

# Start up costs, limited enforcement, and the hidden economy<sup>☆</sup>

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## Abstract

This paper examines how much of the difference in the size of the informal sector and in per capita income across countries can be accounted by regulation costs and enforcement of financial contracts. It constructs and solves numerically a general equilibrium model with credit constrained heterogeneous agents, occupational choices over formal and informal businesses, financial frictions and a government sector which imposes taxes and regulations on formal firms. The benefit from formalization is better access to outside finance. The quantitative exercises suggest that: (i) regulation costs and not the level of enforcement account for differences in the size of the informal sector between United States and Mediterranean Europe; (ii) for a developing country like Peru, however, contract enforcement and regulation costs are equally important in accounting for the size of the informal sector; and (iii) regulation costs and contract enforcement do not account for most of the income differences observed among countries.

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## 1. Introduction

The size of the informal sector as a fraction of GDP is known to vary systematically with a country's level of development. For example, in the United States the informal sector accounts for 10% of total output; in Peru it accounts for roughly 60% of total output; and in Nigeria it accounts for 76% of total output (see Table 1). De Soto (1989) argues that the size of the informal sector is mainly determined by entry barriers, primarily under the form of regulation and bribes. De Soto (2000) further argues that informality works to retard a country's subsequent development because informal entrepreneurs cannot use their wealth as collateral for loans to finance investments in physical and intangible capital.

The purpose of this paper is to assess the quantitative plausibility of De Soto's claims. In particular, we put forth a general equilibrium model and use it to assess how much of the variation in informal sectors and per capita GDP levels are attributed to entry barriers (in the form of regulation costs) and credit market imperfections (in the form of enforcement of financial contracts). Regulation costs should be viewed as the sum of the cost to meet legitimate government regulations plus any bribes and other costs arising from the demands of corrupt officials. The model has three occupational choices (worker, and formal or informal entrepreneur), inequality in wealth and in entrepreneurial

Table 1  
Selected statistics

Country	Informal sector size (%)	Official GDP per capita	Total GDP per capita	Regulation costs	Efficiency of judicial system	Protection against expropriat.	Rule of law	Risk of contract repudiat.	Average enforc.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Denmark	9.4	32,030	35,041	0.1000	10.00	9.67	10.00	9.31	9.74
Canada	14.8	19,320	22,179	0.0145	9.25	9.67	10.00	8.96	9.47
Germany	13.2	25,350	28,696	0.1569	9.00	9.90	9.23	9.77	9.47
France	13.8	23,480	26,720	0.1430	8.00	9.65	9.05	9.19	8.97
United States	10	30,600	33,660	0.0049	10.00	9.98	10.00	9.00	9.74
Belgium	15.3	24,510	28,260	0.0998	9.50	9.63	10.00	9.48	9.65
Portugal	22.1	10,600	12,942	0.1844	5.50	8.90	8.68	8.57	7.91
Spain	22.4	14,000	17,136	0.1730	6.25	9.52	7.80	8.40	7.99
Italy	26	19,710	24,834	0.2002	6.75	9.35	8.33	9.17	8.40
Argentina	21.8	7600	9257	0.1019	6.00	5.91	5.35	4.91	5.54
Brazil	35	4420	5967	0.2014	5.75	7.62	6.32	6.30	6.50
Peru	60	2390	3824	0.1986	6.75	5.54	2.50	4.68	4.87
Nigeria	76	310	546	2.5700	7.25	5.33	2.73	4.36	4.92

*Sources:* Informal sector size is from Schneider and Enste (2000, tables 2, 3 and 7), and Friedman et al. (2000, table 1, first column), and is total production in the informal sector as a share of GDP. Official GDP per capita is the gross domestic product per capita in U.S. dollars in 1999 (The World Bank, 2001). Unofficial GDP per capita is calculated using the first and the second columns above. For countries with a range of informal sector sizes, the upper limit was used. Regulation costs are from Djankov et al. (2002, table 3, column 8). They are direct costs as a fraction of GDP per capita that entrepreneurs face to meet government regulations. Efficiency of the judicial system, protection against expropriation, rule of law, and risk of contract repudiation are from La Porta et al. (1998, table 5, columns 1, 2, 4, and 5). The last column is the average from columns 5 to 8.

ideas,<sup>1</sup> corruption and regulation, and limited enforcement. Occupational choice and the size of each project are determined endogenously. They depend on the agent's "type" (wealth and project), start up costs, and credit market imperfections. Different levels of bureaucracy and limited enforcement generate differences in occupational choice (size of the informal sector) and differences in labor productivity (TFP), which affect development.<sup>2</sup>

We calibrate the steady-state equilibrium of the model to the United States experience. Using the calibrated model, we examine the steady-state effects associated with varying the regulation costs and the level of enforcement. We find that enforcement has a greater effect on the size of the informal sector and on economy's productivity. We also input independent measures of the regulation costs and enforcement costs for individual countries and explore the model's predictions for the size of the informal sector and living standard. Here we find that regulation costs and not enforcement account for most of the differences in the size of the informal sector between the United States and Mediterranean Europe. Additionally, we find that enforcement and regulation costs interact in interesting and non-linear ways so that the steady-state effect of regulation is magnified in the case of low level of enforcement (or high enforcement costs).<sup>3</sup> For countries with low enforcement, we find that regulation and enforcement are equally important in accounting for the size of the informal sector. However, we do not find that differences in regulation costs and enforcement can account for much of the differences in output across countries. In this sense, the channel emphasized by [De Soto \(2000\)](#) as being important for development does not seem plausible. This result is complementary to [Parente and Prescott \(2000\)](#), who show that factor accumulation alone does not explain the huge disparities in international incomes.

There is a large literature in economics which studies the hidden economy both theoretically and empirically. One branch of the literature studies the effects of the informal sector on growth and government policies.<sup>4</sup> Another branch, which is more directly related to this study, investigates the determinants of the hidden economy. Within the empirical literature, [Djankov et al. \(2002\)](#) emphasize that start up costs generate large informal sectors, while [Friedman et al. \(2000\)](#) suggest that it is not tax rates<sup>5</sup> per se that induce entrepreneurs to go underground, but bureaucracy and a weak legal system. Their

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<sup>1</sup>See [Banerjee and Newman \(1993\)](#) and [Lloyd-Ellis and Bernhardt \(2000\)](#). The latter framework is closely related as it studies macroeconomic and distributional dynamics associated with the process of economic development. They develop important tools that we use to characterize the long-run dynamics of our model economy.

<sup>2</sup>[Antunes and Cavalcanti \(2003\)](#) also develop a model economy where capital market imperfections and regulation costs endogenously generate differences in labor productivity.

<sup>3</sup>This result is consistent with [Krasa et al. \(2004\)](#). They construct a model of enforcement in which agents have explicit incentives to default/repay, enforcement is costly, and there is some degree of debtor/creditor protection. They show that under different parameter configurations, different constraints bind, leading to highly non-linear finance.

<sup>4</sup>[Easterly \(1993\)](#) and [Loayza \(1996\)](#) show that growth is negatively related to informal production. [Cavalcanti and Villamil \(2003\)](#) show how the optimal monetary policy and the welfare costs of inflation might be affected by the size of the informal sector through tax evasion.

<sup>5</sup>They argue that the burden of tax rates has two effects: (i) it drives agents into the informal sector to avoid official taxes; but (ii) it can also raise revenue to improve government institutions and enforcement, which leads to a lower informal sector. In general, these two effects offset each other, and the relationship between tax rates and the size of the informal sector across countries is not statistically significant.

findings are consistent with our quantitative results. We contribute to this literature by identifying cases in which start up costs due to government regulations and corruption are the main determinants of the hidden economy.

