

# SELF-INSURANCE IN A LIFE-CYCLE MODEL OF LABOR SUPPLY AND SAVINGS

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December 2004

## Abstract

This paper analyses the incentives to work and to save over the life-cycle in the presence of incomplete markets. In a calibrated, partial equilibrium model, flexibility in hours worked changes asset age-profiles: borrowing when young is greater and saving when middle-aged is greater than when labor supply is fixed. Uncertainty causes individuals to work longer hours and to consume less when young. With flexibility over hours, accumulating precautionary assets incurs less of a utility cost and so the level of saving is greater. Further, allowing for flexibility and uncertainty means simulated hours of work and consumption more closely match the age profiles in the data.

*Keywords:* precautionary saving, life-cycle labor supply

*JEL Classification:* D91, H31

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\*Particular thanks are due to Orazio Attanasio, Richard Blundell, Erzo Luttmer, Costas Meghir and John Rust. Thanks are also due to Kazu Araki, James Banks, Eric Battistin, Ian Crawford, Jayasri Dutta, Amanda Gosling, Ken Judd, Ellen McGrattan, Hashem Pesaran, Luigi Pistaferri, Ian Preston, Steve Satchell, Harald Uhlig and Joachim Winter.

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## 1 Introduction

This paper analyses the effect on precautionary saving and consumption of allowing individuals to vary their labor supply. There is now an extensive literature discussing the way that individuals self-insure against idiosyncratic uncertainty when markets are incomplete. In particular, Deaton (1991) and others show the impact that uncertainty has on consumption paths and on saving assuming labor supply is fixed. This paper shows how allowing for flexibility over labor supply affects these results: first, flexibility allows individuals to work harder before shocks are realized, and thus have more income available to self-insure; second, flexibility allows individuals to react to shocks to wages by changing hours of work, and thus reducing the cost of uncertainty. The main aim of the paper is to show how these effects change the standard results on the extent of precautionary saving and consumption smoothing. A second aim is to show that allowing for precautionary motives in labor supply can give simulated profiles that mimic the data.

Attanasio and Weber (1995) and Blundell, Browning and Meghir (1994) have shown that ignoring labor supply leads to inconsistent estimates both of the elasticity of intertemporal substitution in consumption and of the effects of uncertainty on consumption. Further, Browning and Meghir (1991) have found non-separabilities between consumption and leisure. Even without non-separabilities, flexibility in labor supply can change the amount of income available for smoothing. These results suggest that analyzing consumption behavior ignoring labor supply gives misleading results: choices about labor supply affect the amount of consumption smoothing and saving. The results in this paper show the extent to which the results are misleading and explicitly shows the effect on consumption and saving. Further, evidence for the intertemporal substitution model of labor supply is mixed: labor supply tends to be high early in life when wages are low; but low later in life when wages are high and this is hard to reconcile with intertempo-

ral substitution unless individuals are very patient. The paper shows that uncertainty over wages can explain why individuals work long hours when young despite low wages.

The paper simulates a model of intertemporal choice under uncertainty when individuals choose hours of work and consumption in each period. We calibrate the model to average hours worked over the life-cycle and to median asset holdings. First, in the absence of uncertainty, we show the extent that hours of work will track wages over the life-cycle (Heckman, 1974). This path for hours of work leads to asset life-cycle profiles that differ according to whether labor supply is flexible or not: with flexibility, young individuals borrow more, and the middle-aged save more, than when labor supply is constant. Second, introducing uncertainty holding preference parameters constant, induces individuals to work longer hours and consume less when young relative to the certainty case. Working more when young leads to saving which can act as a buffer against future wage shocks. When individuals are older, and uncertainty is resolved, the accumulated assets allow individuals to reduce labor supply, despite higher wages. The third result is to show the effect of a flexible labor supply (relative to a fixed and constant labor supply) on age profiles when wages are uncertain. Flexibility again generates greater borrowing among the young and greater accumulation in middle-age. The level of asset holdings for a given discount rate also differs: when labor supply is flexible, individuals have greater asset holdings. This arises because flexibility reduces the cost of accumulating assets before the uncertainty is resolved, leading to greater saving. Offsetting this, the benefit of saving is lower because of the ability to react to wage shocks ex-post, but this effect is dominated. Flexibility also generates a more concave path for log consumption.

The paper also carries out sensitivity analysis changing the structure of preferences, changing the intertemporal elasticity of substitution and chang-

ing the elasticity of substitution between consumption and leisure. When consumption and leisure are closer substitutes, paths of consumption and hours of work are more hump shaped. Greater substitutability weakens the precautionary motive.

The underlying idea of this paper is that models of precautionary behavior which ignore labor supply miss a potentially important dimension of behavior. By precautionary behavior we mean the behavioral response to uncertainty: in Gourinchas and Parker (2002) precautionary behavior is interpreted as the effect on expected consumption growth, and in Cagetti (2002) precautionary behavior is interpreted as the effect on wealth accumulation. Since these papers match the consumption profiles or wealth profiles to the data, they will in this sense match observed precautionary behavior, but will misinterpret the structural factors causing this behavior. French (2002) uses a model of labor supply related to the current paper to estimate preference parameters allowing for joint saving-labor supply choices. Implicit in his model is a broad definition of precautionary behavior including labor supply and consumption responses but the focus of that paper is on matching retirement behavior rather than the effects of uncertainty per se.

Section 2 provides data on life-cycle profiles of consumption, labor supply, wages and asset holdings; section 3 outlines the general formulation of the model, and shows approximations to the solution. Section 4 provides simulated life-cycle profiles of hours of work, consumption and saving, showing in turn, the effect of flexibility under certainty, the effect of uncertainty, and the effect of flexibility under uncertainty. Section 5 shows how the profiles change with alternative preferences. Section 6 concludes.

## **2 Data on Life-cycle Profiles**

This section provides data on the life-cycle patterns that this paper is addressing. Figure 1 shows life-cycle profiles of log wages for cohorts of men

and women from the Consumer Expenditure Survey:<sup>1</sup> wages increase in the early stages of life, and then wage growth flattens off as individuals get older. With wages growing, labor supply should be low at the start of life and then increase with age as wages increase. However, figure 1 also shows the path of labor supply over the life-cycle and shows that the predictions of simple intertemporal substitution do not hold: instead, hours of work by men increase sharply with age at the start of life, peak around 40, and then decline towards age 65. The relatively flat profile of hours across a region where wages are rising, and the decline in hours worked when wages are flat has been interpreted as evidence against intertemporal labor supply movements.

Figure 2 shows life-cycle profiles of log income and log consumption. The profiles show the well-known hump shape of consumption, and the apparent tracking of consumption and income across the life-cycle. The fact that this tracking may be due to non-separabilities between labor supply and consumption was first discussed by Heckman (1974) in a model with no uncertainty. If consumption and hours of work are complements and hours of work track wages, consumption would have a similar profile. The apparent lack of intertemporal substitution in labor supply suggests the Heckman (1974) argument has less relevance. Instead, Carroll and Summers (1991) have argued that the co-movement of income and consumption is due to precautionary motives, but in that framework, it is hard to explain why consumption will be hump-shaped and, further, they maintain the counterfactual assumption that labor supply is fixed. The aim of the current paper is to combine these two insights to show that uncertainty about future wages can make individuals work more early in their lives, and defer utility and leisure until older. Thus, uncertainty operates in the opposite direction

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<sup>1</sup>The wages and labor supply for women are included here for completeness. There are two problems with using female data: first, it is clear from the graphs in figure 1 that there are large cohort effects; and second, the participation choice is subsumed by average hours and this is particularly problematic for women (Attanasio et al., 2004).

to intertemporal substitution and can explain the observed profile of labor supply. Further, allowing for uncertainty and flexibility of labor supply can lead to consumption profiles being hump shaped and the tracking of income by consumption observed in figure 2. French (2002) finds a similar result for the profile of life-cycle labor supply by matching simulated profiles to PSID. In particular, he finds estimates of the intertemporal elasticity of substitution that are higher than implied by a model under certainty.

Figure 3 reports asset holdings over the life-cycle, taken from the PSID. Mean asset holdings increase to retirement and then decline somewhat as individuals age. The decline is not steep and it is not present when looking at median asset holdings. This is a well-known empirical finding (Bernheim, 1987), but the focus of the current paper is rather on asset accumulation during working life. Figure 3 does highlight two modelling issues in comparing our simulations to the data: first, a period of retirement is necessary to avoid individuals in the simulations counter-factually running down their assets at the end of their working life. This requires an assumption about available pension wealth. Second, asset holdings are a composite good, partially composed of assets that yield consumption benefit, such as housing, and further, different types of asset have different degrees of liquidity. In our simulations below, we ignore these issues by assuming a single liquid asset is available and thus focus on total asset holdings rather than the composition.

### 3 Life-Cycle Model

Individuals maximize expected utility by choosing consumption and leisure for each period of life. In comparing to the data, we take the choice of labor supply to be male labor supply.<sup>2</sup> Wages are uncertain and uninsurable,

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<sup>2</sup>Male hours for couples are very similar in the data to hours worked by single men. For women, single versus couples makes a significant difference. We assume that women do not work and that this is a unitary model of household choice. Introducing the possibility

but the interest rate is constant. Utility is additively separable across time periods, but nonseparable within period.

$$\max_{c_s, l_s} E_t \left[ \sum_{s=t}^T \beta^{s-t} u(c_s, l_s) \right] \quad (1)$$

subject to,

$$A_{s+1} = (1+r) [A_s + (H - l_s) w_s - c_s] \quad (2)$$

and a terminal condition,  $A_{T+1} = 0$ . Further, leisure in each period is constrained to be feasible:

$$H \geq l_s \geq 0 \quad (3)$$

In each period, total time,  $H$ , is spent on leisure,  $l_t$ , or work;  $c_t$  is consumption,  $A_t$  is the amount of assets held at the start of period  $t$ ;  $w_t$  is the real wage,  $\beta$  is the discount factor,  $r$  the real rate of interest. The life-cycle is divided into an exogenous retirement period of duration  $R$  where  $l_t = H$  and a working-life of duration  $T - R$  where hours of leisure is a choice variable. The terminal condition implies that individuals will not borrow more than the discounted sum of the minimum income they will receive in each remaining period. If there is a positive probability of receiving zero income in each period, individuals will never borrow. Individuals are not allowed to borrow against pension income.

