

# Multiperson Bargaining and Strategic Complexity<sup>‡</sup>

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## Abstract

We investigate the effect of introducing costs of complexity in the  $n$ -person unanimity bargaining game. As is well-known, in this game every individually rational allocation is sustainable as a Nash equilibrium (also as a subgame perfect equilibrium if players are sufficiently patient and if  $n > 2$ ). Moreover, delays in agreement are also possible in such equilibria. By limiting ourselves to a plausible notion of complexity that captures length of memory, we find that the introduction of complexity costs (lexicographically with the standard payoffs) does not reduce the range of possible allocations but does limit the amount of delay that can occur in any agreement. In particular, we show that in any  $n$ -player game, for any allocation  $z$ , an agreement on  $z$  at any period  $t$  can be sustained as a Nash equilibrium of the game with complexity costs if and only if  $t \leq n$ . We use the limit on delay result to establish that, in equilibrium, the strategies implement stationary behaviour. Finally, we also show that “noisy Nash equilibrium” with complexity costs sustains only the unique stationary subgame perfect equilibrium allocation.

**Keywords:** Bargaining, complexity, history independence, equilibrium selection.

**JEL classification:**C7, D0,D7

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## 1. Introduction

In many multistage games, such as repeated games and bargaining with more than two players, a large number of outcome paths can be supported as equilibria. Such outcome paths are often sustained (as equilibria) by the use of non-stationary (history dependent) strategies by the players, which induce different expectations for future play depending on the history preceding the play. These equilibria can be made to satisfy subgame perfection and other stronger refinements.

One approach for dealing with the multiplicity problem is to consider explicitly costs of human computational and storage abilities (bounded rationality) that restrict the extent to which strategies can depend on history. Modelling strategies as being implemented by finite-state machines falls within this category. In this paper, we investigate the effect of introducing complexity considerations in the  $n$ -person unanimity bargaining game and thereby provide a justification in terms of complexity costs for history-independent (stationary) equilibrium strategies. The bulk of the literature on automata theory has dealt with 2-player repeated, normal-form games. (See Kalai (1990) for a survey.) This paper is an exception because first, we are concerned with a single, extensive-form game with  $n \geq 2$  players and, second, we introduce a different (and plausible) notion of complexity that is not found in the previous literature.

We now describe the object of our investigation, the unanimity bargaining game ( $n$ -player, one-cake problem) of Binmore (1985), Herrero (1985) and Shaked (1986). The game is as follows: There are  $n$  players  $(1, \dots, n)$  who have joint access to a ‘cake’ of size unity, if they can agree on how to share it amongst themselves. Player 1 makes the first offer at time  $t = 1$ , offering  $x = (x_1, \dots, x_n)$ , where  $x_i$  is player  $i$ ’s proposed share and  $\sum_i x_i = 1$ . Player 2 to player  $n$  then respond sequentially, each saying either A(ccept) or R(eject) to the proposal. If all responders accept then the game ends with player  $i$  obtaining a payoff of  $x_i$ . A rejection takes the game to the next period, where player 2 now makes an offer and players  $(3, \dots, n, 1)$  sequentially respond. If one of the responders rejects at time  $t = 2$ , then the game goes to the next stage with player 3 making the offer and so on.<sup>1</sup> If the offer  $x$  is accepted by all responders in period  $t$ , the payoff is  $\delta^{t-1}x_i$  to player  $i$ , where  $\delta < 1$  is the common discount factor. There is no exogenously imposed limit on the duration of the game. Absence of agreement, that is bargaining forever, leads to a payoff of 0 for all players. Trivially, one can show that any partition of the cake at any time  $t$  can be sustained as a Nash equilibrium of this game. Shaked showed (see Osborne and Rubinstein (1990) for an exposition) that for the 3-player game every allocation of the cake can be sustained as a subgame perfect equilibrium (SGPE) if  $\delta > \frac{1}{2}$ . Outcomes with delay can also be sustained as SGPE for 3-player games. This is in sharp contrast to the 2-player bargaining

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<sup>1</sup>Thus player  $k$  makes an offer at  $t = k + \tau n$  for every non-negative integer  $\tau$ .

model of Rubinstein (1982) where there is a unique SGPE allocation  $(\frac{1}{1+\delta}, \frac{\delta}{1+\delta})$  with no delay and the property that as  $\delta \rightarrow 1$ , the allocation tends to an equal split. The indeterminacy of the three (or  $n$ ) player game has aroused much interest in the literature, for example Jun (1987) and Krishna and Serrano (1996). These papers have explored alternative extensive forms that give a unique SGPE. Their models have the feature that a player is able to leave with his share before the entire bargaining process is completed, in contrast with the previously-cited work, where the entire allocation has to be unanimously approved before anyone can leave.

The stationary SGPE allocation of the original extensive form has also been proposed as a “focal” equilibrium.<sup>2</sup> It is unique and involves no delay (Herrero, (1985)). Moreover the allocation corresponding to the unique stationary SGPE is  $(\frac{1}{1+\delta+\dots+\delta^{n-1}}, \frac{\delta}{1+\delta+\dots+\delta^{n-1}}, \dots, \frac{\delta^{n-1}}{1+\delta+\dots+\delta^{n-1}})$ , which also tends to the equal split as  $\delta \rightarrow 1$ .

It might be argued, along the lines discussed above, that a player concerned with the cost of implementing complex strategies would choose a stationary strategy, where offers and responses are independent of history.<sup>3</sup>(This would then lend some credence to the “focal” equilibrium.) This paper is an attempt to formalise this intuition by introducing complexity costs lexicographically with the standard payoff into the players preference ordering as in Rubinstein (1986), Abreu and Rubinstein (1988), Piccione and Rubinstein (1993) and others. These earlier papers modelled players as finite-state automata involved in a two-player repeated game. Complexity was modelled as the (arbitrarily small) cost of maintaining an additional “machine” state. In these papers, the number of states in an automaton implementing a strategy can be regarded as a measure of the extent of history-dependence of the strategy. We will also focus on complexity of implementation rather than of computation (see Papadimitriou, (1992)) but we will account for complexity of strategies in a different manner. Informally, our measure of complexity has the property that if two strategies are otherwise identical except that in some instances the first strategy uses more information than that available in the current ‘stage’ of bargaining and the second uses only the information available in the current ‘stage’ then the first strategy is more complex than the second.<sup>4</sup> Our notion of complexity neither implies nor is implied by the ‘counting states’ notion of complexity. In fact, Chatterjee and Sabourian (1998), henceforth referred to as CS, show that with the ‘counting states’ measure of complexity the results of this paper on stationarity and equilibrium selection do not hold in general except

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<sup>2</sup>Almost all models of non-cooperative coalitional bargaining invoke stationarity to get results - see, for example, Chatterjee et.al. (1993). Gul (1989) mentions computational simplicity as a reason for selecting stationary equilibria in these models, but does not elaborate on this statement.

