Mathematics and Economic Reasoning*

by

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Two Girls

Becky's World

Becky, who is 10 years old, lives with her parents and an older brother Sam in a suburban town in mid-West USA. Becky's father works in a law firm specialising in small business enterprises. Depending on the firm's profits, his annual income varies somewhat, but is rarely below 145,000 dollars. Becky's parents met in college. For a few years her mother worked in publishing, but when Sam was born she decided to concentrate on raising a family. Now that both Becky and Sam attend school, she does voluntary work in local education. The family live in a two-storey house. It has four bedrooms, two bathrooms upstairs and a toilet downstairs, a large drawing-cum-dining room, a modern kitchen, and a family room in the basement. There is a small plot of land in the rear, which the family use for leisure activities.

Although their property is partially mortgaged, Becky's parents own stocks and bonds and have a saving account in the local branch of a national bank. Becky's father and his firm jointly contribute to his retirement pension. He also makes monthly payments into a scheme with the bank that will cover Becky's and Sam's college education. The family's assets and their lives are insured. Becky's parents often remark that, federal taxes being high, they have to be careful with money; and they are. Nevertheless, they own two cars, the children attend camp each summer, and the family take a vacation together once camp is over. Becky's parents also remark that her generation will be much more prosperous than they. Becky wants to save the environment and insists on biking to school each day. Her ambition is to become a doctor.

Desta's World

Desta, who is about 10 years old, lives with her parents and 5 siblings in a village in subtropical, southwest Ethiopia. The family live in a two-room, grass-roofed mud hut. Desta's father grows maize and *tef* on half a hectare of land that he owns. Desta's older brother helps him to farm the land and care for the household's livestock, which consist of a cow, a goat, and a few chickens. The small quantity of *tef* produced is sold so as to raise cash income, but the maize is in part consumed by the household as a staple. Desta's mother works a small plot next to their cottage, growing cabbage, onions, and *enset* (an all-year root crop that too serves as a staple). In order to supplement household income, she brews a local drink made from maize. As she is also responsible for cooking, cleaning, and minding the infants, her work day usually lasts 14 hours. Despite the long hours, it wouldn't be possible for her to complete the tasks on her own. (As the ingredients are all raw, cooking alone takes 5 hours or more.) So Desta and her older sister help their mother with household chores and mind their younger siblings. Although a younger brother attends the local school, neither Desta nor her older sister has ever been enrolled there. Her parents can neither read nor write, but they are numerate.

Desta's home has no electricity or running water. Around where they live, sources of water, land for grazing cattle, and the woodlands are communal property. They are shared by people in Desta's village;

but the villagers don't allow outsiders to make use of them. Each day Desta's mother and the girls fetch water, collect fuelwood, and pick berries and herbs from the local commons. Desta's mother frequently observes that the time and effort needed to collect their daily needs has increased over the years.

There is no financial institution nearby to offer either credit or insurance. As funerals are expensive occasions, Desta's father long ago joined a community insurance scheme (*iddir*) to which he contributes monthly. When Desta's father purchased the cow they now own, he used the entire cash he had accumulated and stored at home, but had to supplement that with funds borrowed from friends and kin-folk, with a promise to repay the debt when he had the ability to do so. In turn, when they are in need, his friends and kin-folk come to him for a loan, which he supplies if he is able to. Desta's father says that such patterns of reciprocity he and those close to him practise are part of their "culture", reflecting their norms of social conduct. He says too that his sons are his main assets, as they are the ones who will look after him and Desta's mother in their old age.

Economic statisticians estimate that adjusting for differences in the cost of living between Ethiopia and the United States, Desta's family income is about 5,000 dollars per year, of which 1,000 dollars are attributable to the products they draw from the local commons. However, as rainfall varies from year to year, Desta's family income fluctuates widely. In bad years, the grain they store at home gets depleted well before the next harvest. Food is then so scarce that they all grow weaker, the younger children especially so. It is only after harvest that they regain their weight and strength. Periodic hunger and illnesses have meant that Desta and her siblings are stunted somewhat. Over the years Desta's parents have lost two children in their infancy, stricken by malaria in one case and diarrhoea in the other. There have also been several miscarriages.

Desta knows that she will be married (in all likelihood to a farmer, like her father) when she reaches 18 and will then live on her husband's land in a neighbouring village. She expects her life to be similar to that of her mother.

The Economist's Agenda

That the lives people are able to construct differ enormously across the globe is a commonplace. In our age of travel, it is even a common sight. That Becky and Desta face widely different futures is also something we have come to learn, perhaps also to accept. Nevertheless, it may not be out of turn to imagine that the two girls are intrinsically very similar: they both enjoy eating, playing, and gossiping; they are close to their families; they like pretty things to wear; and they both have the capacity to be disappointed, get annoyed, be happy. Their parents are also alike. They are knowledgable about the ways of their worlds. They also care about their families, finding ingenious ways to meet the recurring problem of producing income and allocating resources among family members - over time and across contingencies. So, a promising route for exploring the underlying causes behind their vastly different conditions of life would be to begin by observing that the constraints the families face are very different; that in some sense Desta's family is far more restricted in what they are able to be and do than Becky's.

Economics in large measure tries to uncover the processes that influence how people's lives come to be what they are. The context may be a village, a district, a state, a country, or the whole world. In its remaining measure, the discipline tries to identify ways to influence those very processes so as to improve the prospects of those who are hugely constrained in what they can be and do. *Modern* economics, by which I mean the style of economics taught and practised in today's graduate schools, does the exercises from the ground up: from individuals, through the household, the village, the district, the state, the country, to the whole world. In various degrees the millions of individual decisions shape the eventualities people all face; for, both theory and evidence inform that there are enormous numbers of unintended consequences of what we all do. But there is also a feedback, in that those consequences in turn go to shape what people subsequently can do and choose to do. For example, when Becky's family drive their cars or use electricity, or when Desta's family create compost or burn woodfuel for cooking, they contribute to global carbon emissions. Their contributions are no doubt negligible, but the millions of such tiny contributions cumulatively sum to a sizable amount, having consequences that people everywhere are likely to experience in different ways.

To understand Becky's and Desta's lives, we need first of all to identify the prospects they face for transforming goods and services into further goods and services - now, and in the future under various contingencies. Secondly, we need to uncover the character of their choices and the pathways by which the choices made by millions of households like Becky's and Desta's go to produce the prospects they all face. Thirdly, and relatedly, we need to uncover the pathways by which the families came to inherit their current circumstances.

