement error has a unit mean.

In addition to the Euler equation (1) we can also consider the Euler equation which links consumption at t and at $t + 2$. Following the same steps used to derive equation (3), we can obtain:

⁶Here, γ is not asymptotically biased because it is not identified from the mean of the moment, but from covariances that do not depend on the mean. However, in the finite sample, measurement error can cause significant bias in this parameter. The proposed estimators in this paper address this issue as well as getting the discount factor right. We are grateful to a referee for pointing this out.

$$\left(\frac{c_{t+2}^o}{c_t^o}\right)^{-\gamma} (1+r_{t+1})(1+r_{t+2})\beta^2 = \varepsilon_{t+1}\varepsilon_{t+2} \left(\frac{\eta_{t+2}}{\eta_t}\right)^{-\gamma} \quad (6)$$

The assumption of a stationary measurement error implies that:

$$E\left(\left(\frac{\eta_{t+2}}{\eta_t}\right)^{-\gamma}\right) = \kappa \quad (7)$$

Under the assumption of lognormality this term is also equal to $\exp\{\gamma^2\nu\}$. Now, let's define

u_{t+1}^1 and u_{t+2}^2 as follows:

$$\begin{aligned} u_{t+1}^1 &= \left[\left(\frac{c_{t+1}^o}{c_t^o}\right)^{-\gamma} (1+r_{t+1})\beta - \kappa \right] \\ u_{t+2}^2 &= \left[\left(\frac{c_{t+2}^o}{c_t^o}\right)^{-\gamma} (1+r_{t+1})(1+r_{t+2})\beta^2 - \kappa \right] \end{aligned} \quad (8)$$

If we assumed that κ is constant across households then we could use these equations to estimate the parameters $\{\beta, \gamma, \kappa\}$ directly by GMM.⁷ However, the parameter γ enters κ in an unknown way so that a homogeneity assumption is a mixture of implicit assumptions on how measurement error distributions vary across households and how this depends on the γ parameter. Instead we prefer to work with known assumptions on primitives and derive two estimators based on Assumptions 2 and 1 respectively.

For the first estimator we invoke Assumption 2 that the errors are lognormally distributed with the same variance for all households. Then κ is given by (5) and we have:

⁷We are grateful to a referee for pointing this out to us.

$$\begin{aligned}
u_{t+1}^1 &= \left[\left(\frac{c_{t+1}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})\beta - \exp\{\gamma^2\nu\} \right] \\
u_{t+2}^2 &= \left[\left(\frac{c_{t+2}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})(1 + r_{t+2})\beta^2 - \exp\{\gamma^2\nu\} \right]
\end{aligned} \tag{9}$$

A constant and a single instrument (for instance a suitably lagged value of the interest rate) would therefore deliver four orthogonality conditions that would over-identify the three parameters of the model $\{\beta, \gamma, \nu\}$. We refer to this estimator as the *GMM-LN* estimator.

Our second estimator, uses the two equation in (8) to difference out κ and obtain orthogonality conditions that involve only β and γ . In this case we only need the assumptions in Assumption 1. Differencing the two equations in 8 gives:

$$\zeta_{t+2} = \left[\left(\frac{c_{t+1}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})\beta \right] - \left[\left(\frac{c_{t+2}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})(1 + r_{t+2})\beta^2 \right] \tag{10}$$

From (8) is clear that ζ_{t+2} has zero mean and is uncorrelated with consumption information available at time $t - 1$ and other information dated t . This residual term can then be used to form orthogonality conditions to identify the parameters of the model. Notice that equation (10) does not depend on the parameters of the measurement error process. We refer to this estimator as *GMM-D*. As equation (10) is a form of double-differencing, we expect a significant decline in precision, as compared to the GMM-LN estimator.

If we assume that we have a homogeneous measurement error distribution then our two estimators are quite similar. In particular, for the just identified case our GMM-D estimator and GMM-LN estimator are the same when κ is set to give the second equation in GMM-LN (9). In the overidentified case, continuously updated GMM would also give the same results asymptotically. However, this equivalence may not be observed in the finite sample; the next section presents some results. Moreover, the equivalence breaks down if one assumes heterogeneity in

measurement error distributions across households.

