Optimal Taxation, Prudence and Risk-Sharing[*]

Hamish Low

University of Cambridge and Institute for Fiscal Studies

Daniel Maldoom

DotEcon Ltd.

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Abstract

This paper analyses optimal income taxation as a trade-off between the incentive effects of increased uncertainty and the welfare benefits of social insurance. Greater prudence increases labor supply because of precautionary incentive effects which reduce the progressivity of the optimal income tax schedule. Increased uncertainty increases progressivity of the income tax schedule because of a greater value of social insurance. Optimal tax progressivity depends on the ratio of prudence to risk aversion: when this ratio is high, incentive effects dominate the social insurance effect, leading to declining optimal income tax rates.

JEL: H21. *Keywords*: prudence, risk sharing, optimal taxation

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1 Introduction

This paper analyses the trade-off between the incentive effect of uncertainty and the welfare benefit of social insurance in the design of optimal tax schedules. Redistributive tax schemes share risk and so reduce the variance of net income. However, uncertainty may induce people to increase their effort for precautionary reasons, and so social insurance may blunt incentives. The main contribution of this paper is to characterise the trade-off between the incentive effect of uncertainty and the welfare benefit of social insurance, partly through numerical solutions. In particular, we show how the optimal amount of social insurance depends on the strength of precautionary motives relative to risk aversion.

The focus of standard optimal tax and principal-agent models has been on how changes to the expected return to effort affects incentives. The key point of this paper is that changes in the variance of returns to effort also have incentive effects. This trade-off between uncertainty and risk sharing underlies the question of how much insurance should be provided through the tax system, as discussed by Varian (1980): progressive taxation provides insurance by reducing the variance of net income, but this insurance reduces the incentive to self-insure through saving. Varian presents some limited comparative static results, holding certain endogenous variables fixed. Strawczynski (1998) further analyses the effect of precautionary motives on optimal tax schedules, assuming a linear tax schedule. Both reach the conclusion that precautionary behaviour leads to more progressive tax schedules. In this paper we show that it is risk aversion that leads to more progressive tax schedules while precautionary motives reduce progressivity. Precautionary motives induce more work and so a progressive tax schedule providing social insurance would reduce this motive for work.

The formulation in this paper uses a framework similar to that of Varian (1980). Unlike Varian, we use a model that includes precautionary effects on effort, an element that was not considered in Vairan's model which is based on a choice between consumption and savings, as opposed to consumption and leisure. A model with both choice between consumption and leisure and income uncertainty was used by Tuomala (1984), but his paper does not consider precautionary effects on labor supply.

We obtain comparative static results on the optimal tax schedule, varying risk aversion and uncertainty. The trade-off at the heart of the model is between inducing particular individuals to exert greater precautionary effort by making their income more uncertain[1] and providing social insurance. Therefore, the ratio of the coefficient of prudence to the coefficient of risk aversion is crucial: when this ratio is large, the incentive effect of precautionary motives dominates and the marginal tax rate falls with income; when this ratio is small, risk aversion dominates and marginal tax rates rise with income. We also show that optimal risk sharing crowds out self-insurance, particularly at high levels of risk aversion. We consider increases in the amount of exogenous uncertainty and show that this has a direct effect on effort, leading to greater effort for precautionary reasons with a given tax schedule, and an indirect effect, altering the tax schedule to provide greater social insurance and reducing effort. The first effect is shown to dominate. Finally, the concavity of the likelihood ratio is important for the progressivity of the marginal tax rate: greater concavity means more progressivity. Assuming concavity in order to satisfy the Jewitt (1988) sufficient conditions for the validity of the first-order approach restricts the class of tax schedules that can be considered.

Section 2 presents the model. Section 3 identifies some properties of the optimal tax schedule analytically, and more complete properties through numerical methods. Section 4 concludes.

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[1] The notion that risk increases effort for precautionary reasons is analysed in Low (1999) and is analogous to the precautionary saving behaviour analysis of Kimball (1990) and others.

2 The model

We consider a simple principal-agent model in which there is a continuum of identical agents. Each agent produces an output that depends on his or her own effort and an idiosyncratic shock. All output risks are potentially fully diversifiable, although it will not be optimal to do so because of incentive issues.

Agents are indexed by $i \in [0,1]$. Agent i's output, x_i, is a function of effort, e_i, and a random shock. The outputs of agents are independently distributed. Let $F(x_i \mid e_i)$ be the distribution function of x_i if effort e_i is expended, with continuously differentiable probability density $f(x_i \mid e_i)$. All agents have identical utility functions, which are additively separable in effort and income; let $u(y)$ be the utility of income y and $g(e)$ be the disutility of effort e.

We consider optimal mechanisms for redistributing output that preserve the veil of ignorance; the allocation mechanism should yield the same income for each agent even if the agents are relabelled (i.e. the indices i are permuted). Such a mechanism may only condition an agent's income on that agent's own output and the *distribution* of all agents' outputs. With an infinite number of agents experiencing independent output shocks, the law of large numbers implies that the distribution of all agents' outputs is non-stochastic. Therefore, we need only consider mechanisms in which each agent's income may be written as a function $s(x)$ of that agent's own output x.[2]

The total payment to agents can be no more than total output in any contingency. We assume that the rule only implements Pareto optimal distributions, so total payments must exactly equal total output. If a tax

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[2] In order to consider the incentive compatibility of the optimal sharing rule, we will need to consider hypothetical changes in effort by a single agent. As there are an infinite number of agents, even under such hypothetical deviations we need only consider the dependency of the sharing rule on an agents' own output and may suppress any dependency on other agents' outputs.

schedule withheld part of the output, a Pareto improving schedule would allocate this surplus to the top earner without worsening incentives.[3]

The optimal tax schedule is found by solving the optimisation problem

$$\max_{s(\cdot),e^*} \int u\left(s\left(x\right)\right)\,dF\left(x \mid e^*\right) - g\left(e^*\right) \tag{1}$$

$$\text{s.t.} \quad e^* \in \arg\max_{e} \int u\left(s\left(x\right)\right)\,dF\left(x \mid e\right) - g\left(e\right) \tag{2}$$

$$\int s\left(x\right)\,dF\left(x \mid e^*\right) = \int x\,dF\left(x \mid e^*\right). \tag{3}$$

where condition (2) is the incentive compatibility constraint and equation (3) is the adding-up condition. This can be interpreted as a standard principal-agent problem of the type first used by Mirrlees (1974).[4]

We will restrict the class of distributions $F\left(x \mid e^*\right)$ such that the first order approach is valid and so the incentive compatibility constraint can be replaced by

$$\int u\left(s\left(x\right)\right)\,dF_e\left(x \mid e^*\right) = g'\left(e\right) \tag{4}$$

The Lagrangian of this problem is

$$\begin{aligned} L &= \int \left[u\left(s\left(x\right)\right) + \mu x - s(x)\right] dF\left(x \mid e^*\right) - g\left(e^*\right) \\ &\quad + \lambda\left\{\int u\left(s\left(x\right)\right)\,dF_e\left(x \mid e^*\right) - g'\left(e^*\right)\right\} \end{aligned} \tag{5}$$

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[3] The standard nature of the framework means that the Holmstrom (1979) sufficient statistic result applies: an optimal sharing rule should not condition on any random variable that does not provide information about an agent's effort. Introducing randomisation to the sharing rule is never optimal.