Within the theoretical literature,<sup>6</sup> Rauch's (1991) model, which is based on Lucas (1978) "span of control framework", suggests that entrepreneurs go underground to avoid the minimum wage, which is greater than the paying market wage.<sup>7</sup> Unlike Rauch's model, ours is a dynamic framework and uses credit constraints in the analysis of occupational choice. In Dessy and Pallage (2003) entrepreneurs become legal because they can use productive infrastructure (which enters in the production function), while in our model the benefit from formalization is better access to outside finance. Quintin (2002) has a similar premium from formalization, but we add bureaucracy costs and bequest transfers to the analysis. He finds that the enforcement level accounts for the documented size of the informal sector in most Latin American nations. We find that enforcement and regulation costs are equally important to determine the size of the informal sector between United States and some Latin American countries.<sup>8</sup>

Finally, our work is also related to the literature on calibrated two-sector growth models and economic development. Parente et al. (2000), for instance, introduce home production, while Restuccia (2004) considers a traditional and a modern sector in a Neoclassical growth model. Similar to our model, these economies can generate larger differences in "official" output levels across countries than standard models for a given policy differential. In our model technologies will be the same in each sector but differences in productivity arise endogenously due to policies that affect each sector differently.<sup>9</sup>

The paper is divided as follows: Section 2 describes the model economy. Section 3 describes the agents' optimal behavior, defines the competitive equilibrium allocations, and presents some analytical results. Section 4 solves the model numerically and conducts policy experiments. The last section provides some concluding remarks and policy implications.

## 2. The model

We consider a small open and infinite horizon economy with one consumption good. Time is discrete ( $t = 0, 1, 2, \dots$ ) and agents are one period lived non-overlapping generations. There are two production sectors: formal and informal. Agents can choose to be either a worker or an (informal or formal) entrepreneur. In order to open a formal/informal business, agents must buy capital in advance to finance their project. Capital markets, however, are imperfect. In order to operate legally entrepreneurs have to pay a start up cost and an employment tax. The premium from formalization is a better access to outside finance. The details of the model economy are described below.

<sup>6</sup>Azuma and Grossman (2002) study a related but different question: Why do governments impose or tolerate burdensome taxes, bribes and bureaucracy that lead many producers to operate in the informal sector?

<sup>7</sup>See Amaral and Quintin (2006) for a model where differences in workers' characteristics and in the wage rate arise endogenously due to better investment opportunities in the formal sector. The question that they address is different from ours. They focus on why informal production emphasizes low-skill work.

<sup>8</sup>We also find that regulation costs and not enforcement costs account for most of the differences in the size of the informal sector between the United States and Mediterranean Europe.

<sup>9</sup>Related to this result is Guner et al. (2004), who consider a model with occupational choice and investigate the effects of size restrictions on productivity.

## 2.1. Household sector

At each date, there is a continuum of agents distributed uniformly on an interval with measure one. Each agent lives for one period, and then is replaced in the population by his offspring so that the population is constant.

### 2.1.1. Preferences

An agent alive in period  $t$  derives utility from his own consumption,  $c_t$ . He also cares about his offspring. Following, Banerjee and Newman (1993), we assume that preferences are over the child's bequests,  $b$ , rather than the child's utility. The utility function of a household at date  $t$  is

$$U = (c_t)^\gamma (b_{t+1})^{1-\gamma}, \quad \gamma \in (0, 1). \quad (1)$$

This utility function implies that agents are risk-neutral with respect to income as the indirect utility function is linear in wealth. This implies that any additive punishment or reward in utility may be measured in terms of income.

### 2.1.2. Endowments

Each household inherits the bequest left by his parent. Each household is also endowed with one unit of time, which he can use to work as a laborer or entrepreneur. Additionally, each household is endowed with some talent,  $x$ , which determines his entrepreneurial ability or span of control (Lucas, 1978). In contrast to bequests, talent is not inherited, but rather a realization from a cumulative probability distribution  $\Gamma(x)$ , where  $x$  belongs to the interval 0 and 1. It follows that at the start of each period, households are distinguished by the pair  $(b, x)$ . We assume that  $(b, x)$  is public information.

## 2.2. Production technologies

There are two production sectors: the informal sector and the formal sector. The production technology is the same for both sectors. An entrepreneur is needed to operate a technology. In addition to the time input of the entrepreneur, the production technology also requires raw labor,  $n$ , and capital,  $k$ . Capital is assumed to depreciate fully at the end of the period. Moreover, capital must be owned by the firm.

The output of an entrepreneur depends on his ability, as well as the inputs of  $k$  and  $n$ . More specifically,

$$y = xk^\alpha n^\beta, \quad \alpha, \beta > 0 \quad \text{and} \quad \alpha + \beta < 1. \quad (2)$$

The formal and informal sectors differ importantly in two ways. The first is that there is a fixed cost,  $\varsigma$ , associated with operating legally. This cost reflects regulations and bribes. Additionally, legal firms are subject to an employment tax,  $\tau$ .

## 2.3. The capital market

All borrowing and lending is done within the period and involves members of the same generation. The only possible borrowers in the economy are entrepreneurs. To simplify the analysis we assume that the financial sector has access to world capital markets and can borrow or lend any amount at the constant gross interest rate  $r$ .

In order to obtain a loan, we assume that a household must use his bequest as collateral. We assume that borrowers cannot commit ex ante to repay their loans. An agent who chooses to default on his loan is assumed to lose the principal and interest on his collateral,  $rb$ . Additionally, we assume that those entrepreneurs who operate legally are subject to a punitive penalty,  $\phi y$ . This cost reflects the strength of contract enforcement in the economy. Those entrepreneurs who operate informally are not subject to this punitive penalty.

### 3. Optimal behavior and equilibrium

In this section, we specify the relevant maximization problems of the agents in the economy and characterize their solutions. We next define a competitive equilibrium.

At the beginning of a period, a household deposits his inheritance,  $b$ , at a financial intermediary. Each household then decides whether to be either a worker, an entrepreneur in the formal sector or an entrepreneur in the informal sector. A household who becomes an entrepreneur chooses the capital and labor inputs for his firm together with the amount of the loan from the financial sector to finance his firm's capital input. Production then occurs, and each entrepreneur chooses whether to default on his loan. At the end of the period, the household chooses how much of his income and deposit he should consume and how much to leave as a bequest to his child.

#### 3.1. Entrepreneurs

We let the letter  $j = I, F$  index whether an entrepreneur operates informally or formally. An entrepreneur who operates legally must pay the fixed cost,  $\varsigma$ , and the payroll tax,  $\tau$ . Let  $\mathcal{I}_F$  be the indicator function that takes on the value of 1 if  $j = F$ , and 0 when  $j = I$ . The indicator function allows us to avoid having to specify informal and formal entrepreneurs' problems separately.

Each entrepreneur chooses the level of capital and labor to maximize his firm profits. We first describe the labor input choice of an entrepreneur with a given level of capital,  $k$ , and ability,  $x$ . Let  $w$  be the competitive wage rate. The relevant maximization problem of an entrepreneur operating in sector  $j$  with ability  $x$  and capital  $k$  is

$$\max_{n_j} \{xk^\alpha n_j^\beta - (1 + \tau \mathcal{I}_F)wn_j\}. \quad (3)$$

Profit maximization yields the following labor demand

$$n_j(k, x; w) = \left( \frac{\beta x k^\alpha}{w(1 + \tau \mathcal{I}_F)} \right)^{1/(1-\beta)}. \quad (4)$$

Substituting (4) into (3) yields the entrepreneur's profit function  $\pi_j(k, x; w)$ ,

$$\pi_j(k, x; w) = [(1 - \beta)xk^\alpha]^{1/(1-\beta)} \left( \frac{\beta}{w(1 + \tau \mathcal{I}_F)} \right)^{\beta/(1-\beta)}. \quad (5)$$