<sup>3</sup>Osborne and Rubinstein (1994) argue, however, that it may be difficult to justify a stationary equilibrium by appealing to its simplicity. They argue as follows: equilibrium should be thought of as equilibrium in beliefs; but then it is not clear why a player should believe that the other players are using stationary strategies after a history of highly non-stationary behaviour by them.

<sup>4</sup>In this paper, a ‘stage’ of bargaining refers to  $n$  periods of play starting with player 1 making the first offer and player  $n$  making the offer in the last period of the stage. The results of this paper also hold if a ‘stage’ refers to a single period of play. (See section 4.)

in 2-player games.<sup>5</sup>

We now briefly describe the rest of the paper. In Section 2, we describe the bargaining game with complexity costs and characterise the set of Nash equilibria in the game. We also contrast our notion of complexity with that found in the automata literature. Our first set of results shows that in any  $n$ -player game, agreement at any period  $t \leq n$  on any partition of the cake can be sustained as a Nash equilibrium of the game with complexity costs. Thus, this result demonstrates that delays in agreement are possible. We also demonstrate that absence of agreement (indefinite delay) can also be sustained as a Nash equilibrium in such a game. Our second set of results shows, however, that there is some bite to the simplicity requirement; agreements beyond period  $n$  cannot be sustained as Nash equilibria in the game with complexity costs. (Without the latter this would not be the case). Thus, all Nash equilibria of the game must entail *either* an agreement by some time period  $t \leq n$  *or* indefinite delay. Further, we show that all such equilibria are ‘stationary’ in the sense they behave in the same way in any ‘stage’ of the bargaining, irrespective of the previous history of the game.<sup>6</sup> Thus, the results of this section demonstrate that all strategies that can be implemented by equilibrium machines must be stationary.

Strategy profiles implemented by Nash equilibria in the game with complexity costs are not necessarily credible ‘off the equilibrium path’. One way of introducing credibility is to extend the concept of SGPE to the game with complexity costs. There is not an obvious way of doing this, since it could potentially involve changing the strategy used after every period, thus making the intuitive notion of choosing to “buy” complexity at the outset of the game (as in Abreu and Rubinstein, 1988) somewhat problematic. An alternative way of capturing credibility of strategies is to introduce noise. A natural way of doing this is to consider the Nash equilibria of a game with complexity costs in which the strategies make errors with arbitrarily small probability. Section 3 introduces this version, where the strategies are not error free. Such errors force the “noisy Nash equilibrium” to be credible on “almost” all histories. The results of section 2 on stationary behaviour can then be used to show that a noisy Nash equilibrium of the game with complexity costs leads to the unique stationary SGPE allocation of the bargaining game.<sup>7</sup> An alternative approach, suggested by a referee, is to require strategies implemented by Nash equilibria in the game with complexity costs to be optimal after *all* histories, rather than after *almost all* histories. We sketch the proof of this slightly weaker version of the result in the paper, since it is shorter than the proof with noise.

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<sup>5</sup>The results of Abreu and Rubinstein on equilibrium selection in 2-player repeated games with the ‘counting states’ measure of complexity do not extend to  $n$ -player repeated games (Bloise 1998).

<sup>6</sup>Thus *stationary strategies* in our set-up refer to strategies that prescribe the same behaviour in every  $n$  periods of play, irrespective of the previous history. Stationarity would have a different meaning if we defined a ‘stage’ as referring to a single period (see footnote 4 and Section 4).

<sup>7</sup>The Shaked argument that generates every allocation as a SGPE does not work in our setup because the strategies used in Shaked’s proof are non-stationary and thus they cannot constitute a Nash equilibrium of the game with complexity costs.

In Section 4, we discuss an alternative formulation of complexity that has the same flavour as the one used in this paper, and conclude the paper with a general discussion.

Binmore, Piccione and Samuelson (1998) also consider complexity (using a definition similar to the ‘counting states’ measure of complexity) and bargaining.<sup>8</sup> Their paper is about the *two-player* Rubinstein model, rather than the  $n(\geq 2)$ -player case, and they obtain their results by using a refinement of Nash equilibrium, modified evolutionary stability, rather than subgame perfectness. In a sense the aim of their paper is to establish the Rubinstein result for 2 players with an evolutionary solution concept. It is therefore quite different in spirit from our model; we concentrate on selection among the set of (subgame perfect) equilibria in the *n-player* game by introducing complexity costs. It is not obvious how the framework of Binmore et al. extends to the more general *n-player* game considered here. Of course, there is no selection issue with subgame perfectness in two-player games; there is with *Nash*.

Baron and Kalai (1993) have considered a majority rule game played by automata. They use undominated Nash equilibrium as the solution concept, and are concerned with the complexity of *equilibria* rather than the complexity of individual strategies.

## 2. The Deterministic Game with Complexity Costs

### 2.1. Basic Definitions

In this subsection, we shall first define the notion of a ‘period’ and a ‘stage’ of the bargaining game and then define the notion of a strategy in the game.

Each player in our game has to play different roles, as proposer or  $k^{\text{th}}$  responder,  $k \geq 1$ , and thus must have different action sets corresponding to these roles. Let  $\Delta^n \equiv \{x = (x_1, \dots, x_n) \mid \sum_i x_i = 1\}$  be the  $(n-1)$ -dimensional simplex. A period refers to a single offer  $x \in \Delta^n$  by one player and responses made sequentially and separately by the others. The set of choices available to a player in the game is denoted by

$$C \equiv \Delta^n \cup A \cup R, \quad \text{where } A \text{ and } R \text{ refer to acceptance and rejection, respectively.}$$

A stage refers to  $n$  consecutive periods with player 1 making the proposal in the first period, player 2 making the proposal in the second period...and player  $n$  making the proposal in the  $n$ -th period. A stage could be terminated by an agreement before  $n$  periods have elapsed. Clearly, the bargaining game is stationary at the beginning of each stage. In any given period, a player is in one of the following  $n$  roles: the proposer or the  $k$ -th responder for some  $k = 1, \dots, (n-1)$ , while in any stage the player must be prepared to play all  $n$  roles. We shall denote a history of outcomes in a stage by  $e$ . Also we shall denote the set of all such possible histories of a stage by  $E$ . Thus a particular history of a stage,  $e = \{(x^1, A, A, R), (x^2, A, A, A, R), \dots, (x^n, R)\} \in E$ , could, for example, consist of an offer  $x^1$  by player 1, an acceptance by the first and

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<sup>8</sup>This paper came to our notice after earlier drafts of our current paper had been written.

the second responder and a rejection by the third responder in the first period of the stage, followed by an offer  $x^2$ , an acceptance by the first three responders and a rejection by the fourth responder in the second period of the stage, ..., and finally an offer  $x^n$  rejected by the first responder in the  $n$ -th period of the stage. A history of a stage is therefore a complete account of what happened in the bargaining in a given stage. Notice that a history of a stage consists of at most  $n \times n$  choices (offers and responses) by the players.