[4] The dual of this problem is a standard single risk-neutral principal and single risk-averse agent problem.

$$\max_{s(x),e^*} \int \left[x - s\left(x\right)\right] dF\left(x \mid e^*\right)$$

$$\text{s.t.} \quad e^* \in \arg\max_{e} \int u\left(s\left(x\right)\right)\,dF\left(x \mid e\right) - e$$

$$\int u\left(s\left(x\right)\right)\,dF\left(x \mid e\right) - e \geq u^*$$

where u^* is the level of utility that solves the problem in the text.

where λ is the Lagrange multiplier on the incentive compatibility constraint and μ that on the adding-up condition. Both μ and λ should be positive (see Mirrlees, 1974; Jewitt, 1988). Then $s(\cdot)$ must be chosen to maximise the integral

$$\int \{[u(s(x)) - \mu s(x)] f(x \mid e) + \lambda\, u(s(x)) f_e(x \mid e)\}\, dx,$$

pointwise maximisation of which gives the first order condition

$$u'(s(x))\,[f(x \mid e) + \lambda f_e(x \mid e)] - \mu f(x \mid e) = 0.$$

This gives a solution for the function $s(\cdot)$ in terms of three real scalar variables e, λ and μ:

$$u'(s(x)) = \frac{\mu}{1 + \lambda h(x \mid e)} \quad \text{where} \quad h(x \mid e) = \frac{f_e(x \mid e)}{f(x \mid e)}. \tag{6}$$

The unknowns in equation (6) are determined by the first-order condition with respect to effort,

$$\int [u(s(x)) + \mu(x - s(x))] f_e(x \mid e)\, dx - g'(e)$$
$$+\lambda \int u(s(x))\; f_{ee}(x \mid e)\, dx - \lambda g''(e) = 0, \tag{7}$$

the adding-up constraint (3) and the incentive compatibility constraint (4). In general, it is not possible to give a closed-form solution for $s(\cdot)$, even with simple utility functions.

Solution (6) corresponds to the solution in Varian (1980). We can interpret λ, the Lagrange multiplier on the incentive compatibility constraint, as measuring the extent to which the optimum deviates from complete insurance $(\lambda = 0)$. Whether an individual does better or worse than under complete insurance depends on the sign of the likelihood ratio, $h(x \mid e)$, which measures the impact of unobservable effort on the log-likelihood of output, as in Holmstrom (1979). As h differs from zero, the impact of additional effort can be more reliably identified and it is optimal for the tax schedule to provide less social insurance.

Milgrom (1981) first discussed how the likelihood ratio h changes as x increases. For an individual's net income to be increasing in x, h must be increasing in x (the monotone likelihood ratio property). Jewitt (1988) imposes concavity of h as one of the sufficiency conditions for the first-order approach to be valid. Additionally, we wish to rule out the possibility of coming arbitrarily close to the first best by putting a very large penalty on the worst outcome, while allowing almost full risk sharing for any other realisation of x (Mirrlees, 1974). This solution would invalidate the interior solution (6) because the sharing rule is discontinuous. This requires that h must be bounded below on the support of output. We discuss these restrictions further in section 3.1 below.

3 Optimal Social Insurance

In this section, we consider how much social insurance is optimal, and how this depends on the strength of the precautionary motive and the extent of uncertainty. We cannot answer these questions without solving explicitly for the optimal level of effort and the value of the two Lagrange multipliers. Varian (1980) does provide some answers to these comparative static questions but treats the Lagrange multipliers as fixed when changing the model parameters. As he recognises, this is at best an approximation.

It is possible, however, to solve for the marginal tax rate and the way the marginal tax rate changes with income, as functions of the endogenous variables, assuming the first order approach to be valid. The marginal tax rate, $1 - s'(x)$, can be found by differentiating solution (6) and rearranging.

$$MTR = 1 - \frac{1}{-\frac{u''}{u'}} \frac{\lambda}{1 + \lambda h} \frac{\partial h}{\partial x} < 1 \tag{8}$$

The MLRP implies $\frac{\partial h}{\partial x} > 0$ and so restricts the marginal tax rate to be less than 100%.

Varian (1980) performs comparative statics on MTR by changing risk aversion, $-\frac{u''}{u'}$, holding e, λ and μ constant and finds that an increase in risk aversion increases marginal tax rates. However, this approach is not in general valid because of the effect of changes in risk aversion on e, λ and μ, as shown by the numerical examples below.

It is possible to derive an expression for how the marginal tax rate changes as income changes. Taking the derivative of equation (8) and rearranging using equation (6),

$$\frac{\partial MTR}{\partial x} = -\frac{1}{-\frac{u''}{u'}} \frac{\lambda}{1+\lambda h} \left\{ \frac{\lambda}{1+\lambda h} \left(\frac{\partial h}{\partial x} \right)^2 [\, -2 + P(s)] + \frac{\partial^2 h}{\partial x^2} \right\} \tag{9}$$

where

$$P(s) = -\frac{u'''(s)}{u''(s)} \bigg/ \frac{u''(s)}{u'(s)}. \tag{10}$$

If $P(s)$ is sufficiently large then $\frac{\partial MTR}{\partial x} < 0$. In the case of the likelihood ratio h being linear in x, $\frac{\partial MTR}{\partial x} < 0$ if and only if $P(s) > 2$. We can interpret $P(s)$ as measuring the relative importance of precautionary incentives and social insurance. If $P(s)$ is sufficiently large, the incentive effects of uncertainty in post-tax income dominate any loss of insurance, and the marginal tax rate declines with income. In other words, stronger precautionary motives leads to a less progressive optimal tax schedule.

The expression $P(s)$ is obtained by taking the derivative of risk tolerance with respect to s,

$$P(s) = 1 + \frac{\partial \left(-\frac{u'(s)}{u''(s)} \right)}{\partial s}.$$

Since the HARA class of utility functions is defined by risk tolerance, $-\frac{u'}{u''}$, being linear in wealth, this implies $P(s)$ is a constant. For isoelastic utility, $P(s) = \frac{1+\gamma}{\gamma}$,which declines as γ, the coefficient of relative risk aversion, increases. If $h(x \mid e)$ is linear in x, the tax schedule will have a declining marginal tax rate if $\gamma < 1$, and an increasing marginal tax rate if $\gamma > 1$ as shown by Tuomala (1984). With logarithmic utility, the marginal tax rate will be constant. If $u'''(s) = 0$ and so there are no prudence effects (for example if utility is quadratic) and if $\frac{\partial^2 h}{\partial x^2} = 0$, it is unambiguous that the marginal tax rate will increase with x. Therefore, allowing for precautionary motives reduces the progressivity of the tax schedule.[5] Strawczynski (1998) compares optimal linear taxes using a logarithmic utility function with optimal taxes using a quadratic utility function, and concludes that marginal tax rates will be greater when the precautionary motive is taken into account (there is no precautionary motive with quadratic utility). However, this comparison varies more than simply the precautionary motive, and so it is hard to attribute the greater marginal tax rates to the existence of prudence. Moreover, his model does not include leisure in the utility function, and thus disregards the incentive effects of uncertainty. In general, higher marginal tax rates are due to greater risk aversion which measures the welfare cost of uncertainty rather than due to prudence which measures the incentive effect of uncertainty.

Equation (9) is valid only if it is appropriate to use the first-order approach. Sufficient conditions for this given by Jewitt (1988) require restrictions on the distribution of output and on the utility function. The Jewitt conditions require the likelihood ratio to be concave in output at any effort level which in turn restricts the optimal sharing rule; other things being equal, the marginal tax rate will be increasing in x. Concavity of h means that when x is high increases in x yield little extra information about effort, and so there is no incentive from rewarding such increases. By contrast, if h is convex, at high values of x changes in x contain more information about effort and so the sharing rule should reward such increases. Imposing concavity of h is unnecessarily restrictive because the Jewitt conditions are suf-

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[5] If we use non-HARA utility, then the ratio $P(s)$ will depend on s. One assumption would be that risk tolerance increases at an increasing rate with consumption, which implies that $P'(s) > 0$. The simplest example where risk tolerance increases with s at an increasing rate is given by: $\frac{\partial\left(-\frac{u'(s)}{u''(s)}\right)}{\partial s} = -1 + ks$. This means that at low wealth, relative risk aversion is high, but as wealth increases, relative risk aversion falls. Again maintaining the assumption that $\frac{\partial^2 h}{\partial x^2} = 0$, equation (9) then suggests that at low values of x the marginal tax rate will be increasing with x, but at high values, the marginal tax rate will be decreasing.

ficient but not necessary and we may, by construction, rule out cases where the marginal tax rate is declining with output. By contrast, in our numerical analysis, we carry out comparative statics on the concavity/convexity of the likelihood ratio, while checking numerically that the first-order approach is still valid. This means we impose weaker restrictions on the likelihood ratio than those required by the Jewitt sufficient conditions.

