

BORROWING CONSTRAINTS, THE COST OF PRECAUTIONARY SAVING AND UNEMPLOYMENT INSURANCE*

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Abstract

When it is costly for individuals to save or to borrow, unemployment insurance (UI) provides an alternative source of liquidity that smooths consumption over time and leads individuals to spend longer unemployed searching for a suitable job. We show in a tractable life-cycle model how the optimal unemployment replacement ratio and the fall in consumption on job loss depend on the cost of self-insurance and the cost of borrowing. This implies that the value of UI depends on age at job loss, consumption needs (such as the presence of children), discount rates, the return on saving, access to credit and the presence of other social insurance programmes. Optimal replacement rates vary substantially with plausible variation in these factors (from less than 20 percent to almost 60 percent).

Keywords: unemployment insurance, savings, credit constraints, life-cycle, consumption

JEL Classification: J65, H53, D91

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I Introduction

This paper addresses the question of when is unemployment insurance likely to be most valuable. As shown by Bailey (1978), optimal unemployment insurance balances the benefit of enhanced consumption smoothing against the costs of the associated moral hazard. The point of this paper is that the consumption smoothing benefits of unemployment insurance (or any other social insurance program) are large when it is costly to self insure, and less important when it is not. Self-insurance is more difficult when the cost of borrowing and the cost of saving are high. Because of its transparency and useful insights, the Bailey model is still used to assess empirical estimates of the costs and benefits of unemployment insurance,¹ but it is not sufficiently rich to explore variations in the cost of self-insurance. We explore these issues in a tractable life-cycle consumption model.

Differences in the cost of saving arise for many reasons. First, individuals may already be making payments to a compulsory state pension, and this makes liquid saving more costly. This highlights the interaction between different social insurance programs: compulsory contributions to a state pension may make the liquidity provided by UI more beneficial. Second, more generally, consumption needs differ over the life-cycle, reflecting for example the presence of children, and this leads to differences in the cost of forgoing consumption in order to save. Third, individuals differ in their expected income growth, and particularly for those who expect the fastest growth, a need to save for precautionary reasons operates against the desire to smooth consumption. Finally, individuals may differ in their impatience or in their willingness to be exposed to consumption fluctuations. These sources of heterogeneity in the cost of saving, and also differences in individuals access to credit, will lead to differences cost of self-insuring and hence in the value of UI. These factors are not present in the Bailey (1978) model in which there are no credit constraints and there is only one motive for saving: precautionary saving against the probability of job loss between two periods.

In our model, job loss is exogenous, the unemployed can invest in subsequent earnings capacity, insurance is partly from public unemployment insurance and partly from private savings. We introduce (i) a retirement savings motive, (ii) variation in the rate of income

¹See for example Gruber (1997) and Chetty (2006).

growth, (iii) variation in age at job loss, and (iv), the possibility of borrowing constraints.

We use this framework to illustrate the connections between credit market imperfections, the cost of precautionary saving and the role of UI. We are able to show theoretically that in the presence of borrowing constraints, UI may have a benefit that derives from providing liquidity that enables consumption smoothing over time, in addition to the benefit in the Bailey model (the latter derives from smoothing over states and from pooling across individuals). Further, the liquidity provided by UI to credit constrained individuals allows those individuals to spend longer unemployed searching for a suitable job: durations may be inefficiently short in the absence of UI. As one might expect, these liquidity benefits can raise the optimal replacement rate. On the other hand, when future income is low, individuals save for life-cycle reasons (ie., for retirement) and so self-insurance is less costly because liquid retirement savings can also serve as a buffer stock to smooth consumption. Thus the liquidity benefit of UI is less important and individuals may remain unemployed more than is efficient. Further, if individuals remain unemployed too long because of unemployment benefits, we show it can be welfare increasing for the government to restrict individual borrowing to induce individuals to return to work more quickly (as in Diamond and Mirrlees, 1979). In our framework, we show that optimal replacement rates may vary substantially with the strength of the retirement saving motive, age at job loss and access to credit (from less than 20 percent to almost 60 percent).

To motivate our analysis, we also present some empirical evidence on variation in asset holdings and in access to credit among job losers. We show that job losers differ significantly in wealth at job loss and access to credit, and that these differences are related to life-cycle considerations. These finding illustrates the heterogeneity in the cost of self-insurance that our model is design to explore.

The paper proceeds as follows. In the the next section we present our empirical evidence on the wealth and credit access of job losers. In section III we develop our life-cycle framework. Section IV outlines the implications of our model for consumption smoothing, asset accumulation and the duration of unemployment. Section V shows how optimal levels of unemployment benefit vary with access to credit, age at job loss, and the extent of other savings motives. Section VI concludes.

II Motivation: Heterogeneity in Consumption Loss, Assets and Credit Market Imperfections

This section assesses the evidence on the extent of observed heterogeneity in the extent of self-insurance and in the importance of capital market imperfections for job losers. In particular, we present evidence from a survey of job losers in Canada, as well as surveying results in the literature. Our data are drawn from the Canadian out of Employment Panel (COEP). Further information on the data and sample are given in the Appendix. We focus on assets at job loss, on consumption loss and in access to credit markets.

Gruber (2001) analyses wealth data from U.S. Survey of Income and Program Participation and documents enormous heterogeneity in wealth holdings around job loss. He reports that the median worker has assets sufficient to finance about two thirds of the income loss from an unemployment spell, but that almost a third of workers cannot finance even 10% of that income loss. We provide further evidence on this heterogeneity in Figure ??, which reports the distribution of net assets among job losers in Canada. Over half of job losers reported that their households had no such resources at the time of job loss or were in debt. A quarter reported that their household had liquid savings of more than three months of usual household income. Figure 2 shows that some of this heterogeneity can be explained by life-cycle considerations. Older individuals hold more liquid wealth, but the extent of their holdings of liquid assets depends on their holdings of illiquid pension wealth.² Similarly, holdings of liquid assets depend on consumption needs, captured here by the presence of children.

The size of asset holdings at job loss is likely to matter for the extent of any consumption fall at job loss. Heterogeneity in this consumption loss is documented by Browning and Crossley (2001) who report that while the mean fall in (total) consumption with unemployment in the COEP is 14%, a quarter of the job-losing households report no fall in consumption and 10% of households report that consumption fell by more than half. Figure 3 provides further evidence where we show the complete distribution of consumption losses following job loss (using a different sample from Browning and Crossley).

²This effect of pension holdings remains statistically significant even when controlling for a range of exogenous and financial characteristics, see Crossley and Low (2006). It is also consistent with the evidence from pension reforms reported in Attanasio and Brugiavini (2003) and Attanasio and Rohwedder (2004).

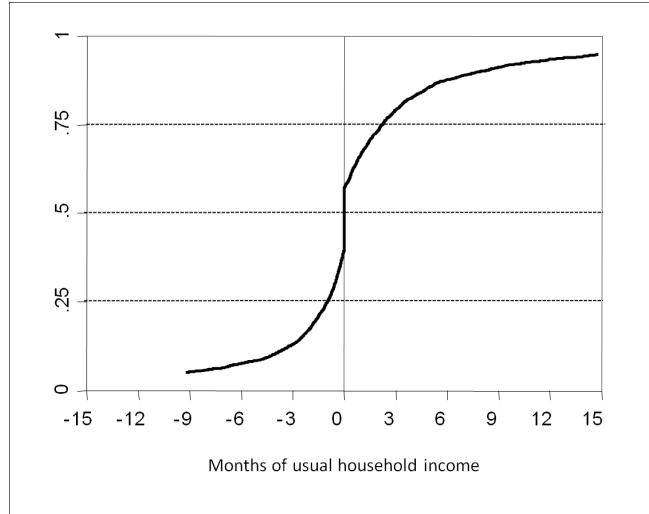


Figure 1: Empirical CDF of Net Asset Position at Job Loss

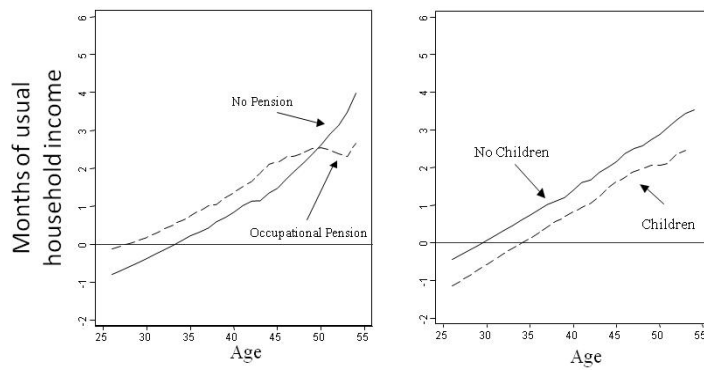
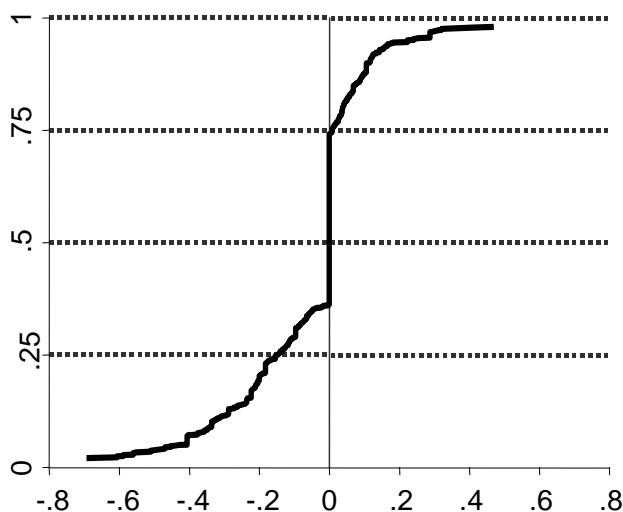


