

Unemployment in an Estimated New Keynesian Model*

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Abstract

We reformulate the Smets-Wouters (2007) framework by embedding the theory of unemployment proposed in Galí (2011a,b). We estimate the resulting model using postwar U.S. data, while treating the unemployment rate as an additional observable variable. Our approach overcomes the lack of identification of wage markup and labor supply shocks highlighted by Chari, Kehoe and McGrattan (2008) in their criticism of New Keynesian models, and allows us to estimate a "correct" measure of the output gap. In addition, the estimated model can be used to analyze the sources of unemployment fluctuations.

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1 Introduction

Over the past decade an increasing number of central banks and other policy institutions have developed and estimated medium-scale New Keynesian DSGE models.¹ The combination of a good empirical fit with a sound, microfounded structure makes these models particularly suitable for forecasting and policy analysis. However, as highlighted by Galí and Gertler (2009) and others, one of the shortcomings of these models is the lack of a reference to unemployment. This is unfortunate because unemployment is an important indicator of aggregate resource utilization and the central focus of the policy debate. Recently, a number of papers have started to address this shortcoming by embedding in the basic New Keynesian model various theories of unemployment based on the presence of labor market frictions (e.g. Blanchard and Galí (2007), Christoffel et al (2007), Gertler, Sala and Trigari (2008), Christiano, Trabandt and Walentin (2009), and de Walque et al (2008)).

The present paper takes a different approach. Following Galí (2011a,b), it reformulates the Smets and Wouters (2007; henceforth, SW) model to allow for involuntary unemployment, while preserving the convenience of the representative household paradigm. Unemployment in the model results from market power in labor markets, reflected in positive wage markups. Variations in unemployment over time are associated with changes in wage markups, either exogenous or resulting from nominal wage rigidities.²

¹See, for example, Smets et al. (2010) for a short description of the two aggregate euro area models used at the ECB. Two of the DSGE models used at the Federal Reserve are described in Edge et al. (2007) and Erceg et al. (2006).

²The general approach builds on Galí (1996). See also Blanchard and Galí (2007),

The proposed reformulation allows us to overcome an identification problem pointed out by Chari, Kehoe and McGrattan (2008; henceforth, CKM) and interpreted by these authors as an illustration of the immaturity of New Keynesian models for policy analysis. Their observation is motivated by the SW finding that wage markup shocks account for almost 50 percent of the variations in real GDP at horizons of more than 10 years. However, without an explicit measure of unemployment (or, alternatively, labor supply), these wage markup shocks cannot be distinguished from preference shocks that shift the marginal disutility of labour. The policy implications of these two sources of fluctuations are, however, very different. Variations in wage markup shocks are inefficient and a welfare-maximising government should be interested in stabilising output fluctuations resulting from those shocks (at least partly). In contrast, output and employment fluctuations driven by preference shocks shifting the labor supply schedule, should in principle be accommodated. Put it differently, the relative importance of those two shocks will influence the extent to which fluctuations in output during a given historical episode should or should not be interpreted as reflecting movements in the welfare relevant output gap (i.e. the distance between the actual and efficient levels of output). By including unemployment as an observable variable, this identification problem can be overcome, and "correct" measures of the output gap can be constructed, as we show in Section 4.

When we estimate the reformulated SW model using unemployment as an observable variable, we find a much diminished role for wage markup shocks

Casares (2010), and Zanetti (2007) for related applications to the New Keynesian model. After having circulated a first draft of the present paper we became aware of Casares, Moreno and Vázquez (2011), which contains an exercise close in spirit (but with substantial differences in details) to the one presented here.

as a source of output and employment fluctuations, even though those shocks preserve a large role as drivers of inflation. Our estimates lead us to classify the multiple shocks in the model in three categories (which we label "demand", "supply", and "labor market" shocks), on the basis of their implied joint comovement among output, employment, the labor force, unemployment, inflation and the real wage, as captured by their associated impulse response functions (IRFs). In addition, we show how the implied measure of the welfare-relevant output gap is to a large extent the mirror image of the unemployment rate, and resembles conventional measures of the cyclical component of log GDP, based on statistical detrending methods (though the correlation is far from perfect).

Our estimates of the reformulated SW model allow us to address a number of additional questions of interest which could not be dealt with using the model's original formulation. Thus, in section 5 we assess quantitatively the relative importance of different shocks as sources of unemployment fluctuations and their role during specific historical episodes, including the recent recession. Also, our approach allows us to uncover a measure of the natural rate of unemployment (i.e. the flexible wage counterfactual) and to study its comovement with actual unemployment. That comovement is shown to be particularly strong at low frequencies, as expected, but the gap between the two caused by wage rigidities is estimated to be large and persistent. We also revisit the evidence on the joint behavior of inflation and unemployment under the lens of our estimated model. This allows us to give a structural interpretation to empirical Phillips curves, both for wage and price inflation. In section 6 we discuss the robustness of our findings to the use of alternative

sample period and data. Section 7 concludes.

