Supplementary Appendix for *Herding and Contrarian Behavior in Financial Markets*

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Abstract

This document contains additional material that was not included in the main version of the paper.

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A Proofs Omitted from the Paper

A.1 Proof of Lemma 1

Observe first that

$$\mathsf{E}[V|S,H^t]-\mathsf{E}[V|H^t]=\mathcal{V}q_2^t\left(\frac{\mathsf{Pr}(S|V_2)}{\mathsf{Pr}(S)}-1\right)+2\mathcal{V}q_3^t\left(\frac{\mathsf{Pr}(S|V_3)}{\mathsf{Pr}(S)}-1\right).$$

The RHS of the the above equality has the same sign as

$$\begin{aligned}
& q_2^t\left(\mathsf{Pr}(S|V_2)\sum_j q_j^t-\sum_j \mathsf{Pr}(S|V_j)q_j^t\right)+2\ q_3^t\left(\mathsf{Pr}(S|V_3)\sum_j q_j^t-\sum_j \mathsf{Pr}(S|V_j)q_j^t\right)\\
= \ & q_1^t q_2^t\left(\mathsf{Pr}(S|V_2)-\mathsf{Pr}(S|V_1)\right)+q_2^t q_3^t\left(\mathsf{Pr}(S|V_2)-\mathsf{Pr}(S|V_3)\right)\\
& +2\ q_3^t\left(q_1^t(\mathsf{Pr}(S|V_3)-\mathsf{Pr}(S|V_1)+q_2^t\left(\mathsf{Pr}(S|V_3)-\mathsf{Pr}(S|V_2)\right)\right)=\text{expression (2).}
\end{aligned}$$

A.2 Proof of Lemma 2

(i) By standard results on MLRP and stochastic dominance it must be that $\mathsf{E}[V|S_l]<\mathsf{E}[V|S_h]$. By a similar reasoning, at any history H^t, $\mathsf{E}[V|S_l,H^t]<\mathsf{E}[V|S_h,H^t]$ if the following MLRP condition holds at H^t: for any $S_l<S_h$ and any $V_l<V_h$

$$\frac{\mathsf{Pr}(S_h|V_h,H^t)}{\mathsf{Pr}(S_l|V_h,H^t)}>\frac{\mathsf{Pr}(S_h|V_l,H^t)}{\mathsf{Pr}(S_l|V_l,H^t)}. \tag{A-1}$$

To show this note first that $\mathsf{Pr}(V|H^t,S)=\mathsf{Pr}(V|S)\mathsf{Pr}(H^t|V)/\sum_{V'\in\mathbb{V}}\mathsf{Pr}(V'|S)\mathsf{Pr}(H^t|V')$. Then we have by the following manipulations that the MLRP condition $\frac{\mathsf{Pr}(S_h|V_h)}{\mathsf{Pr}(S_l|V_h)}>\frac{\mathsf{Pr}(S_h|V_l)}{\mathsf{Pr}(S_l|V_l)}$ implies the MLRP condition (A-1) at any H^t:

$$\begin{aligned}
\mathsf{Pr}(S_l|V_l)\mathsf{Pr}(S_h|V_h) &> \mathsf{Pr}(S_l|V_h)\mathsf{Pr}(S_h|V_l)\\
\Leftrightarrow\ \mathsf{Pr}(V_l|S_l)\mathsf{Pr}(V_h|S_h) &> \mathsf{Pr}(V_h|S_l)\mathsf{Pr}(V_l|S_h)\\
\Leftrightarrow\ \frac{\mathsf{Pr}(V_l|S_l)\mathsf{Pr}(H^t|V_l)}{\sum_{\mathbb{V}}\mathsf{Pr}(V|S_l)\mathsf{Pr}(H^t|V)}\frac{\mathsf{Pr}(V_h|S_h)\mathsf{Pr}(H^t|V_h)}{\sum_{\mathbb{V}}\mathsf{Pr}(V|S_h)\mathsf{Pr}(H^t|V)} &> \frac{\mathsf{Pr}(V_h|S_l)\mathsf{Pr}(H^t|V_h)}{\sum_{\mathbb{V}}\mathsf{Pr}(V|S_l)\mathsf{Pr}(H^t|V)}\frac{\mathsf{Pr}(V_l|S_h)\mathsf{Pr}(H^t|V_l)}{\sum_{\mathbb{V}}\mathsf{Pr}(V|S_h)\mathsf{Pr}(H^t|V)}\\
\Leftrightarrow\ \mathsf{Pr}(V_l|H^t,S_l)\mathsf{Pr}(V_h|H^t,S_h) &> \mathsf{Pr}(V_h|H^t,S_l)\mathsf{Pr}(V_l|H^t,S_h).
\end{aligned}$$

(ii) Suppose contrary to the claim, that an informed trader with signal S_1 does not sell at some history H^t. Then by part (i) no informed trader sells at H^t. This implies that at history H^t, $\mathsf{bid}^t=\mathsf{E}[V|H^t]$. But since, by part (i), $\mathsf{E}[V|H^t]$ exceeds $\mathsf{E}[V|S_1,H^t]$, we have $\mathsf{bid}^t>\mathsf{E}[V|S_1,H^t]$. Hence, an informed trader with signal S_1 sells at H^t. This is a contradiction.

