Imperfect Commitment and the Revelation Principle: the Multi-Agent Case with Transferable Utility

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Abstract

Bester and Strausz (2000) showed that the revelation principle is invalid in a multi-agent, no-commitment setting. We show that if transfers are possible it does apply in their setting unless at least two agents have private information.

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1 Introduction

In many contracting situations, the parties are unable fully to commit to *ex post* actions. For examples of analysis of a variety of such situations, see Laffont and Tirole (1988), Crawford and Sobel (1982), Dewatripont (1989). One difficulty with analyzing such models is that the revelation principle (Dasgupta, Hammond and Maskin, 1979) does not apply. Bester and Strausz (2001), however, showed that a variant of the revelation principle holds in environments without commitment if there is one principal and only one privately informed agent.

In a companion paper (Bester and Strausz, 2000) they showed that this result does not extend to the multi-agent case. The counterexample has two agents ($A1$ and $A2$), and only $A1$ has private information. They exhibit an equilibrium for a mechanism with a message space that contains more messages than types for $A1$ and show that any equilibrium of a mechanism with a smaller message space gives a lower payoff either to the principal (P) or to $A2$.

However, in this example, one can construct an equilibrium of a mechanism with a smaller message space in which both types of $A1$ get the same

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payoff as in the original equilibrium and the sum of P's and $A2$'s payoff is higher. Therefore, if the players have quasi-linear utility for money and they can make *ex ante* transfers, it is possible for all agents to get the same payoff as in the original equilibrium and for P to get more. The case of quasi-linear utility is a leading one in contract theory and it requires negligible commitment to make *ex ante* transfers.

In this paper we show that the Bester-Strausz result extends to the multi-agent case with quasi-linear utilities and *ex ante* transfers if only one agent has private information. We also examine the case in which more than one agent has private information. We provide an example in which $A1$ and $A2$ have two types and in which a mechanism with three messages for $A1$ and two for $A2$ can result in a higher *ex ante* total payoff than any mechanism with two messages each. Therefore, a two-message mechanism must give a lower payoff to at least one player.

The conclusion is that the revelation principle fails to hold in a multi-agent setting without commitment, but only if at least two agents have private information.

2 Model

There is a principal and 2 agents. $A1$ is privately informed about his type $\theta_i \in \Theta = \{\theta_1, ..., \theta_I\}$. The probability that $A1$ is of type θ_i is γ_i and this is assumed common knowledge. $A2$ has no private information. P can choose an action $y \in Y$. Parties can make transfers which enter into payoffs in an additively separable way. Thus, given θ_i we denote P's payoff by $V_i(y) - t^1 - t^2$ and Aj's payoff by $U_i^j(y) + t^j$, where t^j is the payment from P to Aj, $j \in \{1, 2\}$.

A mechanism Γ specifies a message space $M = \{m_1, ..., m_H\}$ for $A1$ and a pair of transfers t. The mechanism induces a two-stage game of incomplete information, in which $A1$ chooses, for each type θ_i, a probability distribution $q_i(\cdot)$ over M. Let $q(\cdot) = (q_1(\cdot), ..., q_I(\cdot))$. On receipt of message m_h, P updates his beliefs about $A1$'s type to $p(m_h) = (p_1(m_h), ..., p_I(m_h))$ and then chooses a (possibly mixed) action $y(m_h)$. The solution concept is Perfect Bayesian Equilibrium.

3 A Revelation Principle

For a given mechanism $\Gamma = [M, t]$, is it possible to obtain the same (or higher) payoffs for all parties with a direct mechanism $\Gamma^d = [\Theta, t^d]$? Bester and Strausz (2001) showed that with only one agent, without transfers, the answer to this question is 'yes'. The Bester and Strausz (2000) example, however, showed that this is not necessarily true if there are at least two agents. The example has $\Theta = \{\theta_1, \theta_2\}, Y = \Re$, $V_1(y) = -y^2$, $V_2(y) = -(2-y)^2$, $U_1^1(y) = -(0.5-y)^2$, $U_2^1(y) = -(1.5-y)^2$ and $U_i^2(y) = -10(1-y)^2$, for $i = 1, 2$. They exhibit an equilibrium for an M with three messages in which P gets -0.5, each type of $A1$ gets -0.25, and $A2$ gets -5 and show that no equilibrium for an M with two messages can replicate or improve on this outcome. However, with two messages, the pooling equilibrium with equal weight on each message gives -1 to P, -0.25 to each type of $A1$ and 0 to $A2$, so if transfers were possible each type of each player could be made at least as well off.