In addition to reformulating the wage equation in terms of unemployment, our model shows a number of small differences with that in SW (2007). First, and regarding the data on which the estimation is based, we use employment rather than hours worked, and redefine the wage as the wage per worker rather than the wage per hour. We do so since the model focuses on variations in labor at the extensive margin, in a way consistent with the conventional definition of unemployment. Given that most of the variation in hours worked over the business cycle is due to changes in employment rather than hours per employee, this change does not have major consequences in itself. We also combine two alternative wage measures in the estimation, compensation and earnings, and model their discrepancy explicitly. Second, we generalise the utility function in a way that allows us to parameterize the strength of the wealth effect on labour supply, as shown in Jaimovich and Rebelo (2009). This generalisation yields a better fit of the joint behavior of employment and the labor force, as we discuss in detail. Third, for simplicity, we revert to a Dixit-Stiglitz aggregator rather than the Kimball aggregator used in SW (2007).

The rest of the paper is structured as follows. Section 2 describes the modified Smets-Wouters model. Next, Section 3 presents the data and estimation. Section 4 contains the discussion of the CKM critique. Section 5 analyses different aspects of unemployment fluctuations which the reformulation of the SW model makes possible. Section 6 presents some robustness exercises and, finally, Section 7 concludes.

2 Introducing Unemployment in the Smets-Wouters Model

2.1 Staggered Wage Setting and Wage Inflation Dynamics

This section introduces a variant of the wage setting block of the SW model, which is in turn an extension of that in Erceg, Henderson and Levin (2000; henceforth, EHL). The variant presented here, based on Galí (2011a,b), assumes that labor is indivisible, with all variations in hired labor input taking place at the extensive margin. That feature gives rise to a notion of unemployment consistent with its empirical counterpart.

The model assumes a (large) representative household with a continuum of members represented by the unit square and indexed by a pair $(i, j) \in [0, 1] \times [0, 1]$. The first dimension, indexed by $i \in [0, 1]$, represents the type of labor service in which a given household member is specialized. The second dimension, indexed by $j \in [0, 1]$, determines his disutility from work. The latter is given by $\chi_t \Theta_t j^\varphi$ if he is employed, zero otherwise, where $\chi_t > 0$ is an exogenous preference shifter (referred to below as a "labor supply shock"), Θ_t is an endogenous preference shifter, taken as given by each individual household and defined below, and $\varphi \geq 0$ is a parameter determining the shape of the distribution of work disutilities across individuals.

Individual utility is assumed to be given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log \tilde{C}_t(i, j) - 1_t(i, j) \chi_t \Theta_t j^\varphi \right)$$

where $\tilde{C}_t(i, j) \equiv C_t(i, j) - h \bar{C}_{t-1}$, with $h \in [0, 1]$. and with \bar{C}_{t-1} denoting (lagged) aggregate consumption (taken as given by each household), and

where $1_t(i, j)$ is an indicator function taking a value equal to one if individual (i, j) is employed in period t , and zero otherwise. Thus, as in SW and related monetary DSGE models, we allow for (external) habits in consumption, indexed by h .

As in Merz (1995), full risk sharing of consumption among household members is assumed, implying $C_t(i, j) = C_t$ for all $(i, j) \in [0, 1] \times [0, 1]$ and t . Thus, we can derive the household utility as the integral over its members' utilities, that is:

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t U_t(C_t, \{N_t(i)\}) &\equiv E_0 \sum_{t=0}^{\infty} \beta^t \left(\log \tilde{C}_t - \chi_t \Theta_t \int_0^1 \int_0^{N_t(i)} j^\varphi dj di \right) \\ &= E_0 \sum_{t=0}^{\infty} \beta^t \left(\log \tilde{C}_t - \chi_t \Theta_t \int_0^1 \frac{N_t(i)^{1+\varphi}}{1+\varphi} di \right) \end{aligned}$$

where $N_t(i) \in [0, 1]$ denotes the employment rate in period t among workers specialized in type i labor and $\tilde{C}_t \equiv C_t - h\bar{C}_{t-1}$.³ We define the endogenous preference shifter Θ_t as follows:

$$\Theta_t \equiv \frac{Z_t}{\bar{C}_t - h\bar{C}_{t-1}}$$

where Z_t evolves over time according to the difference equation

$$Z_t = Z_{t-1}^{1-v} (\bar{C}_t - h\bar{C}_{t-1})^v$$

Thus Z_t can be interpreted as a "smooth" trend for (quasi-differenced) aggregate consumption. Our preference specification implies a "consumption externality" on individual labor supply: during aggregate consumption

³Alternatively, we can take the consumption utility of the household, $\log \tilde{C}_t$, as a "primitive," without making any assumption on how that consumption is distributed among household members, possibly as a function of employment status.

booms (i.e. when $\bar{C}_t - h\bar{C}_{t-1}$ is above its trend value Z_t), individual (as well as household-level) marginal disutility from work goes down (at any given level of employment).

