

Faculty of Economics & Politics
Part2A Paper 3: Microeconometrics and Panel Data
Supervision 2

1. For binary response data in the form $\{y_i, x_i\}$, where y_i is either zero or one, and x_i is a $k \times 1$ vector of independent variables, evaluate the following statements
 - a) Within a standard linear regression model the disturbances cannot be normally distributed.
 - b) The use of an OLS estimator applied to this data is invalid since the disturbances are heteroscedastic.
 - c) Briefly explain why the use of a link (or transformation) function, say $F(x_i\beta)$, where $F(-\infty) = 0$; $F(\infty) = 1$; $dF(x_i\beta)/dx\beta > 0$, is a central component of binary response models.
 - d) Unlike the standard linear regression model, the probit model requires an additional assumption for the purpose of estimation. Namely, instead of assuming, that the errors, say ε , are distributed $\varepsilon \sim (0, \sigma^2)$, the probit model requires $\varepsilon \sim N(0, \sigma^2)$. Why is this the case?
2. Consider the following set of unobserved state-specific utilities for full-time (U_1^*) and part-time employment (U_2^*)

$$\begin{aligned}U_{1i}^* &= \alpha_1 + x_i\beta_1 + u_{1i} \\U_{0i}^* &= \alpha_0 + x_i\beta_0 + u_{0i}\end{aligned}$$

where x_i denotes a common set of exogenous variables, β_0 , and β_1 are unknown parameters, and u_0 , and u_1 denote unobservable taste components.

With reference to the above model, explain how you would specify and estimate a model of binary choice. Which elements of the set of mean parameters $\alpha_1, \beta_1, \alpha_0, \beta_0$ are identifiable.

3. Let Y denote a binary indicator of voting behaviour: $Y = 1$ and $Y = 0$, respectively, represents a vote for Party A and Party B. Let X denote the differential evaluation of the two candidates where $X = X^A - X^B$ and X^A (X^B) represents the ranking of Party A (Party B) candidates. The min (max) values for both X^A and X^B are 0 (100).
 - i) Show that for $P(Y_i = 1|X_i) = E(Y_i|X_i) = \beta_1 + \beta_2 X_i$ to lie within the unit interval the following constraint must be satisfied

$$0 \leq \beta_1 - 100\beta_2, \beta_1 + 100\beta_2 \leq 1$$

- ii) Show that if the constraints are exact, then for every unit increase in the relative liking of Party A compared to Party B, the probability of voting for Party A can increase by no more than 0.005.
4. A data file *phillipines.xls*¹, located in the directory J:\students\mw217, contains the results from a survey of small scale farmers in the Philippines. The focus of the survey is to obtain insight into the determinants of the use of chemical fertilizers. Potential determinants include: land tenure status and the amount of credit obtained. A full list is given below

Using the SIGN function in MicroFit to create a dichotomous variable, FERUSE, indicating whether or not fertiliser is used; this will emulate the use of the indicator function, namely $\mathbf{1}(QFER > 0)$, where *QFER* denotes the *quantity* of chemical fertilizers used, and $\mathbf{1}(\cdot)$ is the indicator function. Estimate two models:

- a linear probability model
- a probit model

using the same set of explanatory variables in each. Discuss your results, interpreting the estimated parameters, and comment on any possible misspecification.

Note: This SIGN function when applied to a variable Y returns the value of 1 when Y is positive, and 0 when Y is zero or negative

Data

The *phillipines.xls* dataset is based upon a cross-section of small-scale farmers in the Philippines. There are 491 observations

Dependent Variable

FERUSE $\mathbf{1}(QFER > 0)$

RHS variables

DMARKET = distance from farm from nearest market

HOURMEET = number of hours farmer met with an agricultural 'expert'

OWNER = 1 if farmer owns some land, 0 otherwise

IRSTAT = 1 if farm irrigated, 0 otherwise

CREDIT = amount (pesos/hectare) of loans obtained by farmer

¹A copy is also on my web page.

5. SECTION B Question 2002

In a random sample of married women 428 report working non-zero hours during a given year. In estimating a labour force participation model using the linear probability model, the following results were obtained.

$$\begin{aligned} \widehat{inlf} = & .586 - .0034nwifeinc + .038educ + .039exper - .00060exper^2 \\ & (.154) \quad (.0014) \quad (.007) \quad (.006) \quad (.00018) \\ & -.016age - .262kidslt6 + .013kidsge6 \\ & (.002) \quad (.034) \quad (.013) \end{aligned}$$

where $inlf$ is a binary variable equal to one if the woman works, $nwifeinc$ denotes nonwife income in thousands, $educ$..., $exper$...Age, education, $()$, $kidslt6$ denotes the number of children less than six years of age, and $kidsge6$ is the number of kids between 6 and 18 inclusive .

- a. Interpret both the economic and statistical significance of these results. Independent of how many young children a woman has, and regardless of the levels of other independent variables, what is the impact of one more child aged between 6 and 18 on labour force participation on labour force participation.
- (a) Carefully explain why there is no guarantee that the predicted values of $inlf$ lie within the unit interval.
- (b) *Although the linear probability model has heteroscedastic error by construction, the form of the heteroscedasticity is known and therefore a weighted least squares estimator may be used for estimation.* Evaluate this statement.
- d. As an alternative to OLS we could have estimated a probit model. Given a resulting log-likelihood value (ℓ^{UN}) using the regressors in (1) of -401.30 and an observed frequency of participation of $\hat{p} = 428/753$, find the value of the log-likelihood for the most restricted model (ℓ^R) with all slope parameters set at zero and calculate the likelihood ratio (LR) statistic, $LR = 2(\ell^{UN} - \ell^R)$. Are the restrictions consistent with the data?

6. SECTION B Question 2004

Consider the linear probability model

$$Y_i = \beta_0 + \beta_1 X_i + u_i, \quad (2)$$

where $\Pr(Y_i = 1|X_i) = \beta_0 + \beta_1 X_i$.

- (a) Show that $E(u_i|X_i) = 0$

- (b) Show that $\text{var}(u_i|X_i) = (\beta_0 + \beta_1 X_i)[1 - (\beta_0 + \beta_1 X_i)]$
- (c) Is u_i heteroscedastic? Would a weighted least squares estimator be more efficient?

Melvyn Weeks
Lent 2008