Figure 2: Liquid Asset Holdings by Age at Job Loss

Figure 3: Empirical CDF of Change in Consumption following Job Loss



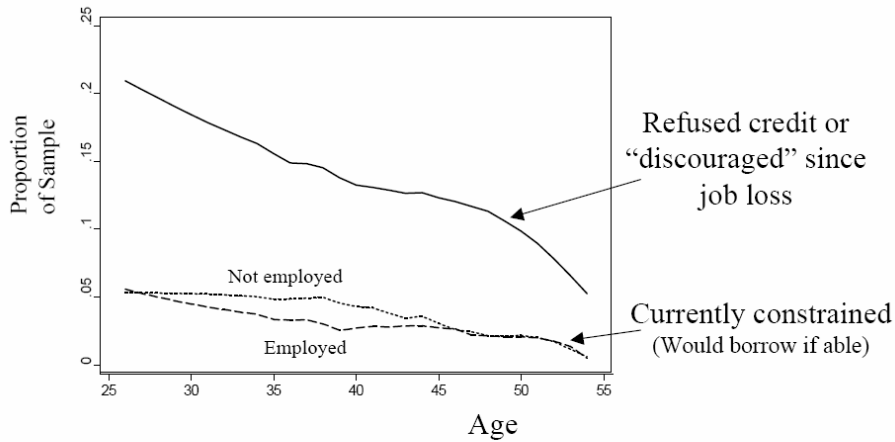
The extent of consumption loss will be affected by the ability of the household to borrow. Some evidence on the presence of borrowing constraints can be inferred from looking at the marginal propensity to consume out of unemployment benefit income. Browning and Crossley (2001) find that the marginal propensity to consume out of unemployment benefit income varies between 0 and .25 for different groups, where the groups are defined by family type and (liquid) wealth. Sullivan (2008) and Bloemen and Stanca (2005), using measures of food consumption in U.S. and U.K. data respectively, also document significant variation in the marginal propensity to consume out of transitory income across job losers with different wealth levels.

The 1995 COEP also provides direct evidence on the presence of borrowing constraints and on the heterogeneity in access to credit among job losers. Individuals are asked a combination of questions including subjective questions about whether they are able to borrow and want to borrow, as well as objective questions on their success at obtaining credit since job loss. Figure 4 reports responses to these questions by age.

Our objective question mirrors the questions in the Survey of Consumer Finance analysed by Jappelli (1990). The questions identify individuals who have either been refused credit or discouraged from applying for credit since job loss.³ Figure 4 shows that 15% of our sample of

³These questions were as follows: (1) *At any time since your job ended on [date of job loss] did you or*

Figure 4: Extent of Credit Constraints by Age



job losers have been constrained by this definition, but that the incidence of this inability to borrow declines markedly with age at job loss. Our subjective questions ask first about whether an individual would be able to borrow if they wanted to, and then about whether or not they want to borrow.⁴ We use responses to define as constrained individuals who report being unable to borrow but wanting to borrow at the interview 9 months after job loss. Since some of these individuals have been reemployed, we show the difference in incidence of the constraint by employment status. Figure 4 shows that by this measure of borrowing constraints, about 4% of job losers face binding credit constraints at the time of interview, but that this fraction declines with age.

Crossley and Low (2006) use the same data to show that those who report a binding

any member of your household apply for a loan at a bank or financial institution, or for credit with any credit company? (Applied). (2) Were any of your requests for credit or a loan turned down? (Declined) (3) Were you, or any member of your household, given as much credit as you applied for? (Not Full Amount) (4) Were you later able to obtain the full amount you requested by reapplying to the same institution or by applying elsewhere? (Got Later) (5) Was there any time since [date of job loss] that you or any member of your household thought of applying for credit at a particular place, but changed your mind because you thought you might be turned down? (Discouraged)

Respondents are classified as being constrained if either of two conditions holds. First, if they “applied” and were “turned down” or “did not receive full amount” and did not subsequently receive the credit (“get later”); or second if they were “discouraged” from applying. See Crossley and Low (2006) for details.

⁴The first question asks the respondent: *If you needed it, could you borrow money from a friend, family, or a financial institution in order to increase your household expenditures?*

If the answer to this question was negative, the respondent was then asked: *Suppose you COULD borrow money from one of these sources at 11% interest per year, to be paid back starting in one year. WOULD you borrow money to increase your weekly spending on household expenses?*

borrowing constraint subsequently exhibit very high consumption growth (relative to those who report not being constrained), consistent with an inability to smooth consumption over time.

To summarize, there is considerable evidence, both in the COEP data and in the literature, that job losers differ significantly in wealth at job loss and access to credit; that these differences are at least in part related to life-cycle considerations; and that these differences translate into substantial differences in the ability to smooth consumption following a job loss.

III Life-Cycle Model

We now set up a model to consider the effects of heterogeneity in self-insurance and the interaction between different savings motives by extending the Bailey framework. Bailey (1978) models the trade-off between consumption smoothing and moral hazard in a partial equilibrium framework. In Bailey's two period model, agents may lose their job between the first and second period. They then choose what portion of the second period to spend out of work. Crucially, utility depends only on total income in the second period: the fact that income may be low while out of work is immaterial. This is inconsistent with the idea that the unemployed face borrowing constraints and may be temporarily in difficult financial circumstances.

We develop that framework as follows. First, we make time continuous in order to introduce a role for borrowing constraints and to vary the age of job loss. Second, we introduce a retirement savings motive. The latter allows us to vary the cost of holding assets for precautionary reasons. Our model is partial equilibrium but closed with a government budget constraint, like the Bailey model.⁵ We allow for heterogeneity by varying the age at which job loss occurs, by varying access to credit and varying the contribution rate into the state pension. We impose the government budget constraint within groups. This precludes risk-pooling across groups with different costs of saving or borrowing and allows us to focus on the liquidity value of UI

⁵We believe that because wealth distributions are so highly skewed, and capital markets open, it is reasonable to model users of unemployment insurance systems as facing an exogenous interest rate. A second potential general equilibrium effect is the effect of unemployment insurance on the vacancy posting behaviour of firms (firms' vacancy decisions do not take into account the positive externality on other firms of creating a "thicker" market). Similarly, we do not capture the negative externality of search on the probability of other people finding jobs. These general equilibrium and externality effects may be important but are beyond the scope of this paper.

for different types of individual.

The generosity of the state pension depends directly on contribution made while working: a greater (compulsory) contribution rate leads to more generous payments in retirement. This means there is no redistributive element to the state pension, whereas in practice such redistribution is a part of the scheme in the UK and elsewhere. We do not include this redistributive element to our characterization of the scheme because our focus on the interaction between insurance and liquidity will be obscured if a state scheme is also redistributive. One consequence of the lack of redistribution in the state scheme is that a compulsory private scheme is equivalent to the state scheme. This means our state pension system is similar to the use of individual accounts for state pensions in Australia and Chile, and proposed elsewhere. Varying the contribution rate changes directly the generosity of the state pension, and we consider a range of contribution rates starting from 0 where there is no state pension at all.

We impose this structure of the state pension system, rather than allowing the government to use a wider set of instruments. For example, it may be that the government could achieve greater efficiency in investment by the unemployed by imposing re-employment taxes to cover unemployment benefits paid out. We have ruled out explicitly the use of re-employment taxes because they are not observed in practice. On the other hand, it is true that such taxes (or deferring all taxation until retirement) might produce a more efficient outcome if the government does not face credit constraints or political constraints.

Exogenous changes to the state pension system change the timing of income over the lifetime, and we use this variation to explore changes in the desire to save or borrow. With high contribution rates and hence a generous state pension, agents face a rising income profile, and would like to borrow, and saving is costly for such agents. This allows us to explore the value of unemployment insurance to agents for whom it is more or less costly to hold additional liquid savings.

There are a number of alternatives to the finite horizon life-cycle model we develop. Hansen and Imrohorglu (1992) model UI in an infinite horizon, calibrated dynamic general equilibrium model. This is less suitable for our purposes of understanding the effects of heterogeneity in the cost of saving because with an infinite horizon, agents must be impatient in order to keep

the problem bounded. In an infinite horizon, partial equilibrium model, Lentz (2009) varies the interest rate and illustrates that the value of UI depends on the cost of saving. The lower the interest rate, the more costly it is for them to hold a buffer stock of savings, and the more valuable social insurance becomes. However, the infinite horizon framework precludes Lentz from considering patient agents and from explicitly introducing life-cycle savings motives. Rendon (2006) carries out a similar exercise to Lentz in a finite horizon, allowing for some life-cycle effects. His focus is on estimating structural parameters rather than on exploring heterogeneity due to life-cycle effects. Costain (1999) also works with a finite horizon model, but allowing for general equilibrium effects. His focus is on the value of UI using a model calibrated to median wealth holdings and so he explicitly ignores the heterogeneity in the data. Further, like Rendon, he does not consider that heterogeneity in characteristics and in wealth may make the value of UI very different for different individuals.