We need to use numerical methods to obtain comparative static results in which the endogenous variables, λ, μ and equilibrium effort vary. In particular, we compute how much social insurance is provided by the tax schedule. However, there are many metrics for insurance. Simply comparing the distributions of pre-tax and post-tax incomes has the problem that the distribution of pre-tax income is itself affected by the amount of insurance self-provided through changes in equilibrium effort. When pre-tax income is endogenous, it is more reasonable to compare the distribution of income under autarky (when every agent consumes their own output) with the distribution of net income under the optimal social insurance. In the numerical simulations below, we report Gini coefficients for the autarkic distribution and the optimal sharing distribution. However, a further problem is that comparing distributions of net income could lead to different results from comparing distributions of utility because effort under autarky differs from effort under optimal social insurance. Thus, standard comparisons of income distributions, such as Gini coefficients, may be misleading because they ignore the welfare cost of different effort levels. Therefore, we also report a measure of the monetary equivalent of the insurance provided by the tax schedule that takes account of the difference in effort. Define ρ by

$$\int u\left(s\left(x\right)-\rho\right)f\left(x\mid e^{s}\right)dx-g\left(e^{s}\right)=\int u\left(x\right)f\left(x\mid e^{a}\right)dx-g\left(e^{a}\right)\qquad(11)$$

where e^{s} is effort under optimal social insurance and e^{a} effort under autarky. ρ is a monetary measure of the benefit to the representative agent of social insurance against income fluctuations.[6] Given the well known problems of

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[6] This measure is appropriate because individuals are ex-ante identical. Clearly the

using single index measures to compare income distributions, we also show diagrams of how net income and the marginal tax schedule change with gross income.

3.1 Parameterisation

The model is analytically intractable in its general form, and so we use a particular, yet flexible, parameterisation.

Preferences We assume that the utility of consumption and disutility of effort have the forms

$$\begin{aligned} u(c) &= \frac{c^{1-\gamma}}{1-\gamma},\ \gamma > 1 \\ g(e) &= e. \end{aligned}$$

The utility function is of the HARA class to aid interpretation of the results.

We specify the disutility of effort to be linear and in the next subsection we specify the way that effort affects output. The relationship between the disutility of effort and output depends on both the function g and the effort-output relationship. Decreasing returns to effort can be expressed either by making the disutility of effort convex or by making the effect of effort on output concave. The two approaches are equivalent and we follow the latter.

The relationship between effort and output The numerical solution for the optimal sharing rule, (6), requires that the first-order approach be valid and that an interior solution exists (in other words, that a Mirrlees-style penalty solution is not optimal). One way of ensuring that the first-order approach is valid would be to choose functional forms which satisfy the Jewitt (1988) sufficient conditions. This has two problems. First, as discussed above, these sufficient conditions rule out certain types of optimal sharing

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inclusion of ρ will change the optimal choice of e^s and $s(x)$, but we ignore these second order effects in this calculation.

rule by construction. In particular, cases where the optimal sharing rule is more likely to be regressive are ruled out by the assumption of concavity of the likelihood ratio, even though this is not a necessary condition for the first order approach to be valid. Second, the Jewitt conditions do not guarantee that an interior solution to the problem exists, only that when such a solution exists the first order approach is valid. For an interior solution to exist, we require that the likelihood ratio is bounded below. Our approach is to specify a class of distributions for which the likelihood ratio is bounded, and then check numerically that the first-order approach is valid.[7] This means that we are not restricted to cases satisfying the Jewitt (1988) sufficient conditions and are able to carry out comparative statics on the degree of concavity or convexity of the likelihood ratio without imposing any unnecessary restrictions on the progressivity of the tax schedule.

We assume that output is distributed over a compact support $x \in [0,1]$ with a differentiable pdf. Changes in effort modify the pdf and this deformation of the output distribution can be characterised by a partial differential equation

$$f_e\left(x|\,e\right) = -\frac{\partial}{\partial x}\left(\tau\left(x|\,e\right) f\left(x|\,e\right)\right). \tag{12}$$

The marginal effectiveness of effort in shifting the distribution of output to the right is given by $\tau\left(x|\,e\right)$. For convenience, we assume that τ takes the multiplicatively separable form $\tau\left(x|\,e\right) = a\left(x\right) b\left(e\right)$, which permits an explicit solution of the PDE, as shown in the appendix. We require that $a\left(x\right) \rightarrow 0$ as $x \rightarrow 0, 1$, so that the solution is a well-defined probability density for all e. The function $b\left(e\right)$ captures the marginal effectiveness of effort and is assumed to be decreasing.

In order to solve the PDE, we require a boundary condition specifying a distribution of output and hence the curvature of the likelihood at a given effort level. In the appendix, we show that both boundedness and

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[7] The validity of the first-order approach is checked by finding a solution assuming the first-order approach and then checking that expected utility is concave in effort at the optimal sharing rule.

monotonocity of the likelihood conveniently depend only on this boundary condition and the function $a(x)$.

Specifically, we take $a(x) = x(1-x)$ and suppose that output is Beta-distributed at a reference effort level.[8] This reference effort level is set by a parameter θ which determines the curvature of the likelihood ratio. For $\theta > 0$, if $e = \theta^{\frac{1}{\eta}}$, then the likelihood ratio is linear and we refer to this effort level as the linear likelihood ratio (LLR) effort level. If effort is greater than this reference level and so $e > \theta^{\frac{1}{\eta}}$, then the likelihood ratio becomes convex; whereas if $e < \theta^{\frac{1}{\eta}}$, the likelihood ratio becomes concave. By varying θ, we can change the curvature of the likelihood ratio at any given effort level. For $\theta < 0$, the likelihood ratio will be convex for all positive effort levels. We show in the appendix that this formulation satisfies the MLRP and has a bounded likelihood ratio.

The parameters of the Beta distribution, q and r, are chosen so that the distribution is skewed right at LLR effort, but as effort increases, probability mass moves to the right, and the distribution becomes increasingly symmetric and ultimately left skewed. It is difficult to use data to motivate these assumptions as observed income distributions reflect differences in ability as well as differences in effort. Therefore, the baseline distribution is not as skewed as observed gross income distributions. The characterisation used here means that effort tends to move individuals up the income distribution, and average output is increasing in effort. Further, an increase in effort can lead to a reduction in the variance of output. Distributions of output given effort are shown in figure 1.

Using this formulation, we solve numerically for e^*, λ and μ using the first-order condition (7), and the constraints (3) and (4). Parameters used are given in table 1. The values of γ considered encompass the range of estimates commonly found in the literature for the degree of risk aversion over consumption (discussed in Attanasio, 1999). The level of uncertainty is

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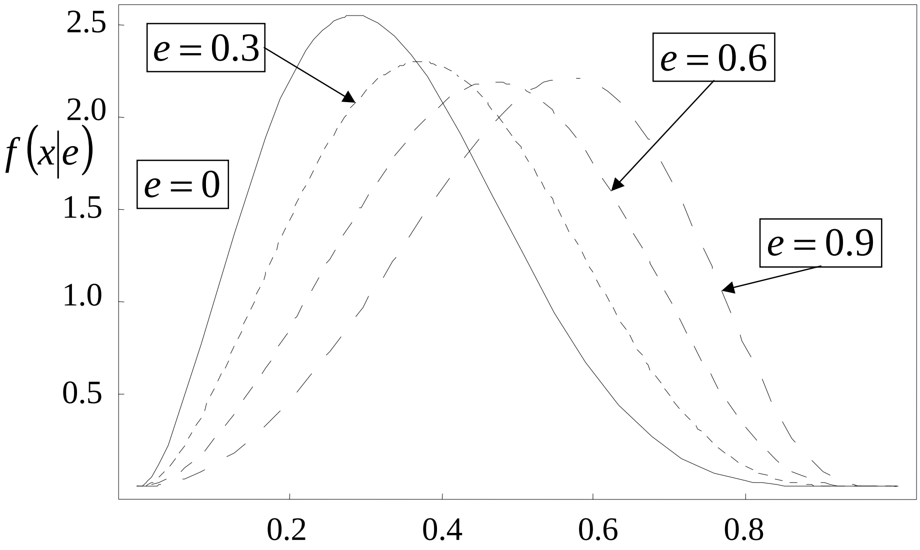
[8] The function $b(e)$ is set so that the first-order approach is valid over the range of parameters considered. In practice, $b(e) = \eta e^{\eta - 1}$ with a value of $\eta = 0.2$.