We also need notation for partial descriptions of choices (partial history) within a stage. We shall denote such a partial history by  $s$  and the set of such partial histories by  $S$ . Examples of  $s \in S$  could be the null (empty) set  $\emptyset$ , an offer  $x$ , an offer  $x$  followed by  $k$  Acceptances for some  $k < n$ , histories of the first  $k$  periods of the stage for some  $k < n$  ..... If  $s$  is the null history, the stage is just beginning and an offer has yet to be made. Thus

$S = \{\emptyset\} \cup \{s = (c^1, \dots, c^r) \in C^r \mid (s, d^1, \dots, d^{r'}) \in E \text{ for some sequence of choices } (d^1, \dots, d^{r'}) \in C^{r'}\}$ , where for any  $\tau$ ,  $C^\tau$  is the  $\tau$ -fold Cartesian product of  $C$ .

Also, we shall denote the information sets (the sets of partial histories) for player  $i$  in any stage by  $S_i$ . Thus  $S_i \equiv \{s \in S \mid \text{it is } i\text{'s turn to play after } s\}$ .

Denote the set of choices available to a player  $i$ , given a partial description  $s \in S_i$ , by  $C_i(s)$ . Thus

$$C_i(s) = \begin{cases} \Delta^n & \text{if } s \text{ is such that } i \text{ is the proposer} \\ \{A, R\} & \text{if } s \text{ is such that } i \text{ is the responder to some offer } x. \end{cases}$$

Let  $E^t$  be the set of all possible histories of outcomes of  $t$  stages, excluding those that have resulted in an agreement. Also let  $E^\infty \equiv \cup_{t=0}^\infty E^t$  be the set of all possible finite histories of stages. ( $E^0$  is assumed to be empty). Then a strategy for player  $i$  is a function  $f_i : E^\infty \times S_i \rightarrow C$ , where  $f_i(e^1, \dots, e^t, s) \in C_i(s)$  for any  $(e^1, \dots, e^t) \in E^t$  and for any partial history  $s \in S_i$ . Also denote the set of strategies for  $i$  by  $F_i$ .

Complexity of strategies can be defined in many different ways. We adopt a definition of complexity that has the property that if two strategies are otherwise identical except that in some instance the first strategy uses more information (memory) than that available in the current stage and the second uses only the information available in the current stage, then the first strategy is more complex than the second.

**Definition 1.** A strategy  $f_i$  is more complex than another  $f'_i$ , denoted by  $f_i \succ f'_i$ , if the strategies  $f_i$  and  $f'_i$  are otherwise identical except that given some partial history  $s' \in S_i$  of a stage,  $f'_i$  always responds the same way to  $s'$  irrespective of the history of the game in previous stages and  $f_i$  responds differently to  $s'$  depending on the history of actions before  $s'$ . Formally, there exists a partial history  $s' \in S_i$  such that

$$\left. \begin{aligned} f_i(h, s) &= f'_i(h, s) & \forall s \neq s' \text{ and } \forall h \in E^\infty \\ f'_i(h, s') &= f'_i(h', s') & \forall h, h' \in E^\infty \\ f_i(h, s') &\neq f_i(h', s') & \text{for some } h \text{ and } h' \in E^\infty \end{aligned} \right\} \quad (2.1)$$

In the above definition, strategies that condition their behaviour at any instance on the history of the game prior to the current *stage* are more complex than those that do not. Thus the (partial) history of the current stage has a special significance in the above definition. We have adopted the above definition of complexity because the structure of the bargaining game is stationary at the beginning of each stage. However, it turns out (see Section 4) that the results of this paper on stationary behaviour and uniqueness with noise hold if we adopt an alternative definition of complexity that has a similar flavour to the one above, except that *stage* in the definition above is replaced by *period*. This alternative definition does not have the ‘game-stationarity’ feature that the current one does.

In the next subsection, we discuss implementation of strategies by automata, in order to facilitate comparisons with the literature on complexity in games. The formal results in the rest of the paper do not depend on Subsection 2.2.

## 2.2. Automata and state-complexity

One can think of a strategy in the bargaining game as an automaton consisting of a set of states (not necessarily finite), an initial state, an output function describing the output of the machine as a function of its current state (and its current input) and a transition function determining the next state of the machine as a function of its current state and the actions of all the players in a given stage. Clearly, if there are no restrictions on the size of the set of states there is no loss of generality in this exercise.

Since the bargaining game has a certain degree of asymmetry built in and each player has to play a different role in different periods, one can choose to specify an automaton (machine) to implement a particular strategy in several different ways. In CS, we represent strategies by automata and assume that the states of the machine do not change during each stage of the bargaining game and transitions from a state to another state in the same player’s machine take place at the end of a stage. Also it is assumed that each state of the machine specifies an action for every role of the player concerned, with the action chosen depending on  $s$  - the partial history of the stage. The formal definition is as follows.

**Definition 2.** A machine  $M_i$  is a four-tuple  $(Q_i, q_i^1, \lambda_i, \mu_i)$ , where  $Q_i$  is a set of states;  $q_i^1$  is a distinguished initial state belonging to  $Q_i$ ;  $\lambda_i : Q_i \times S_i \rightarrow C$ , describes the output function of the machine given the state of the machine and given the partial history that has occurred during the current stage of the bargaining before  $i$ ’s move, such that  $\lambda_i(q_i, s) \in C_i(s)$ ,  $\forall q_i \in Q_i$  and  $\forall s \in S_i$  and  $\mu_i : Q_i \times E \rightarrow Q_i$  is the transition function, specifying the state of the machine in the next stage of the bargaining as a function of the current state and the realised history of the stage.<sup>9</sup>

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<sup>9</sup>We could also have a terminal state  $T$  to indicate that an offer is accepted by all responders and  $M_i$  enters state  $T$  and shuts off.

The basic rationale for using this specification of a machine is again that the bargaining game is identical at the beginning of each stage. Thus the nature of the output and transition maps remain the same in each stage. Other definitions are possible, where the state changes every period, or before a player has to move. (See CS.)

