Cheap Home Goods and Persistent Inequality*

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Abstract

There exists a large literature which shows that public education is favorable for growth because it increases the level of human capital and at the same time it tends to produce a more even income distribution. More egalitarian societies are also associated with less social conflicts and individuals have a lower tendency to report themselves happy when inequality is high. Therefore it is important to study the reasons why the elite opposes the development of a strong public education system. It might be that education is related to social status and a strong public education system might threaten the elite's political power. We show that one aspect of social status is the specialization of skilled workers in high-paid jobs and the abundance of unskilled workers in the production of cheap "home goods" in the market, such as painting and cleaning a house, babysitting and/or cooking. We emphasize the role of general equilibrium price adjustments to show that depending on the level of inequality, the elite might prefer an economy with a positive and "high" cost of education than an economy where skills are freely provided. We show that this result goes through even if the skilled wage is not directly affected by the ratio of skilled to unskilled workers. We also provide empirical evidence consistent with our theory.

JEL Classification: O11; J13

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"...in the traditional caste system groups in the population were condemned for life, and their descendants in perpetuity after them, to such task as the cleaning of latrines and the removal of dead carcasses... Apart from slavery, it is hard to think of a system with greater inequality of opportunity, and the results are also most unequal."

– Mancur Olson (1982: 156)

1 Introduction

When markets function perfectly inequality reflects differences in innate ability to acquire skills, to invest in capital, and/or to manage a labor force. This type of inequality is efficient. In this case, wealth, social status, caste, and/or family connections would not affect individual outcomes. However, as argued by Banerjee (2006), markets do not *"work anywhere close to perfect"*. Empirical evidence, for instance, shows that credit access and borrowing interest rates depend on wealth and social status (see Banerjee (2006) for some examples). In human capital investment, parents cannot borrow against their children's future income. Consequently, poor individuals under-invest in both physical and human capital and inefficient inequality persists over time.[1]

Galor and Zeira (1993) formalized well this idea. Their theory emphasizes the role of credit market imperfections and non-convexity in human capital investment in the persistence of income inequality. Recent empirical evidence (see Easterly (2005)) has shown that *"inequality does cause underdevelopment."* In the model presented in Galor and Zeira (1993), for instance, it is straightforward to design a policy to reduce inequality that is Pareto improving and would also increase development.[2] Contrary to the traditional *efficiency-equity* trade-off, such policies might increase efficiency and improve income distribution. Therefore, a complementary and important question is: Why countries do not adopt policies to improve the functioning of their credit market and/or their educational system?

If the skilled wage depends negatively on the ratio of skilled to unskilled workers,[3] then any increase in this ratio would lower the skilled wage. This direct wage effect explains the opposition of skilled workers to compulsory schooling in, for instance, Doepke and Zilibotti (2005). We present a related but alternative channel to explain why skilled agents might prefer a non-egalitarian education system. In our model, there is one good that can be produced in the market with a linear technology that uses skilled labor as the only input. By assuming a linear technology we rule out this direct wage effect that is present in previous studies. Agents also value another good, which can be produced at home or in the market with skilled or unskilled labor. Agents have similar productivity in the production of the "home" good. When inequality is high, rich families are able to buy home goods in

[1] There is no equality of opportunities when wealth and social status affect outcomes.

[2] Government could issue bonds to finance education. The skilled descendants of unskilled parents would pay the debt. This policy is Pareto improving as long as the tax paid by skilled agents of unskilled descendants is not higher than the difference between the skilled and unskilled wages. Alternatively, multilateral agencies might provide financial aid to developing countries to improve their public education systems to institute free, compulsory education system or to improve the functioning of the credit market.

[3] The production function is Cobb-Douglas in skilled and unskilled labor. For such case, see Doepke (2004) and Doepke and Zilibotti (2005).

the market (i.e., they can hire domestic servants, nannies, house painters, and others) that in more egalitarian societies they would have to produce themselves. Therefore, rich agents might lose from improved access to high quality education, because even if the market price of their skills in unaffected, the lessened availability of cheap home goods would increase their price and the welfare of the initially educated might decline. As a result, skilled agents have a vested interest to erect barriers to school access.

The hypothesis that the ratio of skilled to unskilled workers does not affect directly the skilled wage is supported by the Heckscher-Ohlin-Vanek (HOV) international trade theory. A large fraction of skilled agents works in the tradable sector, which is subject to factor-price equalization, as the HOV theory predicts.[4] Therefore, changes on the ratio of skilled to unskilled workers would not affect skilled wages.[5] On the other hand, the home goods sector is a non-tradable one, which makes the relative price of home goods to be determined endogenously in the economy. Therefore, the independence of the skilled wage from the skilled to unskilled workers ratio could be interpreted as a reduced-form representation of the assumption that the market goods sector is a tradable one.[6] This is consistent with the findings of Cortes (2008) who shows that low-skilled immigration in the United States has reduced the price of immigrant intensive services, such as housekeeping and gardening, but with little effect on the price of traded goods.

The details of our theoretical model are the following. As in Ríos-Rull (1993), there are two differentiated consumption goods: *good* Y, which is produced in the market with a homogenous of degree one technology that uses skilled labor as only the only input (ex., cars, computers and others); and *good* Z, which can be produced at home or in the market with both skilled and unskilled labor (ex., babysitting, cleaning a house, painting a house, and others). As mentioned before, Skilled and unskilled workers have similar productivity in the technology of the "home" good. Parents care about the consumption of both goods and about the average discounted utility of their children. Acquisition of skills is costly and parents cannot borrow against their children's future income. When inequality is high, skilled parents educate their children, specialize in the production of good Y and do not work at home. They buy "cheap" good Z in the market. On the other hand, unskilled parents do not educate their children and do not buy good Z in the market. They instead produce it at home. They will work in the market and at home. Inequality will therefore persist over time, as in Galor and Zeira (1993).

In addition, we show that depending on the initial inequality, skilled parents might prefer an economy with a positive and "high" cost of education than an economy where skills are freely provided.[7] If education is freely provided, then unskilled parents will educate their children and general equilibrium price adjustment implies that future prices of the "home good" (good Z) will increase, while

[4] Empirical results from OECD countries have supported factor price convergence (e.g., Mokhtari and Rassekh (1989)). Although, Freeman and Oostendorp (2000) have shown that wages in the same occupation vary considerably across countries, Trefler (1993) and Zhu and Trefler (2005) show that once productivity differences are considered, the HOV theory explains much of the cross-country variation in factor prices and the recent trend in inequality in developing and industrialized countries.

[5] Indeed, Mayda (2006), using individual level data, shows that more educated agents are more open to immigration.

[6] We thank a referee to have suggested to link the trade literature to the assumption that the skilled wage is independent of the ratio of skilled to unskilled workers.

[7] We show that depending on the level of inequality, there is a threshold value of education costs in which unskilled parents are just indifferent between sending or not their children to school.

the skilled wage will remain unchanged. Since parents value the future utility of their children, the welfare of skilled parents might be reduced. Depending on the political power of skilled agents, they therefore might block any policy that decreases the cost of skill acquisition.

The paper proceeds as follows. Section 2 relates our work to the existing literature and describes some empirical narratives consistent with our results. It also provides some empirical results based on cross-country aggregate data. Section 3 presents the model. Section 4 analyzes the equilibrium and derives the main results. Section 5 presents some possible extensions of our model economy. Section 6 provides some concluding remarks.

2 Related literature and some empirical evidence

This paper is related to two strands of literature. The first literature focuses on the intergenerational transmission of inequality and its persistence. It emphasizes the role of credit market imperfections and some form of non-convexity in human and physical capital investment to show how inequality persists over time (e.g., Aghion and Bolton (1997), Banerjee and Newman (1993), Galor and Zeira (1993), and Ray and Strefert (1993)).[8] Our analysis complements this literature by investigating when it is optimal for some groups in society to block policies and institutions that might increase development and decrease inequality. In this respect, Acemoglu (2005), Acemoglu and Robinson (2000), Bourguignon and Verdier (2000), Doepke and Zilibotti (2005), Grossman and Kim (1999), and Parente and Prescott (1999, 2000) are closest to our work. They all emphasize the conflicts of interest among social classes in the persistence of high inequality or inefficient technology.[9] We differ from this set of papers because we explicitly consider the role of cheap home goods (i.e., general equilibrium price adjustments) in the persistence of income inequality,[10] and we believe we are the first to emphasize and formalize the role of such goods in the persistence of income inequality.

There is also a large literature in economic history with empirical narratives consistent with the view that the "elite' has a vested interest on high costs to skill acquisition. Below, we describe some of them. Sokoloff and Engerman (2000), for instance, show how the "elite" of some countries protected their *status quo* by investing poorly in primary schooling or/and by erecting barriers in the right to vote and other privileges. According to them, the degree of political power of elites was indeed associated to the inequality in wealth and human capital in the society. Using cross-countries data, Easterly (2005) shows that not only inequality cause underdevelopment, but it is also

[8]Interestingly, Moav (2005) shows that inequality might be persistent even when the schooling choice is convex. He differs from the above literature because in his model individuals' productivity as teachers increases with their own human capital. Another related view emphasizes the tendency of the market mechanism and imperfections in the credit market to create inequality. Mookherjee and Ray (2002, 2003) show (see also Ljungqvist (1993)) that if several occupations requiring different levels of skills are necessary, wages must adjust to force separation in choices even if all individuals are *ex-ante* identical. Current individuals will have the same payoffs, but since credit market is imperfect, future generations will have different payoffs and inequality will persist.

[9]Differently, Galor and Moav (2006) show that due to complementarities between physical and human capital in production, capitalists might have incentives to support public education which benefits directly the working class.

[10]Home production has been explicitly treated in general equilibrium models in the study of business cycles (e.g., Benhabib, Rogerson and Wright (1991), Greenwood and Hercowitz (1991), and Ríos-Rull (1993)), and economic development (e.g., Parente, Rogerson, and Wright (2000) and Greenwood and Seshadri (2005)).

a significant and independent barrier to high schooling. Recently, de la Croix and Doepke (2008) argue and show how inequality maps into a segregated education system with poor public schools, which induce rich parents to send their children to private schools. Another empirical evidence is the paternalism, a system of social control in the U.S. South that emerged in the late 19th century and characterized the American South in the first half of the 20th century. Alston and Ferri (1993) argue that the paternalism[11] comprised a variety of laws and practices, such as low level of expenditure on education and the exclusion of blacks and poor whites from the electoral process. Landowners also prevented the appearance of public welfare programs that could substitute this system of social control until the mechanization of the cotton harvest in the 1950's.

In his study of inequality and development, Mancur Olson (1982: 162-163) exemplifies well in a case study of South Africa how the fear of general equilibrium price adjustments might lead skilled agents to erect barriers to the acquisition of skills: "*...The mine owners and management needed labor and naturally preferred to secure it at low wages rather than high wages. Since Africans had few other opportunities outside the traditional sector of African society, they were often available at low wages... European workers were employed in the mines mainly as foreman and skilled and semi-skilled laborers. It was far clear that the far-cheaper African laborers could at very little cost soon be taught the semi-skilled jobs and the employers naturally coveted the savings in labor cost that this would bring.*" However, the Mines and Work Acts of 1911 and 1926 ("Color Bar Acts") constrained employers in their use of African labor in semi-skilled and skilled jobs. "*The denial of various skilled and semi-skilled jobs to Africans not only raised the wages of the European workers, but it also crowded more labor into areas that remained open to Africans, making the wages there lower than they would otherwise be.*"

Lindert (2004) provides similar historical evidence. He shows, for instance, that the Tory opposition view, which was against mass education, was prevalent in England and other European countries in the beginning of the 19th Century. The fear by that time was that education would cause laborers to leave agriculture and other "*laborious employment to which their rank in society had destined them*".[12] According to Lindert (2004), there is also a positive correlation between voting rights, the spread of political voice and schooling. Historical differences in the spread of suffrage explain which countries' children got educated in the nineteenth and twentieth centuries.