The real wage is assumed to have both transitory and permanent shocks which are log-normally distributed and truncated below.

$$\ln w_t = \ln w_0 + \alpha_1 t + \alpha_2 t^2 + \ln w_t^P + \nu_t, \quad \nu_t \sim N \left( -\frac{\sigma_\nu^2}{2}, \sigma_\nu^2 \right) \quad (4)$$

$$\ln w_t^P = \ln w_{t-1}^P + \varepsilon_t, \quad \varepsilon_t \sim N \left( -\frac{\sigma_\varepsilon^2}{2}, \sigma_\varepsilon^2 \right) \quad (5)$$

The log wage at age  $t$  follows a deterministic trend plus the log of the permanent wage plus a plus a transitory shock,  $\nu_t$ . The transitory shock is of female earnings requires assumptions on fixed costs, participation decisions, the return to experience.

assumed to be *i.i.d.* The variances of both the transitory and permanent shocks are assumed to be constant. The log of the permanent wage follows a random walk with permanent shock,  $\varepsilon_t$ . This relatively simple wage process can capture much of the observed wage profiles, particularly if the parameters are conditional on education (MaCurdy, 1981). The wage is bounded below so that the individual will always receive a job offer with a non-zero wage.<sup>3</sup> Finally, the wage process is not conditional on participation, thus the model abstracts from on-the-job human capital accumulation and the wage rate grows with age whether or not individuals participate.

The remaining assumptions are that there is no involuntary unemployment and no fixed costs of work. All periods of non-participation outside the exogenous retirement period are chosen in response to the offered wage. There is only one asset and so we assume explicit saving for retirement is liquid. However, individuals also receive pension income when retired. This is determined by a fraction of their permanent wage on retirement.

The finite horizon makes the value function inherently non-stationary. The first order conditions of the value function are

$$\frac{\partial u(c_t, l_t)}{\partial c_t} = \beta (1 + r) E_t \left[ \frac{\partial u(c_{t+1}, l_{t+1})}{\partial c_{t+1}} \right] \quad (6)$$

$$\begin{aligned} \frac{\partial u(c_t, l_t)}{\partial l_t} &\geq w_t \frac{\partial u(c_t, l_t)}{\partial c_t} \\ l_t &\geq 0 \end{aligned} \quad (7)$$

Since the value function is concave, the solution to these first-order conditions is a maximum. The leisure choice is incorporated by exploiting two-stage budgeting: hours of leisure can be written as a function of consumption through equation (7) and so the solution for how many hours to

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<sup>3</sup>If the distribution were truly log-normal there would be a very small probability of receiving a zero wage offer meaning individuals would never borrow. Since a period in the model is calibrated to be a year, it seems plausible to set a lower bound on the wage so that individuals always receive a non-zero offer.

work is reduced to an intratemporal problem.<sup>4</sup> This means that at a given point in the state space, a guess at optimal consumption in  $t$  pins down optimal leisure without solving a further Euler equation.

### 3.1 Utility Functions

Before analyzing the model, it is useful first to place restrictions on the form of the utility function and to consider the implications of these restrictions.

We make the standard assumptions  $u_c > 0$  and  $u_l > 0$  and that  $u$  is concave ( $u_{cc} < 0$  and  $u_{cc}u_{ll} - u_{cl}^2 > 0$ ). The sign of the cross-partial derivative  $u_{cl}$  determines whether consumption and leisure are complements ( $u_{cl} > 0$ ) or substitutes ( $u_{cl} < 0$ ). If, for example, consumption and leisure are substitutes then the marginal utility of consumption is higher when working and this gives an incentive to move consumption to periods where hours of work are higher. The absolute size of  $u_{cl}$  is bounded by concavity of  $u$  :

$$(u_{cl})^2 < u_{cc}u_{ll}$$

If we further assume that<sup>5</sup>

$$u_{cc} \rightarrow 0 \text{ as } c \rightarrow \infty \text{ (and } u_c \rightarrow 0), \quad (8)$$

then by concavity of  $u$ ,

$$u_{cl} \rightarrow 0 \text{ as } c \rightarrow \infty, \quad (9)$$

and so the effect of increased leisure on the marginal utility of consumption decreases as consumption becomes large.

This means that if  $u_{cl} < 0$ , then  $u_{cl}$  must be increasing in  $c$  (and by analogy  $l$ ), or if  $u_{cl} > 0$ , then  $u_{cl}$  must be decreasing in  $c$  (and  $l$ ). In other

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<sup>4</sup>This would not be possible if there were fixed costs of work, returns to experience or intertemporal non-separabilities in the utility function.

<sup>5</sup>This is a natural assumption to make if marginal utility of consumption is always positive.

words,

$$\left. \begin{array}{l} u_{clc} > 0 \\ u_{ccl} > 0 \end{array} \right\} \text{if } u_{cl} < 0 \qquad \qquad \qquad \left. \begin{array}{l} u_{clc} < 0 \\ u_{ccl} < 0 \end{array} \right\} \text{if } u_{cl} > 0$$

This in turn implies that the marginal utility of consumption is only convex if  $u_{cl} < 0$ : convexity requires that  $u_{ccc} > 0$  and  $u_{ccc} \cdot u_{ccl} - u_{ccl}^2 > 0$  and this in turn requires that  $u_{ccl} > 0$ .

There are two points to stress here: first, specifying whether leisure and consumption are substitutes or complements allows us to sign all third order terms in the utility function, given our assumption of concavity of  $u$  and assumption (8). Second, the convexity of marginal utility and the strength of the precautionary motive is affected by whether leisure and consumption are substitutes.<sup>6</sup>

We have interpreted the cross derivative of within period utility  $u_{cl}$  as reflecting whether consumption and leisure are substitutes or complements. In a dynamic model, the extent of substitution is determined by the size of the Frisch substitution effect,  $\left. \frac{\partial c_t}{\partial w_t} \right|_{\lambda_t}$  where  $\lambda_t$  is the marginal utility of income in  $t$ . This is interpreted as the effect on the path of consumption of wages increasing across time. For a general utility function, assuming an interior solution for leisure,

$$\left. \frac{\partial c_t}{\partial w_t} \right|_{\lambda_t} = -\frac{1}{u_{cc}u_{ll} - u_{cl}^2} u_{cl} \lambda_t \qquad \left\{ \begin{array}{l} < 0 \text{ if } u_{cl} > 0 \\ > 0 \text{ if } u_{cl} < 0 \end{array} \right. \quad (10)$$

where the denominator is positive because  $u$  is concave.<sup>7</sup> The sign of the Frisch substitution effect depends on the sign of cross-derivative,  $u_{cl}$ .

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<sup>6</sup>Huggett and Ospina (2001) show that it is not necessary to have convexity of marginal utility to generate precautionary behaviour, and that liquidity constraints are sufficient.

<sup>7</sup>This term for Frisch substitutability can be derived as follows: the first order condition (7) can be decomposed into two equations:  $u_l(c_t, l_t) = \lambda_t w_t (1+r)$  and  $u_c(c_t, l_t) = \lambda_t (1+r)$ , where  $\lambda_t$  is the Lagrange multiplier on the intertemporal budget constraint

### 3.2 Approximations

In this section, we show approximations to the Euler equation (6). In the simulations in the next section, we solve for the non-linear consumption function numerically. However, some of the insight of the effects of uncertainty on leisure and consumption can be obtained by looking at the second-order expansion of equation (6) around  $c_t$  and  $l_t$ .

We expand  $\frac{\partial u(c_{t+1}, l_{t+1})}{\partial c_{t+1}}$  around  $c_t$  and  $l_t$ , ignoring cross-terms, and then take expectations and replace in equation (6) to give<sup>8</sup>

$$E_t \Delta c_{t+1} \approx -\frac{u_c}{u_{cc}}(r - \delta) + \left(-\frac{u_{cl}}{u_{cc}}\right) E_t \Delta l_{t+1} \quad (11)$$

$$+ \frac{1}{2} \left(-\frac{u_{ccc}}{u_{cc}}\right) E_t [(\Delta c_{t+1})^2] + \frac{1}{2} \left(-\frac{u_{ccl}}{u_{cc}}\right) E_t [(\Delta l_{t+1})^2]$$

where  $\delta = \frac{1}{\beta} - 1$ .

The first two terms on the right hand side of equation (11) are the deterministic components of consumption growth: consumption increases with age if  $r > \delta$  and further, because labour supply is flexible, when leisure is growing, consumption growth will be slower if consumption and hours of leisure are substitutes,  $u_{cl} < 0$  (Heckman, 1974).

(2). Taking the total derivative of these two equations and putting into matrix form, gives

$$\begin{pmatrix} u_{cc} & u_{cl} \\ u_{cl} & u_{ll} \end{pmatrix} \begin{pmatrix} \frac{\partial c_t}{\partial w_t} \\ \frac{\partial l_t}{\partial w_t} \end{pmatrix} = \begin{pmatrix} 0 \\ \lambda_t \end{pmatrix}$$

Inverting the first term on the left hand side and rearranging gives the expression for Frisch substitutability in the text.

<sup>8</sup>In general, we approximate  $f(x, y)$  around  $(\bar{x}, \bar{y})$  by

$$f(x, y) \approx f(\bar{x}, \bar{y}) + \frac{\partial f(\bar{x}, \bar{y})}{\partial x}(x - \bar{x}) + \frac{\partial f(\bar{x}, \bar{y})}{\partial y}(y - \bar{y})$$

$$+ \frac{\partial^2 f(\bar{x}, \bar{y})}{\partial x \partial y}(x - \bar{x})(y - \bar{y}) + \frac{1}{2} \frac{\partial^2 f(\bar{x}, \bar{y})}{\partial^2 x}(x - \bar{x})^2 + \frac{1}{2} \frac{\partial^2 f(\bar{x}, \bar{y})}{\partial^2 y}(y - \bar{y})^2$$

In the text, we ignore the terms capturing the correlation between  $x$  and  $y$ .

The last two terms on the right hand side show the effects of uncertainty, where  $E_t [(\Delta c_{t+1})^2]$  and  $E_t [(\Delta l_{t+1})^2]$  measure the expected variability in consumption and leisure. Greater uncertainty in consumption leads to greater consumption growth, by the precautionary motive. Further, if leisure is uncertain and  $u_{cl} < 0$ , then consumption growth will be greater because of the additional precautionary motive for deferring consumption given by  $u_{cll} > 0$ . If leisure and consumption are complements, then growth in leisure increases consumption growth whereas greater variability in leisure reduces consumption growth. If consumption and leisure are additively separable the second and fourth terms are zero and the equation collapses to the consumption growth equation with a fixed labor supply. However, since the variance of consumption growth may be affected by labor supply choices even in the additively separable case, we cannot infer from the first-order conditions alone whether consumption growth will be greater with a fixed or flexible labor supply.

