Simple Efficient Contracts in Complex Environments

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Abstract

The paper studies a general model of hold-up in a setting encompassing the models of Segal (1999) and Che and Hausch (1999) among others. It is shown that if renegotiation is modeled as an infinite-horizon non-cooperative bargaining game then, with a simple initial contract, an efficient equilibrium will generally exist. The contract is robust in the sense that it does not depend on fine details of the model. The contract gives authority to one party to set the terms of trade and gives the other party a non-expiring option to trade at these terms. The difference from standard results arises because the initial contract ensures that the renegotiation game has multiple equilibria; the multiplicity of continuation equilibria can be used to enforce efficient investment.

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1 Introduction

The classical hold-up problem is that if two individuals who must make relationship-specific investments might subsequently renegotiate the division of the surplus which results, they will under-invest because they will not both be able to realize the full marginal benefit of their investment. The literature on the hold-up problem has been very fruitful in providing explanations for a variety of economic institutions and contracting practices such as vertical integration, property rights and financial structure of firms (Klein, Crawford and Alchian (1978), Williamson (1979), Grossman and Hart (1986), Aghion and Bolton (1992)). Some papers have argued that the hold-up problem can be solved by appropriate contractual design. Most notably, Aghion, Dewatripont and Rey (1994), henceforth ADR (see also Chung (1991)), showed that in some situations the renegotiation game can be altered by contractual devices in such a way as to induce efficient investment, while Nöldeke and Schmidt (1995) showed that option contracts may in certain circumstances achieve efficiency. These papers, however, refer to situations in which bilateral direct externalities do not obtain (for example, in a buyer-seller model, it is not the case that the buyer's investment affects the seller's production cost and the seller's investment affects the value of the good to the buyer, clearly a strong assumption in many applications). Che and Hausch (1999) showed that if bilateral direct externalities are incorporated in the standard model, and if a commitment not to renegotiate the contract is impossible, then it may be that contracts are able to achieve nothing: the optimal contract is the null contract and the inefficiency caused by the hold-up problem is severe. Furthermore, Segal (1999) and Hart and Moore (1999) showed that a similar conclusion obtains if the environment is complex in the sense that there are many potential goods (perhaps not *ex ante* describable) and it is not known *ex ante* how investment will affect the net value of any specific good.

The purpose of this paper is twofold. Firstly, it shows that if production and trade can be indefinitely delayed while renegotiation is taking place then contracts exist which can in fact lead to efficiency even when the environment is complex and when there are bilateral direct externalities. Secondly, it addresses a familiar question in contract theory: why are observed contracts so often simple? Theoretically optimal contracts are often complex. Furthermore, they are often not robust, in the sense that they are sensitive to the details of the model, such as the utility functions, the renegotiation process, or the way in which future surplus depends on the investments. In the ADR analysis, for example, the optimal contract includes a default outcome which has to be calibrated in such a way that at the seller's first-best investment her marginal investment cost is equal to the marginal benefit of investment, assuming the default outcome happens. We show in this paper, however, that a single simple contract can support the efficient investments for a wide range of specifications of the renegotiation process and the environment.

We consider a buyer-seller model with two-sided investments encompassing, among others, settings of the types studied by Che and Hausch (1999), Segal (1999) and Hart and Moore (1999). We adopt the view that there is an exogenously given non-cooperative renegotiation game, which we take to be an infinite-horizon game. The result is that there are simple contracts with the property that, for a large class of plausible non-cooperative renegotiation games, there exists an efficient equilibrium. In addition to simplicity and robustness, the contracts which are used to achieve efficiency have the advantage that they do not assume that future contingencies are describable *ex ante*. They give one party the authority to set *ex post* the nature and terms of trade and they give the other party a non-expiring option whether to trade or not. They may also require one party to post a bond as a guarantee of good behavior.

The logic of the equilibrium is as follows. Suppose that the party with authority is the buyer. After the investments have been made and the uncertainty has been resolved the buyer's equilibrium strategy is to specify an efficient trade. Subsequently, since trading according to the terms of the existing contract is efficient, there will be an equilibrium in which there is no renegotiation of the contract and the parties do indeed trade efficiently on these terms. On the other hand, there will also exist another equilibrium in which, essentially, the seller declines to exercise her option and bargains over a new contract. Since there are multiple equilibria in the continuation game, it is possible to condition the equilibrium played on the investments made in the first stage, giving both parties the incentive to invest efficiently (depending on the parameters, it may also be necessary to use a financial hostage to influence the shares of surplus which the two parties get in renegotiation). Equally, if the buyer abuses his authority by specifying either an inefficient trade or the wrong terms of trade, he can be punished by selection of an appropriate continuation equilibrium. The argument relies on the fact that there are multiple equilibria in the renegotiation game. This explains the difference between the results of this paper and those of earlier papers: typically earlier papers assume (often without an explicit non-cooperative analysis) that the renegotiation game has a unique outcome. Non-cooperative bargaining games (the Rubinstein game, say) do often have a unique equilibrium. However, we have here a game in which there already exists a contract (perhaps a contract which may, depending on the moves which the parties make, give an efficient outcome), but renegotiation can take place according to some given protocol. This is different from a game in which there is no contract and in which the players, using the same protocol, must negotiate an agreement before any surplus can be realized. For most of the standard bargaining protocols, there will necessarily be multiple equilibria in the renegotiation game starting with a contract of the type considered here.

A number of conclusions follow from the analysis. As several of the papers mentioned above have found, contracts which are very incomplete (e.g., authority contracts) may in fact be optimal, particularly when renegotiation is allowed. Our results lend support to this conclusion. Bolton and Rajan (2001) have shown, in an asymmetric information framework with repeated interaction, that employment contracts may be optimal. The contracts in the current paper look very like employment contracts since one party gives instructions and the other has the legal right, in effect, to strike and negotiate for better terms. We find that such contracts are efficient even in a symmetric information framework which is essentially one-shot (in the sense that it is a one-off project - only one good is to be produced). We also find (in Section 6) that if it is not possible for the parties to deposit a financial hostage then it is optimal to give authority to that party which, other things being equal, has relatively high investment cost or low bargaining power. If the 'employer' has high bargaining power then the threat of a 'strike' by the 'employee' is too low to give the former the incentive to invest enough in the relationship. If financial hostages are feasible, then either party could be given authority.

Secondly, an explicitly non-cooperative analysis may yield different conclusions from those of standard contract theory. The latter generally assumes either that, as discussed above, there is a black-box renegotiation process or else that the contract can effectively design an entire extensive form for the parties to play. Here we assume, by contrast, that the contract can only overlay an existing non-cooperative game, an approach which Aghion, Dewatripont and Rey (2002) have called 'partial contracting'. The literature on relational contracting (see Macleod and Malcomson (1989) and Levin (2003)) has a similar outlook. A related point, which has been stressed by Watson (2007) is that if the parties can take costly and irreversible actions as part of the mechanism (rather than simply send messages as in classical contract theory[1]) then the implementable outcomes will be very different.[2]

Most importantly, the fact that there is a contract for which an efficient outcome exists must cast doubt on the standard argument of the incomplete contracts literature that hold-up is a major impediment to efficient contracting. Undoubtedly contracts do often fail to achieve efficient outcomes. The results reported here show that we should look elsewhere for explanations of this fact. Two leading candidates for this explanation are asymmetry of information and failures of *ex post* verifiability.[3]

The remainder of the paper is organized as follows. Section 2 sets out the underlying bilateral monopoly model. In section 3 we outline the essential argument in the context of a simplified model and a particular specification of the renegotiation process. Section 4 sets out a model with a general specification of the renegotiation

[1]See Maskin and Moore (1999) for the standard approach to mechanism design with renegotiation. In this approach, the mechanism delivers an outcome, as a function of messages which are intrinsically costless, which may be inefficient; if so, renegotiation then takes place to an efficient outcome.

[2]Joel Watson also independently discovered the equilibrium described above in which the option is exercised - see the February 2002 discussion paper version (UCSD 2002-04) of Watson (2007).

[3]See Bolton and Dewatripont (2005), chapter 12.

process. The main efficiency result is in Section 5. Section 6 considers the case in which it is not possible to post bonds and analyzes the optimal allocation of authority. Section 7 relates the paper to the existing literature and discusses the realism and plausibility of the analysis. Finally, Section 8 concludes.

2 The Underlying Model

There are two risk-neutral players, a buyer (B) and a seller (S). First S and B each, simultaneously, choose an amount of investment from $[0, \infty)$. S's chosen investment is denoted by i_s, and has cost $\psi_s(i_s)$, and B's by i_b, with cost $\psi_b(i_b)$. ψ_s and ψ_b are both strictly increasing, non-negative functions. After the investments are made a state of the world θ is randomly realized from a set of possible states Θ. S and B both observe each other's investments and θ. After the realization of θ production and trade can take place. For each triple (i_s, i_b, θ), there is a set $A(i_s, i_b, \theta)$ of goods which it is feasible for S to produce. The parties trade at most one of these goods.

Suppose that the investments are (i_s, i_b) and the realized state is θ. Suppose also that, at the first opportunity, S produces good $a \in A(i_s, i_b, \theta)$, and B accepts delivery and pays p to S. Then B's payoff is $v(i_s, i_b, \theta, a) - p - \psi_b(i_b)$ and S's payoff is $p - c(i_s, i_b, \theta, a) - \psi_s(i_s)$, where v, the value of the good to B, and c, the cost of the good's production to S, are non-negative functions. If production and delivery take place at a later date (the time structure of the model will be set out below) then the value, cost and payments are appropriately discounted using a common discount factor δ.

Efficient Actions

Given $(i_s, i_b, \theta) \in \Re_+ \times \Re_+ \times \Theta$, let $a(i_s, i_b, \theta)$, the efficient good, be a solution to the problem

$$\max_{a \in A(i_s, i_b, \theta)} v(i_s, i_b, \theta, a) - c(i_s, i_b, \theta, a)$$

We assume that, for each $(i_s, i_b, \theta) \in \Re_+ \times \Re_+ \times \Theta$, a solution to this problem exists. This would be guaranteed if, for example, $A(i_s, i_b, \theta)$ were a finite set for each (i_s, i_b, θ).