A Model Setup

Life has three stages: youth, middle-age and old age. We use subscripts to denote the life-stage and note that life-stages may be of different lengths. Agents are risk-averse and maximize expected utility. They begin the first stage (which lasts from 0 until T_1) with initial assets $A_0(=0)$. In this stage agents work for a wage, w_1 , and consume continuously. Individuals pay two (proportional) taxes: a contribution to the state pension (τ^r), and an unemployment insurance contribution (τ^u). If they choose to consume less than their net income, they accumulate assets. As in Bailey (1978), at the end of the first stage individuals face an exogenous probability (π) of job displacement. Where necessary, we use superscripts d (displaced) and n (not displaced) to denote states of the world.

In the second stage (from T_1 to T_2) agents consume (and save or possibly borrow). If they are not displaced at the end of the first stage, they continue to earn the wage w_1 . If agents are displaced at the end of the first stage, they can return to work immediately at some wage which is strictly less than the wage in the job from which they were displaced ($w_2(I=0) < w_1$). Alternatively, they may choose to invest for time $I \leq T_2 - T_1$. During this investment period they receive a benefit determined by the replacement rate b . If $I < T_2 - T_1$

they return to work at $T_1 + I$, earning a wage $w_2(I)$ which is increasing in the duration of investment ($w_2(0) \leq w_2(I) \leq w_1$). Individuals pay only retirement taxes on unemployment benefits.

We can interpret investment in a number of alternative ways: first, investment may be search with recall (of previous offers) with longer search leading to a better match;⁶ second, investment may be retraining by the unemployed with wages being higher the longer the training period; third, investment may merely be waiting to be recalled (from temporary layoff) to a job with a high wage (relative to the outside option); finally, if we reinterpret unemployment benefit as a minimum payment to the worker, investment may be thought of as on-the-job training where workers receive a minimum payment during the training period, but a higher wage on completion. The presence of unemployment benefit may distort these investment decisions.⁷

In the final stage of life (from T_2 to T_3), individuals are retired and collect a state pension. At the end of the third stage they die with terminal assets $A_3 = 0$. The amount of resources available for consumption in retirement is determined by their state pension plus liquid asset holdings not consumed in earlier stages. The generosity of the state pension is determined solely by the individual's contributions in the first two stages of life.

In a general intertemporal consumption model, patience (broadly defined as the inclination to save) will be determined by the interest rate, the rate of time preference, the time path of needs, and the time path of income. All may contribute to heterogeneity in patience. For example, recent studies of household wealth (Samwick, 1998) and consumption growth (Alan and Browning, 2010) provide empirical support of heterogeneity in the rate of time preference. Attanasio et al. (1999) show empirically that children make households act as if they are more impatient. For transparency, we choose to model only one determinant of the agents' inclination to save. We assume that there is no discounting or rate of return ($\delta = r = 0$). We also abstract from modelling explicitly changes in needs. This gives us flat desired consumption paths. However, we can vary impatience (again, defined as the inclination to save) by varying

⁶It is possible that wages decline if unemployment is too long. In the current model, there is no uncertainty about job offer arrival, and so we are able to ignore the part of the investment schedule which is declining.

⁷In the Bailey model, moral hazard arises due to the utility of leisure in the second stage. We model moral hazard as arising through the investment decision to allow the possibility that unemployment is productive.

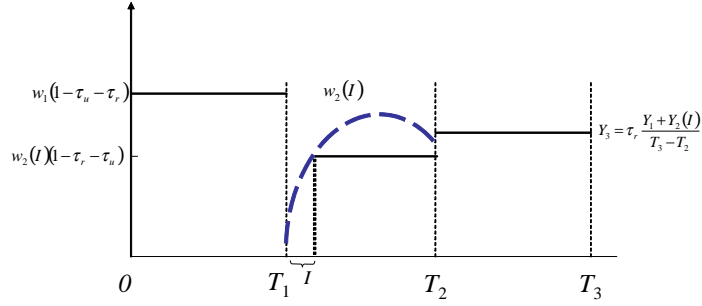


Figure 5: Time Path of Earnings

the growth rate of income.

We alter the growth rate of income through (exogenous) changes to the state pension system. With high contribution rates and hence a generous state pension (large τ^r) agents face a rising income profile. Such agents would like to borrow, and saving is costly for such agents. With lower contributions and a less generous state pension, agents face a falling income profile and wish to save. Varying τ^r will therefore allow us to explore the value of unemployment insurance to agents for whom it is more or less costly to hold additional liquid savings.

Savings motives are not additive: liquid assets held for precautionary reasons (smoothing consumption in the face of a temporary income loss) can be consumed in retirement if the negative shock is not realised. Equally, liquid assets held for retirement purposes may be partially used for precautionary reasons if unexpected shocks occur whereas illiquid retirement saving or contributions to a state pension cannot. It is more costly for an impatient agent (or one with low current net income) to accumulate precautionary balances as the marginal utility of current consumption is high and similarly, resources that become available late in life - if the shock is not realized - have low value. A similar point is also emphasized by Dynan, Skinner and Zeldes (2002) who argue that precautionary savings and savings for a bequest motive cannot be distinguished.

We consider an extreme variation in the cost of borrowing, comparing cases where agents can borrow freely (subject only to the terminal asset condition) with cases where they face an exogenous borrowing limit $A_t \geq -\phi$.

Notation in the model is summarized in Table 1 and timing in Figure 5.

Table 1: Notation and Earnings

c_t : consumption at time t A_s : assets at end of stage s w_s : wage in stage s Y_s : gross income for stage s E_s : gross earnings for stage s	τ^r : social security tax τ^u : unemployment insurance tax b : unemployment benefit (replacement ratio) I : duration of investment
<p style="text-align: center;"><i>Gross Earnings</i></p> Stage 1 $E_1 = w_1 T_1$ Stage 2 $E_2^d = (T_2 - T_1 - I)w_2(I)$ (<i>displaced</i>) Stage 2 $E_2^n = (T_2 - T_1)w_1$ (<i>notdisplaced</i>) Stage 3 $E_3 = 0.0$	<p style="text-align: center;"><i>Gross Income</i> (earnings + benefits)</p> $Y_1 = E_1$ $Y_2^d = E_2 + bY_1 I$ $Y_2^n = E_2^n$ $Y_3^i = \tau^r(Y_1 + Y_2^i)$

All earned income in stages 1 and 2 is subject to tax at a rate $\tau^r + \tau^u$.
Income from unemployment benefit is subject to a tax at rate τ^r .

B Individual Optimization Problem

We now lay out the individual optimization problem, taking b , τ^r , and τ^u as given. The individual maximises

$$V_1 = \max_{c_t, A_1} \int_0^{T_1} u(c_t) dt + \pi V_2^d(A_1) + (1 - \pi) V_2^n(A_1) \quad (1)$$

subject to the budget constraint

$$\int_0^{T_1} c_t dt = -A_1 + Y_1 (1 - \tau^r - \tau^u)$$

and, if present, the credit constraint,

$$A_1 \geq -\phi.$$

The solution to this problem can be characterised by the Euler equation:

$$\frac{\partial V_1}{\partial A_1} = u'(c_1) - \pi \frac{\partial V_2^d}{\partial A_1} - (1 - \pi) \frac{\partial V_2^n}{\partial A_1} + \mu_1 = 0 \quad (2)$$

$$\mu_1 \geq 0, \quad A_1 \geq -\phi. \quad (3)$$

The presence of the credit constraint affects equation (2) in two possible ways: first, it may cause the Euler equation to be violated (ie. μ_1 is strictly positive); second, the constraint may bind in stage 2 and so can affect behaviour in stage 1 through either $\frac{\partial V_2^d}{\partial A_1}$ or $\frac{\partial V_2^n}{\partial A_1}$, even though $\mu_1 = 0$.

In the absence of credit constraints, the solution is simple because the consumption path post-displacement can be separated from the timing of income: individuals displaced in the second stage choose investment simply to maximise income,⁸

$$\max_I w_2(I) (T_2 - T_1 - I) (1 - \tau^u) + bY_1 I$$

This yields the first order condition

$$w_2'(I) (T_2 - T_1 - I) (1 - \tau^u) + bY_1 = w_2(I) (1 - \tau^u) \quad (4)$$

⁸Noting that the pension tax paid in stage 2 is returned in stage 3.

where the left hand side is the marginal benefit of investment and the right hand side is the marginal cost of investment, analogous to the partial equilibrium, linear utility model (Mortensen, 1986). The marginal benefit of investment is increased by the unemployment benefit paid and so a positive replacement rate induces inefficient (over) investment.