The problem of choosing the optimal capital stock is more complex because enforcement of contracts is incomplete. Since entrepreneurs cannot commit to their promises, debt contracts must be self enforcing. Let  $a$  be the amount of capital that is

self-financed (or used as collateral), and let  $l$  be the amount of funds raised in the outside capital market. Let  $V_j(b, x; w)$  denote the income of an entrepreneur operating in sector  $j$  with bequest  $b$  and ability  $x$ . Then the entrepreneur’s income is

$$V_j(b, x; w) = \max_{0 \leq a_j \leq b, l_j \geq 0} \{ \pi_j(a_j + l_j, x; w) - r(a_j + l_j + \varsigma \mathcal{I}_F) \} \tag{6}$$

subject to

$$\pi_j(a_j + l_j, x; w) - r(a_j + l_j + \varsigma \mathcal{I}_F) \geq (1 - \phi \mathcal{I}_F) \pi_j(a_j + l_j, x; w) - ra_j.$$

The above constraint is the incentive compatibility constraint. It guarantees that individual debt obligations will be fulfilled. This constraint can be rewritten as

$$l_j(b, x) \leq \left( \frac{\phi}{r} \pi_j(a_j(b, x; w) + l_j(b, x; w), x; w) - \varsigma \right) \mathcal{I}_F.$$

For the sake of comparison, it is instructive to derive the optimal choice of capital of an entrepreneur in the case where enforcement is perfect, i.e.,  $\phi = 1$ . In such a world, the entire output of an entrepreneur who defaults goes to the financier. The relevant maximization problem of an entrepreneur operating in sector  $j$  is

$$\max_{k_j \geq 0} \{ \pi_j(k_j, x; w) - r(k_j + \varsigma \mathcal{I}_F) \}. \tag{7}$$

This gives the optimal physical capital level:

$$k_j^*(x; w) = \left[ \left( \frac{\beta}{w(1 + \tau \mathcal{I}_F)} \right)^\beta \left( \frac{\alpha}{r} \right)^{1-\beta} x \right]^{1/(1-\alpha-\beta)}. \tag{8}$$

It can be shown that in a world with imperfect enforcement, an entrepreneur working in the formal sector will borrow in excess of his bequest if  $b$  is less than or equal to  $k^*$ . This implies the size of the project of an entrepreneur  $(b, x)$  is such that

$$k_j(b, x; w) \leq b + \left( \frac{\phi}{r} \pi_F(b + l(b, x; w), x; w) - \varsigma \right) \mathcal{I}_F. \tag{9}$$

The following lemma summarizes the value of undertaking each project:

**Lemma 1.** For any  $x \in [\underline{x}, \bar{x}]$ , and  $w > 0$ , the value function  $V_j(b, x; w)$ , and the associated policy function  $l_j(b, x; w)$  for  $j \in \{F, I\}$  have the following properties:

1.  $V_j(b, x; w)$  is continuous and differentiable in  $x$  and  $w$ . If  $x > 0$ , it is also strictly increasing in  $x$  and strictly decreasing in  $w$ .
2. For  $b < k_j^*(x; w)$ ,  $V_j(b, x; w)$  is continuous, differentiable and strictly increasing in  $b$ . For  $b > k_j^*(x; w)$ ,  $V_j(b, x; w)$  is constant in  $b$ . Moreover,  $V_j(b, x; w)$  is continuous at  $b = k_j^*(x; w)$ .
3. For all  $b$  and  $x$ ,  $l_I(b, x; w) = 0$ .  $l_F(b, x; w)$  is strictly increasing for  $b < k_F^*(x; w)$  and  $l_F(b, x; w) = 0$  for  $b > k_F^*(x; w)$ .

**Proof.** See Appendix B.

### 3.2. Occupational choice

The occupational choice of each household determines his income. A household will choose the occupation that gives him the greatest income. Define the sets  $E(w)$ ,  $E^c(w)$ ,  $E_F(w)$  and  $E_I(w)$  as follows:

$$E(w) = \{(b, x) \in [0, \infty) \times [\underline{x}, \bar{x}] : \max\{V_F(b, x; w), V_I(b, x; w)\} \geq w\}, \quad (10)$$

$$E^c(w) = \{(b, x) \in [0, \infty) \times [\underline{x}, \bar{x}] : w \geq \max\{V_F(b, x; w), V_I(b, x; w)\}\}, \quad (11)$$

$$E_F(w) = \{(b, x) \in E(w) : V_F(b, x; w) \geq V_I(b, x; w)\}, \quad (12)$$

$$E_I(w) = \{(b, x) \in E(w) : V_I(b, x; w) > V_F(b, x; w)\}. \quad (13)$$

For a given  $w > 0$ , the set  $E(w)$  is the combination of abilities and bequests for which a household would choose to be an entrepreneur.  $E^c(w)$  is the set of abilities and bequests for which a household chooses to be a worker.  $E_F(w)$  is the set of abilities and bequests for which a household would choose to be a formal entrepreneur and  $E_I(w)$  is the set of abilities and bequests for which a household would choose to be an informal entrepreneur.

Lemma 2 characterizes the occupational choice for a given bequest and entrepreneurial ability.

**Lemma 2.** *Define  $b_e(x; w)$  as the curve in the  $(b, x)$  plane such that  $(b, x) \in [0, \infty) \times [\underline{x}, \bar{x}]$  and  $\max\{V_F(b, x; w), V_I(b, x; w)\} = w$ . Then there exists  $x^*(w)$  such that  $\partial b_e(x; w)/\partial x < 0$  for  $x > x^*(w)$  and  $\partial b_e(x; w)/\partial x = -\infty$  for  $x = x^*(w)$ . Moreover, for all  $x^*(w)$ ,*

1. *If  $b < b_e(x; w)$ , then  $(b, x) \in E^c(w)$ .*
2. *If  $b \geq b_e(x; w)$ , then  $(b, x) \in E(w)$ .*

*In addition, define  $b_s(x; w)$  as the curve in the  $(b, x)$  plane such that  $(b, x) \in [0, \infty) \times [\underline{x}, \bar{x}]$  and  $V_F(b, x; w) = V_I(b, x; w)$ .*

3. *If  $b \geq b_e(x; w)$  and  $b > b_s(x; w)$ , then  $(b, x) \in E_I(w)$ .*
4. *If  $b \geq b_e(x; w)$  and  $b \leq b_s(x; w)$ , then  $(b, x) \in E_F(w)$ .*

**Proof.** See Appendix C.

Fig. 1 illustrates this lemma. It divides the space of  $(b, x)$  into the sets  $E^c(w)$ ,  $E_F(w)$  and  $E_I(w)$  for a given parameterization that will be described in Section 4. It suggests that agents are workers when the quality of their project is low, i.e.,  $x < x^*(w)$  (the lightest shaded area). For  $x$  greater than or equal to  $x^*(w)$ , agents might become entrepreneurs depending if they are credit constrained (notice that for very low bequests agents are workers even though their entrepreneurial ability is higher than  $x^*(w)$ ). The negative association between  $b_e(x; w)$  and  $x$  suggests that managers with better projects need a lower level of initial wealth to run a project. This is rather intuitive since profits are increasing in the quality of the project. Given the low operational costs in the informal sector, unconstrained entrepreneurs will stay illegal. Constrained entrepreneurs will operate in the informal sector only if the premium from formalization (access to outside finance) is not high enough. Since this premium increases with the quality of the project, only high-productivity projects will operate in the formal sector (darkest shaded area). The area