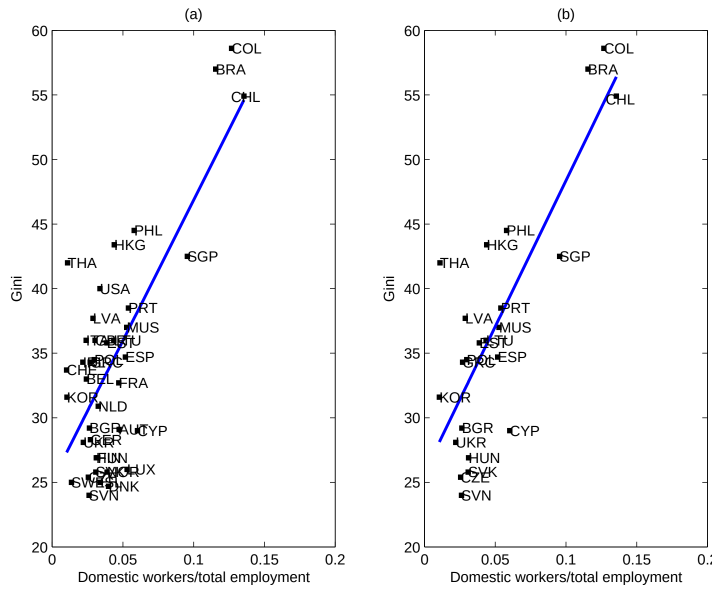
Another empirical evidence related to our theory is India's elitist education policy, which is more biased in favor of the highest educated. Lindert (2003) shows that although most provinces passed compulsory education laws in the 1930s and 1940s, due to the absence of funding and enforcement, such policies were not effective in India. He shows that despite having a democracy, political voice in practice is not spread in India and the traditional caste system seems to be the reason of such political exclusion, since local rule remains concentrated into the elite.

Although the empirical narratives above are consistent with our theory (based on the availability of cheap home goods and services), they are also consistent with other theories of inequality persistence

[11] They define it as an implicit contract in which workers trade "dependable" labor services in exchange for housing, credit and protection.

[12] See Lindert (2004: 100).

Figure 1: Inequality (Gini index) versus share of domestic workers over total employment. Source: Gini index - United Nations (2007). Share of domestic workers over total employment: LABORSTA, International Labor Organization. Countries were selected by data availability. The straight line represents the regression of the Gini index on the share of domestic workers over total employment; t-statistic is in parentheses. Panel (a): 39 industrialized and developing countries; Panel (b): 23 developing countries plus Greece, Spain and Portugal.



based, for instance, on general equilibrium effects on the skill wage premium. We therefore provide a more direct test of our theory based on cross-country aggregate data. Simple correlation shows that indeed the fraction of domestic workers over total employment is positively related to inequality (see Figure 1). Table 1, Panel A, reinforces such result using regression analysis. It shows that the share of domestic workers over total employment is positively correlated with inequality even when we control for the level of development, and whether or not the country is a member of the OECD countries.[13] In Figure 1 we can observe that Latin American countries are outliers with respect to the level of inequality and the share of domestic workers over total employment. Therefore, in order to check whether or not these countries drive our empirical results, we introduce in all regressions a dummy variable for Latin American countries.[14] Notice that the coefficient of the variable "share of domestic workers" is statistically significant at 99 percent of confidence level for all specifications. Moreover, in our most parsimonious specification, in column (1), this variable alone explains about 45 percent of total variability in income inequality.

Table 1, Panel B, also shows that the share of domestic workers over total employment is correlated with the overall quality of education, measured by the students' average score in the math PISA test.[15] Countries with a large share of domestic workers have a low score in the 2006 PISA math

[13]Thirty countries are member of the Organization for Economic Co-operation and Development. They include most developed countries and some emerging market economies.

[14]For instance, in Brazil, which is one of the most unequal country in the world, among the female-dominated occupations, maid is the one with the highest number of workers (Fava and Arends-Kuenning (2008)).

[15]The Programme for International Student Assessment (PISA) is an internationally standardized assessment developed by the OECD to 15 years old students. Students in about 62 countries, including all OECD countries and some developing countries took the test in 2006.

Table 1: How the share of domestic workers over total employment affects inequality and education performance. Source: See Figure 1 for the Gini index and the share of domestic workers over total employment; GDP per capita: Heston, Summers and Aten (2006).

	Panel A: Log Gini Index			Panel B: Log 2006 PISA Score (Math)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Share of domestic workers	4.58***	4.59***	4.40***	-1.90**	-1.81***	-1.76**	-1.27*
	(3.51)	(4.54)	(3.38)	(-2.09)	(-2.85)	(-2.61)	(-1.75)
GDP per capita		$-5.86e^{-06}$*	$-5.15e^{-06}$		$3.54e^{-06}$**	$3.21e^{-06}$*	$2.73e^{-06}$
		(-1.95)	(-1.36)		(2.34)	(1.99)	(1.40)
OECD	No	No	Yes	No	No	Yes	Yes
Latin American dummy	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Gini							-0.003
							(-1.34)
N. of Observ.	39	39	39	34	34	34	34
Adjusted R^2	0.45	0.52	0.52	0.47	0.58	0.59	0.62

All specifications include a constant, not reported. T-Statistics are presented in parentheses, using heterosk.-consistent standard errors. *,**,*** indicate statistical significant at the 90, 95 and 99 percent confidence level, respectively.

test.[16] It is important to observe that the sign and statistical significance of this result is robust to the inclusion of income per capita, OECD dummies, and the Gini index of income. Notice that the Gini index should be correlated with the observed skill wage premium since income inequality is related to how skills are distributed in a given country. Therefore, indirectly the Gini index controls for the skill wage premium. Estimations of Table 1 are therefore consistent with our analytical results and provide direct evidence for our theory. It is worth to emphasize that the causality might also run in the opposite direction. For instance, strong barriers to skill acquisition might lead to a large fraction of domestic workers over total employment. In our theoretical model we are able to investigate both effects and therefore the direction of the causality is not an important issue to test empirically our theory. In addition, there might also be problems of omitted variables. A comprehensive empirical analysis of how cheap home goods affect inequality and the quality of education goes, however, beyond the objective of this paper, which is mainly to provide a political economy argument of how cheap home goods affect the decision of the society to invest or not in a strong public education system.

Finally, some recent studies have provided results that are also consistent with some of our modeling hypotheses. In an interesting article, Cortes and Tessada (2008), using cross-city variation in low-skilled immigrant (c.f., low-skilled immigrants represent a large fraction of the labor employed in domestic services) concentration in the United States, show that very skilled women (those with a professional degree and or Ph.D.) have increased significantly their supply of market worked hours as a result of low skilled immigration, and decreased their time spent in household chores. Interestingly, as in our model, there is no evidence of similar effects for any other education group of the female population.

[16]Results are similar if we use the reading or the science exams instead of the math exam. The dependent variables are in log to make the interpretation of coefficients easier. The statistical significance of the independent variable "share of domestic workers" is robust when we consider the dependent variables in level instead of log.

3 The model

Overlapping generations of agents with differentiated skill levels populate the model economy. For simplicity, there are only two skill levels: unskilled and skilled, $h \in \{S, U\}$. Each household consists of one parent and her children. Fertility differential is exogenous and we assume that skilled parents have a small number of children $(n = P)$ while unskilled parents have a large number of children $(n = G)$, with $(P < G)$. This assumption is based on empirical evidence and the fact that unskilled people have more incentives to have greater number of children because of their lower opportunity time cost (de La Croix and Doepke (2003)).[17] Adults make all decisions and they decide on the education of their children, consumption, and labor supply.

There are two consumption goods in this economy, Y and Z. Good Y is produced in the market with skilled labor only. Good Z, on the other hand, can be produced at home or in the market with both skilled and unskilled labor. Skilled and unskilled agents have similar productivity in the production technology of good Z. Children might either work at home $(e = 0)$ or go to school $(e = 1)$. Working children provide $l \in (0, 1)$ units of unskilled labor at home. When they become adults working children become unskilled workers, while those that went to school become skilled workers. There is an education cost ϕ per child and children that attend school do not supply any labor.

Production technologies

The production side follows closely Ríos-Rull (1993). There are two market technologies in this economy. We assume that there is a continuum of firms in each sector that are competitive in output and factor markets. Since technologies exhibit constant returns to scale profits are zero and firm ownership in unimportant.

Skilled labor sector

Technology in the sector Y uses only skilled labor. The production process in this sector is represented by:

$$Y = AL_S, \quad A > 0, \tag{1}$$

where L_S corresponds to the amount of skilled labor units employed in the production of good Y, and A is a positive productive factor.

Unskilled labor sector

Good Z can be produced in the market or at home, where all agents have the same productivity. The market technology for good Z is represented by

$$Z^M = \delta B L^M, \quad \delta \in (0, 1), \; A > B > 0, \tag{2}$$

[17] In an earlier version of this paper fertility decision was endogenous. However, as pointed out by a referee, all results go through with a constant fertility differential. Since endogenous family size does not add any new insights to the analysis, we therefore abstract from fertility decisions. Some of our results go through even if fertility is the same for skilled and unskilled parents (i.e., $P = G$). As we will show briefly, however, the higher the fertility differential ($\frac{G}{P}$), the higher are the incentives of skilled parents to block policies that improve access to education.

where L^M corresponds to hours employed by the representative firm in the production of good Z, and B is a positive constant.

Agents are more productive at home in the production of good Z than in the market, such that

$$Z^H = BL^H, \quad A > B > 0. \tag{3}$$

The units of Z produced at home, however, cannot be transferred, or used to buy market consumption goods.

Let good Y be the numeraire, q be the market price of good Z, and w_U and w_S be the unskilled and skilled wage. All prices are in terms of the consumption good Y. Profit maximization implies that $w_S = A$, and $\frac{w_U}{q} \geq \delta B$, with equality if $Z^M > 0$. Given the linearity of the production functions, we just need one equilibrium market condition to define w_U and then q. Notice that given that skilled agents can also work in the unskilled sector, it implies that the skilled wage cannot be lower than the unskilled one.

Households:

Let $V_{nh}(\Omega)$ denotes the utility of an adult with n children and skill h and let Ω be the aggregate state of the economy, which is explained below. Parents care about the consumption of goods Y and Z, and about the average discounted utility of their children, which is discounted by $\gamma \in (0,1)$. Let c and z be the household consumption of good Y and Z, respectively. Let a be the time spent at home in the production of good Z. The problem of an adult with n children and skill h is represented by

$$V_{nh}(\Omega) = \max_{e,c,z^M,a} \{\ln c + \alpha \ln z + \gamma[eV_{PS}(\Omega') + (1-e)V_{GU}(\Omega')]\} \tag{4}$$

Subject to

$$c + q(\Omega)z^M + \phi en \leq w_h(\Omega)(1-a), \tag{5}$$

$$z = z^M + Ba + Bn(1-e)l, \tag{6}$$

$$e \in \{0,1\}, c \geq 0, z^M \geq 0, a \in [0,1], nh \in \{PS, GU\}. \tag{7}$$

The budget constraint (equation (5)) states that the sum of expenditures on consumption of good Y and good Z, and on education costs cannot exceed labor income. Equation (6) states that good z can be bought in the market or can be produced at home. The last term on the right hand side of (6) implies that children who do not go to school help their parents in the production of good z at home. Equation (7) shows the constraints on choice variables.

Notice that education is a non-convex choice, $e \in \{0,1\}$, and households do not have the option of educating a fraction of their children.[18] This clearly simplifies the analysis. If we allow households to educate a fraction of their children, then their decisions might depend on current and future states,

[18] In addition, households value each child in the same way. When households educate a fraction of their children, then their descendants might end up with different utilities. For similar approaches, see Doepke and Zilibotti (2005) and Doepke (2004).

which would make it difficult to derive analytically the main results of this paper. With the non-convex assumption, decisions depend only on current state variables (see Appendices A and B). We will, however, consider the case in which a fraction of unskilled households educates their children, while another fraction does not send their children to school.

Assume that initially there is a number X_{GU0} of unskilled parents with a large family size and a number X_{PS0} of skilled parents with a small family size.

Assumption 1: $\alpha > lG$.

Assumption 1 defines an upper bound on the quantity of good Z produced by children.