The latter is the stuff of economic history. In studying history we could, should we feel bold, take the long view - from about the time agriculture came to be settled practice in the Fertile Crescent (roughly, Anatolia) some 11,000 years ago, and try to explain why the many innovations and practices that have cumulatively contributed to the making of Becky's world either didn't reach or didn't take hold in Desta's part of the world.[1] Should we seek a sharper account, we could study, say, the past 600 years and ask how it is that instead of the several regions in Eurasia that were economically promising in about year 1400, it was the unlikely northern Europe that made it and helped to create Becky's world, even while bypassing Desta's.[2] As modern economics is in large measure concerned with the first two sets of enquiries, this essay focuses on them. However, the techniques today's economic historians deploy to answer their questions are much the same as the ones I describe below to study contemporary lives. The techniques involve

[1] Diamond (1997) is an enquiry into this set of questions.

[2] Landes (1998) is an inquiry into that question. Fogel (2004) explores the pathways by which Europe during the past 300 years has escaped permanent hunger.

studying individual and collective choices in terms of maximization exercises. The predictions of the theories are then tested by means of data relating to actual behaviour. Even the theoretical foundations of national economic policies involve maximization exercises: the maximization of social well-being subject to constraints.

The Household Maximization Problem

Both Becky's and Desta's households are micro-economies. Each subscribes to particular arrangements over who does what and when, recognizing that each faces constraints on what its members are capable of doing. For brevity I shall regard the household as the unit of analysis. We imagine that both sets of parents have their respective families' well-being in mind and want to do as well as they can to protect and promote it.[3] Of course, both Becky's and Desta's parents would have a wider notion of what constitutes their families than I have allowed here. Maintaining ties with kin-folk would be an important aspect of their lives, a matter I return to later. One imagines also that Becky's and Desta's parents are interested in their future grandchildren's well-being. But as they recognise that their children will in turn care about *their* children, they are right to conclude by recursion that doing the best for their children amounts to doing the best for their grandchildren, for their great grandchildren, and so on, down the generations.

Personal well-being is made up of a variety of constituents: health, relationships, place in society, and satisfaction at work are but four. Economists and psychologists have identified ways to represent well-being as a numerical measure. To say someone's well-being is greater in situation A than in situation B is to say that her well-being measure is numerically higher in A than B. A family's well-being is an aggregate of its members' well-beings. (Utilitarian philosophers concluded that summation across household members would be the appropriate way to perform that aggregation.) As goods and services are among the determinants of well-being (food, shelter, clothing and medical care are but four), the problem Becky's and Desta's parents face, respectively, is to determine from among those allocations of goods and services that are feasible, the ones that are best for their households. But both pair of parents care not only about today, but the future too. Moreover, the future is uncertain. So the goods and services each household consumes are distinguished by the parents not only by their characteristics (e.g., different types of food and clothing), but also by their date of appearance (food today, food tomorrow, and so on) and the contingency (food the day after tomorrow if rainfall turns out to be bad tomorrow, and so forth). Implicitly or explicitly, both sets of parents convert their experience and knowledge into probabilistic judgments. Some of the probabilities they attach to contingencies are no doubt very subjective, but others are arrived at from long experience (the weather).

[3] A realistic alternative would be to suppose that household decisions are reached by negotiation between the various parties. See McElroy and Horney (1981) and Dasgupta (1993: ch. 11). Qualitatively, nothing much is lost in my assuming optimizing households here.

In subsequent sections we study the way Becky's and Desta's parents allocate goods and services across time and contingencies. So as to keep the exposition simple here, I consider a timeless, deterministic model. Let the vector $\underline{X}_i$ represent the quantities of various the goods and services consumed or supplied by the i^{th} member in a N-member household. $\underline{X}_i$ includes not only various kinds of consumption, but also the services i supplies. By convention, $X_i > 0$ if the good is consumed (food eaten) and $X_i < 0$ if it is supplied (labour). Here we take it that $\underline{X}_i$ is finite dimensional. Let $U_i(\underline{X}_i)$ be the well-being i would enjoy from $\underline{X}_i$ and $W(U_1(\underline{X}_1),...,U_i(\underline{X}_i),...,U_N(\underline{X}_N))$ the corresponding level of household well-being. We take it that U_i is an increasing function of each of the components of $\underline{X}_i$ and that W is an increasing function of U_i for all i. Let $\underline{X} = (\underline{X}_1,...,\underline{X}_i,...,\underline{X}_N)$. We assume that $\underline{X}$ is an element of J, which is interpreted as the set of all potentially feasible $\underline{X}$s. But the household faces constraints (the maximum amount of income it can earn would be one such constraint), meaning that not all elements of J are actually feasible. Let F ($\subseteq$ J) be the (non-empty) set of $\underline{X}$s from which the household can actually choose. F is the household's feasible set.[4] The household chooses $\underline{X} \in F$ so as to maximise $W(U_1(\underline{X}_1),...,U_i(\underline{X}_i),...,U_N(\underline{X}_N))$. This is the household's maximization problem.

As goods and services are not infinitely divisible and the world is bounded, J is a finite set. Therefore, both J and F are compact. But for mathematical convenience, economists assume that F is a continuum and carry the compactness assumption to it. Continuity of W is justified by the same reasoning. Since F is assumed to be compact and W continuous, the household maximization problem has a solution. The theory of non-linear programming can now be used to identify the optimality conditions that the household's choice must satisfy. If F is a convex set and W is a concave function of the $\underline{X}_i$s, those conditions are both necessary and sufficient. The Lagrange multipliers associated with F can be interpreted as *notional prices*: they reflect the worth to the household of relaxing the constraints slightly.