In this section we generalize this result and argue that the Bester-Strausz revelation principle extends to the set-up of section 2. We say that $\tau = (q, p, y, t|M)$ is *incentive-feasible* if (q, p, y) is a Perfect Bayesian Equilibrium of the game induced by the mechanism $[M, t]$. Slightly abusing notation, we denote the expected payoffs in this equilibrium by $V(\tau)$, $U_i^1(\tau)$ and $U^2(\tau)$.

Definition 1 *$\tau = (q, p, y, t|M)$ is incentive-efficient if it is incentive-feasible and there exists no other incentive-feasible $\tau' = (q', p', y', t'|M)$ such that $V(\tau') > V(\tau)$, $U_i^1(\tau) = U_i^1(\tau')$ for all i and $U^2(\tau) = U^2(\tau')$.*

This is a direct extension of the notion of incentive efficiency in Bester and Strausz (2001). Consider another definition:

Definition 2 *$\tau = (q, p, y, t|M)$ is incentive-efficient if it is incentive-feasible and there exists no other incentive-feasible $\tau' = (q', p', y', t'|M)$ such that $V(\tau') + U^2(\tau') > V(\tau) + U^2(\tau)$ and $U_i^1(\tau') = U_i^1(\tau)$ for all i.*

It is easy to see that in our setting, with monetary transfers, these two definitions are equivalent. If τ fails Definition 2 (P and $A2$ can jointly be made better off) then for an appropriate choice of transfer t' it fails Definition 1 (P can be made better off and $A2$ at least as well off).

The variant of the revelation principle that applies to our setting is stated in the following proposition:[3]

Proposition *If $\tau = (q, p, y, t|M)$ is incentive-efficient according to Definition 2, then there exists a direct mechanism $\Gamma^d = (\Theta, t^d)$ and an incentive-feasible $\tau^d = (q^d, p^d, y^d, t^d|\Theta)$ such that τ and τ^d are payoff-equivalent.*

[3] A proof is available at http://www.econ.cam.ac.uk/faculty/reiche/index.html

4 Two privately informed agents

Here we consider an example in which $A1$ and $A2$ are both privately informed about their types. There are two types, a and b, and each agent, independently, has probability 0.5 of being each type. Therefore, there are four states of nature, aa, ab, ba and bb, where the first letter refers to $A1$'s type and the second to $A2$'s. P's action set is $\{y_{aa}, y_{ab}, y_{ba}, y_{bb}, y_{ca}, y_{cb}\}$ and his payoff as a function of the state of nature and his chosen action are given as follows:

	y_{aa}	y_{ab}	y_{ba}	y_{bb}	y_{ca}	y_{cb}
aa	1	0	-1	0	$\frac{3}{4}$	0
ab	0	1	0	-1	0	$\frac{3}{4}$
ba	-1	0	1	0	$\frac{3}{4}$	0
bb	0	-1	0	1	0	$\frac{3}{4}$

The matrix below gives, for each type of each agent, the agent's payoff as a function of P's action.

	y_{aa}	y_{ab}	y_{ba}	y_{bb}	y_{ca}	y_{cb}
$A1a$	1	0	0	-1	$\frac{1}{2}$	$\frac{1}{2}$
$A1b$	0	-1	1	0	$\frac{1}{2}$	$\frac{1}{2}$
$A2a$	0	1	0	1	0	-2
$A2b$	0	1	0	1	-2	0

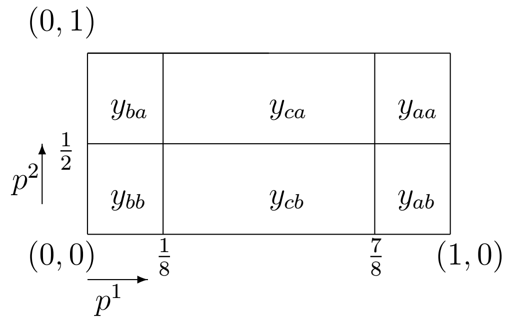
After $A1$ has sent message m_k and $A2$ has sent message m_l P's equilibrium action, if pure, is denoted by $y(m_k, m_l)$, his belief that $A1$ is type a is $p^1(m_k)$ and his belief that $A2$ is type a is $p^2(m_l)$. Mixed strategies of the agents are denoted by $q_i^1(\cdot)$ and $q_j^2(\cdot)$ $(i, j \in \{a, b\})$. For an equilibrium τ, $V(\tau), U_i^1(\tau), U_j^2(\tau)$ refer, respectively, to the expected payoffs of P, type i of $A1$ and type j of $A2$ $(i, j \in \{a, b\})$. All proofs are in the Appendix.