Similarly to equation (11), we can derive an expression for the growth in hours of work by first substituting equation (7) into equation (6), assuming an interior solution for leisure, to give

$$\frac{\partial u(c_t, l_t)}{\partial l_t} \frac{1}{w_t} = \beta (1+r) E_t \left[ \frac{\partial u(c_{t+1}, l_{t+1})}{\partial l_{t+1}} \frac{1}{w_{t+1}} \right] \quad (12)$$

Approximating  $\frac{\partial u(c_{t+1}, l_{t+1})}{\partial l_{t+1}} \frac{1}{w_{t+1}}$  around  $c_t, l_t$  and  $w_t$  and ignoring the cross-terms gives

$$\begin{aligned} \frac{\partial u(c_{t+1}, l_{t+1})}{\partial l_{t+1}} \frac{1}{w_{t+1}} &\approx u_l \frac{1}{w_t} + u_{ll} \frac{1}{w_t} (\Delta l_{t+1}) - u_l \frac{1}{w_t} (\Delta \ln w_{t+1}) \\ &+ u_{lc} \frac{1}{w_t} (\Delta c_{t+1}) \\ &+ u_{lll} \frac{1}{w_t} (\Delta l_{t+1})^2 + u_{ll} \frac{1}{w_t} (\Delta \ln w_{t+1})^2 + u_{lcc} \frac{1}{w_t} (\Delta c_{t+1})^2 \end{aligned}$$

Multiplying this expression through by  $w_t/u_{ll}$ , taking expectations, and substituting back into equation (12),

$$\begin{aligned}
E_t \Delta l_{t+1} \approx & -\frac{u_l}{u_{ll}}(r - \delta) - \left(-\frac{u_l}{u_{ll}}\right) E_t \Delta \ln w_{t+1} + \left(-\frac{u_{cl}}{u_{ll}}\right) E_t \Delta c_{t+1} \\
& + \frac{1}{2} \left(-\frac{u_{lll}}{u_{ll}}\right) E_t [(\Delta l_{t+1})^2] + \left(-\frac{u_l}{u_{ll}}\right) E_t [(\Delta \ln w_{t+1})^2] \\
& + \frac{1}{2} \left(-\frac{u_{lcc}}{u_{ll}}\right) E_t [(\Delta c_{t+1})^2].
\end{aligned} \tag{13}$$

The first three terms show that leisure will increase with age if  $r > \delta$ , if wages are expected to fall, or if consumption is growing and consumption and leisure are substitutes.

The last three terms show the effect of uncertainty. There is the direct precautionary effect of increased variability in leisure leading to leisure being deferred and of increased wage uncertainty leading to leisure being deferred. There is also an indirect effect arising when utility is non-separable: if consumption and leisure are substitutes, growth in leisure will be faster if consumption growth is uncertain. When there is additive separability between consumption and leisure, there is still a precautionary effect leading to a delay in taking leisure because of the direct effect of the variance in the wage rate, and the effect of leisure itself being uncertain.

In summary, the approximations suggest first, that precautionary motives lead individuals to work hard and to give up consumption today, and to work less and consume more in the future; and second, the greater the extent to which consumption and leisure are substitutes, the less consumption is reduced when young, and hence the slower consumption growth. In other words, close substitutability between consumption and leisure in the utility function lessens the precautionary motive.

There are two main problems with these conclusions: first, even if consumption and leisure are substitutes so consumption is greater when young,

we cannot infer what has happened to saving. On the one hand, the substitutability means less consumption is being given up when young, implying less saving, but on the other hand, labor supply is greater when young, implying greater saving for any given consumption level. Second, the variances of consumption and of leisure are clearly endogenous and will themselves depend upon preferences and upon the degree of labor supply flexibility. It is necessary to simulate a calibrated model in order to draw implications for actual behavior. Further, equation (11) is intended to approximate the relationship between consumption and hours of work for an optimizing individual, rather than to approximate the optimal policy rule as a function of the state space. Finally, simulation allows us to consider corner solutions in the choice of hours of work.

## 4 Simulated Effects of Flexible Labor Supply

This section uses numerical solution of the (nonlinear) policy functions to show the extent to which individuals insure themselves through increasing their labor supply when young and through deferring consumption. We begin by specifying the functional forms, parameter values and calibration statistics used. We then present the results in three subsections: first, we show the effect of flexible labor supply compared to a fixed labor supply if wages are certain. Second, we introduce wage uncertainty and show how uncertainty affects behavior. Finally, we show the effect of flexible labor supply compared to a fixed labor supply if wages are uncertain. This division enables us to see separately the effect of flexibility, the effect of uncertainty and then the combined effect.

**Parameters and Calibration** The utility function is assumed to have the isoelastic-Cobb-Douglas form

$$u(c_t, l_t) = \frac{(c_t^\eta l_t^{1-\eta})^{1-\gamma}}{1-\gamma} \quad (14)$$

Using this utility function, it is possible to write the solution for leisure using the intraperiod allocation rule (7)

$$l_t(c_t, w_t) = \min \left[ \frac{c_t (1-\eta)}{w_t \eta}, H \right] \quad (15)$$

and this can be substituted into the Euler equation (6) to give one functional equation in one unknown function,  $c_t(A_t, w_t, w_t^P)$ , for each age  $t$ .

$$\begin{aligned} & c_t(A_t, w_t, w_t^P)^{\eta(1-\gamma)-1} l_t(c_t, w_t)^{(1-\eta)(1-\gamma)} \\ &= \beta(1+r) \int c_{t+1}^{\eta(1-\gamma)-1} l_{t+1}^{(1-\eta)(1-\gamma)} dF(w_{t+1}, w_{t+1}^P | w_t^P) \end{aligned} \quad (16)$$

Once we have substituted in for optimal leisure using (15), the solution method for solving this nonlinear equation follows Deaton (1991). We solve for the consumption function at each  $t$  recursively from the final period by solving the Euler equation (16) at a discrete number of points in the state space. The state space comprises assets, the permanent wage and the actual wage. We transform this into a two state problem by dividing through by the permanent wage. We use linear and cubic splines to interpolate the solution between nodes. Further details of the solution method are discussed in the appendix.

A period in the model is calibrated to be a year in the data. The real interest rate is constant at 1.6%, corresponding to the average annualized real rate on US 3-month treasury bills.<sup>9</sup> The deterministic part of the wage process is estimated from regressing observed log wage growth for men on age, age squared, education and a set of cohort dummies in the CEX. This assumes that the age terms do not interact with education. Log wages

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<sup>9</sup>Estimated from the series from 1960-2000, deflating using CPI.

are concave in age, with log wages decreasing with age after a certain age. The downturn from the maximum at around age 50 to retirement implies wages fall by about 15%. The values for the variance of the permanent and transitory shocks to log earnings are taken from Meghir and Pistaferri (2004) who control for measurement error.<sup>10</sup>

We choose the discount rate,  $\delta$ , and the value of leisure in utility,  $\eta$ , by calibration. We do not match the age profiles, rather we chose parameters to match lifetime averages. The discount rate is chosen so that the ratio of median wealth holdings of those of working age to median income matches the PSID.<sup>11</sup> The value of  $\eta$  is set to give average hours of work of 2080 a year, assuming individuals have 100 hours per week to allocate to work and leisure. This value is chosen to match average male hours of work in the CEX. The baseline value for the coefficient of relative risk aversion,  $\gamma$ , is taken from the estimates of Attanasio and Weber (1995), who condition on labor supply. Attanasio and Weber estimate the conditional elasticity,  $-\frac{u_c}{u_{cc}c}$ , to equal 0.67. Using a value of  $\eta = 0.4$ , this translates into  $\gamma = 2.2$ .<sup>12</sup> The calibrated discount rates and parameter details are given in Table 1.

The specification of the utility function (14) implies

$$u_{cl}(c, l) = (1 - \gamma)\eta(1 - \eta)c^{\eta(1-\gamma)-1}l^{(1-\eta)(1-\gamma)-1}, \quad (17)$$

which is negative if  $\gamma > 1$ , indicating within period consumption and leisure are Frisch substitutes. When  $\gamma < 1$ , consumption and leisure are Frisch

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<sup>10</sup>The process for income in our model is simpler than the one used in Meghir and Pistaferri because we have no MA component to transitory shocks. Further, there is no consideration in the simulations of the ARCH structure that Meghir and Pistaferri estimate.

<sup>11</sup>We use median rather than mean wealth to avoid trying to match the skewness of the wealth distribution (as in Castaneda et al., 2003). We use wealth of the working population to avoid trying to match the slow decline in wealth among the retired.

<sup>12</sup>It would clearly be preferable to use estimates of the unconditional elasticities, giving joint estimates of  $\eta$  and  $\nu$ , but these are hard to obtain because of the participation decision.

complements. In our baseline,  $\gamma = 2.2$  and so consumption and leisure are substitutes in the utility function.

#### 4.1 The Effect of Flexible Labor Supply under Certainty

Figure 4 compares life-cycle profiles when labor supply can vary over time with life-cycle profiles when labor supply is fixed and constant. The figure shows profiles of log consumption, log wages, hours worked and asset holdings under the assumption that there is no uncertainty over wages.

Flexibility means hours of work will be (relatively) low when the wage is low and high when the wage is high. This generates the tracking between hours of work and wages shown in Figure 4(b). The extent of this tracking between hours of work and the wage rate depends on the degree of intertemporal substitution and also the discount rate. Greater intertemporal substitution increases the willingness to exploit wage variation and increases tracking. Greater discounting of the future reduces hours of work early in the life-cycle. The hump shape in hours of work generates the hump shape in consumption shown in Figure 4(a) because consumption and leisure are substitutes in our baseline: in periods when hours of work are high (and leisure is low), the marginal utility of consumption is high and this induces extra consumption in those periods. This is the effect first discussed in Heckman (1974). Further, as shown in the approximations above, consumption growth is faster when labour supply is flexible if hours of work are growing, but consumption growth is slower if hours of work are declining.

There are two main points about the asset profiles: first, the shape of the age profiles differ: with flexibility, borrowing is higher when young, and then saving is higher pre-retirement than when hours are fixed. This difference in shape arises from a *life-cycle smoothing* effect: wealth is being redistributed to periods where hours of work and income are low. When labor supply is constant, income is also smoother over the life-cycle and so there is less desire

for borrowing and saving to smooth consumption. Second, the discount rate reported in Table 1 that is needed to match median asset holdings differs across the two scenarios. The discount rate is greater when labor supply is flexible implying that assets would be higher if the discount rates were identical. This arises because flexibility reduces the cost of accumulating a given amount of assets. The discount rates needed to match the level of assets in this scenario are very low (0.009 when  $r = 0.016$ ), making individuals very patient.

## 4.2 The Effect of Wage Rate Uncertainty

Figures 5 and 6 show the effect of uncertainty over wages on life-cycle profiles when labor supply is flexible. Within each figure, the preference parameters and wage profiles are held constant. The two figures differ as to which model is used to calibrate the preference parameters. In particular, Figure 5 shows the effect of introducing uncertainty to a model calibrated to give median assets under certainty; while Figure 6 shows the effect of removing uncertainty from a model calibrated to give median assets with uncertainty. Both figures show profiles of log consumption, hours worked and asset holdings.