In the literature on repeated games played by automata the number of states of the machine is often used as a measure of complexity - henceforth referred to as state-complexity. The set of states of the machine defines a partition on the set of histories  $E^\infty$  and state-complexity is simply the size of this partition. This definition of complexity has been criticised on many different grounds. In CS we show that irrespective of the definition of an automaton used, state-complexity does not result in stationary behaviour for bargaining games with more than two players (it does with 2-player games). Moreover, we argue that since the states of a machine do not change during a stage of the bargaining game, counting the number of states does not fully capture the complexity of the machine *during a stage*, specifically the complexity of different choices following the same partial history.

More formally, state-complexity measures for each  $s$  the cardinality of the domain of  $\lambda_i(\cdot, s)$ , but it does not capture the cardinality of the range of the mapping  $\lambda_i(\cdot, s)$ . To illustrate the point further, consider the following example.

**Example 1.** *There are two machines for player  $i$ . The first machine  $M'_i$  has two states and always proposes  $x$  in the odd stages of the bargaining game and proposes  $y$  in the even stages. Also, it always rejects any proposal. Machine  $M_i$  also has two states and proposes  $x$  in the odd stages and  $y$  in the even stages. But it responds differently to the same proposal (by conditioning on the two states). In particular, it rejects offer  $z$  in the odd stages, accepts offer  $z$  in the even stages and rejects any other offer.*

The two machines in Example 1 have an equal number of states, despite the fact that the strategy that  $M_i$  implements has the additional complexity of different responses to the same offer. This is not a desirable property. To capture the additional complexity of different behaviour after the same partial history, CS propose the following weak partial order condition in addition to state-complexity.

**Definition 3.** *(Response-complexity) A machine  $M_i = \{Q_i, q_i^1, \lambda_i, \mu_i\}$  is more  $r$ -complex than another machine  $M'_i = \{Q'_i, q'_i^1, \lambda'_i, \mu'_i\}$  if the machines  $M_i$  and  $M'_i$  are otherwise identical except that given some non-empty partial history  $s' \in S_i$ ,  $M'_i$  always responds the same way to  $s'$  irrespective of the state of the machine and  $M_i$  responds differently to  $s'$  depending on the state of the machine. Formally,  $Q_i = Q'_i, q_i^1 = q'_i^1, \mu_i = \mu'_i$  and there exists a non-empty partial history  $s' \in S_i$  such that*

$$\left. \begin{array}{l} \lambda_i(q_i, s) = \lambda'_i(q_i, s) \quad \forall s \neq s' \text{ and } \forall q_i \in Q_i = Q'_i \\ \lambda'_i(q_i, s') = \lambda'_i(q'_i, s') \quad \forall q_i, q'_i \in Q_i, \\ \lambda_i(q_i, s') \neq \lambda_i(q'_i, s') \quad \text{for some } q_i, q'_i \in Q'_i. \end{array} \right\} \quad (2.2)$$

Note that in the above definition we require the existence of a *non-empty*  $s'$  such that condition (2.2) holds. It is clear that if in the above Definition we allow  $s'$  to be *empty* as well as non-empty then the above definition is equivalent to our definition of complexity in Definition 1 expressed in terms of strategies. CS demonstrates that the results of this paper hold if one assumes response-complexity (as in Definition 2) in addition to state-complexity. Therefore, response-complexity together with either state-complexity (as in CS) or with requiring the above definition of response-complexity to hold for *empty*  $s'$  at the beginning of each stage (as in this paper) are sufficient to prove the results of this paper.

### 2.3. Equilibrium in the machine game

We now describe the game with complexity costs. At time 0, players 1,...,n simultaneously and independently choose their strategies. At time period 1, the sequential bargaining game, described in the previous section, begins, with the prescribed order being 1,...,n. Let  $\pi_i(f_i, f_{\sim i})$  be the expected payoff to  $i$  in the bargaining game if the strategy profile  $(f_i, f_{\sim i})$  is chosen. Thus  $\pi_i(f_i, f_{\sim i}) = \delta^{t-1}z_i$  if there is an agreement  $z \in \Delta^n$  in period  $t$ , and zero otherwise.

Next we define *Nash Equilibrium* of the game with complexity cost.

**Definition 4.** A strategy profile  $f = (f_1, \dots, f_n)$  constitutes a Nash equilibrium with complexity cost (denoted by NEC) if

1.  $\pi_i(f_i, f_{\sim i}) \geq \pi_i(f'_i, f_{\sim i}) \quad \forall f'_i \in F_i$
2.  $\nexists f'_i \in F_i$  such that  $\pi_i(f_i, f_{\sim i}) = \pi_i(f'_i, f_{\sim i})$  and  $f_i \succ f'_i$ .

Complexity costs are treated lexicographically in the definition of NEC above. We could also have considered the Nash equilibrium of a new game with the payoff to player  $i$  defined as  $\pi_i(f_i, f_{\sim i}) - c \sum_s \Gamma(s)$ , where  $c > 0$  is some fixed cost and  $\Gamma(s) \equiv |\{f(h, s) | h \in E^\infty\}|$ . Such a payoff function would induce at least as much economy as the lexicographic criterion. Our results on stationarity are also valid for this new game.

Before stating our first proposition we define stationary behaviour.

**Definition 5.** A strategy  $f_i$  is stationary if

$$f_i(h, s) = f_i(h', s) \quad \forall h, h' \in E^\infty \text{ and } \forall s \in S_i$$

Clearly, in each stage a stationary strategy will implement the same action, following different histories of the preceding stages, provided that the partial history within the current stage is the same. Thus, the behaviour of such strategies at any time may depend on the past outcomes in the current stage but not on the previous history of the game before the current stage. It is in this sense that the strategy is stationary.

Note that if  $f_i$  is stationary then it is minimally complex in the sense that there does not exist another strategy  $f'_i$  such that  $f_i \succ f'_i$ . Thus a stationary Nash equilibrium strategy profile of the bargaining game is a NEC of the game with complexity cost.<sup>10</sup>

**Proposition 1.** *Any agreement  $z \in \Delta^n$  in period one can be sustained as a NEC.*

**Proof.** Consider the stationary strategy  $f_i$  defined as:

$$f_i(h, s) = \begin{cases} z & \text{if } s \text{ is such that } i \text{ is the proposer} \\ A & \text{if } s \text{ is such that } i \text{ is a responder, } y \text{ has been proposed and } y_i \geq z_i \\ R & \text{if } s \text{ is such that } i \text{ is a responder, } y \text{ has been proposed and } y_i < z_i \end{cases}$$

Thus strategy  $f_i$  always offers  $z$  as a proposer, and rejects anything that gives player  $i$  less than  $z_i$ , if called on to respond.