First, consider a mechanism with no transfers, $\Gamma = [M^1, M^2]$, in which $A1$'s message space M^1 contains three messages, m_a, m_b and m_c, and $A2$'s message space M^2 contains two messages, m_a and m_b.

Claim 1 *The mechanism Γ has an equilibrium in which $A1a$ sends messages m_a and m_c with equal probability, $A1b$ sends messages m_b and m_c with equal probability, $A2a$ sends message m_a and $A2b$ sends message m_b. The total ex ante expected equilibrium payoff is $\frac{13}{8}$.*

In the remainder of the section we show that any mechanism Γ' that uses just two messages for each agent must, in any equilibrium, have an *ex ante* total payoff below $\frac{13}{8}$. Suppose that $M^1 = M^2 = \{m_a, m_b\}$. Since the meaning of messages is arbitrary and the prior is uniform we assume without loss of generality that $p^i(m_a) \geq \frac{1}{2}$ and $p^i(m_b) \leq \frac{1}{2}$ for $i = 1, 2$.

The diagram below represents P's optimal action as a function of his posterior beliefs (p^1, p^2). For example, y_{aa} is optimal if and only if $p^1 \geq \frac{7}{8}$ and $p^2 \geq \frac{1}{2}$.



First, consider equilibria in which the action is in the set $\{y_{ca}, y_{cb}\}$ with probability 1. There are many such equilibria, with $A2$ pooling or separating and $A1$ pooling or partially separating. In each of them, $A1$ gets $\frac{1}{2}$, $A2$ gets at most 0, and P gets at most $\frac{3}{4}$, so the total ex ante expected payoff is at most $\frac{10}{8}$.

Next, we look for equilibria with strictly positive probability on the set $\{y_{aa}, y_{ab}, y_{ba}, y_{bb}\}$. In that case, $p^1(m_a) \in (\frac{1}{2}, 1]$ since, if $p^1(m_a) = \frac{1}{2}$, either $p^1(m_b) = \frac{1}{2}$ or both types of $A1$ send m_a with probability 1, which would imply $pr(\{y_{ca}, y_{cb}\}) = 1$. We distinguish the cases $p^1(m_a) \in (\frac{1}{2}, 1)$ and $p^1(m_a) = 1$.

Claim 2 *In any equilibrium of* Γ' *with* $pr(\{y_{aa}, y_{ab}, y_{ba}, y_{bb}\}) > 0$ *and* $p^1(m_a) \in (\frac{1}{2}, 1)$,

(i) *$A1a$ sends message m_a for sure and $A1b$ sends each message with strictly positive probability,*

(ii) *if $p^2(m_a) > \frac{1}{2} > p^2(m_b)$, $A2a$ sends message m_a for sure, $A2b$ sends message m_b for sure and the total payoff is less than $\frac{11}{7} < \frac{13}{8}$,*

(iii) *if $p^2(m) = \frac{1}{2}$ for any positive probability m, the total payoff is less than $\frac{12}{8}$.*

Turning to $p^1(m_a) = 1$, note that by the symmetry of the game, we have already considered all cases in which $p^1(m_b) \in (0, \frac{1}{2}]$. So it remains to consider the case $p^1(m_a) = 1$, $p^1(m_b) = 0$.

Claim 3 *In any equilibrium of Γ' with $pr(\{y_{aa}, y_{ab}, y_{ba}, y_{bb}\}) > 0$, if $A1a$ sends m_a and $A1b$ sends m_b for sure, $p^2(m) = \frac{1}{2}$ for any positive probability m, and the total payoff is less than $\frac{12}{8}$.*

In the equilibrium of the three-message mechanism Γ in Claim 1 $A2$ separates. To achieve this separation P needs to put positive probability on the set $\{y_{ca}, y_{cb}\}$ since $A2$'s types have the same preferences over the other actions. If P cannot commit he will not play these actions unless he is unsure about $A1$'s type. Hence, $A1$ must pool to some extent. However, pooling by $A1$ is costly and the optimal degree of separation requires three messages rather than two. It is therefore the interplay between the incentives of the two privately informed agents that leads to a failure of the revelation principle.