The message from panel (b) in each figure is similar: uncertainty induces individuals to work harder when young and less hard when old than if wages had been certain. The desire for intertemporal substitution and impatience is being offset by precautionary behavior. The greater hours worked when young are at an age of low wages when intertemporal substitution would generate low hours worked. The relatively flat profile of hours of work observed in the data and mimicked by the profile under uncertainty in Figure 6(b) may reflect strong intertemporal substitution (captured by the certainty line) being offset by precautionary motives when uncertainty is controlled for. This is similar to the result in Figure 4. The difference between the two figures is in the level of the discount rate: introducing uncertainty has a

similar effect to making individuals more patient, and so a higher discount rate is needed to keep median assets constant.

Similarly panel (a) in Figures 5 and 6 show consumption is lower when young and greater when old than under certainty. Greater hours worked and lower consumption when young mean asset accumulation is greater when we introduce uncertainty.

When we show the effect of removing uncertainty in Figure 6, the difference in behavior partly reflects a difference in borrowing opportunities: individuals are able to borrow up to the amount they can repay with certainty. As discussed in Huggett and Ospina (2001), uncertainty changes households possibilities for borrowing by reducing worst-case income. This reduced borrowing opportunity means greater leisure when young would require a fall in consumption and so it is not clear whether the increased hours of work when young are due to the altered opportunity set or a precautionary motive.

The simulated pattern of hours of work is similar to that shown in the data in Figure 1: hours of work increase at the start of working life but the hours profile is much flatter than the wages profile, until later in life when hours decline more sharply than wages.<sup>13</sup> The consumption age-profiles in Figure 6 show the same hump shape that is seen in the data, Figure 2. Asset age-profiles during working life in the data, Figure 3, are flatter than in the simulations, but have a broadly similar shape with the flattening out of accumulation before retirement.

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<sup>13</sup>In the data, part of the decline in labor supply may reflect early retirement rather than reduced hours: this may reflect a greater discreteness in the available choice of hours than assumed in the simulations, where participation in the baseline is always over 95%. However, the implication is the same and individuals have reduced labor supply despite relatively high wages.

### 4.3 The Effect of Flexible Labor Supply under Uncertainty

We now turn to the effect that flexibility over labor supply has compared to a fixed labor supply when wages are uncertain. Figure 7 compares life-cycle profiles when labor supply is flexible with life-cycle profiles when labor supply is fixed and constant. The figure shows profiles of log consumption, hours worked and asset holdings under the assumption that wages are uncertain. In making this comparison, the discount rate in each scenario is chosen so that median asset holdings match the data. Other preference parameters and the wage process are held constant across the two scenarios.

The consumption profile when labor supply is flexible is markedly more concave than when fixed. Consumption growth when young is greater when flexible and consumption growth when old is less when flexible. This is due to the age-profile of leisure which is captured by the  $\Delta l_{t+1}$  term in equation (11): when young, leisure is falling and this increases consumption growth; when old, leisure is increasing and this decreases consumption growth.

There are two main points about the asset profiles in Figure 7: first, the discount rate that is needed to match median asset holdings (reported in Table 1) differs across the two scenarios with a higher discount rate when labor supply is flexible implying that saving would be higher if the discount rates were identical. Flexibility has two effects. First, flexibility means individuals can accumulate assets through working longer hours rather than just through lower consumption and this reduces the cost of accumulating a given amount of assets. Therefore, since individuals work harder before shocks are realized, they have more income available to self-insure (a *pre-emptive* effect). Second, flexibility means individuals can adjust *ex-post* to a bad wage shock. This ability to exploit wage variation means individuals work harder when wages are higher than expected and work less when lower than expected (or vice-versa if the income effect dominates the substitution effect) and this reduces the cost of uncertainty and the value of holding pre-

cautionary assets. The net effect of flexibility on the discount rate and hence the level of saving under uncertainty is ambiguous, but in these calibrated simulations the preemptive effect dominates

The second point is that the shape of the age profiles differ: with flexibility, saving is lower when young, and then saving is higher pre-retirement than when hours are fixed. This difference in shape partly reflects the *life-cycle smoothing* effect discussed above: wealth is being redistributed to periods where hours of work and income are low. The difference is partly because when labor supply is flexible, individuals can adjust hours of work in response to a bad shock, to pay back debt. Hence for a given level of life-cycle asset holdings, it is less costly for the individual to delay accumulating assets (or even to borrow) when young and then to accumulate more when middle-aged.

Part of the difference observed in the life-cycle path of assets is due to the different path of hours of work. This variation in hours of work generates the life-cycle smoothing motive for saving. To understand how much of the difference in the shape of asset profiles between the fixed and flexible scenarios is due to this motive, we simulate the fixed labor supply economy holding labor supply at each age fixed at average hours worked at that age when labor supply is flexible. We compare, in Figure 8, the resulting asset accumulation profile with the fixed and flexible labor supply cases just discussed. Realized income will still differ between scenarios because when labor supply is flexible for individuals of a given age, those with high wages can work more hours than those with low wages, rather than everyone working average hours.<sup>14</sup>

The age profiles for assets show that part of the difference between the “fixed” and “flexible” cases is due to the life-cycle smoothing effect: this

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<sup>14</sup>If we performed the same exercise for the economy without uncertainty, then the “fixed at flexible” and “flexible” scenarios would like identical.

effect is removed by fixing labor supply at age-specific average hours worked (shown in the “fixed at flexible” line), and the difference in age profile between this and the fully flexible case is then small. Some difference in age profiles still remains with the truly flexible case because the ability to vary labor supply away from the age-specific average means wage variation can be exploited, leading to more debt when young and then greater asset accumulation pre-retirement.

The calibrated discount rates show that when labor supply is fixed at age-specific average hours worked, the discount rate is greater than when labor supply is constant over the life-cycle, but less than when labor supply is flexible. This highlights that part of the difference between the fixed (and constant) hours and flexible hours cases arises due to the fall in hours when old, generating a greater need for asset accumulation to smooth consumption.

## 5 Alternative Preference Specifications

The aim of this section is to show that the results of section 4 are robust to alternative specifications of preferences. We first consider assuming consumption and leisure are additively separable; second, we consider alternative values of risk aversion, maintaining the functional form (14); and finally, we consider varying the degree of substitutability between consumption and leisure through using a general constant elasticity of substitution utility function. Table 2 presents calibrated discount rates for the scenarios considered in this section.

### 5.1 Additive Separability

In our baseline case, we allow for non-separability between consumption and leisure because of the empirical evidence of Browning and Meghir (1991).

The point of this subsection is to show which of the main results depend on the non-separability assumption.

We use the additive specification for utility:

$$u(c, l) = \eta \ln c + (1 - \eta) \ln l$$

which is a special case of the utility function (14) with  $\gamma = 1$ .

Figure 9 shows profiles for log consumption, hours of work and asset holdings under log utility when wages are uncertain, comparing fixed labor supply with a flexible labor supply. Figure 9(b) shows that the pattern for hours of work is similar to the baseline case shown in Figure 7. In particular, hours of work start high despite the relatively low wage, and then decline in old age despite the relatively high wage. As discussed above, this is simply the precautionary motive operating on leisure. This motive operates whether or not preferences are additive in consumption and leisure. However, the path of consumption shown in Figure 9(a) differs from the baseline because the level of leisure no longer has a direct effect on the marginal utility of consumption. This means that log consumption does not fall towards the end of the working life as hours of work declines. The difference in the level of log consumption across the two scenarios highlights the extra income that individuals receive *ex post* from being able to work more hours when wages are high.

The comparison of age profiles for asset accumulation shown in Figure 9 is similar to the comparison in the baseline case. When labor supply is flexible, young individuals borrow more and then accumulate more assets in middle age. This is partly driven by the desire to move resources to times of low hours and hence income.

## 5.2 Varying the Elasticity of Intertemporal Substitution

The additive specification of utility above shows that the qualitative results are not necessarily dependent on the utility specification. It is clear however

that life-cycle profiles in the baseline case depend on the underlying preferences of individuals. In particular, the parameter  $\gamma$  determines the elasticity of intertemporal substitution and the degree of precautionary behavior. The aim of this subsection is to show how profiles change with different values of  $\gamma$ .<sup>15</sup>

Using the utility function (14),

$$u_{cl}(c, l) = (1 - \gamma)\eta(1 - \eta)c^{\eta(1-\gamma)-1}l^{(1-\eta)(1-\gamma)-1}, \quad (18)$$

which is negative if  $\gamma > 1$ , indicating within period consumption and leisure are Frisch substitutes. When  $\gamma < 1$ , consumption and leisure are Frisch complements.

An increase in  $\gamma$  has three main effects: first, it strengthens the precautionary motive; second, it reduces the (Frisch) intertemporal elasticity; and third, it increases the Frisch substitutability of leisure and consumption. The first effect implies that as  $\gamma$  becomes high, individuals become increasingly averse to fluctuations in utility and so work more when young and accumulate precautionary balances. The second effect reduces willingness to substitute leisure over time. The third effect means that, if  $\gamma$  is high, individuals get similar utility levels from low consumption and high leisure (chosen if the realized wage is low) as from high consumption and low leisure (chosen if the realized wage is high), and this reduces the benefit of precautionary balances.

Figure 10 shows the effect of varying  $\gamma$  on labor supply. Higher values of  $\gamma$  (a lower intertemporal substitution elasticity) mean hours of work are flatter and the variation in wages over the life-cycle is not exploited as much.

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<sup>15</sup>Empirical estimates of the elasticity of intertemporal substitution for consumption vary considerably depending on whether aggregate or micro data is used, and whether estimates condition on demographics, labor supply etc. Sensitivity over the range of  $[0.25, 1]$  covers the range of values commonly found, allowing for the interdependency of consumption and leisure.

Figure 11 shows the implications of varying  $\gamma$  for consumption and asset accumulation. Figure 11 also compares the implications when labor supply is fixed. Increases in  $\gamma$  lead to greater deferral of consumption for precautionary reasons, but the extent of this deferral is offset by the greater substitutability of consumption and leisure. The hump shape in hours worked generates a hump shape in consumption when consumption and leisure are substitutes, with the hump in consumption being more pronounced as the degree of substitutability increases ( $\gamma$  increases). When consumption and leisure are complements, the hump shape in hours worked generates more of a U-shape in consumption. This effect of hours worked on consumption choices is shown by comparing panel (a) with panel (b) where labor supply is fixed and constant and so there is only the direct effect of increasing  $\gamma$  causing a deferral in consumption.

Asset profiles are affected by changing  $\gamma$ : increases in  $\gamma$  lead to flatter asset accumulation paths, holding median asset holdings constant. This partly reflects a weaker intertemporal substitution effect and a weaker life-cycle smoothing effect when  $\gamma$  is high since hours of work and hence income vary less markedly over the life-cycle. Calibrated discount rates (shown in Table 2) also differ by  $\gamma$ : a higher discount rate is needed to match median assets when  $\gamma$  is higher and the precautionary motive stronger.

