Problem Set 1: Vector-Matrix Operations

Exercise 1 *Let* $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} -\frac{1}{2} \\ -1 \end{bmatrix}$

(a) Find graphical representation of $\mathbf{a}, \mathbf{b}, \mathbf{c}$*.*

(b) Compute vector $\mathbf{u} = \mathbf{a} + 2\mathbf{b} - 4\mathbf{c}$*.*

(c) Which vectors are orthogonal?

(d) Are vectors linearly independent?

(e) Which subset of the set of vectors $\mathcal{V} = \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ *form a basis of* $\mathbb{R}^2$*? How many different basis can you construct from this set?*

(f) Which one of the vectors is the longest? Which one is the shortest?

Exercise 2 *Find rank of following matrices* $\mathbf{A}, \mathbf{B}$ *and* $\mathbf{C}$*:*

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ -1 & 0.5 \end{bmatrix}, C = \begin{bmatrix} 3 & 0.5 \\ -9 & -1.5 \end{bmatrix}$$

Exercise 3 *Find the sums and differences of following matrices*

(a)

$$\mathbf{A} = \begin{bmatrix} 4 & -1 \\ 6 & 9 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & 3 \\ 3 & -2 \end{bmatrix}$$

(b)

$$\mathbf{A} = \begin{bmatrix} 6 & -2 & 7 \\ 5 & 3 & -2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} -1 & 3 & 2 \\ 2 & -1 & 1 \end{bmatrix}$$

Exercise 4 *Find following scalar products*

$$3\begin{bmatrix} 4 & -1 \\ 6 & 9 \end{bmatrix}, 9\begin{bmatrix} 0 & 3 \\ 3 & -2 \end{bmatrix}$$

Exercise 5 *Find following product matrices*

(a)

$$\begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 4 \\ 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 0 & 1 \\ 3 & 5 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 3 & 2 & 0 \\ 4 & 2 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(d)

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} 7 & 0 \\ 0 & 2 \\ 1 & 4 \end{bmatrix}$$

Exercise 6 *Given* $\mathbf{A} = \begin{bmatrix} 2 & 8 \\ 3 & 0 \\ 5 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 2 & 0 \\ 3 & 8 \end{bmatrix}, C = \begin{bmatrix} 7 & 2 \\ 6 & 3 \end{bmatrix}$

(a) Is $\mathbf{AB}$ *defined? If yes, calculate* $\mathbf{AB}$*.*

(b) Is $\mathbf{BA}$ *defined? If yes, calculate* $\mathbf{BA}$*.*

(c) Is $\mathbf{BC} = \mathbf{CB}$*?*

Exercise 7 *Find following Kronecker Products*

(a)

$$\begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 4 \\ 2 & 3 & 0 \end{bmatrix} \otimes \begin{bmatrix} 8 & 0 \\ 0 & 1 \\ 3 & 5 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 6 & 5 & 1 \\ 3 & 0 & 4 \end{bmatrix} \otimes \begin{bmatrix} 4 & -1 \\ 5 & 2 \\ 0 & 1 \end{bmatrix}$$

Exercise 8 *Find following Hadamart products*

(a)

$$\begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 4 \\ 2 & 3 & 0 \end{bmatrix} \odot \begin{bmatrix} 8 & 0 & 5 \\ 0 & 1 & 1 \\ 3 & 5 & 8 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 4 & 7 & 1 \\ -2 & 1 & -6 \\ 2 & 3 & 0 \end{bmatrix} \odot \begin{bmatrix} -8 & -1 & 5 \\ 0 & 2 & 1 \\ 7 & 5 & -2 \end{bmatrix}$$

Exercise 9 *Given*

$$\mathbf{A} = \begin{bmatrix} 3 & -2 \\ 9 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 4 & 1 \\ -2 & 7 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 4 \\ 2 & 3 & 0 \end{bmatrix}$$

Find the transpose of $\mathbf{A}, \mathbf{B}$ *and* $\mathbf{C}$*.*