The proof that informed traders with signal S_3 always buy is analogous.

(iii) First we show that $\mathsf{Pr}(S_1|V_1)>\mathsf{Pr}(S_1|V_3)$. Suppose otherwise; thus $\mathsf{Pr}(S_1|V_1)\leq$

$\mathsf{Pr}(S_1|V_3)$. Then the two MLRP conditions $\mathsf{Pr}(S_1|V_1)\mathsf{Pr}(S_2|V_3) > \mathsf{Pr}(S_2|V_1)\mathsf{Pr}(S_1|V_3)$ and $\mathsf{Pr}(S_1|V_1)\mathsf{Pr}(S_3|V_3) > \mathsf{Pr}(S_3|V_1)\mathsf{Pr}(S_1|V_3)$ imply respectively that $\mathsf{Pr}(S_2|V_1) < \mathsf{Pr}(S_2|V_3)$ and $\mathsf{Pr}(S_3|V_1) < \mathsf{Pr}(S_3|V_3)$. Hence, since $\mathsf{Pr}(S_1|V_1) \leq \mathsf{Pr}(S_1|V_3)$ we have $\sum_{i=1}^3 \mathsf{Pr}(S_i|V_3) > \sum_{i=1}^3 \mathsf{Pr}(S_i|V_1)$. But this contradicts $\sum_{i=1}^3 \mathsf{Pr}(S_i|V_j) = 1$ for every j.

The same argument can be applied to show that $\mathsf{Pr}(S_1|V_1) > \mathsf{Pr}(S_1|V_2)$ and $\mathsf{Pr}(S_1|V_2) > \mathsf{Pr}(S_1|V_3)$, and also in the reverse direction for $\mathsf{Pr}(S_3|V_1) < \mathsf{Pr}(S_3|V_2) < \mathsf{Pr}(S_3|V_3)$.

A.3 Proof of Lemma 3

This follows from Lemma 1: By the symmetry assumption on the priors ($q_1^1 = q_3^1$), the RHS of (2) is negative (positive) at $t = 1$ if and only if $(\mathsf{Pr}(S|V_3) - \mathsf{Pr}(S|V_1))(q_2^1 + 2q_1^1)q_3^1$ is less (greater) than 0; the latter is equivalent to S having a negative (positive) bias.

A.4 Proof of Lemma 4

The claim follows from $\mathsf{E}[V|H^t] - \mathsf{E}[V] = \mathcal{V}[(1 - q_1^t - q_3^t) + 2q_3^t] - \mathcal{V} = \mathcal{V}(q_3^t - q_1^t)$.

A.5 Proof of Lemma 6

The proof is analogous to the derivation in the proof of Lemma 1. To show (i) note that

$$\mathsf{E}[V|S, H^t] - \mathsf{ask}^t = \mathcal{V}q_2\left(\frac{\mathsf{Pr}(S|V_2)}{\mathsf{Pr}(S)} - \frac{\beta_2}{\mathsf{Pr}(\mathsf{buy}|H^t)}\right) + 2\mathcal{V}q_3\left(\frac{\mathsf{Pr}(S|V_3)}{\mathsf{Pr}(S)} - \frac{\beta_3}{\mathsf{Pr}(\mathsf{buy}|H^t)}\right).$$

The RHS of the above has the same sign as

$$\begin{aligned}
& q_2\left(\mathsf{Pr}(S|V_2)\sum_j \beta_j q_j - \beta_2 \sum_j \mathsf{Pr}(S|V_j)q_j\right) + 2\ q_3\left(\mathsf{Pr}(S|V_3)\sum_j \beta_j q_j - \beta_3 \sum_j \mathsf{Pr}(S|V_j)q_j\right) \\
= \ & q_1 q_2\left(\beta_1\mathsf{Pr}(S|V_2) - \beta_2\mathsf{Pr}(S|V_1)\right) + q_2 q_3\left(\beta_3\mathsf{Pr}(S|V_2) - \beta_2\mathsf{Pr}(S|V_3)\right) \\
& +2\ q_3\left(q_1\left(\beta_1\mathsf{Pr}(S|V_3) - \beta_3\mathsf{Pr}(S|V_1)\right) + q_2\left(\beta_2\mathsf{Pr}(S|V_3) - \beta_3\mathsf{Pr}(S|V_2)\right)\right) = \text{expression (3)}.
\end{aligned}$$

The proof of (ii) is analogous to that of (i).