### 5.3 Constant Elasticity of Substitution

The Cobb-Douglas structure for within period preferences means that the elasticity of substitution between consumption and leisure is 1. In this subsection, we isolate the effect of the change in substitutability by considering a CES within period utility function and varying the elasticity of substitution between consumption and leisure through varying parameter  $\rho$ ,

$$u(c, l) = \frac{(\theta c^\rho + l^\rho)^{\frac{1-\gamma}{\rho}}}{1-\gamma}.$$

The Cobb-Douglas case presented in our baseline is the  $\rho = 0$  case. The value of  $\theta$  is calibrated with the discount rate in order to keep average hours at 2080 per year and the median asset to median income ratio at 1.84.<sup>16</sup>

Figure 12 shows how consumption, hours of work and asset holdings vary over the life-cycle for different values of  $\rho$ . Relative to our baseline case where  $\rho = 0$ , as  $\rho$  becomes increasingly negative, consumption and leisure become less close substitutes. This leads to consumption and hours of work profiles becoming flatter (when  $\rho = -3.0$ ), rather than having high leisure and low consumption followed by low leisure and high consumption (when  $\rho = 0.5$ ). For all values of  $\rho$ , we observe a decline in hours of work pre-retirement. This result suggests that even if our baseline overestimates the elasticity between consumption and leisure (and so  $\rho < 0$ ), the result that precautionary motives can partially explain why hours of work decline with age despite wages remaining relatively high is robust. The key point is that a higher  $\rho$  means individuals are better able to exploit the intertemporal substitution opportunities offered by wage growth.

The difference in consumption and hours of work profiles by  $\rho$  translates into differences in asset profiles. The relatively flat consumption and leisure profiles associated with greater complementarity between consumption and leisure and a negative value of  $\rho$  implies a need to reallocate resources over the life-cycle through borrowing when young followed by building large asset holdings in middle-age. By contrast, the close substitutability captured by  $\rho = 0.5$  does not require the same reallocation over the life-cycle and asset paths are flatter. There is also a level effect on asset accumulation: in order

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<sup>16</sup>Equation (10) shows that the sign of the Frisch elasticity is determined by the sign of  $u_{cl}$  where we now have

$$u_{cl}(c, l) = (1 - \gamma - \rho) (\theta c^\rho + l^\rho)^{\frac{1-\gamma-2\rho}{\rho}} \theta (cl)^{\rho-1}.$$

This implies that consumption and leisure are Frisch substitutes and  $\frac{\partial c_t}{\partial w_t} \Big|_{\lambda_t} > 0$  if  $1 - \gamma < \rho$ .

to match the data, a larger discount rate is needed as  $\rho$  becomes increasingly negative. Close substitutability of consumption and leisure weakens the precautionary motive for saving, as suggested by the approximations (11) and (13).

## 6 Conclusion

The approximations and simulations in this paper show the effect on life-cycle behavior of uncertainty and of flexibility in labour supply. The elasticity of intertemporal substitution induces individuals to choose hours of work to track wages, but this is offset by the effect of uncertainty inducing individuals to work more while young (and wages are low). The simulated age profiles for hours taking account of these two factors come close to generating the patterns of life-cycle labor supply that we observe in the data. Simulated consumption profiles also come close to generating the patterns observed in the data: consumption is concave, with the degree of concavity depending on the substitutability between consumption and leisure.

Flexibility changes the age profiles of asset accumulation: individuals borrow more when young, then save more in middle age than if hours are fixed and constant. This effect of flexibility is present with and without uncertainty. The change in the age profile arises because under flexibility, hours of work fall with age and so more saving is needed to smooth consumption. Flexibility also changes the level of asset holding: the discount rate needed to match the asset level in the data is greater with flexible labor than when labor supply is fixed. This implies greater asset holdings with flexibility and this arises because the utility cost of accumulating assets is lower if both increased labor supply and reduced consumption can be used to increase savings. Flexibility does reduce the value of holding assets, but this effect is dominated across in our calibrated simulations.

The sensitivity analysis varying the structure of preferences shows these

results are robust to a number of changes and highlights that closer substitutability between leisure and consumption firstly leads to more opportunity to exploit intertemporal incentives and secondly, reduces the need to hold precautionary balances.

To conclude, individuals who are constrained to work fixed hours through their lives will have accumulated fewer assets, have flatter asset age-profiles and have faster consumption growth than those with flexibility. Ignoring the effects of labor supply therefore may lead us to overestimate the amount of consumption growth due to uncertainty and to underestimate the proportion of wealth held for precautionary motives.

## APPENDIX

Section 4 uses standard techniques to solve for the optimal policy functions which give leisure and consumption as functions of the actual wage, permanent wage and the asset level for that age, conditional on the assumed wage process:

$$\begin{aligned} c_t &= c_t(A_t, w_t, w_t^P | \sigma^2) \\ l_t &= l_t(A_t, w_t, w_t^P | \sigma^2) \end{aligned}$$

Since  $l_t$  can be written as a function of  $c_t$  as in equation (15), the problem reduces to solving for  $c_t$ . The solution is found, as in Deaton (1991), through backward induction, starting with the terminal constraint. Once the policy functions for every age have been found, the model is then simulated starting from the initial period to give a profile of how consumption, hours of work and assets evolve across the lifetime. Profiles are averages of 10000 profiles of individuals who are ex-ante identical but with different realizations of the shocks.

The expectations operator in equation (6) and the non-linear form of marginal utility mean that it is not possible to find analytic solutions to the policy functions and so numerical solution methods are necessary. There are three state variables, the asset level,  $A_t$ , the wage rate and the permanent wage. As in Deaton (1991), we reduce the numerical problem to one with two state variables by dividing by

$w_t^p$ , and substituting  $c_t/w_t^p$  in place of actual consumption. This gives the Euler equation that is actually solved as

$$\begin{aligned} & \frac{c_t}{w_t^p} \left( \frac{A_t}{w_t^p}, \frac{w_t}{w_t^p}, 1 \right)^{\eta(1-\gamma)-1} l_t \left( \frac{c_t}{w_t^p}, \frac{w_t}{w_t^p} \right)^{(1-\eta)(1-\gamma)} \\ &= \beta(1+r) \int \left\{ \frac{c_{t+1}}{w_{t+1}^p} \frac{w_{t+1}^p}{w_t^p} \right\}^{\eta(1-\gamma)-1} l_{t+1} \left( \frac{c_{t+1}}{w_{t+1}^p}, \frac{w_{t+1}}{w_{t+1}^p} \right)^{(1-\eta)(1-\gamma)} dF(w_{t+1}, w_{t+1}^p | w_t^p) \end{aligned}$$

and this equation is solved recursively for  $\tilde{c}_t = c_t/w_t^p$ .

The solutions to each age-specific functional equation are found by solving at a number of discrete nodes and then interpolating using a local approximation method (results were unchanged between using linear splines and cubic splines). The grid for assets is defined to avoid any extrapolation.<sup>17</sup>

To check that the solution is sufficiently accurate, we use simulated profiles for consumption and leisure to calculate, for each  $t$ ,

$$\beta(1+r) \sum_i \frac{c_{t+1}^{\eta(1-\gamma)-1} l_{t+1}^{(1-\eta)(1-\gamma)}}{c_t^{\eta(1-\gamma)-1} l_t^{(1-\eta)(1-\gamma)}}.$$

This is the realized values of the Euler equation and should equal 1 when averaged across individuals in the absence of explicit liquidity constraints. The numerical solution proceeds by setting this equal to 1 in expectation, but the check uses the realization of simulated consumption and leisure paths. The condition does not deviate from 1 by more than three decimal places for any  $t$  for any of the results presented in the text. Further, increasing the number of nodes substantially (increasing the asset grid to 2000) does not change the simulated profiles.

There remain some implementation issues: to impose a minimum value on the wage rate, it was assumed that the shock to log wages,  $\varepsilon$ , is within five standard deviations of the mean. Shocks are multiplicative, so this is a restriction on the proportion by which wages can change between  $t$  and  $t+1$ .<sup>18</sup> Since the wage is

<sup>17</sup>Fortran code is available from the author on request. The code uses the NAG library to solve the underlying nonlinear equations.

<sup>18</sup>In Carroll (1992) and others, the worst outcome in a given period is an income of zero, and this occurs with probability  $p$ , which both papers set equal to 0.005. This

non-stationary, the state space is defined across  $A_t/w_t^P$  and  $w_t/w_t^P$  (i.e. the transitory wage). The grid spacing for  $A_t/w_t^P$  needs to be more dense in areas where the gradient of the policy function changes quickly (e.g. close to  $\underline{A}_t$ , the minimum feasible asset stock in  $t$ ). There are 400 nodes for the asset grid in each age. The transitory shock is discretised into three values. In calculating the numerical integral in equation (??) for the permanent shock, we use 12 point Gaussian Hermite quadrature. In the simulations, the permanent shock is a continuous random variable. Interpolation is still only in one dimension, however, because we combine the two continuous state variables (the asset stock and the permanent wage) to give the transformed state variable and solve for the transformed policy rule.

The lifetime budget constraint imposes a restriction on the extent of spending in any one period. As discussed in Attanasio *et al.* (1999), this is information available to the individual and so should be exploited in the solution method. To be specific, the individual cannot spend more than minimum income for remaining periods, where this minimum income is determined by the minimum wage in each period. This is because the marginal utility of having zero within period income to spend is infinite and the individual is constrained not to die in debt. Hence, the solution for within period spending lies between 0 and minimum income.

In section 5.3, a modified technique is used because the utility function is not homothetic. However, since the value function is still concave, it is relatively straightforward to solve recursively with the only conceptual difference being the addition of an extra state variable for the permanent wage and the solution is for the level of consumption rather than the ratio of consumption to the wage. The finite horizon nature of the model mean that we can use a finite size grid to approximate this non-stationary wage process but this is somewhat slower computationally. In practice, an 18 state Markov transition matrix is used for the permanent wage. The nodes are chosen to match the age-specific unconditional moments of the true process. For example, the unconditional variance of realized wages should increase linearly with age with a slope equal to the conditional variance and we ensure our discrete process matches this.

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means individuals never borrow because the probability of the worst outcome occurring is set at a significant level. Borrowing in the current paper will be very limited because minimum income is very low, although not zero.