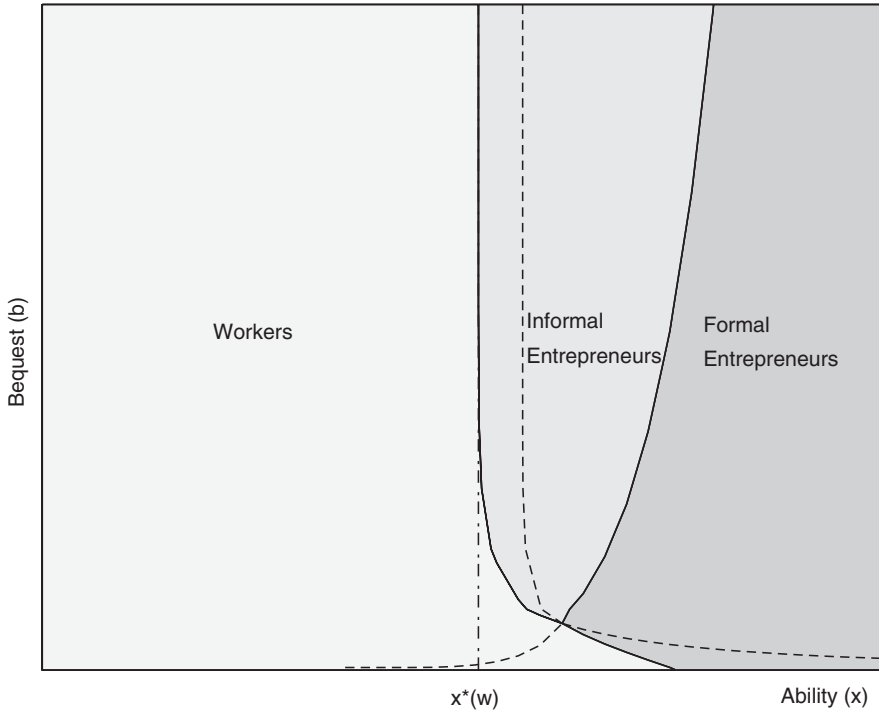


Fig. 1. Firm size distribution in the formal and informal sectors.

in between the darkest and lightest shaded areas corresponds to managers in the informal sector.

The size of the informal sector depends on the institutional and policy parameters  $\tau$ ,  $\varsigma$  and  $\phi$ , as well as on distribution  $\Gamma$ .

### 3.3. Consumers

Let  $Y_t$  be the sum of the occupational income of a household and the value of his assets. Given  $(b_t, x_t)$ ,  $Y_t$  is

$$Y_t = Y(b_t, x_t; w_t) = \max\{w_t, V_F(b_t, x_t; w_t), V_I(b_t, x_t; w_t)\} + rb_t. \tag{14}$$

Given  $Y_t$ , a household chooses consumption  $c_t$  and bequest  $b_{t+1}$  to maximize expression (1) subject to the constraint that the sum of consumption and bequests is less than or equal to  $Y_t$ . The Cobb–Douglas utility function implies that the household leaves fraction  $1 - \gamma$  of  $Y_t$  as a bequest and consumes fraction  $\gamma$ .

### 3.4. Competitive equilibrium

Let  $Y_t$  be the bequest distribution at period  $t$  which evolves endogenously across periods. The initial bequest distribution,  $Y_0$ , is exogenously given. In a competitive equilibrium, agents optimally solve their problems and all markets clear. The agents’ optimal behavior

was previously described in detail. It remains, therefore, to characterize the market equilibrium conditions. The interest rate  $r$  is determined exogenously and the consumption good is the numeraire. Therefore, it is necessary only one market clearing condition to determine the wage rate. The labor market equilibrium condition states that labor demand equals to labor supply in every period:

$$\begin{aligned} & \int \int_{z \in E_F(w_t)} n_F(x; w_t) Y_t(db_t) \Gamma(dx_t) + \int \int_{z \in E_I(w_t)} n_I(x; w_t) Y_t(db_t) \Gamma(dx_t) \\ & = \int \int_{z \in E^c(w_t)} Y_t(db_t) \Gamma(dx_t). \end{aligned} \quad (15)$$

Given that the only connection between periods is the bequest it is essential to provide the law of motion for the distribution of bequests to fully characterize the competitive equilibrium. Define  $P_t(b_t, A) = \Pr\{b_{t+1} \in A | b_t\}$  as a non-stationary transition probability function, which assigns a probability for a bequest in  $t + 1$  for the descendant of an agent that has bequest  $b_t$ .<sup>10</sup> The law of motion of the bequest distribution is

$$Y_{t+1} = \int P_t(b_t, A) Y_t(db_t). \quad (16)$$

A steady-state competitive equilibrium is one where the distribution of abilities and bequests over the population is constant over time. The following proposition states that there exists a unique stationary equilibrium. Additionally, it states that an economy converges to this equilibrium from any initial condition.

**Proposition 3.** *There exists a unique stationary equilibrium with  $0 < w < \infty$  and invariant distribution  $Y$ . In addition, for any initial bequest distribution  $Y_0$  and stationary government policies and institutions, the bequest distribution converges to  $Y$ .*

**Proof.** See Appendix D.

In the calibration and quantitative experiments we will study the economy in this particular equilibrium and therefore we will consider the long-run impact of changes in policies and institutions.

#### 4. Quantitative results

In this section we examine the quantitative properties of the model in order to assess the plausibility of De Soto's (2002) hypothesis. We first calibrate the model economy to the U.S. observations. Next we explore how the equilibrium properties of the model change with variations in the cost of regulation and contract enforcement. Next we examine the predictions of the model for Mediterranean Europe and Peru using independent estimates of the cost of regulation and contract enforcement. Lastly, we consider the sensitivity of the results to alternative parameterizations of the model economy.

<sup>10</sup>See the definition of  $Y_t$  and  $P_t(b_t, A)$  in Appendix D.

Table 2  
Parameter values, baseline economy

Parameters	Values	Comment/observations
$\gamma$	0.8	Value estimated by <a href="#">Laitner and Juster (1996)</a>
$\beta$	0.55	Labor income share based on <a href="#">Gollin (2002)</a>
$\alpha$	0.35	Capital income share based on <a href="#">Gollin (2002)</a>
$r$	2	Yearly real interest rate of 2%
$\tau$	0.33	Payroll tax based on <a href="#">Jones et al. (1993)</a>
$\varsigma$	0.0004	Start up costs based on estimates of <a href="#">Djankov et al. (2002)</a>
$\phi$	0.25	10% informal sector size reported by <a href="#">Schneider and Enste (2000)</a>
$\varepsilon$	6	9% of entrepreneurs over the total population based on <a href="#">Quadrini (1999)</a>

#### 4.1. Model calibration

The general strategy of the calibration is to restrict the model economy so that the steady-state competitive equilibrium matches some key U.S. observations in the postwar period. Before assigning parameter values, it is first necessary to specify the functional form for the distribution of abilities,  $\Gamma(x)$ . We choose  $\Gamma(x) = x^{1/\varepsilon}$ . The motivation for this choice is that [Chatterjee et al. \(2002\)](#) have shown that this functional form can generate an earnings distribution that is similar to the U.S. distribution.

The parameters of the model whose values must be assigned are:  $\varepsilon$ ,  $r$ ,  $\gamma$ ,  $\alpha$ ,  $\beta$ ,  $\phi$ ,  $\varsigma$ , and  $\tau$ . [Table 2](#) lists the value of each parameter and includes a comment on how each was selected. The gross real interest rate is set to 2. Given that the empirical counterpart of a period is 35 years, this value is consistent with the historical annual average real interest rate in the United States equal to 2%. We set  $\alpha$  and  $\beta$  such that about 55% of income is paid to labor, 35% is paid to the remuneration of capital, and 10% are profits.<sup>11</sup> The payroll tax is set to equal the average tax rate on labor income in the United States ([Jones et al., 1993](#)). The share of bequests in the instantaneous utility function,  $1 - \gamma$ , was taken to be 0.2, which is consistent with those estimated by [Laitner and Juster \(1996\)](#). We choose parameters  $\varepsilon$ ,  $\phi$  and  $\varsigma$  such that the size of the informal sector was 10%, the percentage of entrepreneurs in the steady-state equilibrium was 9%, and regulation costs were around 0.5%<sup>12</sup> of GDP per capita.

The model matches the U.S. economy fairly well along a number of dimensions that were not used in the calibration. [Table 3](#) compares the predictions of the model with income inequality, and the capital to output ratio. The Gini coefficient for the model economy is 0.34 whereas it is between 0.40 and 0.44 for the United States. Obviously, if the model allowed for heterogeneity in the quality of labor inputs, the model would generate

<sup>11</sup>[Gollin \(2002\)](#) argues that it is important to adjust factor income shares by the entrepreneurial income, which is often treated incorrectly as capital income share. If we input entrepreneurial profits as labor income as suggested by Gollin, then the effective labor and capital income shares will be 0.65 and 0.35, respectively. These income shares are those that map our model to those observed in national accounts. If we use another adjustment also suggested by Gollin, which assume that entrepreneurial income is a mix of labor and capital income as the rest of the economy, then the effective labor and capital income shares will roughly be 0.61 and 0.39, respectively. In any case, the effective labor income share will be in the range estimated by Gollin, which goes from 0.60 to 0.80.