A.6 Proof of Theorem 3

We first prove that any informed type buys initially if it has a negative bias and if there are enough noise traders.

Lemma II *Let S be negatively biased. Then $\mathsf{E}[V|S] < \mathsf{E}[V]$. Hence, there exists $\mu^{in} \in (0, 1]$ such that S sells at the initial history if $\mu < \mu^{in}$.*

Proof of Lemma II: Without loss of generality, we present the proof only for the case when the number of states $n > 2$ is even so that $n = 2k$ for some integer k. Then by the symmetry of the prior $\mathsf{E}[V] = (2k-1)/2$. Also, $\mathsf{E}[V|S] = \sum_{i=1}^{2k}(i-1)\mathsf{Pr}(V_i|S)$. Thus, we need to show

$$\sum_{i=1}^{2k}(i-1)\mathsf{Pr}(V_i|S) < \frac{2k-1}{2}. \tag{A-2}$$

Next note that by $\mathsf{Pr}(S|V_i) > \mathsf{Pr}(S|V_{n+1-i})$ together with the symmetry of the initial prior, we have $\mathsf{Pr}(V_i|S) > \mathsf{Pr}(V_{n+1-i}|S)$ for all $i < (2k+1)/2$. Using this and $\sum_{i=1}^{n}\mathsf{Pr}(V_i|S) = 1$, we have $\sum_{i=1}^{k}\mathsf{Pr}(V_i|S) > \frac{1}{2} > \sum_{i=k+1}^{2k}\mathsf{Pr}(V_i|S)$. Therefore

$$(k-1) + \sum_{i=k+1}^{2k}\mathsf{Pr}(V_i|S) < (k-1) + \frac{1}{2} = \frac{2k-1}{2}. \tag{A-3}$$

Then by (A-2) it is sufficient to show that

$$\sum_{i=1}^{k}(i-1)\mathsf{Pr}(V_i|S) + \sum_{i=k+1}^{2k}(i-1)\mathsf{Pr}(V_i|S) < (k-1) + \sum_{i=k+1}^{2k}\mathsf{Pr}(V_i|S). \tag{A-4}$$

But the second term on the left hand side of (A-4) is

$$\begin{aligned}
&\sum_{i=k+1}^{2k}(i-1)\mathsf{Pr}(V_i|S) = \sum_{i=k+1}^{2k}\mathsf{Pr}(V_i|S) + (k-1)\mathsf{Pr}(V_{k+1}|S) + \ldots + (2k-2)\mathsf{Pr}(V_{2k}|S) \\
< \; &\sum_{i=k+1}^{2k}\mathsf{Pr}(V_i|S) + (k-1)\mathsf{Pr}(V_{k+1}|S) + [(k-1)\mathsf{Pr}(V_{k+2}|S) + \mathsf{Pr}(V_{k-1}|S)] \\
&+[(k-1)\mathsf{Pr}(V_{k+3}|S) + 2\mathsf{Pr}(V_{k-2}|S)] + \ldots + [(k-1)\mathsf{Pr}(V_{2k}|S) + (k-1)\mathsf{Pr}(V_1|S)] \\
= \; &\sum_{i=k+1}^{2k}\mathsf{Pr}(V_i|S) + (k-1)\sum_{i=k+1}^{2k}\mathsf{Pr}(V_i|S) + \sum_{j=1}^{k}(k-j)\mathsf{Pr}(V_j|S).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\text{LHS of (A-4)} \; &< \; \sum_{i=1}^{k}(i-1)\mathsf{Pr}(V_i|S) + \sum_{i=k+1}^{2k}\mathsf{Pr}(V_i|S) + (k-1)\sum_{i=k+1}^{2k}\mathsf{Pr}(V_i|S) + \sum_{i=1}^{k}(k-i)\mathsf{Pr}(V_i|S) \\
&= \; (k-1) + \sum_{i=k+1}^{2k}\mathsf{Pr}(V_i|S) = \text{RHS of (A-4)}.
\end{aligned}$$

This demonstrates that $\mathsf{E}[V|S] < \mathsf{E}[V]$. To complete the proof of the lemma we also need to show that there exists $\mu^{in} \in (0,1]$ such that $\mathsf{E}[V|S] < \mathsf{bid}^1$ if $\mu < \mu^{in}$. As in Lemma 5 this follows immediately from $\mathsf{E}[V|S] < \mathsf{E}[V]$ and from $\lim_{\mu\to 0}\mathsf{E}[V] - \mathsf{bid}^1 = 0$. This completes the proof of Lemma II.