We assume also that at least one element of A gives a non-negative surplus. This is essentially without loss of generality because we can always define an arbitrarily fixed sum of money as one of the "goods" in $A(i_s, i_b, \theta)$. In that case the "produced" good generates a zero surplus. Where the investments and state (i_s, i_b, θ) are understood, we will sometimes denote the value and cost of a good $a \in A(i_s, i_b, \theta)$ by $v(a)$ and $c(a)$ respectively.

Let $\sigma(i_s, i_b, \theta) = v(a(i_s, i_b, \theta)) - c(a(i_s, i_b, \theta))$. This is the *ex post* surplus available if the investments and state of the world are (i_s, i_b, θ), gross of investment costs and evaluated at a time when production is feasible. We assume that the expected gross surplus $E_\theta \sigma(i_s, i_b, \theta)$ exists for each (i_s, i_b) and is strictly increasing in i_s and i_b, and that a solution exists for the problem

$$\max_{i_s \in \Re_+, i_b \in \Re_+} E_\theta \sigma(i_s, i_b, \theta) - \psi_s(i_s) - \psi_b(i_b)$$

This would be ensured, for example, if $E_\theta \sigma(i_s, i_b, \theta)$ were bounded and concave and ψ_s and ψ_b were unbounded and convex.

We denote the first-best investment levels, the solution to the above problem, by (i_s^*, i_b^*) and we make the following assumption, which guarantees that the problem is not trivial.

$$u^* = E_\theta \sigma(i_s^*, i_b^*, \theta) - \psi_s(i_s^*) - \psi_b(i_b^*) > 0. \tag{1}$$

Relation to the Literature

Before completing the description of the game let us relate the model to the existing literature. The model is general enough to incorporate as special cases many of the models found in the literature on incomplete contracts.

(a) Suppose that there is a single indivisible good to be traded, there is no exogenous uncertainty, v is an increasing function of the buyer's investment and c is a decreasing function of the seller's investment. This is a version of the classic hold-up model (for an early treatment of the hold-up problem see, e.g., Williamson (1983)). The model above reduces to this if Θ and A are singletons, $v(i_s, i_b, \theta, a)$ depends only on i_b and $c(i_s, i_b, \theta, a)$ depends only on i_s.

(b) Suppose that the model is as in (a) except that $\Theta \subseteq \Re^n$ is not a singleton, v varies with θ and i_b but not with i_s and c varies with θ and i_s but not with i_b. Then we have a version of the model of Hart and Moore (1988).[4] In this case the value and cost, conditional on the investments, are random variables.

(c) Letting v vary with i_s as well as i_b and θ and letting c vary with i_b as well as with i_s and θ gives, *inter alia*, a version of the "cooperative" investment model of Che and Hausch (1999). In this case the buyer's expected value for the good may be increased by an increase in the seller's investment and, similarly, the seller's cost of production may be reduced by an increase in the buyer's investment. That is, there are bilateral externalities involved in the choice of investments.

(d) Suppose, as in Segal (1999) and Hart and Moore (1999), that one indivisible good will be traded but that there are many possible versions of it ("widgets") and it is not known *ex ante* which of these widgets should be traded if total surplus is to be maximized. This can be accommodated by letting $A = W$ where W is the set of widgets, and letting $v(i_s, i_b, \theta, a)$ depend only on (i_b, θ, a) and $c(i_s, i_b, \theta, a)$ only on (i_s, θ, a). Indeed, in the model set out here, the parties may not know in advance what the feasible set will be.

(e) The model also allows the parties to produce and consume variable quantities of the good rather than a single indivisible good (as in, for example, Edlin and Reichelstein (1996)), since different elements of A might be different quantities of the same good, or indeed bundles of goods (but we assume that a bundle must be produced all at once).

Most of the results in fact would apply equally in a model considerably more general than the one above but we concentrate on this one in order to simplify the exposition. For example, one could consider, rather than a model of production and trade, one in which the parties each take general multi-dimensional actions. Further-

[4]As we discuss below, however, Hart and Moore make different assumptions about contractibility.

more, the results of this paper extend to the case of multi-dimensional investments as well as to the case in which the parties are risk-averse.

3 Outline of the Argument

The model set out in the previous section needs to be augmented in a number of ways. Firstly, we need to specify what contracts are enforceable. Secondly, we need a description of how contracts are renegotiated. Thirdly, we have to specify the dates at which it is feasible to produce and deliver a good. The approach adopted here is that bargaining over new contracts, and the interaction of this bargaining with decisions about production and acceptance of delivery, should be modeled as a dynamic non-cooperative game. In section 4 we will analyze a general class of such games. It may be helpful, however, to set out the basic argument in the context of a specific game and a simplified version of the general model. We do that here.

For the purposes of this section, assume that $\sigma(i_s, i_b, \theta) + c(i_s, i_b, \theta, a)$ is uniformly bounded above by some number P. Assume also that there exists $\varepsilon > 0$ such that, for all $i_b \leq i_b^*$,

$$E_\theta \sigma(i_s^*, i_b^*, \theta) - \psi_s(i_s^*) - \psi_b(i_b^*) > \frac{1}{2} E_\theta \sigma(i_s^*, i_b, \theta) - \psi_b(i_b) + \varepsilon. \quad (2)$$

That is, the total available surplus exceeds what B can get by investing $i_b \leq i_b^*$, assuming that S invests i_s^* and B subsequently gets half of the gross surplus.

Consider the following game, played over a potentially infinite sequence of periods $0, 1, 2, \ldots$ by B and S, who have a contract α_0 at the outset of the game. In period 0 there is the following sequence of moves.

1. B and S simultaneously choose investments i_b and i_s respectively.

2. A state of the world $\theta \in \Theta$ is realized and observed by both players.

3. B chooses a public message from a set $M(i_s, i_b, \theta)$ to be described below.

4. S proposes a contract from a set C to be described below, or else makes no proposal.

5. B accepts or rejects S's proposed contract.

6. S produces (and trades) a good a from the set $A(i_s, i_b, \theta)$, or else chooses not to produce. If S produces a good, the game is over. If not, it continues to the next period.

In period 1, B proposes a contract in C, S accepts or rejects it, and then, as in the previous period, S chooses whether to produce and, if so, which good. That is, steps 4, 5 and 6 of period 0 are repeated, but with a different proposer. If there is no production, then, in period 2, S proposes again, B accepts or rejects, and S gets another chance to produce. Play proceeds in this way, with the identity of the proposer alternating, until S produces a good and the game ends. Each player always observes all the previous moves of the other player.

At any stage of the game there will be a ruling contract, or contract in force: either α_0 or, if a player has accepted a contract renegotiation proposal, the last such accepted contract. Contracts are enforced by a court. This court observes the public message in period 0 and, in each period, observes whether S has produced a good and, if so, what that good (i.e., its name) is. It also observes what the contract in force is. Nothing else is observed by the court: in particular, the investments and θ are not verifiable, and the court does not know which good is efficient. A *verifiable history* is that part of the history of play which is observable to the court. Any contract will specify money transfers as a function of the verifiable history. These transfers are assumed to be automatically enforced by the court at the time that they are triggered by the verifiable history.

The set C of contracts which the parties are allowed to propose during renegotiation consists of all contracts of the form 'if S produces good $\hat{a}$ in this period then B must pay p to S; if S does not produce $\hat{a}$ in this period then S must pay $P - p$ to B'. Here P is the upper bound of $\sigma + c$, p is a real number and $\hat{a}$ is a specific, described good. 'This period' refers to the period in which the contract is agreed.

The set $M(i_s, i_b, \theta)$ of messages includes, at least, all messages of the form (a, p) where a is a feasible good and p is a price.

In this game B does not have a choice as to whether to accept delivery of the produced good, so if S produces a in period t then S gets payoff $-c(a)\delta^t - \psi_s(i_s)$ and

B gets $v(a)\delta^t - \psi_b(i_b)$ (excluding any transfers). Any transfers which are enforced by the court enter the payoff functions in an additively separable way, appropriately discounted.

Denote this game by $g(\alpha_0)$. The question is: does there exist a contract α_0 which is enforceable (i.e., specifies transfer payments as a function of verifiable histories) and is such that $g(\alpha_0)$ has an efficient subgame-perfect equilibrium? The conventional wisdom would be that the answer is no, since the game merely adds a specific infinite-horizon non-cooperative renegotiation game to a model which in general exhibits a severe hold-up problem when renegotiation is conventionally modeled.

But consider the following simple contract, $\tilde{\alpha}_0$, which combines the features of an option contract (S has a non-expiring option to supply) with a partial authority contract (B is given the right to set the terms of the option).

$\tilde{\alpha}_0$: At step 3 of period 0 (i.e., after learning θ), B nominates a good a_b and a price p. If at any subsequent time S produces good a_b then B must pay p to S. If S produces a different good, or no good, then no payments are due by either party.

We will show that $g(\tilde{\alpha}_0)$ has an efficient equilibrium if δ is close enough to 1.

First, note that, in any equilibrium, once a renegotiation proposal (for good $\hat{a}$) has been accepted, then S must immediately produce $\hat{a}$, since otherwise the penalty outweighs any conceivable future benefit. Therefore we can regard a renegotiation proposal as equivalent to a proposed allocation of the available surplus (an inefficient allocation if $\hat{a}$ is not the efficient good).

Consider a subgame starting at step 4 of period 0 in which B has nominated the efficient good $a(i_s, i_b, \theta)$ and price $p = c(a(i_s, i_b, \theta)) + b\sigma(i_s, i_b, \theta)$, where $b < \delta^2(1+\delta)^{-1}$. That is, the price would give S a share b of the available surplus. There is an equilibrium of this subgame in which S never produces until a new contract is agreed, and both parties adopt the bargaining strategies of the standard Rubinstein equilibrium. Clearly, given these bargaining strategies, it is always optimal for S not to exercise her option since doing so gives her a share b, which is less than she gets if she waits (either $\delta(1+\delta)^{-1}$ or $\delta^2(1+\delta)^{-1}$, depending on who proposes next). Equally, since S will not exercise the option, the option is irrelevant and the game effectively reduces to the Rubinstein bargaining game. Hence the stated strategies are always optimal. This would be the unique equilibrium if there were no pre-existing contract.

However, the following is a second equilibrium. As long as the game has not ended and no new contract has been agreed, S *always* exercises the option. In the bargaining over a new contract, B always demands the whole surplus (σ) and accepts if and only if offered at least a share $(1-b)$, while S always demands the whole surplus and accepts if and only if offered a share of at least b. Since S will exercise the option, it is clearly optimal for B not to accept anything less than $(1-b)$ or offer more than b. Equally, given B's strategy, S must exercise the option.