The previous specification generalizes the preferences assumed in SW by allowing for an exogenous labor supply shock, χ_t , and by introducing the endogenous shifter Θ_t , just described. The main role of the latter is to reconcile the existence of a long-run balanced growth path with an arbitrarily small *short-term* wealth effect. The latter's importance is determined by the size of parameter $v \in [0, 1]$. As discussed below in detail, that feature is needed in order to match the joint behavior of the labor force, consumption and the wage over the business cycle. That modification is related to, but not identical, to the one proposed by Jaimovich and Rebelo (2009) as a key ingredient in order to account for the economy's response to news about future productivity increases.⁴

Note that under the previous preferences, the household-relevant marginal rate of substitution between consumption and employment for type i workers in period t is given by:

$$\begin{aligned} MRS_t(i) &\equiv -\frac{U_{n(i),t}}{U_{c,t}} \\ &= \chi_t \Theta_t \tilde{C}_t N_t(i)^\varphi \\ &= \chi_t Z_t N_t(i)^\varphi \end{aligned}$$

⁴In particular, and leaving aside the presence of habits, our specification assumes that the period utility is separable in consumption and employment, in contrast with that in Jaimovich and Rebelo (2009). This facilitates aggregation of individual utilities into the household utility, and simplifies the analysis by implying equalization of consumption across individuals in the presence of risk sharing within each household.

where the last equality is satisfied in a symmetric equilibrium with $\bar{C}_t = C_t$.

Using lower case letters to denote the natural logarithms of the original variables, we can derive the average (log) marginal rate of substitution $mrs_t \equiv \int_0^1 mrs_t(i) di$ by integrating over all labor types:

$$mrs_t = z_t + \varphi n_t + \xi_t$$

where $n_t \equiv \int_0^1 n_t(i) di$ is (log) aggregate employment and $\xi_t \equiv \log \chi_t$.

We assume nominal wages are set by "unions," each of which represents the workers specialized in a given type of labor, and acting in an uncoordinated way. As in EHL, and following the formalism of Calvo (1983), we assume that the nominal wage for a labor service of a given type can only be reset with probability $1 - \theta_w$ each period. That probability is independent of the time elapsed since the wage for that labor type was last reset, in addition to being independent across labor types. Thus, and by the law of large numbers, a fraction of workers θ_w do not reoptimize their wage in any given period, making that parameter a natural index of nominal wage rigidities. Furthermore, all those who reoptimize their wage choose an identical wage, denoted by W_t^* , since they face an identical problem. Following SW, we allow for partial wage indexation between re-optimization periods, by making the nominal wage adjust mechanically in proportion to past price inflation. Formally, and letting $W_{t+k|t}$ denote the nominal wage in period $t+k$ for workers who last reoptimized their wage in period t , we assume

$$W_{t+k|t} = W_{t+k-1|t} \Pi^x (\Pi_{t-1}^P)^{\gamma_w} (\Pi^P)^{1-\gamma_w}$$

for $k = 1, 2, 3, \dots$ and $W_{t,t} = W_t^*$, and where $\Pi_t^P \equiv P_t/P_{t-1}$ denotes the (gross) rate of price inflation, Π^P is its corresponding steady state value, Π^x is the

steady state (gross) growth rate of productivity, and $\gamma_w \in [0, 1]$ measures the degree of wage indexation to past inflation.

When reoptimizing their wage in period t , workers (or the union representing them) choose a wage W_t^* *in order to maximize their respective households utility* (as opposed to their individual utility), subject to the usual sequence of household flow budget constraints, as well as a sequence of isoelastic demand schedules of the form $N_{t+k|t} = (W_{t+k|t}/W_{t+k})^{-\epsilon_{w,t}} N_{t+k}$, where $N_{t+k|t}$ denotes period $t+k$ employment among workers whose wage was last reoptimized in period t , and where $\epsilon_{w,t}$ is the period t wage elasticity of the relevant labor demand schedule.⁵ We assume that elasticity varies exogenously over time, thus leading to changes in workers' market power.