Next, we turn to the switching of behavior. Before doing that note that with MLRP the probability of a buy is increasing in the liquidation values and probability of a sale is decreasing in the liquidation values. To show this assume without any loss of generality that $S_1 < S_2 < \ldots < S_n$. Then we have the following.

Lemma III *When signals satisfy the MLRP, $\beta_1^t < \beta_2^t < \ldots < \beta_n^t$ and $\sigma_1^t > \sigma_2^t > \ldots > \sigma_n^t$.*

Proof of Lemma III: We will show only $\beta_1^t < \beta_2^t < \ldots < \beta_n^t$, the result on sales σ_i follows analogously. To show the former, observe that with MLRP signals, expectations are ordered in signals: for $i > j$, $\mathsf{E}[V|H^t, S_i] > \mathsf{E}[V|H^t, S_j]$. Thus, if signal type S_k buys, so will all $S_l > S_k$. Thus for $i > j$, $\beta_i - \beta_j$ has the same sign as

$$\begin{aligned}\sum_{l=m}^{n} \mathsf{Pr}(S_l|V_i) - \sum_{l=m}^{n} \mathsf{Pr}(S_l|V_j) &= 1 - \sum_{l=1}^{m-1} \mathsf{Pr}(S_l|V_i) - \left(1 - \sum_{l=1}^{m-1} \mathsf{Pr}(S_l|V_j)\right) \\ &= \sum_{l=1}^{m-1} \mathsf{Pr}(S_l|V_j) - \sum_{l=1}^{m-1} \mathsf{Pr}(S_l|V_i), \text{ for some } m \leq n.\end{aligned}$$

This latter expression is positive since MLRP implies First Order Stochastic dominance. This completes the proof of Lemma III.

Proof of part (a) of Theorem 3: Fix S. Analogously to Lemma 6, by simple calculations, it can be shown that $\mathsf{E}[V|S, H^t] - \mathsf{ask}^t$ has the same sign as

$$q_n^t q_{n-1}^t \cdot \sum_{j=1}^{n-1} \sum_{i=1}^{n-j} j \cdot \frac{q_i^t q_{i+j}^t}{q_{n-1}^t q_n^t} (\beta_i^t \mathsf{Pr}(S|V_{i+j}) - \beta_{i+j}^t \mathsf{Pr}(S|V_i)). \tag{A-5}$$

Next consider the infinite path consisting of only buys at every date. By MLRP and Lemma III, $\beta_1^t < \beta_2^t < \ldots < \beta_n^t$. Then we have that q_i^t/q_n^t converges to zero for all $i < n$. But this implies that along this path

$$\lim_t \mathsf{E}[V|H^t] = V_n > \mathsf{E}[V] \tag{A-6}$$

Also, since for any $i < n-1$ and $j \geq 1$ and for any t,

$$\frac{q_i^{t+1} q_{i+j}^{t+1}}{q_{n-1}^{t+1} q_n^{t+1}} = \frac{\beta_i^t \beta_{i+j}^t}{\beta_{n-1}^t \beta_n^t} \frac{q_i^t q_{i+j}^t}{q_{n-1}^t q_n^t} < \frac{q_i^t q_{i+j}^t}{q_{n-1}^t q_n^t},$$

we have that $(q_i^t q_{i+j}^t)/(q_{n-1}^t q_n^t)$ converges to zero along this infinite path of buys. Thus, as $t \to \infty$

$$\sum_{j=1}^{n-1} \sum_{i=1}^{n-j} j \cdot \frac{q_i^t q_{i+j}^t}{q_{n-1}^t q_n^t} (\beta_i^t \mathsf{Pr}(S|V_{i+j}) - \beta_{i+j}^t \mathsf{Pr}(S|V_i)) \to \lim_t [\beta_{n-1}^t \mathsf{Pr}(S|V_n) - \beta_n^t \mathsf{Pr}(S|V_{n-1})]. \tag{A-7}$$