This level of investment determines income post-displacement. Since there is no discounting, individuals choose consumption to be constant in any particular state. Once we know income, we know how consumption post-displacement will vary with A_1 . We can then solve for A_1 by using the envelope theorem to replace $\frac{\partial V_2^n}{\partial A_1}$ and $\frac{\partial V_2^d}{\partial A_1}$ in equation (2) by the marginal utility of consumption in each state.

The presence of credit constraints introduces an interaction between the investment decision and the consumption decision because, if the constraint binds, longer investment means a longer period at lower consumption levels. The choice of investment depends on the consumption level in the investment period and this in turn means investment will depend on asset holdings, A_1 . To solve the problem with the credit constraint, we have to solve simultaneously the asset allocation equation (2) and for optimal investment (equation (8) below).

After displacement, the individual chooses the length of investment and the amount of consumption to solve

$$V_{2I}^d(A_1) = \max_{c_{tI}, A_{1+I}, I} \int_{T_1}^{T_{1+I}} u(c_{tI}) dt + V_{2E}^d(A_{1+I}, I) \quad (5)$$

subject to

$$\int_{T_1}^{T_{1+I}} c_{tI} dt = A_1 - A_{1+I} + bY_1 I (1 - \tau^r) \quad (6)$$

$$A_{1+I} \geq -\phi \quad (7)$$

where $V_{2E}^d(A_{1+I}, I)$ is the value from reemployment after investment I with remaining assets A_{1+I} . The recursive structure of the problem means we can solve (5) using Lagrange multipliers from future stages, recognising that the credit constraint may bind after reemployment if impatience is high enough (see appendix).

To determine investment, we need to use the first-order condition from maximising equation (5) with respect to I .

$$\frac{\partial V_{2I}^d}{\partial I} = 0 = u(c_{2I}) + \frac{\partial V_{2E}^d}{\partial I} + \lambda_{2I} b Y_1 (1 - \tau^r) - \lambda_{2I} c_{2I}$$

where $\partial V_{2E}^d / \partial I$ is the marginal benefit of further investment realised once reemployed and λ_{2I} is the Lagrange multiplier on the budget constraint in the investment stage in stage 2, equation (6). Using the solution for $\partial V_{2E}^d / \partial I$ derived in the appendix, and rearranging,

$$\begin{aligned} w'(I) (T_2 - T_1 - I) [\lambda_{2E} (1 - \tau^r - \tau^u) + \lambda_3 \tau^r] + b Y_1 [\lambda_{2I} (1 - \tau^r) + \lambda_3 \tau^r] \\ = w(I) [\lambda_{2E} (1 - \tau^r - \tau^u) + \lambda_3 \tau^r] + \Psi \end{aligned} \quad (8)$$

The left hand side of equation (8) is the marginal benefit of investment and the right hand side is the marginal cost of investment, analogous to condition (4). The marginal benefit of investment includes unemployment benefit and the resulting increase in the future wage. Here (and in contrast to condition 4) both are weighted by marginal utility terms which are share weighted averages of the marginal utilities in the stages in which the relevant resources will be realized.

The first term in the marginal cost is the (forgone) wage, again valued at a share weighted average of the marginal utilities in the stages in which it is received. The second term Ψ can be approximated by

$$\Psi \approx \gamma \Delta c_{2E} u'(c_{2E}), \quad (9)$$

where γ is the coefficient of relative risk aversion which captures the degree of aversion to fluctuations in consumption. The term Ψ is a utility cost term associated with the failure to smooth consumption between the investment and earnings periods of stage 2.⁹ The presence of Ψ increases the marginal cost of investment because consumption is no longer smoothed over substages in a way that would have occurred if there had been no credit constraints. The size of this cost is increasing in the degree of fluctuation aversion and would be zero if utility were linear. This reduces investment below the level of investment that would occur if unconstrained. Investment when constrained may potentially fall below the level which would maximise earned income. In this case, increasing unemployment benefits can induce a more efficient level of search.

⁹If the credit constraint is not binding, $\Psi = 0$ and $\lambda_{2I} = \lambda_{2E} = \lambda_3$ and so equation (8) reduces to equation (4).

For given values of τ^r, τ^u and b , we can now solve for A_1 and I using equations (2) and (4) if unconstrained, or equations (2) and (8) if constrained. We solve for τ^u to balance the budget as discussed in the next subsection.

C Government Budget Constraints

Unemployment benefit is financed in our model by the tax τ^u and we set τ^u to balance the government budget constraint. Ignoring the government budget constraint would mean increases in unemployment duration associated with more generous benefits do not introduce extra costs.

The budget constraint for the unemployment insurance system is:

$$\tau^u (w_1 T_1 + \pi w_2 (I^*) ((T_2 - T_1) - I^*) + (1 - \pi) w_2 (T_2 - T_1)) = \pi I^* b Y \quad (10)$$

This implies that the budget is set to balance across individuals and there is redistribution from workers to the unemployed. Because there is no aggregate risk, we can alternatively say that the budget balances in expectation and so insurance is actuarially fair.

Budget balance in the state pension system is imposed by each individual receiving the sum of their earlier contributions as retirement income: $Y_3 = \tau^r (Y_1 + Y_2 (I))$. This implies that the pension system contains no element of redistribution between individuals and no notion of insurance.¹⁰

We assume that the government cannot defer all taxation until retirement. Such a deferral of taxation could be partially captured by considering negative values of τ^r and this might be more efficient if the government is able to borrow when individuals are not. However, such a structure of taxation is not observed in practice, perhaps because of political constraints or perhaps because it would require commitment to enforcing individuals to save to pay taxes in retirement. Our aim is to model the key elements of existing pension systems, and to use variation in the generosity of the state pension to capture variation in the timing of income and the cost of saving.

¹⁰If there were only one government budget constraint, pension provision could contain an element of redistribution by providing “pension credits” for periods in unemployment. Similarly we do not consider redistribution across individuals who face different job loss risk, π , or different loss of potential earnings. Our focus is the on the non-redistributive aspects of unemployment insurance.

IV Implications

In this section, we use our model to show the implications of varying the cost of saving and introducing borrowing constraints for individual saving, consumption smoothing and investment behaviour in response to unemployment risk. In the next section, we use these implications to derive optimal unemployment benefit levels which vary with the cost of saving and borrowing. Implications of the model are demonstrated partly analytically and partly numerically.

For the numerical analysis we assume CRRA utility,

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

and a simple investment function, $w(I) = I^\eta$. Parameters used are summarised in Table 2. We explore variation in replacement rates, variation in the timing of layoff and variation in the patience of agents. As noted above, the latter is controlled by the pension contribution rate (τ^r) which controls the growth rate of expected income. With low τ^r agents anticipate a low state pension and so low income in the future and save for their retirement in the liquid asset; with high τ^r agents anticipate high income in retirement and would like to borrow.

Table 2: Parameters for Numerical Solution

<i>Parameter</i>	<i>Baseline</i>	<i>Alternatives</i>
γ	1.5	
η	0.5	
w_1	1.0	
T_1	1.0	[0.25,1.75]
T_2	1.0	[0.25,1.75]
T_3	1.0	
b	0.2	[0, 0.7]
τ^r	0.05, 0.30, 0.45	

Total length of life remains fixed at 3 units. Variation in age at job loss through varying T_1 means age at job loss varies from age 25 through to age 55, if we interpret 1 unit of time as being 20 years.

A Consumption and Saving

From the first-order conditions of the individual optimisation problem, in the absence of borrowing constraints, or if the constraints do not bind, marginal utility is smoothed over time

(at least in expectation):

$$\lambda_1 = \pi \lambda_{2d} + (1 - \pi) \lambda_{2n}$$

$$\lambda_{2I} = \lambda_{2E} = \lambda_{2d} = \lambda_{3d}$$

$$\lambda_{2n} = \lambda_{3n}$$

but not smoothed over states:

$$\lambda_{2n} \neq \lambda_{2d}$$

where $\lambda_{ti} = \partial u / \partial c_i$ with $t \in \{1, 2, 3\}$ indicating the stage and $i \in \{n, d, I, E\}$ indicating the state within the second and third stages, corresponding, respectively, to the non-displaced state, the displaced state, the investment state post job loss and the earnings state after returning to work. The finiteness of life means that households cannot perfectly self-insure even in the absence of borrowing constraints. The “insurance” benefit of UI is that it helps to smooth marginal utility across states, reducing $\lambda_{2d} - \lambda_{2n}$ which is the “permanent shock” of job loss. This is the benefit of UI that operates in the Bailey model. This benefit is also present in our framework where the finite horizon limits the possibility of self-insurance. In general, the size of the insurance benefit of UI will depend on the persistence of the unemployment shock relative to the horizon of the individual. When unemployment durations are short, as in the US, for example, where the median duration is 3 months, the insurance value of UI is likely to be small.