The first order condition associated with the wage-setting problem can be written as:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ \left(\frac{N_{t+k|t}}{C_{t+k}} \right) \left(\frac{W_{t+k|t}^*}{P_{t+k}} - \mathcal{M}_{w,t+k}^n MRS_{t+k|t} \right) \right\} = 0 \quad (1)$$

where, in a symmetric equilibrium, $MRS_{t+k|t} \equiv \chi_t Z_t N_{t+k|t}^\varphi$ is the relevant marginal rate of substitution between consumption and employment in period $t+k$, and $\mathcal{M}_{w,t}^n \equiv \frac{\epsilon_{w,t}}{\epsilon_{w,t}-1}$ is the natural (or desired) wage markup in period t , i.e. the one that would obtain under flexible wages.

Under the above assumptions, we can write the aggregate wage index $W_t \equiv \left(\int_0^1 W_t(i)^{1-\epsilon_{w,t}} di \right)^{\frac{1}{1-\epsilon_{w,t}}}$ as follows:

$$W_t \equiv \left[\theta_w (W_{t-1} \Pi^x (\Pi_{t-1}^p)^{\gamma_w} (\Pi^p)^{1-\gamma_w})^{1-\epsilon_{w,t}} + (1-\theta_w) (W_t^*)^{1-\epsilon_{w,t}} \right]^{\frac{1}{1-\epsilon_{w,t}}} \quad (2)$$

Log-linearizing (1) and (2) around a perfect foresight steady state and combining the resulting expressions, allows us to derive (after some algebra)

⁵Details of the derivation of the optimal wage setting condition can be found in EHL (2000).

the following equation for wage inflation $\pi_t^w \equiv w_t - w_{t-1}$:

$$\pi_t^w = \alpha_w + \gamma_w \pi_{t-1}^p + \beta E_t \{ \pi_{t+1}^w - \gamma_w \pi_t^p \} - \lambda_w (\mu_{w,t} - \mu_{w,t}^n) \quad (3)$$

where $\alpha_w \equiv (1 - \beta)((1 - \gamma)\pi^p + \pi^x)$, $\lambda_w \equiv \frac{(1 - \beta\theta_w)(1 - \theta_w)}{\theta_w(1 + \epsilon_w\varphi)}$, $\mu_{w,t}^n \equiv \log \mathcal{M}_{w,t}^n$ is the (log) natural wage markup, and

$$\mu_{w,t} \equiv (w_t - p_t) - mrs_t \quad (4)$$

is the (log) average wage markup, i.e. the log deviation between the average real wage and the average marginal rate of substitution. As equation (3) makes clear, variations in wage inflation above and beyond those resulting from indexation to past price inflation are driven by deviations of average wage markup from its natural level, because those deviations generate pressure on workers currently setting wages to adjust those wages in one direction or another.

One might argue that the previous model provides, if interpreted literally, an unrealistic description of wage setting in the U.S. We view it instead as a simple modelling device, consistent with the labor market block of the medium-scale DSGE models currently used for policy analysis (as exemplified by the SW model), and embedding three features of actual labor markets: (i) nominal wage rigidities, (ii) staggered wage-setting, and (iii) the presence of average wage levels above their perfectly competitive counterparts, resulting from different sources of market power by workers which prevent their underbidding by the unemployed.

2.2 Introducing Unemployment

Consider an individual specialized in type i labor and with disutility of work $\chi_t \Theta_t j^\varphi$. Using household welfare as a criterion, and taking as given current labor market conditions (as summarized by the prevailing wage for his labor type), that individual will find it optimal to participate in the labor market in period t if and only if

$$\left(\frac{1}{\widetilde{C}_t}\right) \left(\frac{W_t(i)}{P_t}\right) \geq \chi_t \Theta_t j^\varphi$$

Evaluating the previous condition at the symmetric equilibrium, and letting the marginal supplier of type i labor be denoted by $L_t(i)$, we have:

$$\frac{W_t(i)}{P_t} = \chi_t Z_t L_t(i)^\varphi$$

Taking logs and integrating over i we obtain

$$w_t - p_t = z_t + \varphi l_t + \xi_t \tag{5}$$

where $l_t \equiv \int_0^1 l_t(i) di$ can be interpreted as the (log) aggregate participation or labor force.

Following Galí (2011a,b), we define the *unemployment rate* u_t as:

$$u_t \equiv l_t - n_t \tag{6}$$

Note that under our assumptions, the unemployed thus defined include all the individuals *who would like to be working* (given current labor market conditions, and while internalizing the benefits that this will bring to their households) *but are not currently employed*. It is in that sense that one can

view unemployment as involuntary.⁶

Combining (4) with (5) and (6), the following simple linear relation between the average wage markup and the unemployment rate can be derived

$$\mu_{w,t} = \varphi u_t \tag{7}$$

which is also graphically illustrated in Figure 1.

