

# What Does $q$ Predict?\*

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## Abstract

The lack of success of empirical measures of “Tobin’s  $q$ ” as a predictor of investment is now quite well-established, if still something of a puzzle. We examine this puzzle from a new angle. If Tobin’s  $q$  is properly measured there are strong *a priori* reasons to expect it to mean-revert; and these priors are consistent with data for Tobin’s  $q$  over the course of the twentieth century. If  $q$  mean-reverts, but does not predict investment, it must predict something else. We show that it has some predictive power for dividends, but strongly predicts stock market returns. This result is broadly consistent with well-known evidence that the dividend-price ratio predicts returns. If, however, there is unit root measurement error in  $q$  (consistent with claims by Hall (2000) and others) then measured  $q$  may not predict anything. But the implied measurement error in recent years would need to have been on a massive scale.

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## 1. Introduction

The lack of success of empirical measures Tobin's  $q$  as a predictor of investment is now quite well-established, if still something of a puzzle.<sup>1</sup> As with all empirical puzzles, there has been no shortage of possible explanations. A range of different authors have attributed this failure to: capital market imperfections (*e.g.* Hubbard, 1998); non-convex adjustment costs (*e.g.* Caballero, 1997); concavity of the value function (*e.g.*, ); “noise” in share prices (*e.g.*, Blanchard *et al.*, 1993; Bond & Cummins, 2000); or gross mis-measurement of true capital (Hall, 2000).

But whatever the explanation may be, there is another puzzle associated with Tobin's  $q$  (hereafter referred to simply as  $q$ )<sup>2</sup> that has not received attention. In the absence of nonstationary measurement error, we show that there are strong *a priori* grounds for expecting  $q$  to mean-revert. We also show, using a long dataset for aggregate Tobin's  $q$ , over the course of the entire twentieth century (Wright, 2001), that, subject to the usual statistical caveats,  $q$  does indeed appear to be mean-reverting.<sup>3</sup> If  $q$  both mean-reverts, and does not predict investment, it must predict something else.

Meanwhile there is an almost entirely separate (and massive) literature in finance that has generally concluded, that stock market valuation indicators (most notably dividend-price ratios) have predictive power for stock market returns.<sup>4</sup>

This paper shows that both literatures can be seen as two different aspects of the same puzzle. We show, using a log-linear framework very close to that of Campbell & Shiller (1988), that  $q$  can be expressed as a forward sum of future changes in the capital stock, the rate of profitability, and future returns. Given mean reversion of  $q$ , it must “Granger-Cause”, or predict, at least one of these. In line with the investment literature, our econometric analysis shows that there is barely any evidence that  $q$  predicts investment; some evidence that it predicts future profitability (as captured by dividends/capital); but there is strong evidence that it predicts future returns. In a companion paper (Robertson & Wright 2002a) we show that, under certain circumstances, mean reversion of  $q$ , and its predictive power for returns, will be matched by the same features of the dividend-price ratio, as has generally been found in the finance literature. But we show that mean reversion of  $q$  is logically prior to that of the dividend yield. In this paper

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<sup>1</sup>See *eg* Chirinko, 1993 for a literature survey on empirical studies.

<sup>2</sup>We follow standard practice in letting lower case letter letters denote natural logarithms. Hence whenever we refer to  $q$  we are referring to the log of the levels ratio  $Q$ . We shall usually assume a log-linear framework so most of our results relate to  $q$ .

<sup>3</sup>Or, more precisely, certainly seemed so up until around the start of the 1990s. The last decade of the century is something of a puzzle to everybody, and weakens the statistical evidence.

<sup>4</sup>Campbell, Lo and MacKinlay (1997) provide an excellent review of the bulk of the literature. For more recent survey, see also Campbell (2000).

we show that this ranking is also evident empirically. Evidence of mean reversion of standard measures of the dividend yield is very weak; though there is much stronger evidence of mean reversion if dividends are adjusted for new issues and buybacks.

These results are conditional on mean reversion of  $q$ , but although this is consistent with the data, the evidence is not overwhelmingly strong. If  $q$  does not mean-revert (which we shall argue can only arise with unit root measurement error in capital) then it may not predict anything except itself. This is at least qualitatively consistent with claims made by Hall (2000) and others that capital has been grossly under-recorded in the recent past. We find that this interpretation of the data cannot be convincingly rejected on econometric grounds; but we do find other grounds for scepticism.

The paper is structured as follows. In Section 2 we examine the time series properties of  $q$  over the course of the twentieth century and conclude that there is reasonably strong (but not overwhelming) evidence that  $q$  does mean-revert. In Section 3 we flesh out the analytical basis for our statement of what  $q$  must predict if it mean-reverts and for our strong priors that  $q$  will indeed mean-revert. In Section 4 we examine the impact of allowing for leverage, new issues and buybacks. In Section 5 we present econometric results that are consistent with the assumption that  $q$  mean-reverts (as a cointegrating relation in a system of non-stationary variables), and allow us to account for what  $q$  does predict. We also discuss an alternative interpretation of the data, in which  $q$ , as measured, does not mean-revert. Section 6 concludes the paper.

## 2. Data

### 2.1. Data Sources and Construction

Sources and methodology for all data used in this paper are provided in a separate paper (Wright, 2001). All data relate to the total nonfinancial US corporate sector. Precise definitions of all series used in this paper, in terms of the underlying dataset, are provided in Appendix A.<sup>5</sup>

### 2.2. Tobin's $q$ over the Twentieth Century

To motivate the analysis of this paper, we look first at a century's worth of data for  $q$ . Figure 2.1 shows three alternative definitions of  $q$  ( $\equiv \ln Q$ ) for the nonfinancial sector as a whole over the course of the twentieth century. Although precise definitions vary, in general terms Tobin's  $q$  can be defined by:

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<sup>5</sup>The underlying data can be downloaded from [www.econ.bbk.ac.uk/faculty/wright](http://www.econ.bbk.ac.uk/faculty/wright).

Measures of  $q$  ( $=\ln Q$ ) for the US Nonfinancial Corporate Sector

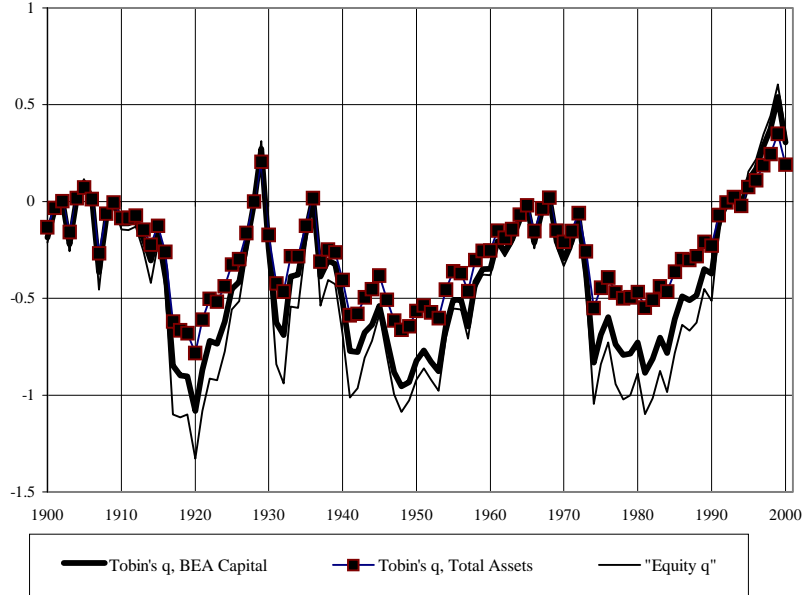


Figure 2.1:

$$Q = \exp(q) = \frac{\text{market value of equities} + \text{market value of debt}}{\text{total assets}} \quad (2.1)$$

Where the numerator is the total market value of the nonfinancial corporate sector, and the denominator is the underlying asset value. In measured terms, there is some scope for ambiguity of the precise definitions, depending on the extent to which debt is measured on a net or gross basis (and, by implication, whether assets are measured including or excluding financial assets). In our empirical work described below, we follow standard practice in measuring debt in net terms and the denominator as physical capital. Figure 2.1 shows, however, that the alternative measures have very similar properties<sup>6</sup>.

<sup>6</sup>The first compares total liabilities (with debt in gross terms) with total assets (including financial assets); the second measures “equity  $Q$ ” =  $\frac{\text{market value of equities}}{\text{net worth}}$ . The closeness of the link reflects the adding-up constraint that any addition or subtraction from the numerator implies a corresponding addition or subtraction from the denominator, for given net worth figures. In measured terms, therefore, “equity  $q$ ”, rather than Tobin’s  $q$  is the fundamental series. The necessary link between adjustments to numerator and denominator is often ignored in the common claim that the value of the denominator of  $q$  has been understated in recent years. For this not to imply a similar understatement of the denominator would require a

Figure 2.1 also shows that all the measures were for most of the century below zero, implying that the underlying ratio in levels was less than unity<sup>7</sup>. On the face of it this is somewhat surprising, given that it is usually assumed that the levels ratio should have an equilibrium value of unity. We discuss this issue at some length in Section 3.3, and point to two alternative (though possibly complementary) explanations.

Is Tobin's  $q$  a mean-reverting process? By mean reversion here we wish to capture two key features: first, that point forecasts converge, as the forecast horizon lengthens, to a fixed value (or a close neighbourhood thereof); and second, that forecast variance tends to a finite limit (or grows so slowly as to be indistinguishable from this). These are the properties that would make  $q$  useful in forecasting. We shall show below, in Section 3.3 that our quite strong prior would be that, as long as  $q$  is measured correctly, it should be mean-reverting in this sense. As is often the case, however, the data do not provide a definitive answer. As figure 2.1 makes evident, the behaviour of  $q$  in the last decade of the century took it outside its previous norms.<sup>8</sup> Table 1 (see Appendix E) shows that conventional tests of the null hypothesis that  $q$  has a unit root (*i.e.* is not mean-reverting) would have rejected fairly strongly if carried out over a sample excluding the 1990s; but that this rejection is less marked if all data are included.

It is worth stressing that this feature is in no sense unique to  $q$ . Alternative commonly used valuation criteria such as the dividend yield or the price-earnings multiple also moved out of their historic ranges in this period by at least as much as did  $q$ . This common feature has a common explanation: the behaviour of stock market returns over this period was quite exceptional. In Robertson & Wright (2002a), we show that data for returns, for  $q$  and other valuation ratios all point to this period as being well into the tail of any underlying distribution, and conclude that at such times we would expect unit root tests to yield rather weak results.