By contrast, if credit constraints bind, then from the first-order conditions,

$$\lambda_1 = \pi \lambda_{2I} + (1 - \pi) \lambda_{2n} + \mu_1$$

$$\lambda_{2I} = \lambda_{2E} + \mu_{2I} = \lambda_{3d} + \mu_{2I} + \mu_{2E}$$

$$\lambda_{2n} = \lambda_{3n} + \mu_{2n}$$

Marginal utility is again only partially smoothed over states ($\lambda_{2I} \neq \lambda_{2n}$), but in addition, marginal utility is only partially smoothed over time after job loss ($\lambda_{2I} \neq \lambda_{2E} \neq \lambda_{3d}$). By reducing $\lambda_{2I} - \lambda_{2E}$ UI can have another benefit (beyond the insurance benefit noted above): it helps to smooth consumption over time after job loss. This consumption smoothing benefit of UI is absent in the Bailey (1978) model because post-displacement, consumption is independent

of labour market state.¹¹ To the extent that unemployment reflects a transitory shock only, this liquidity benefit provides the rationale for UI: self-insurance is not enough to smooth consumption over time.

Figure 6: Asset and Consumption Paths

The left hand column presents consumption and asset paths when able to borrow, the right hand column paths when unable to borrow. The three rows represent different levels of impatience. C is consumption, A is the asset stock at a point in time, t is age, which ranges from 0 to 3.

¹¹Gruber (1997) and Browning and Crossley (2001) both estimate regression equations of the form:

$$\Delta \ln c_t = X\beta + \alpha b + e$$

where legislative variation (across time, or time and jurisdictions) is used to estimate α . Gruber interprets his estimate of α as an estimate of the insurance benefits of unemployment insurance (and uses that estimate in optimal benefit calculations based on the Bailey model). In contrast, Browning and Crossley set out an Euler equation framework in which α captures the effect on consumption growth of a binding credit constraint. In terms of the model presented here, Gruber interprets α as $\lambda_{2d} - \lambda_{2n}$, while Browning and Crossley interpret $\alpha = \lambda_{2I} - \lambda_{2E} = \mu_I$.

Figure 6 displays the time paths of assets and consumption for simulations of our model with different parameter values. The left hand side panel present time paths for agents who are able to borrow; the right hand side panels present time paths for agents who are unable to borrow. Moving from top to bottom the panels are differentiated by a decreasing cost of saving. In the top panels a very high value for pension withholdings is chosen which has the effect of making additional savings costly and agents very impatient (they would like to bring resources forward from the future). In the bottom panels illiquid pension contributions are very low, the income profile is downward sloping, agents have a strong life-cycle (retirement) savings motive, and hence are patient. The middle panels present an intermediate case.

When agents are able to borrow, consumption is equalized across time (after the shock is realised) and the consumption path is independent of the timing of income. However, because time diversification is limited by the finiteness of life, consumption is not completely equalized across states. Patient agents (row *iii* in Figure 6) smooth by saving and their holdings of liquid assets increase with age until retirement, while impatient agents (row *i*) smooth by borrowing and their borrowing increases with age until retirement. This implies that as the cost of saving increases, individuals save less, and then borrow if the cost of saving becomes high enough.

The right hand column of Figure 6 shows that a similar results holds when individuals are unable to borrow: as the cost of saving increases, individuals save less, and then want to borrow if the cost of saving becomes high enough. Because patient agents have sufficient liquid wealth to smooth without borrowing, their time paths of consumption are unaffected by their inability to borrow (row *iii*). By contrast, impatient agents who cannot borrow do not fully smooth consumption across time after job loss and consumption rises at reemployment (rows *i* and *ii*).

B Effects of Varying the Replacement Rate

We next show the effects of varying the replacement rate on savings, consumption loss on unemployment and investment behaviour.

Figure 7 shows the extent of asset holdings at the point of job loss (A_1) and Figure 8 shows the size of the consumption drop ($\Delta \ln c_t$) following job loss, for different replacement rates

and for different costs of saving and borrowing. The three graphs in each Figure different costs of saving, and in each graph we show the case where agents are able to borrow and the case where agents are unable to borrow.

Figure 7: Asset Accumulation by Replacement Rate

The graphs show asset holdings, normalised by the wage rate, at job loss on the y-axis and the unemployment benefit replacement rate on the x-axis. The three graphs represent different levels of impatience. In the graph showing patient agents, the “unable to borrow” and “able to borrow” cases coincide.

Figure 7 reinforces that the extent of liquid asset holdings and the ability to self-insure depends on the cost of saving: greater forced retirement saving or greater impatience lead to lower liquid asset holdings. This result holds whether or not individuals are able to borrow. However, Figure 7 shows that the inability to borrow leads to greater asset holdings relative to the case where individuals are able to borrow. Further, the middle graph in Figure 7 shows that borrowing constraints can lead to greater asset holdings even if asset holdings are positive in the unconstrained case.

Asset accumulation in this model is partly for precautionary reasons and partly to fund consumption in retirement. Assets not needed for precautionary reasons can instead be consumed in retirement. In this context, an increase in UI will crowd out liquid asset holdings,¹² but the extent of the crowd-out will depend on the substitutability between asset motives: crowd-out is greater when liquid assets are not used for consumption in retirement (as in the

¹²Engen and Gruber (2001) estimate the extent unemployment insurance crowds out precautionary saving.

Figure 8: Consumption Loss by Replacement Rate

The graphs show the consumption fall on job loss on the y-axes and the unemployment benefit replacement rate on the x-axes. The three rows represent different levels of impatience. In the graph showing patient agents, the “unable to borrow” and “able to borrow” cases coincide.

first graph in Figure 7).

Figure 8 shows that consumption loss decreases as benefits increase, but among the impatient (the left hand graph) and intermediate agents (the middle graph) the loss is greater and the relationship is steeper when borrowing is restricted. In other words, unemployment is more costly and unemployment benefit provides more insurance when saving and borrowing are costly.

Equation (8) in section B shows how the marginal benefit of investment depends on the presence of borrowing constraints. This is illustrated by the simulations presented in Figure 9. Each graph plots the duration of investment against the replacement rates. The graphs differ by the assumed patience of the agent.¹³ Among the impatient agents and agents of intermediate patience, borrowing constraints lead to under-investment, and efficient search durations are induced by positive replacement rates. As we saw in the preceding analysis of consumption smoothing, the very patient agents are unaffected by borrowing constraints (because they have considerable liquid savings). As with consumption, differences in the state

¹³Across the cases with no borrowing constraints, the average elasticity of durations with respect to the benefit rate is 0.9, which matches the estimate of Meyer (1991), but is higher than Chetty (2006).

Figure 9: Investment by Replacement Rate

The length of investment (or duration of unemployment as a fraction of the second stage) is shown on the y-axis and the unemployment benefit replacement rate on the x-axis. The three graphs represent different levels of impatience.

pension contribution rate (or heterogeneity in impatience) only matters for search behaviour in our model if individuals are unable to borrow. If we added further capital market imperfections to the model, heterogeneity in impatience would matter more generally.

V Optimal Unemployment Insurance

We have shown that the cost of saving and the ability to borrow matter for understanding how individuals behave in response to UI. In this section, we address the natural next question of how optimal unemployment insurance depends on the cost of saving and the ability to borrow. We show first the dependence of the optimal replacement rate on the ability to borrow analytically ignoring the retirement stage and the retirement tax. We then show numerically that a common replacement rate across individuals of differing age and with different costs of saving and borrowing leads to very different degrees of consumption loss on job loss. We end the section by showing numerically how this translates into optimal benefit rates which differ by the cost of saving and borrowing.

A Marginal Benefit of UI

We calculate the marginal benefit of increasing the replacement rate, $\partial V_1/\partial b$, from equation (1) and evaluate this at optimal choices for investment and saving by the individual to give

$$\frac{\partial V_1}{\partial b} = -\lambda_1 Y_1 \frac{\partial \tau^u}{\partial b} + \pi \left(\lambda_I Y_1 I - \lambda_E E_2^d(I) \frac{\partial \tau^u}{\partial b} \right) - (1 - \pi) \frac{\partial \tau^u}{\partial b} \lambda_n Y_2^n \quad (11)$$

Because of the envelope theorem we can ignore indirect effects of changing benefits operating through optimised values of I, A_1, A_{1+I} .

We set this expression equal to 0, use the approximation $\lambda_i \approx \lambda_n - u''(c_n)(c_n - c_i)$ and rearrange to give

$$\begin{aligned} & \pi Y_1 I \left\{ 1 + \gamma \left[\frac{(c_n - c_E)}{c_n} \right] + \gamma \left[\frac{(c_E - c_I)}{c_n} \right] \right\} \\ &= \frac{\partial \tau^u}{\partial b} \left\{ Y_1 \left(1 + \gamma \frac{(c_n - c_1)}{c_n} \right) + (1 - \pi) Y_2^n + \pi E_2^d(I) \left(1 + \gamma \frac{(c_n - c_E)}{c_n} \right) \right\} \end{aligned} \quad (12)$$

The left hand side represents the marginal benefit of an increased replacement rate if job loss occurs. The right hand side represents the marginal costs which arise due to a higher tax rate. Higher taxes impose a cost in the first stage. They also impose a cost throughout the second stage if no job loss occurs and after return to work if job loss occurs. We want to focus, however, on the gross marginal benefit. The marginal benefit depends on the consumption differences in the square brackets: the first is the difference in consumption between the “no job loss” state and the “reemployed” state, the second is the difference in consumption at the time of search and consumption in the future after reemployment. The first of these terms is the benefit of smoothing over states, the second is the benefit of smoothing over time. If there were no credit constraints, this second benefit would be absent because consumption would be smooth after job loss. Both terms are multiplied by γ : this represents the utility cost of consumption not being smooth. With a more general utility function, the term on the first consumption difference would be risk aversion, whereas the term on the second would be fluctuation aversion.