Before turning to small sample properties of our estimators it is useful to compare them to those proposed by Chioda (2004) and Ventura (1994). Chioda's first estimator assumes that the expectation conditional on the set of information prior to time t is equal to an unknown constant for any given household and identifies the coefficient of relative risk aversion by first differencing two adjacent conditional moment conditions. The estimator yields the following orthogonality condition:

$$\lambda_{t+1} = \left(\frac{c_{t+1}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})\beta - \left(\frac{c_t^o}{c_{t-1}^o} \right)^{-\gamma} (1 + r_t)\beta \quad (11)$$

where λ_{t+1} has zero mean and is uncorrelated with the information set at time $t - 2$. The lack of identification for the discount factor is clear from the above equation. Her second estimator further assumes that the measurement error is identically distributed across individuals and uses the fact that exact Euler equation constitutes a valid moment condition even in the presence of measurement error as long as instruments are chosen wisely. The idea is that the GMM on exact Euler equation yields asymptotically unbiased γ estimates since its identification does not come from the mean of the moment. It should be noted however that the situation could be very different in the small sample as we demonstrate in the next section. Also, note that the discount factor is still not identified (as we also show in the next section). The estimator proposed by Ventura (1994) is our GMM-LN estimator without the second equation (the two period apart Euler equation):

$$u_{t+1}^1 = \left[\left(\frac{c_{t+1}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})\beta - \exp\{\gamma^2 \nu\} \right] \quad (12)$$

Therefore, even though she assumes a lognormally distributed measurement error as we do for our first estimator, her estimator cannot identify β and ν separately.

In this section we have developed two GMM estimators for the consumption Euler equation

if consumption is contaminated by stationary multiplicative measurement error. The first estimator, GMM-LN (see (8)), makes use of the lognormality assumption. The second, GMM-D (see (10)), does not require any supplementary assumptions. We have also described alternative estimators proposed by Ventura (1994) and Chioda (2004) in the same context. We turn now to the small sample properties of four estimators: the linearized (approximate) GMM equation (AGMM), the exact GMM (EGMM, with no allowance for measurement error), GMM-LN and GMM-D.

3 Small Sample Results.

3.1 Baseline results.

The simulated data used in our experiments are generated by solving a standard intertemporal utility maximization problem under interest rate and labor income uncertainty. The details of the model and its solution are given in the Appendix. The parameter values for the coefficient of relative risk aversion and the discount factor are set to 4 and 0.95 respectively. The interest rate series for each agent is a stationary $AR(1)$ process with a mean of 0.05 and an AR coefficient of 0.6; thus the discount factor multiplied by the mean gross interest rate is very close to unity. We solve the model for 100 periods and drop the initial 19 and final 18 observations to avoid starting and ending effects. For periods 20 to 81 the optimal consumption functions are all very similar and consumption growth is effectively stationary. To construct the data used in the experiments below we first generate consumption paths using the optimal program and then add multiplicative lognormal measurement error. The measurement error has a unit mean and a variance of 0.004. The latter is chosen so that 75% of the period to period variance in consumption growth is due to noise. This is at the upper end of estimates of the amount of noise in the real world data.

The experiments are performed to obtain the sampling distribution of the estimators devel-

oped in the previous section. For comparison, we also document the sampling distributions of conventional estimators based on the exact nonlinear Euler equation (EGMM) and its log-linear approximation (AGMM)⁸. We experiment with different panel lengths and we set the number of households for each estimation to 100. To be as close to real data availability as possible, we experiment with panel lengths of 15 and 25. We use a single instrument (coupled with a constant), lagged real rates so that all systems are just identified. For our first estimator, the constant is applied to both equations while the lagged interest rate is applied only to the first equation to make the system just identified. For the weighting matrix (although irrelevant due to just identifiability) we allow for heteroskedasticity and cross sectional correlation across households. The latter is particularly important when we experiment with households that face common interest rates.