### 3. What $q$ Predicts: Some Simple Analytics

#### 3.1. $q$ , Investment, Dividends and Returns

We first show how, by exploiting a log-linearised identity, we can relate  $q$  to future returns, dividends and investment, and hence that, if  $q$  is a mean-reverting

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similar mis-statement of corporate net worth, and hence an extremely large cumulative error in measuring profits: a necessary link that has been acknowledged by serious academic studies of the mis-measurement claim, such as Hall (2000).

<sup>7</sup>The three series have geometric means in levels of 0.68, 0.76 and 0.62 respectively.

<sup>8</sup>After our sample period ended,  $q$  continued to move back towards, but remained well above, more normal levels. Latest revisions to data for 1999 and 2000 imply somewhat higher values than in our dataset but with the same general pattern, and show a further fall in 2001.

process, it will predict future values of at least one of these.

To simplify the analysis we assume a world in which Miller-Modigliani conditions hold, and hence we can ignore the impact of debt finance (we reintroduce this complication later, before we proceed to the econometric analysis). We therefore make the standard assumptions a) that firms are solely equity-financed, and b) that all new investment is financed from retained profits. Under these conditions, firms will make no new issues, and hence the value of the stock market will be proportional to the stock price. By normalising the equity issue to unity, we can re-express (2.1) as:

$$Q_t = \frac{P_t}{K_t} \quad (3.1)$$

or, in logarithms,

$$q_t = p_t - k_t \quad (3.2)$$

Following Campbell and Shiller (1988) we have an approximation for the log stock return:

$$\begin{aligned} r_{t+1} &\equiv \ln(1 + R_{t+1}) = \ln\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right) \\ &\simeq \varphi + \Delta p_{t+1} + (1 - \rho)(d_{t+1} - p_{t+1}) \end{aligned} \quad (3.3)$$

wherein  $\rho = \frac{1}{1 + \exp(\overline{d-p})}$  and  $\varphi = \ln(1 + \exp(\overline{d-p})) - (1 - \rho)\overline{d-p}$  and  $\overline{d-p}$  is the mean dividend yield. Substituting from (3.2) into (3.3) to eliminate  $p_t$ , solving for  $q_t$ , and iterating forwards, subject to the transversality condition  $\lim_{i \rightarrow \infty} \rho^i q_{t+i} = 0$ , we obtain:

$$q_t \simeq \frac{\varphi}{1 - \rho} + \sum_{i=1}^{\infty} \rho^{i-1} f_{t+i} \quad (3.4)$$

$$\text{where } f_t = \Delta k_t + (1 - \rho)(d_t - k_t) - r_t$$

Now if  $q$  is a mean-reverting process,  $f$ , the linear combination of variables on the right-hand side must also be a mean-reverting process: either each of the three elements must be individually mean-reverting, or there must be cointegrating relations amongst them. Further, since  $q$  is expressed as a weighted sum of future values of  $f$ , mean reversion of  $q$  also implies that  $q$  Granger-causes, or predicts, at least one of the three component series, or some combination thereof.<sup>9</sup>

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<sup>9</sup>This forward sum holds by definition *ex-post*, and, with rationality, also *ex-ante*. Here we are only concerned with a description of the data relationships and use the *ex-post* specification. See the discussion in Campbell, Lo and McKinlay (1997) pp262-263.

The investment literature has typically found that  $q$  is a poor predictor of investment ( $\Delta k$ ), which in itself has been puzzling. But in fact the puzzle has been deeper since, if  $q$  mean-reverts and does not predict investment, then it must predict something else: as (3.4) makes clear this must be either future profitability, as proxied by the ratio of dividends to capital, or future returns.

### 3.2. Tobin's vs Marginal $Q$ : Hayashi Revisited

So far we have done no more than manipulate log-linear approximations of dynamic accounting identities, on the assumption that  $q$  is a mean-reverting process. We now reinforce the empirical evidence by setting out the process determining Tobin's  $q$ , and relating the observable series to its underlying (and largely unobservable) determinants. We then discuss whether these processes in turn would be expected to be mean-reverting.

We consider the investment problem of a representative unleveraged firm, whose dividends and capital stock equal the aggregate figures by normalisation. Our analysis is a simplified, discrete time version of Hayashi(1982), and is close to that of Abel and Blanchard (1986) and many others.

Let  $\Omega_t$  represent information available to the firm at the end of period  $t$ . The firm chooses investment  $\{I_{t+i}\}_{i=1}^{\infty}$  to maximise the value of the firm:<sup>10</sup>

$$V_t = E_t [M_{t+1} (D_{t+1} + V_{t+1}) | \Omega_t] \quad (3.5)$$

The firm chooses an optimal sequence of investments and simultaneously prices its resulting maximised value with reference to  $M_{t+1}$ , the one-period ahead stochastic discount factor from the end of period  $t$  to the end of period  $t + 1$  (note that  $M_{t+i} \notin \Omega_t$  for  $i \geq 1$ ).<sup>11</sup> In standard fashion, we assume that the representative firm treats the discount factor as exogenous. Note that the firm's information set is likely to be greater than the public information set; hence the firm is in a unique position to assess its own underlying value. It does so, however, using a market-based measure of (stochastic) opportunity cost.

At the end of period  $t$  the firm has inherited capital stock  $W_t$  (which we distinguish explicitly from the recorded capital stock,  $K_t$  to account for the possibility of measurement error). In the next period ( $t + 1$ ) it uses this, and other factors of production, assumed optimally chosen within the period, to produce dividends  $D_{t+1}$  given investment choice  $I_{t+1}$ . Dividends are given by

$$D_{t+1} = D(W_t, I_{t+1}, \theta_{t+1}) = \Pi(W_t, \theta_{t+1}) - I_{t+1} - G\left(\frac{I_{t+1}}{W_t}\right) W_t \quad (3.6)$$

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<sup>10</sup>We work in terms of end-of-period stocks for consistency with published data.

<sup>11</sup>We thus differ from *e.g.* Abel and Blanchard (1986), who discount the first future period's dividend with a non-stochastic discount factor.

wherein  $\Pi(\cdot)$  is the maximised profit function, net of depreciation. Dividends are assumed to be stochastic due to shifts in the profit function through some iid shock  $\theta_{t+1}$ . Investment (measured on a net basis) lowers dividends both directly, and indirectly, due to adjustment costs,  $G(\cdot)W_t$ .

Capital accumulates as<sup>12</sup>

$$W_{t+1} = W_t + I_{t+1} \quad (3.7)$$

Defining  $\mu_t = \frac{\partial V_t}{\partial W_t}$  the optimal policy must satisfy

$$E_t (M_{t+1}\mu_{t+1}) = E_t M_{t+1} \left( 1 + G' \left( \frac{I_{t+1}}{W_t} \right) \right) \quad (3.8)$$

The derivative of the value function with respect to capital,  $\mu_t$ , is usually called “marginal  $Q$ ” in the investment literature. Equation (3.8) implies that the expected present value of marginal  $Q$  at the end of period  $t + 1$  is equated, at the optimum, to the expected present value of the (known) costs of investment. Note that, in contrast to the continuous time case, or discrete time cases where the one-period-ahead discount factor is non-stochastic (*e.g.* Abel and Blanchard, 1986) the discount factor does not drop out of the first order condition, although it affects the optimum only through covariance terms.

Equation (3.8) implicitly defines the investment function. More crucially, for our purposes, we shall see that it also determines the mean level of marginal  $Q$ .

Under certain (highly restrictive) conditions, unobservable marginal  $Q$  will be equal to observable Tobin’s  $Q$ . These can be summarised, in our framework, by three conditions:

$$\mu_t = \frac{V_t}{W_t} \quad (3.9)$$

$$P_t = V_t \quad (3.10)$$

$$K_t = W_t \quad (3.11)$$

Hayashi (1982) showed that the first of these conditions, (3.9) will be satisfied under conditions of constant returns to scale and competitive product and labour markets.<sup>13</sup> Of the remaining two conditions, (3.10) requires a very strong form of stock market efficiency, and (3.11) requires that recorded capital must equal true underlying capital.

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<sup>12</sup>Most other analysis formulates the problem in terms of gross, rather than net investment, but the key results are unaffected by this.

<sup>13</sup>We verify the Hayashi result for our model in Appendix B.

For the general case, in which one or all of conditions (3.9), (3.10) and (3.11) may *not* hold, we can write

$$Q_t = \mu_t \left( \frac{V_t/W_t}{\mu_t} \right) \left( \frac{P_t}{V_t} \right) \left( \frac{W_t}{K_t} \right)$$

or, taking logs of both sides

$$q_t = \log \mu_t + h_t + e_t + b_t \tag{3.12}$$

Thus measured Tobin's  $q$  will differ from log marginal  $Q$  to the extent that three hypotheses fail to hold in the data, which we can refer to as the Hayashi hypothesis; the Efficient Market Hypothesis, and the "BEA" hypothesis (*i.e.* that statisticians do their job properly). Each hypothesis implies that  $h_t, e_t$  and  $b_t$  are respectively equal to zero for all  $t$ . All three of these have in effect been considered in past research: the inclusion of profits in investment regressions (*e.g.* Abel and Blanchard, 1986), for example, can be rationalised as proxying concavity of the value function; others have investigated the distortions caused by "noise" in stock prices (*e.g.*, Bond & Cummins, 2000; Blanchard *et al*, 1990); and recently a number of investigations, *e.g.*, Hall (2001), McGrattan and Prescott (2001), Nakamura (2001) have proceeded on the assumption that capital has been significantly mis-measured in recent years.<sup>14</sup>

It would be very surprising if all three of the elements in (3.12) that cause Tobin's  $q$  to differ from log marginal  $Q$  were to be identically zero. However, our primary concern is not whether they are zero, but whether they have stable mean values. If they do, and if log marginal  $Q$  does too, then Tobin's  $q$  will be a mean-reverting process, which is all that we require if the forward sum representation in (3.4) is to have useful informational content.

Our secondary concern will be to find an explanation for the observation, noted in Section 2.2 above, that in the data Tobin's  $q$  appears to have a mean significantly below zero.<sup>15</sup>

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<sup>14</sup>This does not of course provide a comprehensive list of theoretically possible distortions. We have for example not explicitly taken into account those due to taxation, which are simply subsumed within  $h_t$ .

<sup>15</sup>A third potential area of investigation is the nature of the dynamic processes of the elements in (3.12), but we do not pursue this in this paper. It should be noted however that it would be very suprising if these processes were mutually uncorrelated, or were uncorrelated with other variables in the economy. Thus we should be very cautious in interpreting econometric results (whether ours or those of others) - as implying rejection of any of the three hypotheses in particular.