These two types of continuation equilibrium can be exploited to give both parties the right investment incentives at the outset. By (2), there exist β_1, β_2 and $\bar{\delta} \in (0,1)$ such that, for all $\delta > \bar{\delta}$, $\beta_1 < \delta^2(1+\delta)^{-1}$,

$$\beta_1 - \frac{\psi_s(i_s^*)}{E_\theta \sigma(i_s^*, i_b^*, \theta)} > \beta_2 > 0,$$

and

$$E_\theta(1-\beta_1)\sigma(i_s^*, i_b^*, \theta) - \psi_b(i_b^*) > \frac{\delta}{(1+\delta)} E_\theta \sigma(i_s^*, i_b, \theta) - \psi_b(i_b) \tag{3}$$

for all $i_b < i_b^*$.

The following strategy profile is efficient and subgame-perfect if $\delta > \bar{\delta}$. S invests i_s^* and B invests i_b^*. At step 3 of period 0, B nominates the efficient good $a(i_s, i_b, \theta)$. If S has not under-invested then B nominates a share β_1 for S, i.e., he nominates an option price $c(a(i_s, i_b, \theta)) + \beta_1 \sigma(i_s, i_b, \theta)$; and otherwise, i.e., if $i_s < i_s^*$, he nominates a share β_2 for S. From step 4 onwards: (i) if B has not under-invested ($i_b \geq i_b^*$) and has nominated as described above, they play the equilibrium above in which the option is always exercised; (ii) if B's nominated good is infeasible they play the Rubinstein equilibrium; (iii) if B has deviated by under-investing or nominating the wrong good or wrong option price, then, letting x_s be S's payoff if she exercises the option, (a) if $x_s \leq \frac{\delta^2}{(1+\delta)}\sigma$ they play the Rubinstein equilibrium and (b) otherwise they play an arbitrary continuation equilibrium in which, of course, S gets at least x_s since she can always exercise the option.[5]

To see that this is an equilibrium, note first that neither party has an incentive to invest more than the first-best amount (the surplus shares do not change as a result of such a deviation). If S chooses $i_s < i_s^*$ then she gets an expected payoff of

$$\beta_2 E_\theta \sigma(i_s, i_b^*, \theta) - \psi_s(i_s) < \beta_2 E_\theta \sigma(i_s^*, i_b^*, \theta)$$

compared with $\beta_1 E_\theta \sigma(i_s^*, i_b^*, \theta) - \psi_s(i_s^*)$ if she invests i_s^*. By the definition of β_2, i_s^* is better. The Rubinstein bargaining outcome is worse for B than the shares $1 - \beta_1$ or $1 - \beta_2$ that he would get by following the given strategy profile, so B will not want to deviate from the specified nominations. If B invests $i_b < i_b^*$, his expected payoff is $\delta(1+\delta)^{-1} E_\theta \sigma(i_s^*, i_b, \theta) - \psi_b(i_b)$, compared with $E_\theta(1-\beta_1)\sigma(i_s^*, i_b^*, \theta) - \psi_b(i_b^*)$ if he invests i_b^*. By (3), i_b^* is better.[6]

The equilibrium works because, given the initial contract $\tilde{\alpha}_0$, there will be multiple equilibria in the contract renegotiation stage. These multiple continuation equilibria are used to punish each party for investing suboptimally. In addition, the problem that the efficient actions are unverifiable and unknown in advance is solved by giving one party authority to specify them. This party can then be deterred from acting opportunistically by means of the same multiple continuation equilibria. Note that since the game is not repeated (production takes place at most once) this result is not due to Folk Theorem considerations.

The model of this section is special in a number of ways. Firstly, inequality (2) is restrictive. In the equilibrium B is punished for under-investing by reversion to the Rubinstein equilibrium. The role of inequality (2) is to ensure that this punishment is large enough. However, as we show in section 4, if the inequality is not satisfied, efficiency can be achieved by a simple variant of $\tilde{\alpha}_0$: at the outset of the game B is required to deposit a sum of money which is returnable to B if and only if S

[5] Details are omitted for reasons of space, but, for any such nominated option, it is straightforward to construct an equilibrium in which S exercises the option, in the absence of agreement or previous production, in periods $1, .., T, .., 2T-1, ..$ where T depends on the option.

[6] Since each player's equilibrium payoff is strictly positive, a similar construction would work, at least for some parameters, if one or both parties had a preference for over-investment, i.e., negative investment cost.

produces a good. The deposit has the effect of increasing the surplus over which the parties bargain, and therefore increasing S's share (and reducing B's) relative to the production surplus σ.

Secondly, one might wonder whether the result is an artifact of the precise extensive-form used. We have assumed not only an alternating-offers bargaining protocol, but also a particular way in which the timing of the production opportunities relates to the timing of offers and responses. Furthermore, the set of contracts which the players are allowed to propose during renegotiation is restrictive. We provide a general analysis in section 4, but make a brief observation here.

Suppose that in each period, instead of having an opportunity to produce after the proposal and rejection of a new contract, S has this opportunity only *before* the renegotiation opportunity. That is, S can produce at step 4 of period 0 and at step 1 of subsequent periods. Then the surplus available for bargaining over during renegotiation is $\delta\sigma$ rather than σ because production can only take place after a delay of one period. Apart from this, however, the analysis is unchanged. It is easy to see that the Rubinstein equilibrium remains. But there is still an equilibrium in which S always exercises the option. B and S both always demand all the surplus. B accepts an offer if and only if it gives him at least $\delta(1-b)\sigma$. S accepts if and only if it gives her at least $\delta b\sigma$. As we show below, the key point is that exercising the option is *efficient*.[7] Furthermore, if renegotiation were not possible, the option would, after any history, be exercised in equilibrium. Therefore, when renegotiation is possible, there will always be an equilibrium in which there is no renegotiation and the above efficient equilibrium is played, and this conclusion holds essentially regardless of the precise details of the renegotiation process.

4 General Analysis

The disadvantage of a non-cooperative game-theoretic analysis is that the results may be sensitive to the precise formulation of the extensive form. Therefore, rather

[7]Because the option is efficient, the equilibrium discussed here is different from the one in Shaked (1994). See Section 7.

than setting out a particular extensive form, we analyze here a large class of extensive forms, varying in the fine detail of precisely when proposals can be made, how these dates relate to dates at which production decisions can be made and verifiable messages sent, how the proposer is chosen, and what contracts can be proposed. A generic game in this class is denoted by $G(\alpha_0)$, α_0 being the initial contract.

Like the game $g(\alpha_0)$ analyzed in section 3, $G(\alpha_0)$ begins with simultaneous investments and the extensive form subsequently consists of singleton nodes extending over an infinite number of discrete periods. Payoffs are as in $g(\alpha_0)$ except where stated below. Each node is either a *renegotiation* node (proposal or response), a *real action* node, or a *randomization* node. At a proposal node, the relevant player may or may not make a proposal. Each proposal is immediately (in the same period) followed by a response by the other player (we assume that no action can be taken between a proposal and a response, but it is easy to see, for example, that this assumption is not necessary in some variants of $g(\alpha_0)$). There is at least one proposal node per period,[8] and at least one of the proposal nodes in period 0 arrives after θ is realized. The identity of the proposer at a proposal node, and the precise position of such a node relative to other nodes, are functions (possibly random, via the randomization nodes mentioned above) of the previous actions of the players.

Real action nodes are either public message nodes (for either player), production nodes (for S) or delivery acceptance nodes (for B). There is at least one public message node for each player in period 0, after realization of θ, but there may be others in later periods. There is at least one production node per period (the earliest is after the realization of θ and the subsequent message nodes), but there may be more. After production, B has at least one opportunity to accept delivery per period, including the period in which production took place, and obtains consumption value $v(a)$ only if and when he accepts delivery. The game ends only when B does so. Acceptance and non-acceptance of delivery are both verifiable. Other verifiability assumptions, and descriptions of feasible actions at each real action node, are as in $g(\alpha_0)$.

[8]Here, and below, this means at least one node per period *on any path*.

As above, a contract specifies payments to be made by one party to the other, as a function of verifiable events. The sets of contracts which the parties may propose[9] in the renegotiation process are referred to as C_1 (for renegotiation proposals made before θ is realized) and C_2 (for proposals made after this). We assume that after θ is realized the elements of A are describable (this is the assumption commonly made in the incomplete contracts literature) and that C_2 includes at least all those in C, for arbitrary quantities P (see section 3 above). In effect, such contracts are *specific performance contracts*, as allowed, for example, by ADR and Nöldeke-Schmidt. Since contracts in C condition on whether, and what, S has produced, our contractibility assumptions are different from those of Hart and Moore (1988), who assume that the court can observe whether trade took place but not who was responsible for a failure of trade. We do not assume that before θ is realized the goods are describable.[10] Therefore, unlike contracts in C_2, contracts in C_1 might not be able to include references to the potential goods. The pre-existing contract α_0 is in C_1 and $C_1 \subseteq C_2$, so it is always possible to propose the current ruling contract (i.e., "no renegotiation").

We assume that contractual payments cannot depend on previous ruling contracts. We also assume that it is possible in period 0 for one or both parties to deposit a sum of money which will be returned to that party if and only if certain verifiable events have happened. One can suppose, for example, that the deposit is held by the court, or by a court-appointed lawyer; alternatively, it might be held by the other party. For simplicity, we assume that a deposit can be released only at dates when production of goods is feasible.

Given $K \geq 0$, let $C^K \subseteq C_1$ be the set of contracts in C_1 which require that B make a non-interest-bearing deposit K at the beginning of the game, this sum being repayable to B as soon as S produces a good (any good) and not otherwise. $\emptyset^K \in C^K$ is the contract which has no other provisions. Thus $\emptyset^0$ is equivalent to the null contract.

[9]The proposer proposes a single contract. Alternatively, we could assume that the proposer offers a menu of contracts for the responder to select from.

[10]For discussions of the idea that incompleteness of contracts is related to inability to describe contingencies in advance, and that this indescribability problem leads to inefficiency, see Maskin and Tirole (1999a) and Hart and Moore (1999).

The rules of the renegotiation process, and the nature of the extensive form generally, are constrained by two assumptions (Assumptions 1 and 2).