The strategies  $f_i$  are clearly best against each other. Since they are stationary, they are minimally complex. ■

We can also demonstrate that delays in agreement are possible.

**Proposition 2.** *Any agreement  $z \in \Delta^n$  at any period  $t \leq n$  can be sustained as a NEC.*

**Proof.** We exhibit stationary strategies that constitute a NEC and induce agreement in period  $t \leq n$  of the bargaining game at some allocation  $z \in \Delta^n$ .

Consider the  $n$ -tuple of stationary strategies  $f = (f_1, \dots, f_n)$  defined as follows. The strategy  $f_t$  always offers  $z$  as a proposer and rejects all offers as a responder. For any  $i \neq t$ , the strategy  $f_i$  always proposes unit vector  $1_i \in \Delta^n$ , where  $1_i$  stands for player  $i$  receiving 1 and others receiving 0, and accepts an offer if and only if  $z$  is proposed by player  $t$ .

Clearly, if  $f$  is implemented, at any period  $t' < t$  the proposal is  $1_{t'}$  and is rejected by the first responder. At  $t$  the offer made is  $z$  and will be accepted by all. Since all strategies are stationary they are minimally complex. Therefore, to show that  $f = (f_1, \dots, f_n)$  constitute a NEC, we only need to show that the strategies are best responses to each other on the equilibrium path.

At any  $t' < t$ , the first responder  $t' + 1$  cannot make himself better off by deviating and accepting the offer  $1_{t'}$  because either the other responders reject or in the game with 2 players the game ends and he will receive zero. At period  $t$ , no responder  $i \neq t$  can make himself better off by deviating and rejecting  $z$  because, given the strategy of others,  $i$  can obtain at most  $z_i$  in the future.

No proposer  $i \neq t$  can make himself better off by making a different offer from  $1_{t'}$  because it will be rejected by the first responder. Also player  $t$  cannot make himself better off by making a proposal different from  $z$  because it will be rejected. ■

It is also possible to generate no agreement or indefinite delay.

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<sup>10</sup>Note that although stationary strategies are minimally complex, our equilibrium notion does not trivially imply stationary behaviour. This is because in a NEC there is no restriction on the choice of strategies and complexity appears lexicographically; thus, each player prefers a stationary strategy to a non-stationary one if and only if the former guarantees at least the same payoff as the latter.

**Proposition 3.** *Indefinite delay can be sustained as a NEC outcome.*

**Proof.** Again, we exhibit stationary strategies that constitute a Nash equilibrium and induce no agreement.

Each stationary strategy  $f_i$  for player  $i$  always offers  $1_i$  as a proposer and rejects all offers as a responder. Clearly, the strategies  $(f_1, \dots, f_n)$  are best against each other. Since they are stationary, they are minimally complex. ■

It appears therefore that the introduction of complexity considerations really has no effect in this context. However, the next two results show that this is not true. Although complexity considerations do not restrict the equilibrium allocation of payoffs, they *do* restrict the amount of delay that can occur, if an agreement is reached.

## 2.4. Delay and Stationarity

The main result of this subsection is that no delay in agreement beyond period  $n$  can be sustained as a NEC.

**Proposition 4.** *Let  $(z, t)$  be an agreement  $z \in \Delta^n$  at period  $t < \infty$  in a  $n$ -player bargaining game. Suppose that  $(z, t)$  is induced by a NEC. Then  $t \leq n$ .<sup>11</sup>*

**Proof.** Suppose not. Then there exists a profile of strategies  $f = (f_1, \dots, f_n)$  which constitutes a NEC and results in an agreement  $(z, t)$  with  $t > n$ . Suppose that on the equilibrium path the last stage of the game consists of  $L$  sequential moves (thus  $n \leq L \leq n^2$ ). Let us denote these sequential moves by  $a^1, \dots, a^L$  with  $a^1$  being the first and  $a^L$  being the last move in the last stage of the equilibrium path. For any  $l \leq L$ , we refer to the player that makes the  $l$ -th move in the last stage by  $l^\#$ . Thus in period  $t$ , for example, player  $(L - n + 1)^\#$  is the proposer and makes the offer  $z = a^{L-n+1}$  and player  $(L - n + i + 1)^\#$  is the  $i$ -th responder for any  $i < n$  and accepts the offer ( $a^{L-n+i+1} = A$ ). Also, for any  $l \leq L$  define  $s^l$  to be the partial history of the equilibrium path in the last stage of the game before the  $l$ -th move. Thus  $s^1$  is empty and  $s^l = (a^1, \dots, a^{l-1})$  for all  $l > 1$ .

The rest of the proof of the above Proposition is in several steps.

**Step 1:**  $f_{L^\#}(h, s^L) = a^L = A$  for all  $h \in E^\infty$

Suppose not. Now consider a strategy  $f'_{L^\#}$  for player  $L^\#$  which is identical to the equilibrium strategy  $f_{L^\#}$  except that it always chooses  $A$  in response to the partial history  $s^L$ . By the hypothesis and the definition of complexity,  $f'_{L^\#}$  is less complex than  $f_{L^\#}$ . Now we demonstrate a contradiction by showing that  $\pi_{L^\#}(f'_{L^\#}, f_{\sim L^\#}) \geq \pi_{L^\#}(f_{L^\#}, f_{\sim L^\#})$ . Consider two separate cases. In the first case  $s^L$  occurs on the equilibrium path of  $(f_1, \dots, f_n)$  before the last stage. Let  $\tau$  be the first stage on the

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<sup>11</sup>In CS we provide a counter-example to show that the analogous result to Proposition 4 with state-complexity does not hold for games with more than two players. The counter-example is a 3-player game with an equilibrium resulting in an agreement in 10 stages or 30 periods. We could not find a counter-example with fewer stages for 3-player games.

equilibrium path at which  $s^L$  occurs. Then clearly  $(f'_{L\#}, f_{\sim L\#})$  results in agreement  $z$  in stage  $\tau$ , thus contradicting the definition of a *NEC*. In the second case,  $s^L$  occurs only in the last stage. In this case  $(f'_{L\#}, f_{\sim L\#})$  implements the same outcome path as  $(f_{L\#}, f_{\sim L\#})$  resulting in an agreement  $z$  at period  $t$ . Since  $f'_{L\#}$  is less complex than  $f_{L\#}$ , this also contradicts the definition of *NEC*.

**Step 2:**  $\forall l > 1$ , if for all players  $i = (l + 1)^\#, (l + 2)^\#, \dots, L^\#$  we have  $f_i(h, s^i) = a^i$  for all  $h \in E^\infty$  then  $f_{l\#}(h, s^l) = a^l$  for all  $h \in E^\infty$ .