5 Appendix

Proof of Claim 1: Suppose P's beliefs and action rule are:

	(m_a, m_a)	(m_a, m_b)	(m_b, m_a)	(m_b, m_b)	(m_c, m_a)	(m_c, m_b)
(p^1, p^2)	$(1, 1)$	$(1, 0)$	$(0, 1)$	$(0, 0)$	$(\frac{1}{2}, 1)$	$(\frac{1}{2}, 0)$
y	y_{aa}	y_{ab}	y_{ba}	y_{bb}	y_{ca}	y_{cb}

Clearly the beliefs satisfy Bayes' rule given the agents' strategies and the action rule is optimal for P given the beliefs. If $A1$ sends message m_k, $k = a, b, c$, he expects P to choose y_{ka} or y_{kb}, with equal probability. Thus, $A1a$ gets expected payoff $\frac{1}{2}$ from m_a, $-\frac{1}{2}$ from m_b and $\frac{1}{2}$ from m_c, so is willing to randomize between m_a and m_c. The argument for $A1b$ is symmetric. If $A2$ sends message m_l, $l = a, b$, he expects P to choose y_{al} and y_{bl} each with probability $\frac{1}{4}$, and y_{cl} with probability $\frac{1}{2}$. Thus, $A2a$ gets 0 from m_a and $-\frac{1}{2}$ from m_b while $A2b$ gets -1 from m_a and $\frac{1}{2}$ from m_b. Therefore, the given strategies form an equilibrium. In each state, P is equally likely to get 1 or $\frac{3}{4}$, so his expected payoff is $\frac{7}{8}$. $A2a$'s is 0, $A2b$'s is $\frac{1}{2}$ and both types of $A1$ get $\frac{1}{2}$. This establishes the claim.

Proof of Claim 2:

(i) If neither type of $A1$ sends m_a then $p^1(m_b) = \frac{1}{2}$ and $pr(\{y_{ca}, y_{cb}\}) = 1$. So $pr(A1 \text{ sends } m_a) > 0$. Similarly, $pr(A1 \text{ sends } m_b) > 0$. If $q_b^1(m_a) = 0$ then $p^1(m_a) = 1$. Hence $q_b^1(m_a) > 0$. Also, we know that $q_b^1(m_b) > 0$ because

$pr(A1 \text{ sends } m_b) > 0$ and by assumption $q^1_b(m_b) \geq q^1_a(m_b)$. Let $U^i_j(m)$ be Aij's expected payoff from sending m, for $i = 1, 2$ and $j = a, b$.

Since $pr(\{y_{aa}, y_{ab}\}|\ A1 \text{ sends } m_b) = pr(\{y_{ba}, y_{bb}\}|\ A1 \text{ sends } m_a) = 0$,

$$U^1_a(m_a) \geq U^1_b(m_a) = U^1_b(m_b) \geq U^1_a(m_b)$$

with at least one inequality strict since $pr(\{y_{aa}, y_{ab}, y_{ba}, y_{bb}\}) > 0$. Therefore $U^1_a(m_a) > U^1_a(m_b)$. It follows that $A1a$ does not send m_b.