Two sets of experiments are performed. In the first set we use simulated data that are generated allowing for correlation of expectational errors across households. For this, we simply assume that every group of 10 households face the same interest rates and estimations are performed by pooling 10 of these groups together (to give a total of 100 households). The second set of experiments uses simulated data generated allowing for cross-sectional variation in interest rates so that every household faces different series (although the underlying interest rate process is the same for every household, *ex-post* series are different). Again, estimations are performed by pooling 100 households together. Notice that in both experiments individual households experience independent (across households) wage shocks. We set the number of Monte Carlo replications for both sets to 1000.

Table 1 presents the results for the first set of experiments in which the some of the households face the same interest rate series. The table presents the means, medians and standard deviations of the sampling distributions for all four estimators. As can be seen, all estimators

⁸In the case of log-linearized Euler Equation estimation, we attempt to identify only the coefficient of relative risk aversion since the discount factor is buried in the constant.

improve in estimating the coefficient of relative risk aversion (γ) as the sampling period (T) increases. Both AGMM and EGMM display significant bias in the estimates of γ and our two new estimators have less bias. The same holds for the estimates of the discount factor.

Table 2 presents the results for the second set of experiments in which the pooled households all face different interest rate series. Both of our estimators, GMM-LN and GMM-D, still perform better than the other two estimators and capture the true value of γ and β fairly well. The difference between our estimators and AGMM and EGMM is again substantial for small T . Both EGMM and AGMM display upward biases (in means) although both estimators greatly improve as T becomes larger. However, the medians of the sampling distributions of γ for EGMM and AGMM are very close to the true value which is not the case for our proposed estimators. As was the case for the common interest rate experiments, both our new estimators do an excellent job in capturing the true value of β , while EGMM performs poorly

Overall, the results suggest that the type of measurement error assumed in the literature causes a serious downward bias in the discount factor estimates if it is ignored and this bias does not go away as the panel length increases. The bias in the coefficient of relative risk aversion is not as severe and it seems to become smaller as the panel length increases (particularly in the common interest rate case). It is interesting that with a shorter panel of 15 years, the log-linearized Euler equation yields almost the same result as the exact one, whether interest rate series are common or there is a cross section variation. Recall that the main reason for using the log-linearized version of the Euler equation is that linearizing the exact Euler equation removes the measurement error by shifting it to the expectation errors. The log-linear version seems to be superior to the exact version only as the panel length becomes (unrealistically) long.

Turning to our estimators, they both perform very well in estimating the parameters of interest. The most striking feature of our estimators is that they outperform the other two estimators especially when the panel length is short. This is clearly an important feature since panel data on consumption are rare, let alone long panels. These estimators' advantage over

traditional approaches is more pronounced in the case where blocks of households face the same interest rate series (which is perhaps a more realistic assumption than cross sectional variation in interest rates). Chioda’s (2004) results (using the same life cycle model) are consistent with our findings; while her first method tends to produce outliers, her second method performs quite well in terms of estimating the coefficient of relative risk aversion even for T as low as 15.

3.2 Robustness Check.

In the previous subsection we took the same distribution to generate the measurement as we assume for GMM-LN; here we check the small sample consequences of misspecifying the underlying measurement error distribution. We are particularly interested in the consequences of misspecifying the underlying tail thickness of the measurement error distribution so we take data generating process based a $\chi^2(1)$. For this we first simulate $\chi^2(1)$ random draws, ξ_{it} . These have unit mean and variance 2. We then generate measurement errors by:

$$\eta_{it} = 1 + \sqrt{0.004} \frac{(\xi_{it} - 1)}{\sqrt{2}} \tag{13}$$

This has a unit mean and a variance of 0.004.