As in the previous step, suppose not. Now consider a strategy  $f'_{l\#}$  for player  $l^\#$  which is identical to the equilibrium strategy  $f_{l\#}$  except that it always chooses  $a^l$  in response to the partial history  $s^l$ . By the supposition and the definition of complexity,  $f'_{l\#}$  is less complex than  $f_{l\#}$ . Now we demonstrate a contradiction by showing that  $\pi_{l\#}(f'_{l\#}, f_{\sim l\#}) \geq \pi_{l\#}(f_{l\#}, f_{\sim l\#})$ . Consider two separate cases. In the first case  $s^l$  occurs on the equilibrium path of  $(f_1, \dots, f_n)$  before the last stage. Let  $\tau$  be the first stage on the equilibrium path at which  $s^l$  occurs. Then clearly  $(f'_{l\#}, f_{\sim l\#})$  results in agreement  $z$  in stage  $\tau$  before period  $t$ , given that for all players  $i = (l + 1)^\#, (l + 2)^\#, \dots, L^\#$  we have that  $f_i(h, s^i) = a^i$  for all  $h \in E^\infty$ . This contradicts the definition of a *NEC*. In the second case,  $s^l$  occurs only in the last stage. In this case  $(f'_{l\#}, f_{\sim l\#})$  implements the same outcome path as  $(f_{l\#}, f_{\sim l\#})$  resulting in an agreement  $z$  at period  $t$ . Since  $f'_{l\#}$  is less complex than  $f_{l\#}$ , this also contradicts the definition of *NEC*.

**Step 3:**  $\forall l \geq 1$  we have  $f_{l\#}(h, s^l) = a^l$  for all  $h \in E^\infty$ .

This step follows by backwards induction from the previous two steps.

**Step 4: A contradiction.**

It follows from Step 3 that for all  $h \in E^\infty$  we have  $f_1(h, s_1 = \emptyset) = a^1$ ,  $f_2(h, s_2 = a^1) = a^2$ , ... Therefore the outcome  $(a^1, \dots, a^L)$  occurs in the first stage of the game resulting in an agreement  $z$  in the first stage. But this contradicts the hypothesis that the equilibrium path results in an agreement at period  $t > n$ . ■

Note that the argument for proving Proposition 4 uses the following two features of the game. First, it uses the feature that the final payoff to a player depends on the final agreement allocation  $z$ ; the history of the game up to period  $t$  does not affect the payoff. Thus, if in some stage prior to the final stage (on the equilibrium path) the same non-null partial history arises as in the final stage, and if a responder  $i$  could ensure that the game ends in an agreement  $z$  by accepting the offer, he would do so in order to reduce complexity. This implies, for example, that the partial history on the equilibrium path in the last stage before the last acceptance could not have occurred in an earlier stage. Second, as in other extensive-form games, only actual plays are observed, not strategies. If a history of play is off-the-equilibrium path, the action after this history can be arbitrary without a deviation being observed. This, together with complexity, ensures that if a non-empty partial history of a stage happens on the equilibrium path *only once*, then the action taken after such a partial history must be the same irrespective of the previous history of the game. In the proof of Proposition 4, by appealing to the above two features of the bargaining game and by using backwards induction reasoning on the sequence of moves in the last stage of the

game, we show that *each player's* response to any partial history of the last stage is always the same and therefore agreement would be reached in the first stage.

The last result demonstrates that the game with complexity cost must entail agreement by  $t = n$ . We now show that any equilibrium strategies with agreement at  $t \leq n$  or no agreement *must* be stationary.

**Lemma 1.** *Suppose  $f = (f_1, \dots, f_n)$  constitute a NEC. Then each  $f_i$  is stationary if either  $f$  results in an agreement at some  $t \leq n$  or if  $f$  results in no agreement.*

**Proof.** Let us consider each case.

**Case 1: Agreement is reached at some  $t \leq n$ .** The essential idea in this case is that if player  $i$ 's moves on the equilibrium path at every period  $\tau \leq t$  is  $a^\tau$  then it follows from  $t \leq n$  that he could implement the same equilibrium play by using a stationary strategy  $f'_i$  that always plays  $a^\tau$  in every  $\tau - th$  period of a stage. Since  $f'_i$  obtains the same payoff as  $f_i$ , the equilibrium strategy  $f_i$  must also be stationary.

**Case 2: No agreement is reached in equilibrium.** Suppose not; then for some  $i$  the strategy  $f_i$  is not stationary. Thus for some  $s \in S_i$  we have  $f_i(h, s) \neq f_i(h', s)$  for some  $h$  and  $h' \in E^\infty$ . Now consider the strategy  $f'_i$  that is identical to  $f$  except that  $f_i(h, s) = f_i(h', s)$  for all  $h$  and  $h' \in E^\infty$ . Clearly,  $f'_i$  is less complex than  $f_i$  and guarantees player  $i$  a payoff of at least 0. Since we have no agreement it follows that  $\pi_i(f_1, \dots, f_n) = 0$ . But this contradicts the hypothesis that  $(f_1, \dots, f_n)$  constitutes a NEC. ■

Finally, we can state the main result of this section.

**Proposition 5.** *If the profile  $f = (f_1, \dots, f_n)$  is a NEC then each strategy  $f_i$  is stationary.*

**Proof.** This follows immediately from Proposition 4 and Lemma 1. ■

**Remark 1.** *Notice that the proof of Proposition 4 and Lemma 1 did not assume that the discount factor  $\delta$  is less than 1. Therefore, Proposition 5 on stationary behaviour holds even for the case of no discounting.*

The last proposition demonstrates that complexity considerations result in ‘stationary’ behaviour. However, Nash equilibrium with complexity does not generate uniquely the stationary **subgame perfect allocation** of the bargaining game. All partitions of the cake and delayed agreements up to  $t = n$  can also be generated. The arguments in the proof of Propositions 1 and 2 explain why this is the case. For example, in games with more than 2 players, if there are two rejections anticipated for any non-equilibrium offer, it does not pay any single proposer or responder to deviate unilaterally from the equilibrium path. (In 2-player games, the same is true if ‘off the equilibrium path’ the players always ask for the whole pie and always reject all offers). However, if the responders make mistakes, so that a rejection could be

an acceptance with some small probability, the preceding reasoning would not hold. Thus the result is a consequence of the assumption that the strategies are error-free. In the next section, we allow each player to tremble when performing their tasks and thereby introduce ‘credibility’ off the equilibrium path.