Proposition 1 *Consider the problem of an unskilled parent with a large family size,* $nh = GU$*. Under assumption 1:*

i. $z_{GU}^{M} = 0$ *and* $a_{GU} > 0$.

Proof. See appendix A ■

Proposition 1 suggests that unskilled parents with a large family size do not buy good Z in the market. They instead produce it at home. Therefore, unskilled parents work in the market either to buy consumption good Y or/and to pay for their children's education. Assumption 1 guarantees that unskilled parents with working children will also work at home. If the productivity of their children were too high ($lG > \alpha$) then only children will produce good Z at home.

In order to investigate the problem of skilled parents, let's assume that education costs are small relatively to the skilled market wage. Otherwise, there will be no skilled agents in the economy.

Assumption 2: $A > \phi P$.

Proposition 2 *Consider the problem of a skilled parent with a small family size,* $nh = PS$*. It can be shown that under assumption 2.*

i. If $q(\Omega) < \frac{A}{B}$*, then* $a_{PS} = 0$*,* $z_{PS}^{M} > 0$*. Otherwise,* $a_{PS} > 0$*,* $z_{PS}^{M} = 0$.

ii. If $e_{GU} = 1$*, then* $e_{PS} = 1$.

Proof. See appendix B ■

Item (i) implies that if the market price of good Z is small relatively to the skilled wage, i.e., if $q(\Omega) < \frac{w_S}{B} = \frac{A}{B}$, then skilled parents will not produce good Z at home, they will instead buy it in the market. Therefore, in order to have market demand for good Z its price cannot be "too" high. Otherwise skilled agents will produce it at home. Notice that this also implies an upper bound for the unskilled wage, $w_U(\Omega) < \delta w_S$. The intuition is straightforward: If the price of good Z is low enough, it is optimal for skilled agents to specialize in the market production of good Y and do not produce good Z at home. Item (ii) suggests that if unskilled parents educate their children, then it is also optimal for skilled agents to educate their children. Therefore, in order to have production of good Y skilled agents must educate their children. Recall that X_{GU0} and X_{PS0} are the initially number of unskilled agents with a large family size and skilled agents with a small family size, respectively. The number of unskilled agents and the number of skilled agents evolves according to:

$$X'_{GU} = GX_{GU}(1 - e_{GU}), \tag{8}$$

$$X'_{PS} = PX_{PS} + GX_{GU}e_{GU}. \tag{9}$$

The state variable, Ω, that describes the position of the economy in each period of time corresponds to the ratio of unskilled and skilled workers, $\Omega = \frac{X_{GU}}{X_{PS}}$. As we will show shortly, the unskilled wage depends negatively on Ω. The skill wage premium ($\frac{w_S}{w_U(\Omega)}$), on the other hand, depends positively on the state variable. Therefore, we can view Ω as a measure of income inequality. From (8) and (9), we have that

$$\Omega' = \frac{X'_{GU}}{X'_{PS}} = \frac{GX_{GU}(1 - e_{GU})}{PX_{PS} + GX_{GU}e_{GU}} = \frac{G(1 - e_{GU})}{P + G\Omega e_{GU}}\Omega. \tag{10}$$

Given the law of motion of the aggregate state variable, we define the recursive competitive equilibrium as follows.

Definition: *A Recursive Competitive Equilibrium for this economy consists of value functions* V_{nh}*, policy function (*e_{nh}*,* c_{nh}*,* z^M_{nh}*,* a_{nh}*), for* $nh \in \{GU, PS\}$*, price function (*w_S*,* w_U*,* q*), and a law of motion* $\Omega' = F(\Omega)$ *for the aggregate state variable such that*

i. V_{nh} *satisfy Bellman equation (4) and* e_{nh}*,* c_{nh}*,* z^M_{nh}*,* a_{nh} *are the associated policy functions.*

ii. $w_S = A$ *and* $\frac{w_U}{q} \geq \delta B$*. Moreover, good* Y *market clears*

$$X_{GU}c_{GU} + X_{PS}c_{PS} + \phi GX_{GU}e_{GU} + \phi PX_{PS}e_{PS} = AX_{PS}(1 - a_{PS}) \tag{11}$$

iii. *Law of motion* $F(\Omega)$ *satisfies equation (10).*

The equilibrium condition (11) states that the demand for good Y should be equal to the supply of good Y. The demand consists of total consumption of good Y and education expenditures by skilled and unskilled households. Recall that education costs are in terms of good Y.

4 Equilibrium Analysis

Now let's analyze the equilibrium with fixed policies. There might be three equilibria for this economy:[19]

i. Just skilled parents educate their children.

ii. Skilled and unskilled parents educate their children.

[19]Notice that item (ii) of Proposition 2 rules out the equilibrium in which only unskilled parents educate their children.

iii. Skilled and a fraction of unskilled parents educate their children.

Just skilled parents educate their children.

Let's consider the first equilibrium in which only skilled parents educate their children. See Appendix C, part I, for a full characterization of this equilibrium path. In this equilibrium, there will be market production for good Z in all periods. Skilled workers will specialize in the production of good Y, devoting a zero fraction of their time endowment to the home production of good Z. They will instead buy good Z in the market. Using condition (11), it can be shown that the unskilled wage will be given by:

$$w_U(\Omega) = \min\{\delta w_S, \frac{\alpha(A - \phi P)}{\Omega(1 + lG)}\}. \tag{12}$$

The ratio of unskilled to skilled workers will grow at rate $\frac{G}{P} - 1 > 0$, i.e., $\Omega' = \frac{G}{P}\Omega$. Therefore, at some time period $\bar{t} \geq 0$, the measure of unskilled agents relatively to skilled agents will be sufficiently large, i.e., $\Omega = \frac{X_{GU}}{X_{PS}} > \frac{(A-\phi P)}{\delta A} \times \frac{\alpha}{(1+lG)}$, such that $w_U(\Omega') = \frac{P}{G} w_U(\Omega)$. Since $G > P$, we have that the skill wage premium ($\frac{w_S}{w_U(\Omega)}$) and therefore inequality will increase over time. The market price of good Z will, on the other hand, decrease over time. Inequality will be persistent for two reasons: (i) Unskilled parents will have a larger family size than skilled agents; and (ii) they will not educate their descendants, increasing therefore the skill wage premium. Notice that there are two effects here: First, when the skill premium increases, the benefits of education for unskilled parents also increase. In addition, the price of consumption relatively to $w_U(\Omega)$ also increases. This increases the cost of education for unskilled parents. Observe that, if parents have the same family size ($G = P$), then inequality will still be persistent, but the skilled wage premium would in this case be constant. It can be shown that the growth rate of output per capita will be lower the higher is the differential in fertility ($\frac{G}{P}$) between unskilled and skilled parents,[20] which is consistent with the theory and data presented by de la Croix and Doepke (2001).

Appendix C, part I, shows that we can define a $\phi G < w_U^*$, such that for all $w_U(\Omega) < w_U^*$ it is optimal for an unskilled parent to not educate their children when all other unskilled parents are not sending their children to school. For some parameter values, we can also guarantee that w_U^* is finite and $w_U^* < \delta w_S$. In the case that $w_U^* > \delta w_S$, we have that independent of the level of inequality, it is always optimal for unskilled parents to not send their children to school.

Skilled and unskilled parents educate their children.

Let's consider the second equilibrium in which both skilled and unskilled agents educate their children. See Appendix C, part II, for a full characterization of this equilibrium path. In this case, there will be market production of good Z only in the first period. From the second period on, all

[20]Gross domestic output (GDP) per worker will be $Y + q(\Omega)Z^M = [A + \frac{\alpha(A-\phi P)}{(1+\alpha)}]\frac{1}{1+\Omega}$. Notice that here, as in the National Accounts, we are considering only the value of market production. If we also consider the value of home production, then GDP per worker will be $Y + q(\Omega)(Z^M + Z^H) = [A + \frac{\alpha(A-\phi P)}{(1+\alpha)}(1+\frac{\alpha}{\delta})]\frac{1}{1+\Omega}$, which also decreases over time. We, however, could have assumed that the productivity factors (A and B) and education costs ϕ are all increasing at some exogenous rate μ, and then output would grow over time as in the standard neoclassical growth model. This, however, would not add any new insights in our analysis.

agents will be skilled and all will devote a positive fraction of their time endowment to the home production of good Z.[21] It can be shown that the initial unskilled wage is:

$$w_U(\Omega) = \min\{\delta w_S, \frac{\alpha(A-\phi(P+\Omega G))}{\Omega}\}. \tag{13}$$

Since from the second period on there will be only skilled agents, we have that $\Omega' = 0$.[22] Inequality will vanish and skilled agents will have to work at home in the production of good Z. From the second period on, output per worker will be constant over time.[23] Therefore, as in Banerjee and Newman (1993) and Galor and Zeira (1993), the initial conditions determine final outcomes.

Appendix C, part II, shows that depending on parameter values there exists a $\phi G < w_U^{\bullet} < \delta w_S$, such that for all $w_U(\Omega) > w_U^{\bullet}$ it is optimal for unskilled parents to send their children to school when all other parents are also sending their children to acquire skills. Moreover, we can also show that $w_U^* < w_U^{\bullet}$. Therefore, for some parameter values, we have that if the initial inequality is "too" high, such that $w_U(\Omega) < w_U^*$, then the economy will be characterized by the first equilibrium with persistent and increasing inequality in which unskilled parents do not educate their children. On the other hand, if the initial inequality is relatively low, such that $w_U(\Omega) > w_U^{\bullet}$, then the equilibrium path will be consistent with an economy with decreasing inequality and high level of income per capita. There is also an equilibrium in which just a fraction of unskilled parents sends their children to acquire skills. See below.

Skilled and a fraction of unskilled parents educate their children.

In this equilibrium, a fraction of unskilled parents educate their children. Since parents give the same weight in the utility function to each child, we must have that unskilled parents are indifferent to send or not their children to school. Let η be the fraction of unskilled parents that educate their children. In this case, the unskilled wage is given by:

$$w_U(\Omega) = \frac{\alpha(A-\phi(P+\eta\Omega G))}{\Omega(\eta+(1-\eta)(1+lG))}. \tag{14}$$

The state variable evolves according to:

$$\Omega' = \frac{(1-\eta)G}{P+\eta\Omega G}\Omega. \tag{15}$$

For analytical purposes, let's focus on a stationary equilibrium in which the ratio of unskilled to skilled parents is constant (i.e., $\Omega' = \Omega$). There might be other equilibrium paths in which, for instance, the share of unskilled parents educating their children oscillates over time. The fraction of unskilled

[21] In a recent article, Ramey (2009) shows that across the entire population, per capita time spent in home production in the United States remained roughly constant over the last century.

[22] The skill wage premium will be at its lowest value $\frac{w_S}{w_U(\Omega')} = \frac{1}{\delta}$.

[23] Output per worker in the initial period is $Y + q(\Omega)Z^M = [A + \frac{\alpha(A-\phi P)}{(1+\alpha)}]\frac{1}{1+\Omega}$, while in the second period on it will be $Y = \frac{A+\alpha\phi P}{1+\alpha}$.

parents that sends their children to school is:

$$\eta = \frac{G - P}{(1 + \Omega)G}. \tag{16}$$

This equilibrium will be similar to the first equilibrium, except that inequality will be constant instead of increasing. See Appendix C, part III, for a characterization of this equilibrium path. For some parameter values, it can be shown that there exists a $w^{***} \in (w_U^*, w_U^\bullet]$, such that if $w_U(\Omega) = w^{***}$, then the economy will be represented by this equilibrium with constant inequality. There will be market production for good Z in all periods and skilled parents will be specialized in the production of good Y, devoting a zero fraction of their time endowment to the home production of good Z. The market price of good Z and the skill wage premium will be constant over time.

We now investigate whether it is optimal for skilled parents to erect barriers to the acquisition of skills.