Finally, combining (3) and (7) we obtain an equation relating wage inflation to price inflation, the unemployment rate and the wage markup.

$$\pi_t^w = \alpha_w + \gamma_w \pi_{t-1}^p + \beta E_t \{ \pi_{t+1}^w - \gamma_w \pi_t^p \} - \lambda_w \varphi u_t + \lambda_w \mu_{w,t}^n \tag{8}$$

Note that in contrast with the representation of the wage equation found in SW and related papers, the error term in (8) captures exclusively shocks to the wage markup, and *not* preference shocks (even though the latter have been allowed for in our model). That feature, made possible by reformulating the wage equation in terms of the (observable) unemployment rate, allows us to overcome the identification problem raised by CKM in their critique of New Keynesian models. We turn to this issue below, when we discuss our empirical findings.

Finally, note that we can define the *natural* rate of unemployment, u_t^n , as the unemployment rate that would prevail in the absence of nominal wage rigidities. Under our assumptions, that natural rate will vary exogenously

⁶As noted by one of our discussants, unemployed individuals will enjoy a higher utility ex-post, since their consumption will be the same but won't experience any disutility from work. This is, of course, an unavoidable consequence of our assumption of full consumption risk-sharing within the household. Under the latter assumption, and given the infinitesimal weight of each individual in the household, not internalizing the benefits to the latter of an individual's employment would unavoidably lead to no participation.

in proportion to the natural wage markup, and can be determined using the simple relation:

$$\mu_{w,t}^n = \varphi u_t^n \tag{9}$$

The remaining equations describing the log-linearized equilibrium conditions of the model are presented in the appendix. Those equations are identical to a particular case of the specification in SW (2007), corresponding to logarithmic consumption utility. In addition to the wage markup and labor supply shocks discussed above, the model includes six additional shocks: a neutral, factor-augmenting productivity shock, a price markup shock; a risk premium shock, an exogenous spending shock, an investment-specific technology shock and a monetary policy shock.

3 Data and Estimation

3.1 Data

We estimate our model on US data for the sample period 1966Q1-2007Q4 using Bayesian full-system estimation techniques as in SW (2007). We end our estimation period in 2007Q4 to prevent our estimates from being distorted by the non-linearities induced by the zero lower bound on the federal funds rate and binding downward nominal wage rigidities during the most recent recession.⁷ In Section 5 below we nevertheless use the estimated model to interpret the behaviour of unemployment in the recent recession, i.e. beyond

⁷For some discussion on how downward nominal wage rigidity may distort the the estimates of the New Keynesian wage Phillips curve, see Gali (2011a).

the estimated period. Section 7 on robustness discusses briefly the impact of estimating our model over an extended sample period ending in 2010Q4.

Five of the seven data series used by SW (2007) are also used here: GDP, consumption, investment, GDP deflator inflation, and the federal funds rate, with the first three expressed in per capita terms and log differenced. As the SW model is reformulated in terms of employment (given our interest in explaining unemployment), we use per capita employment rather than hours worked. The main results are not affected if we use hours instead, as discussed in Section 7. In addition, we experiment with two wage concepts. The first one is total compensation per employee obtained from the BLS Productivity and Costs Statistics.⁸ The second one is "average weekly earnings" from the Current Employment Statistics. Finally, we add the unemployment rate as an additional observable variable. In the following section, we systematically compare the model estimated with and without the latter variable as an observable variable.

The properties of both wage series are quite different.⁹ This is illustrated in Figure 2, which plots their quarterly nominal growth rates. First, average wage inflation based on compensation per employee is significantly higher than that based on earnings per employee (1.24 versus 1.02). Given average price inflation, the compensation series appears more compatible with a balanced growth path in which real wages grow at the same rate as real output, consumption and investment. Second, the compensation series is much more volatile than the earnings series, especially over the past two decades. The

⁸Note that SW (2007) used compensation per hour instead, in a way consistent with their model specification.

⁹See Abraham, Spletzer and Stewart (1999) and Mehran and Tracy (2001) for a discussion about the sources of some of those difference.

standard deviation of wage inflation based on compensation is 0.70, compared to 0.56 for the earnings-based series. Finally, the correlation between both wage inflation measures is surprisingly low at 0.60.

For our baseline estimation, we use both wage series as imperfect measures of the model-based wage concept. This is done by adding measurement error to the corresponding measurement equations and allowing for a separate, smaller trend in the earnings series.¹⁰ In the section on robustness, we briefly discuss the estimation results when we only use the compensation series. In the rest of the paper, we focus on the model with both wage concepts and measurement error.

3.2 Estimation Results

Table 1 compares the estimated structural parameters of the model obtained with and without unemployment being used as an observable variable. As discussed above, adding unemployment allows us to separately identify wage markup and labour supply shocks. In addition, it allows us to exploit the model's prediction of proportionality between the unemployment rate and the wage markup (see equation (7)), in order to identify and estimate the elasticity of substitution between different labor types, which in turn determines the steady-state wage markup. In the model without unemployment this parameter is not identified; instead, we calibrate it to be very similar to the mean of the estimate in the model with observable unemployment.