Put another way, borrowing constraints limit the time diversification of risk. In particular, in our model they prevent the optimal allocation of resources (over time) in the bad state. As a consequence, they exacerbate the difference (in marginal utility) between the two states of the world, and raise the value of the benefit provided by the UI system.

This equation is similar to equation (2) in Chetty (2006) except that we have a benefit of smoothing over time, rather than just over states, and because taxes are paid in the first stage in our model in addition to the second stage. However, the important contribution of Chetty (2006) is to show that many similar marginal benefit - marginal cost trade-offs (reflecting different economic environments) lead to a common Bailey-type optimal benefit formula when expressed in terms of observable elasticities.¹⁴ We do not present such a reduced form expression here because expressing the optimal benefit condition in terms of primitives of the model makes clearer the role and source of heterogeneity, which is our main focus.

B Consumption Loss

Equation (12) shows the potential role of credit constraints in affecting the optimal level of UI. Heterogeneity in the cost of saving or access to credit among individuals is likely to lead to different values for the optimal level of UI. To show the effect of this heterogeneity, Figure 10 reports the simulated change in consumption following job loss. To introduce heterogeneity, we vary the age at which job loss may occur and we vary the state pension contribution rate. All individuals face a 50% replacement rate. The figure shows three different regions: (i) a region where the liquidity constraint is not binding; (ii) a region where the liquidity constraint binds after job loss; and (iii) a region where the liquidity constraint binds even before job loss, and continues to bind after job loss. This latter region where the constraint binds both before and after job loss provides a floor on the consumption loss: in this region consumption post displacement is equal to benefit income.

When job loss is at older ages, the fall in consumption is less because individuals have had longer to accumulate assets and so self-insurance is more effective. The effect of increasing the retirement tax (or the cost of saving) depends on whether the borrowing constraint is binding:

¹⁴The key requirement for this result is that wages and the supply of jobs be independent of benefit level, as they are in our model.

Figure 10: Simulated Consumption Loss

The replacement rate is set at 0.55. Age at job loss is changed by varying T_1 : age 25 corresponds to $T_1 = 0.25$ while age 55 corresponds to $T_1 = 1.75$. Age at retirement is set at 60 by T_2 and remains constant. Fixed parameters: probability of job loss = 0.1, $\gamma = 1.5$.

when it does not bind, changes in the tax do not affect consumption loss. When the constraint binds, consumption loss increases as the tax increases.

Consumption changes in Figure 10 range from -13% to -45%. While our model is clearly very stylized, these numbers are within the range observed in the data (reported in Section II).

C Optimal Replacement Rates

In Figure 11 we solve numerically for the optimal replacement rate. As in Figure 10, we introduce heterogeneity through varying the age at job loss and the retirement tax and we allow the optimal replacement rate to vary with this heterogeneity. The most striking point in Figure 11 is the extent of heterogeneity in the optimal replacement rates: the optimal replacement rate varies from 0.17 to 0.59 even without preference heterogeneity in risk aversion. This heterogeneity mirrors the differences in consumption loss shown above. Optimal replacement rates fall with age of job loss when self-insurance is more effective. The effect of increasing the cost of saving on the replacement rate depends on whether the borrowing constraint is binding: when it does not bind, changes in the cost of saving do not affect the optimal replacement rate. When the constraint binds after job loss, increases in the cost of saving lead to an increased replacement rate. When the constraint binds both before and after job loss, individuals become unwilling even to pay the insurance premium in stage 1 and the optimal replacement rate falls.

Figure 11 highlights the complementarity between government programs. In particular, we can interpret the effect of increasing τ^r leading to an increased optimal replacement rate as showing that UI has more value for agents who have made substantial contributions to a state pension, and hence do not wish to accumulate additional assets; but has less value for agents who are privately saving for retirement and hence have liquid savings available.

A final implication in considering the value of UI is that for some parameterisations (for example with $\tau^r = 0.3$) credit constraints can raise welfare. The reason is that the displaced agent does not internalize the negative externality that her search behaviour has through the government budget constraint. Since the borrowing constraint reduces search, it mitigates the moral hazard cost of UI and leaves the government able to offer more insurance. Another

Figure 11: Optimal Replacement Rates

The optimal benefit ranges from 0.17 to 0.59. Age at job loss is changed by varying T_1 : age 25 corresponds to $T_1 = 0.25$ while age 55 corresponds to $T_1 = 1.75$. Age at retirement is set at 60 by T_2 and remains constant. Fixed parameters: probability of job loss = 0.1, $\gamma = 1.5$.

way to think about this is that in a second best world, the ability to control borrowing would give the government a second instrument to reduce moral hazard, analogous to the result in Diamond and Mirrlees (1979).

VI Conclusions

Unemployment insurance is more valuable when self-insurance is more difficult. Self-insurance is more difficult when the cost of borrowing and the cost of saving are high. The cost of borrowing depends on credit markets, and in particular, is effectively infinite for households facing a binding borrowing constraint. The cost of savings depends on the timing of income and the timing of needs, as well as private and market discount rates. Heterogeneity in these factors could lead to the empirical heterogeneity in asset holdings, and in marginal propensities to consume, that has been documented among job losers. Heterogeneity in these factors would also imply differences in the value of UI.

We developed a simple life-cycle model to illustrate these connections. Our model illustrates that in the presence of borrowing constraints UI may have an additional benefit that derives from smoothing consumption over time after job loss. Borrowing constraints can also dampen the moral hazard. These effects of borrowing constraints can raise the optimal replacement rate. In addition, optimal replacement rates vary substantially with the age at job loss and the generosity of the state pension. In our model, these are the factors that determine the cost of accumulating and holding a buffer stock.

In our model, there is only one unemployment shock, but it has a substantial effect. In reality, unemployment shocks are more transitory, and the risk of unemployment is ongoing through the life-cycle. To the extent that shocks are transitory, then the value of smoothing across individuals is diminished and the value of UI comes from smoothing consumption over time for liquidity constrained individuals. In this case, the argument in support of UI is more an argument for unemployment insurance savings accounts (Feldstein and Altman, 1998). In a finite horizon model like ours, however, the cost of using savings accounts is that there is no pooling across individuals.

Models with ongoing uncertainty (regarding the arrival of job offers, or future job losses)

are unlikely to yield many analytical results, but can, of course, be approached numerically. Nevertheless, the insights from models such as ours are quite general. Carroll (1997) has emphasized the behavioural effects of ongoing uncertainty are very similar to the effects of a borrowing constraint. Thus the borrowing constraint in our model can be interpreted as a proxy for more general capital market imperfections. More generally, Chetty (2006) emphasises the common structure of this class of models in thinking about optimal UI.

The life-cycle approach to unemployment insurance taken in this paper suggests a number of important policy implications. An obvious implication is that the design of state pensions and public unemployment insurance systems are interdependent. To the extent that state pensions mean that workers retirements savings are not available to smooth a temporary income shock (either directly or as collateral) they may make unemployment insurance more valuable.

At a more general level, our analysis demonstrates that optimal replacement rates are very sensitive to the costs of saving and borrowing. Optimal UI results will always depend, in part, on what is being assumed about borrowing possibilities, the strength of other savings motives, and the liquidity of the assets held because of those motives savings.

However, the most important implication of our analysis is that models that ignore the kind of heterogeneity we have documented are likely to provide an incomplete guide to policy. In light of our model, the data presented in Section 2 suggest that there are many workers for whom unemployment insurance has little value. Because circumstances or other savings motives makes it easy for them to hold a buffer of liquid assets, or because they have good access to credit markets, self-insurance is a reasonable option. For other workers, this is not the case. Even without redistribution between groups, the optimal single unemployment insurance system must depend on the weight society places on the welfare of different groups. Moreover, differential benefits by age or family type (for example, higher benefits to families with children) may be desirable on insurance grounds alone. This is because the cost of self-insurance, and hence the value of unemployment insurance, may differ across ages and family types. Of course, this needs to be balanced against differences across groups in the extent of moral hazard.

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I Appendix: Solving the Model

This appendix describes explicitly the recursive solution of the model. We solve for the optimal choices of consumption at each stage and investment after job loss. The tax rate, τ^u , is determined by the government revenue equation in the text. The system reduces to 3 nonlinear equations determining A_1, I and τ^u which we solve numerically.

Stage 3: In the third (retirement) stage of life, the value function is

$$V_3(A_2; I) = \max_{c_t} \int_{T_2}^{T_3} u(c_t) dt$$

Subject to:

$$\int_{T_2}^{T_3} c_t dt = A_2 + \tau^r (Y_1 + Y_2(I)) \quad (13)$$

where $I = -1$ indicates the individual was not displaced in stage 2. Note that the borrowing constraint, if present, is irrelevant because the constraint that terminal assets are zero and the assumption that $\delta = 0$ mean consumption is spread evenly through the stage. Associating λ_3 with the budget constraint (13) and using the envelope theorem,

$$\frac{\partial V_3}{\partial A_2} = \lambda_3$$

$$\frac{\partial V_3}{\partial I} = \lambda_3 \cdot \tau^r \cdot \left(\frac{\partial E_2}{\partial I} + bY_1 \right)$$

These expressions are used in solving for assets and investment in earlier stages.