To save space, we only present results for the environment in which groups of 10 households face *ex-post* common interest rate series; see Table 3. As in baseline experiments each estimation is conducted for 100 households observed for 15 and 25 periods. We use the same instruments and all systems are just identified. The results for the conventional estimators (AGMM and EGMM) are very similar to the baseline experiments. AGMM, as expected, seems to do well in estimating γ under all measurement error distributions, especially when the sampling period is long. As in the baseline cases, the discount factor estimates with EGMM exhibit serious downward bias. What is surprising in these experiments is that GMM-LN does very well even though we have misspecified the measurement error distribution; this is particularly the case for

the discount factor but the coefficient of relative risk aversion is also reasonably well estimated. GMM-LN also recovers the true variance of the measurement error accurately. The GMM-D estimator (which does not rely on a particular distributional assumption) shows less bias for the γ regardless of the sampling period. This is the reverse of the previous subsection. This lower bias is offset by a higher dispersion of the parameter estimates for GMM-D. The general picture mirrors that of the previous subsection: both GMM-LN and GMM-D have good small sample properties and both are significantly better than EGMM for the discount factor.

4 An Empirical Application

In this section we present an empirical application of our GMM estimators using the Panel Study of Income Dynamics (PSID). The survey contains annual information on food at home and food at restaurants. Despite its shortcomings (no expenditure variable other than food, large measurement error, representativeness issues due to attrition) we choose to work with the PSID because it is the longest available panel survey on a large item of consumption and it has been used extensively for Euler equation estimation previously. Our sample covers the periods between 1974 and 1987. Although the actual panel length is much longer, some of the food variables are hard to interpret prior to 1974 and food related questions were suspended for several years after 1987.

Our sample excludes households that did not report food expenditure for at least five consecutive years. We treat split-ups as separate household units. We also exclude any household that may be liquidity constrained in the sample period; we do this by keeping only the households that reported some savings⁹. We exclude single headed households, households whose marital status changed over the sample period and those whose head is less than 22 in 1974 and more

⁹Note that the Euler equation estimation is based on the implicit assumption that the equation holds every period for a given household. If a household carries forward some liquid assets from one period to the next than the relevant Euler equation for that household should hold. Even though this is not a perfect way of handling binding and/or expected to bind liquidity constraints, it seems to be a widely accepted one.

than 60 in 1987. Our final unbalanced panel (a minimum of 5 years and a maximum of 14 years) has a total of 1,906 households (19,317 observations). We assume that all households face a common interest rate series calculated based on the U.S. three-month treasury bill rates and the consumer price index. This amounts to using only the "time variation" in the intertemporal price.

Following Attanasio *et al* (1999), we specify the within period utility function in the following way:

$$U(C_t, Z_t) = \frac{1}{1-\gamma} C_t^{1-\gamma} \exp(\theta' Z_t) \quad (14)$$

where γ is the coefficient of relative risk aversion, Z variables are demographics such as family composition, the number of children and labor supply variables. It is important to incorporate them into empirical specifications since they are likely to affect the level of utility for a given level of consumption. The resulting Euler equation is

$$E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} (1 + r_{t+1}^i) \beta \exp(\theta \Delta Z_{t+1}) \right] = 1 \quad (15)$$

We are now interested in estimating γ and β controlling for changes in demographics. The first estimator, GMM-LN, can be re-written as follows

$$\begin{aligned} u_{t+1}^1 &= \left[\left(\frac{c_{t+1}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1}) \beta \exp(\theta \Delta Z_{t+1}) - \exp\{\gamma^2 \nu\} \right] \\ u_{t+2}^2 &= \left[\left(\frac{c_{t+2}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})(1 + r_{t+2}) \beta^2 \exp(\theta \Delta Z'_{t+2}) - \exp\{\gamma^2 \nu\} \right] \end{aligned} \quad (16)$$

where $\Delta Z'_{t+2} = Z_{t+2} - Z_t$

The second estimator is based on:

$$\zeta_{t+2} = \left[\left(\frac{c_{t+1}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1}) \beta \exp(\theta \Delta Z_{t+1}) \right] - \left[\left(\frac{c_{t+2}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})(1 + r_{t+2}) \exp(\theta \Delta Z'_{t+2}) \beta^2 \right] \quad (17)$$

Note that we now have an additional parameter vector (θ) to estimate. In practice, we assume that the only relevant demographic variable is ‘family size’. We assume that changes in this are perfectly foreseen so that we use ΔZ_{t+1} as an instrument.