### 3.3. Will Tobin's $q$ Mean-Revert?

#### 3.3.1. Mean Reversion of Marginal $Q$ ( $\mu_t$ )

We have a strong prior that  $\log \mu_t$  will be mean-reverting, since it is straightforward to show that the mean value of marginal  $Q$  is invariant to most structural parameters of the economy.

Assuming log-normality, taking logs of both sides of (3.8) and taking unconditional expectations by repeated application of the law of iterated expectations implies

$$\overline{\log \mu} \simeq \overline{G'} - C \quad (3.13)$$

where, for any variable  $x$ ,  $\bar{x} = E(x_{t+1})$ , and  $C = cov(\log \mu_{t+1}, \log M_{t+1}) + \frac{1}{2}var(\log(\mu_{t+1}))$

The assumption that mean marginal  $Q$  will be precisely unity in levels typically arises from assuming that  $\overline{G'} = 0$  and a non-stochastic environment. A common specification of  $G(\cdot)$  (*e.g.* Kiley, 2000; Bond and Cummins, 2000; Lafourcade, 2001) assumes it to be quadratic in deviations of the investment rate from its steady state value: in our net investment framework, it would thus be assumed that  $\overline{G'} = G'(\bar{g}) = 0$ , where  $\bar{g}$  is the mean growth rate. In a non-stochastic world, mean marginal  $Q$  would therefore be unity in levels terms.

Arguably this standard specification arises from the *assumption* that mean marginal  $Q$  must be unity. Although the assumption of mean zero marginal installation costs might appear counter-intuitive,<sup>16</sup> it is perhaps easier to rationalise in a general equilibrium context than for an atomistic firm. In general equilibrium, the marginal cost involved is the marginal cost of converting consumption into investment (or vice versa) close to the equilibrium investment/output or consumption/output ratio. A zero marginal cost implies that consumption and investment are perfectly fungible at this point. If nothing else, this assumption is implicit in all national accounts and capital stock statistics.

In general, however, whether or not this assumption is valid, in a non-stochastic environment, the mean value of marginal  $q$  will be determined only by the capital installation technology, and will thus be invariant to all other parameters of the economy. In a stochastic environment, mean marginal  $Q$  will also be affected by the terms in  $C$ ; but as long as the covariance structure of the economy is reasonably stable,<sup>17</sup> these terms will be constant.

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<sup>16</sup>It implies that marginal installation cost are negative when net investment is negative, and is therefore inconsistent with, for example, external adjustment costs. *cf*, for example, Abel and Blanchard, 1983, who assume  $\overline{G'} > 0$ , implying (in their deterministic model)  $\overline{\log \mu} > 0$ .

<sup>17</sup>Which of course we need to assume if we are to use most standard time series econometric techniques.

### 3.3.2. The “Hayashi Hypothesis” ( $h$ )

Under the standard Hayashi assumptions of constant returns to scale and competitive product and labour markets (which in turn imply linearity of the value function) condition (3.9) will always hold, implying that  $h = 0$ . However, it is important to note that these assumptions, which are highly restrictive, are sufficient, but not necessary conditions for (3.9) to hold. It may also hold under a wider set of conditions, that are particularly relevant to equilibrium positions, and hence mean values. To see this, note that in any steady state in which marginal  $Q$  is expected to be constant, (3.9) can be rewritten, using (3.8), as:

$$V = (1 + G')W$$

which says that, at any such point, the representative firm’s value function is simply equal to the opportunity cost of its installed capital. This condition can hold in a distinctly wider set of circumstances. Thus, for example, Lafourcade (2001) examines a general equilibrium model of imperfect competition with increasing returns to scale, resulting in a concave value function for every individual firm. If there are permanent barriers to entry, he shows that  $\frac{V}{W}$  will be greater than  $\mu$ , implying  $h > 0$ . But Lafourcade also shows, that, with free entry to the corporate sector,  $h = 0$  in steady state (though not in transition to that steady state), even in the presence of increasing returns and monopoly power, since, in a zero profit equilibrium, fixed costs will absorb monopoly profits.

Thus whatever the transitional dynamics may be, the degree of stability of  $\bar{h}$  is ultimately dependent on the extent to which monopoly profits - as distinct from monopoly power - are vulnerable to the impact of new entry. To the extent that some element of monopoly profits is permanent, we would expect  $\bar{h} > 0$ , but, as long as the power of entry to limit monopoly profits remains reasonably stable, we would expect  $h$  to have a stable mean, and to be mean reverting in our sense.

### 3.3.3. Pricing Efficiency( $e$ )

The assumption that (3.10) will hold at all points in time, implying  $e = 0$ , while very commonly made,<sup>18</sup> is an extremely strong one. It will hold if the stock market has precisely the same information set as the firm and stock market valuations are free from bubbles (rational or otherwise) or “noise”. But, since Grossman & Stiglitz (1980), it has been accepted that such a strong form of efficiency is a logical impossibility when information is costly. Indeed in such circumstances the

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<sup>18</sup>Though, of course, not universally: see, for example, Blanchard *et al* (1990); Bond and Cummins (2000).

managers of our representative firm would effectively be entirely redundant as far as investment choices are concerned.

We can however form at least some priors about the nature of possible deviations of markets from such perfect efficiency. In particular we can show that this variable ( $e$ ) must itself be mean-reverting, under fairly general conditions. Under more restrictive conditions, we can also form priors about whether its mean will be greater or less than zero.

The basis of the value function (3.5) is the assumption that the firm effectively prices its own risk using the unique market stochastic discount factor. Given its superior information, *only* the representative firm can price its own internal return: and since its response to this valuation is only revealed with a lag, market participants cannot infer the full information set from the firm's actions.<sup>19</sup> At the same time, the stock market return is priced by the market using the same SDF, but on the basis of public information,  $S_t$ . Hence we have the standard pricing condition:

$$E_t \left[ M_{t+1} \left( \frac{D_{t+1} + V_{t+1}}{V_t} \right) | \Omega_t \right] = E_t [M_{t+1} (1 + R_{t+1}) | S_t] = 1 \quad (3.14)$$

In a perfectly efficient market this condition will automatically be satisfied by the equality of  $P$  with  $V$ , and hence of the stock return with the firm's internal return. More generally when the two returns may differ, the pricing condition will provide a crucial link between stock market valuations and the underlying value function.

Define  $i_{t+1} = \log \left( \frac{D_{t+1} + V_{t+1}}{V_t} \right)$  as the log internal return. For values of  $\bar{e}$  sufficiently close to zero, the Campbell-Shiller approximation, as in (3.3) will hold equally well for  $i_{t+1}$  and for  $r_{t+1}$ , implying that we can write:

$$\begin{aligned} r_{t+1} &\simeq i_{t+1} + \varepsilon_{t+1} \\ \text{where } \varepsilon_{t+1} &= \Delta e_{t+1} - (1 - \rho)e_t \end{aligned} \quad (3.15)$$

We can refer to  $\varepsilon_{t+1}$  as "noise" - although we do not wish necessarily to imply thereby that it arises from any irrationality (as in, for example, the "noise trader" model of De Long, et al, 1990): the above analysis suggests that it may arise solely from imperfect information. Nor would we necessarily expect  $\varepsilon_{t+1}$  to be a serially independent process (indeed, if markets are assumed to infer past value from revealed actions of firms, we would positively expect the contrary).

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<sup>19</sup>There is a large literature in finance on mechanisms by which markets may attempt to infer such information; managers may also exploit this for signalling purposes.

Taking logs and unconditional expectations of (3.14), using (3.15) and assuming  $\overline{\Delta v} = \overline{\Delta p}^{20}$  implies

$$\bar{e} \equiv \overline{p - v} \simeq \frac{1}{1 - \rho} \left[ \text{cov}(\varepsilon, m) + \text{cov}(\varepsilon, i) + \frac{\sigma_\varepsilon^2}{2} \right] \quad (3.16)$$

Thus  $\bar{e}$ , the average deviation of  $p$  from  $v$ , will be determined by the signs, and relative magnitudes of the noise variance and its covariances with the stochastic discount factor and with the internal return. In a covariance-stable world we would therefore expect  $e$  to have a stable mean. Thus we have the result that  $e$  must be mean-reverting (hence  $p$  and  $v$  must be cointegrated), by making the minimal assumption that the market and the representative firm use the same stochastic discount factor.<sup>21</sup> However,  $\bar{e}$ , will only be precisely zero if the right-hand side of (3.16) is zero. Note also that it will be perfectly possible for  $\bar{e}$  (and hence  $\bar{q}$ ) to have a value less than zero.<sup>22</sup>

We conclude therefore, that we have a strong prior that  $e_t$  will be a mean-reverting process. While it is less easy to form priors on whether its mean value will differ from zero, the above analysis suggests that there may be grounds to expect a negative mean - a possible explanation of the negative mean of measured  $q$  itself.

### 3.3.4. Mis-Measurement of Capital ( $\bar{b}$ )

Probably the weakest long-run prior relates to the stability, and mean value (if any) of  $b_t$ , which captures mis-measurement of capital. However seriously statisticians may take their job, there is no obvious mechanism that penalises them if they systematically mis-measure the capital stock - most especially if this mis-measurement arises from under- or over-estimation of the true rate of economic

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<sup>20</sup>While this standard assumption is reasonable over sufficiently long samples, it should be noted that it is not entirely innocuous over the sorts or sample lengths actually available. Thus, for example, if (taking an admittedly extreme position) all of the rise in  $q$  in the last few years of the twentieth century were to be attributed to a bubble in  $e$ , the magnitude of that rise would be sufficient to drive quite a significant wedge between mean stock returns and mean implied internal returns, even taken over the entire century. In levels terms Tobin's  $Q$  at end-2000 was 1.68 times its level at end-1900 - which would imply a difference between the two average returns for the century as a whole of around 0.5% per annum. Clearly over the course of a shorter period the wedge would be very much larger.

<sup>21</sup>If  $\bar{e} \neq 0$ , then, to the extent that the linearisation coefficients are sufficiently different, there will be an additional term in the mean dividend yield, so the result will still hold as long as the dividend yield mean-reverts. Nor, even, would deterministic bubbles invalidate this result, as long as  $\overline{\Delta p} - \overline{\Delta v} \equiv \overline{\Delta e}$  has a stable mean.

<sup>22</sup>See Appendix D for a rough calibration, on further, quite restrictive assumptions. Note that a negative mean value of  $e$  is a feature that can also arise out of De Long *et al's* (1990) "noise trader" model of market inefficiency, albeit with a different rationale.

depreciation (mis-measurement of gross investment is arguably both less likely to occur, and more likely to be penalised, given the attention paid to GDP figures).