Let us conduct an exercise to test the power of the modern economist's way of studying choice. Assume that W is a symmetric and concave function of the Us (as would be the case if W were additive in the Us). Suppose now that household members are identical (i.e., U_i is independent of i, say, U); that U is a strictly concave function of the $\underline{X}_i$s (marginal well-being declines as consumption increases); and that F is non-empty, convex, and symmetric (i.e., $(\underline{X}_1,...,\underline{X}_i,...,\underline{X}_j,...,\underline{X}_N) \in F$ if and only if $(\underline{X}_1,...,\underline{X}_j,...,\underline{X}_i,...,\underline{X}_N) \in F$ for all $i,j \in \{1,...,N\}$). It follows that household members would be treated equally: at the optimum all would receive the same bundle of goods and services. At low levels of consumption, however, the hypothesis that F is convex is unreasonable. For example, 60-75 percent of the daily energy intake of someone in nutritional balance goes toward maintenance, while the remaining 25-40 percent is expended in discretionary activities (work and leisure). The 60-75 percent is rather like a fixed cost: over the long run a person needs it as a minimum no matter what he or she does. The simplest way

[4] Presently we will see why we need to distinguish J from F.

to uncover the implications of such fixed costs is to continue to suppose that F is convex (say, there is a fixed quantity of food for allocation among household members), but that U is a strictly convex function at low intakes of food and a strictly concave function thereafter. It is easy to confirm that a poor household in such a world would optimally allocate food unequally among its members, while a rich household would be able to afford the luxury of equal treatment and would choose to distribute food equally. Suppose, to take a very stylized example, that energy requirement for daily maintenance is 1,500 Kcal and a household of four can obtain at most 5,000 Kcal for consumption. Then equal sharing would mean that no one would have sufficient energy for any work. On the other hand, if the household is able to obtain more than 6,000 Kcal, it can share the food equally without jeopardizing its future.

There are empirical correlates of this finding. When food is very scarce, the younger and weaker members of Desta's household are given less to eat than the others, even after allowance is made for differences in their ages. In good times, though, Desta's parents can afford to be egalitarian. In contrast, Becky's household can always afford enough food. Her parents therefore allocate food equally every day.

Social Equilibrium

Household transactions in Becky's world are carried out mostly in markets. The terms of trade are the quoted market prices. To illustrate, let us introduce time into the model and imagine that Becky's parents have decided on how much of resources to set aside for the future and are now planning the pattern of consumption for the present period only.[5] Let $\underline{P}$ be the vector of market prices for current goods and services and S the amount of income they save (or borrow) for the future. S is evaluated at market prices and is positive if the household is a net saver, but negative if it is a net borrower. Recalling our convention that goods consumed are of positive sign and goods supplied are of negative sign, define $\underline{Z} = \Sigma\underline{X}_i$. Then F is the set of $\underline{X}$s that satisfy the "budget" constraint $\underline{P}.\underline{Z} + S \leq 0$.

The income Becky's household earns from the assets it supplies to the market is determined by market prices (Becky's father's salary, interest rates on bank deposits, returns on shares owned). Those prices in turn depend on the size and distribution of all capital assets in the economy and on household needs and preferences. They depend too on the ability and willingness of institutions, such as private firms and the government, to make use of the rights they in turn have been awarded. These functional relationships explain why Becky's father's skills as a lawyer (itself an asset, termed 'human capital' by economists) would not be worth much in Desta's village, even though they are much valued in the USA. In fact it was a firm belief that lawyers would continue to prove valuable in the USA that encouraged Becky's father to *be* a lawyer.

Although Desta's household does transact in markets (when her father sells *tef* or her mother sells the liquor she has brewed), it undertakes many transactions directly with nature in the local commons and

[5] Later we will study how Becky's parents would decide how much to put aside for the future.

in farming, and in non-market relationships with others in the village. Therefore the F that Desta's household faces isn't defined simply by a linear budget inequality, as in the idealised model we have constructed to display Becky's world, but it also reflects the constraints nature imposes (soil productivity, rainfall), the assets it has access to, and the terms and conditions involving transactions with others in the village via non-market relationships, a matter I come to below. The constraints imposed by nature are felt by Becky's household too, but through market prices. For example, should there be a general drought and world cereal production consequently low, Becky's household would notice it from the fact that the price of cereal is high. Desta's household in contrast would notice it directly from the reduced harvest from their field.

Desta's household assets include the family home, livestock, agricultural implements, and the half hectare of land. The skills Desta's family members have accumulated in farming, managing livestock, and collecting resources from the local commons are part of their human capital. Those skills don't command much return in the global marketplace, but they do shape the household's feasible set F and are vital to the family's well-being. Desta's parents learnt those skills from their parents and grandparents, just as Desta and her siblings have learnt them from their parents and grandparents. Desta's family can also be said to own a portion of the local commons: in effect her household shares their ownership with others in the village. Difficulties in reaching and enforcing agreement with neighbours over the use of the local commons are less severe than they are in the case of global commons, such as the atmosphere as a sink for carbon emissions. This is not only because the required negotiations involve far fewer people when the commons are local, but also because there is likely to be greater congruence of opinions and interests among the users. It helps too that the parties are able to observe whether the agreements they made over the use of local commons are being kept. (See below in our discussion of insurance arrangements in Desta's world.)

There are thus feedbacks between the choices people make and the choices they all are able to make. In a market economy, the feedback is in large part transmitted in prices. In non-market economies the feedback is transmitted in the terms at which households are able to negotiate with one another.

Imagine an economy of H households (h = 1,...,H). For ease of exposition, I now write household well-being W directly in terms of the household's allocation $\underline{X}$. Let $\underline{X}_h$ be the vector of goods and services to be 'consumed' by household h (with the usual sign convention); let J_h be the set of potentially feasible vectors $\underline{X}_h$; and let $W_h(\underline{X}_h)$ be h's well-being. We now model the feedback by recognising that h's feasible consumption vectors depend on the choices of other households. Let $F_h(\underline{X}_{-h})$ be the set of feasible $\underline{X}_h$ s, where $\underline{X}_{-h}$ denotes the allocation of goods and services in all households other than h (i.e., $\underline{X}_{-h} = (\underline{X}_1,...,\underline{X}_{h-1},\underline{X}_{h+1},...,\underline{X}_H)$) and F_h is a mapping from $\underline{X}_{-h}$ into subsets of J_h. Household h's economic problem is to choose $\underline{X}_h$ in $F_h(\underline{X}_{-h})$ so as to maximize $W_h(\underline{X}_h)$. The optimum choice depends on h's beliefs about $\underline{X}_{-h}$ and the correspondence $F_h(\underline{X}_{-h})$.