Table 4 presents the estimates from the PSID. As in the simulations, we also present the results based on the approximate GMM (AGMM) and the exact GMM (EGMM) for comparison. For all estimators, the parameters are estimated by allowing for conditional heteroskedasticity of the disturbances but not serial correlation. Moreover, the weighting matrices used do not allow for cross section correlation of the disturbances. In the case of GMM-LN, we allow for correlation across equations. The values for the elasticity of intertemporal substitution (γ) given in Table 4 are all within a familiar range. The AGMM estimation results in the highest estimate while EGMM yields the lowest. Although the estimates of the coefficient of relative risk aversion we obtain from GMM-LN and GMM-D are similar (around 1.5), the discount factor estimates are quite different. GMM-LN yields a reasonably precisely estimated value of 0.99 (corresponding to a discount rate of 1%) whereas GMM-D yields an estimate of 0.93 (a discount rate of 7.5%) albeit less precisely. In comparison, the discount factor estimate obtained from EGMM is implausibly low (0.87 corresponding to a discount rate of 15%). The estimated measurement error variance using GMM-LN is 0.085. This value corresponds to 62% of the variation in consumption growth being noise. In all cases the parameter for the effect of family size (θ) is positive but imprecisely estimated for all models. The fact that the estimates for GMM-LN and GMM-D are so similar statistically suggests that the lognormal assumption is a reasonable one. If this is the case then the conclusion is that our best estimate of the mean coefficient of relative risk aversion is 1.45 (with a standard error of 0.3) and the mean discount

factor is 0.99 (with a standard error of 0.1).

5 Conclusion

In this paper we have proposed two new GMM estimators for the Euler equation that take account of the measurement error in consumption. A feature of dealing with measurement error in nonlinear models is that the solutions are specific to the context and that is the case here. In particular, we have exploited the iso-elastic assumption which gives a ratio of consumptions in the Euler equation. If some other functional form were to be used then different assumptions will be needed to achieve identification. Monte Carlo simulations suggest that when the panel length is short both of our estimators perform significantly better than the linearized Euler equation or the exact GMM estimator that ignores measurement error. The estimator that makes a specific lognormal assumption, GMM-LN, performs better when that assumption is correct but is likely to be less robust than the estimator that simply assumes stationarity, GMM-D. The empirical application of the proposed estimators yields plausible estimates of the coefficient of relative risk aversion and discount rate. In particular, the estimate of the elasticity of intertemporal substitution is 1.45 with a standard error of 0.3.

References

- [1] Alan, S. and M. Browning (2003): “Estimating Intertemporal Allocation Parameters Using Simulated Residual Estimation”, CAM Working Paper 2003-03, University of Copenhagen.
- [2] Amemiya, T. (1985): “Instrumental Variable Estimator for the nonlinear Errors-in-Variables model.” *Journal of Econometrics*, 28(3), 273-289.
- [3] Altonji, J. G. and A. Siow (1987): “Testing the Response of Consumption to Income Changes with Noisy Panel Data”, *Quarterly Journal of Economics*, 102, 293-328.
- [4] Attanasio, O.P. (1999): “Consumption” in Taylor and Woodford (eds.): *Handbook of Macroeconomics*, North Holland.
- [5] Attanasio, O. P. , J. Banks, C. Meghir, G. Weber (1999): “Humps and Bumps in Lifetime Consumption”, *Journal of Business and Economic Statistics*, 17(1), 22-35.
- [6] Attanasio, O.P. and M. Browning (1995): “Consumption over the Life Cycle and over the Business Cycle ” , *American Economic Review*, 86(5), 1118-1137.
- [7] Attanasio, O. P. and H. Low (2004): “Estimating Euler Equations”, *Review of Economic Dynamics*, 7, 405-435.
- [8] Browning, M. and T. Crossley (2000): “Luxuries are easier to postpone: a proof”, *Journal of Political Economy*, 108(5), 1022-1026.
- [9] Browning, M., A. Deaton, and M. Irish (1985): “A Profitable Approach to Labor Supply and Commodity Demand over the Life Cycle”, *Econometrica*, 53, 503-544.
- [10] Browning, M. and A. Lusardi (1996): “Household Saving: Micro Theories and Micro Facts”, *Journal of Economic Literature*, December, 1797-1855.
- [11] Chioda, L. (2004): “Estimating Euler Equations with Measurement Error: A Non-parametric Approach”, manuscript.