<sup>12</sup>See [Table 1](#), column (4).

Table 3  
Basic statistics, U.S. and baseline economy

	U.S. economy	Baseline economy
Income Gini	40–44	34
Capital to output ratio	2.5	2.4

Sources: Quadrini (1999), and Maddison (1995); figures for the income Gini in percentage.

Table 4  
Policy experiments

	Informal sector size, % output	Total output per capita, % of baseline	Official output per capita, % of baseline	% of entrepreneurs	Income Gini
Baseline	10	100	100	9	34
<i>Part (a): Regulation costs parameter, <math>\zeta_{\text{base}} = 0.0004</math></i>					
$\phi = \frac{1}{2} \phi_{\text{base}}$	29	79	67	10.8	33
$\phi = \frac{1}{4} \phi_{\text{base}}$	59	69	40	11.5	30
$\phi = \frac{1}{8} \phi_{\text{base}}$	90	60	19	11.6	25
<i>Part (b): Enforcement parameter, <math>\phi_{\text{base}} = 0.25</math></i>					
$\zeta = 2\zeta_{\text{base}}$	10.5	99.6	99.2	9.1	33
$\zeta = 4\zeta_{\text{base}}$	11.1	99	98	9.2	33
$\zeta = 8\zeta_{\text{base}}$	13.5	97.8	95.2	9.3	33
<i>Part (c)</i>					
$\phi = \frac{1}{2} \phi_{\text{base}}; \zeta = 2\zeta_{\text{base}}$	31	77	64	11	33
$\phi = \frac{1}{4} \phi_{\text{base}}; \zeta = 4\zeta_{\text{base}}$	64	66	47	11.8	30
$\phi = \frac{1}{8} \phi_{\text{base}}; \zeta = 8\zeta_{\text{base}}$	94	55	15	11.8	25

$\phi_{\text{base}}$  and  $\zeta_{\text{base}}$  denote the baseline parameter values.

more inequality.<sup>13</sup> The model also matches the capital to output ratio, which is about 2.5 in the data (Maddison, 1995) and 2.45 in the model.

Next, we will run some quantitative experiments. We will change regulation costs,  $\zeta$ , and the level of enforcement,  $\phi$ , separately to provide their long-run impact on the share of production generated outside the realm of government regulation, productivity, and percentage of entrepreneurs. We will also run experiments by changing these two parameters simultaneously.

<sup>13</sup>The income Gini coefficient when we consider only entrepreneurs' income is higher in the data. For instance, data in Quadrini (1999) imply a value around 45% for the U.S. economy. In our model the Gini index for entrepreneurs' income is roughly 49%, which is close to the data.

#### 4.2. Contract enforcement and regulation costs

We now explore how changes in regulation costs and contract enforcement affect the model's predictions. We first consider arbitrary changes in the value of these parameters. We then use independent estimates of these parameters for Mediterranean Europe and Peru.

Table 4 describes the model's predictions along five dimensions as the value of the regulation cost parameter and the contract enforcement parameter are changed. These five dimensions are: Informal sector output as a fraction of total output, total output as a fraction of the U.S. economy's total output, GDP per capita as a fraction of the U.S. economy's per capita GDP, percentage of population that are entrepreneurs, and the Gini coefficient. Observe that all statistics correspond to the steady-state equilibrium of the model.

We begin by decreasing the value of the contract enforcement parameter, holding the value of the regulation cost parameter at the U.S. level. We then increase the value of the regulation cost parameter, holding the contract enforcement parameter fixed at the U.S. value. We then change both proportionally.

Observe (Table 4a) that when enforcement decreases by a factor of 2 from the baseline economy, the informal sector size increases by 19 percentage points and measured output decreases by about 33 percentage points. A larger decrease in  $\phi$  by a factor of 4 increases the informal sector by 49 percentage points, while measured output decreases by 60 percentage points. When  $\phi$  decreases by a factor of 8, then roughly 90% of the total output is produced in the informal sector. Official output is only 19% of the baseline level. Notice that as the level of enforcement decreases the percentage of entrepreneurs over the total population increases. This is consistent to the empirical evidence, which shows that less developed countries tend to have more but less productive entrepreneurs (Lucas, 1978; Tybout, 2000).

We now verify the impacts of regulation costs,  $\zeta$ , on the size of the informal sector and on productivity (Table 4b). Compared to the enforcement parameter, regulation costs have a weaker effect on the informal sector size and output differences. For instance, when regulation costs increase by a factor of 8, the informal sector size increases by only 3.5 percentage points and the official output is about 95.2% of its baseline level. Therefore, it is necessary higher variations in regulation costs than enforcement to generate observed differences in the informal sector size across countries. In fact, when we change both parameters in the same proportion (Table 4c) we verify that the numbers follow closely the previous results when we only changed the enforcement parameter.

#### 4.3. Mediterranean Europe and Peru

As an additional test, we feed in independent estimates of regulation costs and contract enforcement for Mediterranean Europe and Peru and compare the model's predictions with the relevant data from these two regions. The estimates are based on the data presented in Table 1. The size of the regulation cost parameter for Peru is the cost of regulation reported by Djankov et al. (2002). The regulation cost for Mediterranean Europe is the average of the Djankov et al. (2002) estimates for Portugal, Spain and Italy. Estimates of the values of the contract enforcement parameter for Peru and Mediterranean Europe are based on the average enforcement estimates in Table 1 derived from La Porta

Table 5  
Empirical data and model's predictions for reference economies

	$\phi$	$\frac{\xi}{y}$	Informal sector size, % of official output	Output per capita, % of U.S. level	
				Total	Official
Baseline case	0.25	0.005	10	100	100
Mediterranean Europe (data)	0.209	0.18	24	62	55
Model's predictions					
(1) Enforcement	0.209	0.005	13.4	94	91
(2) Regulation	0.25	0.18	25	94	82
(3) Enforcement and regulation	0.209	0.18	31	87	72
Peru (data)	0.13	0.20	60	11	8
Model's predictions					
(1) Enforcement	0.13	0.005	28	81	68
(2) Regulation	0.25	0.2	27	93	79
(3) Enforcement and regulation	0.13	0.20	63	74	41

Mediterranean Europe comprises Italy, Portugal and Spain.

et al. (1998). To arrive at the parameter value for  $\phi$ , we multiply the ratio of the average enforcement of a country/region to the U.S. value by the value of  $\phi$  used in the benchmark calibration. For Mediterranean Europe, the corresponding value of  $\phi$  is 0.209 and for Peru, the corresponding value of  $\phi$  is 0.13.

Table 5 reports the model's predictions for the size of the informal sector and per capita GDP for Mediterranean Europe and Peru. For Mediterranean Europe, the model over predicts the size of the informal sector. Whereas the output of the informal sector is 24% of total output in Mediterranean Europe, the model predicts a slightly larger figure of 31%. Regulation costs in Mediterranean Europe are far more important in accounting for the size of the informal sector compared to contract enforcement.

For Peru, the model matches the size of the informal sector fairly well. The model predicts that output of the informal sector in Peru is 63% of total output. In the data, output of the informal sector is 60% of total output. For Peru, contract enforcement and regulation costs are equally important in accounting for the size of the informal sector.

The model fails to account for the differences in outputs observed across these countries/regions and the United States. For Mediterranean Europe, the model predicts that the official GDP per capita is 72% of the U.S. level. This is far below the 1999 difference of 45% observed in the data. For Peru, the disparity between the model's predictions and the actual data is even wider. The model predicts per capita GDP in Peru equal to 41% the U.S. level. In the data, Peru's per capita GDP is 8% of the U.S. level.