Let S_i be the lowest type that buys at t. Then with MLRP all S_j with $j \in \{i, \ldots, n\}$ will buy. This implies that

$$\begin{aligned}
\beta_{n-1}^t \mathsf{Pr}(S|V_n) - \beta_n^t \mathsf{Pr}(S|V_{n-1}) &> 0 \\
\Leftrightarrow (\gamma + \mu \textstyle\sum_{j=i}^n \mathsf{Pr}(S_j|V_{n-1}))\mathsf{Pr}(S|V_n) &> (\gamma + \mu \textstyle\sum_{j=i}^n \mathsf{Pr}(S_j|V_n))\mathsf{Pr}(S|V_{n-1}) \\
\Leftrightarrow (\mathsf{Pr}(S|V_n) - \mathsf{Pr}(S|V_{n-1})) &> \tfrac{\mu}{\gamma} \textstyle\sum_{j=i}^n \{\mathsf{Pr}(S_j|V_n)\mathsf{Pr}(S|V_{n-1}) - \mathsf{Pr}(S_j|V_{n-1})\mathsf{Pr}(S|V_n)\}.
\end{aligned} \tag{A-8}$$

Since $\mathsf{Pr}(S|V_n) > \mathsf{Pr}(S|V_{n-1})$, the left hand side of the last inequality in (A-8) is positive. Therefore, by (A-8), for μ sufficiently small, $\beta_{n-1}^t \mathsf{Pr}(S|V_n) - \beta_n^t \mathsf{Pr}(S|V_{n-1}) > 0$. Since there is a finite number of types, it then follows from (A-5), (A-6), and (A-7) that there exist a critical level of informed trading $\mu^{ch} > 0$ and a history H^t along the infinite path of buys such that $\mathsf{E}[V|S, H^t] - \mathsf{ask}^t > 0$ and $\mathsf{E}[V|H^t] > \mathsf{E}[V]$. This together with Lemma II completes the proof.

Proof of part (b) of Theorem 3: Analogously to (a), we can rewrite $\mathsf{E}[V|S, H^t] - \mathsf{ask}^t =$

$$q_1 q_2 \cdot \sum_{j=1}^{n-1} \sum_{i=1}^{n-j} j \cdot \frac{q_i q_{i+j}}{q_1 q_2} (\beta_i^t \mathsf{Pr}(S|V_{i+j}) - \beta_{i+j}^t \mathsf{Pr}(S|V_i)). \tag{A-9}$$

Now consider the infinite path consisting of only sales at every date. By MLRP and Lemma III, $\sigma_1^t > \sigma_2^t > \ldots > \sigma_n^t$. Then we have that q_i^t / q_1^t converges to zero for all $i > 1$. But this implies that

$$\lim_t \mathsf{E}[V|H^t] = V_1 < \mathsf{E}[V] \tag{A-10}$$

Also, since for any $i, j \geq 1$ such that either i or $j > 1$, and any t,

$$\frac{q_i^{t+1} q_{i+j}^{t+1}}{q_1^{t+1} q_2^{t+1}} = \frac{\sigma_i^t \sigma_{i+j}^t}{\sigma_1^t \sigma_2^t} \frac{q_i^t q_{i+j}^t}{q_1^t q_2^t} < \frac{q_i^t q_{i+j}^t}{q_1^t q_2^t},$$

we have that $(q_i^t q_{i+j}^t)/(q_1^t q_2^t)$ converges to zero along this infinite path of buys. Thus, as $t \to \infty$

$$\sum_{j=1}^{n-1} \sum_{i=1}^{n-j} j \cdot \frac{q_i q_{i+j}}{q_1 q_2} (\beta_i \mathsf{Pr}(S|V_{i+j}) - \beta_{i+j} \mathsf{Pr}(S|V_i)) \to \lim_t [\beta_1 \mathsf{Pr}(S|V_2) - \beta_2 \mathsf{Pr}(S|V_1)]. \tag{A-11}$$

Let S_i be the highest type that buys at t. Then with MLRP all S_j with $j \in \{i, \ldots, n\}$ will buy at t. This implies that $\beta_1^t \mathsf{Pr}(S|V_2) - \beta_2^t \mathsf{Pr}(S|V_1) > 0$ if and only if

$$\Leftrightarrow (\mathsf{Pr}(S|V_2) - \mathsf{Pr}(S|V_1)) > \frac{\mu}{\gamma} \sum_{j=i}^n \{\mathsf{Pr}(S_j|V_2)\mathsf{Pr}(S|V_1) - \mathsf{Pr}(S_j|V_1)\mathsf{Pr}(S|V_2)\}. \tag{A-12}$$

Since $\mathsf{Pr}(S|V_2) > \mathsf{Pr}(S|V_1)$, the left hand side of (A-12) is positive. Thus, for μ sufficiently small, $\beta_1^t \mathsf{Pr}(S|V_2) - \beta_2^t \mathsf{Pr}(S|V_1) > 0$. As there are finite number of types, it then follows from (A-9), (A-10), and (A-11) that there exist a critical level of informed trading $\mu^{ch} > 0$

and a history H^t along the infinite path of sales such that $\mathsf{E}[V|S,H^t] - \mathsf{ask}^t > 0$ and $\mathsf{E}[V|H^t] < \mathsf{E}[V]$. This together with Lemma II completes the proof.