How can we unravel the feedbacks? One way would be to ask people to disclose their beliefs about the feedbacks. Fortunately, economists avoid that route. So as to anchor their investigation, economists study *equilibrium* beliefs; that is, beliefs that are self-confirming. The idea is to identify states of affair where the choices people make on the basis of their beliefs about the feedbacks are precisely those that give rise to those very feedbacks. We call any such state of affairs a *social equilibrium*. Formally, $\underline{X}^* \equiv (\underline{X}_1^*,...,\underline{X}_h^*,...,\underline{X}_H^*)$ is a social equilibrium if, for all h, $\underline{X}_h^*$ maximizes $W_h(\underline{X}_h)$ in the set $F_h(\underline{X}_{-h}^*)$.

Does a social equilibrium exist? Assume that for all h, J_h is non-empty, convex, and compact. It can be shown that if W_h is continuous and quasi-concave[6] and if, for all $\underline{X}_h$, the correspondence $F_h(\underline{X}_h)$ ($\subseteq J_h$) is non-empty, compact, convex, and continuous, a social equilibrium exists.[7] There is no presumption though that a social equilibrium is just or collectively good. Moreover, excepting for the most artificial examples, social equilibrium is not unique - meaning that a study of equilibria per se would leave open the question of which social equilibrium we should expect to observe. In order to probe that question, economists study disequilibrium behaviour and analyse the stability properties of the resulting dynamic processes. However, this has proved to be such a formidable problem that there is yet no consensus on how best to model the processes.[8] Therefore, we shall continue to study social equilibria here.

Public Policy

So far it has been assumed that households consume and supply what economists call 'private goods'; meaning that if X is the economy wide consumption of such a good and X_h the consumption of that good in household h, then $X = \Sigma X_h$. In sharp contrast to private goods are goods that are (i) jointly consumable and (ii) non-excludable. They are called 'public goods'. If G is the quantity of a public good produced and G_h is the quantity consumed by household h, (i) and (ii) imply that $G = G_h$ for all h. National security, the law, the distribution of income and wealth, and even the state, are public goods. The atmosphere is a global public good. Samuelson (1954) showed that the supply of public goods by private individuals involves the Prisoners' Dilemma and concluded that the dilemma would be resolved effectively,

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[6] $W_h(\underline{X}_h)$ is quasi-concave if for all $\underline{\bar{X}}_h \in J_h$, the set of all $\underline{X}_h \in J_h$ such that $W_h(\underline{X}_h) \geq W_h(\underline{\bar{X}}_h)$ is convex.

[7] The proof is straightforward: For each $\underline{X}_{-h}$ let $\underline{\Psi}_h(\underline{X}_{-h})$ be the set of optimum choices for h. From the assumptions made on $W(\underline{X}_h)$ and $F_h(\underline{X}_{-h})$ we know that $\underline{\Psi}_h(\underline{X}_{-h})$ is a non-empty, convex, compact, and upper-semi-continuous correspondence. Write $\underline{X} = (\underline{X}_1,...,\underline{X}_h,...,\underline{X}_H)$. Define $\underline{\Psi}(\underline{X}) \equiv \Pi\underline{\Psi}_h(\underline{X}_{-h}) \subseteq \Pi F_h(\underline{X}_{-h})$. It follows from the Kakutani Fixed Point Theorem that there exists $\underline{X}^* \in \underline{\Psi}(\underline{X}^*)$. The classics on the existence of social equilibrium are Nash (1950), Debreu (1952), and Arrow and Debreu (1954). In Becky's world, a social equilibrium is called a *market equilibrium*.

[8] The basic idea is to hypothesise the way people form beliefs about the way the world works, track the consequences of those patterns of learning, and check them against data. It would be reasonable, though, to limit any such study by considering only those learning processes that converge to a social equilibrium in stationary environments. Initial beliefs would then dictate which equilibrium is reached in the long run. See, for example, Evans and Honkapohja (2000).

not by markets, but by politics.[9] It is now conventional in political theory that government should be charged with imposing taxes, subsidies, and transfers, and engaged in supplying public goods. The state is also the natural agency to supply infrastructure, such as roads, ports, and electrical cables, requiring as they do investments that are huge in comparison with individual incomes. We now extend our earlier model to include public goods and infrastructure so as to study the state's economic task.

Imagine that social well-being is a numerical aggregate of household well-beings. If V is social well-being, we write it as $V(W_1,...,W_h,...,W_H)$. We would want to postulate that V is an increasing function of W_h, for all h. (Utilitarian philosophy would prescribe that $V = \Sigma W_h$.) Let $\underline{G}$ and $\underline{I}$ be vectors denoting the quantities of various public goods and infrastructure commodities, respectively. If $\underline{T}_h$ is the transfer of goods and services the government imposes on household h, let $K(\underline{T})$ be the set of feasible $\underline{G}$s and $\underline{I}$s for each $\underline{T}$ ($\equiv (\underline{T}_1,...,\underline{T}_h,...,\underline{T}_H)$). As we have introduced a new set of goods, the domain of household well-being functions has to be enlarged. So, in the obvious notation, we write $W_h = W_h(\underline{X}_h,\underline{G},\underline{I},\underline{T}_h)$. Moreover, h's feasible set F_h is now dependent also on $\underline{G}$, $\underline{I}$, and $\underline{T}_h$; so we write the set of feasible $\underline{X}_h$s as $F_h = F_h(\underline{G},\underline{I},\underline{T}_h,\underline{X}_{-h})$. To model optimum public policy, imagine a two-stage game where the government has the first move of choosing $\underline{G}$, $\underline{I}$, and $\underline{T}$. Households are the second movers, simultaneously choosing their optimal $\underline{X}_h$s from their feasible sets $F_h(\underline{G},\underline{I},\underline{T}_h,\underline{X}_{-h})$. Assume that the optimum is unique. Let $\underline{X}^*$ ($\equiv (\underline{X}_1^*,...,\underline{X}_h^*,...,\underline{X}_H^*)$) be a social equilibrium. Clearly, $\underline{X}_h^* = \underline{X}_h^*(\underline{G},\underline{I},\underline{T}_h)$ for all h. (We imagine that if there are multiple equilibria, the government is able to select among them by resorting to public signals.) An intelligent and benevolent government will anticipate $\underline{X}_h^*(\underline{G},\underline{I},\underline{T}_h)$ and choose $\underline{T}$, and $\underline{G}$ and $\underline{I}$ from $K(\underline{T})$, so as to maximize $V(W(\underline{X}_1^*),...,W(\underline{X}_h^*),...,W(\underline{X}_H^*))$.