Figure 1: The Effect of Effort on the Distribution of Output



The graph shows how effort changes the probability distribution. The effort level giving a linear likelihood ratio is set at 0 (ie. $\theta = 0.0$) so the line "e=0" is the distribution if LLR effort is expended This baseline distribution is given by the Beta Distribution with parameters $q = 3$ and $r = 6$.

the variance of output at LLR effort: our baseline case has a Gini coefficient of 0.26. This is less than the level of inequality typically observed: for example, for 1998, the Census Bureau report a pre-tax Gini for the US of 0.46. However, in our model individuals are ex-ante identical, and baseline inequality is due only to different realisations of uncertainty, and so we set a lower value than the observed Ginis.

Table 1: Parameters for Numerical Solution

Parameter	*Baseline*	*Alternatives*
γ	1.5	1.2, 3.0, 5.0
σ^2	0.022 (0.26)	0.044(0.36), 0.011(0.18), 0.0022(0.08)
θ	0.0	-1.0, 1.0

Values of the variance are for the distribution at effort equal to the linear likelihood ratio level with $\theta = 0.0$. Corresponding Gini coefficients are given in brackets. Changes in the value of the variance are introduced through changing the parameters of the Beta distribution (q and r), keeping the mean constant.

3.2 Comparative Statics on Social Insurance

We now consider the extent to which the optimal tax provides social insurance. The section first discusses social insurance at the baseline parameterisation. We then present comparative statics changing γ, the variance of the distribution of output and finally, the curvature of the likelihood ratio. There is no presumption that the optimal tax schedule should provide insurance at all income levels; for example, it might be optimal to increase uncertainty over payoffs relative to autarky by having negative marginal tax rates at some income levels. For each comparative static exercise, we compare income distributions, and present results on the marginal tax rates across the entire income distribution.

The baseline parameterisation Results for the baseline parameterisation are shown in bold in table 2 and in the unbroken lines in figure (2). The first two columns in table (2) compare effort under autarky with effort under the optimal tax schedule: allowing for optimal social insurance reduces effort relative to autarky. When individuals are prudent, uncertainty increases expected marginal utility and causes individuals to put in extra effort. Since social insurance reduces uncertainty, this reduces the need for precautionary effort. This difference in effort levels is a measure of the extent to which social insurance reduces self-insurance. The table also reports Gini coefficients: under autarky, self-insurance reduces the Gini from 0.26 to 0.20, while social insurance reduces the Gini substantially more to 0.09. Further, individuals are willing to pay 8.4% of mean output to avoid having to self-insure (the value of ρ, the monetary value of social insurance defined in equation 11). The increase in the gross income Gini under social insurance (0.22) relative to the Gini under autarky (0.20) shows the extent to which self-insurance is crowded out. This is in contrast to Sinn (1996), who shows that social insurance can increase post-tax inequality by encouraging risk-taking activities. In this baseline parameterisation, marginal tax rates are declining with output, from 65% to 40%, as shown in figure (2). Marginal tax rates are high for all output levels relative to estimates from models of adverse selection (Mirrlees, 1971).

Increasing γ The effects of increasing γ are shown in table 2 and in figure 2. Both under autarky and under social insurance, a higher value of γ leads to greater effort. Under autarky, a higher γ means individuals are more prudent, and so are more willing to put in effort. This direct incentive effect of increased prudence occurs under social insurance, but in this case there is also an indirect effect through changes to the optimal amount of social insurance. A higher γ means a fall in the ratio of prudence to risk aversion, $P(s)$, and so the welfare cost of uncertainty is greater relative to the incentive effect, increasing the optimal amount of social insurance. This greater social insurance reduces effort, offsetting the direct incentive effect of increased prudence. Table 2 shows that the direct effect of increased prudence dominates.

Table 2: Extent of Social Insurance varying γ

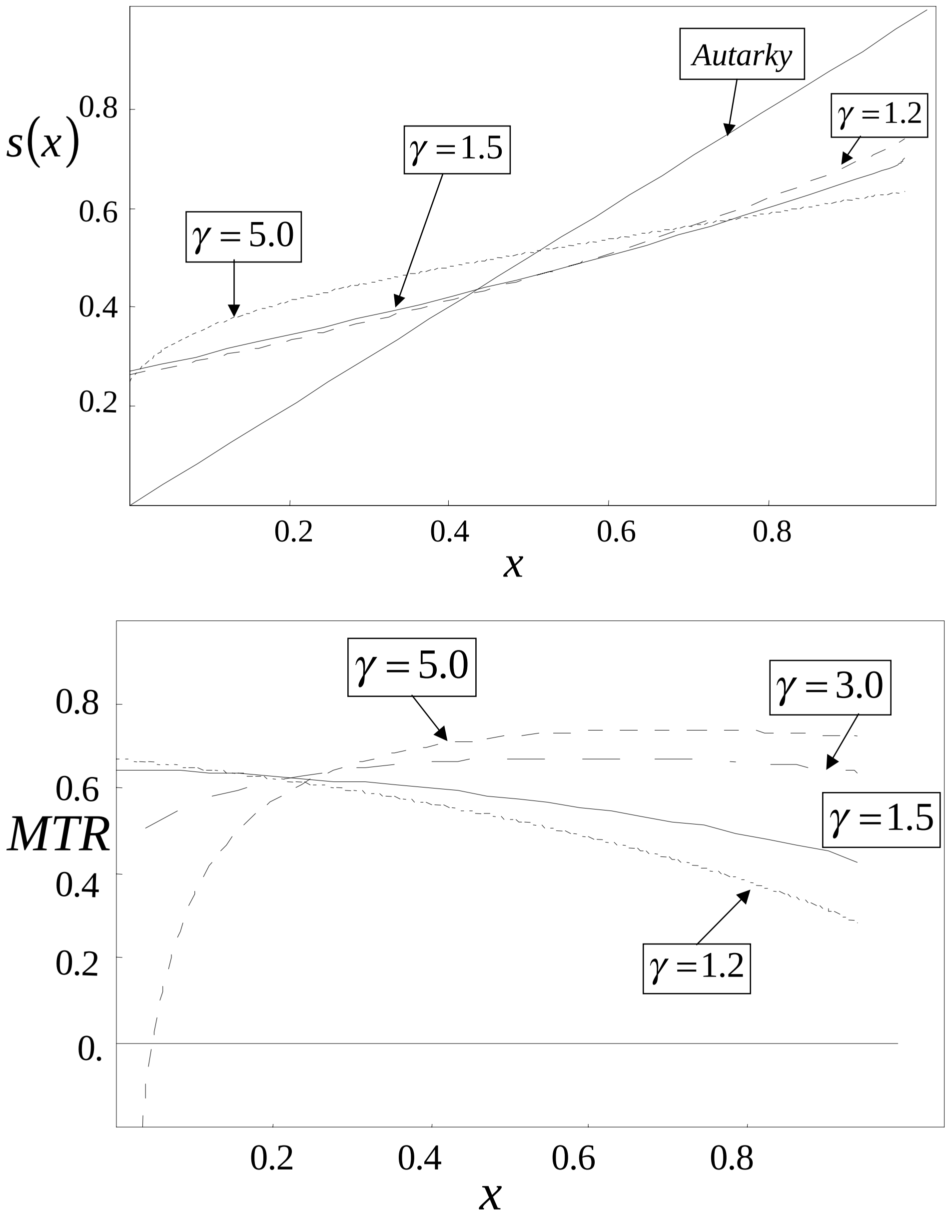
	Effort		*Gini Coeff*				$\frac{\rho}{mean\ output}$
Model	*Aut*	*Soc Ins*	*Aut*	*Soc Ins*	*Gross*	*LLR*	
$\gamma = 1.2$	0.59	0.47	0.21	0.098	0.22	0.26	0.057
$\gamma = 1.5$	**0.65**	**0.50**	**0.20**	**0.088**	**0.22**	**0.26**	**0.084**
$\gamma = 3.0$	0.96	0.63	0.18	0.069	0.21	0.26	0.205
$\gamma = 5.0$	2.74	0.82	0.06	0.054	0.19	0.26	0.488

Columns with *Aut* give results under autarky, columns with *Soc Ins* give results under optimal social insurance, the column with *Gross* gives the Gini for the distribution of output before the tax schedule reallocates, and the column with *LLR* gives the gross Gini coefficient at the linear likelihood ratio effort level, $e = 0.0$. Other parameter values: $q = 3.0, r = 6.0, \theta = 0.0$.