#### 4.4. Sensitivity analysis

In this section we explore the sensitivity of our results to the parameter values used in the baseline. In particular, we consider how the predictions change when a higher value for the capital share parameter is used. The motivation for using a higher capital share parameter is that intangible capital is not recognized as an important input in the production process.

Table 6  
Higher capital share ( $\alpha = 0.45$ ). Empirical data and model's predictions for reference economies

	$\phi$	$\frac{\zeta}{y}$	Informal sector size, % of official output	Output per capita, % of U.S. level	
				Total	Official
Baseline case	0.25	0.005	10	100	100
Mediterranean Europe (data)	0.209	0.18	24	62	55
Model's predictions					
(1) Enforcement	0.209	0.005	12.9	84	83
(2) Regulation	0.25	0.18	20.4	87	80
(3) Enforcement and regulation	0.209	0.18	25.5	75	62
Peru (data)	0.13	0.20	60	11	8
Model's predictions					
(1) Enforcement	0.13	0.005	27	65	55
(2) Regulation	0.25	0.2	23	79	71
(3) Enforcement and regulation	0.13	0.20	54	54	32

Mediterranean Europe comprises Italy, Portugal and Spain.

The national accounts fail to measure investments in intangible capital as well as the income from this input. This suggests that capital's share of income is much larger than the value of 0.35 used in the baseline.

In light of this, we recalibrate the model using a higher capital share value equal to 0.45.<sup>14</sup> The labor share parameter is reduced to a value of 0.45. The other parameter values are reset so that the model matches the same set of observations listed in Table 2.<sup>15</sup> The recalibrated model continues to match the income inequality observed in the United States. It predicts a higher capital to output ratio equal to 3.5, since capital in the model consists of both an intangible component and a tangible capital component.

Table 6 examines the predictions of the recalibrated model for the size of the informal sector and output for Mediterranean Europe and Peru. The recalibrated model continues to match the size of the informal sector in Mediterranean Europe and Peru fairly well. It predicts larger differences in per capita output. However, these differences are still too small relative to the data. For Mediterranean Europe, measured per capita output is predicted to be 62% of the U.S. level, and for Peru, per capita GDP is predicted to be 32% of the U.S. level.

#### 4.5. Policy implications

In order to investigate the policy implications of our results it is important to understand the real counterpart of parameters  $\zeta$  and  $\phi$ . Parameter  $\zeta$  measures the *barriers to legality*. It corresponds to costs and procedures to comply with government requirements to start a business. These costs deter entrepreneurs from becoming formal and are most of them sunk. Djankov et al. (2002), for instance, show that in Italy entrepreneurs on average need

<sup>14</sup>If we use Gollin's (2002) adjustment and assume that for entrepreneurial income the mix of labor and capital is equal to that of the rest of the economy, then the effective capital income share will roughly be 0.5.

<sup>15</sup>In this case,  $\zeta = 0.00017$ ,  $\alpha = 0.45$ ,  $\beta = 0.45$ ,  $\phi = 0.25$ , and  $\varepsilon = 5.33$ .  $\gamma$ ,  $\tau$ , and  $r$  have the same values as before.

to follow 16 procedures, pay US\$ 3946 in fees, and wait roughly 62 business days to acquire legal status. In Canada, however, the process needs only 2 procedures, takes two days and costs around US\$280 in fees. Parameter  $\phi$ , on the other hand, is a proxy for *legal failures* in financial markets. When financial markets operate poorly, capital is misallocated and lucrative investment opportunities are forgone. According to De Soto (2000), capital markets fail in developing countries for two main reasons. First, the majority of residents in developing countries do not have legal tender of their property and therefore cannot use their assets as collateral for a loan to convert it into capital—notice that this is related to parameter  $\zeta$ . Second, even when their assets are formally registered their properties and financial contracts are not well enforced (*idem*, parameter  $\phi$ ).

Policies should therefore simplify the entry process, decrease bureaucracy and provide legal tender to informal assets. Moreover, a set of complementary institutional reforms to secure properties and debt contracts are needed, such as reforms of bankruptcy laws, banking regulation, and judiciary and enforcement systems. Such policies are consistent to those proposed by two recent reports from the McKinsey Global Institute (Farrell, 2004; Capp et al., 2005) about the deleterious effects of the hidden economy. These policies, however, are not easy to implement since they might change the status quo of politicians and bureaucrats who are the main beneficiaries from high regulation costs (Djankov et al., 2002) and a weak legal system. Path dependence in economic institutions is an important issue, but it is not the objective of this paper.

## 5. Concluding remarks

This paper contributes to the literature by characterizing how government policies and institutions interact with the distribution of wealth and entrepreneurial ability in a general equilibrium model with formal and informal sectors, regulation costs, and contractual imperfections. Formal entrepreneurs have better access to outside finance. The quantitative exercises suggest that:

- i. Compared to the degree of enforcement of financial contracts, regulation costs have a weaker effect on the informal sector size and productivity. Therefore, it is necessary higher variations in regulation costs than enforcement to explain observed differences in the informal sector size across countries.  
However, when we use independent estimates of enforcement and regulation costs we show that:
- ii. Regulation costs rather than limited enforcement in financial contracts account for the difference in the size of the shadow economy between Mediterranean Europe and the United States.
- iii. In countries with very weak enforcement, such as Peru, credit market imperfection explains roughly 50% of the difference in the informal sector size, while regulation costs explain the other half.

Regarding differences in productivity, the model fails to account for the differences in outputs observed across these countries/regions and the United States. For Peru, for instance, the model predicts a per capita GDP of 41% the U.S. level. In the data, Peru's per capita official GDP is 8% of the U.S. level. This is also an important lesson that contradicts De Soto's (2000) hypothesis that firms' inability to raise capital when they

operate in the informal sector is the main determinant of per capita output differences among countries. Our result is complementary to Parente and Prescott (2000), who show that factor accumulation alone does not explain the huge disparities in international incomes. Therefore, there must be other elements such as differences in total factor productivity (TFP) (Parente and Prescott, 2000) or tax code that must be relevant to determining output differences. Intuitively, for instance, a tax on financial intermediation might not only affect the size of the projects and then productivity, but also have an effect on the occupational choice of the agents. In this paper we do not learn how barriers to technology adoption or tax structure affect the informal sector size and then productivity. Those, however, are certainly relevant topics for future work.

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### Appendix A. Kuhn–Tucker conditions for problem (6)

The Lagrangean associated with problem (6) is

$$L_j = \pi_j(a_j + l_j, x; w) - r(a_j + l_j + \zeta \mathcal{J}_F) + \lambda_j[\mathcal{J}_F \phi \pi_j(a_j + l_j, x; w) - r(l_j + \zeta \mathcal{J}_F)] + \chi_j[b - a_j].$$

The Kuhn–Tucker conditions are:

$$\frac{\partial L_j}{\partial l_j} = \pi_{j1}(a_j + l_j, x; w) - r + \lambda_j[\mathcal{J}_F \phi \pi_{j1}(a_j + l_j, x; w) - r] \leq 0, \tag{17}$$

$$\frac{\partial L_j}{\partial a_j} = \pi_{j1}(a_j + l_j, x; w) - r + \lambda_j[\mathcal{J}_F \phi \pi_{j1}(a_j + l_j, x; w)] - \chi_j \leq 0, \tag{18}$$

$$\lambda_j[\mathcal{J}_F \phi \pi_j(a_j + l_j, x; w) - r(l_j + \mathcal{J}_F \zeta)] = 0, \tag{19}$$

$$\chi_j[b - a_j] = 0,$$

$$l_j \geq 0, \quad \frac{\partial L_j}{\partial l_j} l_j = 0, \quad a_j \geq 0, \quad \frac{\partial L_j}{\partial a_j} a_j = 0, \quad \lambda_j \geq 0, \quad \chi_j \geq 0, \tag{20}$$

along with the incentive compatible constraint and the upper limit on  $a_j$ . If the entrepreneur is credit constrained,  $\lambda_j > 0$ , that is, he would be better off if the credit constraint were eased. Notice first that, from (19),  $l_j = 0$ . We know that  $\pi_{F1}(k_F^*(x; w), x; w) = r$  and  $\pi_{F1}(a_F +$

$l_F(x; w)$  is decreasing with  $l_F$ . Notice that  $a_F + l_F \leq k_F^*(x; w)$ , since  $k_F^*(x; w)$  is the unconstrained optimal level of capital. Then, Eq. (18) implies  $\chi_F > 0$ , which implies by (20) that  $a_F = b$ .