Given our focus on mean reversion, mis-measurement in itself need not necessarily be of any great significance, if the measurement error is itself mean-reverting. However, since capital is measured by accumulation of the estimated underlying flows, it is also difficult, on *a priori* grounds, to rule out a unit root in measurement error.

These features of the measurement process have two implications.

First, they provide an alternative candidate explanation for the low historic mean value of  $q$ , as possibly due to over-estimation of the capital stock on a sustained basis. Wright (2001) presents some supporting evidence that this in turn may have been due to under-recording of depreciation, a relatively small amount of which could fairly easily explain all of the deviation of mean  $q$  from zero.<sup>23</sup>

Second, in qualitative terms, at least, these features are consistent with claims from Hall (2000), and others cited above, that capital has in recent years become increasingly *under*-stated due to a permanent shift towards intangibles. We shall return to this issue in Section (5.6) below.<sup>24</sup>

### 3.3.5. Implications for Mean Reversion of Tobin's $q$

The analysis of the last four sub-sections can be summarised as implying that, in the absence of measurement error, there are strong *a priori* grounds for expecting mean values of Tobin's  $q$  to be both stable over time, and insensitive to shifts in most underlying structural parameters of the economy.

This stability arises because each of the components of "true" Tobin's  $q$ , namely,  $\log \mu$ ,  $h$  and  $e$ , can be viewed as capturing some process which can be very loosely termed arbitrage. The deviation of marginal  $Q$  from its mean value captures arbitrage via the investment process; movements in  $h$  will ultimately be determined by zero profit conditions in product markets; and movements in  $e$  by arbitrage via information-processing in financial markets.

Since Tobin's  $q$  will be invariant to, *inter alia*, the investor preference and other parameters that determine the equilibrium dividend yield this implies that mean reversion of Tobin's  $q$  is also logically prior to that of the dividend yield, in

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<sup>23</sup>The supporting evidence comes from: a) examination of "equity  $q$ " derived from quoted stock prices, which also appears to have the property of a mean significantly below zero; b) the observation that the mean earnings yield (on reported earnings) has historically been higher than the mean stock return.

<sup>24</sup>We should note at this point that some arguments (including those of Hall) for the importance of intangible assets in the New Economy are implicitly arguments about shifts in monopoly power, and thus, in our framework, relate, not to  $b$ , but to  $h$ .

the sense that there may in principle exist circumstances in which the dividend yield does not have a stable mean, but in which  $q$  does.<sup>25</sup>

Measurement error will not affect the stability of the mean of measured Tobin's  $q$ , as long as measurement error is itself mean-reverting. Thus, by implication, in the absence of unit root measurement errors, we have a strong prior that Tobin's  $q$  should mean-revert. As a corollary, if we wish to claim that measured Tobin's  $q$  does *not* mean-revert, our strong prior would be that this must be due to unit root measurement error.

We also have two possible, and plausible explanations for the observed mean value of  $q$  being less than zero: informational asymmetries ( $e$ ) or measurement errors ( $b$ ). Hence the historic mean value of Tobin's  $q$  is not of itself necessarily evidence of measurement error.

## 4. Allowing for Leverage, Equity Issues & Buybacks

In our empirical investigations, we follow most other research in focussing on stock markets. A significant fraction of aggregate firm value is however represented by debt (around 20-25% if measured in net terms, and around 50% in gross terms). This has an impact on our measurement both of  $q$  and dividends. An additional problem is that empirical measures of dividends do not correspond directly with the theoretical measures used above, which assumed zero new issues and buybacks. The former problem is rather easier to deal with than the latter.

### 4.1. Tobin's $q$ in terms of Total Corporate Value

We shall assume (as implicitly does most research using Tobin's  $q$ ) that the Miller-Modigliani conditions hold, such that the value of a firm does not depend on the source of its financing. If we continue for now to rule out net new issues, with non-zero debt Tobin's  $Q$  will be given in levels terms by:

$$Q_t = \frac{P_t + L_t}{K_t} \quad (4.1)$$

Where  $L_t$  are corporate liabilities, and the stock price,  $P_t$  is again assumed equal to stock market value (the equity issue is normalised to unity). We can derive a log-linearised approximation for  $q$  (expanding around the log mean level of leverage) as:

$$q_t \simeq \xi + (1 - \zeta)(p_t - k_t) + \zeta(l_t - k_t) \quad (4.2)$$

where  $\xi = \ln(1 + \exp(\bar{l} - \bar{p}))$ , and  $\zeta = \exp(\bar{l} - \bar{p}) / (1 + \exp(\bar{l} - \bar{p}))$

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<sup>25</sup>For further discussion of this issue, see Robertson and Wright, 2002a.

If we redefine dividends to be all net income payments to equity holders only (postponing any discussion of the measurement issue, and any complications due to new issues and buybacks, until the next section), we can still derive the log-linearised forward sum (as in (3.4))

$$p_t - k_t \simeq \frac{\varphi}{1 - \rho} + \sum_{i=1}^{\infty} \rho^{i-1} (-r_{t+i} + (1 - \rho)(d_{t+i} - k_{t+i}) + \Delta k_{t+i}) \quad (4.3)$$

where now  $p_t$  is the equity price and  $d_t$  the equity dividend and  $\rho$  and  $\varphi$  are redefined as in (3.3) accordingly. Substituting from (4.2) into (4.3), and rearranging, we obtain<sup>26</sup> (ignoring constants):

$$q_t \simeq \sum_{i=1}^{\infty} \rho^{i-1} f_{t+i} \quad (4.4)$$

where now  $f_t = \Delta k_t + (1 - \rho)[(1 - \zeta)(d_t - k_t) + \zeta(l_t - k_t)] - \zeta \Delta l_t - (1 - \zeta)r_t$

So, with leverage, mean-reversion of  $q$  implies that it must predict some or all of: a linear combination of the dividend-capital and debt-capital ratios; changes in capital; changes in debt; or returns.

A sufficient (though not necessary) condition for this to hold is that returns, dividend-capital and leverage are all individually mean-reverting. In these circumstances the equity dividend yield,  $d - p$  would also be a mean-reverting variable (i.e. a cointegrating relation). However, we have very little reason, *a priori*, to expect leverage to have a stable mean,<sup>27</sup> and (as we shall show below) not much evidence that it does from the data, either. A less strong condition, if returns and growth rates are all mean-reverting, but leverage is a unit root process, is that the dividend-capital ratio must be cointegrated with the debt-capital ratio, with the same cointegrating coefficients as in the approximation for  $q$  (i.e. the term in square brackets in (4.4) must be mean-reverting).<sup>28</sup> The intuition for this is straightforward: the higher is leverage, the lower the proportion of total profits paid out to equity owners, since the remainder must be paid out to bond-holders.

<sup>26</sup>Using the identity  $z_t = \sum_{i=1}^{\infty} \rho^{i-1} ((1 - \rho)z_{t+i} - \Delta z_{t+i})$

<sup>27</sup>In a world of strict Miller-Modigliani equivalence of debt and equity, the leverage ratio would be indeterminate. Models do of course exist which result in determinate leverage ratios; but it would not be at all surprising if the determinants of equilibrium leverage were subject to permanent shifts.

<sup>28</sup>Although we log-linearise  $q$  around the sample mean of  $l - p$ , we do not need the true mean to be constant to apply the approximation - it just becomes increasingly inaccurate over long samples. In our sample the squared correlation coefficient between the log-linearised approximation for  $q$  and the log of  $q$  itself is 0.998, so we do not regard this as a major problem

## 4.2. Dividends, New Issues and Buybacks.

There are more thorny complications in deriving empirical measures of dividends that have been surprisingly neglected in most of the empirical literature. Both national income and quoted stock index measures of dividends measure only payments of dividends *per se*; yet the theoretical measures discussed above should represent the total net cashflow from firms to shareholders. It is this measure, not dividends alone, that, in discounted terms, must equal the stock market value of the corporate sector. Miller and Modigliani's (1961) original critique of the Gordon Growth Model on these grounds has been widely ignored in most of the literature, although there have been important exceptions (see, for example Mehra, 1998).

Figure 4.1 (using data from Wright, 2001) shows that there was a distinct apparent downward drift in the (conventionally defined) dividend yield in the postwar period, that has been well-documented elsewhere.<sup>29</sup> Although the downward drift was accentuated by the rise in the stock market in the 1990s, the chart makes clear that the process began distinctly earlier.

There are two fairly obvious, and linked explanations for this, that we discuss in more detail in Robertson and Wright (2002a). The first was a distinct shift, from the 1960s onwards, away from new issues of equity, as firms increased leverage (see Wright, 2001 for evidence of this); the second, from the 1980s onwards, was the switch to buybacks instead of direct dividend payments.<sup>30</sup> The impact of both was almost certainly to lower the recorded dividend yield on a sustained (and rationally explicable) basis.

One way to deal with this, following Mehra (1998) (and consistent with Miller & Modigliani's (1961) own treatment) is to treat dividends, buybacks and new issues as equivalent. Wright (2001) constructs a time series for "adjusted dividends" on this basis over the course of the twentieth century. Figure 4.1 shows that the resulting adjusted dividend yield appears more consistent with mean reversion (this is borne out by statistical tests discussed in Section 5.4); but with considerably greater volatility of the implied dividend series (both yields in Figure 4.1 have the same denominator).

The adjustment to dividends implies a corresponding adjustment to the stock price series, with the adjusted series being proportional to stock market value (see Robertson and Wright (2002a) for further details).

Having incorporated leverage and buybacks into the theoretical model, we now proceed to our empirical analysis. In what follows we investigate the properties of datasets using both adjusted and unadjusted dividends.<sup>31</sup>

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<sup>29</sup>For example Fama and French (2001). This drift was even more marked in the dividend yield on quoted stocks.

<sup>30</sup>Microsoft, for example, has announced its intention never to pay a dividend.