Assumption 1 (Weakly Exogenous Renegotiation): The rules governing when proposals can be made, and the identity of the proposer at a proposal node, are independent of past contract proposals and of past and current ruling contracts.

This implies that the parties cannot write contracts which alter the underlying renegotiation game. It still, however, allows for a good deal of flexibility in the modeling of the renegotiation process. For example, the renegotiation protocol (who makes the next proposal and when) may vary with θ, with the production decision made, if any, with past verifiable messages, with past proposal dates and the identity of the proposer at those dates (although not the proposals made), and indeed with the investments made at the outset of the game.

Before stating Assumption 2 we need some further notation and terminology.

For any history (i.e., node) h of $G(\alpha_0)$, the contract in force is $\alpha(h)$, $a(h)$ is the good that has been produced (if no good has been produced yet then $a(h) = \emptyset$), and the investments and state (if realized) are $i_s(h), i_b(h)$, and $\theta(h)$.

Efficient Outcomes Given any history h of $G(\alpha_0)$ in which θ has been realized, a continuation path is *ex post efficient* if the following conditions hold: (i) if $a(h) = \emptyset$ then the efficient good $a(i_s(h), i_b(h), \theta(h))$ is produced and accepted at the earliest opportunity; (ii) if $a(h) = a$ then B accepts delivery of a at the earliest opportunity; and (iii) the deposits, if any, are returned at the earliest opportunity to the players (and there are no payments to third parties). A continuation equilibrium[11] is efficient if it gives probability 1 to *ex post* efficient continuation paths. Note that an efficient continuation path need not be efficient *ex ante* because the investments made in h may have been suboptimal, or the wrong good may have been produced. At a node at which θ has been realized, an *ex post efficient forcing contract*[12] is a contract in C such that efficient production takes place at the earliest opportunity in any

[11] Throughout, the term *equilibrium* will refer to subgame-perfect equilibrium (SPE).

[12] See Watson (2007).

continuation equilibrium (i.e., the named good is efficient and P is large enough to force production). A pure strategy profile for the whole game (starting at $t = 0$) is efficient if the outcome of the profile has investments (i_s^*, i_b^*), has no payments to third parties and gives probability 1 to efficient continuation paths after θ is realized.

Available Surplus The *available surplus* at h, denoted by $\mu(h)$, is the sum of the continuation payoffs which the two players get in an *ex post* efficient continuation path (gross of sunk payments, i.e., investment costs, sunk production costs and payments of deposits). Thus, if, for example, no good has been produced, there are outstanding deposits of value K (jointly under the control of the two parties[13]), and production is feasible without delay, then the available surplus is $\sigma(i_s(h), i_b(h), \theta(h)) + K$.

Our second key assumption about $G(\alpha_0)$ is as follows.

Assumption 2 (Efficient Renegotiation): For some $\lambda_b \in (0,1)$ the following is true. For any $K \geq 0$ there is an equilibrium of $G(\emptyset^K)$ such that at any history h in which θ has been realized but there has not yet been any renegotiation or production and there is no outstanding proposal, (i) agreement is reached, at the first renegotiation opportunity, on an ex post efficient forcing contract; and (ii) B's expected payoff is less than or equal to $\lambda_b \mu(h)$.

The requirement of efficiency is unexceptionable: the normal presumption in a complete information framework is that the parties will negotiate to the Pareto frontier. All that we require here is that the complete information bargaining game has at least one equilibrium with this property.[14] The second clause says that at any node h in the stated class the buyer's equilibrium share of the available surplus is no more than λ_b. The assumption therefore rules out a bargaining game in which B gets all the surplus (say, because B makes all the offers); we deal with that case separately in Section 5 below (see Proposition 3).

Equivalent Histories A history h_1 of $G(\alpha_0)$ and a history h_2 of $G(\alpha_0')$ are defined

[13] For example, the parties have not previously signed the rights away to a third party.

[14] *Full* efficiency might not be achieved. For example, it may be that production is still feasible in the current period, but the next opportunity to renegotiate will only arrive next period.

to be *equivalent* if they have the same investments, θ, outcomes at randomization nodes, real actions, verifiable messages and available surplus. Two equivalent histories therefore differ only insofar as the bargaining moves (proposals and/or acceptances) and ruling contracts differ.

Two contracts $\alpha(h_1)$ and $\alpha(h_2)$ are defined to be *equivalent* if (i) h_1 and h_2 are equivalent histories and (ii) for any pair of equivalent histories $\tilde{h_1}$ and $\tilde{h_2}$ such that, for $i \in \{1, 2\}$, $\tilde{h_i}$ is subsequent to h_i and there has been no renegotiation since h_i, any payments mandated by $\alpha(\tilde{h_1})$ and $\alpha(\tilde{h_2})$ are equal. These two contracts may not be the same: for example, they may have provisions which are stated relative to the date of agreement and they may have been agreed at different times. Nevertheless, for the given histories, they induce the same map from future verifiable actions to payments.

A history h_1 of $G(\alpha_0)$ and a history h_2 of $G(\alpha_0')$ are defined to be *fully equivalent* if they are equivalent and, in addition, $\alpha(h_1)$ is equivalent to $\alpha(h_2)$. By Assumption 1, and the fact that a contract cannot condition payments on previous ruling contracts, the subgame following h is then identical to the subgame following h', up to interchange of $\alpha(h_1)$ and $\alpha(h_2)$: i.e., 'no renegotiation' means a proposal of $\alpha(h_1)$ in the first game, and $\alpha(h_2)$ in the second.

$G^c(\alpha_0)$ (c standing for commitment) refers to a game which is the same as $G(\alpha_0)$ except that renegotiation is not allowed, so the ruling contract is always α_0. Equivalence and full equivalence between a history h of $G(\alpha_0)$ and a history $\tilde{h}$ of $G^c(\alpha_0')$ are defined as above. In particular, $\alpha(h)$ is equivalent to $\alpha_0'(\tilde{h})$. In this case, of course, the following subgames are not the same because in only one of them is renegotiation permitted.

Preliminary Results

In constructing equilibria, one needs to be sure that equilibrium exists in every subgame. Since a specific extensive form has not been explicitly defined (including, in particular, the set of allowable contracts) we cannot appeal to standard existence theorems. However, as the following Lemma will imply, existence is guaranteed by Assumptions 1 and 2.

LEMMA 1: *For any* $\alpha_0 \in C_1$ *and any history* h' *of* $G(\alpha_0)$ *such that a renegotiation proposal has just been accepted, there exists a continuation equilibrium in the ensuing subgame.*

PROOF: *see Appendix.*

The next result says that if in some equilibrium the current contract is always *ex post* efficient then there is an equilibrium in which this contract is not renegotiated.

PROPOSITION 1: *Take any history* $h \in G^c(\alpha_0)$ *at which* θ *has been realized and for which there is a pure strategy continuation equilibrium* $s(h)$*, starting from* h*, such that (i) the outcome path of* $s(h)$ *is ex post efficient, and (ii) the continuation outcome path of* $s(h)$ *at every history subsequent to* h *is ex post efficient. Let* h' *be a history of* $G(\alpha'_0)$ *in which there is no outstanding proposal and which is fully equivalent to* h*. Then there is a continuation equilibrium* $s'(h')$ *of* $G(\alpha'_0)$ *starting from* h' *in which there is no renegotiation and the continuation payoffs for each player in* $s(h)$ *are equal to the corresponding payoffs in* $s'(h')$*.*

PROOF: *see Appendix.*

The idea of the proof is straightforward. Take the game in which renegotiation is not allowed and consider a history h in which θ has been realized. Suppose that there is a continuation equilibrium s which is (*ex post*) efficient and which also prescribes efficient continuation play at every subsequent possible history reachable from h. Then, in the game in which renegotiation is allowed, at a history which is equivalent to h, it is open to the players to play exactly as in s, as far as production, acceptance of delivery and verifiable messages are concerned, and to refuse to renegotiate unless offered something which gives a higher payoff than s does. Since s is always efficient, it would then be impossible to offer a renegotiation which is both profitable to the offeror and acceptable to the responder. Hence there is an equilibrium which is equivalent to s.[15]

[15]This does not contradict the familiar result that prohibiting renegotiation makes it easier to

Proposition 1 shows that the continuation equilibrium of the game $g(\tilde{\alpha}_0)$ in section 3 in which the option is always exercised is an example of a general phenomenon. At any production node in the relevant subgame, it is efficient to exercise the option, and, if renegotiation were not permitted, there would be an equilibrium in which this happens at every production node. Therefore there is an equivalent equilibrium in the game in which renegotiation is allowed. It is likely that Proposition 1 applies in a substantially wider class of bargaining processes than considered here: indeed it should apply in any extensive form which respects the voluntary character of renegotiation.

Though simple, Proposition 1 has strong implications which have been overlooked in the literature. It implies that bargaining games in the class considered here will have multiple equilibria when there is already a contract a place, if it is an option contract. This is incompatible with the usual assumption that there is a unique outcome of contract renegotiation.

5 Efficient Equilibrium

We now show that there will be an efficient subgame-perfect equilibrium of the game $G(\alpha_0)$ as long as the initial contract α_0 is appropriately designed. Assume that B has liquid wealth of at least K available to leave as a deposit, where

$$K \geq \frac{\psi_s(i_s^*)}{\delta(1-\lambda_b)}. \tag{4}$$

In many standard bargaining games (e.g. variants of the alternating offers game) λ_b can be taken to be approximately $\frac{1}{2}$ if δ is close to 1. In that case, the required lower bound is at most approximately $2\psi_s(i_s^*)$.

The contract used to generate the first-best equilibrium will be denoted α_b^K and is defined as follows.

α_b^K: B deposits K at the start of the game. After learning θ, B nominates a good

implement the first-best. Prohibiting renegotiation has bite only if the efficient equilibrium has some continuation equilibrium (necessarily off the equilibrium path) which is inefficient; at such a contingency the parties would renegotiate if they could.

a_b and a price p. If at any subsequent date t, S produces good a_b then B must pay p to S. In that event, B's deposit is returned to him. If S produces $a \neq a_b$, B gets the deposit back and no other payments are due. If S never produces anything, B does not get his deposit back and no payments are due.