- [12] Colera, E. V. (1994): “A Note on Measurement Error and Euler Equations”, *Economics Letters*, 45, 305-308.
- [13] Dellavigna, S. and M. D. Paserman (2005): “Job Search and Impatience”, *Journal of Labor Economics*, 23(3), 527-588.
- [14] Dynan, K. (2000): “Habit Formation in Consumer Preferences: Evidence from Panel Data”, *American Economic Review*, 90(3), 391-402.
- [15] Hall, R. H. (1978): “The Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence ”, *Journal of Political Economy*, 86, 971-987.
- [16] Hall, R. H. and F. S. Mishkin (1982): “The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households.”, *Econometrica*, 50(2), 461-481.
- [17] Hansen, L.P. and K.J. Singleton (1982): “Generalized Instrumental Variables Estimation of Non-Linear Rational Expectation Models ”, *Econometrica*, 50, 1269-1286.
- [18] Hausman, J.(2001): “Mismeasured Variables in Econometric Analysis: Problems from the Right and Problems from the left ” *Journal of Economic Perspectives*, 15(4), 57-67.
- [19] Hausman, J., H. Ichimura, W. Newey, J. Powell (1991): “Identification and estimation of polynomial errors-in-variables models ” *Journal of Econometrics*, 50, 273-295.
- [20] Hong, H. and E. Tamer (2003): “A Simple Estimator for Nonlinear Error in Variable Models” *Journal of Econometrics*, 117, 1-19.
- [21] MaCurdy, T. E. (1981): “An Empirical Model of Labor Supply in a Life-Cycle Setting”, *Journal of Political Economy*, 89(6), 1059-1085.
- [22] Runkle, D. E. (1991): “Liquidity Constraints and The Permanent Income Hypothesis”, *Journal of Monetary Economics*, 27, 73-98.

- [23] Samwick, A. A. (2006): “Saving for Retirement: Understanding the Importance of Heterogeneity”, *Business Economics*, 41(1), 21-27.
- [24] Schennach, S. M. (2004): “Estimation of Nonlinear Models with Measurement Error”, *Econometrica*, 72, 33-75.
- [25] Shapiro, M. D. (1984): “The Permanent Income Hypothesis and the Real Interest Rate”, *Economics Letters*, 14, 93-100.
- [26] Venti, S. F. and Wise, D. A. (1998): “The Cause of Wealth Dispersion at Retirement: Choice or Chance?”, *American Economic Review*, 88(2): 185-191.
- [27] Wansbeek, Tom (2001): “GMM Estimation in Panel Data Models with Measurement Error.” *Journal of Econometrics*, 104, 259-268.
- [28] Zeldes, S. P. (1989): “Consumption and Liquidity Constraints: An Empirical Investigation.”, *Journal of Political Economy*, 97(2), 305-346

Estimator	γ		β		ν	
	$T = 15$	$T = 25$	$T = 15$	$T = 25$	$T = 15$	$T = 25$
AGMM	4.71	4.61	–	–	–	–
	[3.75]	[4.01]	–	–	–	–
	(4.48)	(2.18)	–	–	–	–
EGMM	4.56	4.49	0.87	0.88	–	–
	[3.69]	[3.96]	[0.91]	[0.89]	–	–
	(2.98)	(1.53)	(0.12)	(0.05)	–	–
GMM-LN	4.33	4.05	0.95	0.95	0.004	0.004
	[3.69]	[3.80]	[0.96]	[0.95]	[0.004]	[0.004]
	(2.16)	(1.17)	(0.02)	(0.01)	(0.001)	(0.001)
GMM-D	3.83	4.27	0.96	0.95	–	–
	[3.51]	[3.97]	[0.96]	[0.95]	–	–
	(1.71)	(1.65)	(0.01)	(0.01)	–	–

Note: Values are means, medians (in square brackets) and standard deviations (in brackets) of sampling distributions. $N = 100$.

Number of Monte Carlo Simulations: 1000.