Proposition 3 *Let Ω^* be the measure of unskilled to skilled agents such that*

$$\alpha \ln(1 + lG) + \frac{\gamma}{1-\gamma} \ln(\frac{P}{G}) + \gamma \ln(\frac{\alpha}{\Omega}) = 0. \tag{17}$$

Then, for every $\Omega > \Omega^*$ $\exists$ a unique $\bar{\phi}(\Omega) \in [0, \frac{A}{P + \frac{(1+\alpha)}{\alpha}\Omega G})$, such that:

i. $\phi < \bar{\phi}(\Omega) \Rightarrow e_{GU} = 1.$

ii. $\phi > \bar{\phi}(\Omega) \Rightarrow e_{GU} = 0.$

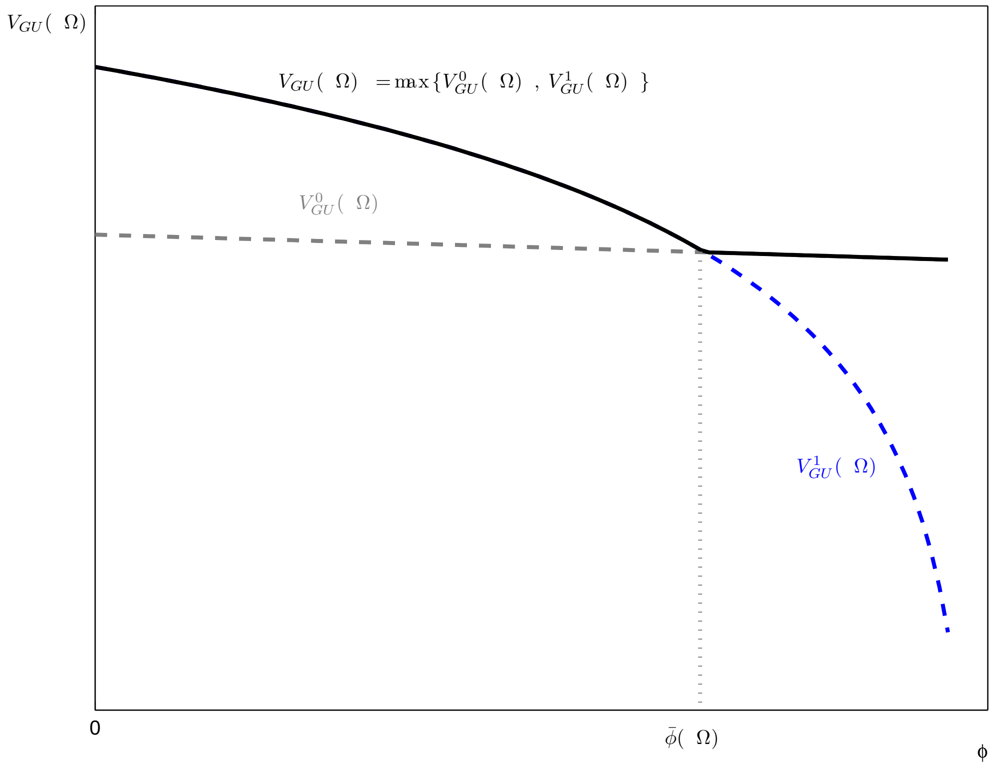
Proof. See appendix D ■

Equation (17) guarantees that if $\Omega > \Omega^*$, then $\lim_{\phi \to 0} V_{GU}^1 > \lim_{\phi \to 0} V_{GU}^0$. The intuition of equation (17) is as follows: From the point of view of a parent even if the direct cost of education is zero ($\phi = 0$), there still exists an opportunity cost (i.e., the foregone income from child labor) of sending their children to school. This explains the first term on the right hand side of (17). This opportunity cost increases with the child labor relative productivity lG. In addition, the higher the fertility differential ($\frac{G}{P}$), the higher is the incentive for unskilled parents to educate their children. This is because, if they do not educate their children, then the equilibrium unskilled wage evolves according to $w_U(\Omega') = \frac{P}{G} w_U(\Omega)$,[24] and future unskilled market income will decrease over time, as well as the future utility of their descendants. In addition, the indirect utility of unskilled parents decreases with the ratio of unskilled to skilled agents, Ω. When unskilled parents educate their children, then such measure affects negatively their value function in the first period only.

Therefore, item (i) and (ii) from Proposition 3 and Figure 2 illustrate the decision of unskilled parents to educate or not their children. They show that, for each level of inequality, it is optimal for all unskilled parents to send their children to school only if the cost of education is sufficient small ($\phi < \bar{\phi}(\Omega)$).

[24]This explains the second term on the right hand side of (17).

Figure 2: Optimal schooling choice: Unskilled Parents



For skilled parents there are two opposing effects associated with an increase in education costs ϕ. First, a higher direct cost of education implies a lower present consumption and therefore lower present utility. On the other hand, a higher education cost implies a higher demand for good Y, and therefore an increase in the skill wage premium $\frac{w_S}{w_U(\Omega)}$. This allows skilled agents to purchase a higher amount of good Z in the market. When unskilled agents do not send their children to school ($\phi > \bar{\phi}(\Omega)$), the effect on the skilled wage premium is small, and the value function associated with a skilled parent $V^1_{PS}(\Omega)$ is continuous and strictly decreasing in $\phi \in (\bar{\phi}(\Omega), \frac{A}{P})$.

When unskilled parents decide to send their children to school ($\phi < \bar{\phi}(\Omega)$) this reinforces the increase in the demand for good Y and therefore the effect of ϕ on the skill wage premium. This effect is stronger when inequality (Ω) is high. This effect will, however, be present only in the first period. From the second period on, there will be only skilled agents and for future generations, utility will be strictly decreasing with education costs ϕ. Therefore, it can be shown that, in an equilibrium when both skilled and unskilled parents send their children to acquire skills, there exists a sufficient large altruism factor γ, such that the value function of skilled parents $V^{1,a'>0}_{PS}(\Omega)$ is continuous and strictly decreasing with $\phi \in [0, \bar{\phi}(\Omega))$ where $\bar{\phi}(\Omega) \in [0, \frac{A}{P+\frac{(1+\alpha)}{\alpha}\Omega G})$. Define $\bar{\phi}(\Omega)_{\text{sup}} = \frac{A}{P+\frac{(1+\alpha)}{\alpha}\Omega G}$. It can be shown that if inequality is sufficiently high, skilled agents strictly prefer an education cost $\phi = \bar{\phi}(\Omega)_{\text{sup}} > 0$, than a policy in which the acquisition of skill is freely provided ($\phi = 0$).

Proposition 4 *Let Ω^{**} be the measure of unskilled to skilled agents defined implicitly by*

$$\alpha\gamma \ln(\frac{\delta\Omega}{\alpha}) + \alpha \ln(1 + lG) + \frac{\alpha\gamma}{1-\gamma} \ln(\frac{G}{P}) = \ln(1 + \frac{P}{\frac{(1+\alpha)}{\alpha}\Omega G}). \tag{18}$$

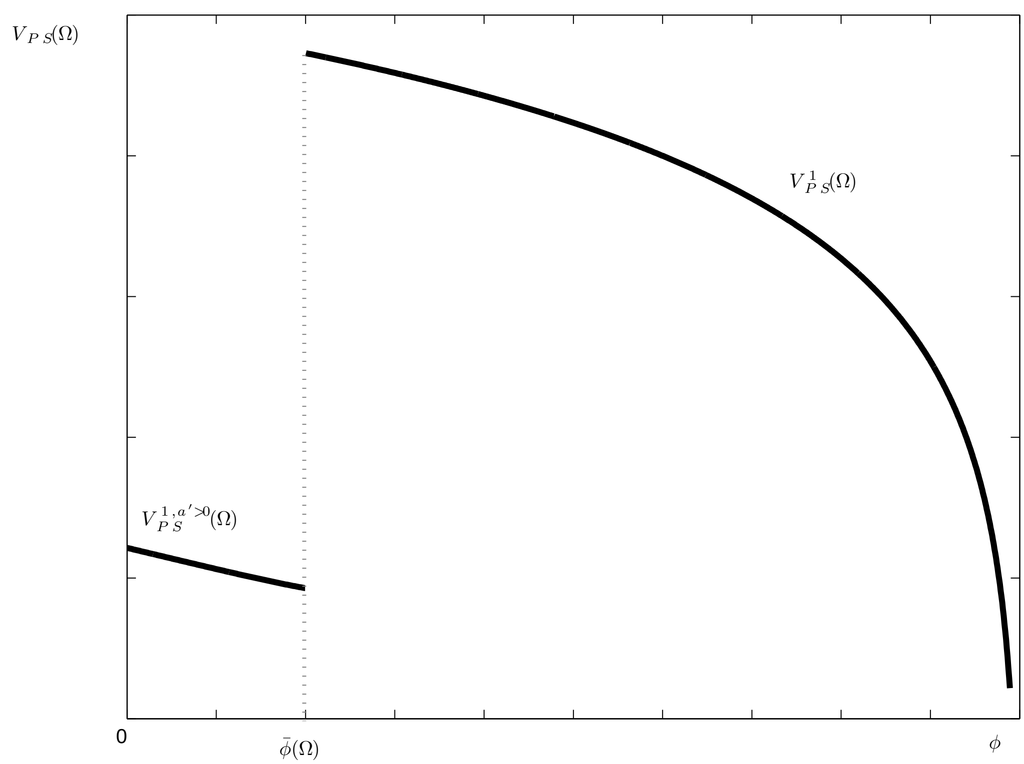
Then, for any $\Omega > \max\{\Omega^{*}, \Omega^{**}\}$ *skilled agents strictly prefer education costs* $\phi = \bar{\phi}(\Omega)_{\sup}$ *than* $\phi = 0$.

Proof. See appendix D ■

The left hand side of equation (18) corresponds to the benefits for skilled parents of having a high educational cost, such that unskilled parents do not send their children to school. Such benefits are increasing with inequality Ω, child labor lG and with fertility differential $\frac{G}{P}$.[25] The right hand side corresponds to the negative effect on utility of a "high" cost of education.

Figure 3 describes well Proposition 4. It shows that the value function of skilled parents is not necessarily continuous at $\bar{\phi}(\Omega)$. In addition, skilled parents might prefer an education cost $\phi > \bar{\phi}(\Omega)$, such that unskilled parents do not educate their children than a lower education cost $\phi < \bar{\phi}(\Omega)$, such that unskilled parents send their children to acquire skills. If $\phi < \bar{\phi}(\Omega)$, then future generations of skilled parents will have to work at home in the production of good Z, since future market prices of good Z will be at its maximum value (δw_S) and it is cheaper to produce good Z at home. This implies that depending on the level of inequality it is a vested interest of skilled agents to erect barriers to the acquisition of skills.

Figure 3: Skilled Parents Value Function



Consider, for instance, that multilateral agencies provide financial aids for developing countries to improve their public education systems. These financial aids might help to institute a free, compulsory education or might be in a form of subsidies to poor families to educate their children.[26] What

[25] Recall that the unskilled wage and therefore the market price of "home goods" are decreasing with Ω and lG. Moreover, in an equilibrium in which only skilled parents educate their children $w_U(\Omega') = \frac{P}{G} w_U(\Omega)$.

[26] Notice that this policy does not require tax increases.

Proposition 4 and Figure 3 show is that, depending on the level of inequality, this policy is not necessarily welfare improving under the Pareto criterion. General equilibrium price adjustments implies that future skill premiums would reduce as the children of unskilled parents attend schools. In addition, the future price of the "home good" (Z) will increase. Future skilled agents might have to work at home to produce good Z instead of buying it in the market. Since parents value the future utility of their children, the welfare of skilled parents might be reduced.[27]

Observe, however, that this policy is Pareto improving in a model similar to the one presented by Galor and Zeira (1993).[28] In their model, parents' utility depends on the size of bequest and not on their children's utility. The policy analyzed above is, however, also Pareto improving in the Galor and Zeira model even if parents utility depends on the utility of their children. The reason is that as long as the skilled current and future wages do not change, the policy will be Pareto improving, even though the skilled wage premium is reduced. There is no "home good" in the Galor and Zeira model. In our model, a reduced skilled wage premium implies a higher market price of good Z.

5 Extensions

In order to derive the main results of the last section, the model was kept at a very simple level. There are, however, some extensions that would enrich some of our results, especially those concerning how societies decide to change public education. Firstly, we abstract from capital accumulation. Capital accumulation is, however, key in the analysis of economic development. If, for instance, the skilled labor were more complementary to capital than the unskilled one, then the skilled wage premium would increase with capital accumulation. Skilled agents would therefore have an additional motive to block policies that provide education at low costs, reinforcing some of our results.