The public policy problem we have just constructed, involving as it does double optimization, is technically very difficult. It transpires, for example, that for each $\underline{X}_{-h}$, $F_h(\underline{G},\underline{I},\underline{T}_h,\underline{X}_{-h})$ is non-convex even in some of the simplest model economies one can imagine; meaning that the $\underline{X}_h^*$s cannot be guaranteed to be continuous functions of $\underline{G}$, $\underline{I}$, and $\underline{T}_h$ (Mirrlees, 1984). This in turn means that the government's optimization exercise is not solvable by the standard Kuhn-Tucker techniques. In fact, of course, even 'double optimization' is a huge simplication. The government chooses; people respond by trading, producing, consuming; the government chooses again; people respond once again; and so forth in an unending series of moves and counter-moves. Identifying optimum public policy involves severe computational difficulties.

Matters of Trust: Laws and Norms

The previous examples stress that the fundamental problem facing people who would like to transact with one another concerns *trust*. For example, the extent to which parties trust one another shapes

[9] The Prisoners' Dilemma is a game in which each party has a strategy that is best for him/her regardless of what strategies the other parties choose, even though there is another set of strategies, one per party, that they all prefer.

the sets F_h and K. Therefore, if the parties don't trust one another, what could have been mutually beneficial transactions won't take place. But what grounds does a person have for trusting someone to do what he promises to do under the terms of an agreement? They would have grounds if promises were credible. Societies everywhere have constructed solutions to the credibility problem, but in different ways. What all solutions have in common, however, is their insistence that those failing to comply with agreements without cause will suffer punishment.

How does that common solution work?

In Becky's world the rules governing transactions are embodied in the law. The markets Becky's family enters are supported by an elaborate legal structure (a public good). Becky's father's firm, for example, is a legal entity; as are the financial institutions he deals with in order to accumulate his retirement pension, to save for Becky's and Sam's education, and so on. Even when someone in the family goes to the grocery store, the purchases (paid in cash or by card) involve the law, which provides protection for both parties (the grocer, in case the cash is counterfeit or the card is void; the purchaser, in case the product turns out on inspection to be sub-standard). The law is enforced by the coercive power of the state. Transactions involve legal contracts backed by an *external enforcer*, namely, the state. It is because Becky's family and the grocery store's owner are confident that the government has the ability and willingness to enforce contracts (i.e. to continue to supply the public good in question) that they are willing to transact.

What is the basis of that confidence? After all, the contemporary world has shown that there are states and there are states. Why should Becky's family trust the government to carry out its tasks in an honest manner? A possible answer is that the government in her country worries about its *reputation*: a free and inquisitive press in a democracy helps to sober the government into believing that incompetence or malfeasance would mean an end to its rule, come the next election. Notice how the argument involves a system of interlocking beliefs about one another's abilities and intentions. The millions of households in Becky's country trust their government (more or less!) to enforce contracts, because they know that government leaders know that not to enforce contracts efficiently would mean being thrown out of office. In their turn, each side of a contract trusts the other to refrain from renaging (again, more or less!), because each knows that the other knows that the government can be trusted to enforce contracts. And so on. Trust is maintained by the threat of punishment (a fine, a jail term, dismissal, or whatever) for anyone who breaks a contract. Once again, we are in the realm of equilibrium beliefs, held together by their own bootstraps. Mutual trust encourages people to seek out mutually beneficial transactions and engage in them.

Although the law of contracts exists also in Desta's country, her family cannot depend on it because the nearest courts are far from their village. Moreover, there are no lawyers in sight. As transportation is enormously costly, economic life is shaped outside a formal legal system. In short, crucial public goods and infrastucture are either unavailable or at best in short supply. But even though there is no external enforcer, Desta's parents do transact with others. Credit (not dissimilar to insurance in her village) involves saying, "I lend you now with your promise that you will repay me when you can". Saving for funerals involves saying, "I agree to abide by the terms and conditions of the*jiddir*". And so on. But why should the parties be sanguine that the agreements won't turn sour on account of malfeasance?

They would be sanguine if agreements were *mutually enforced*. The basic idea is this: a credible threat by members of a community that stiff sanctions would be imposed on anyone who broke an agreement could deter everyone from breaking it. The problem then is to make the threat credible. In Desta's world the solution to the credibility problem is achieved by recourse to social norms of behaviour.

By a *social norm* we mean a rule of behaviour followed by members of a community. A rule of behaviour ('strategy' in economic parlance) reads like, "I will do X if you do Y", "I will do P if Q happens", and so forth. For a rule of behaviour to *be* a social norm, it must be in the interest of everyone to act in accordance with the rule if all others act in accordance with it. Social norms are equilibrium rules of behaviour. We will now see how social norms work and how transactions based on them compare with market-based transactions, by studying insurance as a commodity.

Insurance

To insure oneself against a risk is to act in ways to reduce that risk. (Formally, distribution $\tilde{X}$ is said to be more risky than distribution $\tilde{Y}$ if $\tilde{Y}$ dominates $\tilde{X}$ second-order stochastically.) So long as the cost involved in so acting isn't too high, risk-averse households would wish to purchase insurance. In fact, avoiding downside risk would seem to be a universal urge. To formalise these notions, consider an isolated village, such as Desta's. Households there are H in number and are identical. If household h's food consumption is X_h (a scalar), its well-being is $W(X_h)$. We take it that $W'(X_h) > 0$ and $W''(X_h) < 0$ (strict concavity). We confirm below that strict concavity of W implies, and is implied by, risk-aversion.