The remaining columns in table 2 show the effect on self-insurance and social insurance of changes in γ. Increases in γ increase the amount of self-insurance substantially: at $\gamma = 5.0$, self-insurance reduces the Gini from 0.26 to 0.06, compared to a reduction of 0.26 to 0.20 at the baseline. Increases in γ also increase the amount of social insurance, reducing the Gini to 0.088 at the baseline and to 0.054 at $\gamma = 5.0$. There is also substantial crowding-out of self-insurance by social insurance at $\gamma = 5.0$: effort falls to 30% of its autarkic level and the Gini for gross income is 0.19, compared to 0.06 under autarky. Further, for high γ, self-insurance can provide similar levels of insurance as social insurance, but the high effort required has a large welfare cost: at $\gamma = 5.0$, individuals would be willing to pay almost 50% of gross income to avoid having to self-insure.

One problem with the results in table 2 is that single index measures of

Figure 2: Social Insurance Varying γ



The first graph shows how the net output varies with gross output for different values of γ. The second graph shows how the marginal tax rate varies with gross output. Other parameters: $q = 3, r = 6$ and $\theta = 0.0$

the distribution give little indication of the progressivity or otherwise of the tax schedule. Therefore, figure 2 shows both the social insurance schedule and the marginal tax rate as output changes. The graph of marginal tax rates shows a result similar to that in equation (9) above:[9] when the ratio $P(s)$ is high (i.e. γ is low), prudence effects dominate and the marginal tax rate falls with output. However, when γ is large, the incentive effects of prudence are less important relative to risk aversion, and marginal tax rates rise with output. Further, the effect of increasing γ on marginal tax rates varies across the income distribution: marginal tax rates are lower with higher values of γ at the bottom of the distribution, but higher with higher values of γ at the top of the distribution. This is in contrast to the partial equilibrium approaches in Varian (1980) and in equation (8) above that hold e, μ and λ constant, where an increase in γ leads to an increase in the marginal tax rate across the entire distribution. The graph of social insurance shows that when γ is high, individuals are incentivised through penalties at low output with significant insurance elsewhere; however, when individuals are less risk averse, they are incentivised through rewards for high output with greater insurance elsewhere.

Increasing uncertainty The effects of increasing underlying uncertainty are shown in table 3 and in figure 3. Under autarky, effort increases as the variance of gross output increases. This is simply the precautionary motive at work: greater uncertainty increases expected marginal utility and so individuals increase effort. This precautionary effect applies under social insurance, but there is also an indirect effect through changes in the amount of social insurance. Greater uncertainty means that a given realisation of x gives less information about effort expended; and greater uncertainty increases the welfare benefit of insurance. These two indirect effects lead to

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[9] If effort was at the linear likelihood ratio level, we could use equation (9) to determine how marginal tax rates change with output. In all cases shown however, optimal effort is higher than LLR effort and so the likelihood is convex and equation (9) is ambiguous.

more social insurance, and this reduces effort, offsetting the direct effect on effort of greater precautionary effort. In table 3, the indirect effect of increased insurance dominates and effort under the optimal tax schedule falls as uncertainty increases. Further, insurance increases as the underlying uncertainty increases. In particular, the difference in the Gini coefficients between autarky and social insurance are greatest (and the value of ρ is also greatest) when uncertainty is greatest. These conclusions are reiterated in the graphs showing marginal tax rates and the social insurance schedule in figure 3. Social insurance and marginal tax rates are greater when underlying uncertainty is higher.

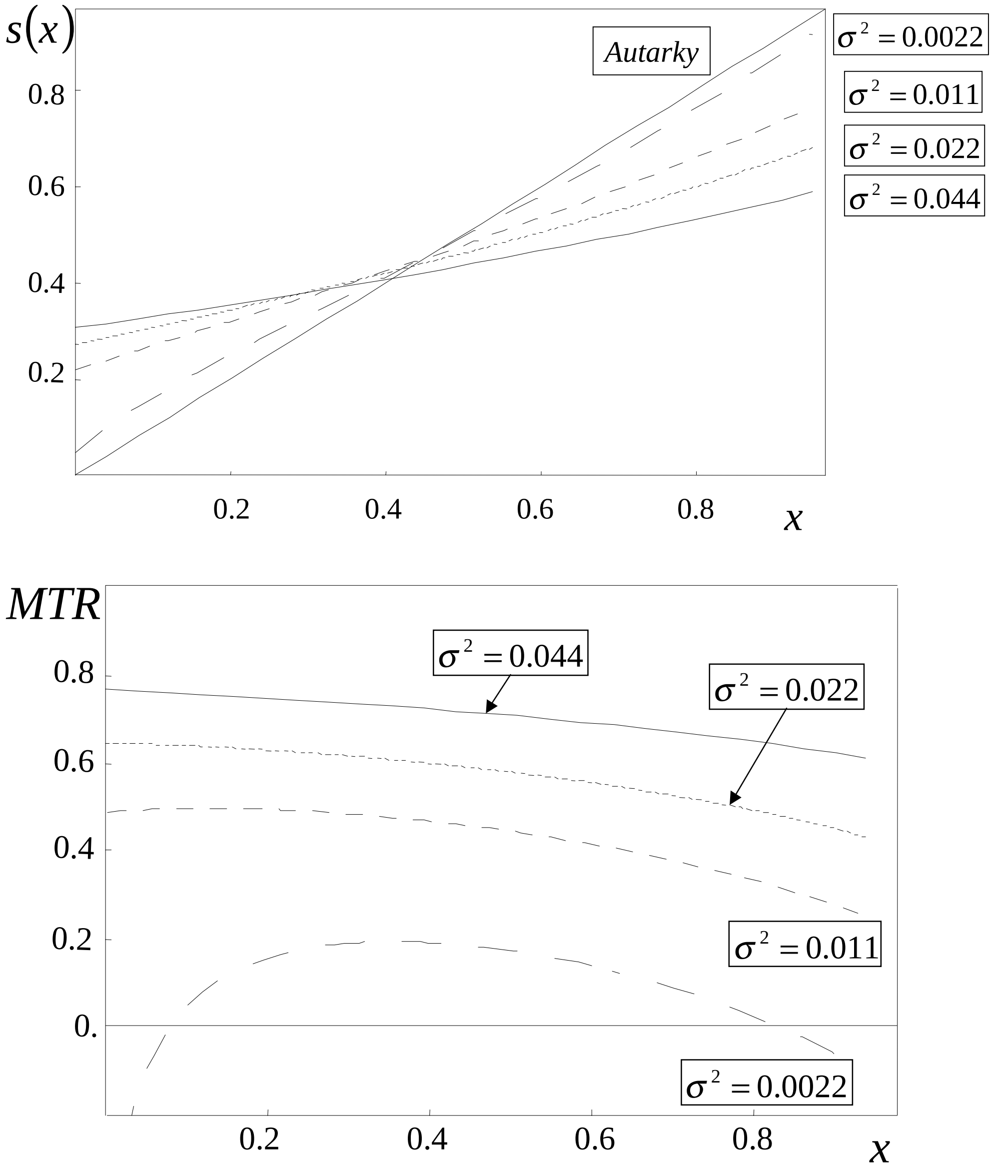
Table 3: Extent of Social Insurance Changing the Variance

	Effort		*Gini Coeff*				$\frac{\rho}{mean\ output}$
Model	*Aut*	*Soc Ins*	*Aut*	*Soc Ins*	*Gross*	*LLR*	
$\sigma^2 = 0.0022$	0.62	0.60	0.06	0.052	0.07	0.08	0.002
$\sigma^2 = 0.011$	0.63	0.54	0.14	0.080	0.15	0.18	0.027
$\boldsymbol{\sigma^2 = 0.022}$	**0.65**	**0.50**	**0.20**	**0.088**	**0.22**	**0.26**	**0.084**
$\sigma^2 = 0.044$	0.69	0.45	0.29	0.089	0.32	0.36	0.281

Columns with *Aut* give results under autarky, columns with *Soc Ins* give results under social insurance, the column with *Gross* gives the Gini for the distribution of output before the tax schedule reallocates, and the column with *LLR* gives the Gini coefficient at $e = 0.0$ where the likelihood ratio is linear. Parameters q and r are varied to keep the mean of the Beta distribution at LLR effort constant as the variance changes. Other parameter values: $\gamma = 1.5, \theta = 0.0$.