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**Table 1: Calibrated Discount Rates**

Scenario		$\frac{Median[A_{i,t}]}{Median[y_{i,t}]}$	$\frac{\sum_i \sum_t A_{i,t}}{\sum_i \sum_t y_{i,t}}$	$1 + \delta$
PSID Data <sup>†</sup>		1.84	4.64	-
<i>Certainty</i>	Flexible	1.84	2.22	1.009
	Fixed*	1.84	2.08	1.008
<i>Uncertainty</i>	Flexible	1.84	2.14	1.032
	Fixed*	1.84	2.05	1.028
	Fixed at flexible*	1.84	2.06	1.030

Other parameters:  $\gamma = 2.2$ ,  $\eta = 0.4$ ,  $H = 5200$ ,  $r = 0.016$ . Deterministic wage growth is given by  $\alpha_1 = 0.0561$  and  $\alpha_2 = -0.000599$ . When wages are uncertain,  $\sigma_\varepsilon^2 = 0.031$  and  $\sigma_v^2 = 0.030$ . The social security replacement rate is 0.55.

<sup>†</sup>Wealth data statistics are calculated from the PSID 1995 wealth supplement and using the PSID weights.

\*The “Fixed” case fixes hours of work at the same level for each age where the level is determined by the life-cycle average. The “fixed at flexible” case fixes hours of work at each age according to the average hours worked at that age when labour supply is flexible.

**Table 2: Discount Rates for Alternative Preferences**

Scenario		$\frac{\text{Median}[A_{i,t}]}{\text{Median}[y_{i,t}]}$	$\frac{\sum_i \sum_t A_{i,t}}{\sum_i \sum_t y_{i,t}}$	$1 + \delta$
PSID Data <sup>†</sup>		1.84	4.64	-
<i>Flexible Labor Supply</i> $u(c, l) = \frac{(e^{\eta} l^{1-\eta})^{1-\gamma}}{1-\gamma}$	$\gamma = 0.5$	1.84	2.92	1.023
	$\gamma = 1.0^{\ddagger}$	1.84	2.50	1.025
	$\gamma = 1.2$	1.84	2.39	1.026
	$\gamma = 2.2$	1.84	2.14	1.032
	$\gamma = 4.0$	1.84	1.88	1.050
<i>Fixed Labor Supply*</i> $u(c, l) = \frac{(e^{\eta} l^{1-\eta})^{1-\gamma}}{1-\gamma}$	$\gamma = 0.5$	1.84	2.59	1.021
	$\gamma = 1.0^{\ddagger}$	1.84	2.35	1.022
	$\gamma = 1.2$	1.84	2.28	1.023
	$\gamma = 2.2$	1.84	2.05	1.028
	$\gamma = 4.0$	1.84	1.87	1.044
<i>Flexible Labor Supply</i> $u(c, l) = \frac{(\theta c^{\rho} + l^{\rho})^{\frac{1-\gamma}{\rho}}}{1-\gamma}$	$\rho = 0.5$	1.84	2.61	1.019
	$\rho = 0.0^{\dagger\dagger}$	1.84	2.14	1.032
	$\rho = -3.0$	1.84	2.12	1.054

Other parameters:  $H = 5200$ ,  $r = 0.016$ . Deterministic wage growth is given by  $\alpha_1 = 0.056$  and  $\alpha_2 = -0.000599$ . All scenarios assume wages are uncertain, with  $\sigma_{\epsilon}^2 = 0.031$  and  $\sigma_{\nu}^2 = 0.030$ .

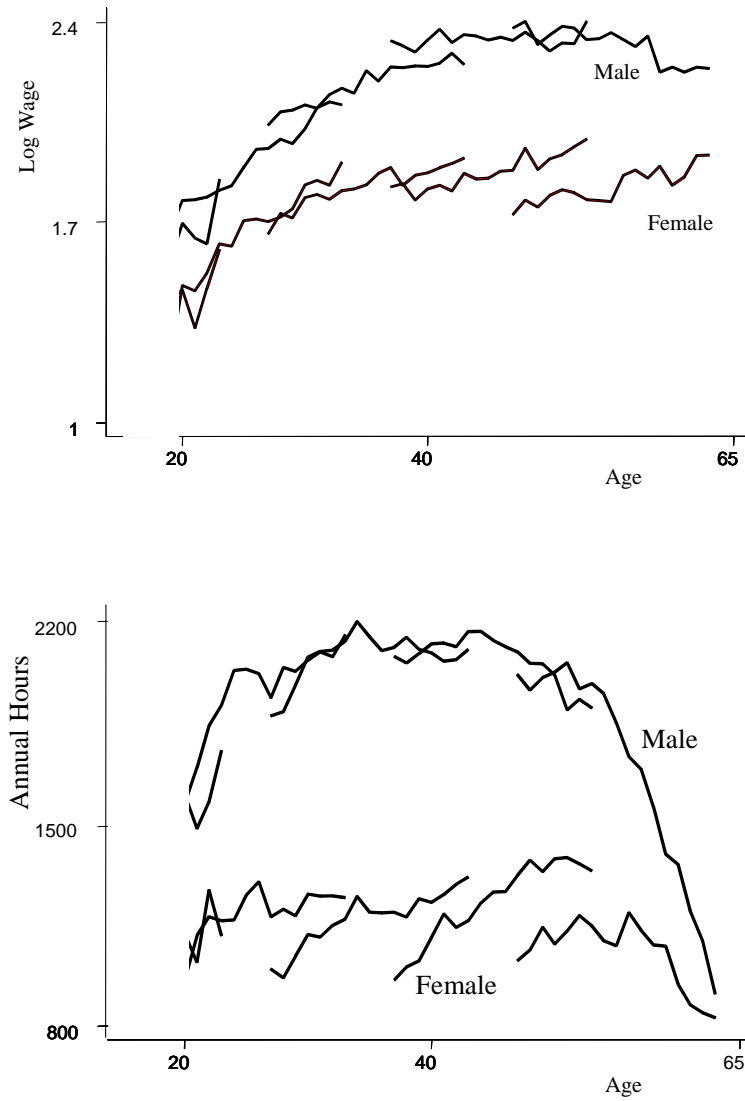
<sup>†</sup>Wealth data statistics are calculated from the PSID 1995 wealth supplement and using the PSID weights.

<sup>‡</sup>When  $\gamma = 1.0$ , we have log utility and additive separability between consumption and leisure.

\*The “Fixed” case fixes hours of work at the same level for each age where the level is determined by the life-cycle average.

<sup>††</sup>When  $\rho = 0.0$  the CES utility function becomes the baseline CRRA Cobb-Douglas function. The value of  $\theta$  varies with  $\rho$  to keep average hours of work constant: when  $\rho = 0.5$ ,  $\theta = 0.37$ , when  $\rho = -3.0$ ,  $\theta = 15.82$ .

**Figure 1: Wage and Labor Supply Life-Cycle Profiles in the CEX**



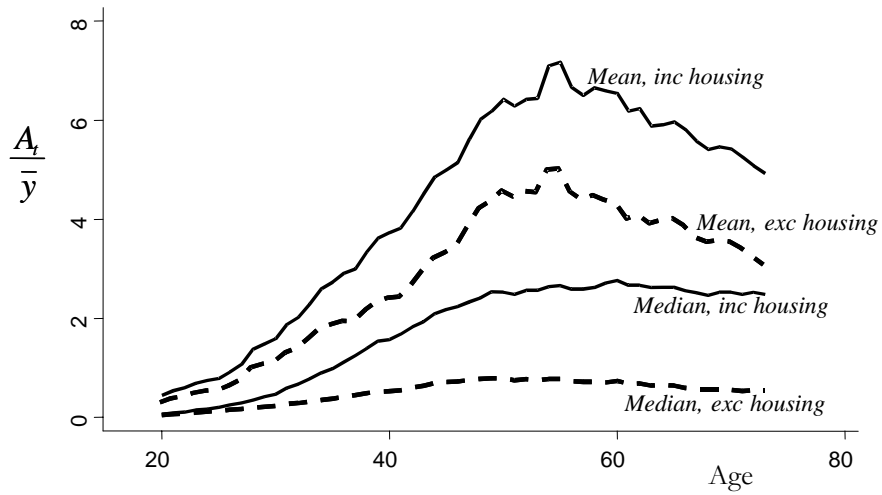
The first graph shows average log wage by cohort and by age, split by sex. The second shows average hours of work by cohort and by age, split by sex. Cohorts are defined by date-of-birth in 10 year divisions. Data is from the Consumer Expenditure Survey, 1980 - 1992.

**Figure 2: Income and Consumption across the Life-Cycle in the CEX**



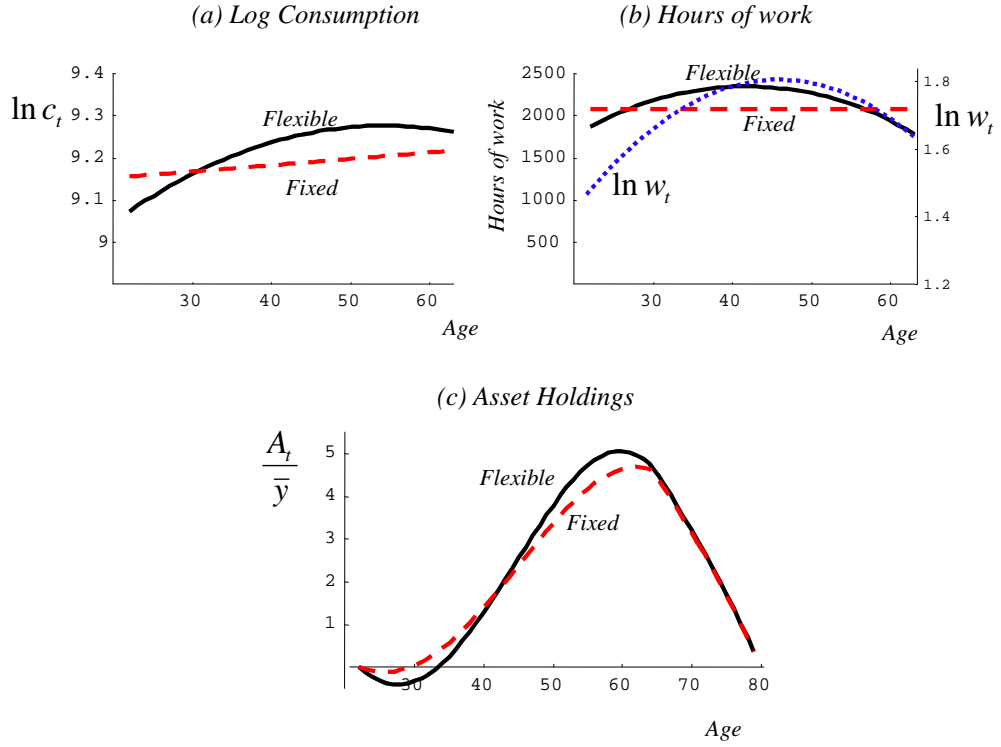
Log income and log consumption, by age and cohort, averaged over households. Consumption data is for non-durables. Cohorts are defined by date-of-birth in 10 year divisions. Data is from the Consumer Expenditure Survey, 1980 - 1992.