B Additional Results

B.1 Proposition 3a

(i) *Suppose that S buy herds and there is at most one U shaped signal.*
Then $\mu < \min\{\mu_s^{in}, \mu_2^{ch}\}$.

(ii) *Suppose that S sell herds and there is at most one U shaped signal.*
Then $\mu < \min\{\mu_b^{in}, \mu_1^{ch}\}$.

(iii) *Suppose that S acts as a buy contrarian and there is at most one hill shaped signal.*
Then $\mu < \min\{\mu_s^{in}, \mu_1^{ch}\}$.

(iv) *Suppose that S acts as a sell contrarian and there is at most one hill shaped signal.*
Then $\mu < \min\{\mu_b^{in}, \mu_2^{ch}\}$.

We shall prove (i); the proof of $(ii) - (iv)$ are analogous.

Since S sells initially it follows from Lemma 5 that $\mu < \mu_s^{in}$. To show that $\mu < \mu_2^{ch}$ first note that by Proposition 1, S must be nU shaped. Next consider the different possibilities separately.

Case A. There is no signal $S' \neq S$ such that $\mathsf{Pr}(S'|V_3) > \mathsf{Pr}(S'|V_2)$. Then it must be that $\mu_2^{ch}(S') = 1$ for all S' and therefore it must be that $\mu < \mu_2^{ch} = 1$.

Case B. There is a signal $S' \neq S$ such that $\mathsf{Pr}(S'|V_3) > \mathsf{Pr}(S'|V_2)$. Since S is U shaped it must be that $\mathsf{Pr}(S|V_3) > \mathsf{Pr}(S|V_2)$ and $\mathsf{Pr}(S''|V_3) \leq \mathsf{Pr}(S''|V_2)$ for $S'' \neq S, S'$. This implies that $\mu_2^{ch}(S'') = 1$ and hence, $\mu_2^{ch}(S') = \mu_2^{ch}$.

Now there are two cases. First, if $\mu_2^{ch}(S')$ also equals 1 then clearly $\mu_2^{ch} = 1$ and the claim is trivially true.

Second, assume that $\mu_2^{ch}(S') = \mu_2^{ch} < 1$. Since S buy herds at H^t, to show that $\mu < \min\{\mu_s^{in}, \mu_2^{ch}\}$ it suffices to show that S' also buys whenever S buys (the alternative is that S' does not buy so that $\mu^{ch} = 1 > \mu_2^{ch}$). When S' buys, $\mathsf{E}[V|S',H^t] - \mathsf{ask}^t > 0$. Suppose S' does not buy. As the sign of $\mathsf{E}[V|S',H^t] - \mathsf{ask}^t$ is given by equation (3), it must then hold that

$$\begin{aligned} q_1q_2\left[\beta_1\mathsf{Pr}(S'|V_2) - \beta_2\mathsf{Pr}(S'|V_1)\right] + q_2q_3[\beta_2\mathsf{Pr}(S'|V_3) - \beta_3\mathsf{Pr}(S'|V_2)] \\ +2\ q_1q_3\left[\beta_1\mathsf{Pr}(S'|V_3) - \beta_3\mathsf{Pr}(S'|V_1)\right] \quad \leq 0. \end{aligned} \tag{B-13}$$

Also, since there is at most one U shaped signal it must be that

$$\mathsf{Pr}(S'|V_3) > \mathsf{Pr}(S'|V_2) \geq \mathsf{Pr}(S'|V_1). \tag{B-14}$$

By Proposition 1 this implies that S' does not sell. By supposition S' does not buy and therefore S is the only buyer at H^t (S'' is selling). Since S is nU shaped we must also have $\beta_1^t > \beta_3^t \geq \beta_2^t$. This, together with (B-14) imply that the first and the third term in (B-13) are positive. Furthermore, the second term equals

$$\gamma(\mathsf{Pr}(S'|V_3) - \mathsf{Pr}(S'|V_2)) + \mu(\mathsf{Pr}(S|V_2)\mathsf{Pr}(S'|V_3) - \mathsf{Pr}(S|V_3)\mathsf{Pr}(S'|V_2)). \tag{B-15}$$

By (B-14) the first term in the last expression is positive; furthermore, since S is nU, we have $m^2 = \mathsf{Pr}(S|V_3) > \mathsf{Pr}(S|V_2)$. Since $\mu_2^{ch}(S') < 1$ we must have that $M^2(S') < 1$ is negative. But $-\mu M^2(S')$ is the second term in the last expression and it is thus positive. Consequently, (B-15) is positive. Therefore, the second term in (B-13) must also be positive. Therefore, S' must be buying at any H^t at which S buys and thus $\mu_2^{ch} < 1$ is unique.