(ii) Note that in this case $y(m_b, m_a) = y_{ba}$ and $y(m_b, m_b) = y_{bb}$. First, $A2b$ strictly prefers y_{bb} to y_{ba} and $pr(A1 \text{ sends } m_b) > 0$. Second, $A2b$ (weakly) prefers any mixture of y_{cb} and y_{ab} to any mixture of y_{ca} and y_{aa}. So, $U^2_b(m_b) > U^2_b(m_a)$. This shows that $A2b$ only sends m_b. Because $A1b$ is indifferent between m_a and m_b we know $p^1(m_a) \leq \frac{7}{8}$, otherwise he would prefer m_b. If $p^1(m_a) < \frac{7}{8}$, $U^1_b(m_a) = \frac{1}{2}$ and therefore also $U^1_b(m_b) = \frac{1}{2}$. This implies that $A2a$ must send m_a for sure. If $p^1(m_a) = \frac{7}{8}$, by Bayes' rule we must have $q^1_b(m_a) = \frac{1}{7}$ and $q^1_b(m_b) = \frac{6}{7}$. Because $U^2_a(m_a) = 0$ we need $U^2_a(m_b) = 0$ for $A2a$ to be mixing between m_a and m_b. Therefore, after message pair (m_a, m_b) P must put weights $\frac{7}{12}$ and $\frac{5}{12}$ on actions y_{cb} and y_{ab} respectively. But this contradicts that $A1b$ is indifferent between m_a and m_b because $U^1_b(m_b) = \frac{1-\gamma}{2}$, where $\gamma = q^2_a(m_b)$, and $U^1_b(m_a) \leq \frac{1+\gamma}{2}((\frac{7}{12})(\frac{1}{2}) - (\frac{5}{12})) + (\frac{1-\gamma}{2})(\frac{1}{2})$. This shows that $A2a$ only sends m_a. It follows that $U^1_b(m_b) = U^1_b(m_a) = \frac{1}{2}$, which is possible only if $p^1(m_a) \leq \frac{7}{8}$, $y(m_a, m_a) = y_{ca}$ and $y(m_a, m_b) = y_{cb}$. Let $\alpha = q^1_b(m_a)$. By Bayes' rule, $\alpha \geq \frac{1}{7}$. So we have $U^2_a = 0$, $U^2_b = \frac{1}{2}(1-\alpha)$ and $V = \frac{1}{2}(\frac{3}{4}) + \frac{1}{2}[(1-\alpha)(1) + \alpha(\frac{3}{4})]$. The total expected payoff is $\frac{13}{8} - \frac{3\alpha}{8} < \frac{13}{8}$.

(iii) Since $A2$'s strategy is pooling, P's action is uncorrelated with $A2$'s type, so P cannot get a payoff of more than $\frac{1}{2}$. Let $pr(y_{bb}|\ A1 \text{ sends } m_b) = p_{bb}$, so $pr(y_{ba}|\ A1 \text{ sends } m_b) = 1 - p_{bb}$. Also, let $pr(y_{aa}|\ A1 \text{ sends } m_a) = p_{aa}$ and $pr(y_{ab}|\ A1 \text{ sends } m_a) = p_{ab}$.

Then $U^1_a = \frac{1}{2}(1 - p_{aa} - p_{ab}) + p_{aa}$ and $U^1_b = \frac{1}{2}(1 - p_{aa} - p_{ab}) - p_{ab}$ (since $A1b$ sends m_a with positive probability in equilibrium). Therefore $A1$'s *ex ante* expected payoff is $\frac{1}{2} - p_{ab}$. Conditional on $A1$ sending m_b, $U^2_a = U^2_b = p_{bb}$. Conditional on $A1$ sending m_a, $U^2_a \leq p_{ab}$ and $U^2_b \leq p_{ab}$. Letting $q^1_b(m_b) = \beta$, $A2$'s expected payoff is therefore at most $\frac{1}{2}\beta p_{bb} + [\frac{1}{2}(1-\beta) + \frac{1}{2}]p_{ab}$. Thus the sum of $A1$'s and $A2$'s *ex ante* expected payoffs is at most $\frac{1}{2} + \frac{\beta}{2}(p_{bb} - p_{ab}) \leq \frac{1}{2} + \frac{\beta}{2}$. It follows that the sum of expected payoffs is less than $\frac{12}{8}$.

Proof of Claim 3: Suppose first that $p^2(m_a) > \frac{1}{2} > p^2(m_b)$. Then $y(m_b, m_a) = y_{ba}$ and $y(m_b, m_b) = y_{bb}$. In that case both types of $A2$ strictly prefer m_b so $p^2(m_b) = \frac{1}{2}$ (contradiction). Therefore $p^2(m) = \frac{1}{2}$ for any positive probability m. As in the previous proof, P gets at most $\frac{1}{2}$. Using the same notation as in the previous proof, $A1a$ gets p_{aa}, $A1b$ gets $p_{ba} = 1 - p_{bb}$, $A2$ gets $\frac{1}{2}p_{bb} + \frac{1}{2}p_{ab} = \frac{1}{2}p_{bb} + \frac{1}{2}(1 - p_{aa})$. Therefore the total expected payoff is at most $\frac{3}{2}$.

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