**Figure 3: Asset Holdings in the PSID**



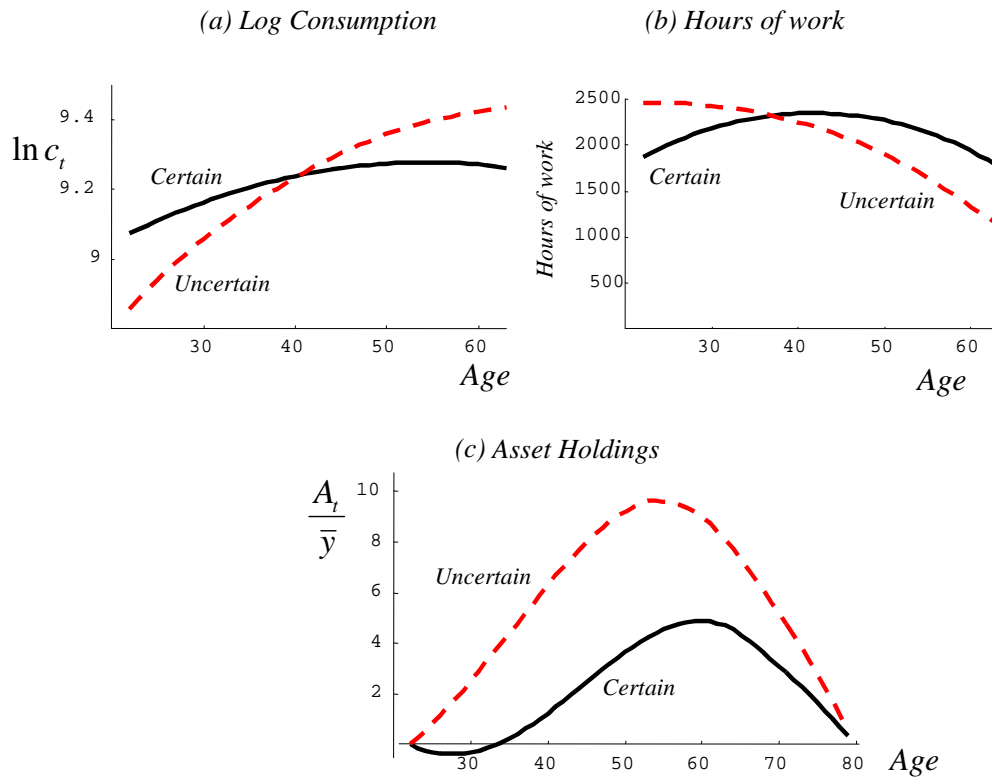
The graph shows mean and median asset holdings over the life-cycle, including and excluding housing wealth. Data is from the PSID wealth supplements in 1985,1990,1995. For lines showing median holdings, wealth is divided by median income over the life-cycle, for lines showing mean holdings, wealth is divided by mean income.

**Figure 4: The Effect of Flexible Labor Supply under Certainty**



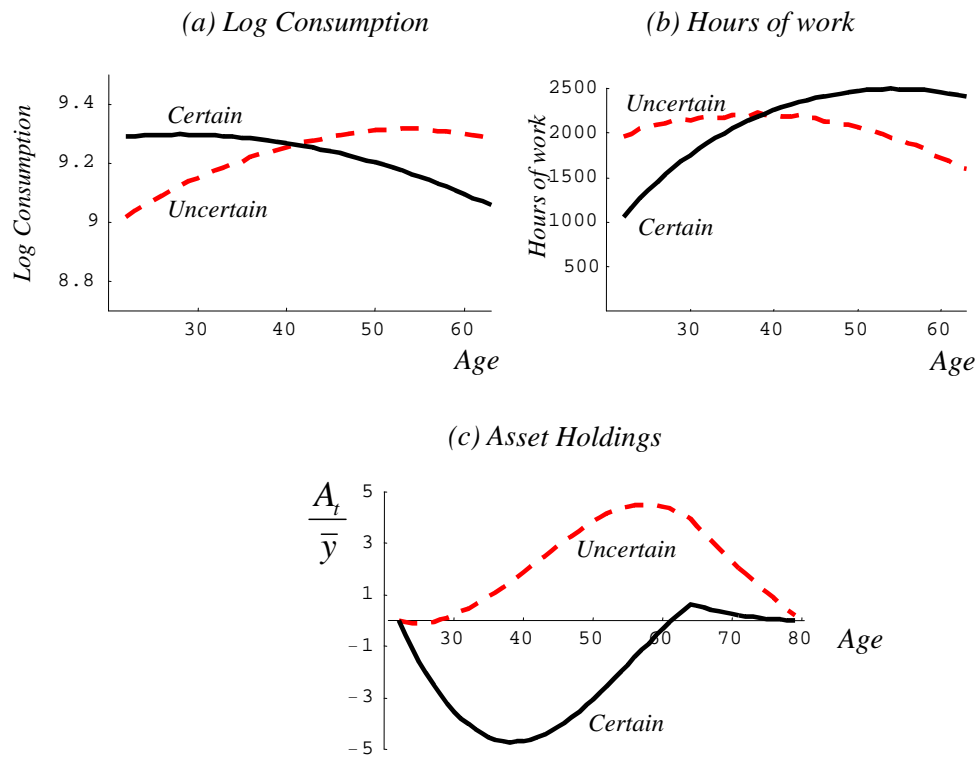
Graph (a) shows simulated log consumption averaged over individuals at each age. Graph (b) shows average hours of work and log wages by age and Graph (c) shows median asset holding by age divided by median earnings (across individuals of working age). Each graph shows profiles assuming a flexible labor supply (the solid lines) and assuming labor supply is fixed at a constant value determined by life-cycle average hours (the dashed line). Wages are certain. The discount rate is calibrated separately for each scenario so that median assets match the data.

**Figure 5: The Effect of Uncertainty on Life-Cycle Profiles**



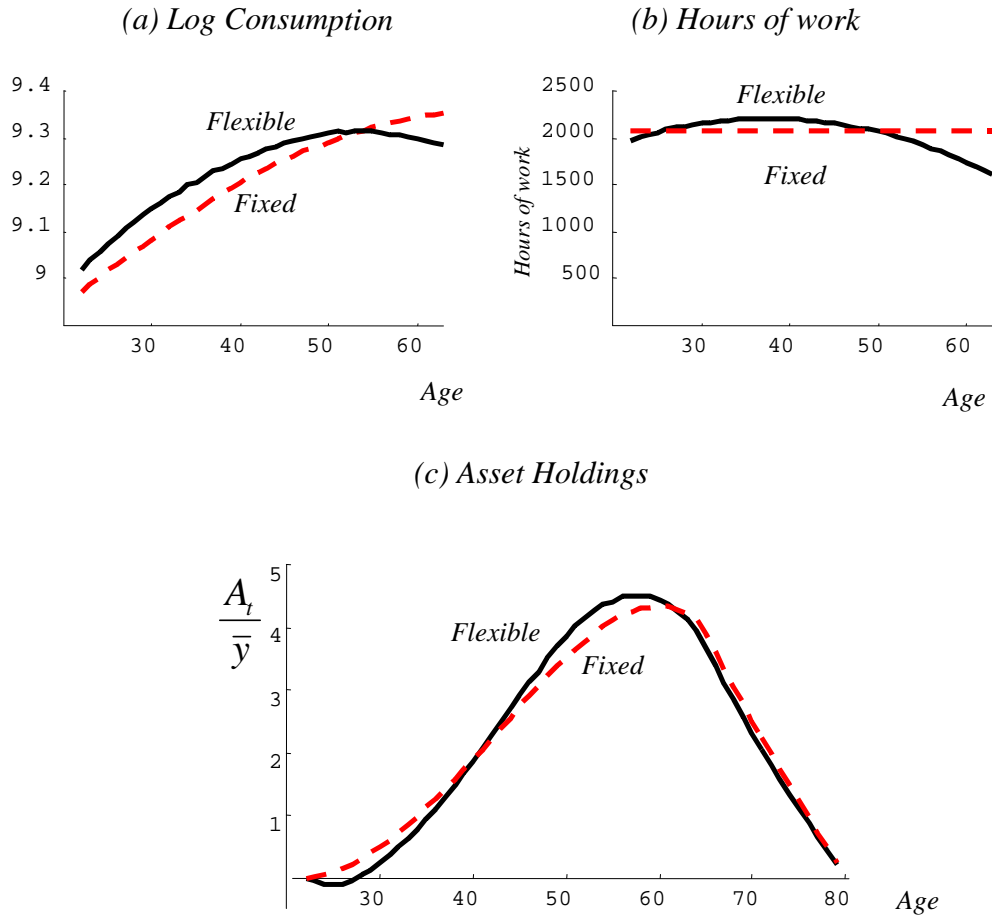
Graph (a) shows average log consumption, (b) shows average hours of work and (c) shows median asset holding by age divided by median income. Solid lines show life-cycle profiles if there is no uncertainty, dashed lines when wages are uncertain. The discount rate is calibrated so that median assets under certainty match the data. Wage rate uncertainty is then introduced holding preference parameters constant.

**Figure 6: The Effect of Removing Uncertainty on Life-Cycle Profiles**



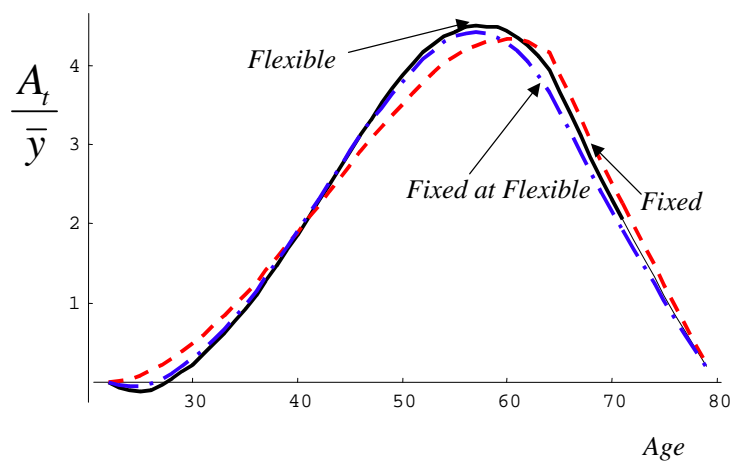
Graph (a) shows average log consumption, (b) shows average hours of work and (c) shows median asset holding by age divided by median income. Solid lines show life-cycle profiles if there is no uncertainty, dashed lines when wages are uncertain. The discount rate is calibrated so that median assets under uncertainty match the data. Wage rate uncertainty is then removed, holding preference parameters constant.