α_b^K is the same as $\tilde{\alpha}_0$ of section 3 except that the party with authority (B) has to post a bond (in effect, give a hostage); this will have the effect of giving him the incentive to behave in the appropriate way. Note that this contract does not include any descriptions of goods, so that it can be implemented even if the goods are not describable until θ is realized. It also has the important advantage of simplicity.

First we prove two Lemmas which exhibit two kinds of continuation equilibria obtainable when the contract is α_b^K. In the first, derived from Proposition 1, the option, which is efficient in the subgame under consideration, is always exercised. The second, derived from Assumption 2, corresponds to the Rubinstein equilibrium in the example set out in section 3.

LEMMA 2: *Consider any history h of $G(\alpha_b^K)$ such that (i) there has been no renegotiation of α_b^K, (ii) B has nominated the efficient good $a = a(i_s(h), i_b(h), \theta(h))$ and a price $p \geq c(a)$, (iii) no good has been produced and (iv) there is no outstanding renegotiation offer. There is an ex post efficient continuation equilibrium beginning at h in which the payoffs are $\delta^\tau(p - c(a))$ for S and $\delta^\tau(K + v(a) - p)$ for B, where τ is the time until the next production opportunity.*

PROOF: Take a history h' of $G^c(\alpha_b^K)$ which is fully equivalent to h. Since renegotiation is not possible there is a pure strategy continuation equilibrium in which S produces a at the next opportunity, because delay can only reduce her payoff. Similarly, B accepts delivery at the first opportunity since $v(a) \geq 0$ so, since a is the efficient good, this equilibrium is *ex post* efficient after every history. So, by Proposition 1, there is an efficient continuation equilibrium of $G(\alpha_b^K)$ beginning at h in which there is no renegotiation and S produces a at the first opportunity, giving the payoffs as described. Q.E.D.

Lemma 3: *Consider any history h of $G(\alpha_b^K)$ such that (i) α_b^K has not been renegotiated, (ii) the node at which B is supposed to nominate a good has passed and B has either specified a good a and a price p such that*

$$p \leq c(a) + \delta(1 - \lambda_b)(K + \sigma(i_s(h), i_b(h), \theta(h))),$$

or else has not specified any good, (iii) S has not yet produced a good, and (iv) there is no outstanding offer. There exists an ex post efficient continuation equilibrium $\tilde{s}(h)$ in which B's payoff $\pi_b(\tilde{s}(h))$ satisfies

$$\pi_b(\tilde{s}(h)) \leq \lambda_b[K + \sigma(i_s(h), i_b(h), \theta(h))]$$

and S's payoff $\pi_s(\tilde{s}(h))$ satisfies

$$\pi_s(\tilde{s}(h)) \geq \delta(1 - \lambda_b)[K + \sigma(i_s(h), i_b(h), \theta(h))]\,.$$

Proof: *see Appendix.*

The idea behind Lemma 3 is simple. Take a history h' of $G(\emptyset^K)$ which is equivalent to h but $\emptyset^K$ is still the ruling contract. By Assumption 2 there is an efficient continuation equilibrium $s'(h')$. But the game following h is the same as the game following h' except that in the former S has the option of producing B's good and getting p. Since p is low, there is an equilibrium in which S always declines to exercise the option and they play according to $s'(h')$. In this equilibrium the available surplus to be bargained over (of which B's share is no more than λ_b) includes the deposit K since, if S adopts this strategy, B will not get the deposit back until agreement is reached.

The continuation equilibria in Lemma 2 and Lemma 3 can be used to construct an *ex ante* efficient equilibrium, as the next Proposition shows.

Proposition 2: *If the initial contract is α_b^K and the renegotiation process satisfies Assumptions* 1 *and* 2 *then there is an equilibrium in which both parties invest and trade* *efficiently. That is, $G(\alpha_b^K)$ has an efficient subgame-perfect equilibrium.*

PROOF: *see Appendix*

As in the efficient equilibrium of $g(\tilde{\alpha}_0)$ in section 3, S's investment incentives derive from the fact that she expects the continuation equilibrium to be as set out in Lemma 2, with a higher option price if she invests i_s^* than if she invests less. In this case, the option price on the equilibrium path is $c(a)+\psi_s(i_s^*)$ and, if she underinvests, the price is $c(a)$. Similarly, B's incentive to name the correct option price comes from the fact that if he names a lower one play reverts to the equilibrium set out in Lemma 3, giving him at most $\lambda_b(K+\sigma)$, whereas if he does not deviate he gets $K+\sigma-\psi_s(i_s^*)$, which is greater by (4). The equilibrium differs from that of section 3 in that B's investment incentive comes from the fact that he is residual claimant: in equilibrium he gets all the *ex ante* expected surplus.

Our assumptions about the renegotiation process have ruled out games with continuous time, simultaneous moves and open offers (that is, a responder cannot wait before deciding whether to accept an offer of a new contract, and can observably and verifiably reject an offer, thereby killing it), as well as games in which the parties have to decide which contract to show to the court. It seems probable, however, that the results above are robust to some, at least, of these extensions. A renegotiation game in which the equilibrium outcome is unique (when the contract is α_b^K) would presumably have to have the feature that at some date t the seller is not able unilaterally to exercise the option in the current period, but she can offer B an agreement generating a joint surplus at t. For example, in the Rubinstein model uniqueness of equilibrium derives from the property that if a responder accepts a proposal then surplus is created at t, but if this proposal is rejected then no surplus can be created until some time after t. But if there is no technological obstacle to producing at t (and how otherwise could S offer a payoff pair at t?) there can be no technological reason why the option could not be exercised at t. Once one allows exercise of the option at any date at which surplus could be realized under a renegotiated contract, one gets equilibria[16] of the type described in Proposition 2.

6 Liquidity Constrained Agents

The contract α_b^K used above to obtain the first-best equilibrium (Proposition 2) required the buyer to make a deposit. Suppose now that it is not possible to use deposits, perhaps because of liquidity constraints. Under what circumstances is it possible to achieve the first-best? It turns out that it will still be possible if the surplus is large enough relative to the size of the investments or if the bargaining powers are relatively unequal (in particular if one party has all the bargaining power full efficiency is achievable).

Suppose that a version of Assumption 2 holds for S, with parameter $\lambda_s \in (0,1)$. That is, the game without a contract has an efficient continuation equilibrium and in this equilibrium S's share of the surplus is always[17] no more than λ_s. Define a contract α_s^0, symmetric to α_b^0, as follows.

α_s^0: After learning θ, S may specify a price p and good a. If she does not do so, no payments are due. If at the first production node, S produces the nominated good a and B subsequently accepts delivery then B must pay p to S, while if B never accepts delivery then no payments are due. If S does not produce the nominated good at the first production node then she pays a large penalty to B.

S is not required to make any deposit under this contract. Like α_b^K, it gives a non-expiring option to one party (in this case the option to accept delivery at a specified price) and it gives the other party the right to set the terms of the option (i.e. the price). We then have the following result.

[16] Another extensive form bargaining game in which there are multiple equilibria is one in which the players have the option of leaving the relationship at any time. Such an equilibrium would, however, not be renegotiation-proof because after a party has opted out they could create surplus by opting back in.

[17] At least if there is no outstanding offer.

PROPOSITION 3: *(i) Suppose that there exists* $\varepsilon > 0$ *such that, for all* $i_b \leq i_b^*$,

$$E_\theta \sigma(i_s^*, i_b^*, \theta) - \psi_s(i_s^*) - \psi_b(i_b^*) > \lambda_b E_\theta \sigma(i_s^*, i_b, \theta) - \psi_b(i_b) + \varepsilon. \tag{5}$$

Then, if δ *is close enough to 1,* $G(\alpha_b^0)$ *has an efficient equilibrium.*

(ii) Suppose that there exists $\varepsilon > 0$ *such that, for all* $i_s \leq i_s^*$,

$$E_\theta \sigma(i_s^*, i_b^*, \theta) - \psi_s(i_s^*) - \psi_b(i_b^*) > \lambda_s E_\theta \sigma(i_s, i_b^*, \theta) - \psi_s(i_s) + \varepsilon. \tag{6}$$

Then, if δ *is close enough to 1,* $G(\alpha_s^0)$ *has an efficient equilibrium.*

PROOF: *see Appendix.*

Note that (5) holds if

$$E_\theta \sigma(i_s^*, i_b^*, \theta) > \frac{\psi_s(i_s^*) + \psi_b(i_b^*)}{1 - \lambda_b}$$

since $E_\theta \sigma(i_s^*, i_b^*, \theta) \geq E_\theta \sigma(i_s^*, i_b, \theta)$ for all $i_b \leq i_b^*$. In the alternating-offers game, λ_b and λ_s are both approximately equal to 0.5 if δ is close to 1. In that case Proposition 3 implies that the first-best is achievable if the first-best gross expected surplus is more than twice the cost of the optimal investments. This condition becomes less restrictive as the bargaining powers become more asymmetric. For example, if B has all the bargaining power ($\lambda_s = 0$) then, by (1) and Proposition 3(ii), $G(\alpha_s^0)$ has an efficient equilibrium if δ is high.[18]

In the equilibrium of Proposition 3(i) both parties invest efficiently; B nominates the efficient good and gives S a profit margin of $\psi_s(i_s^*)\sigma(i_s^*, i_b, \theta)(E_\theta \sigma(i_s^*, i_b^*, \theta))^{-1}$, which, in expectation, compensates S for her investment. If S under-invests then S is punished by getting zero gross profit. As in Proposition 2, B is punished for making the wrong nomination by a renegotiation of the shares of the surplus, but in this case he is punished in the same way for under-investing. (5) ensures that this

[18] In fact, as the proof shows, δ does not have to be high in this case.

punishment is strong enough. The equilibrium of Proposition 3(ii) is symmetric to this. In equilibrium S nominates the efficient good and a price which gives B enough surplus to compensate him, in expectation, for his investment. If S deviates then a lower price is negotiated.

Proposition 3 enables us to draw some conclusions about which party should be given authority. In the model of the previous section, with either party able to make a sufficiently high deposit, authority could be given to either party since a result analogous to Proposition 2 can be established with initial contract α_s^K, which is the same as α_s^0 except that S makes a deposit of K which is returned only when B accepts S's good. However, when neither party is able to make a deposit, (5) and (6) show that the buyer should have authority if λ_b is low or if $\psi_s(i_s^*)$ is low and, conversely, S should have authority when λ_s or $\psi_b(i_b^*)$ is low. If the seller has a sufficiently low cost of optimal investment then the buyer will have a lower incentive to under-invest and take a share λ_b instead of compensating the seller for her investment and taking the residual surplus. If the buyer has a low bargaining power then the threat of renegotiation if he deviates is stronger, so giving him authority is less costly.