### 3. Noisy Games

We now introduce strategies that make mistakes with a small probability. The noisy strategy  $f_i(\epsilon_k)$  for player  $i$  is now a behavioural strategy given by

$$f_i(\epsilon_k) : E^\infty \times S \rightarrow \Delta_{\epsilon_k}(C_i(s)),$$

where, as before,  $C_i(s)$  is the set of actions corresponding to  $s$ , that is  $A$  or  $R$  for a responder or  $\Delta^n$  for a proposer. The set  $\Delta_{\epsilon_k}(C_i(s))$  is the set of completely mixed probability distributions on  $C_i(s)$  with  $\epsilon_k$  being the probability mass associated with non-desired choices. That is, if a player’s strategy chooses a particular action, this is played only with probability  $1 - \epsilon_k$ . We will consider the effects of taking an infinite sequence of  $k$ , such that  $\epsilon_k \rightarrow 0$ .

More explicitly, where  $C_i(s) = \{A, R\}$ ,  $\Delta_{\epsilon_k}(C_i(s))$  is the set of probability vectors that accord at least  $\epsilon_k$  probability to each choice. Of course,  $1 - \epsilon_k > \epsilon_k$ , which is supposed to be “small”. If  $C_i(s)$  describes the set of offers, then  $\Delta_{\epsilon_k}(C_i(s))$  consists of probability distributions  $\Psi(\cdot)$  with an absolutely continuous part with specified density  $\psi(x) > 0, \forall x$  in the feasible set, and  $\int_{\sum x_i=1} \psi(x) dx_1 dx_2 dx_3 \dots dx_n = \epsilon_k$  and a spike of mass  $1 - \epsilon_k$  at a particular value  $x \in \Delta^n$ .

Thus all strategies  $f_i(\epsilon_k)$  make errors of magnitude  $\epsilon_k$ . *Note that the errors made by each player’s strategy in each period are independent of each other- independent of errors made by other players’ strategies and independent of errors made by the same player’s strategies in other time periods.*

We now discuss convergence of the behavioural strategies  $(f_1(\epsilon_k), \dots, f_n(\epsilon_k))$  as  $k \rightarrow \infty$ , given that the errors are independent. We shall require that each  $f_i(\epsilon_k)$  be a best response to  $f_{\sim i}(\epsilon_k)$  along the sequence, *in the payoff sense only, without considering costs of complexity.*

The behavioural strategies involved are those of a proposer or one of the responders. The proposer has to put a positive density on all feasible offers, but a mass of only  $\epsilon_k$ . The remaining  $1 - \epsilon_k$  can be put on any offer or any set of offers. Convergence as  $k \rightarrow \infty$  is then used in the sense of weak convergence of the probability distributions of offers. It turns out that the mass  $1 - \epsilon_k$  will be placed on the largest offer (from the point of view of the proposer) that is acceptable with ‘high’ probability, so that the entire machinery of weak convergence need not be brought into play. For the responders, an offer  $x$  is accepted or rejected with at least  $\epsilon_k$  probability. It also turns out that as  $\epsilon_k \rightarrow 0$ , the acceptance probability for a given  $x$  converges to either 1 or 0.

It is easily shown that if the  $f_i(\epsilon_k)$  are best responses to the  $f_{\sim i}(\epsilon_k)$  for all  $\epsilon_k$ , the limiting strategy  $f_i$  is a best response to the limiting strategies  $f_{\sim i}$ . We therefore

have a Nash equilibrium in payoffs and now invoke complexity costs on the limiting strategy. Effectively this construction assumes that the cost of additional complexity is of a higher order than the error probability.

We write down now a definition of a ‘noisy’ Nash equilibrium of the game with complexity costs.

**Definition 6.** Any profile of strategies  $(f_i, f_{\sim i})$  is said to be a noisy Nash equilibrium of the game with complexity costs (NNEC) if

1. For some sequence  $\epsilon_k \rightarrow 0$ , there exists a strategy profile  $(f_i(\epsilon_k), f_{\sim i}(\epsilon_k))$  s.t.  $f_i(\epsilon_k)$  is a best response to  $f_{\sim i}(\epsilon_k)$  for all  $i$ , and all  $k$ , and  $f_i(\epsilon_k) \rightarrow f_i$  as  $k \rightarrow \infty$  (and as  $\epsilon_k \rightarrow 0$ ).
2.  $\nexists f'_i \in F_i$  such that  $\pi_i(f_i, f_{\sim i}) = \pi_i(f'_i, f_{\sim i})$  and  $f_i \succ f'_i$ .

Part (1) of the definition captures “extensive form perfectness” and part (2) the cost of complexity. Note that a ‘noisy Nash equilibrium’, is also a NEC; this follows directly from the definition and the properties of perfectness. Therefore, the results of the previous section can be applied. In particular, each noisy equilibrium must be stationary and agreement (if any) must take place by  $t = n$  (in the limiting case where  $\epsilon = 0$ ).

**Proposition 6.** In  $n$ -player games, any NNEC must have an agreement at  $t = 1$  (no delay) and give players  $1, 2, \dots, n$  the following payoffs  $\frac{1}{\sum_{i=1}^n \delta^{i-1}}, \frac{\delta}{\sum_{i=1}^n \delta^{i-1}}, \dots, \frac{\delta^{n-1}}{\sum_{i=1}^n \delta^{i-1}}$  respectively.

**Proof.** See CS. ■

The above result demonstrates that the unique noisy NEC is the stationary subgame perfect equilibrium (SGPE) allocation, and agreement occurs in the first period.

Such errors in outputs of the machines ensure that credibility is restored for *almost* all subgames.<sup>12</sup> This is sufficient to obtain our uniqueness result in Proposition 6. A more direct way of introducing credibility and obtaining our uniqueness result is to require the NEC to be a SGPE of the bargaining game. Clearly, such a requirement ensures optimal behaviour after every history. We give below a sketch of the proof that any NEC with subgame perfect behaviour in the bargaining game implements the unique stationary, subgame perfect allocation. This is similar to the proof for noisy Nash equilibrium, but easier to describe. However, note that the more direct interpretation of requiring any NEC to be a SGPE does not treat the payoff in the bargaining game and the complexity cost in a similar fashion. Strategies are required

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<sup>12</sup>Note that since the number of possible paths is uncountable (the number of offers is not countable), in any noisy NEC it is not necessary for strategies to choose optimal responses for all possible subgames. The noise, however ensures that the set of subgames after which non-optimal behaviour occurs is of measure zero.

to be optimal in terms of the payoff after every history; but minimal in terms of complexity (among strategies that give the same payoff) *only* at the beginning of the game. Thus with this interpretation we require players to consider the complexity of strategies at the beginning of the game assuming that they are not permitted to change later, but still require these strategies to be optimal in every subgame.<sup>13</sup> Noise in the output of the strategies seems a more natural way of inducing credibility in games with complexity because both optimality and minimal complexity are required to hold only at the beginning of the game; however, it does take limits in a particular way.