We also abstract from improvements in technology in the market and at home. Improvements in the technology in the "home sector", such as the introduction and development of home appliances, that save time in household chores would increase the opportunity cost of buying the "home good" in the market.[29] This would decrease the market price of home goods and the unskilled wage. Unskilled parents will have higher incentives to send their children to school, since their future income will be lower.

As in Doepke and Zilibotti (2005), the cost of acquiring skills is exogenous in our model economy. The education cost ϕ per child could be endogenous as, for instance, in de la Croix and Doepke (2003), where this cost is defined in units of time of teachers who have the average human capital

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[27] Notice that with this policy the skill wage premium in the first period might increase, since, as argued before, there is a higher demand for good Y. This is because education costs ϕ is in terms of the consumption good Y. In addition, skilled parents do not have to work at home in the period. They can still buy it in the market.

[28] In the model described by Doepke (2004), this policy is also not Pareto improving. In his model there is only one good, which can be produced with skilled or unskilled labor with just one technology (i.e., $Y = F(L_S, L_U)$). Changes in the supply of unskilled labor will then affect the skilled wage. Notice, however, that in Doepke's model the unskilled wage might be higher than the skilled one.

[29] This is emphasized in a model by Greenwood, Seshadri and Yorukoglu (2005) who investigates the rise in female labor force participation rates in the last century. See Cavalcanti and Tavares (2008) for some empirical evidence on this issue.

in the population. For our purpose, however, it is important to consider not only the direct cost of acquiring skills, but also some indirect costs, such as availability of schools and entrance policies in universities. Sokoloff and Engerman (2000: 230) states that "*where there existed elites who were sharply differentiated from the rest of the population on the basis of wealth, human capital, and political influence, they seem to have used their standing to restrict competition.*"

In addition, the political power is also not endogenous in our model (see, for instance, Bourguignon and Verdier (2000)). Notice that if the political power depends on the vote of the median agent, then countries would in some point in time improve their public education system and inequality will decrease. This is because unskilled parents have a higher fertility than skilled ones, then in some point in time political power will be in the hands of unskilled parents. However, if the political power is also unequal and its concentration depends on income and education (see Engerman and Sokoloff (2005)), then improvements in the public education system will be slow and inequality will persist. Like in Bernabou (1996, 2005), if in our model the pivotal agent is an agent with some rank λ^* of income, which is not necessarily equal to the median ($\lambda = \frac{1}{2}$),[30] then as inequality changes over time, due to differences in fertility, political power might change hands and inequality might decrease.[31] As emphasized by Lindert (2004), it was the unequal distribution of political voice and not the lack of intellectual leaders that explain why countries did not invest in public education in the eighteenth century.

Finally, in our model the elite opposes policies that improve access to skill acquisition to guarantee a cheap supply of "home" services. Another way of getting such services at low price is by importing workers from developing countries. This point is emphasized, for instance, in Kremer and Watt (2006). They show that many "new rich" countries such as Hong Kong and Singapore issue special visas for foreigner women to work as private household workers. In this case, more high-skilled native women are therefore available to join the labor market, which can drive down relative wages among high-skilled workers and reducing the disparity in wages between low and high skilled workers. Therefore, a cheap supply of "home" goods does not necessarily depend on suppressing public education. Indeed, the correlation of domestic workers and foreign born over total population is about 43 percent, and it remains statistically significant when we control for the level of development. In the sample with 39 developing and developed countries used in Figure 1, Singapore, for instance, has one of the largest fraction of domestic workers over total employment. In addition, according to international standardized tests, Singapore, however, performs well in the area of education and is one of the countries with the highest stock of foreign born population and net migration rate.[32] Therefore, if we consider the possibility of international migration in our model, then more liberal policies for unskilled labor immigration may endogenously lead to more political support for a strong domestic

[30] A perfect democracy corresponds to $\lambda = \frac{1}{2}$. When political power is correlated with income or education, we have that $\lambda > \frac{1}{2}$.

[31] In Acemoglu and Robinson (2000), the rich elite in power might choose to enlarge the income franchise because of a threat of insurrection when inequality is high.

[32] In the Trends in International Mathematics and Science Study (TIMSS) 2003 report, Singapore ranks first in Math and Science. Moreover, according to the World Development Indicators, Singapore has in the recent years a net migration rate that is higher than what is observed in all OECD countries. Data form the United Nations Statistics Division also shows that the ratio of foreign born over total population is about 42 percent in Singapore, which is about 3 times that in the United States.

public education system, while more restrictive immigration policies may lower the prospects for improvements in education. Such analysis is consistent for the case of countries with positive net migration. If there is more emigration of unskilled than skilled workers, then the elite will have stronger incentive to block policies to improve public education. The interaction of immigration and education policies is certainly an interesting topic that we plan to investigate.

Though all extensions are important to understand the evolution of policies and institutions, they are not crucial to show how some cheap home goods benefit part of the society and why some individuals might block policies that are beneficial for aggregate growth and development.

6 Concluding Remarks

There exists a large literature which shows that public education is favorable for growth because it increases the level of human capital and at the same time it tends to produce a more even income distribution.[33] More egalitarian societies are also associated with less social conflicts and individuals have a lower tendency to report themselves happy when inequality is high (e.g., Alesina, DiTella, and MacCulloch (2004)). Therefore it is important to study the reasons why the elite opposes the development of a strong public education system. It might be that education is related to social status and a strong public education system might threaten the elite's political power. We contribute to this literature by showing that one of this social status might be the specialization of skilled workers in high-paid jobs and the abundance of unskilled workers in the production of some cheap "home goods" in the market, such as painting and cleaning a house, babysitting and/or cooking. Unskilled workers will, however, have to work at home and in the market with low-paid jobs to acquire "market goods". We emphasize the role of general equilibrium price adjustments to show why the elite might erect barriers to policies that improve public education. The higher the unskilled to skilled labor ratio, the lower is the relative price of the "home good". We show that, depending on the level of inequality, the elite might oppose policies that improve the education system even if there is no tax increases to finance such policies and their wages are not directly affected. We also provide some direct empirical evidence and narratives consistent with our theory. Finally, we discuss the case of Singapore and other new riches countries, which guarantees the cheap supply of "home" goods by importing workers from developing countries, without suppressing public education and how more liberal policies for unskilled labor immigration may endogenously lead to more political support for a strong domestic public education system.

References

Acemoglu, D. (2005). "The Form of Property Rights: Oligarchic vs. Democractic Societies," *Mimeo*, MIT.

[33]See Galor and Zeira (1993). For a recent reference, see Doepke (2004).

Acemoglu, D and J. Robinson (2000). "Why Did the West Extend the Franchise? Democracy, Inequality, and Growth in Historical Perspective," *Quarterly Journal of Economics*, **115** (4), 1167-1199.

Aghion, P. and P. Bolton (1997), "A Theory of Trickle-Down Growth and Development", *Review of Economic Studies*, **64** (2), 151-172.

Alesina, A., R. DiTella and R. MacCulloch (2004). "Inequality and Happiness: Are Europeans and Americans Different?" *Journal of Public Economics*, **88**, 2009-2042.

Alston, L. and J. Ferrie (1993). "Paternalism in Agricultural Labor Contracts in the U.S. South: Implications for the Growth of the Welfare State," *American Economic Review*, **83** (4), 852-876.

Banerjee, A. and A. Newman (1993). "Occupational Choice and the Process of Development," *Journal of Political Economy*, **101**, 274-298.

Banerjee, A. (2006). "Inequality and Investment." In: *World Development Report 2006*, Eds. F. Ferreira and M. Walton, Washington DC: World Bank.

Bernabou, R. (1996). "Inequality and Growth," In: *NBER Macroeconomics Annual*, Eds. B. Bernanke and J. Rotemberg, 11-74.

Bernabou, R. (2005). "Inequality, Technology, and the Social Contract," *Handbook of Economic Growth*, Eds. P. Aghion and S. N. Durlauf, vol. 1B, 1595-1638.

Bourguignon, F. and T. Verdier (2000). "Oligarchy, Democracy, Inequality, and Growth," *Journal of Development Economics*, **62** (2), 285-313.

Cavalcanti, T. and J. Tavares (2008). "Assessing the "Engines of Liberation": Home Appliances and Female Labor Force Participation," *Review of Economics and Statistics*, **90** (1), 81-88.

Cortes, P. (2008). "The Effect of Low-Skilled Immigration on U.S. Prices: Evidence from CPI data," *Journal of Political Economy*, **116** (3), 381-422.

Cortes, P. and J. Tessada (2008). "Cheap Maids and Nannies: How Low-Skilled Immigration Is Changing the Labor Supply of High-Skilled American Women," *Mimeo, University of Chicago*.

de La Croix, D. and M. Doepke (2003). "Inequality and Growth: Why Differential Fertility Matters," *American Economic Review*, **93**(4), 1091-1113.

de La Croix, D. and M. Doepke (2008). "To Segregate or to Integrate: Education Politics and Democracy," *Forthcoming Review of Economic Studies.*

Doepke, M. (2004). "Accounting for Fertility Decline During the Transition to Growth," *Journal of Economic Growth*, **9**, 347-383.

Doepke, M. and F. Zilibotti (2005). "The Macroeconomics of Child Labor Regulation," *American Economic Review*, **95** (5), 1492-1524.

Easterly, W. (2005). “Inequality Does Cause Underdevelopment,” *Mimeo*, New York University.

Engerman, S. L. and K. L. Sokoloff (2005). “The Evolution of Suffrage Institutions in the New World,” *Journal of Economic History*, **65**, 891-921.

Fava, A. C. P. and M. Arends-Kuenning (2008). “Maid’s Services as a Substitute Factor in Home-Production,” *Mimeo, University of Illinois*.

Freeman, R. and R. H. Oostendorp (2000). “Wages Around the World: Pay Across Occupations and Countries,” *NBER Working Paper 8058*.

Galor, O. and O. Moav (2006). “Das Human-Kapital: A Theory of the Demise of the Class Structure,” *Review of Economic Studies*, **73** (1), 85-117.

Galor, O. and J. Zeira (1993). “Income Distribution and Macroeconomics,” *Review of Economics Studies* **60**, 35-52.

Greenwood, J. and A. Seshadri (2005). “Technological Progress and Economic Transformation,” Eds. P. Aghion and S. N. Durlauf *Handbook of Economic Growth*, Vol 1B, Amsterdam: Elsevier North-Holland, 1225-1273.

Greenwood, J., A. Seshadri and Yorukoglu, M. (2005), “Engines of liberalization,” *Review of Economic Studies* **72** (1), 109–133.

Grossman, H. and M. Kim (1999). “Educational Policy: Egalitarian or Elitist?,” *Mimeo*, Brown University.

Heston, A., R. Summers and B. Aten (2006). “Penn World Table Version 6.2”, *Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania.*

Kremer, M. and S. Watt (2006). “The Globalization of Household Production,” *Mimeo*, *University of Harvard.*

Lindert, P. (2003). “Voice and Growth: Was Churchill Right?”, *Journal of Economic History* **63** (2), 315-350.

Lindert, P. (2004). “*Growing Public: Social Spending and Economic Growth Since the Eighteenth Century*,” Cambridge University Press.

Ljungqvist, L. (1993). “Economic Undervelopment: The Case of Missing Market for Human Capital,” *Journal of Development Economics* **40**, 219-239.

Mayda, A. M. (2006). “Who Is Against Immigration? A Cross-Country Investigation of Individual Attitudes Toward Immigrants, *Review of Economic and Statistics* **88** (3), 510-530.

Mokhtari, M. and Rassekh, F. (1989). “The Tendency towards Factor Price Equalization among OECD Countries”, *Review of Economics and Statistics* **71** (4), 636-642.

Moav, O. (2005). “Cheap Children and the Persistence of Poverty,” *Economic Journal*, **115** (1), 88-110.

Mookherjee, D. and D. Ray (2002). “Is Equality Stable?” *American Economic Review Papers and Proceedings*, **92** (2), 253-259.