7 Discussion and Relation to the Literature

Robust Renegotiation-Proofness

As argued above, the efficient equilibrium of this paper is renegotiation-proof in a robust sense: the same contract, with broadly the same type of equilibrium behavior, will work for an essentially arbitrary choice of non-cooperative bargaining game. There are other ways in which an efficient equilibrium could be constructed, but they would not be robustly renegotiation-proof in the same way.

For example, the argument that the Rubinstein bargaining game has multiple equilibria if one party has a non-expiring option is reminiscent of Shaked (1994). However, the efficient option equilibrium used in this paper is different.[19] In Shaked

[19]The equilibrium of Ponsati and Sákovics (1998) should also be mentioned. However, that equilibrium relies on the fact that both parties have an outside option, and, in the case where the outside options are efficient, equilibrium is unique in their model.

(1994) the multiplicity arises because one party can credibly 'burn money' by taking an inefficient outside option. The threat to do so is credible because if this player does not opt out then play reverts to a different equilibrium, which punishes him. Avery and Demsky (1994) examine a variety of papers, including Shaked (1994), which exhibit multiplicity and delay in infinite-horizon complete information bargaining games. They show that these papers all rely on a version of this money-burning argument. In the equilibria of the current paper, by contrast, there is no money-burning threat of this kind because the option is efficient.[20]

It is known that in contracting situations efficiency can generally be achieved if money-burning is possible[21] (see, e.g., Maskin and Sjöstrom (2003)) but it is usually argued in the incomplete contracts literature that mechanisms which rely on money-burning are suspect because they are not renegotiation-proof. If the argument of this paper depended on the Shaked equilibrium then it would be vulnerable to a similar objection. One might expect that the parties would renegotiate before the money-burning action is taken, and indeed Shaked shows that his multiplicity result does not hold if a player can take his outside option only after he rejects a proposal (i.e. if the other player can make an offer just before he exits). To put the same point another way, the argument would be contingent on a particular specification of the renegotiation process.

Rubinstein and Wolinsky (1992), in the context of a buyer-seller model with a single tradable good, use the following definition: a choice rule (mapping pairs (v, c) to prices) is renegotiation-proof-implementable (over time) if there exists an implementing dated game form such that, at every node, it is not the case that the equilibrium outcome is inferior, for both parties, to some outcome one period in the future.[22] They show that one can construct an infinite-horizon mechanism which implements, in this

[20]Note that Shaked (1994, p.428) and Avery and Demsky (1994, p.155) both claim, wrongly as shown above, that there is a unique equilibrium if the outside option is efficient.

[21]Contractual payments to a third party have an effect similar to that of money-burning. For example, Maskin and Tirole (1999b) construct an efficient mechanism for a classic property-rights model which uses options combined with third-party payments. Some authors (e.g., Hart and Moore (1999)) have argued that third-party payments are not renegotiation-proof or collusion-proof. For a contrary view, see Baliga and Sjöstrom (2005).

[22]The idea is that it takes one period for the parties to renegotiate.

sense, a relatively large set of rules (though not large enough to obtain efficiency in hold-up models of the kind considered in the current paper). This approach too lacks robustness in the above sense. In the mechanism there are announcements of values and costs and possible challenges to the announcements; after each possible challenge a specific infinite horizon bargaining game is played, the rules of which are laid down by the mechanism. It is effectively assumed that certain kinds of renegotiation moves are impossible, or prevented by the contract. For example, the mechanism stipulates at certain nodes that the buyer makes an offer which is below some value v_s. If the buyer can secretly make an offer above v_s at the same time, or fractionally later in the same period, then the argument will not work. The court seems to have, in this analysis, too much ability to constrain renegotiation.[23]

Multiple Equilibria and Efficient Investment

A number of other authors have noted that multiple continuation equilibria can be used to create good investment incentives. Ellingsen and Robles (2002), Tröger (2002) and Carmichael and MacLeod (2003) all show,[24] in the context of a simple hold-up model, that if the investment stage is followed by a version of the one-shot Nash demand game then efficient investment is an equilibrium outcome, without any contract. Ellingsen-Robles and Tröger (who both assume one-sided investment) also show that this efficient equilibrium is selected by stochastic stability criteria. Ellingsen-Robles conclude that the existence, or non-existence, of a hold-up problem depends on, among other things, the nature of the game, since only minimal investment can occur in subgame-perfect equilibrium if the bargaining process takes the form of an ultimatum game. However, in each of these analyses the bargaining game is special and not renegotiation-proof in the sense discussed above. If the two par-

[23]At the very least, it is assumed that the parties know in advance what the renegotiation game will be, since the contractually-specified intervals between offers must not be longer than those of the renegotiation process.

[24]Muthoo (1998) is superficially similar, because he finds that efficient investment can result, for some parameters, in a model in which, after investment, one party has an outside option whose value depends on the size of his investment, and bargaining proceeds according to the Shaked game discussed above. However, the argument is somewhat different. Efficiency obtains because only one party invests and equilibrium selection is such that this party always gets all the surplus.

ties fail to reach agreement in the Nash demand game they will presumably re-open negotiations rather than walk away.

A similar point can be made about the following scheme (based on one suggested by a referee). After the investment stage, the two parties play a simultaneous-move coordination game (Game A).

	A_s	B_s	C_s
A_b	a, a	$-K, -K$	$-K, -K$
B_b	$-K, -K$	b, b	$-K, -K$
C_b	$-K, -K$	$-K, -K$	c, c

Game A

The column player is S and the row player is B. In addition to the gross payoffs above there may be monetary transfers: if (A_b, A_s) is played then S has to pay f to B and if (C_b, C_s) is played then B has to pay f to S, where $a - f > -K, c - f > -K$, and $b > -K$. There are three Nash equilibria of this game: $(A_b, A_s), (B_b, B_s)$ and (C_b, C_s). If the parameters of the game, including f, are chosen appropriately then two of the equilibria can be used to punish the players for under-investing and so to enforce the first-best. If neither deviates at the investment stage, they play (B_b, B_s), if B deviates, they play (C_b, C_s), and if S deviates they play (A_b, A_s).

Suppose first that the actions in the game are not directly payoff-relevant: for example, they may be messages which the parties simultaneously send to the court, and the payoffs given above derive from actions which are specified by the court, contingent on the messages sent. In that case the parties will renegotiate after the message game is played, which implies that the game (including the renegotiation stage) is constant-sum. Therefore the game has a unique equilibrium and the scheme will not work. The actions $\{A_i, B_i, C_i\}$ must, therefore, be real actions. For example, it may be that, in addition to the good that it is efficient for S to produce, there is a second good that S can verifiably produce, this good comes in three varieties, for each of these there is a complementary verifiable investment that B must make (beyond the initial investment in the relationship), and the cost of mis-coordination between the good and the investment is high. The game would then not be constant-sum[25] and the scheme sketched above would give the right incentives. There are grounds, however, for doubting the plausibility and general applicability of such a scheme. It may be that there are no available actions with the required characteristics. Simultaneity of the actions would be essential: a party which intends to under-invest would have an incentive to commit early to its action in the coordination game. Above all, the advantage of the option contract described in the previous section is that has a simplicity, a realism and an independence of the fine details of the situation which the above scheme lacks.

A somewhat different, but related, argument is made by Che and Sákovics (2004). They assume that the parties are able to delay their investment indefinitely and that bargaining over the surplus goes on while, possibly, investment gradually takes place. Their result is that even if there is no contract there will be an asymptotically efficient equilibrium, as the discount factor goes to one. The difference from the current paper is that we assume, in common with most of the rest of the hold-up literature, that the investment of the two parties has to be made, simultaneously, at the outset of the game. It may be, for example, that if the investment is delayed then other firms will come in and take the market, so that substantial sums have to be committed at the outset if the project is to have a positive net present value.

Full Implementation

The game analyzed above has multiple equilibria, not all of which give the first-best outcome. Thus, the option contract implements the first-best only in a weak sense. What reason is there to think that the efficient equilibrium will be played?

It is fairly straightforward, using techniques from the implementation literature,[26] to augment the contract so as to ensure that every equilibrium gives the efficient

[25] We should assume that the players are able to renegotiate before the coordination game is played. They would not, however, renegotiate away the consequences of mis-coordination, because they would not expect it to happen.

[26] See Baliga and Sjöstrom (2005).

outcome. Consider the following mechanism. Before the game is played, each player, simultaneously and verifiably, announces a non-negative integer. We should assume that after that, but before the investment stage, there is an opportunity to renegotiate. Let $\underline{u}_s$ (resp. $\underline{u}_b$) be the worst equilibrium payoff for S (resp. B) in the game beginning with this renegotiation stage, assuming that there is no initial contract. Denote by (u_b^*, u_s^*) the equilibrium payoffs in an efficient equilibrium of $g(\tilde{\alpha}_0)$, as described in Section 3. If both integers are zero, then the contract is $\tilde{\alpha}_0$ and $g(\tilde{\alpha}_0)$ is played. If at least one integer is strictly positive then, if B's integer is weakly larger than S's, S pays $u_b^* - \underline{u}_b$ to B, while, if S's integer is strictly larger than B's, B pays $u_s^* - \underline{u}_s$ to S; the game is then played without a contract. There is an equilibrium of this mechanism in which the payoffs are (u_b^*, u_s^*). Each announces zero and they then play the efficient equilibrium of $g(\tilde{\alpha}_0)$; if one player deviates by announcing a strictly positive number then the worst continuation for that player is played. Moreover, in any equilibrium, S (resp. B) can get u_s^* (resp. u_b^*) by announcing a sufficiently high integer, so the unique equilibrium payoff pair is (u_b^*, u_s^*).

Formally, therefore, the efficient outcome can be fully implemented. However, the argument of this paper is that the efficient outcome can be implemented by a contract and post-contract behavior which are simple, realistic and independent of the fine details of the problem. The above scheme fails these tests.