<sup>31</sup>Since our focus in this paper is on  $q$ , we analyse the issue of the adjustment to dividends

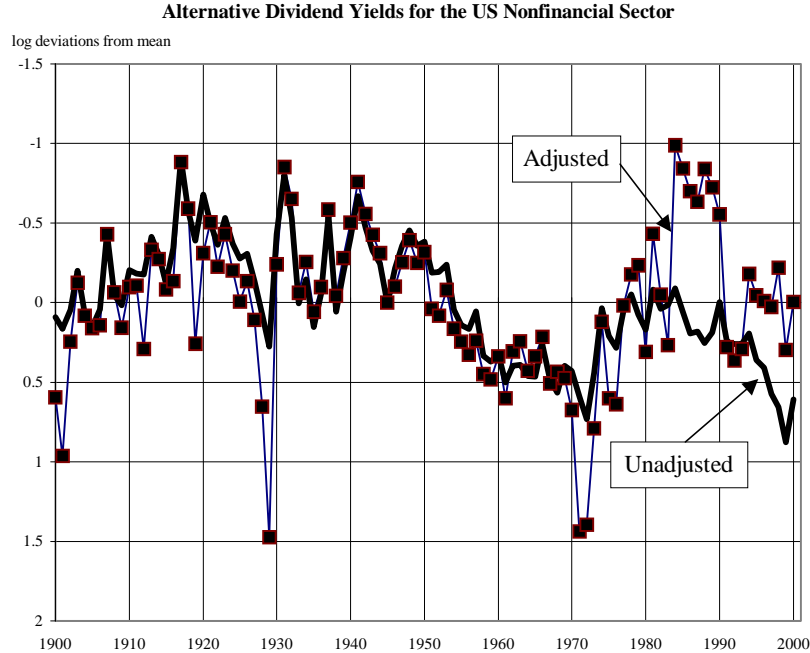


Figure 4.1:

## 5. What $q$ Predicts: Econometric Evidence

### 5.1. The Cointegrating VAR Representation

Equation (4.4) expressed  $q$  as a forward sum of dividends, capital, liabilities and returns. Since returns can be expressed, through (3.3) as a linear combination of prices and dividends, (4.4) has an equivalent representation in terms of dividends, capital, liabilities and the stock price. Mean reversion of  $q$ , taken together with the forward sum representation, must imply that (log-linearised)  $q$  should be a cointegrating relation between these four variables.

The empirical version must include at least these four variables: we therefore examine a vector autoregressive (VAR) representation of  $\mathbf{x}_t = [p_t \ d_t \ k_t \ l_t]'$ . Dividends, debt and capital are all measured on a real, per share basis, by dividing constant price dollar values by an index of the number of shares issued (analogous to the “divisor” used by the S&P indices), as described in Wright (2001).<sup>32</sup>

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more thoroughly in a separate paper (Robertson & Wright, 2002b). In this we show that, in terms of their predictive power for prices and returns, dividends, new issues and buybacks can be treated as equivalent.

<sup>32</sup>Precise definitions in terms of the underlying dataset are provided in Appendix A.

We investigate in parallel systems using the unadjusted and adjusted price and dividend series. Since, by construction, the equity issue is assumed constant in the latter case, all four elements in the system are affected by the adjustment. Where it is necessary to distinguish between them we denote the unadjusted and adjusted series by  $\mathbf{x}_{ut}, p_{ut}, \dots$  etc., and  $\mathbf{x}_{at}, p_{at}, \dots$  etc.

Standard lag order testing procedures suggest a VAR(2) representation. Unit root tests, unsurprisingly, do not reject the hypothesis that all four series contain a unit root but are difference-stationary.<sup>33</sup> We therefore estimate all systems with the general form:

$$(\Delta \mathbf{x}_t - \mathbf{g}) = \Phi(\Delta \mathbf{x}_{t-1} - \mathbf{g}) + \alpha(\beta' \mathbf{x}_{t-1} - \mathbf{e}) + \varepsilon_t \quad (5.1)$$

where  $\mathbf{g}$  is a vector of growth rates,  $\Phi$  is a full rank ( $4 \times 4$ ) matrix of coefficients, and  $\alpha$  and  $\beta$  are both  $4 \times r$ , where  $r$  is the number of cointegrating relations;  $\mathbf{e}$  is a vector of  $r$  cointegrating constants (estimated mean values of the  $r$  cointegrating relations) and  $\varepsilon_t$  a white noise error process. In general, cointegration implies that  $\mathbf{g}$  must satisfy  $\beta' \mathbf{g} = \mathbf{0}$ , but we also test the additional restriction

$$\mathbf{g} = g\boldsymbol{\iota} \quad (5.2)$$

where  $\boldsymbol{\iota}$  is a unit vector and  $g$  a scalar, so that a common deterministic growth rate is imposed, ruling out deterministic bubbles in any of the underlying ratios (this restriction is always easily accepted).

## 5.2. Candidate Cointegrating Relations

Our analysis suggests that  $q$  should be a cointegrating relationship, so a minimum value of  $r$ , the number of cointegrating relations, should be unity. The existence of a unit root in all four series means that  $r$  can be at most three. The forward sum representation of  $q$  (4.4) will be consistent with mean reversion of  $q$  if the dividend-capital and debt-capital ratios are individually mean-reverting. This would give three cointegrating relations - note that in this case the dividend yield and returns will both also be mean reverting.

If the debt-capital ratio (and therefore leverage) does not have a stable mean, then our theory suggests that the weighted combination of the dividend-capital and debt-capital ratio in (4.4) should be stationary, and this together with  $q$  represents the theoretical  $r = 2$  case. Call this weighted combination  $cr_d$ . The dividend yield would again be mean-reverting in this representation (derived as  $d - p = \frac{1}{1-\zeta}(cr_d - q)$ ), and so would be returns. Indeed it is this latter characteristic

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<sup>33</sup>Details of both sets of tests can be obtained from the authors on request.

that motivates us to view this case as of particular interest, since it is effectively driven by the joint assumption of mean reversion of  $q$  and of returns.<sup>34</sup>

In terms of the VAR specification, we therefore wish to test for the number of cointegrating relations, and whether the estimated  $\beta$  matrix can be restricted in accordance with our theoretical discussions. That is for  $r = 2$  or  $r = 3$  we wish to test

$$\beta'_2 = \begin{pmatrix} 1 - \zeta & 0 & -1 & \zeta \\ 0 & 1 - \zeta & -1 & \zeta \end{pmatrix} \text{ or } \beta'_3 = \begin{pmatrix} 1 - \zeta & 0 & -1 & \zeta \\ 0 & 1 - \zeta & -1 & \zeta \\ 0 & 0 & -1 & 1 \end{pmatrix} \quad (5.3)$$

respectively.

We shall also investigate the  $r = 1$  case, in which  $q$  is *not* a cointegrating relation, but the dividend yield is (which we require for stationary returns). Thus we investigate as well:

$$\beta'_1 = \begin{pmatrix} -1 & 1 & 0 & 0 \end{pmatrix} \quad (5.4)$$

Given our strong priors that true (as opposed to measured) Tobin's  $q$  must mean-revert, we show below that imposing this specification implicitly tests the hypothesis of a unit root measurement error in capital.

### 5.3. Cointegration Tests

We first investigate tests, shown in Table 2 for the rank of the system. A non-zero value of  $r$  implies cointegration. Due to the unusual nature of the final decade of the sample we present all test results both including and excluding this period.

As for univariate tests on  $q$ , the results are not clear-cut. There is quite strong evidence of cointegration in some form. Using the adjusted dataset the hypothesis that  $r = 0$  is rejected fairly conclusively (the rejection is at the 5% level over the sample as a whole); with the unadjusted series the rejection is somewhat less clear-cut.

There is however no clear indication from the test statistics of whether the rank of the system is greater than unity. Inspection of the eigenvalues does though suggest at least two significant values, as would the Akaike model selection criterion, and as, indeed, would our priors, since  $r = 2$  corresponds in principle to mean reversion of both  $q$  and returns.

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<sup>34</sup>See Robertson and Wright (2002a) for a discussion of the assumption that returns are stationary.

## 5.4. Testing Restrictions on $\beta$ and Growth Rates

The test results in Table 2 do not, of course, constrain the cointegrating parameters in  $\beta$ ; nor do they impose a common deterministic growth rate. We find in all tests that the restriction to a common growth rate is easily accepted and has a uniformly trivial impact on other test statistics. We report therefore only tests where the common growth restriction has been imposed. Table 3 gives likelihood ratio tests of the alternative restricted specifications, consistent with theory,  $\beta_k$ ,  $k = 1, 2, 3$  (as defined in equations (5.3) and (5.4) against the unrestricted form of each rank  $k$ .<sup>35</sup> Given that each test is predicated on a given rank, likelihood ratio tests follow standard  $\chi^2$  distributions. For reference we also show the impact of imposing the common growth rate restriction in isolation (against the unrestricted  $r = 0$  specification).

The restrictions on  $\beta$  display a striking difference between the adjusted and unadjusted datasets. When the unadjusted dataset is used, virtually all restrictions on  $\beta$  are strongly rejected; when the adjusted dataset is used, virtually all are easily accepted.

### 5.4.1. Restrictions on Unadjusted Dataset

The strength of the rejections using the unadjusted dataset might on the face of it appear surprising, given the much more modest difference in tests for cointegration, shown in Table 2. The explanation, however, lies in the restrictions that, effectively, impose the dividend yield as an implicit cointegrating relation, rather than simply a linear combination of dividends and prices; they do not relate to  $q$ .

Thus, in both the  $r = 3$  and  $r = 2$  cases, the restriction that  $q$  be a cointegrating relation is easily imposed, as long as there is a single free parameter in the cointegrating relation for dividends (the second row of  $\beta_3$  or  $\beta_2$ ). There is still, it should be stressed, a cointegrating relation between dividends and capital (and hence, via  $q$ , between dividends and prices), but the data reject the restriction that the coefficients are consistent with the dividend yield being stationary.<sup>36</sup> These results are not altogether surprising given the apparently permanent shifts in the behaviour of buybacks and new issues discussed above, which in turn have implied permanent shifts in the unadjusted dividend yield.

We conclude, therefore, that there are major problems with using the system with unadjusted dividends, since the unadjusted dividend yield cannot be imposed

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<sup>35</sup> All estimation was carried out using the *views* full information maximum likelihood routine to estimate the parameters in restricted versions of (5.1) in a single stage.

<sup>36</sup> The joint restriction of  $\beta_1$  and the common growth rate is marginally acceptable at conventional significance levels over the truncated sample, but is strongly rejected over the full sample.

as a cointegrating relation.

### 5.4.2. Restrictions on Adjusted Dataset

In contrast, using the adjusted dataset, all restrictions are easily accepted. We thus find that the data are consistent with mean reversion of  $q$  and of the adjusted dividend yield (and hence of returns). This result holds whether or not we assume that there is also a stable equilibrium leverage ratio.<sup>37</sup>

## 5.5. What $q$ Predicts: Granger-Causality Tests for the ( $r = 2, 3$ ) Cases

We have established that the data are consistent with (log-linearised)  $q$  being one out of two, or possibly three, cointegrating relations between the set of variables in our cointegrating VAR - whether the unadjusted or adjusted dataset is used. If  $q$  is a cointegrating relation, it must have predictive power for (it must “Granger-Cause”) at least one of these variables. We first examine this predictive power in terms of these variables themselves; we then go on to examine predictive power for the elements in the forward sum representation, (4.4) which can be derived as functions (whether exact, or log linearised) of the VAR variables.