**Appendix B. Proof of Lemma 1**

Continuity of  $V_j(b, x; w)$  follows from the *Maximum Theorem* and differentiability from Theorem 4.11 of *Stokey and Lucas (1989)*. From the envelope theorem it is easily seen that, provided  $x > 0$ ,

$$V_{j2}(b, x; w) = \pi_{j2}(b + l_j(b, x; w), x; w)(1 + \lambda_j \phi \cdot \mathcal{I}_F) > 0,$$

$$V_{j3}(b, x; w) = \pi_{j3}(b + l_j(b, x; w), x; w)(1 + \lambda_j \phi \cdot \mathcal{I}_F) < 0.$$

If  $b \leq k^*(x; w)$ , then

$$V_{j1}(b, x; w) = \pi_{j1}(b + l_j(b, x; w), x; w)(1 + \lambda_j \phi \cdot \mathcal{I}_F) > 0.$$

When  $b > k_j^*(x; w)$ , then by definition of  $k_j^*(x; w)$  the net income from entrepreneurship cannot increase and  $V_{j1}(b, x; w) = 0$ .  $l_j(b, x; w) = 0$  since there is no borrowing in the informal sector. For  $b > k_F^*(x; w)$  it is also obvious that  $l_F(b, x; w) + a_F(b, x; w) = k_F^*(x; w)$ . When agents are credit constrained, the incentive compatible constraint holds with equality and

$$\phi \pi_F(b + l_F(b, x; w), x; w) = r(l_F(b, x; w) + \varsigma).$$

Thus,

$$\frac{\partial l_F(b, x; w)}{\partial b} = \frac{\phi \pi_{F1}(k_F, x; w)}{r - \phi \pi_{F1}(k_F, x; w)}.$$

By condition (17), we have that  $r - \phi \pi_{F1}(k_F, x; w) = \pi_{F1}(k_F, x; w) - r/\lambda_F$ . Since this is for constrained agents,  $\lambda_F > 0$  and, as we have seen previously,  $\pi_{F1}(k_F, x; w)$  is greater than  $r$ . Therefore,

$$\frac{\partial l_F(b, x; w)}{\partial b} = \lambda_F \frac{\phi \pi_{F1}(k_F, x; w)}{\pi_{F1}(k_F, x; w) - r} > 0. \quad \square$$

**Appendix C. Proof of Lemma 2**

If agents have sufficiently high  $b$  and

$$\max_{j \in \{I, F\}} \{V_j(b, x; w)\} \geq w,$$

there is  $x^*(w)$  such that for  $x < x^*(w)$  agents prefer to be workers rather than managers:

$$x^*(w) = \min_{j \in \{I, F\}} \left\{ \left( \frac{r}{\alpha} \right)^\alpha \left( \frac{w(1 + \tau \cdot \mathcal{I}_F)}{\beta} \right)^\beta \left( \frac{w + r \varsigma \cdot \mathcal{I}_F}{1 - \alpha - \beta} \right)^{1 - \alpha - \beta} \right\}.$$

$x^*(w)$  is independent of  $b$ . For constrained agents with  $x \geq x^*(w)$ , we have that

$$\max_{j \in \{I, F\}} \{V_j(b, x; w)\} = w$$

defines  $b_e(x; w)$  such that

$$\frac{\partial b_e(x; w)}{\partial x} = -\frac{V_{j2}(b, x; w)}{V_{j1}(b, x; w)},$$

where  $j = \arg \max_{j \in \{I, F\}} \{V_j(b, x; w)\}$ , in all points where  $b_e(x; w)$  is differentiable. This is negative from Lemma 1. Define

$$G(b, x; w) = V_F(b, x; w) - V_I(b, x; w).$$

Provided  $G_1(b, x; w) \neq 0$ , by the implicit function theorem  $G(b, x; w) = 0$  defines  $b_s(x; w)$ , where

$$\frac{\partial b_s(x; w)}{\partial x} = -\frac{V_{F2}(b, x; w) - V_{I2}(b, x; w)}{V_{F1}(b, x; w) - V_{I1}(b, x; w)}.$$

We have

$$\begin{aligned} V_{F2}(b, x; w) - V_{I2}(b, x; w) &= x^{1/(1-\beta)} \left(\frac{\beta}{w}\right)^{\beta/(1-\beta)} \\ &\quad \times \left[ \left(\frac{(b+l)^x}{(1+\tau)^\beta}\right)^{1/(1-\beta)} (1 + \lambda_F \phi) - (b^x)^{1/(1-\beta)} \right], \end{aligned}$$

$$\begin{aligned} V_{F1}(b, x; w) - V_{I1}(b, x; w) &= \alpha x^{1/(1-\beta)} \left(\frac{\beta}{w}\right)^{\beta/(1-\beta)} \\ &\quad \times \left[ \left(\frac{(b+l)^x}{(1+\tau)^\beta}\right)^{1/(1-\beta)} \frac{(1 + \lambda_F \phi)}{b+l} - \frac{(b^x)^{1/(1-\beta)}}{b} \right]. \end{aligned}$$

Notice that  $V_F(b, x; w) \geq V_I(b, x; w)$  implies that

$$x^{1/(1-\beta)} \left(\frac{\beta}{w}\right)^{\beta/(1-\beta)} \left[ \left(\frac{(b+l)^x}{(1+\tau)^\beta}\right)^{1/(1-\beta)} - (b^x)^{1/(1-\beta)} \right] \geq \frac{r(l+\zeta)}{1-\beta} > 0.$$

Since  $\lambda_F \geq 0$ , this implies that the numerator of  $\partial b_s(x; w)/\partial x$  is always positive for  $V_F(b, x; w) \geq V_I(b, x; w)$ . For a given  $b, l$  increases and  $\lambda$  decreases with  $x$ , which implies that for sufficiently high  $x$  the denominator is negative and  $\partial b_s(x; w)/\partial x$  is positive (see Fig. 1).  $\square$

### Appendix D. Proof of Proposition 3

First we need to show that, for every bequest distribution, there exists a finite equilibrium wage rate that clears the labor market. Given the bequest and ability distributions,  $\Upsilon$  and  $\Gamma$ , define the excess demand function  $ED(w)$  by

$$\begin{aligned} ED(w) &= \int \int_{z \in E_F(w)} n_F(x; w) \Upsilon(db) \Gamma(dx) \\ &\quad + \int \int_{z \in E_I(w)} n_I(x; w) \Upsilon(db) \Gamma(dx) - \int \int_{z \in E^c(w)} \Upsilon(db) \Gamma(dx). \end{aligned} \tag{21}$$

The excess demand  $ED(w)$  is continuous since both  $n_j(x; w)$  and  $V_j(b, x; w)$  are continuous in  $w$  (see Eq. (4) and Lemma 1). In addition,  $n_j(x; w)$  and  $V_j(b, x; w)$  are also strictly decreasing in  $w$ . Notice that as  $w$  goes to zero, no agent wants to become a worker,  $V_j(b, x; w)$  is unbounded and  $ED(w) > 0$ . Analogously, when  $w$  increases, then  $ED(w) < 0$ . Therefore, by continuity of  $ED(w)$  there must be some  $w^*$  such that  $ED(w^*) = 0$ .