There is, however, a natural refinement which selects a unique, and efficient, equilibrium. Suppose that before the game is played the parties negotiate a contract and also engage in cheap-talk negotiation about what strategies they will play in the post-contract game. There will of course exist equilibria in which the cheap-talk is ignored, but a natural approach is to consider only equilibria in which it is not: that is, if the parties agree on a contract and strategies which jointly form a subgame-perfect equilibrium in the continuation, then their continuation strategies indeed specify that they then play in the agreed way. A similar refinement is used, for example, in Farrell (1987), and the same idea is implicit in the definition of Weakly-Renegotiation-Proof Equilibrium (Farrell and Maskin (1989)).

To be specific, consider the following game, g'. First, S proposes a contract α_0

and also an element ϕ_s of her strategy set in the game $g(\alpha_0)$. B then either accepts or rejects and, if he accepts, announces an element ϕ_b of his strategy set in the game $g(\alpha_0)$. If B has accepted, they then play $g(\alpha_0)$, and if B has rejected, they play $g(\emptyset)$, i.e. without a contract. We say that a strategy profile ϕ' of g' is *negotiation-consistent* if (a) it is subgame-perfect, and (b) for any triple $(\alpha_0, \phi_s, \phi_b)$, if (ϕ_s, ϕ_b) is a SPE of $g(\alpha_0)$, then, whenever B has accepted (α_0, ϕ_s) and announced ϕ_b, the continuation strategy profile specified by ϕ' (for the game $g(\alpha_0)$) is (ϕ_s, ϕ_b).

For simplicity, assume that $g(\emptyset)$ has a unique subgame-perfect equilibrium payoff pair, (u_s, u_b).

Claim: Any negotiation-consistent equilibrium of g' is efficient, and the equilibrium payoffs are $u^* - u_b$ for S and u_b for B.

Proof: For any sufficiently small $\epsilon > 0$, there is an efficient equilibrium $(\phi_b(\epsilon), \phi_s(\epsilon))$ of $g(\tilde{\alpha}_0)$ corresponding to $\beta_1 = \frac{\psi_s(i_s^*)+\epsilon}{E_\theta \sigma(i_s^*, i_b^*, \theta)}$. This gives payoff $u^* - \epsilon$ to B and ϵ to S. Suppose S were to propose the contract $\tilde{\alpha}_0$ combined with an unconditional payment of $u^* - u_b - 2\epsilon$ from B to S, and announce strategy $\phi_s(\epsilon)$. If B agrees to this and announces $\phi_b(\epsilon)$ then his net payoff will be $u_b + \epsilon$, whereas if he rejects he will get u_b. Therefore he must accept. This implies that S can get a payoff of at least $u^* - u_b - 2\epsilon$ (since S gets at least zero in the continuation if B agrees and announces a different strategy). Since ϵ is arbitrary, and B must get at least u_b in any equilibrium, this establishes the claim.

How Plausible is the Equilibrium?

How realistic is it that the parties would write contracts such as $\tilde{\alpha}_0$ and subsequently behave according to the equilibrium described in section 3? Note first that each of the main features of the contract - delegated authority, options to supply, and (in the contracts below) bonds - is common in contracts which are actually observed in practice. As noted in the Introduction, the contract is similar in some ways to a standard employment contract - the employer has the right to specify the task and the pay, but the employee is also free to withhold supply.

As for the postulated equilibrium, one might ask whether it is plausible that the seller would, after deviating by under-investing, accept a continuation equilibrium which is comparatively bad for him (worse than if he had not under-invested). The alternative view, which is related to the idea of subgame-consistency (Harsanyi and Selten (1988)), is that, since investments are sunk, the equilibrium shares in the renegotiation game should be independent of them. This is the standard view in the incomplete contracts literature; according to this viewpoint, bargaining in the context of an incomplete contract is the same as bargaining in isolation. This view is not, of course, generally accepted for other contexts in which there are long-run relationships: subgame-consistency would imply, for example, that the only possible outcome in the infinitely-repeated Prisoners' Dilemma is repetition of the Nash equilibrium of the stage game.

There is evidence (see, for example, Macauley (1963)) that economic agents do indeed enter into informal agreements, explicit or implicit, which are characterized by reciprocity - that is, the agreement stipulates that if one party violates the agreement, the other party will react in a way which is to the violator's disadvantage. Of course, the stipulated behavior needs to be self-enforcing, i.e., subgame-perfect. Many authors have argued that it is the possibility of such flexible enforcement that accounts for the ubiquity of incomplete contracts. For example, Bernheim and Whinston (1998) point out that it may be desirable to leave some aspects of performance unspecified in the contract even if they are verifiable - because, for example, discretion over salary gives an employer flexibility in encouraging non-verifiable types of effort. According to Macauley (1963), 'business executives often seem to prefer contracts that provide parties with considerable discretion' and they 'often prefer to rely on "a man's word" ..or "common honesty and decency" - even when the transaction exposes them to serious risks'. The explanation may be that there is a self-enforcing social norm in which each party does indeed keep his word.[27] The literature on relational contracting (Macleod and Malcomson (1989), Levin (2003)) is also based on the same premise - that contracting parties frequently use multiple continuation equilibria to

[27] See also the Kreps (1990) theory of corporate culture.

provide incentives.

The experimental evidence too is incompatible with the subgame-consistency thesis. In the hold-up experiments of Hackett (1994), negotiated shares do depend on sunk investments, with low investment leading to a lower share. See also Gantner et al (1998), Ellingsen and Johannesson (2000), Kahneman, Knetsch and Thaler (1986) and Borges and Knetsch (1997).

8 Concluding Remarks

We have analyzed a general buyer-seller hold-up model in which there is symmetric information and ex post contractible production and trade. The main result is that if an explicitly non-cooperative approach is taken to the renegotiation process (allowing the seller to delay production) then there will generally be an efficient equilibrium even in those settings (direct externalities, complex environments) in which it has been thought that contracts can achieve nothing. Furthermore, the contracts used to generate the efficient equilibrium are simple and robust (in the sense that they do not have to be finely adapted to the cost and payoff functions), and they are similar to contracts which are frequently observed in practice.

Appendix: Proofs of Propositions

Proof of Lemma 1: Let the amount of deposit which is under the joint control of the parties just before h' be K. Consider a history $\tilde{h}$ of $G(\emptyset^K)$ which is the same as h' in every respect except for bargaining moves: in $\tilde{h}$ there has been no renegotiation until the final two nodes, when a contract $\alpha(\tilde{h})$ equivalent to $\alpha(h')$ has been proposed and accepted. Thus, $\tilde{h}$ and h' are fully equivalent: they have the same investments, θ, verifiable history, and available surplus (including deposit), together with equivalent ruling contracts. Furthermore, by Assumption 1, the renegotiation game after h' is the same as the renegotiation game after $\tilde{h}$. Therefore the subgame beginning at h' is the same as the subgame beginning at $\tilde{h}$. But $\tilde{h}$ is subsequent to a history h

as described in Assumption 2. There is, by Assumption 2, a SPE in the subgame beginning at h; the profile induced by this equilibrium in the game following $\tilde{h}$ is then a SPE of this game, hence a SPE in the game starting at h'. Q.E.D.

Proof of Proposition 1: Define a continuation strategy profile $s'(h')$ as follows. Consider three types of subgame (reachable from h'): (i) those in which there has been no change of contract since h' (so the contract is $\alpha(h')$) and there is no outstanding offer; (ii) those in which there has been no change of contract since h' but a new contract has just been proposed; and (iii) those at which a renegotiation proposal has just been accepted for the first time since h'. For any subgame of type (iii), there exists a SPE by Lemma 1. Select an arbitrary such equilibrium profile and set $s'(h')$ equal to this profile in this subgame. For histories of type (i), $s'(h')$ is defined as 'play according to $s(h)$ at non-bargaining nodes, and at proposal nodes always propose $\alpha(h')$ (i.e., no renegotiation)'. For any history of type (ii), the continuation payoff of the responder if he accepts the proposal is determined by (iii) above, and, if he rejects the proposal, by (i) above (i.e., the payoffs are given by $s(h)$). At this history, the responder's action specified by $s'(h')$ is: 'accept if and only if the continuation payoff for 'accept' is higher than that for 'reject' '. This defines $s'(h')$. If the players adopt this profile, there is no renegotiation and the outcome is the same as the outcome of $s(h)$. By construction, $s'(h')$ forms an equilibrium after any renegotiation. Clearly, neither player can benefit before that by a one-shot deviation at a response node. To show that $s'(h')$ is an equilibrium, we need to show that neither player can deviate profitably at a history of type (i). No deviation which does not involve a renegotiation proposal can be profitable because $s(h)$ is an equilibrium in the game without renegotiation. Consider a deviation in which a player makes a proposal not equal to $\alpha(h')$ (i.e., proposes a new contract). Without this deviation, this player's continuation payoff would be as in $s(h)$. If the proposer benefits by this deviation, it must be that the responder is worse off, by efficiency of $s(h)$. But, since the responder can reject the proposal and obtain the continuation payoff corresponding to $s(h)$, that is impossible. This shows that $s'(h')$ is an equilibrium. Q.E.D.

Proof of Lemma 3: Take a history h' of $G(\emptyset^K)$ which is equivalent to h, and there has been no renegotiation, so that $\alpha(h') = \emptyset^K$. Thus, the verifiable actions, available surplus and cost and value of each good are the same as in h. By Assumption 1, the game-form after h is the same as that after h'. The only difference between the subgame at h and the subgame at h' is that, as long as the current contract has not been renegotiated, in the former S gets paid p if she produces B's nominated good, whereas in the latter she gets paid nothing for producing any good, although in both cases B gets his deposit of K returned in that event.

By Assumption 2, there exists a continuation equilibrium profile $s'(h')$, starting at h', in which agreement is reached at the earliest opportunity on an efficient forcing contract and in which B's share of the available surplus is no more than λ_b at this point and also at any subsequent point at which no contract has been negotiated and there is no outstanding contract offer.