For simplicity, imagine that household food production, which is subject to chance (weather), involves no effort. Let h's uncertain food output be a random variable $\tilde{X}_h$, with expected value μ (> 0). Write by E the expectation operator. Then, if households were autarkic, h's expected well-being would be $E(W(\tilde{X}_h))$. But $W(\mu) > E(W(\tilde{X}_h))$, which means that h prefers a sure consumption level to a risky one with a mean equal to that sure consumption level. In short, h is risk-averse. Define $\bar{\mu}$ by $W(\bar{\mu}) = E(W(\tilde{X}_h))$. Then $(\mu-\bar{\mu})$ is the cost of risk bearing under autarky. Notice that the greater is the 'curvature' of W, the greater is the cost of risk bearing associated with $\tilde{X}_h$.[10] To see how households could gain by pooling their risks, suppose $\tilde{X}_h = \mu + \tilde{\epsilon}_h$, where $\tilde{\epsilon}_h$ is a random variable with zero mean, σ^2 variance, and finite support. The probability distributions underlying the $\tilde{\epsilon}_h$s are assumed to be symmetrically arranged over the states of nature. Let $cov(\tilde{\epsilon}_h,\tilde{\epsilon}_{h'})/\sigma^2 = \rho$ for all h and h′. So long as $\rho < 1$, households would be able to reduce their

[10] A useful measure of curvature is $-XW''(X)/W'(X)$. We will make use of this measure when discussing intertemporal choices.

own risks by agreeing to share their realised outputs. Suppose households are able to observe one another's outputs. Given the underlying symmetry of the probability distributions, the obvious insurance scheme involves equal sharing of realized outputs. In this scheme, h's uncertain food consumption would be $\Sigma \tilde{X}_{h'}/H$, which would be an improvement on autarky, because $E(W(\Sigma \tilde{X}_{h'}/H)) > E(W(\tilde{X}_h))$. The problem is that without an enforcement mechanism, the agreement to share would not stick, because once food outputs are realised, all but the unluckiest households would wish to renage. (Why?) Since households would know this in advance, the only social equilibrium would be autarky: there would be no risk pooling.

To see how the insurance scheme could be made viable if a suitable social norm were to emerge, assume that the villagers face the same set of risks period after period, indefinitely, and that the risks are independent across time. Denote time by t (t = 0, 1, 2, ...). We are to imagine that in each period, once food outputs are realised, households decide independently of one another whether to abide by the agreement to share their produce equally or whether to renage. We call the insurance game just studied the 'stage game', for which the unique social equilibrium was shown to be autarkic. Assume that δ (> 0) is the constant rate at which households discount future expected well-beings. We now show that, provided δ is sufficiently small, there is a social equilibrium where households abide by the agreement.

If $\tilde{Y}_h(t)$ is h's uncertain income at t, expected well-being at t = 0 is ${}_0\Sigma^{\infty}(E(W(\tilde{Y}_h(t)))/(1+\delta)^t$. Under the agreement $\tilde{Y}_h(t) = \mu + (\Sigma\tilde{\epsilon}_{h'})/H$ at all $t \geq 0$. Consider the following strategy for h: Begin by participating in the insurance scheme and continue to participate so long as no household has renaged on the agreement; but withdraw from the scheme in all periods following the first violation of the agreement by some household. Game theorists have christened it the 'grim strategy', or simply *grim*, because of its unforgiving nature. Let us see how grim could support the original agreement to share aggregate output equally at every date.[11]

The basic idea is simple: As renaging on the agreement is the unique social equilibrium of the stage game, the threat by a household to defect permanently following the first defection by anyone is credible if all other households play grim. If δ is sufficiently small, no household could do better than to play grim if all others play grim; because to be the first to renage would mean a loss of future benefits from cooperation. We conclude that grim is an equilibrium strategy in the repeated game; meaning that it can function as a social norm.

How low does δ have to be if grim is to function as a social norm? To answer, consider household h. All other households are assumed to play grim. Once outputs have been realised at t = 0, household h wonders whether to break the agreement. As all other households play grim, h knows they will all cooperate at t = 0. Let X_{max} and X_{min} be the largest and smallest values of the support of $\tilde{X}_h$. Notice that the

[11] For a general account of repeated games and the variety of social norms that can sustain agreements, see Fudenberg and Maskin (1986).

incentive for h to renage would be greatest if its own output was to be X_{max} and every other household's output X_{min}. Since the average of that is $(X_{max} + (H-1)X_{min})/H$, the one-period gain h would enjoy by defecting is $W(X_{max}) - W((X_{max} + (H-1)X_{min})/H)$. However, as all other households are known to be playing grim, h knows that if it were to defect now, the per period loss it would suffer from t = 1 would be $E(W(\Sigma X_{h'}/H)) - E(W(\bar{X}_h))$; the present discounted value of which over the indefinite future, when evaluated at t = 0, is $[E(W(\Sigma X_{h'}/H)) - E(W(\bar{X}_h))]/\delta$. It follows that h won't renage on the agreement if

$$\delta < [E(W(\Sigma X_{h'}/H)) - E(W(\bar{X}_h))]/[W(X_{max}) - W((X_{max} + (H-1)X_{min})/H)]. \quad (1)$$

We usually reserve the term 'society' to denote a collective that has managed to equilibrate at a mutually beneficial outcome. Notice however that autarky is also a social equilibrium of the repeated game: If everyone was to believe that all others would break the agreement from the start, then everyone would break the agreement from the start. Non-cooperation would involve each household selecting the strategy: renage on the agreement. Failure to cooperate could be due simply to a collection of unfortunate, self-confirming beliefs, nothing else. It is also easy to show that autarky is the only social equilibrium of the repeated game if

$$\delta > [(E(W(\Sigma X_{h'}/H)) - E(W(\bar{X}_h))]/[W(X_{max}) - W((X_{max} + (H-1)X_{min})/H)]. \quad (2)$$

We now have in hand a tool for understanding how a community can slide from cooperative to non-cooperative behaviour. For example, political instability (in the extreme, civil war) can mean that households are increasingly concerned that they will be forced to disperse from their village. This would translate into an increase in δ. But from (1) and (2) we know that if δ increases sufficiently, cooperation ceases. The model therefore offers an explanation for why in recent decades cooperation at the local level has declined in the unsettled regions of sub-Saharan Africa. Social norms work only when people have reasons to value the future benefits of cooperation.

Unlike Desta's parents, Becky's parents have access to an elaborate set of insurance markets that pool the risks of hundreds of thousands of households across the country (even the world, if the insurance company is a multi-national). Pooling the risks of households that live at some distances from one another helps to reduce individual risks more than Desta's parents are able to enjoy, because spatially distant risks are likely to be uncorrelated. To illustrate, imagine that there are a large number of households elsewhere that are identical to household h in the village we have been studying. Suppose too that the risks are independent of one another. By the law of large numbers, equal sharing among all such households would guarantee μ to each household. This is an advantage of markets, backed by the coercive power of the state as an external enforcer: in a competitive market insurance contracts are available, enabling people who don't know one another to transact business through third parties (the insurance companies).