**Figure 7: The Effect of Flexible Labor Supply under Uncertainty**



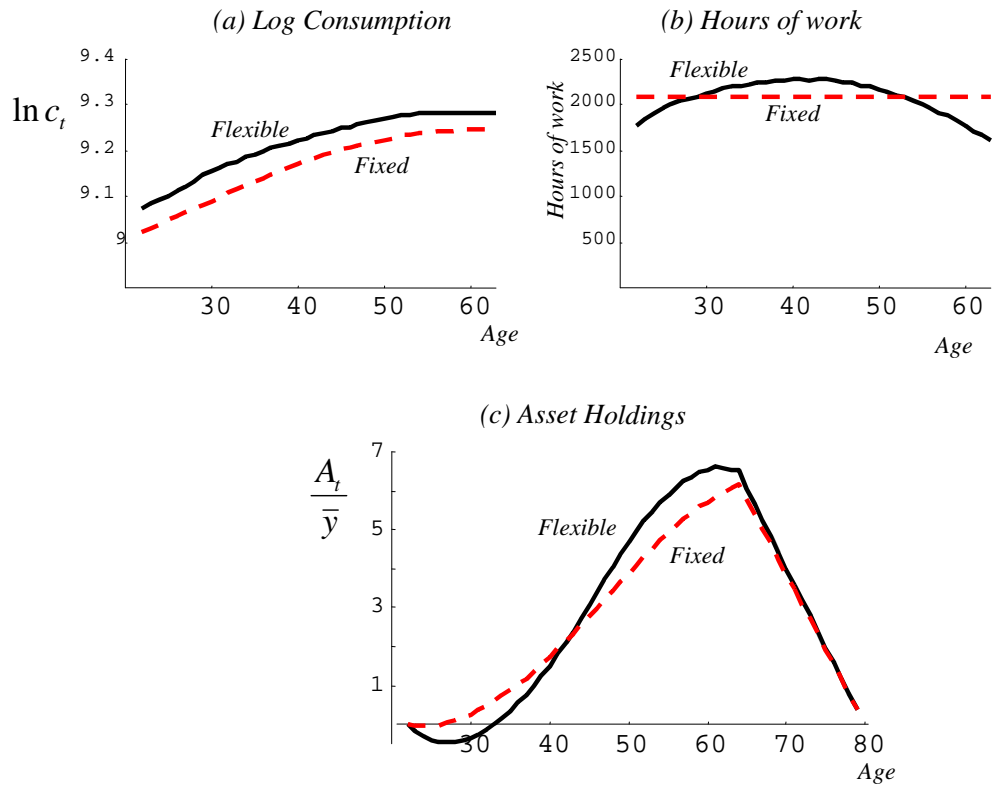
Graph (a) shows average log consumption, (b) shows average hours of work and (c) shows median asset holding by age divided by median income. Solid lines show life-cycle profiles if labor supply is flexible, dashed lines when labor supply is fixed at the life-cycle average. Wages are uncertain in both cases. The discount rate is calibrated for each scenario so that median assets match the data.

**Figure 8: Partial Decomposing of Saving Motives**



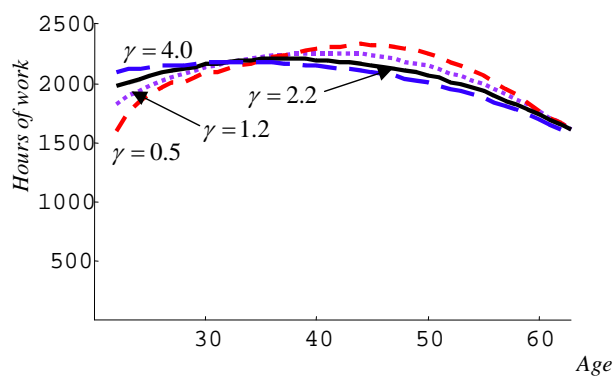
The discount rate is calibrated so that median assets under uncertainty match the data in each scenario. The solid line marked “Flexible” has hours of work varying across individuals and over time. The dashed line marked “Fixed” has hours of work fixed at a constant level where the level is determined by the life-cycle average. The dashed-dotted line marked “fixed at flexible” has hours of work fixed at each age according to the average hours worked at that age when labour supply is flexible.

**Figure 9: The Effect of Flexible Labor Supply under Additive Preferences**



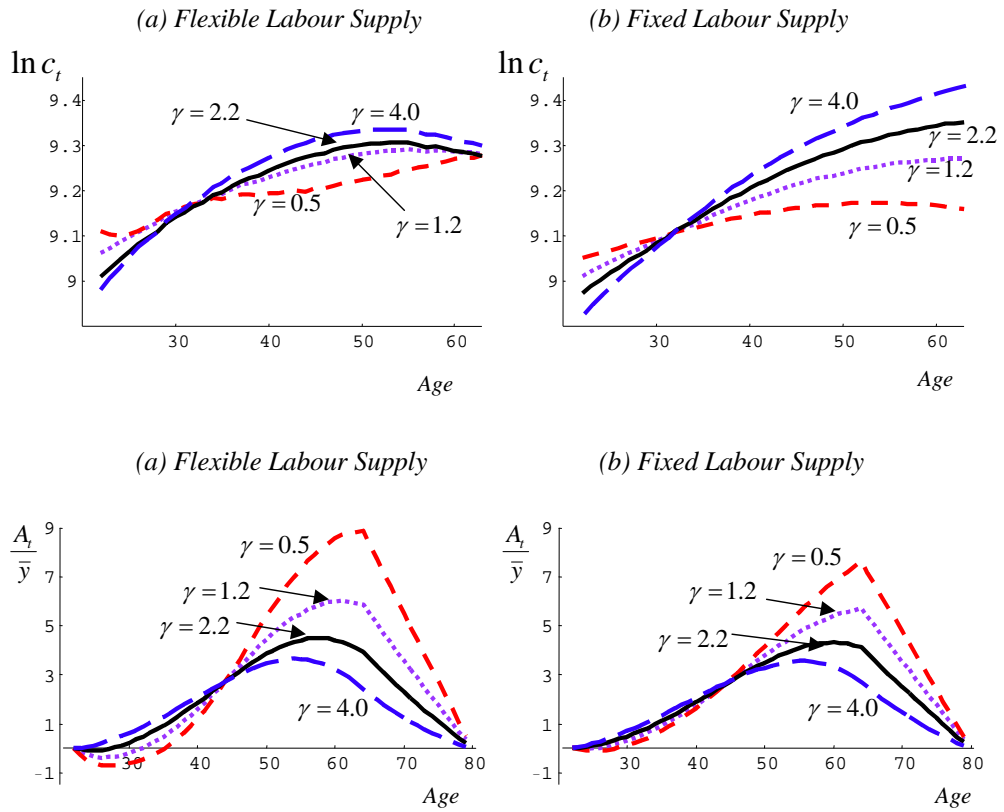
Utility is assumed to be logarithmic  $U = \eta \ln c + (1 - \eta) \ln(l)$ . Graph (a) shows average log consumption, (b) shows average hours of work and (c) shows median asset holding by age divided by median income. Solid lines show life-cycle profiles if labor supply is flexible, dashed lines when labor supply is fixed at the life-cycle average. Wages are uncertain in both scenarios. The discount rate is calibrated for each scenario so that median assets match the data.

Figure 10: Labor Supply, Varying  $\gamma$



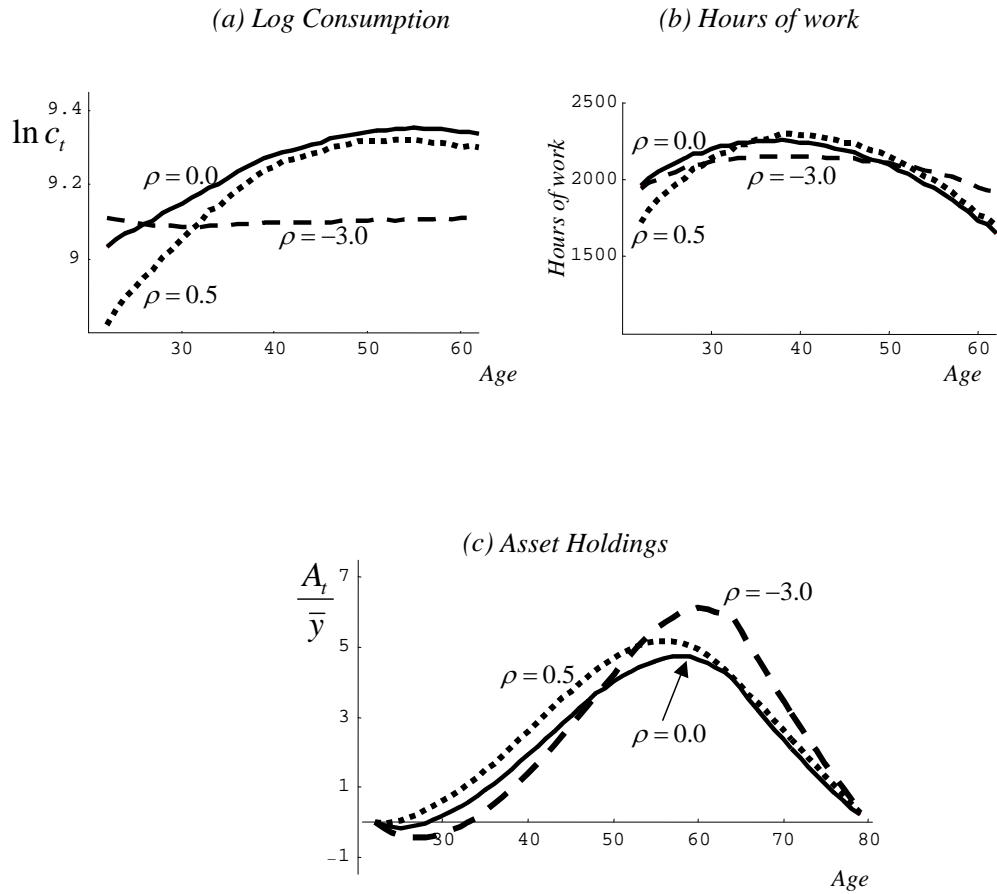
The figure compares life-cycle paths for hours worked assuming labor supply is flexible, varying the coefficient of relative risk aversion. Wages are uncertain in all cases. The discount rate is calibrated so that median assets match the data in each scenario.

Figure 11: Consumption and Asset Profiles, Varying  $\gamma$



The figure compares life-cycle paths for log consumption in the first row and for asset holdings in the second row. The left-hand column reports paths when labor supply is flexible, the right hand column reports paths when labor supply is fixed at a constant rate where the level is determined by the life-cycle average. Wages are uncertain in all cases. The discount rate is calibrated so that median assets match the data in each scenario.

Figure 12: Life-Cycle Profiles Varying the Elasticity of Substitution



Graph (a) shows average log consumption, (b) shows average hours of work and (c) shows median asset holding by age divided by median income. The figure shows profiles for different values of  $\rho$  where  $\rho$  is the parameter in the CES utility function determining the elasticity of substitution between leisure and consumption. The discount rate is calibrated in each scenario so that median assets match the data.