Varying θ and the curvature of the likelihood ratio We now consider the effect of varying the concavity of the likelihood ratio by changing the parameter θ. For $\theta > 0$, the likelihood ratio is linear at $e = \theta^{\frac{1}{\eta}}$. For $\theta < 0$,

Figure 3: Social Insurance Changing the Variance



The first graph shows how net output varies with gross output for different values of the underlying variance. The second graph shows how the marginal tax rate varies with gross output. Parameters q and r are varied to keep the mean of the Beta distribution at LLR effort constant as the variance changes. Other parameters: $\gamma = 1.5$, and $\theta = 0.0$

the likelihood ratio is convex for all effort levels.[10] The comparison is made holding the mean and variance of output constant for a fixed level of effort equal to optimal effort in the baseline case. Thus differences in the extent of social insurance or marginal tax rates are not due to differences in the mean or variance of output, but rather are due largely to differences in the curvature of the likelihood ratio. This approach is reasonable provided optimal effort under alternative values of θ does not differ too far from the optimal e in the baseline.

Figure 4 shows the likelihood ratio for different values of θ. We choose a wide spread for θ so we consider cases where the likelihood ratio is concave, respectively convex. These differences in the likelihood ratio affect the shape of the insurance schedule and marginal tax rates shown in figure 5. When $e^{\eta} < \theta$ at equilibrium, the likelihood ratio is concave and little is learned about effort at high values of x. This means the marginal tax rate is initially low, but is increasing in output. When $e^{\eta} > \theta$ at equilibrium, the likelihood ratio is convex and little is learned about effort at low values of x and the marginal tax rate starts high, but is decreasing in output. These results confirm the implication of equation (9) that a more concave likelihood ratio implies marginal tax rates will be rising with output. Table 4 shows that equilibrium effort in the three cases is almost identical. This arises because we are imposing the same mean and variance on the distribution at an effort level close to the optimum. Given the effort, mean and variance are almost identical in the three models, the amounts of insurance in equilibrium are similar. The curvature of the likelihood ratio (about which it is hard to make empirical statements) does not have a large effect on the amount of insurance in equilibrium, once we hold the mean and variance constant. Similarly, the shape of the insurance schedule close to mean output is little affected. On the other hand, the curvature of the likelihood ratio makes a

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[10] We have assumed a finite domain for output and so for any value of θ, there will be some effort level at which the likelihood ratio becomes convex (and the Jewitt (1988) conditions for the first-order approach will be violated).

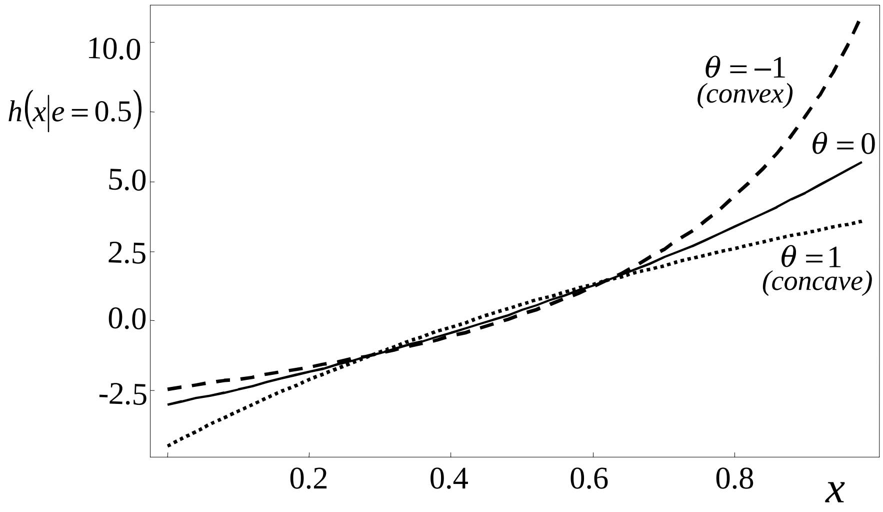
big difference to the extremal properties of the marginal tax rate: when the likelihood ratio is concave marginal tax rates rise from 0 to 80%; whereas when it is convex, marginal tax rates fall from 80% to -80%.

Table 4: Extent of Social Insurance varying the curvature of the likelihood ratio

	Effort		*Gini Coeff*				$\frac{\rho}{mean\ output}$
Model	*Aut*	*Soc Ins*	*Aut*	*Soc Ins*	*Gross*	*LLR*	
$\theta = 1.0$	0.65	0.50	0.20	0.085	0.22	0.17	0.073
$\boldsymbol{\theta = 0.0}$	**0.65**	**0.50**	**0.20**	**0.088**	**0.22**	**0.26**	**0.084**
$\theta = -1.0$	0.65	0.50	0.20	0.088	0.22	$--$	0.095

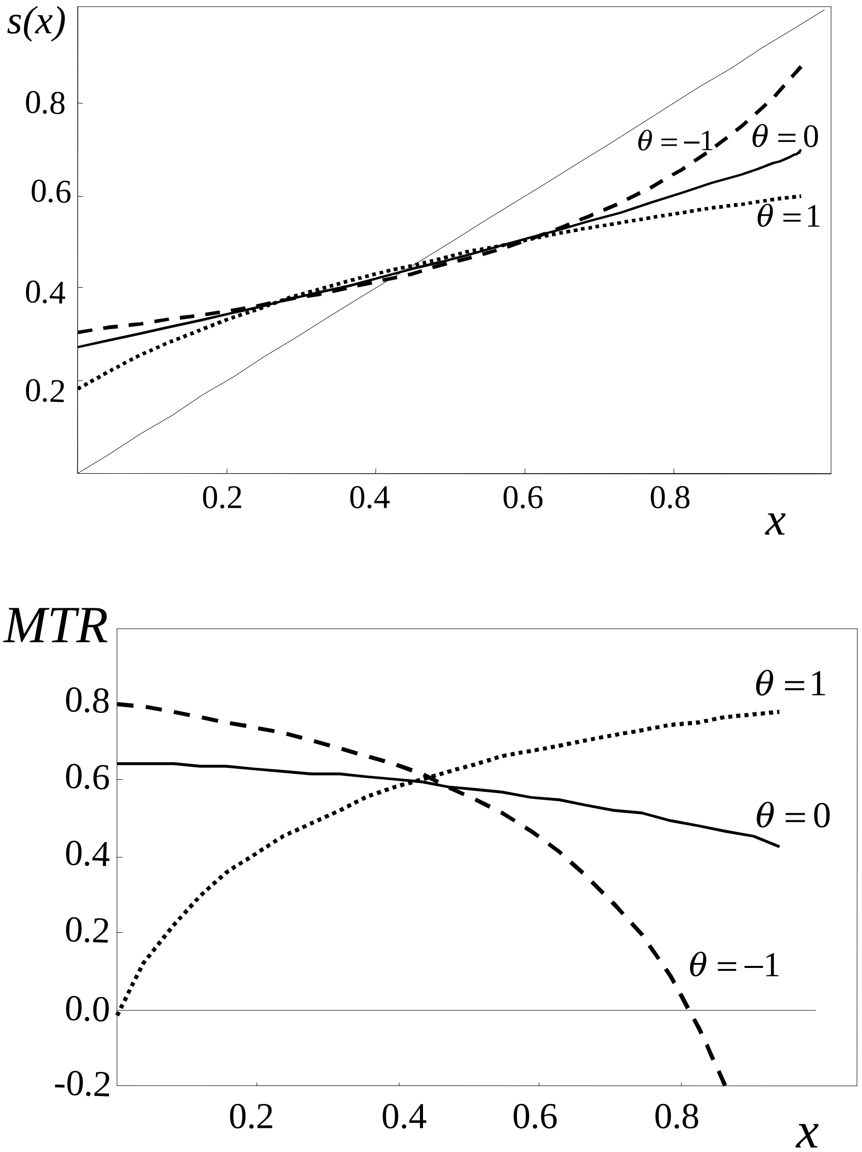
Columns with *Aut* give results under autarky, columns with *Soc Ins* give results under social insurance, the column with *Gross* gives the Gini for the distribution of output before the tax schedule reallocates, and the column with *LLR* gives the Gini coefficient at $e = \theta^{\frac{1}{\eta}}$. When $\theta = 1.0$, optimal effort is less than *LLR* effort, and so the gross Gini is greater than the *LLR* Gini. $\gamma = 1.5$.