Overall, most of the estimated structural parameters are very similar in

¹⁰A similar strategy is followed by Justiniano, Primiceri and Tambalotti (2011). They show how using a single series (compensation) and not allowing for measurement error implies a standard deviations for the estimated wage markup shocks that is six times higher than in their baseline model.

the two models.¹¹ Focusing on the parameters that are important for the labour market, a number of findings are worth emphasizing.¹² First, the estimated labour supply elasticity is quite similar whether one uses unemployment or not as an observable variable: the inverse of the Frisch elasticity increases slightly from 3.3 to 4.0 as one includes unemployment. In the latter case, the steady state wage markup is identified and estimated to be slightly below 20 percent, which is consistent with an average unemployment rate of about 5 percent.

Turning to some of the other parameters that enter the wage Phillips curve, the estimated degree of wage indexation is relatively small (around 0.15) and robust across the two models. The estimated Calvo probability of unchanged wages falls somewhat from 0.61 to 0.47, suggesting relatively flexible wages with average contract durations of 2 quarters. Overall, the introduction of unemployment as an observable variable leads to a somewhat steeper wage Phillips curve.

Third, the parameter, ν , governing the short-run wealth effects on labour supply, changes quite dramatically from 0.73 to 0.02. Roughly speaking this amounts to a change from preferences close to those in King, Plosser and Rebelo (1988; henceforth, KPR), characterized by strong short-run wealth effects on labor supply, to a specification closer to that in Greenwood, Hercowitz and Huffman (1988). In the latter case, wealth effects are close to

¹¹A robust feature of the model with observed unemployment is that the labour preference shock and the productivity shock are positively correlated. Allowing for such a correlation further improves the fit of the model, but does not affect the estimation results discussed below.

¹²Unless otherwise noted, we will consistently refer to the mode of the posterior probability distribution when discussing estimates. Table 1 also reports the mean and 5 and 95 percentiles of the posterior distribution.

zero in the short run. As discussed below, this helps ensure that not only employment, but also the labour force moves procyclically in response to most shocks.¹³

Finally, it is worth pointing out that the monetary policy reaction coefficient to the output gap (defined as the deviation relative to the constant markup output), doubles from 0.07 to 0.15. As discussed below, this is mainly due to the lower volatility of the output gap once unemployment is used to identify wage markup shocks.

3.3 Impulse Responses

Figures 3 to 5 show the estimated impulse responses of output, inflation, the real wage, the interest rate, employment, the labour force, the unemployment rate, and the output gap to the eight structural shocks. Figure 3 focuses on the four "demand" shocks, which include the investment-specific technology shock, the risk premium shock, the exogenous spending shock and the monetary policy shock. We use the label "demand" to refer to those shocks because they all imply a positive comovement between output, inflation and the real wage. It is particularly noteworthy that employment and the labour force comove positively in response to all those shocks. Note, however, that the size of the labour force response is typically much smaller than that of employment, so that unemployment fluctuations are mostly driven by changes in employment. This is consistent with the unconditional second moments of detrended data (see, e.g., Galí (2011c)), as well as the empirical VAR evidence

¹³Jaimovich and Rebelo (2009) have argued that small short-run wealth effects on labour supply are necessary to generate a positive response of output to favorable news about future productivity.

on the effects of monetary policy shocks as shown in Christiano et al. (2010).

Figure 4 reports the dynamic responses to the labour supply and markup shocks, which we group under the heading of "labor market" shocks. These shocks generate a negative comovement of inflation and the real wage with output. An adverse wage markup shock has a sizeable positive impact on price inflation and unemployment and a negative one on output, employment and the output gap, thus generating a clear trade-off for policy makers. On the other hand, an adverse labor supply shock has similar negative effects on output, employment and the output gap (and positive effects on inflation), but instead leads to a rise in the output gap and a drop in the unemployment rate, so that no significant policy trade-off arises. It is this different effect on unemployment and the output gap associated with the two labour market shocks that makes their separate identification so important from a policy perspective, as further discussed below.

Figure 5 displays the estimated model's implied impulse responses to a positive neutral technology shock and a (negative) price markup shock. We refer to those shocks as "supply" shocks, their distinctive feature being that they generate simultaneously a procyclical real wage response and a countercyclical response of inflation. It is worth noting, that, in line with much of the empirical evidence (e.g. Galí (1999), Barnichon (2010)), in our estimated model a positive technology shock leads to a short-run decline in employment and a rise in the unemployment rate. This is in contrast with the predictions of conventionally calibrated real business cycle or search and matching models. Secondly, and in a way analogous to wage markup shocks, we see that price markup shocks also create a policy trade-off between

stabilizing inflation and the output gap. This is not the case for technology shocks, since they drive both these variables in the same direction.