It remains to show that  $w^* \in [\underline{w}, \bar{w}]$ , where  $\underline{w} > 0$  and  $\bar{w} < \infty$ . Let us consider an initial bequest distribution that assigns zero bequest to all agents. Set  $E_F(w)$  is the measure of all agents for sufficiently small  $w$  as long as  $\phi > 0$ . In this case, the equilibrium wage rate,  $\underline{w}$ , is positive and finite, as stated in the previous paragraph. Since the wage rate is positive, next periods' bequests will all be positive. Therefore, the set of possible occupational choices cannot shrink, and might even expand. This implies that for the previous wage rate  $\underline{w}$ , the excess demand is non-negative,  $ED(w) \geq 0$ , which in turn means that for this new bequest distribution the wage rate that clears the labor market is  $w' \geq \underline{w}$ . Consequently,  $\underline{w} > 0$  is the lowest equilibrium wage rate for any initial distribution.

Now suppose an initial bequest distribution that assigns  $\bar{b}_0$  to all agents such that  $\bar{b}_0 \geq k_j^*(\bar{x}; \underline{w})$ . By the first argument in this proof, there exists a positive and finite equilibrium wage rate,  $\bar{w} < \infty$ . In this case, no agent is credit constrained. Either the smallest bequest,  $(1 - \gamma)(\bar{w} + r\bar{b}_0)$ , is higher than  $\bar{b}_0$ , in which case the next periods' equilibrium wage rate will be the same; or it is smaller than  $\bar{b}_0$  and the set of occupation choices might shrink. Therefore, for this new wealth distribution  $ED(\bar{w}) \leq 0$ . In this case the new equilibrium wage rate is  $w' \leq \bar{w}$ .

We can thus conclude that  $w_t \in [\underline{w}, \bar{w}]$  for all  $t$ . The maximum possible bequest is thus  $\bar{b}$  such that

$$\bar{b} = (1 - \gamma) \left( \max_{j=F,I} \{V_j(\bar{b}, \bar{x}; \underline{w})\} + r\bar{b} \right), \tag{22}$$

where we assume that  $(1 - \gamma)r < 1$ . On the other hand the minimum bequest is

$$\underline{b} = (1 - \gamma)(\underline{w} + r\underline{b}). \tag{23}$$

Define  $Z = [\underline{b}, \bar{b}]$  and  $z_t = (b_t, x_t)$ .  $Z$  is compact. Define the measurable space  $(Z, \mathcal{B})$ , where  $\mathcal{B}$  is the Borel algebra for the set. Define  $\Lambda(Z, \mathcal{B})$  as the set of all possible probability measures defined on the measurable space  $(Z, \mathcal{B})$ . For instance,  $Y_t$ , which specifies the probability of each event in  $\mathcal{B}$  at time  $t$ , belongs to  $\Lambda(Z, \mathcal{B})$ . Measure  $Y_t$  defines a non-stationary transition probability function,

$$P_t(b_t, A) = \Pr\{b_{t+1} \in A | b_t\},$$

for any  $(b_t, A)$  in  $(Z, \mathcal{B})$ . Function  $P_t$  assigns a probability to event  $A$  for the descendant of an agent that has bequest  $b_t$  but does not know yet  $x_t$ . We want to show that the operator  $T^* : \Lambda(Z, \mathcal{B}) \rightarrow \Lambda(Z, \mathcal{B})$  defined as

$$(T^* Y_t)(A) = \int P_t(b_t, A) Y_t(db_t), \tag{24}$$

where  $P_t$  is the transition function defined above, has a unique fixed point  $T^* Y = Y$  for any Borel subset  $A \in \mathcal{B}$ , given the initial bequest distribution  $Y_0$ .  $(T^* Y_t)(A)$  can be interpreted as the probability that the next period's state lies in  $A$  according to the present period's distribution. Of course,  $T^* Y_t = Y_{t+1}$ . Notice first that  $w_t$  is well defined for every distribution  $Y_t$ , as we argued previously. Second, we know that  $b_{t+1} = h(z_t; w_t)$ , where

$h(z_t; w_t) = (1 - \gamma)Y(z_t; w_t)$  (see equation 14), is increasing in  $z_t$  for any  $w_t$ , and  $Z$  is compact. Operator  $(Tf)(b_t) = \int f(b_{t+1})P_t(b_t, db_{t+1})$ , defined for any bounded function  $f : B(Z) \rightarrow B(Z)$ , where  $B(Z)$  is the set of real-valued bounded functions defined on  $Z$ , is the conditional expectation of function  $f$  at  $t + 1$  given that the state at  $t$  is  $b_t$ . Since, for any wage rate  $w_t \in [\underline{w}, \bar{w}]$ ,  $h(z_t, u_t)$  is bounded and increasing in  $b_t$ , and  $x_{t+1}$  is independent of  $b_t$ , the conditional expectation of  $f(b_{t+1})$  on  $b_t$  is also increasing and bounded provided that  $f$  is increasing. Intuitively, this means that, given the equilibrium wage rate  $w_t$ , an agent's descendant would never be worse off in terms of the expected value of  $b_{t+1}$  if, for any  $\varepsilon > 0$ , the agent's state were  $b_t + \varepsilon$  instead of  $b_t$ . As function  $Tf$  is increasing,  $T^*$  is increasing and  $P_t$  is a monotonic transition function.<sup>16</sup> By Corollary 2 of Hopenhayn and Prescott (1992), there is a fixed point for map  $T^*$ .

It remains to show that  $P_t$  satisfies the monotone mixing condition (MMC). First, define  $P_{t+n}(b_t, A) = \Pr\{b_{t+n} \in A | b_t\}$ . This is the  $n$ -step transition function beginning at  $t$ . We must show that the transition function  $P_{t+n}$  satisfies, for all  $t$ ,

$$P_{t+N}(\underline{b}, [b_a, \bar{b}]) > \varepsilon \quad \text{and} \quad P_{t+N}(\bar{b}, [\underline{b}, b_a]) > \varepsilon$$

for some  $b_a \in Z$ ,  $\varepsilon > 0$ , and  $N \in \mathbb{N}$ . Let us, for simplicity and without loss of generality, omit subscript  $t$ . Let  $w$  be the wage rate associated with the fixed point of map  $T^*$ ,  $Y$ . Define the minimum stationary bequest  $b_l$  such that  $b_l = (1 - \gamma)(w + rb_l)$ . Let  $b_a = (1 - \gamma)(w + rb_l) + \varrho$  for some small  $\varrho > 0$ . We now show that there is a positive probability that the  $N$ th descendent of an agent with  $b = \underline{b}$  receives a bequest above  $b_a$ . Notice first that the agent's descendants will have bequest in the vicinity of  $b_l$  in finite time because they will all be workers. Since the measure of sets  $E(w)$  and  $E^c(w)$  is non-zero and constant (as the labor market clears with wage in  $[\underline{w}, \bar{w}]$ ), and ability is independent across generations, there is a positive probability that a worker becomes entrepreneur and vice versa. Suppose that agents with ability in the vicinity of  $\bar{x}$  and bequest in the vicinity of  $b_l$  cannot have descendants that become entrepreneurs. Since all agents' descendants face a positive probability of having bequest in the vicinity of  $b_l$  in finite time (as they can have successive low  $x$ 's), this implies that the measure of agents (workers) in the vicinity of  $b_l$  is 1, a contradiction to the fact that  $E(w)$  has non-zero measure. Therefore, agents with ability in the vicinity of  $\bar{x}$  and bequest in the vicinity of  $b_l$  have descendants that become entrepreneurs. Moreover, they can become so in the following generation. This implies that they can also have bequest higher than  $b_a > b_l$  as long as they have a sufficiently high  $x$ , in which case they have high credit limits. Starting from  $b = \bar{b}$  is easier: a succession of low  $x$ 's leaves the agent's descendants with bequest lower than  $b_a$ , as they will become workers and remain so until one of them gets a sufficiently high  $x$ . Therefore, by Theorem 2 of Hopenhayn and Prescott (1992), there exists a unique time invariant distribution  $Y$  and associated equilibrium wage  $w$ , such that from any initial distribution  $Y_0$ , the operator  $T^* Y_t$  converges to  $Y$ .  $\square$

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<sup>16</sup>See Stokey and Lucas (1989, p. 220, 379).

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