We show these test results solely for the adjusted dataset, given the rejection of the restrictions on  $\beta$  on the unadjusted dataset.

### 5.5.1. Granger Causality to VAR Variables

Table 4 summarises the predictive power of  $q$  for the VAR variables themselves. The relevant coefficients are simply the elements in the first row of  $\alpha$ ,  $\alpha_{1i}$ ,  $i = 1, \dots, 4$ . The test results show that, in line with most of the findings of the  $q$  investment literature,  $q$  has virtually no predictive power for the change in the capital stock (albeit that the estimated coefficients are of the correct sign). However,  $q$  has strong predictive power for both stock prices and dividends.

### 5.5.2. Granger Causality to Returns, (Proxied) Profitability and Leverage

The forward sum representation of  $q$  in (4.4) showed that, given mean reversion,  $q$  must predict one or more of the elements in the forward sum. Predictive power for two of the elements,  $\Delta k$  and  $\Delta l$ , can be examined directly from the VAR, and is shown in Table 4: as noted already, there is no evidence of Granger causality. Predictive power for the remaining elements can however only be estimated indirectly, as a function of the VAR coefficients. In Appendix C we show how to derive

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<sup>37</sup>For discussion of the  $\beta_1$  restriction, see Section 5.6.

the relevant causality tests. Table 5 shows the implied test statistics. It shows that  $q$  has statistically significant predictive power for (proxied) profitability (*i.e.*,  $d - k$  in the  $r = 3$  case, and  $cr_d$  in the  $r = 2$  case. Most strikingly, it shows that predictive power for returns is highly significant;

Thus, we find that, consistent with the  $q$  investment literature,  $q$  has no discernible predictive power for investment. But, as we noted at the start of the paper, if  $q$  mean-reverts, it must predict something else. Its predictive power for returns closely parallels that of the dividend yield in the predictability literature. Thus both literatures are effectively two sides of the same coin. We have argued however that the predictive power of  $q$  is logically prior to that of the dividend yield; and our econometric results suggest that, empirically,  $q$  dominates standard (ie unadjusted) measures of the dividend yield.

### 5.6. But Does $q$ Predict Anything Except Itself?

The Granger Causality tests of the previous section are, it should be stressed, predicated on the assumption that  $q$  mean-reverts, as one of two, or possibly three cointegrating relations in the system. The data are clearly consistent with this assumption. But it must be acknowledged that the power of these results is effectively only as strong as the evidence for mean reversion of  $q$ . We have already noted that this evidence is not overwhelmingly powerful when we looked at the univariate properties of  $q$  in Section 2.2. The counterpart to this result in terms of system properties is the ambiguity as to whether there is more than a single cointegrating relation.

Table 3 shows that if we restrict the system to have a single cointegrating relation, the restrictions implied by  $\beta_1$ , while strongly rejected for the unadjusted dataset, are accepted on the adjusted dataset. Thus the data are also consistent with the adjusted dividend yield being the sole cointegrating relation. As noted above, given our strong priors that true (as opposed to measured)  $q$  must mean-revert, this specification would imply unit root measurement error in capital.

This is most easily understood by comparison with the restricted  $r = 2$  system. The dividend yield can be expressed as a combination of the two cointegrating relations in that system, as  $d - p = \frac{1}{1-\zeta}(cr_d - q)$ , and so it arises as a cointegrating relation in both the restricted  $r = 2$  and  $r = 1$  systems. The finding that, using the adjusted dataset, we can accept the restriction that it be the *only* cointegrating relation is consistent with two possibilities:

- Both  $q$  and  $cr_d$  are individually mean-reverting, but the data do not provide sufficiently strong evidence to identify statistically significant differences in adjustment parameters.<sup>38</sup>

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<sup>38</sup>*i.e.* in the  $r = 2$  case using the adjusted dataset the first and second columns of  $\alpha$  have

- Neither  $q$  nor  $cr_d$  is individually mean-reverting, but they are cointegrated with cointegrating vector  $(1, -1)$ .

Our discussion of the determinants of the mean of Tobin's  $q$  in Section 3.3 would imply, if we rule out unit root measurement error in capital, the first of the two explanations must be correct, and the observation that the adjusted dividend yield appears mean-reverting must arise from both  $q$  and returns being mean-reverting (which, as we noted in Section 5.2, is equivalent to mean reversion of  $cr_d$ ).

The second case can therefore only arise as a result of unit root measurement error in capital. It is internally consistent, since, given the structure of both  $\beta_2$  and  $\beta_3$ , any such measurement error would feed symmetrically into the first and second rows, and would therefore indeed imply a unit cointegrating coefficient. It would also be qualitatively consistent with recent claims that capital has been systematically understated. This alternative representation of the data provides a quite drastically different answer to the question in the title of this paper. If measurement error means that measured  $q$  is a unit root process, then  $q$ , as measured, need not predict anything except itself.<sup>39</sup> The contrast between this answer and the answer if  $q$  mean-reverts is particularly marked if viewed in the context of data at the end of our sample period. This can be demonstrated most easily with reference to two charts.

Figure 5.1 provides a comparison of  $q$  with the adjusted dividend yield.<sup>40</sup> For most of the twentieth century, the two series gave very similar signals; albeit that the signal from the adjusted dividend yield was typically more accentuated than that from  $q$ . However, in the final three decades, the two series became increasingly divergent. The most striking contrast is in the last few years of the century. Whereas, as already noted,  $q$  was close to an historic high, the adjusted dividend yield ended the century almost precisely at its historic average.

Since, as we showed in Section 5.2, the adjusted dividend yield can in turn be expressed as the difference between  $q$  and the second possible cointegrating relation,  $cr_d$ , which determined the level of dividends in relation to capital (taking into account the impact of leverage) the implication is that *both* these series were at exceptionally high levels in historic terms. This is confirmed by the second chart, Figure 5.2,

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coefficients which are sufficiently close to being equal but opposite in sign.

<sup>39</sup>In principle, in a 2nd or higher order VAR, lagged values of  $\Delta q$  may have predictive power, but in our system to the extent that  $q$  has predictive power, it is the level of  $q$ , not its rate of change that counts.

<sup>40</sup>Both series are shown as log deviations from mean, with the sign switched on the adjusted dividend yield for ease of comparison.

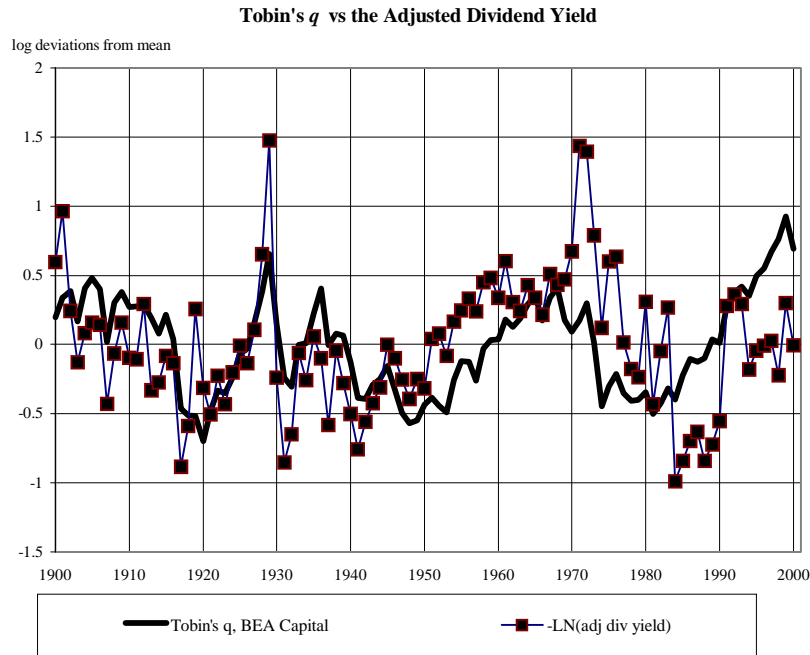


Figure 5.1:

The issue of what  $q$  was predicting at the end of the twentieth century comes down to what the two lines in Figure 5.2 can be expected to do in the next few years. If both  $q$  and  $cr_d$  are mean-reverting, both must be predicted to fall, and the only possible outcome is very poor returns. If, however, neither series is mean-reverting (due to unit root measurement error in capital) all that we need to look at is their relative magnitude (given by the adjusted dividend shown in Figure 5.1), which was at an historically normal level. On this argument, there was no reason to expect subsequent returns to deviate from historic averages.

The economic behaviour underlying this statistical argument is that, once account is taken of buybacks, the US corporate sector returned very significant amounts of cash to its shareholders during the 1990s. If the arguments of Hall (2000) and others are taken seriously, this could in principle reflect the fact that firms actually had very much higher levels both of capital and profits than recorded, and were in effect signalling this fact via buyback behaviour.

How seriously should we take these arguments? We have already acknowledged that, on strictly econometric grounds, it is hard to reject them. However, we should note two further implications, if such arguments are followed through, that provide other grounds for scepticism.