Before turning to several interesting questions that can be addressed with our estimated model, we wish to emphasize the importance of departing from conventional KPR preferences in order to match certain aspects of the data. Note that under standard KPR preferences ($\nu = 1$) the labor supply equation (5) can be written as

$$w_t - p_t = c_t + \varphi l_t + \xi_t$$

where habit formation is omitted to simplify the argument. As emphasized by Christiano et al. (2010) the previous equation is at odds with their empirical estimates of the effects of monetary policy shocks, which show a countercyclical response of $w_t - p_t - c_t$ coexisting with a procyclical response of the labor force l_t . Instead, under the assumed preferences, a procyclical response of the labor force is consistent with the model as long as the short run wealth effect is sufficiently weak, implying a small adjustment of z_t and hence a procyclical response of $w_t - p_t - z_t$. This is illustrated in Figure 6 which compares the impulse responses of employment, the labor force and the unemployment rate to a monetary policy shock under (i) our baseline estimated model and (ii) an otherwise identical model with KPR preferences (corresponding to $\nu = 1$). Note that in the latter case, and in contrast with the evidence, the labor force indeed falls significantly following an easing of monetary policy, amplifying the response of the unemployment rate and becoming as important a driver of the latter as employment.

4 Wage Markup vs. Labour Supply Shocks: Addressing the CKM Critique

In this section we address one of the CKM criticisms pointing to an implausibly large variance of wage markup shocks and a large contribution of the latter to output and employment fluctuations, often implied by estimated DSGE models (e.g. SW (2007)). As argued by CKM, that evidence cannot be of much use to policymakers since the SW model is not able to distinguish between wage markup and labor supply shocks. They are effectively "lumped together" as a residual in the wage equation, even though—as discussed above—they have very different policy implications.

As discussed above, that problem of incomplete identification is overcome by our reformulation of the SW model using the unemployment rate as an observable variable.¹⁴ In particular, the estimated parameters of the ARMA(1,1) process for the exogenous wage markup reported in Table 1 imply the latter's standard deviation drops from 23 to 12 percent once unemployment is included as an observable. Based on equation (7) and the estimated inverse labour supply elasticity, this implies a standard deviation of the natural unemployment rate of the order of 3%. This estimate is relatively high, but not unreasonable, especially given that much of that volatility is concentrated at low frequencies, unrelated to business cycles.

How important are wage markup shocks in driving output and employ-

¹⁴Justiniano, Primiceri and Tambalotti (2010) seek to overcome that problem by assuming a different stochastic structure for both driving forces: purely transitory in the case of markup shocks, and potentially persistent (as allowed for by an AR(1) process) for the labor supply shock. Their assumption of a white noise wage markup shock is at odds with our estimated process for that shock, which displays an important low frequency component.

ment fluctuations in our estimated model? Table 2 presents the variance decomposition of the forecast errors of the eight observable variables at the 10 quarter and 10 year horizons. The first entry in each cell gives the percent contribution of each shock to fluctuations in each variable in the model with unemployment as an observable, whereas the second entry given the corresponding share in the model without unemployment. CKM argue that the contribution of the wage markup shocks to output and employment fluctuations (about 50 and 80 percent at the 10 year horizon in the model without unemployment) was too high to be plausible. Distinguishing labour supply shocks from wage markup shocks by introducing unemployment helps address this issue. From Table 2 it is clear that the contribution of the wage markup shocks to output (employment) fluctuations at the 10 year horizon drops substantially, from 45 (77) percent to 17 (39) percent, in the model with unemployment. Furthermore, in the latter labor supply shocks (which are now separately identified) account for about 17, 40 and 89 percent of fluctuations in output, employment and the labor force respectively (instead they are ignored in the model without unemployment, as in SW (2007)).

As discussed by CKM, the identification of wage markup and labor supply shocks has implications for monetary policy, since those two shocks have very different effects on the efficient level of output and thus on the welfare-relevant output gap. Figure 7 plots the output gap, defined as the log deviation between actual output and the level of output that would prevail with constant mark-ups and flexible prices and wages. Two versions of the same variable are shown, as implied by the estimated models with and without un-

employment, respectively.¹⁵ Figure 7 shows that the separate identification of labor supply shocks allowed by our reformulation has a substantial impact on the estimated output gap, which now looks considerably more stationary.