**Proposition 7.** *In  $n$ -player games, the unique payoff allocation that is both a NEC and a SGPE of the original bargaining game is  $(\frac{1}{\sum_{i=1}^n \delta^{i-1}}, \frac{\delta}{\sum_{i=1}^n \delta^{i-1}}, \dots, \frac{\delta^{n-1}}{\sum_{i=1}^n \delta^{i-1}})$ . This allocation is obtained without delay at  $t = 1$ .<sup>14</sup>*

**Sketch of Proof:** Consider any *NEC* profile  $f = (f_1, \dots, f_n)$  that is also a SGPE of the bargaining game. Since the profile  $f$  is a *NEC*, each  $f_i$  is stationary. Therefore, at the beginning of each stage, the outcome path induced by  $f$  in each stage is unique. Let the payoffs associated with this outcome path be  $(v_1, \dots, v_n)$  respectively.

Now we show that the continuation equilibrium payoff profile to the players at the beginning of the each period  $t \leq n$  is also unique and involves an agreement. This is done by backward induction as follows. In any period  $t \leq n + 1$ , if it is reached, suppose that for each player  $i$  the continuation equilibrium payoff is unique and is equal to  $v_i^t$ . Then in period  $t - 1$  an offer  $x = (x_1, \dots, x_n) \in \Delta^n$  will be accepted by all responders if and only if  $x_i \geq \delta v_i^t$  for all  $i$ . Therefore, because of discounting, it is optimal for the proposer in  $t - 1$  to offer  $\delta v_i^t$  to each  $i$ ; and this offer will be accepted. Thus each  $i$ 's continuation equilibrium payoff at  $t - 1$  is unique and is equal to

$$v_i^{t-1} = \begin{cases} \delta v_i^t & \text{if } i \neq t - 1 \\ 1 - \sum_{i \neq t-1} \delta v_i^t & \text{otherwise} \end{cases} \quad (3.1)$$

Since the equilibrium payoff profile at the beginning of the second stage  $v_i^{n+1}$  is unique and equal to  $v_i$  for each  $i$ , it follows by backwards induction from above that the game must end in agreement in period 1 and the payoff to player  $i$  at the beginning of each period  $t \leq n + 1$  is unique and is equal to  $v_i^t$  satisfying condition (3.1).

It also follows from the stationarity of the equilibrium strategies that  $v_i = v_i^1 = v_i^{n+1}$ . This together with condition (3.1) imply that  $v_i = \frac{\delta^{i-1}}{1 + \delta + \delta^2 + \dots + \delta^{n-1}}$ . ■

The above result is identical and the nature of the proof is similar to the noisy *NEC* case. In fact, if we restricted the set of proposals of the strategies to be finite then any strategy profile induced by any 'noisy' *NEC* would be subgame perfect in the bargaining game.

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<sup>13</sup>A stronger refinement would be to consider strategies satisfying both *NEC* conditions (optimality and complexity) after each history. (See Neme and Quintas, 1995.)

<sup>14</sup>Herrero (1985) also considers stationary SGPE. However, the proof of Proposition 7 does not follow from her result because our notion of stationary behaviour is different (and weaker) from that assumed in Herrero.

## 4. Conclusions and Discussion of Alternative Specifications

As mentioned before, an alternative specification of complexity that has the same flavour as that in Definition 1 would be one that replaces *stage* by *period* in that definition. First some notation. Let  $\overline{E}^t$  be the set of all possible histories of the game up to period  $t$  and let  $\overline{E}^\infty = \cup_{t=0}^\infty \overline{E}^t$ . Denote the set of partial histories for player  $i$  in any *period* (the set of all outcomes in a period before  $i$ 's turn to play) by  $\overline{S}_i$ . Then for any history of play  $h \in \overline{E}^t$  up to period  $t$  and for any  $s \in \overline{S}_i$ , the pair  $(h, s)$  is said to be a feasible path if  $s$  constitutes a partial history at period  $t$ . Now, a strategy  $f_i$  for player  $i$  in the game prescribes an action  $f_i(h, s)$  for any feasible path  $(h, s) \in \overline{E}^\infty \times \overline{S}_i$ .

**Definition 7.** A strategy  $f_i$  is more period-complex than  $f'_i$  if the strategies  $f_i$  and  $f'_i$  are otherwise identical except that given some partial history of a period  $s' \in \overline{S}_i$ ,  $f'_i$  always responds the same way to  $s'$  irrespective of the previous history of the game and  $f_i$  responds differently to  $s'$  depending on the history of actions in preceding periods.<sup>15</sup> Formally, there exists a partial history of a period  $s' \in \overline{S}_i$  such that

$$\left. \begin{aligned} f_i(h, s) &= f'_i(h, s) \quad \text{for any feasible path } (h, s) \in \overline{E}^\infty \times \overline{S}_i \text{ such that } s \neq s' \\ f'_i(h, s') &= f'_i(h', s') \text{ for any } h \text{ and } h' \in \overline{E}^\infty \text{ such that } (h, s') \text{ and } (h', s') \text{ are feasible} \\ f_i(h, s') &\neq f'_i(h', s') \text{ for some } h \text{ and } h' \in \overline{E}^\infty \text{ such that } (h, s') \text{ and } (h', s') \text{ are feasible} \end{aligned} \right\}$$

It can be shown (using techniques similar to those in the previous sections) that the results of this paper on stationarity (suitably redefined) and uniqueness with ‘noise’ hold if the period-complexity criterion is adapted. With period-complexity, we define stationary strategies as those strategies that depend only on the partial history of outcomes in the current period.<sup>16</sup>

In conclusion, this paper has attempted to explore the consequences of incorporating costs of complexity in the sequential moves, multiperson unanimity game. Essentially, such costs yield the result that the strategies used in equilibrium will be stationary. This is not sufficient to give the stationary, subgame perfect equilibrium allocation as the unique equilibrium; this requires an additional property of robustness against errors. It is interesting to note that not all definitions of complexity give us stationarity<sup>17</sup>; the intuition behind stationarity relies on a plausible kind of complexity that considers strategies that condition their behaviour only on the information available in the current stage, less complex than those that do not.

It would be of interest to explore the issue of stationarity in other contexts where multiplicity of equilibria is prevalent. This might be true especially in other bargaining games, which share the special feature that no payoffs are obtained unless there is an agreement. (For example, see Sabourian (1999)).

<sup>15</sup>Thus, this definition captures the complexity of conditioning behaviour on the history of the game prior to the current *period*.

<sup>16</sup>For discussion on other specifications in the automata framework, see CS.

<sup>17</sup>For example, counting the number of states in the automata representation, when the number of players is greater than 2.

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