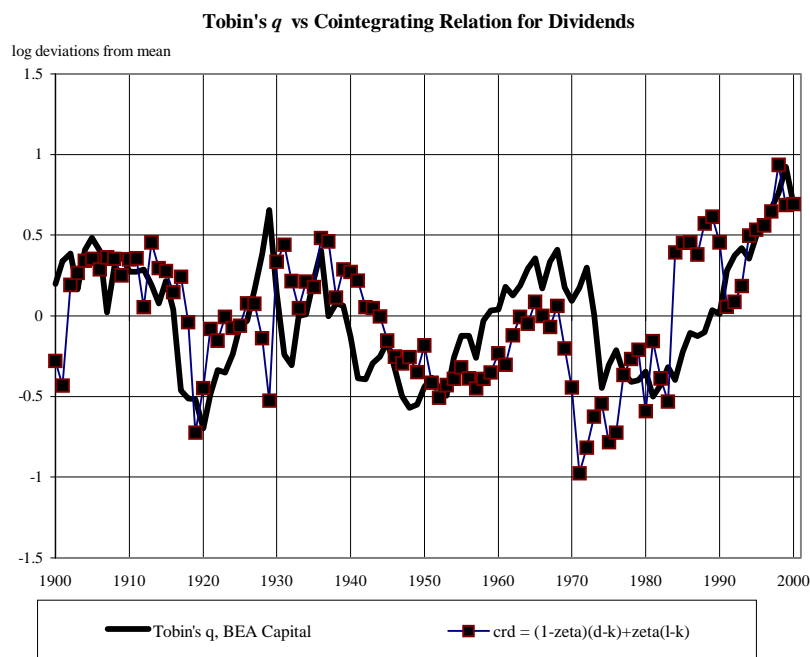


Figure 5.2:

The first is to note the sheer scale of the implied measurement error in  $q$ , which must imply that in recent years statisticians have made extremely large errors, not just in measuring capital, but also in their primary objective of measuring GDP and profits. Thus, Pickford, Smithers and Wright (2000) show that Hall's attribution of the rise in measured  $q$  in the 1990s to missing capital (which he christens "e"-capital) would have implied an under-recording of corporate GDP by at least 15% by the end of the 1990s<sup>41</sup>, with the implied "true" figure for profits being at least twice the recorded figure in the national accounts.

If true profits were being understated to this extent it might however also have been expected that companies would have signalled underlying profits, not just by buybacks, but by their reported profits. But Wright (2001) shows that P/E multiples on reported profits and on national income profits were very similar at the end of the century. By implication, underlying rates of profit on recorded capital were near-normal on both bases - in strong contrast to cash payments to shareholders. As a result, Wright also shows that, using adjusted dividends,

<sup>41</sup> On Hall's own figures, that assume an arguably very low rate of depreciation of "e-capital". A higher figure would imply an even greater understatement of gross investment, and hence GDP.

retained profits were negative in the late 1990s, giving added reason to suspect that adjusted dividends were unsustainably high at the end of the twentieth century.

## 6. Conclusions

Our key analytical and empirical findings are mutually consistent. Our theoretical analysis provided strong prior grounds for expecting that, in the absence of unit root measurement error, Tobin's  $q$  will be a mean-reverting process; but we have weaker priors about the nature of any measurement error. Our empirical results are consistent with mean reversion, but not sufficiently strongly that we can entirely rule out the possibility of unit root measurement error.

If any measurement error in  $q$  is mean-reverting, and  $q$  is therefore itself a mean-reverting process, it must predict something. We find, consistent with the empirical investment literature that Tobin's  $q$  does not, to any significant extent, predict investment. This remains a puzzle, but our approach has we hope made it clear that this does not necessarily imply a rejection of the (marginal)  $q$  theory of investment, *per se*, given the range of factors that can drive a wedge between marginal and Tobin's  $q$ . It is not the aim of this paper to attempt any resolution of this particular puzzle, although we hope that our framework may provide some pointers for future research.

If  $q$  is mean-reverting, and does not predict investment, it must predict something else. At current, still historically extreme, values of  $q$  the implication is pretty clear-cut:  $q$  predicts a prolonged period of poor returns.

If, however,  $q$  is not being properly measured, and is therefore not mean-reverting,  $q$  as measured need not be predicting anything. Some recent research (most notably Hall, 2000 and McGrattan & Prescott, 2001) has expressed scepticism as to the reliability of the data for  $q$ . We are sceptical as to the basis for this scepticism, but acknowledge that it should be taken seriously, and that this view is not inconsistent with some of our econometric results.

# Appendices

## A. Data Definitions

All data are taken from the dataset described in Wright (2001). We refer interested readers to that paper for information on sources and definitions of underlying data. The table below gives definitions in terms of variable codes given out in the appendix to the same paper.

variable	Unadjusted Dataset	Adjusted Dataset
$P$	SP/PCEY/E	SPTILDE/PCEY/ETILDE
$D$	DIV/PCEY/E	(DIV - NI)/PCEY/ETILDE
$K$	KBEA/PCEY/E	KBEA/PCEY/ETILDE
$L$	NLMBEA/PCEY/E	NLMBEA/PCEY/ETILDE
$Q$	(MV+NLMBEA)/KBEA	

where:

SP = Nonfinancial Share Price, 1945=1

PCEY= End-year CPI<sup>42</sup>

E = MV/SP

where MV = Market Value of Nonfinancial Equities, \$bns

SPTILDE= Adjusted Nonfinancial Share Price, 1945 = 1

ETILDE = MV/SPTILDE (= constant, by construction, = MV in 1945)

DIV = Nonfinancial dividends, \$bns

NI = New Issues, net of Buybacks, \$bns

KBEA = Capital Stock<sup>43</sup>

NLMBEA = Net Liabilities<sup>44</sup>

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<sup>42</sup>We deflate by the end-year CPI because this is the most reliable and consistent price index to span the entire period (it is also most appropriate when we approximate real returns). Results using a (less consistently defined) measure of the nonfinancial GDP deflator (PGDP, in Wright's (2001) dataset) are however very similar, and are available from the authors on request. Note that all series must of necessity be deflated by the same price index if underlying cointegrating relations ( $q$ ,  $d-p$ , and  $l-p$ ) are to be invariant to the method of deflation. As a result, however, implied changes in the "real" capital stock measure, effectively, changes in the real opportunity cost of the value of the capital stock, in terms of foregone consumption.

<sup>43</sup>We use a consistent measure of capital, taken (where available) direct from the BEA, rather than the series from the Fed's flow of funds, in which there is a distinct discontinuity from 1990 onwards(see Wright, 2001 for further discussion).

<sup>44</sup>Adjusted to ensure consistency with both Fed net worth figures and BEA capital stock figures. The adjustment only applies after 1989. See Wright (2001) for further discussion.

## B. The Hayashi Conditions: Linearity of the Representative Firm's Value Function

Differentiating the value function, given by (3.5) with respect to  $W_t$  at the optimal choice ( $I_{t+1}^*$ ), and using the envelope condition,

$$\begin{aligned}
\mu_t &\equiv \frac{\partial V(W_t, \theta_{t+1})}{\partial W_t} \\
&= E_t \left\{ M_{t+1} \left( \frac{\partial D_{t+1}}{\partial W_t} + \mu_{t+1} \right) \right\} \\
&= E_t \left\{ M_{t+1} \left( \frac{\partial \Pi_{t+1}}{\partial W_t} - G_{t+1} + \frac{I_{t+1}^*}{W_t} G'_{t+1} + \mu_{t+1} \right) \right\} \\
&= E_t \left\{ M_{t+1} \left( \frac{\partial \Pi_{t+1}}{\partial W_t} - G_{t+1} - \frac{I_{t+1}^*}{W_t} (\mu_{t+1} - G'_{t+1}) + \mu_{t+1} \frac{W_{t+1}}{W_t} \right) \right\} \\
&= E_t \left\{ M_{t+1} \left( \frac{\partial \Pi_{t+1}}{\partial W_t} - G_{t+1} - \frac{I_{t+1}^*}{W_t} + \mu_{t+1} \frac{W_{t+1}}{W_t} \right) \right\} \tag{B.1}
\end{aligned}$$

Where the last line of (B.1) is derived from the penultimate line by using (3.8).

Following Hayashi(1982), under the assumption of linear homogeneity of the production function, and competitive product and labour markets, the representative firm's profits are linear in capital.<sup>45</sup> Linearity of the profit function is a sufficient (but not necessary) condition for:

$$\frac{\partial \Pi_{t+1}}{\partial W_t} = \frac{\Pi_{t+1}}{W_t} \tag{B.2}$$

which in turn implies, using both (B.2) and (3.6) in (B.1),

$$\mu_t = E_t \left\{ M_{t+1} \left( \frac{D_{t+1}}{W_t} + \mu_{t+1} \frac{W_{t+1}}{W_t} \right) \right\} \tag{B.3}$$

The equivalent expression for  $\mu_{t+i}$ , can be written as:

$$\mu_{t+i} = E_{t+i} \left\{ M_{t+i+1} \left( \frac{D_{t+i+1}}{W_{t+i}} + \mu_{t+i+1} \frac{W_{t+i+1}}{W_{t+i}} \right) \right\} \tag{B.4}$$

Substituting recursively into (B.3), and applying the law of iterated expectations, implies

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<sup>45</sup>It should be stressed that this assumption, if acceptable, can only apply to the single atomistic representative firm. At the aggregate level, clearly profits must be concave in capital.

$$\begin{aligned}\mu_t &= \frac{E_t \{ \sum_{i=1}^{\infty} \beta_{t+i} D_{t+i} \}}{W_t} \\ &= \frac{V_t}{W_t}\end{aligned}$$

where  $\beta_{t+i} \equiv \prod_{j=1}^i M_{t+j}$  and the second line follows from (3.5) solved forwards.

### C. Deriving Granger Causality Tests on Returns and Other Cointegrating Relations

Dealing first with returns, the Campbell-Shiller approximation in (3.3) can be written as:

$$\begin{aligned}r_t &= \mathbf{e}'_1 \Delta \mathbf{x}_t + (1 - \rho) \mathbf{J} \boldsymbol{\beta}' \mathbf{x}_t \\ &= \mathbf{e}'_1 [\boldsymbol{\Psi} - \mathbf{A}_2 \Delta \mathbf{x}_{t-1} + (A_1 + A_2 - I) \mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t] \\ &\quad + (1 - \rho) \mathbf{J} \boldsymbol{\beta}' [\boldsymbol{\Psi} - \mathbf{A}_2 \Delta \mathbf{x}_{t-1} + (\mathbf{A}_1 + \mathbf{A}_2) \mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t] \\ &= \mathbf{e}'_1 [\boldsymbol{\Psi} - \mathbf{A}_2 \Delta \mathbf{x}_{t-1} + \boldsymbol{\alpha} \boldsymbol{\beta}' \mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t] + (1 - \rho) \mathbf{J} \boldsymbol{\beta}' [\boldsymbol{\Psi} - \mathbf{A}_2 \Delta \mathbf{x}_{t-1} + (\mathbf{I} + \boldsymbol{\alpha} \boldsymbol{\beta}') \mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t] \\ &= \left[ \mathbf{e}'_1 + (1 - \rho) \mathbf{J} \boldsymbol{\beta}' \right] \boldsymbol{\Psi} - \left[ \mathbf{e}'_1 + (1 - \rho) \mathbf{J} \boldsymbol{\beta}' \right] \mathbf{A}_2 \Delta \mathbf{x}_{t-1} \\ &\quad + \left[ \mathbf{e}'_1 \boldsymbol{\alpha} + (1 - \rho) \mathbf{J} + (1 - \rho) \mathbf{J} \boldsymbol{\beta}' \boldsymbol{\alpha} \right] \boldsymbol{\beta}' \mathbf{x}_{t-1} + \left[ \mathbf{e}'_1 + (1 - \rho) \mathbf{J} \boldsymbol{\beta}' \right] \boldsymbol{\varepsilon}_t\end{aligned}$$

where,  $\mathbf{e}'_1 = [1 \ 0 \ 0 \ 0]$  and  $\mathbf{J}$  is a matrix that selects the dividend yield as a linear combination of the cointegrating vectors (if the dividend yield is stationary this must always be possible)<sup>46</sup>. The test of non-causality from  $q$  to returns can be derived as follows.