True values: $\gamma = 4$, $\beta = 0.95$, $\nu = 0.004$

Table 1: Monte Carlo Results: Common interest rates

Estimator	γ		β		ν	
	$T = 15$	$T = 25$	$T = 15$	$T = 25$	$T = 15$	$T = 25$
AGMM	4.64	4.38	–	–	–	–
	[4.03]	[3.97]	–	–	–	–
	(6.85)	(3.19)	–	–	–	–
EGMM	4.49	4.38	0.87	0.88	–	–
	[3.95]	[3.94]	[0.89]	[0.89]	–	–
	(2.30)	(1.69)	(0.09)	(0.07)	–	–
GMM-LN	3.93	3.99	0.95	0.95	0.004	0.004
	[3.71]	[3.82]	[0.95]	[0.95]	[0.004]	[0.004]
	(1.19)	(0.98)	(0.004)	(0.01)	(0.001)	(0.001)
GMM-D	4.39	3.75	0.95	0.95	–	–
	[3.90]	[3.53]	[0.95]	[0.95]	–	–
	(1.86)	(1.56)	(0.01)	(0.005)	–	–

Note: Values are means, medians (in square brackets) and standard deviations (in brackets) of sampling distributions. $N = 100$.

Number of Monte Carlo Simulations: 1000.

True values: $\gamma = 4$, $\beta = 0.95$, $\nu = 0.004$

Table 2: Monte Carlo Results: Cross section interest rate variation

Common Interest Rates, $\chi^2(1)$ process

Estimator	γ		β		ν	
	$T = 15$	$T = 25$	$T = 15$	$T = 25$	$T = 15$	$T = 25$
AGMM	4.70	4.36	—	—	—	—
	[3.92]	[4.17]	—	—	—	—
	(2.28)	(1.22)	—	—	—	—
EGMM	4.22	4.22	0.90	0.90	—	—
	[3.67]	[3.98]	[0.91]	[0.91]	—	—
	(1.49)	(1.23)	(.05)	(0.04)	—	—
GMM-LN	3.69	3.90	.96	0.95	0.004	0.004
	[3.55]	[3.78]	[0.96]	[0.95]	[0.004]	[0.004]
	(0.98)	(0.92)	(0.01)	(0.004)	(0.001)	(0.0003)
GMM-D	4.14	4.04	0.96	0.95	—	—
	[4.00]	[3.94]	[0.96]	[0.95]	—	—
	(1.77)	(1.71)	(0.01)	(0.004)	—	—

Note: Values are means, medians (in square brackets) and standard deviations (in brackets) of sampling distributions. $N = 100$. True values: $\gamma = 4$, $\beta = 0.95$, $\nu = 0.004$

Table 3: Robustness Checks

Estimator	γ	β	ν	θ
AGMM	2.33	—	—	0.045
	(0.77)			(0.03)
EGMM	1.15	0.87		0.186
	(0.23)	(0.05)		(0.47)
GMM-LN	1.45	0.99	0.085	0.894
	(0.27)	(0.10)	(0.05)	(0.51)
GMM-D	1.53	0.93		0.729
	(0.66)	(0.71)		(0.51)

Table 4: Estimates from the PSID (1974-1987). Standard errors in parantheses.

A The Simulation Model.

We assume that the utility function is intertemporally additive and the sub-utilities are iso-elastic. The problem of the generic household is:

$$\max_{c_t} E_t \sum_{t=0}^T \beta^t u(C_t)$$

$$s.t. X_{t+1} = (1 + r_{t+1})(X_t - C_t) + Y_{t+1}$$

where C_t is nondurable consumption in period t , X_t is cash-on-hand (total financial and non-financial wealth) and Y_t is current labor income. We assume that durable consumption and leisure are separable from the nondurable consumption. The income process is assumed as follows:

$$Y_{t+1} = P_{t+1}U_{t+1}$$

$$P_{t+1} = P_t N_{t+1}$$

P_t is permanent income which is subject to lognormally distributed shocks N_t with mean unity and variance $\exp(\sigma_n^2) - 1$, current income Y_t equals permanent income multiplied by a transitory shock, U_{t+1} , which is distributed lognormally with mean one and variance $\exp(\sigma_u^2) - 1$. The interest rate series is assumed to be generated by a stationary first order autoregressive process with long-run mean μ and autoregressive coefficient ρ . Interest rates shocks ϵ_{t+1} are assumed to be white noise with variance σ_ϵ^2 . The process is