Mookherjee, D. and D. Ray (2003). “Persistent Inequality,” *Review of Economic Studies*, **70**, 369-393.

Olson, M. (1982). *“The Rise and Decline of Nations: Economic Growth, Stagflation, and Social Rigidities”*, Yale University Press.

Parente, S. and E. Presott (1999). “Monopoly Rights: A Barrier to Riches,” *American Economic Review*, **89** (5), 1216-1233.

Parente, S. and E. Prescott (2000). *“Barriers to Riches”*, The MIT Press.

Parente, S., R. Rogerson and R. Wright (2000). “Homework in Development Economics: Household Production and the Wealth of Nations,” *Journal of Political Economy*, **108** (4), 680-687.

Ramey, V. A. (2009). “Time Spent in Home Production in the Twentieth-Century United States: New Estimates from Old Data,” *Journal of Economic History*, **69** (1), 1-47.

Ray, D. and P. Streufert (1993). “Dynamic Equilibria with Unemployment Due to Undernourishment,” *Economic Theory*, **3** (1), 61-85.

Ríos-Rull, J-V. (1993). “Working in the Market, Working at Home, and the Acquisition of Skills: A General Equilibrium Approach,” *American Economic Review*, **83** (4), 893-907.

Sokoloff, K. and S. Engerman (2000). “Institutions, Factor Endowments, and Paths of Development in the New World,” *Journal of Economic Perspectives*, **14** (3), 217-232.

Trefler, D. (1993). “International Factor Price Differences: Leontief was Right,” *Journal of Political Economy*, **101** (3), 961-987.

Zhu, S. C. and D. Trefler (2005). “Trade and Inequality in Developing Countries: a General Equilibrium Analysis,” *Journal of International Economics*, **65** (1), 21-48.

United Nations (2007). “*Human Development Report 2007/2008: Fighting Climate Change*,” Palgrave Macmillan, New York.

A Unskilled Households

Consider the problem of a unskilled household with a large family size.

$$
\begin{aligned}
V_{GU}(\Omega) &= \max_{e\in\{0,1\},z^M,a}\{\ln(w_U(\Omega)(1-a)-q(\Omega)z^M-\phi eG)+\alpha\ln(z^M+Ba+BlG(1-e)) \\
&+ \gamma[eV_{PS}(\Omega')+(1-e)V_{GU}(\Omega')]\}.
\end{aligned}
$$

The optimal conditions for a and z^M are

$$
\frac{\partial V_{GU}}{\partial a} = -\frac{w_U(\Omega)}{c}+\alpha\frac{B}{z}\leq 0,\ a\geq 0,\ \frac{\partial V_{GU}}{\partial a}a=0, \tag{19}
$$

$$
\frac{\partial V_{GU}}{\partial z^M} = -\frac{q(\Omega)}{c}+\alpha\frac{1}{z}\leq 0,\ z^M\geq 0,\ \frac{\partial V_{GU}}{\partial z^M}z^M=0. \tag{20}
$$

Notice that if $z^M > 0$ this implies that $\frac{w_U(\Omega)}{q(\Omega)} \geq B$, which is a contradiction since $\frac{w_U(\Omega)}{q(\Omega)} = \delta B$ and $\delta \in (0,1)$. This implies that $z^M_{GU} = 0$ and

$$
a_{GU} = \max\{0, \frac{w_U(\Omega)(\alpha - lG(1-e)) - \alpha\phi eG}{w_U(\Omega)(1+\alpha)}\}.
$$

If $e_{GU} = 0$

$$
a^0_{GU} = \max\{0, \frac{\alpha - lG}{1+\alpha}\},
$$

and

$$
V^0_{GU}(\Omega) = \ln(w_U(\Omega)(1-a^0_U)) + \alpha\ln(Ba^0_U + BlG) + \gamma V_{GU}(\Omega').
$$

Assumption 1 states that $\alpha \geq lG$. Then

$$
V^0_{GU}(\Omega) = \ln(\frac{w_U(\Omega)(1+lG)}{1+\alpha}) + \alpha\ln(\frac{B\alpha(1+lG)}{1+\alpha}) + \gamma V_{GU}(\Omega'). \tag{21}
$$

Now, suppose that $e_{GU} = 1$, in this case:

$$
a^1_{GU} = \max\{0, \frac{\alpha(w_U(\Omega) - \phi G)}{w_U(\Omega)(1+\alpha)}\}.
$$

Then, as long as education costs are not too high relative to the unskilled wage[34], i.e., $w_U(\Omega) > \phi G$, a^1_{GU} is positive and

$$
V^1_{GU}(\Omega) = \ln(\frac{w_U(\Omega) - \phi G}{1+\alpha}) + \alpha\ln(\frac{B\alpha(w_U(\Omega) - \phi G)}{w_U(\Omega)(1+\alpha)}) + \gamma V_{PS}(\Omega'). \tag{22}
$$

[34]Notice that since parents weight their children in the same way, we have that if $w_U(\Omega) < \phi G$, then $e_{GU} = 0$ and the value function of unskilled parents is represented by equation (21)

B Skilled Households with Small Family

Now let's consider the problem of a skilled parent with a small family size:

$$\begin{aligned} V_{PS}(\Omega) &= \max_{e\in\{0,1\},z^M,a} \{\ln(w_S(\Omega)(1-a) - q(\Omega)z^M - \phi eP) + \alpha\ln(z^M + Ba + BlP(1-e)) \\ &+ \gamma[eV_{PS}(\Omega') + (1-e)V_{GU}(\Omega')]\}. \end{aligned}$$

The optimal conditions for a and z_M are

$$\frac{\partial V_{PS}}{\partial a} = -\frac{w_S(\Omega)}{c} + \alpha\frac{B}{z} \leq 0,\ a \geq 0,\ \frac{\partial V_{PS}}{\partial a}a = 0, \tag{23}$$

$$\frac{\partial V_{PS}}{\partial z^M} = -\frac{q(\Omega)}{c} + \alpha\frac{1}{z} \leq 0,\ z^M \geq 0,\ \frac{\partial V_{PS}}{\partial z^M}z^M = 0. \tag{24}$$

It is straightforward to show that:

i. When $q(\Omega) < \frac{A}{B}$, then $a = 0$, $z^M > 0$.

ii. When $q(\Omega) = \frac{A}{B}$, then $a \geq 0$, $z^M \geq 0$.

iii. When $q(\Omega) > \frac{A}{B}$, then $a > 0$, $z^M = 0$.

Therefore, in order to have market demand for good Z its price cannot be "too" high. Otherwise agents will produce good Z at home.

Assumption 2 states that education cost is not "too" high relatively to the skilled wage, i.e., $w_S = A > \phi P$. Otherwise, skilled parents will never educate their children. Let's first assume that there is initially market production of good Z, therefore we are under case 1 above. We have that $a_{PS} = 0$. If $e_{PS} = 1$, then

$$z^M_{PS} = \frac{\alpha(w_S - \phi P)}{q(\Omega)(1+\alpha)},$$

as long as $w_S = A > \phi P$, which is true by assumption. Therefore,

$$V^1_{PS}(\Omega) = \ln(\frac{w_S - \phi P}{1+\alpha}) + \alpha\ln(\frac{\alpha(w_S - \phi P)}{(1+\alpha)q(\Omega)}) + \gamma V_{PS}(\Omega'). \tag{25}$$

Now, if $e_{PS} = 0$, then

$$z^M_{PS} = \frac{\alpha w_S - q(\Omega)BlP}{(1+\alpha)q(\Omega)}.$$

Therefore,

$$V^0_{PS}(\Omega) = \ln(\frac{w_S + q(\Omega)BlP}{1+\alpha}) + \alpha\ln(\frac{\alpha(w_S + qBlP)}{(1+\alpha)q(\Omega)}) + \gamma V_{GU}(\Omega').$$

Now, let's consider case (iii). We have that $a_{PS} > 0$ and $z^M_{PS} = 0$. If $e_{PS} = 1$, then

$$V^{1,a>0}_{PS}(\Omega) = \ln(\frac{w_S - \phi P}{1+\alpha}) + \alpha\ln(\frac{B\alpha(w_S - \phi P)}{(1+\alpha)w_S}) + \gamma V_{PS}(\Omega'). \tag{26}$$

Notice that under case (iii) there exists no market demand for good Z. Let's assume initially that there is market production for good Z, and the economy is under case (i). In this case, skilled parents educate their children if and only if

$$V^1_{PS}(\Omega) - V^0_{PS}(\Omega) > 0.$$

This requires that

$$\gamma[V_{PS}(\Omega') - V_{GU}(\Omega')] > \ln(\tfrac{w_S + q(\Omega)BlP}{1+\alpha}) + \alpha \ln(\tfrac{\alpha(w_S + qBlP)}{(1+\alpha)q(\Omega)}) - [\ln(\tfrac{w_S - \phi P}{1+\alpha}) + \alpha \ln(\tfrac{\alpha(w_S - \phi P)}{(1+\alpha)q(\Omega)})].$$

Therefore,

$$\gamma[V_{PS}(\Omega') - V_{GU}(\Omega')] > (1+\alpha)\ln(\frac{w_S + q(\Omega)BlP}{w_S - \phi P}). \tag{27}$$

In case (ii), skilled agents are indifferent between working at home or buying the "home" good in the market. It can be therefore represented by either case (i) or (iii), or any solution in which $a > 0$ and $z^M > 0$ and all constraints are satisfied.

From equations (21) and (22), we have that unskilled agents send their children to school if and only if

$$V^1_{GU}(\Omega) - V^0_{GU}(\Omega) > 0.$$

This requires that

$$\gamma[V_{PS}(\Omega') - V_{GU}(\Omega')] > \ln(\tfrac{w_U(\Omega)(1+lG)}{1+\alpha}) + \alpha \ln(\tfrac{B\alpha(1+lG)}{1+\alpha}) - [\ln(\tfrac{w_U(\Omega) - \phi G}{1+\alpha}) + \alpha \ln(\tfrac{B\alpha(w_U(\Omega) - \phi G)}{w_U(\Omega)(1+\alpha)})].$$

Or

$$\gamma[V_{PS}(\Omega') - V_{GU}(\Omega')] > (1+\alpha)\ln(\frac{w_U(\Omega)(1+lG)}{w_U(\Omega) - \phi G}). \tag{28}$$

In order to prove that $e_{GU} = 1$ implies $e_{PS} = 1$ it is sufficient to show that the right hand side of (28) is greater than the right hand side of (27). We have to show that

$$(1+\alpha)\ln(\frac{w_U(\Omega)(1+lG)}{w_U(\Omega) - \phi G}) > (1+\alpha)\ln(\frac{w_S + q(\Omega)BlP}{w_S - \phi P}).$$

Or

$$\frac{w_U(\Omega)(1+lG)}{w_U(\Omega) - \phi G} > \frac{w_S + q(\Omega)BlP}{w_S - \phi P}.$$

Recall that $q(\Omega) = \frac{w_U(\Omega)}{\delta B}$, which implies that

$$\frac{w_U(\Omega)(1+lG)}{w_U(\Omega) - \phi G} > \frac{\delta w_S + w_U(\Omega)lP}{\delta(w_S - \phi P)}.$$

Rearranging the above equation yields

$$w_U(\Omega)[\delta w_S G - w_U(\Omega)P]l + \delta\phi[w_S G - w_U(\Omega)P] + w_U(\Omega)GlP\phi[1-\delta] > 0,$$

which is clearly positive, since $w_U(\Omega) < \delta w_S$, $G > P$, and $\delta \in (0,1)$.

C Characterization of the Steady State

We can have the following equilibrium paths:

i. Just skilled parents educate their children.

ii. Skilled and unskilled parents educate their children.

iii. Skilled and a fraction of unskilled parents educate their children.

Part I: Just skilled parents educate their children.