Figure 4: Changing the Curvature of the Likelihood Ratio



The graph shows the likelihood ratio varying θ evaluated at $e = 0.5$. Parameters q and r are varied to keep the mean and variance of the Beta distribution at $e = 0.5$ constant as θ changes. $\gamma = 1.5$

Figure 5: Social Insurance changing Concavity of the Likelihood



The first graph shows how the net output varies with gross output for different values of θ. The second graph shows how the marginal tax rate varies with gross output for different values of of θ. Parameters q and r are varied to keep the mean and variance of the Beta distribution at $e = 0.5$ constant as θ changes. $\gamma = 1.5$

4 Conclusions

This paper provides a characterisation of the trade-off between the welfare benefit of social insurance and the incentive effect of uncertainty in optimal tax schedules. Social insurance through redistributive taxation increases social welfare, but reduces equilibrium effort, firstly by reducing the average marginal product of effort and secondly by weakening the precautionary motive for effort. The main contribution of this paper is to highlight the effect of social insurance on precautionary motives, and to show how this precautionary motive can be exploited in designing optimal tax schedules. We provide a way of specifying principal-agent problems that permits tractable numerical solutions. We use this method to characterise the nature of the trade-off between insurance and the incentive effects of risk. By using numerical methods we are able to perform full comparative statics rather than a partial equilibrium approach holding endogenous variables fixed.

The level of marginal tax rates in our numerical solutions are generally high: individuals are willing to face high marginal tax rates because of the uncertainty they face. When choices are made after uncertainty is resolved (as in Mirrlees, 1971), individuals are less willing to face high marginal tax rates.

Effort increases with the coefficient of relative risk aversion, γ, and increases with uncertainty, holding the tax schedule constant. Effort falls if the tax schedule provides greater risk-sharing: public insurance crowds out self-insurance. Increases in γ reduce the ratio of prudence to risk aversion, P, leading to an increase in optimal social insurance and an offsetting fall in effort. The overall effect of increasing γ varies across the income distribution, lowering marginal tax rates at the bottom of the distribution, but increasing them at the top. Similarly, an increase in uncertainty increases optimal social insurance, leading to an offsetting fall in effort.

Whether marginal tax rates fall or rise with income depends on the ratio, P. When the ratio is large, the incentive effects of prudence dominate and the marginal tax rate falls with output. When the ratio is small, the risk sharing benefit dominates and the marginal tax rate rises with output. Finally, we show that increasing the concavity of the likelihood ratio, holding the mean and variance of the distribution of output constant, has a strong effect on optimal marginal tax rates, causing marginal tax rates to increase with output.

APPENDIX

Deformation method for modelling the effort-output relationship
Output is distributed over a finite support $[\underline{x}, \bar{x}]$. As effort increases the probability distribution of x shifts toward higher values. Consider the probability mass on a small range of outcomes $(x, x+\delta x)$, which is approximately $f(x|\, e)\, \delta x$, and then suppose that effort increases by δe. Probability mass moves to the right, with the *proportionate* rate of movement being given by $\tau(x|\, e)\, \delta e$. Thus, the range $(x, x+\delta x)$ receives an inflow of probability from the left of $\tau(x|\, e)\, f(x|\, e)\, \delta e$ and produces an outflow to the right of $\tau(x+\delta x|\, e)\, f(x+\delta x|\, e)\, \delta e$. Therefore, the change in probability mass on $(x, x+\delta x)$ is

$$(f(x|\, e+\delta e) - f(x|\, e))\, \delta x = -(\tau(x+\delta x|\, e)\, f(x+\delta x|\, e) - \tau(x|\, e)\, f(x|\, e))\, \delta e$$

Taking the limits of δx and δe,

$$f_e(x|\, e) = -\frac{\partial}{\partial x}(\tau(x|\, e)\, f(x|\, e)) \tag{13}$$

In order for $f(x|\, e)$ to be a properly defined family of density functions, no mass must flow out of the edges of the support and so $\tau(\underline{x}, e) = \tau(\bar{x}, e) = 0$. Thus, if

$$\int_{\underline{x}}^{\bar{x}} f(x \mid e)\, dx = 1$$

for any particular e, this is true for all e.

The partial differential equation (13) for the family of probability distributions f can be explicitly solved in certain simple cases. First, integrating (13) with respect to x gives that

$$\int_{\underline{x}}^{x} f_e\left(s \mid e\right) ds = -f\left(x \mid e\right)\tau\left(x \mid e\right)$$

and so introducing the cumulative distribution F we obtain that

$$F_e = -F_x \tau \tag{14}$$

This PDE can be solved explicitly if we assume that τ takes the separable form $\tau\left(x \mid e\right) = a\left(x\right) b\left(e\right),$ where $a\left(\underline{x}\right) = a\left(\bar{x}\right) = 0$ and a and b are continuously differentiable. In this case we can separate (14) to obtain

$$-\frac{F_e\left(x \mid e\right)}{b\left(e\right)} = F_x\left(x \mid e\right) a\left(x\right).$$

This has an 'almost additive' general solution of the form

$$F\left(x \mid e\right) = \phi\left(A\left(x\right) + B\left(e\right)\right)$$

where ϕ is any function and

$$A'\left(x\right) = \frac{1}{a\left(x\right)} \qquad B'\left(e\right) = -b\left(e\right).$$

Thus the general solution of (14) is

$$F\left(x \mid e\right) = \phi\left[\int_{x^*}^{x} \frac{1}{a\left(s\right)} ds - \int_{\bar{e}}^{e} b\left(s\right) ds\right]. \tag{15}$$

where x^* is some fixed point in the domain $\left(\underline{x}, \bar{x}\right)$. The function ϕ is determined by providing a boundary condition, for example specifying the distribution of output at a particular effort level, $\bar{e}$, where $B\left(\bar{e}\right) = 0$

$$F\left(x \mid \bar{e}\right) = \phi\left[\int_{x^*}^{x} \frac{1}{a\left(s\right)} ds\right].$$

Notice that ϕ has the whole real line as its domain. Since a goes to zero at $\underline{x}$ and $\bar{x}$ we have that

$$A\left(\bar{x}\right) = \int_{x^*}^{\bar{x}} \frac{1}{a\left(s\right)} ds = +\infty \qquad \text{and} \qquad A\left(\underline{x}\right) = \int_{x^*}^{\underline{x}} \frac{1}{a\left(s\right)} ds = -\infty.$$

Proposition 1 *If* $a(x)\frac{f_x(x|\bar{e})}{f(x|\bar{e})}$ *is bounded on* $[\underline{x},\overline{x}]$*, then the likelihood ratio* $h(x \mid e)$ *is bounded on* $[\underline{x},\overline{x}]$ *for any given* e*.*

Proof: Differentiating (15) gives

$$f(x \mid e) = \phi'\left[\int_{x^*}^{x}\frac{1}{a(s)}ds - \int_{\bar{e}}^{e} b(s)\,ds\right]\frac{1}{a(x)}$$

and so

$$f_e(x \mid e) = -\phi''\left[\int_{x^*}^{x}\frac{1}{a(s)}ds - \int_{\bar{e}}^{e} b(s)\,ds\right]\frac{b(e)}{a(x)}.$$

Thus

$$h(x \mid e) = -\frac{\phi''\left[\int_{x^*}^{x}\frac{1}{a(s)}ds - \int_{\bar{e}}^{e} b(s)\,ds\right]}{\phi'\left[\int_{x^*}^{x}\frac{1}{a(s)}ds - \int_{\bar{e}}^{e} b(s)\,ds\right]}b(e) \tag{16}$$

Thus, $h(x \mid e)$ is bounded if $\phi''(y)/\phi'(y)$ is bounded.