Since the game-forms at h and at h' are identical, $s'(h')$ is a valid continuation strategy profile for the game $G(\alpha_b^K)$, beginning at h. Let $\tilde{s}(h) = s'(h')$. Under this profile, S does not produce until a new contract is negotiated (otherwise her payoff in $G(\emptyset^K)$ would be at most zero, contradicting Assumption 2). We will show that it is an equilibrium profile. Clearly it is an equilibrium after any proposal is accepted (including α_b^K and $\emptyset^K$) because the subgame is then the same as the corresponding subgame in $G(\emptyset^K)$, since the ruling contract, payoff structure and verifiable history will all be the same. It is only necessary to show that neither player can deviate profitably at a history h'' when there has been no renegotiation and there is no outstanding offer. If the deviation does not involve producing B's nominated good then it cannot be profitable because, if it were, then the same deviation would be profitable in $G(\emptyset^K)$, contradicting the fact that $s'(h')$ is an equilibrium. If S produces B's nominated good a then S gets a payoff of $p - c(a)$, whereas if she plays the strategy given by $s'(h')$ she gets at least $(1-\lambda_b)\mu(h'')$ by Assumption 2. But $p - c(a) \leq \delta(1-\lambda_b)[K + \sigma(i_s(h''), i_b(h''), \theta(h''))] \leq (1-\lambda_b)\mu(h'')$ so producing a is worse for S than not doing so. Therefore $\tilde{s}(h)$ is an equilibrium. The bounds on the equilibrium payoffs follow from Assumption 2 and the fact that at most one period can elapse before renegotiation and production (and the return of the deposit) take place. Q.E.D.

PROOF OF PROPOSITION 2: Define a strategy profile $\hat{s}$ as follows. Let t_1 refer to the node at which B is supposed to nominate a good.

(a) At date 0, B invests i_b^* and S invests i_s^*.

(b) At any renegotiation proposal node before t_1 at which no renegotiation proposal has been accepted before, the proposer proposes α_b^K, i.e., 'no renegotiation'.

(c) At t_1, given that there has been no renegotiation so far,

(i) if S has invested i_s^* and B has invested i_b, B specifies the efficient good $a(i_s^*, i_b, \theta)$ and price $p = c(a(i_s^*, i_b, \theta)) + \psi_s(i_s^*)$;

(ii) if the investments were (i_s, i_b) for $i_s \neq i_s^*$, B specifies the efficient good $a(i_s, i_b, \theta)$ and price $p = c(a(i_s, i_b, \theta))$;

(d) Consider three types of history following those in (c) above:

(i) if B has followed the rule given in (c), play the continuation strategy profile specified in Lemma 2.

(ii) if B has deviated from the rule in (c) and has named a good a and price $p \leq c(a) + \psi_s(i_s^*)$ or else has not named any good or price, play the continuation strategy profile specified in Lemma 3.

(iii) if B has deviated in any other way from the rule in (c), play an arbitrary continuation equilibrium of the subgame in which an equivalent contract has just been accepted: this exists by Lemma 1.

(e) At any history before t_1 at which a renegotiation proposal has just been accepted for the first time, play an arbitrary continuation equilibrium profile: this exists by Lemma 1.

(f) At any response node before t_1, at which no previous renegotiation proposal has been accepted, the responder accepts the proposal if and only if doing so gives the responder strictly higher continuation payoff than does rejecting, where the continuation payoffs are defined implicitly by (b)-(e) above.

This completes the description of $\hat{s}$. The outcome path of $\hat{s}$ is that both players invest the first-best amount, there is no renegotiation, B nominates the efficient good and a price which covers S's investment cost, and S then produces this good for this price, B's deposit then being returned. Clearly $\hat{s}$ is efficient.

The continuation strategy profiles described in (d)(i) are equilibria by Lemma 2. Those in (d)(ii) are equilibria by Lemma 3, since $\psi_s(i_s^*) < \delta(1-\lambda_b)(K+\sigma(h))$ by (4). Those in (d)(iii) and (e) are equilibria by construction. The responder at a node as described in (f) cannot profit by a one-shot deviation.

Consider a history h as described in (c)(i). B's payoff if he conforms to $\hat{s}$ is $[K+\sigma(i_s^*, i_b(h), \theta(h)) - \psi_s(i_s^*)$. If B deviates as in (d)(ii) his payoff is at most $\lambda_b(K+\sigma(i_s^*, i_b(h), \theta(h)))$, which is less by (4), given that $\lambda_b < 1$ and $\sigma \geq 0$. If B deviates as in (d)(iii) S gets at least as much as if B had not deviated, since she has the option of producing and getting at least as high a profit margin; therefore B cannot benefit. At a history h as described in (c)(ii), B obtains all the surplus if he conforms to $\hat{s}$ and he cannot deviate in such a way that S gets a negative continuation payoff. Therefore deviation is not profitable.

At a renegotiation node as described in (b) the continuation equilibrium is efficient if there is no renegotiation, and unaffected by any rejected proposal. Therefore, by (f), a proposal will only be accepted if it strictly reduces the payoff of the proposer and so the behavior described in (b) is optimal.

It is optimal for S to invest i_s^* because, if she does so, her net payoff, given any θ, will be $(p - c(a(i_s^*, i_b^*, \theta))) - \psi_s(i_s^*) = 0$ by (a), (b), (c)(i) and (d), whereas if she deviates, she can get at most zero, by (b), (c)(ii) and (d). Now consider B's investment decision. If B invests i_b, then his expected payoff is

$$E_\theta \sigma(i_s^*, i_b, \theta) - \psi_s(i_s^*) - \psi_b(i_b) + K$$

since he will get all the available surplus (including the return of his deposit) less the price $\psi_s(i_s^*)$ which he will pay for the efficient good. Since i_b^* maximizes the net social surplus, this is his optimal choice.

This establishes that $\hat{s}$ is an equilibrium. Q.E.D.

Proof of Proposition 3(i) Define a strategy profile s^1 as in the proof of Proposition 2 except in the following respects.

(a) After $i_b \geq i_b^*$ and $i_s = i_s^*$, the option gives S a profit margin of $\psi_s(i_s^*)\sigma(i_s^*, i_b, \theta)(E_\theta \sigma(i_s^*, i_b^*, \theta))^{-1}$.

(b) After $i_b < i_b^*$ and $i_s = i_s^*$, given no renegotiation so far:

(i) if B has nominated no good, or has nominated a good and a profit margin for S less than or equal to $\delta(1-\lambda_b)\sigma$, play the continuation strategy profile specified in Lemma 3.

(ii) if B has nominated a profit margin for S greater than $\delta(1-\lambda_b)\sigma$, play an arbitrary continuation equilibrium profile.

(iii) given (i) and (ii) above, B chooses a good and price (or no nomination) to maximize his expected continuation payoff.

This describes the profile s^1. Note that, for high δ,

$$\frac{\psi_s(i_s^*)\sigma(i_s^*, i_b, \theta)}{E_\theta \sigma(i_s^*, i_b^*, \theta)} < \delta(1-\lambda_b)\sigma(i_s^*, i_b, \theta) \tag{7}$$

by (5) (with $i_b = i_b^*$). In (a), if B conforms to s^1, S gets (valued at a production node) $\psi_s(i_s^*)\sigma(i_s^*, i_b, \theta)(E_\theta \sigma(i_s^*, i_b^*, \theta))^{-1}$. If he makes no nomination, or nominates a lower margin, S gets at least $\delta(1-\lambda_b)\sigma(i_s^*, i_b, \theta)$, which is greater by (7), so B must be worse off. He must also be worse off if he nominates a higher margin.

If $i_s \neq i_s^*$, , B gets all the surplus if he conforms. Since S must get a non-negative continuation payoff after any deviation, B cannot profitably deviate.

S's expected *ex post* expected surplus if she invests i_s^* is $\psi_s(i_s^*)$ as in the proof of Proposition 2, so that i_s^* is the optimal choice for her. It remains to show that it is optimal for B to invest i_b^*. If $i_b \geq i_b^*$ then B's expected payoff is

$$E_\theta \sigma(i_s^*, i_b, \theta) - \psi_b(i_b) - \frac{\psi_s(i_s^*) E_\theta \sigma(i_s^*, i_b, \theta)}{E_\theta \sigma(i_s^*, i_b^*, \theta)}$$

$$\leq E_\theta \sigma(i_s^*, i_b, \theta) - \psi_b(i_b) - \psi_s(i_s^*)$$

since $E_\theta\sigma$ is increasing in i_b, and, if he invests i_b^* his payoff is $E_\theta\sigma(i_s^*, i_b^*, \theta) - \psi_s(i_s^*) - \psi_b(i_b^*)$, which maximizes the RHS of the above inequality, so i_b^* is optimal for B in this range. If $i_b < i_b^*$ then S gets, *ex post*, at least $E_\theta\delta(1-\lambda_b)\sigma(i_s^*, i_b, \theta)$ so B gets, *ex ante*, at most $E_\theta(1-\delta+\lambda_b\delta)\sigma(i_s^*, i_b, \theta) - \psi_b(i_b)$. If he does not deviate, he gets $E_\theta\sigma(i_s^*, i_b^*, \theta) - \psi_s(i_s^*) - \psi_b(i_b^*)$, which is greater by (5), for high δ. Therefore it is optimal for him to invest i_b^*. Q.E.D.

Proof of Proposition 3(ii) (a) Consider a history at which there has been no renegotiation, there is no outstanding offer, S has named a price p and good a, and produced a, where $p \leq v(a)$. There is an *ex post* efficient continuation equilibrium in which there is no renegotiation and B accepts delivery of a at the first opportunity, paying price p. This follows from Proposition 1, since if renegotiation were not possible, there would be an efficient continuation equilibrium equivalent to the above.

(b) Consider a history at which there has been no renegotiation, there is no outstanding offer, θ has been realized and no good has been produced. From Assumption 2, using arguments analogous to those in the proof of Lemma 3, there is an *ex post* efficient continuation equilibrium in which S gets a share at most (approximately) λ_s of the surplus. In this equilibrium:

(i) if there is no nomination, there is an *ex post* efficient continuation in which the parties negotiate as if there is no contract (and S gets no more than λ_s of the surplus).

(ii) If S nominates (p, a) such that $v(a) - p \leq \delta(1-\lambda_s)v(a)$, then B does not exercise his option and in the renegotiation B gets at least $\delta(1-\lambda_s)v(a)$.

(iii) if S nominates (p, a) such that $v(a) - p > \delta(1-\lambda_s)v(a)$, then there is an arbitrary continuation equilibrium in which B gets at least $\delta(1-\lambda_s)v(a)$ (since he can always exercise his option).

The equilibrium of the whole game is analogous to the one in the proof of Proposition 3(i). The continuation equilibrium in (b) above is used to punish S for deviation (including under-investment). The equilibria in (a), with appropriate prices, are used to punish B for under-investing. Remaining details are omitted. Q.E.D.

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