B.2 Proof of the statement in condition (7) in Section 7 of the main text

First, note that, by (33) in the proof of Proposition 7, we have

$$\begin{aligned} \sigma_3\sigma_{2,o} - \sigma_{3,o}\sigma_2 &= -\mu^2\rho_{12}^{23} + \mu\gamma(\mathsf{Pr}(S|V_2) - \mathsf{Pr}(S|V_3)) < 0 \\ \sigma_2\sigma_{1,o} - \sigma_{2,o}\sigma_1 &> \sigma_3\sigma_{1,o} - \sigma_{3,o}\sigma_1 \end{aligned} \tag{B-16}$$

Also, since for herding we require $\mathsf{E}[V|S,H^1] < \mathsf{bid}^1$, it follows from Lemma 6 and (32) that

$$q_2^1 q_1^1[\sigma_2\sigma_{1,o} - \sigma_{2,o}\sigma_1] + q_3^1 q_2^1[\sigma_3\sigma_{2,o} - \sigma_{3,o}\sigma_2] + 2q_3^1 q_1^1[\sigma_3\sigma_{1,o} - \sigma_{3,o}\sigma_1] > 0.$$

But then by (B-16) we have

$$\sigma_2\sigma_{1,o} - \sigma_{2,o}\sigma_1 > 0. \tag{B-17}$$

Since $\mathsf{E}[V|H^t] - \mathsf{E}_o[V|H^t]$ has the same sign as the expression in (29), by simple expansion of this expression we have that if $b = 0$ then $\mathsf{E}[V|H^t] - \mathsf{E}_o[V|H^t]$ has the same sign as

$$\begin{aligned} & q_2^r q_1^r \left\{ (\sigma_2\sigma_{1,o} - \sigma_{2,o}\sigma_1) \sum_{\tau=0}^{s-1} (\sigma_2\sigma_{1,o})^{s-1-\tau}(\sigma_{2,o}\sigma_1)^\tau \right\} \\ & + q_3^r q_2^r \left\{ [(\sigma_3\sigma_{2,o}) - (\sigma_{3,o}\sigma_2)] \sum_{\tau=0}^{s-1} (\sigma_3\sigma_{2,o})^{s-1-\tau}(\sigma_{3,o}\sigma_2)^\tau \right\} \\ & + 2q_3^r q_1^r \left\{ (\sigma_3\sigma_{1,o} - \sigma_{3,o}\sigma_1) \sum_{\tau=0}^{s-1} (\sigma_3\sigma_{1,o})^{s-1-\tau}(\sigma_3\sigma_1)^\tau \right\}. \end{aligned}$$

Rearranging, we have that for $b = 0$, $\mathsf{E}[V|H^t] - \mathsf{E}_o[V|H^t]$ has the same sign as

$$\begin{aligned} & q_2^r q_1^r \frac{\sum_{\tau=0}^{s-1}(\sigma_2\sigma_{1,o})^{s-1-\tau}(\sigma_{2,o}\sigma_1)^\tau}{\sum_{\tau=0}^{s-1}(\sigma_3\sigma_{2,o})^{s-1-\tau}\ (\sigma_{3,o}\sigma_2)^\tau}[\sigma_2\sigma_{1,o} - \sigma_{2,o}\sigma_1] + q_3^r q_1^r[\sigma_3\sigma_{2,o} - \sigma_{3,o}\sigma_2] \\ + \ & 2q_3^r q_1^r \frac{\sum_{\tau=0}^{s-1}(\sigma_3\sigma_{1,o})^{s-1-\tau}(\sigma_3\sigma_1)^\tau}{\sum_{\tau=0}^{s-1}(\sigma_3\sigma_{2,o})^{s-1-\tau}(\sigma_{3,o}\sigma_2)^\tau}[\sigma_3\sigma_{1,o} - \sigma_{3,o}\sigma_1]. \end{aligned} \tag{B-18}$$