As the insurance Desta's parents are able to obtain within their village is very limited, they adopt additional risk-reducing strategies, such as diversifying their crops. Desta's parents plant maize, *tef*, and *enset* (an inferior crop), with the hope that even if maize were to fail one year, *enset* wouldn't let them down. That the local resource base in Desta's village is communally owned probably also has something to do with a mutual desire to pool risks. Woodlands are spatially non-homogeneous ecosystems. In some year one group of plants bears fruit, in another year some other group does. If the woodland were divided into private parcels, each household would face a greater risk than it would under communal ownership. The reduction in individual household risks owing to communal ownership may be small; but as average incomes are very low, household benefits from communal ownership are large.[12]

The Reach of Transactions and the Division of Labour

Payments in Becky's world are made in money, expressed in US dollars. Money wouldn't be required in a world where everyone is known to be utterly trustworthy and people don't incur computational costs: simple IOUs, stipulating repayment in terms of specific good and services, would suffice in that world. However, we don't live in that world. A debt in Becky world involves a contract specifying that the borrower is to receive a certain number of dollars and that he promises to repay the lender dollars in accordance with an agreed upon schedule. When signing the contract the relevant parties entertain certain beliefs about the dollar's future value in terms of goods and services. Those beliefs in part are based on their confidence in the US government to manage the value of the dollar. Of course, the beliefs are based on many other things besides; but the important point remains that money's value is maintained only because people believe it will be maintained.[13] Similarly if, for whatever reason, people feared that the value won't be maintained, then it won't be maintained. Currency crashes, as the one that occurred in Weimar Germany in 1931, was an illustration of how a loss in confidence can be self-fulfilling. Bank runs share that feature, as do stock market bubbles and crashes. To put it formally, there are multiple social equilibria, each supported by a set of self-fulfilling beliefs.

The use of money enables transactions to be anonymous. Becky frequently doesn't know the sales persons in the department stores of her town's shopping mall, nor do the sales persons know Becky. When Becky's parents borrow from their bank, the funds made available to them come from unknown depositors. Literally millions of transactions occur each day between people who have never met in the past and won't ever meet. The problem of creating trust is solved in Becky's world by building confidence in the medium of exchange: money. The value of money is maintained by the state, which has an incentive to maintain it because, as we saw earlier, it wishes not to destroy its reputation and be thrown out of office.

In the absence of infrastructure, markets are unable to penetrate Desta's village. Becky's suburban town, in contrast, is embedded in a gigantic world economy. Becky's father is able to specialise as a lawyer only because he is assured that his income can be used to purchase food in the supermarket, water from the tap, and heat from cooking ovens and radiators. Specialization enables people to produce more in total

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[12] For a fuller account of the management of local commons in poor countries, see Dasgupta (1993).

[13] The classic on this is Samuelson (1958).

than they would be able to were they each required to diversify their activities. Adam Smith famously remarked that the division of labour is limited by the size of the market. Earlier we noted that Desta's household does not specialize, but produces pretty much all its daily requirements from a raw state. Moreover, the many transactions it enters into with others, being supported by social norms, are of necessity personalised, thus limited. There is a world of a difference between laws and social norms as the basis of economic activities.

Borrowing, Saving, and Reproducing

Purchasing insurance helps to smooth consumption across contingencies. We will see presently that the human desire to so smooth consumption is related to our common desire to smooth consumption across time: they are both a reflection of the strict concavity of W. But the flow of income over a person's lifetime is not necessarily smooth. Borrowing and saving are ways to transfer consumption across time. Borrowing for current consumption transfers future consumption to the present; saving achieves the reverse. Since capital assets are productive, they can earn positive returns if they are put to good use. This is a reason why in Becky's world borrowing involves having to pay interest, while saving and investing earn positive returns.

Becky's parents took out a mortgage on their house, because at the time of purchase they didn't have sufficient funds to finance it. The house itself serves as a guarantee that the transaction won't turn sour on the bank. The mortgage represents a debt. On the other hand, by paying into a pension fund, Becky's parents transfer present consumption to their retired future. Moreover, they don't expect the children to repay them for the investment they will have made in their education. The direction of resource transfer is from parents to children in Becky's world. Children are a direct source of parental well-being, they are not investment goods.

A simple way to formulate the problem Becky's parents face in arranging transfers of resources across time is to imagine that they view themselves as part of a dynasty. The analysis is notationally tidiest if we assume that time is a continuous variable. At t (≥ 0), let K(t) denote household wealth and X(t) the consumption rate, which I take to be a scalar aggregate, constructed out of market prices. To concentrate on intertemporal issues, we consider a deterministic model. Suppose the market rate of return on investment is a constant r (> 0). The dynamical equation describing the dynasty's consumption options over time is then

$$dK(t)/dt = rK(t) - X(t), \qquad K(t), X(t) \geq 0, \tag{3}$$

where K(0) is the wealth that Becky's parents have inherited from the past. K(t) is a state variable. Earlier we assumed that the household's decision criterion for allocating consumption across contingencies is expected well-being. The corresponding criterion for allocating consumption across time is

$$\int_0^{\infty} W(X(t))e^{-\delta t}dt, \qquad \delta > 0, \tag{4}$$

where, as before, $W'(X) > 0$ and $W''(X) < 0$. In (4) δ, a constant, is the rate of pure time preference; it is the rate at which future well-beings are discounted, owing to myopia, risk of death, or whatever. In Becky's world it makes empirical sense to suppose that $r > \delta$. For concreteness, suppose $-XW''(X)/W'(X) = \alpha > 0$. Becky's parents' problem at $t = 0$ is to maximize (4) by suitable choice of $X(t)$, subject to (3).[14]

This is a problem in the calculus of variation. But it is of a somewhat unusual form: the horizon is infinite and there is no boundary condition at infinity. However, assuming for the moment that a solution exists, we know it must satisfy the Euler-Lagrange equation

$$\alpha dX(t)/dt = (r - \delta)X(t), \qquad t \geq 0. \tag{5}$$

There is an intuitive way to arrive at (5): Consider an arbitrary $X(t)$ that satisfies (3). Since r is the rate of return on investment, it is the rate at which consumption can be feasibly transfered across time. However, from (4) we can deduce that the rate at which the household would be *willing* to substitute consumption between two 'adjacent' dates is $(\delta + \alpha(dX(t)/dt)/X(t))$. If during any interval of time the two rates were unequal, an appropriate reallocation of consumption across time would increase (4). The two rates must therefore be equal at all t if $X(t)$ is an optimum.