Let's characterize the first equilibrium in which just skilled parents educate their children. In this case there will be market production of good Z in every period and skilled parents will not work at home. In this case, the wage rate is given by

$$w_U(\Omega) = \min\{\delta w_S, \frac{\alpha(A-\phi P)}{\Omega(1+lG)}\}.$$

The ratio of unskilled to skilled workers will grow at rate $\frac{G}{P} - 1 > 0$, i.e., $\Omega' = \frac{G}{P}\Omega$. Therefore, at some time period $t \geq 0$, the measure of unskilled agents relatively to skilled agents will be sufficiently large, i.e., $\Omega = \frac{X_{GU}}{X_{PS}} > \frac{(A-\phi P)}{\delta A} \times \frac{\alpha}{(1+lG)}$, such that $w_U(\Omega') = \frac{P}{G} w_U(\Omega)$. Moreover, $q(\Omega) = \frac{w_U(\Omega)}{\delta B}$, which will also grow at gross rate $\frac{P}{G}$. Using $q(\Omega)$ into equation (25) and solving it forward yield:

$$V^1_{PS}(\Omega) \;=\; \frac{\ln(\frac{w_S - \phi P}{1+\alpha}) + \alpha \ln(\frac{\alpha(w_S-\phi P)}{(1+\alpha)q(\Omega)})}{1-\gamma} - \frac{\alpha\gamma}{(1-\gamma)^2}\ln(\frac{P}{G}). \tag{29}$$

Unskilled parents will not educate their children and will also not buy good Z in the market. Thus

$$V^0_{GU}(\Omega) = \ln(w_U(\Omega)(1-a^0_U)) + \alpha\ln(Ba^0_U + BlG) + \gamma V_{GU}(\Omega').$$

Since unskilled parents will not send their children to school they will not pay the cost associated to education and will receive income from child labor. The family size enters positively in the indirect utility function:

$$V^0_{GU}(\Omega) = \frac{(1+\alpha)\ln(\frac{1+lG}{1+\alpha}) + \alpha\ln\alpha B + \ln w_U(\Omega)}{1-\gamma} + \frac{\gamma}{(1-\gamma)^2}\ln(\frac{P}{G}). \tag{30}$$

In order to fully characterize the equilibrium it remains to find equilibrium prices. We know that $w_S = A$ and $\frac{w_U(\Omega)}{q(\Omega)} = \delta B$. Using the equilibrium in the goods market, we can show that

$$\frac{X_{GU}}{X_{PS}} c_{GU} + c_{PS} + \phi P = A,$$

and

$$w_U(\Omega) = \frac{\alpha(A - \phi P)}{\Omega(1 + lG)},$$

as long as $w_U(\Omega) < \delta w_S$. Recall that $\Omega' = \frac{G}{P}\Omega$, and the initial state Ω_0 is given. It is also important to show that if a parent deviates from its optimal choice, then he gets a lower utility. Consider first an unskilled parent that chooses to deviate from this equilibrium. This parent will send their children to school when all other unskilled parents are not educating their children. It is important to observe that since parents value each child in the same way, and cannot use their children's future income as a collateral in a loan to finance their children education, then the choice of unskilled parents to send their children to school is only available if $w_U(\Omega) > \phi G$. Therefore, if $w_U(\Omega) < \phi G$, then unskilled parents do not have the choice to send their children to school and $V_{GU}(\Omega)$ is the optimal value function for unskilled parents. Future descendants will have a small family size and educate their children. The utility of this parent is:

$$\begin{aligned}\tilde{V}_{GU}(\Omega) &= \ln(\frac{w_U(\Omega) - \phi G}{1 + \alpha}) + \alpha \ln(\frac{B\alpha(w_U(\Omega) - \phi G)}{w_U(\Omega)(1 + \alpha)}) \\ &+ \gamma[\frac{\ln(\frac{w_S - \phi P}{1+\alpha}) + \alpha \ln(\frac{\alpha(w_S - \phi P)}{(1+\alpha)q(\Omega)}) - \frac{\alpha}{1-\gamma}\ln(\frac{P}{G}))}{1 - \gamma}].\end{aligned}$$

Therefore, as long as

$$V^0_{GU}(\Omega) - \tilde{V}_{GU}(\Omega) \geq 0,$$

it is optimal for an unskilled parent to not send their children to school when all other unskilled parents are not educating their children. It can be shown that this condition is satisfied when

$$\begin{aligned}(1 + \alpha)\ln(w_U(\Omega)) - (1 - \gamma)(1 + \alpha)\ln(w_U(\Omega) - \phi G) \quad &> \\ \gamma(1 + \alpha)\ln(w_S - \phi P) + \gamma\alpha \ln \delta - (1 + \alpha)\ln(1 + lG) - \frac{(1 + \alpha)\gamma}{1 - \gamma}\ln(\frac{P}{G}).\end{aligned} \tag{31}$$

The left hand side (LHS) of (31) attains a minimum at $w_U(\Omega) = \frac{\phi G}{\gamma}$ and for all $\phi G < w_U(\Omega) < \frac{\phi G}{\gamma}$ the LHS is decreasing in $w_U(\Omega)$. Moreover, it is continuous in $w_U(\Omega) > \phi G$ and it goes to infinity as $w_U(\Omega) \rightarrow_+ \phi G$. On the other hand, the right hand side (RHS) of (31) is independent of $w_U(\Omega)$. Therefore, we can define value for the parameters of the model, such that $\phi G < w^*_U \leq \min\{\delta A, \frac{\phi G}{\gamma}\}$ such that for any $\phi G < w_U(\Omega) < w^*_U$, it is optimal for unskilled parents to not educate their children.[35] In the case that $\phi G > w_U(\Omega)$, $V^0_{GU}(\Omega)$ is the optimal value function.

[35] For some parameters value we can have $w^*_U > \delta w_S$, which implies that is always optimal for unskilled parents to not educate their children.

Now, consider a skilled parent that choose to not educate their children when all skilled parents are sending their descendants to school. Future descendants choose to have a large family size and do not educate their children. The utility of this parent is

$$\tilde{V}_{PS}(\Omega) = (1+\alpha)\ln(\frac{w_S + q(\Omega)BlP}{1+\alpha}) + \alpha\ln(\frac{\alpha}{q(\Omega)}) + \frac{\gamma}{1-\gamma}[(1+\alpha)\ln(\frac{1+lG}{1+\alpha}) + \ln w_U(\Omega) + \alpha\ln\alpha B + \frac{1}{1-\gamma}\ln(\frac{P}{G})].$$

Therefore, as long as $V^1_{PS}(\Omega) - \tilde{V}_{PS}(\Omega) > 0$, it is optimal for a skilled parent to educate their children when all other skilled parents are also sending their children to school and unskilled parents are not educating their children. This condition is satisfied when

$$(1+\alpha)\ln(w_S - \phi P) + \gamma\alpha\ln\delta - \gamma(1+\alpha)\ln(1+lG) - \frac{(1+\alpha)\gamma}{1-\gamma}\ln(\frac{P}{G}) > (1-\gamma)(1+\alpha)\ln(w_S + \frac{w_U(\Omega)}{\delta}lP) + \gamma(1+\alpha)\ln(w_U(\Omega)). \quad (32)$$

The LHS of (32) is independent of $w_U(\Omega)$, while the RHS is continuous and increasing in $w_U(\Omega)$. Moreover, as $w_U(\Omega) \to 0$ the $RHS \to -\infty$, and when $w_U(\Omega) \to \infty$, then the $RHS \to \infty$. Therefore, there exists a $w^{**}_U > 0$, such that for any $w_U(\Omega) < w^{**}_U$, it is optimal for skilled parents to educate their children. Define $\bar{w}_U = \min\{w^*_U, w^{**}_U\}$. This implies that for any $w_U(\Omega) \leq \bar{w}_U$ it is optimal for unskilled parents to not send their children to school and it is optimal for skilled parents to educate their children.

Part II: Skilled and unskilled parents educate their children.

Now, let's characterize the equilibrium when skilled and unskilled parents educate their children. In this equilibrium there is market production of good Z only in the first period. The value function of a skilled worker is:

$$V^{1,a'>0}_{PS}(\Omega) = \frac{(1+\alpha)\ln(\frac{w_S-\phi P}{1+\alpha}) + \alpha\ln(\alpha B) - \gamma\alpha\ln w_S}{1-\gamma} + \alpha\ln\delta - \alpha\ln w_U(\Omega). \quad (33)$$

Unskilled agents with a large family size will educate their children. Their descendants will then have a small family size and will also educate their children. The value function of an unskilled parent is:

$$V^1_{GU}(\Omega) = (1+\alpha)\ln(\frac{w_U(\Omega) - \phi G}{1+\alpha}) + \frac{1}{1-\gamma}\alpha\ln(\alpha B) - \alpha\ln(w_U(\Omega)) + \gamma\frac{(1+\alpha)\ln(\frac{w_S-\phi P}{1+\alpha}) - \alpha\ln w_S}{1-\gamma}. \quad (34)$$

Now, it is important to show that individual deviations from the optimal policy yield a lower payoff. Given item (ii) of proposition 2 it is sufficient to show only the condition that guarantees that it is optimal for unskilled parents to educate their children. Suppose that an unskilled parent decides to not educate their children when all other parents (skilled and unskilled) are sending their descendants to attend school. In this case, there are two possibilities: their descendants choose to educate or not their children. Notice, however, that in the next period, their children will be the only unskilled agents in the economy. If they are a small measure of the total population, then the unskilled wage will be at its maximum value,[36] $w_U(\Omega') = \delta w_S$. Under the interesting case, where $w_U^* < \delta w_S$, their descendants will choose to educate their children. Then, the value associated with this deviation is:

$$\hat{V}_{GU}(\Omega) = (1+\alpha)\ln(\frac{1+lG}{1+\alpha}) + \ln w_U(\Omega) + \alpha \ln \alpha B +$$
$$\gamma[(1+\alpha)\ln(\frac{\delta w_S - \phi G}{1+\alpha}) + \alpha \ln(\frac{\alpha B}{\delta w_S})] + \frac{\gamma^2}{1-\gamma}[(1+\alpha)\ln(\frac{w_S - \phi P}{1+\alpha}) + \alpha \ln(\frac{\alpha B}{\delta w_S})]$$

As long as $V_{GU}^1(\Omega) - \hat{V}_{GU}(\Omega) > 0$ it is optimal for unskilled parents send their children to school when all other parents are also educating their descendants. This is true as long as

$$(1+\alpha)\ln(\frac{w_U(\Omega)}{w_U(\Omega)-\phi G}) < \gamma(1+\alpha)\ln(w_S - \phi P) - (1+\alpha)\ln(1+lG) - \gamma[(1+\alpha)\ln(\delta w_S - \phi G) - \alpha \ln \delta]. \quad (35)$$

The LHS of (35) is continuous and decreasing in $w_U(\Omega)$. Moreover, when $w_U(\Omega) \to \phi G$, then the $LHS \to \infty$, and when $w_U(\Omega) \to \infty$, then the $LHS \to 0$. Therefore, as long as the RHS is positive, there exists a $w_U^\bullet$, such that for any $w_U(\Omega) > w_U^\bullet$, condition (35) is satisfied.

Observe that condition (31) can be rewritten as:

$$(1+\alpha)\ln(\frac{w_U(\Omega)}{w_U(\Omega)-\phi G}) > \gamma(1+\alpha)\ln(w_S - \phi P) - (1+\alpha)\ln(1+lG) - \frac{(1+\alpha)\gamma}{1-\gamma}\ln(\frac{P}{G}) - \gamma[(1+\alpha)\ln(w_U(\Omega) - \phi G) - \alpha \ln \delta]. \quad (36)$$

Since

$$\frac{(1+\alpha)\gamma}{1-\gamma}\ln(\frac{G}{P}) + \gamma(1+\alpha)\ln(\frac{\delta w_S - \phi G}{w_U(\Omega)-\phi G}) > 0,$$

it can be shown that the RHS of (36) is larger than the RHS of (35), which implies that for some parameters value in which w_U^* and $w_U^\bullet$, we have that $\phi G < w_U^* < w_U^\bullet$.

Part III: Skilled and a fraction of unskilled parents educate their children.