Stage 2 (not displaced): In the second stage there are two cases: displaced (d) or not (n).

If the agent is not displaced, her value function is

$$V_2^n(A_1) = \max_{c_t, A_2} \int_{T_1}^{T_2} u(c_t) dt + V_3(A_2; I = -1)$$

subject to

$$\int_{T_1}^{T_2} c_t dt = A_1 - A_2 + Y_2^n (1 - \tau^r - \tau^u) \quad (14)$$

As with stage 3, consumption will be constant within the stage. Associating the multiplier λ_{2n} with constraint (14) gives the Euler equation

$$\begin{aligned} \frac{\partial V_2^n}{\partial A_2} &: & -\lambda_{2n} + \frac{\partial V_3}{\partial A_2} + \mu_{2n} &= 0 \\ \mu_{2n} &\geq 0, & A_{2n} &\geq -\phi. \end{aligned} \tag{15}$$

If there is no borrowing constraint, or the constraint is not binding, $\mu_{2n} = 0$ and consumption will be smooth between stages 2 and 3.

Stage 2 (displaced): No credit constraint We consider optimal choices after displacement first for the case with no credit constraint and in the next subsection for the case with credit constraints.

If the worker is displaced her value function is

$$V_2^d(A_1) = \max_{c_t, A_2, I} \int_{T_1}^{T_2} u(c_t) dt + V_3(A_2, Y_2(I))$$

subject to

$$\int_{T_1}^{T_2} c_t dt = A_1 - A_2 + bY_1 I (1 - \tau^r) + E_2^d(I) (1 - \tau^r - \tau^u) \tag{16}$$

$$0 \leq I \leq T_2 - T_1 \tag{17}$$

As before, consumption will be constant within the stage. Associating the multiplier λ_{2d} with constraint (16) gives the Euler equation

$$\frac{\partial V_2^d}{\partial A_2} : \quad \frac{\partial V_3}{\partial A_2} - \lambda_{2d} = 0 \tag{18}$$

Since we know consumption in the final stage, we can solve directly for consumption in period 2 and for λ_{2d} .

Turning to investment behaviour, the absence of a credit constraint means that the choice over I can be considered independently from the choice of A_2 .

Stage 2 (displaced): With credit constraint

The presence of the credit constraint means the timing of income within the second stage may matter. The length of investment and the path of consumption will be jointly determined

rather than being separable decisions as in the absence of credit constraints. Therefore, it is useful to divide the stage into an earnings and an investment substage.

Earnings sub-stage

$$V_{2E}^d(A_{1+I}, I) = \max_{c_{tE}, A_2} \int_{T_{1+I}}^{T_2} u(c_{tE}) dt + V_3(A_2, Y_2(I))$$

subject to

$$\int_{T_{1+I}}^{T_2} c_t dt = A_1 - A_2 + E_2^d(I) (1 - \tau^r - \tau^u) \quad (19)$$

$$A_2 \geq -\phi \quad (20)$$

Associating the multiplier λ_{2E} with the first constraint and μ_{2E} with the second constraint,¹⁵ gives first-order conditions:

$$\begin{aligned} \frac{\partial V_{2E}^d}{\partial A_2} & : \quad \frac{\partial V_3}{\partial A_2} - \lambda_{2E} + \mu_{2E} = 0 \\ \mu_{2E} & \geq 0, \quad A_2 \geq -\phi \end{aligned} \quad (21)$$

Using the envelope theorem,

$$\begin{aligned} \frac{\partial V_{2E}^d(A_{1+I}, I)}{\partial A_{1+I}} & = \lambda_{2E} \\ \frac{\partial V_{2E}^d(A_{1+I}, I)}{\partial I} & = -u(c_{2E}) + \frac{\partial V_3}{\partial I} + \lambda_{2E} \left(\frac{\partial E_2^d}{\partial I} (1 - \tau^r - \tau^u) + c_{2E} \right) \end{aligned} \quad (22)$$

Investment sub-stage

$$V_{2I}^d(A_1) = \max_{c_{tI}, A_2, I} \int_{T_1}^{T_{1+I}} u(c_{tI}) dt + V_{2E}^d(A_{1+I}, I) \quad (23)$$

subject to

$$\int_{T_1}^{T_{1+I}} c_{tI} dt = A_1 - A_{1+I} + bY_1 I (1 - \tau^r) \quad (24)$$

$$A_{1+I} \geq -\phi \quad (25)$$

¹⁵ As before, consumption must be constant within each sub-stage because no new information arrives within each sub-stage and the interest rate equals the discount rate. This in turn means that if the constraint binds at all in a sub-stage, it must bind at the end of that sub-stage.

Associating the multiplier λ_{2I} with the first constraint and μ_{2I} with the second constraint, gives the first-order condition for savings:

$$\begin{aligned} \frac{\partial V_{2I}^d}{\partial A_{1+I}} & : & \frac{\partial V_{2E}^d}{\partial A_{1+I}} - \lambda_{2I} + \mu_{2I} + 0 \\ \mu_{2I} & \geq 0, & A_{1+I} \geq -\phi \end{aligned} \quad (26)$$

The equation determining investment is given by the first-order condition from maximising equation (5) with respect to I .

$$\frac{\partial V_{2I}^d}{\partial I} = 0 = \frac{\partial V_{2E}^d}{\partial I} + u(c_{2I}) + \lambda_{2I} b Y_1 (1 - \tau^r) - \lambda_{2I} c_{2I}$$

Substituting in from equation (22) and rearranging,

$$\begin{aligned} \frac{\partial E_2^d}{\partial I} [\lambda_3 \tau^r + \lambda_{2E} (1 - \tau^r - \tau^u)] + b Y_1 [\lambda_{2I} (1 - \tau^r) + \lambda_3 \tau^r] \\ = u(c_{2E}) - u(c_{2I}) + \lambda_{2I} c_{2I} - \lambda_{2E} c_{2E}. \end{aligned}$$

Using the definition of E_2^d , this can be rearranged as:

$$w'(I) (T_2 - T_1 - I) [\lambda_3 \tau^r + \lambda_{2E} (1 - \tau^r - \tau^u)] + b Y_1 [\lambda_{2I} (1 - \tau^r) + \lambda_3 \tau^r] \quad (27)$$

$$= w(I) [\lambda_3 \tau^r + \lambda_{2E} (1 - \tau^r - \tau^u)] + \Psi \quad (28)$$

where we define Ψ by

$$\Psi = [u(c_{2E}) - u(c_{2I})] - [\lambda_{2E} c_{2E} - \lambda_{2I} c_{2I}].$$

Taking a first-order approximation to $u(c_{2E})$ around c_{2I} and substituting gives

$$\begin{aligned} \Psi & = u'(c_{2I}) c_{2E} - \lambda_{2E} c_{2E} \\ & = (u'(c_{2I}) - u'(c_{2E})) c_{2E} \end{aligned}$$

Taking an approximation for $u'(c_{2E})$ around c_{2I}

$$\begin{aligned} \Psi & = -u''(c_{2E}) (c_{2E} - c_{2I}) c_{2E} \\ & = \left[-\frac{u''(c_{2E})}{u'(c_{2E})} c_{2E} \right] (c_{2E} - c_{2I}) u'(c_{2E}) \end{aligned}$$

which gives

$$\Psi \approx \gamma \Delta c_{2E} u'(c_{2E}),$$

where γ is the coefficient of relative risk aversion which captures the degree of aversion to fluctuations in consumption.

II Appendix: Deriving Optimal Benefit

In the text we provide brief derivation of an expression to show the trade-off in setting the optimal benefit. This appendix provides the details of the derivation.