Integrating (5) yields

$$X(t) = X(0)e^{(r-\delta)t/\alpha}. \tag{6}$$

$X(0)$ is a choice variable. It transpires (Koopmans, 1965) that $X(t)$ in (6) is optimal if $W'(X(t))K(t)e^{-\delta t} \to 0$ as $t \to \infty$. It transpires also that for the model in hand, there is a value of $X(0)$ - write it as $X^*(0)$ - such that (3) and the above asymptotic condition are satisfied by (6). This implies that $X^*(0)e^{(r-\delta)t/\alpha}$ is the unique optimum. The larger is the productivity of investment (r), the higher is the optimum rate of growth of consumption, other things being the same. In contrast, the larger is α, the lower is the rate of growth, other things being the same. α, being a measure of the curvature of W, is an index of the desire on the dynasty's part for intergenerational equity in consumption.

Let us conduct a simple exercise with our finding. Suppose the annual market rate of return is 4 per cent (i.e., $r = 0.04$ per year), that $\delta \approx 0$, and that $\alpha = 2$.[15] Then we can conclude from (6) that optimum consumption will grow at an annual rate of 2 per cent; meaning that it will double every 36 years - roughly, every generation. The figure is close to the post-War growth experience in the United States.

In contrast, Desta's parents are heavily constrained in their ability to transfer consumption across time. For example, they have no access to capital markets from which they can earn a positive return.

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[14] The problem originated in a classic by Ramsey (1928). Ramsey insisted that $\delta = 0$ and devised an ingenious argument to show that an optimum exists despite the non-convergence of the integral in (4). For simplicity, I am assuming $\delta > 0$. As $W(X)$ is bounded above and $r > 0$ (meaning that it is feasible for $X(t)$ to grow indefinitely), we should expect (3) to converge if $X(t)$ were allowed to rise fast enough.

[15] None of the assumptions is *ad hoc*. For example, empirical studies of savings behaviour in the USA have revealed that α is in the range 2-4. See Hall (1988).

Admittedly, they invest in their land (clearing weeds, leaving portions fallow, and so forth), but that is to prevent the productivity of land from declining. Moreover, the only way they are able to draw on the maize crop following each harvest is to store it. Let us see how Desta's household would ideally wish to consume that harvest over the annual cycle.

Let K(0) be the harvest, measured, say, in kilo calories. As rats and moisture are a potent combination, stocks depreciate. If X(t) is the planned rate of consumption and γ (> 0) the rate of depreciation of the maize stock, then the stock at t satisfies the equation

$$dK(t)/dt = -X(t) - \gamma K(t), \; K(t) \geq 0. \qquad (7)$$

Imagine that Desta's parents regard their household's well-being over the year to be ${}_0\int^1 W(X(t))dt$. As with Becky's household, let $-XW''(X)/W'(X) = \alpha > 0$. Then it is simple to show that optimum maize consumption *declines* over time at the rate γ/α. This explains why Desta's family consume less and become physically weaker as the next harvest grows nearer. But Desta's parents have realised that the human body is a more productive bank. So, the family consume much maize during the months following each harvest so as to accumulate body mass, but draw on that reserve during the weeks before the next harvest, when maize reserves have been depleted. Across the years maize consumption assumes a saw-tooth pattern.

As Desta and her siblings contribute to daily household production, they are economically valuable assets. Her male siblings, however, offer a higher return to their parents, because the custom (itself a social equilibrium!) is for girls to leave home on marriage and for boys to inherit the family property and offer security to their parents in old age. Because of an absence of capital markets and state pension, male children are an essential form of investment. The transfer of resources in Desta's household, in contrast to Becky's, will be from the children to their parents.[16]

The under-5 mortality rate in Ethiopia was until relatively recently in excess of 300 per 1000 births. So, parents had to aim at large families if they were to have a reasonable chance of being looked after by a male child in their old age. But fertility is not entirely a private matter, people are influenced by others' choices. This gives rise to a certain stickiness in household behaviour even under changing circumstances; which is why even though the under-5 mortality rate has fallen in Ethiopia in recent decades, Becky has 5 siblings.[17] High population growth has placed additional pressure on the local

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[16] See Caldwell (1982).

[17] See Dasgupta (1993) for the use of interdependent preferences to explain fertility behaviour. In the notation of the section on social equilibria, we are to suppose that household h's well-being has the form $W_h(\underline{X}_h, \underline{X}_{-h})$, where one of the components of $\underline{X}_{-h}$ is the number of births in the household, and that the higher is the fertility rate among other households in the village, the larger is the desired number of children in h. The theory based on interdependent preferences interprets transitions from high to low fertility rates as bifurcations (Dasgupta, 2003). Fertility rates are expected to decline even in Ethiopia. Interdependent preferences are currently being much studied by economists. See Durlauf and Young (2001).

ecosystem, meaning that the local commons that were managed in a sustainable manner previously are no longer harvested in a sustainable way. That it isn't is reflected in Desta's mother's complaint that the daily time and effort required to collect from the local commons has increased in recent years.

Differences in Economic Life

In this essay, I have used Becky's and Desta's experiences to show how it can be that the lives of essentially very similar persons can become so different.[18] Desta's life is one of poverty. In her world people don't enjoy food security, don't own many assets, are stunted and wasted, don't live long (life expectancy at birth in Ethiopia is under 50 years), can't read or write, are not empowered, can't insure themselves well against crop failure or household calamity, don't have control over their own lives, and live in unhealthy surroundings. The deprivations reinforce one another, so that the productivity of labour effort, ideas, physical capital, and of land and natural resources are all very low and remain low. The rate of return on investment is zero, perhaps even negative (γ, in the storage example). Desta's life is filled with *problems* each day.

Becky suffers from no such deprivation (for example, life expectancy at birth in the USA is nearly 80 years). She only faces what her society calls *challenges*. In her world, the productivity of labour effort, ideas, physical capital, and of land and natural resources are all very high (r, in the savings example) and continually increasing; for success in meeting each challenge reinforces the prospects of success in meeting further challenges.

We have seen, however, that despite the enormous differences between Becky's and Desta's lives, there is a unified way to view them, and that mathematics is an essential language for analysing them. No doubt it sounds deep and knowing to pronounce that life's essentials cannot be reduced to mere mathematics; in fact, mathematics is essential in economic reasoning. It is essential because in economics we deal with quantifiable objects of vital interest to people.

[18] For further elaboration, see Dasgupta (2004).

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