In this equilibrium, a fraction of unskilled parents educate their children. Since parents give the same weight in the utility function to each child, we must have that unskilled parents are indifferent to send or not their children to school. Let η be the fraction of unskilled parents that educate their children. In this case, the unskilled wage is given by:

$$w_U(\Omega) = \frac{\alpha(A - \phi(P + \eta\Omega G))}{\Omega(\eta + (1-\eta)(1+lG))}. \quad (37)$$

[36] The next period state variable will be $\Omega' = \frac{G}{G(X_{GU}-1)+PX_{PS}}$.

The state variable evolves according to:

$$\Omega' = \frac{(1-\eta)G}{P+\eta\Omega G}\Omega. \tag{38}$$

In order to have an analytical solution for this case, let's focus on a stationary equilibrium in which the ratio of unskilled to skilled parents is constant[37] (i.e., $\Omega' = \Omega$), we have that:

$$\eta = \frac{G-P}{(1+\Omega)G}, \tag{39}$$

and $w_U(\Omega') = w_U(\Omega)$. In addition:

$$\begin{aligned}
V_{PS}(\Omega) &= \frac{\ln(\frac{w_S-\phi P}{1+\alpha}) + \alpha\ln(\frac{\alpha(w_S-\phi P)}{(1+\alpha)q(\Omega)})}{1-\gamma}, \\
V_{GU}^0(\Omega) &= \frac{\ln(\frac{w_U(\Omega)(1+lG)}{1+\alpha}) + \alpha\ln(\frac{B\alpha(1+lG)}{1+\alpha})}{1-\gamma}, \\
V_{GU}^1(\Omega) &= \ln(\frac{w_U(\Omega)-\phi G}{1+\alpha}) + \alpha\ln(\frac{B\alpha(w_U(\Omega)-\phi G)}{w_U(\Omega)(1+\alpha)}) + \gamma V_{PS}(\Omega).
\end{aligned}$$

Moreover, we must have $V_{GU}^0(\Omega) = V_{GU}^1(\Omega)$, this implies that:

$$\begin{aligned}
(1+\alpha)\ln(w_U(\Omega)) - (1-\gamma)(1+\alpha)\ln(w_U(\Omega)-\phi G) &= \\
\gamma(1+\alpha)\ln(w_S-\phi P) + \gamma\alpha\ln\delta - (1+\alpha)\ln(1+lG).&
\end{aligned} \tag{40}$$

The LHS of (40) is identical to the LHS of (31). The RHS of (40) is also similar to the RHS of (31), except for the last part of equation (31), which has a term with $ln(\frac{P}{G})$, that is not in the equation above because the ratio of unskilled to skilled parents is now constant. This implies that the RHS of (31) is larger than the RHS of (40). Therefore, we can define a w_U^{***}, which is larger than w_U^* such that for $w_U(\Omega) = w_U^{***} < \delta w_S$, unskilled parents are indifferent about educating or not their children and the ratio of unskilled to skilled agents is constant over time. In addition, by rearranging equation (40) and comparing it with (35), we can show that $w_U^{***} < w^\bullet$. Notice that by Proposition 2, we have that if $e_{GU} = 1$, then $e_{PS} = 1$.

D Main Results

Proof of Proposition 3

The equilibrium value function when unskilled parents do not educate their children is:

$$V_{GU}^0(\Omega) = \frac{(1+\alpha)\ln(\frac{1+lG}{1+\alpha}) + \alpha\ln\alpha B + \ln w_U(\Omega)}{1-\gamma} + \frac{\gamma}{(1-\gamma)^2}\ln(\frac{P}{G}).$$

[37] This does not rule out other types of equilibrium in which just a fraction of unskilled parents educate their children.

In this equilibrium, the unskilled wage is:

$$w_U(\Omega) = \frac{\alpha(A - \phi P)}{\Omega(1 + lG)}.$$

Notice that $V^0_{GU}(\Omega)$ is clearly continuous in $\phi \in [0, \frac{A}{P})$. Moreover, for $\phi \in [0, \frac{A}{P})$

$$\frac{\partial V^0_{GU}(\Omega)}{\partial \phi} = \frac{\frac{\partial w_U(\Omega)}{\partial \phi}}{w_U(\Omega)(1-\gamma)} < 0.$$

In addition:

$$\lim_{\phi \to \frac{A}{P}} V^0_{GU}(\Omega) = -\infty,$$

and

$$\lim_{\phi \to 0} V^0_{GU}(\Omega) = \frac{(1+\alpha)\ln(\frac{1+lG}{1+\alpha}) + \alpha \ln \alpha B + \ln(\frac{\alpha A}{\Omega(1+lG)})}{1-\gamma} + \frac{\gamma}{(1-\gamma)^2}\ln(\frac{P}{G}).$$

The equilibrium value function when unskilled parents educate their children is:

$$V^1_{GU}(\Omega) = (1+\alpha)\ln(\frac{w_U(\Omega) - \phi G}{1+\alpha}) + \frac{\alpha \ln \alpha B}{1-\gamma} - \alpha \ln w_U(\Omega) + \gamma \frac{(1+\alpha)\ln(\frac{w_S - \phi P}{1+\alpha}) - \alpha \ln w_S}{1-\gamma},$$

where

$$w_U(\Omega) = \frac{\alpha(A - \phi(P + \Omega G))}{\Omega}.$$

The value function $V^1_{GU}(\Omega)$ is continuous in $\phi \in [0, \frac{A}{P + \frac{1+\alpha}{\alpha}\Omega G})$ and

$$\frac{\partial V^1_{GU}(\Omega)}{\partial \phi} = \frac{\partial w_U(\Omega)}{\partial \phi}[\frac{w_U(\Omega) + \alpha\phi G}{w_U(\Omega)(w_U(\Omega) - \phi G)}] - \frac{(1+\alpha)G}{w_U(\Omega) - \phi G} - \frac{\gamma(1+\alpha)P}{(1-\gamma)(A - \phi P)} < 0.$$

We also have that:

$$\lim_{\phi \to \frac{A}{P + \frac{1+\alpha}{\alpha}\Omega G}} V^1_{GU}(\Omega) = -\infty,$$

and

$$\lim_{\phi \to 0} V^1_{GU}(\Omega) = \ln\frac{\alpha A}{\Omega} + \frac{\alpha}{1-\gamma}\ln \alpha B + \frac{\gamma}{1-\gamma}\ln A - \frac{1}{1-\gamma}(1+\alpha)\ln(1+\alpha).$$

Condition (17) guarantees that $\lim_{\phi \to 0} V^1_{GU}(\Omega) > \lim_{\phi \to 0} V^0_{GU}(\Omega)$. Since $\frac{A}{P} > \frac{A}{P + \frac{1+\alpha}{\alpha}\Omega G}$, this proves item (i) and item (ii) of proposition 3.

Proof of Proposition 4

In equilibrium, skilled parents always educate their children. Therefore:

$$V^1_{PS}(\Omega) \;=\; \ln(\frac{w_S - \phi P}{1+\alpha}) + \alpha \ln(\frac{\alpha(w_S - \phi P)}{(1+\alpha)q(\Omega)}) + \gamma V_{PS}(\Omega').$$

If $\phi \in (\bar{\phi}(\Omega), \frac{A}{P})$ and $\bar{\phi}(\Omega) \in [0, \frac{A}{P + \frac{(1+\alpha)}{\alpha}\Omega G})$, then unskilled parents will not educate their children and skilled parents will not work at home. Then

$$V^1_{PS}(\Omega) = \frac{\ln(\frac{w_S-\phi P}{1+\alpha}) + \alpha\ln(\frac{\alpha(w_S-\phi P)}{(1+\alpha)q(\Omega)})}{1-\gamma} - \frac{\alpha\gamma}{(1-\gamma)^2}\ln(\frac{P}{G}),$$

with $w_S = A$, $q(\Omega) = \frac{w_U(\Omega)}{\delta B}$ and $w_U(\Omega) = \frac{\alpha(A-\phi P)}{\Omega(1+lG)}$. It is straightforward to show that $V^1_{PS}(\Omega)$ is continuous in $\phi \in (\bar{\phi}(\Omega), \frac{A}{P})$ and

$$\frac{\partial V^1_{PS}(\Omega)}{\partial \phi} = -\frac{1}{1-\gamma}\frac{P}{A-\phi P} < 0.$$

Notice also that

$$\lim_{\phi\to\frac{A}{P}} V^1_{PS}(\Omega) = -\infty.$$

If $\phi \in [0, \bar{\phi}(\Omega))$, then unskilled parents will educate their children and there exists market production of good Z only in the first period. The value associated with a skilled parent problem is

$$V^{1,a'>0}_{PS}(\Omega) = \frac{(1+\alpha)\ln(\frac{w_S-\phi P}{1+\alpha}) + \alpha\ln(\alpha B) - \gamma\alpha\ln w_S}{1-\gamma} + \alpha\ln\delta - \alpha\ln w_U(\Omega),$$

with $w_S = A$ and $w_U(\Omega) = \frac{\alpha(A-\phi(P+\Omega G))}{\Omega}$. Moreover:

$$\frac{\partial V^{1,a'>0}_{PS}(\Omega)}{\partial\phi} = -\frac{1+\alpha}{1-\gamma}\frac{P}{A-\phi P} + \alpha\frac{P+\Omega G}{A-\phi(P+\Omega G)}.$$

Notice that there are two effects in opposing directions. It can be shown that for a large altruism factor $\frac{1}{1-\gamma} > \frac{\alpha}{1+\alpha}\frac{A-\phi P}{A-\phi(P+\Omega G)}\frac{P+\Omega G}{P}$, we have that $\frac{\partial V^{1,a'>0}_{PS}(\Omega)}{\partial\phi} < 0$.

Notice that if $\phi = 0$ and $\Omega > \Omega^*$, then

$$V^{1,a'>0}_{PS}(\Omega, \phi = 0) = \frac{1+\alpha}{1-\gamma}\ln(\frac{A}{1+\alpha}) + \frac{\alpha}{1-\gamma}\ln(\alpha B) - \frac{\gamma\alpha}{1-\gamma}\ln A + \alpha\ln(\frac{\delta\Omega}{\alpha A}).$$

Recall that $\bar{\phi}(\Omega) \in [0, \frac{A}{P+\frac{(1+\alpha)}{\alpha}\Omega G})$. Define $\bar{\phi}(\Omega)_{\sup} = \frac{A}{P+\frac{(1+\alpha)}{\alpha}\Omega G}$. If $\phi = \bar{\phi}(\Omega)_{\sup}$ and $\Omega > \Omega^*$, then unskilled parents will not educate their children and

$$V^1_{PS}(\Omega, \phi = \bar{\phi}(\Omega)_{\sup}) = \frac{1}{1-\gamma}\ln(\frac{A}{1+\frac{\alpha P}{(1+\alpha)\Omega G}}) + \frac{\alpha}{1-\gamma}\ln(\delta B\Omega(1+lG)) - ...$$
$$\frac{1+\alpha}{1-\gamma}\ln(1+\alpha) + \frac{\alpha\gamma}{(1-\gamma)^2}\ln\frac{G}{P}.$$

It can be shown that $V^1_{PS}(\Omega, \phi = \bar{\phi}(\Omega)_{\sup}) > V^{1,a'>0}_{PS}(\Omega, \phi = 0)$ if and only if $\Omega > \Omega^*$ and

$$\alpha\gamma\ln(\frac{\delta\Omega}{\alpha}) + \alpha\ln(1+lG) + \frac{\alpha\gamma}{1-\gamma}\ln\frac{G}{P} > \ln(1+\frac{\alpha P}{(1+\alpha)\Omega G}). \quad (41)$$

The LHS of (41) is continuous and increasing in $\Omega > 0$. Moreover, as $\Omega \to 0$, then $LHS \to -\infty$ and as $\Omega \to \infty$, then $LHS \to \infty$. On the other hand, the RHS of (41) is continuous and decreasing in $\Omega > 0$. In addition, as $\Omega \rightarrow 0$, then $RHS \rightarrow \infty$ and as $\Omega \rightarrow \infty$, then $LHS \rightarrow 0$. Therefore, there exists an $\Omega^{**} > 0$, such that if $\Omega > \max\{\Omega^{**}, \Omega^{*}\}$, then skilled agents strictly prefer an education cost $\phi = \bar{\phi}(\Omega)_{\text{sup}}$ than a zero education cost ($\phi = 0$).