Further manipulations show that

$$\left(\frac{\sigma_1}{\sigma_3}\right)^s > \frac{\sum_{\tau=0}^{s-1}(\sigma_2\sigma_{1,o})^{s-1-\tau}(\sigma_{2,o}\sigma_1)^\tau}{\sum_{\tau=0}^{s-1}(\sigma_3\sigma_{2,o})^{s-1-\tau}(\sigma_{3,o}\sigma_2)^\tau} \Leftrightarrow \sum_{\tau=0}^{s-1}\sigma_2{}^{s-1-\tau}\sigma_{2,o}{}^\tau(\sigma_1\sigma_3)^\tau\left((\sigma_1\sigma_{3,o})^{s-1-\tau}-(\sigma_3\sigma_{1,o})^{s-1-\tau}\right)>0.$$

Also, by assumption we have $\frac{\sigma_1}{\sigma_3} > \frac{\sigma_{1,o}}{\sigma_{3,o}}$. Therefore, we must have

$$\left(\frac{\sigma_1}{\sigma_3}\right)^s > \frac{\sum_{\tau=0}^{s-1}(\sigma_2\sigma_{1,o})^{s-1-\tau}(\sigma_{2,o}\sigma_1)^\tau}{\sum_{\tau=0}^{s-1}(\sigma_3\sigma_{2,o})^{s-1-\tau}(\sigma_{3,o}\sigma_2)^\tau}. \tag{B-19}$$

Similar manipulations show that

$$\left(\frac{\sigma_1}{\sigma_2}\right)^s < \frac{\sum_{\tau=0}^{s-1}(\sigma_3\sigma_{1,o})^{s-1-\tau}(\sigma_3\sigma_1)^\tau}{\sum_{\tau=0}^{s-1}(\sigma_3\sigma_{2,o})^{s-1-\tau}(\sigma_{3,o}\sigma_2)^\tau} \Leftrightarrow \sum_{\tau=0}^{s-1}\sigma_3{}^{s-1-\tau}\sigma_{3,o}{}^\tau(\sigma_1\sigma_2)^\tau\left((\sigma_2\sigma_{1,o})^{s-1-\tau}-(\sigma_1\sigma_{2,o})^{s-1-\tau}\right)>0.$$

This together with (B-17), implies that

$$\left(\frac{\sigma_1}{\sigma_2}\right)^s < \frac{\sum_{\tau=0}^{s-1}(\sigma_3\sigma_{1,o})^{s-1-\tau}(\sigma_3\sigma_1)^\tau}{\sum_{\tau=0}^{s-1}(\sigma_3\sigma_{2,o})^{s-1-\tau}(\sigma_{3,o}\sigma_2)^\tau}. \tag{B-20}$$

Also, since $\mathsf{E}[V|S,H^t]-\mathsf{bid}^t>0$, by Lemma 6, we have that if $b=0$ then

$$q_2^r q_1^r\left(\frac{\sigma_1}{\sigma_3}\right)^s[\sigma_2\sigma_{1,o}-\sigma_{2,o}\sigma_1]+q_3^r q_2^r[\sigma_3\sigma_{2,o}-\sigma_{3,o}\sigma_2]$$
$$+2q_3^r q_1^r\left(\frac{\sigma_1}{\sigma_2}\right)^s[\sigma_3\sigma_{1,o}-\sigma_{3,o}\sigma_1] \;<\; 0. \tag{B-21}$$

Then it follows from (B-21), together with $\frac{\sigma_1}{\sigma_3} > \frac{\sigma_{1,o}}{\sigma_{3,o}}$, (B-17), (B-19) and (B-20), that the expression in (B-18) is negative. Thus $\mathsf{E}[V|H^t]-\mathsf{E}_o[V|H^t]<0$ and (7) follows.

C The Parameters used for Figure 1

Herding Example

$\mathsf{Pr}(S\|V)$	V_1	V_2	V_3
S_1	$\frac{601}{1000}$	$\frac{270}{1000}$	0
S_2	$\frac{399}{1000}$	$\frac{180}{1000}$	$\frac{245}{1000}$
S_3	0	$\frac{550}{1000}$	$\frac{755}{1000}$

$\mathbb{V}=(0,10,20)$

$\mathsf{Pr}(V)=(1/100,98/100,1/100)$

$\mu_b^{ch}=0.9496,\ \mu_b^{in}=0.4294$

$\Rightarrow \mu=0.4294-0.0001$

Contrarian Example

$\mathsf{Pr}(S\|V)$	V_1	V_2	V_3
S_1	$\frac{7}{10}$	$\frac{1}{4}$	0
S_2	$\frac{3}{10}$	$\frac{1}{2}$	$\frac{3}{20}$
S_3	0	$\frac{1}{4}$	$\frac{17}{20}$

$\mathbb{V}=(0,10,20)$

$\mathsf{Pr}(V)=(1/4,1/2,1/4)$

$\mu_b^{ch}=0.4706,\ \mu_b^{in}=0.1922$

$\Rightarrow \mu=0.1922-0.001$