We now need sufficient conditions for this. Since ϕ is defined in terms of a and the boundary condition $F(x \mid \bar{e})$, we need to impose conditions on these functions. Now

$$f_x(x \mid \bar{e}) = \phi''\left[\int_{x^*}^{x}\frac{1}{a(s)}ds\right]\frac{1}{a(x)^2} - \phi'\left[\int_{x^*}^{x}\frac{1}{a(s)}ds\right]\frac{a'(x)}{a(x)^2}$$

and so

$$\frac{f_x(x \mid \bar{e})}{f(x \mid \bar{e})} = \frac{\phi''}{\phi'}\frac{1}{a(x)} - \frac{a'(x)}{a(x)}.$$

Thus we obtain

$$\frac{\phi''}{\phi'} = a'(x) + a(x)\frac{f_x(x \mid \bar{e})}{f(x \mid \bar{e})}. \tag{17}$$

a' is bounded above and below on $[\underline{x},\bar{x}]$ as a is continuously differentiable. Thus provided that

$$a(x)\frac{f_x(x \mid \bar{e})}{f(x \mid \bar{e})}$$

is bounded for all x in the domain, then $h(x \mid e)$ is bounded, proving proposition 1. ■

Proposition 2 *If* $a'(x)+a(x)\frac{f_x(x|\bar{e})}{f(x|\bar{e})}$ *is monotone decreasing in* x*, then the MLRP is satisfied.*

Proof: It follows directly from equation (16) that $h(x \mid e)$ will be monotone increasing in x and the MLRP satisfied providing that $\phi''(y)/\phi'(y)$ is monotone decreasing in y. From equation (17) it follows that a necessary and sufficient condition for the MLRP is that $a'(x) + a(x)\frac{f_x(x|\bar{e})}{f(x|\bar{e})}$ is decreasing in x.$\blacksquare$

Particular Parameterisation The easiest way to specify an example which satisfies these conditions is to define the mapping $A(x) : (\underline{x}, \bar{x}) \to \mathbf{R}$ such that $\underline{x}$ goes to $-\infty$ and $\bar{x}$ goes to $+\infty$. We do this in the numerical solution in section 3 by setting $\underline{x} = 0$, $\bar{x} = 1$,

$$\begin{aligned} A(x) &= \ln\left(\frac{x}{1-x}\right) & a(x) = x(1-x) \\ B(e) &= -e^{\eta} + \theta & b(e) = \eta e^{\eta-1} \end{aligned} \tag{18}$$

For the boundary condition, we specify the distribution of output at $e = \theta^{\frac{1}{\eta}}$, which is the effort level where the likelihood ratio is linear (which we call LLR effort, $\bar{e}$) as having a Beta Distribution. In other words,

$$F(x \mid \bar{e}) = \phi\left[\int_{x^*}^{x} \frac{1}{a(s)} ds\right] = CDF\left[\beta_{q,r}(x)\right] \tag{19}$$

$$f(x \mid \bar{e}) = \phi'\left[\int_{x^*}^{x} \frac{1}{a(s)} ds\right] \frac{1}{a(x)} = \beta_{q,r}(x) \tag{20}$$

In general,

$$f(x \mid e) = \phi'\left[A(x) + B(e)\right] \frac{1}{a(x)}$$

Using boundary condition (20), we can write

$$f(x \mid e) = z(1-z)\frac{1}{a(x)}\beta_{q,r}(z)$$

where we have defined $z = \frac{\exp(A(x)+B(e))}{1+\exp(A(x)+B(e))}$. The transform of $A(x) + B(e)$ maps from the domain of ϕ (the real line) back into the domain of F and β (i.e. in the interval $[0, 1]$).

With this formulation, we have that

$$\frac{f_x(x \mid \bar{e})}{f(x \mid \bar{e})} = \frac{q}{x} - \frac{r}{1-x}$$

so

$$a(x)\frac{f_x(x \mid \bar{e})}{f(x \mid \bar{e})} = q(1-x) - rx$$

which is bounded on $[0,1]$, implying that the likelihood ratio h is bounded.

As effort moves away from the LLR effort level, the likelihood ratio is given by

$$h(x \mid e) = -b(e)\left[q+1-(q+r+2)z\right]$$

At LLR effort, $z = x$, and so $h(x \mid e)$ is linear in x. Further,

$$\frac{\partial h(x \mid e)}{\partial x} = [q+r+2]\, b(e)\frac{z(1-z)}{x(1-x)} > 0 \tag{21}$$

and so the MLRP is satisfied.

The curvature of the likelihood is given by

$$\frac{\partial^2 h(x \mid e)}{\partial x^2} = [q+r+2]\, b(e)\frac{2z(1-z)}{x^2(1-x)^2}(x-z) \qquad \begin{array}{ll} > 0 & if\ x > z \\ < 0 & if\ x < z \end{array} \tag{22}$$

The condition $x > z$ requires $B(e) > 0$ which means that $e^{\eta} > \theta$, and so the likelihood is convex when effort is greater than LLR effort or $\theta < 0$. If $x < z$, $B(e)$ is negative, $e^{\eta} < \theta$ and the likelihood is concave. If $\theta < 0$, $B(e)$ must be positive and the likelihood is convex for any e.

Computational Issues There are three unknowns in solving for optimal social insurance: optimal effort, e^*, and the Lagrange multipliers, μ and λ. These are defined by the adding-up constraint (3), the incentive compatibility constraint (4), and the first order condition with respect to effort (7). We construct the distribution of output as above to ensure an internal solution for the optimal tax schedule in equation (6) and we check the validity of the first-order approach once we have a solution for the three unknowns. In practice, we solve for the three unknowns by solving for the optimal values of the Lagrange multipliers for a given effort level and then solving for the optimal effort level. Each of these steps is carried out using nonlinear equation solvers written in Fortran. We check the validity of the first order approach by calculating expected utility for an individual varying that individual's effort level while holding the optimal social insurance scheme and everyone else's effort constant. There are two checks we then carry out: first, we check that effort is greater at the optimum than at zero effort; second, we check that there is only one turning point of expected utility as effort increases. Since effort has a diminishing effect on output, expected utility at high effort levels necessarily becomes very low.

References

[1] Attanasio, O. (1999), Consumption. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1, chapter 11. North-Holland.

[2] Eaton, J. and Rosen, H. S. (1980), Labor supply, uncertainty and efficient taxation. *Journal of Public Economics*, 14:365–374.

[3] Holmstrom, B. (1979), Moral hazard and observability. *Bell Journal of Economics*, 10:74–92.

[4] Jewitt, I. (1988), Justifying the first order approach to principal-agent problems. *Econometrica*, 56:1177–1190.

[5] Kimball, M. (1990), Precautionary saving in the small and in the large. *Econometrica*, 58:53–73.

[6] Low, H. (1999), Self-insurance and unemployment benefit in a life-cycle model of labor supply and savings behaviour. *Institute for Fiscal Studies Working Paper*, #99-24.

[7] Milgrom, P. (1981), Good news and bad news: Representation theorems and applications. *Bell Journal of Economics*, 12:380–391.

[8] Mirrlees, J. A. (1971), An exploration in the theory of optimum income taxation. *Review of Economic Studies*, 38:175–208.

[9] Mirrlees, J. A. (1974), Notes on welfare economics, information and uncertainty. In Balch, M., McFadden, D., and Wu, S., editors, *Essays in Equilibrium Behaviour and Uncertainty.* North-Holland.

[10] Sinn, H.-W. (1996), Social insurance,incentives and risk-taking. *International Tax and Public Finance*, 3:259–80.

[11] Strawczynski, M. (1998), Social insurance and the optimum piecewise linear income tax. *Journal of Public Economics*, 69:371–388.

[12] Tuomala, M. (1984), Optimal degree of progressivity under income uncertainty. *Scandinavian Journal of Economics*, 86:184–193.

[13] Varian, H. (1980), Redistributive taxation as social insurance. *Journal of Public Economics*, 14:49–68.