Define the third term in square brackets as:

$$\boldsymbol{\tau} = \left[ \mathbf{e}'_1 \boldsymbol{\alpha} + (1 - \rho) \mathbf{J} + (1 - \rho) \mathbf{J} \boldsymbol{\beta}' \boldsymbol{\alpha} \right]$$

$\boldsymbol{\tau}$  is a row vector with  $r$  elements,  $\tau_1, \dots, \tau_r$ . Given the structure of  $\boldsymbol{\beta}_2$  and  $\boldsymbol{\beta}_3$ , as given in (5.3) we wish to test  $H_0 : \tau_1 = 0$ . This is equivalent to

$$H_0 : q_t \not\Rightarrow r_{t+1} : \rho \alpha_{11} + (1 - \rho) \alpha_{21} = \frac{1 - \rho}{1 - \zeta}$$

<sup>46</sup>We need  $\mathbf{J} = \left[ -\frac{1}{1-\zeta} \quad \frac{1}{1-\zeta} \quad 0 \right]$  for the  $r = 3$  case, and  $\mathbf{J} = \left[ -\frac{1}{1-\zeta} \quad \frac{1}{1-\zeta} \right]$  for the  $r = 2$  case.

The forward sum also implies that  $q$  may predict the remaining cointegrating relations. In the  $r = 3$  case, we can test separately whether  $q$  Granger-causes  $d - k$  and  $l - k$ ; in the more plausible  $r = 2$  case we test whether it predicts  $cr_d = (1 - \zeta)(d_t - k_t) + \zeta(l_t - k_t)$ . The implied restrictions are:

$$\begin{aligned} H_0 & : q_t \not\Rightarrow d_{t+1} - k_{t+1} : \alpha_{21} - \alpha_{31} = 0 \\ H_0 & : q_t \not\Rightarrow l_{t+1} - k_{t+1} : \alpha_{41} - \alpha_{31} = 0 \\ H_0 & : q_t \not\Rightarrow cr_{d,t+1} : (1 - \zeta)\alpha_{21} - \alpha_{31} + \zeta\alpha_{41} = 0 \end{aligned}$$

## D. Calibrating the Variance of “Noise”

We can attempt to calibrate the implied variance of  $\varepsilon$ , the “noise” process, in stock prices, consistent with the observed mean value of  $q$ , by reference to the observed historical equity premium. In the absence of a clear resolution of the puzzle identified by Mehra and Prescott (1985) that the premium appears to imply implausibly high risk aversion if priced off aggregate consumption, we can nonetheless work on the assumption that the observed historical equity premium does capture the true covariance between the excess stock return and the (unobservable, and still inexplicable) stochastic discount factor. Thus, we have the pricing relation:

$$E_t\{(1 + R_{t+1})M_{t+1}\} = (1 + R_{t+1}^s)E_tM_{t+1}$$

where  $R_{t+1}^s$  is the safe return. Assuming log-normality, taking logs of both sides and taking unconditional expectations, we can follow Campbell, Lo and Mackinlay (1997), and find an expression for the log of the mean equity premium in terms of the covariance of the stock return with the unobservable stochastic discount factor:

$$\eta \equiv \log E \left( \frac{1 + R_{t+1}}{1 + R_{t+1}^s} \right) = -cov(r_{t+1}, m_{t+1})$$

If we are prepared to assume that  $\varepsilon$  represents pure noise so that  $cov(\varepsilon, i) = 0$ , and under the further strong assumptions that there is no measurement error, and that  $\overline{\log \mu} = \bar{h} = 0$ , then

$$\bar{q} = \bar{e} \simeq \frac{1}{1 - \rho} \left[ cov(\varepsilon_{t+1}, m_{t+1}) + \frac{\sigma_\varepsilon^2}{2} \right]$$

but using (3.15),:

$$\begin{aligned} cov(r_{t+1}, m_{t+1}) &= cov(\varepsilon_{t+1}, m_{t+1}) + cov(i_{t+1}, m_{t+1}) \\ &= \beta_{m,\varepsilon} var(\varepsilon_{t+1}) + \beta_{m,i} var(i_{t+1}) \end{aligned}$$

where  $\beta_{m,\varepsilon}$  and  $\beta_{m,i}$  are the responses of the SDF to unit changes in  $\varepsilon$  and  $i$  respectively. Since the two elements of the stock return are by assumption observationally indistinguishable using the market information set, we can on *a priori* grounds set

$$\beta_{m,\varepsilon} = \beta_{m,i} = \beta_{r,i} = \frac{-\eta}{var(r_{t+1})}$$

which is directly measurable. Thus, on these assumptions:

$$\bar{q} = \frac{1}{1-\rho} var(\varepsilon_{t+1}) \left( -\frac{\eta}{var(r_{t+1})} + \frac{1}{2} \right)$$

Rather conveniently, for the purposes of our calculations, a mid-range estimate of  $\eta$  is of the order of  $0.04 \simeq 1-\rho \simeq var(r_{t+1})$ . Thus, with  $\bar{q} \simeq -.35$ , the implication is that, for “noise” to be the sole explanatory factor of the non-zero mean of  $q$  would require the variance of the “noise” process to be around two thirds of the variance of the log stock return.

## E. Tables

Table.1 Unit Root Tests for Alternative Measures of  $q$  ( $\equiv \ln Q$ )

		Tobin's $q$ (net debt)	Tobin's $q$ (gross debt)	“Equity $q$ ”
1900-1990	ADF	-3.12**(4)	-3.18**(4)	-3.13**(4)
	PP	-2.84*	-2.90 **	-2.96**
1900-2000	ADF	-2.71*(4)	-2.57(4)	-2.58*(4)
	PP	-2.46	-2.38	-2.44

Figures in parentheses after ADF statistics show number of lagged difference terms in ADF regression, chosen by Akaike Information Criterion

\* (\*\*) Rejection at 10% (5%) significance levels

Table 2 Cointegration (Maximal Eigenvalue) Tests

System with Unadjusted Dividends ( $\mathbf{x}_{ut}$ )					
Sample:	1902-1990		1902-2000		
$r$	LR	Eigenvalue	LR	Eigenvalue	90% C.V.
0 vs 1	23.80	0.215	25.24	0.180	25.0
1 vs 2	13.22	0.167	9.32	0.127	19.0
2 vs 3	4.52	0.084	5.12	0.075	13.0
3 vs 4	0.12	0.001	1.44	0.017	6.5
System with Adjusted Dividends ( $\mathbf{x}_{at}$ )					
0 vs 1	25.04	0.221	29.54	0.205	25.0
1 vs 2	8.84	0.109	10.16	0.136	19.0
2 vs 3	5.50	0.096	4.10	0.055	13.0
3 vs 4	1.38	0.016	0.14	0.001	6.5

Table 3 Likelihood Ratio Tests of Restrictions on  $\beta$  and Growth Rates

System with Unadjusted Dividends ( $\mathbf{x}_{ut}$ )					
Sample:		1902-1990		1902-2000	
	d.o.f.	LR	$p$ -value	LR	$p$ -value
$\beta_3, g$	3	18.08	.000	19.26	.000
$\beta_2, g$	5	17.36	.004	20.20	.001
$\beta_1, g$	5	9.84	.080	15.85	.007
$g$	3	0.51	.917	0.23	.972
System with Adjusted Dividends ( $\mathbf{x}_{at}$ )					
$\beta_3, g$	3	1.07	.783	1.69	.640
$\beta_2, g$	5	4.66	.459	9.76	.082
$\beta_1, g$	5	2.22	.817	4.19	.523
$g$	3	0.36	.948	0.24	.971

Notes:  $g$  denotes common growth rate restriction,  $\beta_k$  denotes the restriction of the  $\beta$  matrix in the  $r = k$  system to the form discussed in the text. The form of  $\beta_3$  imposes the common growth restriction automatically.

Table 4 Granger causality from  $q$  to VAR Variables

System with Adjusted Dividends ( $\mathbf{x}_{ut}$ )				
Sample:	1902-1990		1902-2000	
$r = 3$	$\hat{\alpha}_{1i}$	$p$ -value	$\hat{\alpha}_{1i}$	$p$ -value
$H_0 : q \not\Rightarrow \Delta p$	-0.267	0.002	-0.230	0.002
$H_0 : q \not\Rightarrow \Delta d$	0.207	0.042	0.311	0.001
$H_0 : q \not\Rightarrow \Delta k$	0.014	0.157	0.006	0.479
$H_0 : q \not\Rightarrow \Delta l$	0.084	0.076	-0.013	0.762
$r = 2$	$\hat{\alpha}_{1i}$	$p$ -value	$\hat{\alpha}_{1i}$	$p$ -value
$H_0 : q \not\Rightarrow \Delta p$	-0.298	0.000	-0.236	0.027
$H_0 : q \not\Rightarrow \Delta d$	0.260	0.010	0.297	0.030
$H_0 : q \not\Rightarrow \Delta k$	0.011	0.244	0.013	0.353
$H_0 : q \not\Rightarrow \Delta l$	0.069	0.139	0.039	0.524

Table 5 Granger causality from  $q$  to Returns and Other Cointegrating Relations

System with Adjusted Dividends ( $\mathbf{x}_{ut}$ )				
Sample:	1902-1990		1902-2000	
$r = 3$	$\chi^2(1)$	$p$ -value	$\chi^2(1)$	$p$ -value
$H_0 : q \not\Rightarrow r$	14.23	0.000	11.61	0.001
$H_0 : q \not\Rightarrow d - k$	3.69	0.055	6.34	0.012
$H_0 : q \not\Rightarrow l - k$	2.12	0.146	0.97	0.325
$r = 2$	$\chi^2(1)$	$p$ -value	$\chi^2(1)$	$p$ -value
$H_0 : q \not\Rightarrow r$	17.47	0.000	13.98	0.000
$H_0 : q \not\Rightarrow cr_2$	6.80	0.009	10.49	0.001

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