We assume there is no third stage in the model and so no retirement tax and no assets are held at the end of the second stage.

$$V_1 = \max_{c_t, A_1} \int_0^{T_1} u(c_t) dt + \pi V_2^d(A_1) + (1 - \pi) V_2^n(A_1)$$

Substituting in optimal values for consumption, investment and saving at each stage:

$$\begin{aligned} V_1 = & u\left(\frac{Y_1(1 - \tau^u) - A_1^*}{T_1}\right) T_1 \\ & + \pi \left(u\left(\frac{A_1^* + bY_1 I^* - A_{1+I}^*}{I^*}\right) I^* + u\left(\frac{A_{1+I}^* + E_2^d(I^*)(1 - \tau^u)}{T_2 - T_1 - I^*}\right) (T_2 - T_1 - I^*) \right) \\ & + (1 - \pi) u\left(\frac{A_1^* + Y_2^n(1 - \tau^u)}{T_2 - T_1}\right) (T_2 - T_1) \end{aligned}$$

We take the derivative with respect to the replacement rate, b , and substitute $\lambda_i = \partial u / \partial c_i$, where $i \in \{1, n, I, E\}$ correspond, respectively, to stage 1, the non-displaced stage, the investment sub-stage post job loss and the earnings sub-stage after returning to work. We use the envelope theorem to ignore indirect effects of changing benefits operating through optimised values of I, A_1, A_{1+I} . This gives equation (11) in the text.

$$\frac{\partial V_1}{\partial b} = -\lambda_1 Y_1 \frac{\partial \tau^u}{\partial b} + \pi \left(\lambda_I Y_1 I - \lambda_E E_2^d(I) \frac{\partial \tau^u}{\partial b} \right) - (1 - \pi) \frac{\partial \tau^u}{\partial b} \lambda_n Y_2^n$$

We set $\partial V_1 / \partial b = 0$, and rearrange

$$\pi \left(Y_1 I \lambda_I - E_2^d(I) \frac{\partial \tau^u}{\partial b} \lambda_E \right) = (1 - \pi) Y_2^n \frac{\partial \tau^u}{\partial b} \lambda_n + Y_1 \frac{\partial \tau^u}{\partial b} \lambda_1$$

We take first-order approximations of marginal utility around consumption in the state with no job loss, c_n ,

$$\lambda_i \approx \lambda_n - u''(c_n)(c_n - c_i)$$

$$\begin{aligned} & \pi \left(Y_1 I \lambda_n - Y_1 I u''(c_n)(c_n - c_I) - E_2^d(I) \frac{\partial \tau^u}{\partial b} (\lambda_n - u''(c_n)(c_n - c_E)) \right) \\ = & (1 - \pi) Y_2^n \frac{\partial \tau^u}{\partial b} \lambda_n + Y_1 \frac{\partial \tau^u}{\partial b} \lambda_n - Y_1 \frac{\partial \tau^u}{\partial b} u''(c_n)(c_n - c_1) \end{aligned}$$

Dividing through by λ_n

$$\begin{aligned} & \pi \left(Y_1 I - Y_1 I \frac{u''(c_n)}{u'(c_n)}(c_n - c_I) - E_2^d(I) \frac{\partial \tau^u}{\partial b} \left(1 - \frac{u''(c_n)}{u'(c_n)}(c_n - c_E) \right) \right) \\ = & (1 - \pi) Y_2^n \frac{\partial \tau^u}{\partial b} + Y_1 \frac{\partial \tau^u}{\partial b} - Y_1 \frac{\partial \tau^u}{\partial b} \frac{u''(c_n)}{u'(c_n)}(c_n - c_1) \end{aligned}$$

Replacing

$$\gamma = -\frac{u''(c_n)}{u'(c_n)} c_n$$

$$\begin{aligned} & \pi \left(Y_1 I - E_2^d(I) \frac{\partial \tau^u}{\partial b} + \gamma \frac{(c_n - c_I)}{c_n} Y_1 I - \gamma \frac{(c_n - c_E)}{c_n} \frac{\partial \tau^u}{\partial b} E_2^d(I) \right) \\ = & (1 - \pi) Y_2^n \frac{\partial \tau^u}{\partial b} + Y_1 \frac{\partial \tau^u}{\partial b} + Y_1 \frac{\partial \tau^u}{\partial b} \gamma \frac{(c_n - c_1)}{c_n} \end{aligned}$$

Adding and subtracting $\pi Y_1 I \gamma (c_n - c_E) / c_n$ from the left hand side,

$$\begin{aligned} & \pi \left(\begin{array}{l} Y_1 I - E_2^d(I) \frac{\partial \tau^u}{\partial b} + \gamma \frac{(c_n - c_I)}{c_n} Y_1 I - \gamma \frac{(c_n - c_E)}{c_n} \frac{\partial \tau^u}{\partial b} E_2^d(I) \\ + \gamma \frac{(c_n - c_E)}{c_n} Y_1 I - \gamma \frac{(c_n - c_E)}{c_n} Y_1 I \end{array} \right) \\ = & (1 - \pi) Y_2^n \frac{\partial \tau^u}{\partial b} + Y_1 \frac{\partial \tau^u}{\partial b} + Y_1 \frac{\partial \tau^u}{\partial b} \gamma \frac{(c_n - c_1)}{c_n} \end{aligned}$$

$$\begin{aligned} & \pi \left(\begin{array}{l} Y_1 I - E_2^d(I) \frac{\partial \tau^u}{\partial b} + \gamma \frac{(c_n - c_E)}{c_n} (Y_1 I - \frac{\partial \tau^u}{\partial b} E_2^d(I)) \\ - \gamma Y_1 I \left(\frac{(c_n - c_E)}{c_n} - \frac{(c_n - c_I)}{c_n} \right) \end{array} \right) \\ = & (1 - \pi) Y_2^n \frac{\partial \tau^u}{\partial b} + Y_1 \frac{\partial \tau^u}{\partial b} + Y_1 \frac{\partial \tau^u}{\partial b} \gamma \frac{(c_n - c_1)}{c_n} \end{aligned}$$

$$\begin{aligned}
& \pi \left(Y_1 I - E_2^d(I) \frac{\partial \tau^u}{\partial b} + \gamma \left[\frac{(c_n - c_E)}{c_n} \right] \left(Y_1 I - \frac{\partial \tau^u}{\partial b} E_2^d \right) + \gamma \left[\frac{(c_E - c_I)}{c_n} \right] Y_1 I \right) \\
= & (1 - \pi) Y_2^n \frac{\partial \tau^u}{\partial b} + Y_1 \frac{\partial \tau^u}{\partial b} + Y_1 \frac{\partial \tau^u}{\partial b} \gamma \frac{(c_n - c_1)}{c_n}
\end{aligned}$$

Finally, putting the marginal benefit terms on the left hand side and the marginal cost terms (through higher taxes) on the right hand side gives equation (12) in the text:

$$\begin{aligned}
& \pi Y_1 I \left(1 + \gamma \left[\frac{(c_n - c_E)}{c_n} \right] + \gamma \left[\frac{(c_E - c_I)}{c_n} \right] \right) \\
= & \frac{\partial \tau^u}{\partial b} \left\{ (1 - \pi) Y_2^n + Y_1 \left(1 + \gamma \frac{(c_n - c_1)}{c_n} \right) + \pi E_2^d(I) \left(1 + \gamma \left[\frac{(c_n - c_E)}{c_n} \right] \right) \right\}
\end{aligned}$$

III Appendix: Data and Sample

The empirical analysis in this paper is based on the 1995 Canadian Out of Employment Panel (COEP). The Canadian Out of Employment Panels are a series of surveys commissioned by Human Resources Development Canada for the purposes of evaluating legislative changes to the Canadian unemployment insurance system. The flows of job separations within certain time windows formed the sampling frames for these surveys. We focus on the 1995 edition of this survey because it posed the detailed questions on the ability and desire to borrow that we refer to in Section 2.

The respondents in the 1995 survey lost their jobs in the first half of 1995, and were interviewed twice, in the third and fifth quarters after job loss. Thus the respondents were first interviewed in the last quarter of 1995 and first quarter of 1996. Information was collected pertaining to their circumstances at the interview dates and retrospectively about their circumstances prior to the end of the relevant job, and over the intervening period. Information was collected about work, training, and job search, about household composition, consumption, income and finances, and about benefit receipt.

These data offer a number of advantages. First, the data reports on assets and debts, consumption, and borrowing constraints for the same households. Second, the COEP is unusual in collecting a measure of total consumption, not just food. Third, because it is a survey

specifically of job losers, the data contain a large sample of unemployed individuals. Fourth, because the COEP survey is designed around the job loss, the timing of information is ideally suited to our purposes. For example, there is information on assets and debts at exactly the time of job loss. With a regular panel survey such as the PSID or SIPP, we would have to use information collected at the last interview prior to the beginning of an unemployment spell, and with administrative data, such as that employed by Lentz (2009), information is typically annual. Finally, a number of other data sets suffer from ambiguities with respect to the time period to which information in the data pertains (see, for example, the discussion of the timing of the PSID consumption information in Dynarski and Gruber, 1997). The COEP data do not suffer from such ambiguities.

There are 7818 respondents to the 1995 COEP. The COEP samples job separations of various types, including quits, dismissals, separations due to illness, and temporary and permanent layoffs. In the selection of a sample for analysis, we discard 18 respondents who did not report a separation reason. We also discard 464 individuals who, although they lost a job, reported continuing employment in a second job. Next, we delete from the sample 665 respondents who reported that they quit to take another job. These individuals experienced little or no unemployment and are outside the scope of our interest. Finally we delete 1091 individuals age 25 or younger and 474 individuals over age 55, to focus on prime age workers.

Of the remaining 5015 observations, we focus on those 2922 who lived in a nuclear family (alone, with a spouse, or spouse and children) and were the primary earner in their household. Past experience with this data suggests that the quality of the survey responses on household finances is lower among respondents in other family types (for example, living with their parents or with unrelated adults.) The job loss of primary earners is of particular interest, and in focusing on primary earners, we are following much of the previous literature (for example, Dynarski and Gruber, 1997). Of these 2922 respondents, 1659 were employed at the time of the first interview. The other 1263 were not working at the time of interview, though some of these had spells of employment between the initial job loss and the interview.