$$r_{t+1} = (1 - \rho)\mu + \rho r_t + \epsilon_{t+1}$$

The intertemporal model described above does not have an analytical solution due to the assumed income uncertainty. Therefore we utilized the standard numerical dynamic programming methods to obtain a solution. Since the utility function is additive over the life cycle we solved the model recursively starting from the last period of life. We assume away any bequest motive so that consumption in period T is:

$$c_T(x_T) = x_T$$

The problem is solved via policy function iteration using the terminal value condition. Having a nonstationary income process makes the problem harder to solve since the range of possible income values is too large. Instead, we redefine all relevant variables in terms of their ratios to permanent income and solve for the consumption to income ratio. By doing this we reduced the number of state variables to two, namely the cash on hand to income ratio and the interest rate. Moreover, we obtain an iid income process which can be approximated by standard Quadrature methods. Given the redefinition of the variables, the Euler equation can be written as

$$\theta_t(w_t, r_t)^{-\gamma} - \frac{1}{(1 + \delta)} E_t \left[(1 + r_{t+1}) \theta_{t+1}(w_{t+1}, r_{t+1})^{-\gamma} n_{t+1}^{-\gamma} \right] = 0$$

where $\theta_t = \frac{C_t}{P_t}$, $w_t = \frac{X_t}{P_t}$. At the terminal date T , consumption to income ratio is a function of only the cash on hand to income ratio and since the bequest motive is assumed away it follows that $\theta_T = w_T$.

For the income process, we use 10 point Gaussian Quadrature and we approximate the interest rate process by forming a 10 point first order discrete Markov process. We use a cubic spline to approximate the consumption function at each iteration. The agent is allowed to borrow the amount he can pay back with certainty. In practice this constraint will never bind because the functional form of the utility function implies that zero consumption results in infinite marginal utility. Since we do not assume an explicit borrowing limit, the consumption

Parameter	Value
Coefficient of risk aversion (γ)	4
Discount factor (β)	.95
Discount rate ($\delta = \frac{1}{\beta} - 1$)	.053
Standard deviation of permanent income shocks (σ_n)	.02
Standard deviation of transitory income shocks (σ_u)	.1
Unconditional mean of interest rate process (μ)	.05
AR(1) coefficient of interest rate process (ρ)	.6
Standard deviation of interest rate process (σ_ϵ)	.025
Variance of log measurement error (σ_η)	.004 (75% noise)

Table 5: Parameter Values

functions are continuously differentiable. In fact, in our case where agents have iso-elastic preferences and income uncertainty, consumption functions are strictly concave.

In order to solve the problem, we define an exogenous grid for the cash on hand to income ratio: $\{x_j\}_{j=1}^J$. It is important to adjust the grid as the solution goes back in time. The algorithm finds the consumption level that makes the standard Euler equation hold for each value of x and r . In practice, we took 100 points for x and 10 points for r . The grid for x is finer at lower levels in order to capture the curvature of the consumption function. After solving for the consumption function of a generic household for 100 periods, we simulate consumption paths for 100,000 ex-ante identical households facing different interest rate realizations. Then, using the same consumption function, we simulate paths for 100,000 ex-ante identical households where groups of 100 households face the same realized interest rate series. We use only 15 and 25 periods for the estimations and these observations are taken from the middle of each path. Table 5 presents the assumed parameter values for our experiments. The discount rate and the mean interest rate are chosen to be close in order to prevent consumers to quickly go towards the borrowing constraint¹⁰.

¹⁰When the discount rate is large relative to the interest rate, consumers borrow close to the maximum possible amount. Then the movement of consumption is largely driven by income and the identification of interest rate impact on consumption growth becomes very difficult. See Attanasio and Low (2004) and Alan and Browning (2003) for a detailed discussion on the problems of identification and impatience.