How does our estimated output gap relate to other variables often used as cyclical indicators? Figure 8 shows that our estimate of the output gap is to a large extent the mirror image of the unemployment rate. The correlation between the two is -0.95 . This finding suggests that variations in wage markups, whether exogenous or induced by wage rigidities, are a key factor underlying inefficient output fluctuations.¹⁶ That finding is consistent with the evidence in Galí, Gertler and López-Salido (2007).¹⁷

Finally, Figure 9 emphasizes that the model-based output gap resembles conventional measures of the cyclical component of log GDP, based on a variety of statistical detrending methods (HP filter, band-pass filter and quadratic detrending, as well as the CBO measure).¹⁸ There are, however, periods such as the 2005-2006 boom period with substantial deviations from the conventional measures. The output gap correlation with each of the four measures lies in the $0.6 - 0.8$ range, with quadratic detrending showing the

¹⁵Note that, under the assumptions of the model, the output gap thus defined will differ from the gap relative the efficient level of output by an additive constant.

¹⁶See also the analysis in Galí (2011b) in the context of a much simpler model. A similar qualitative finding is uncovered in Sala, Soderstrom, and Trigari (2010), though their approach is subject to the CKM critique.

¹⁷It would also appear to be consistent with the evidence on the so-called "labor wedge" (e.g. Chari, Kehoe and McGrattan (2007), Shimer (2010)). Note, however, that the concept of "labor wedge" often used in the literature refers to the gap between the marginal rate of substitution and the marginal product of labor (as opposed to the wage). As a result (and despite its name) it captures variations in goods markets distortions, like price markups, in addition to labor market ones.

¹⁸Justiniano, Primiceri and Tambalotti (2011) obtain a qualitatively similar finding, using an approach that does not exploit the connection between unemployment and wage markups, assuming instead a particular stochastic structure for the latter (white noise).

highest value.

5 Understanding Unemployment Fluctuations

In the present section we use our estimated model to analyze different aspects of unemployment fluctuations, which the reformulation of the SW model makes possible.

First, we can assess the role of wage rigidities as a factor underlying observed unemployment fluctuations by comparing the observed unemployment rate to its estimated *natural* counterpart, where the latter is defined as the unemployment rate that would be observed in the absence of nominal wage rigidities, as determined by equation (9). Figure 10 shows the time series for both variables, together with the gap between the two. The figure makes clear that the natural rate of unemployment accounts for a large fraction of the low-frequency movements in the observed unemployment rate. Yet, it is clear that the natural rate cannot account for the bulk of unemployment fluctuations at business cycle frequencies, which are captured by the unemployment gap. Those fluctuations should thus be attributed to the presence of wage rigidities, interacting with the different shocks.

The variance decomposition reported in Table 1 shows that about 50 percent of unemployment fluctuations at the 10-quarter horizon is due to "demand " shocks, with a prominent role attributed to risk premium shocks. The other half is mostly due to wage mark-up shocks. In the longer run (10-year horizon), the contribution of demand shocks drops to 17 percent and wage markup shocks become the dominant driving force. Interestingly, those wage markup shocks also explain a dominant share of the fluctuations in

price and wage inflation at all horizons. In contrast, labor supply and other supply shocks have only a limited impact on unemployment. The labor force instead is mostly driven by labor supply shocks, with most other shocks having a very limited impact on that variable.

The importance of demand and wage markup shocks in driving unemployment can also be illustrated by means of the historical decomposition depicted in Figure 11. The secular rise of unemployment and inflation in the 1970s and early 1980s is mostly driven by cost-push factors coming from increasing wage markups. This is reversed in the mid 1980s. On the other hand, most of the unemployment fluctuations at business cycle frequencies are seen to be driven by demand shocks. This is particularly the case since the early 1990s. Both the 2001 and 2007-2008 recessions are driven by negative demand shocks. Figure 12 zooms in on the most recent recession, displaying the contribution of each individual shock to the rise of unemployment over this period. We see that about three quarters of the 5 percentage point increase in the unemployment rate is due to demand factors, with adverse risk premium shocks playing a large role at the start of the crisis, thus capturing the tightening of financial conditions. As of 2009 our estimates identify an "effective" tightening of monetary policy, which we attribute to the attainment of the zero lower bound on the federal funds rate, and which is shown to contribute about 1 to 2 percentage points to the rise in the unemployment rate. Finally, it is also worth noting that our estimates suggest a significant contribution of wage markup shocks to the recent rise in the unemployment rate. As conjectured by Gali (2011a), this may be due to downward nominal wage rigidities interacting with very low inflation, which may have prevented

the average real wage from adjusting as much as it would be warranted by the decline in inflation and the rise in unemployment.

Finally, we can use the estimated model to interpret the observed comovements between the unemployment rate and measures of wage and price inflation. With that objective, Figure 13 displays the joint variation in wage inflation and the unemployment rate conditional on each shock, as well as their unconditional joint variation (bottom-right diagram). The evidence makes clear that whatever Phillips-curve-like negative comovement between wage inflation and unemployment can be found in the data it is largely the result of the four demand shocks. By contrast, wage markup shocks generate what looks like a positive lower frequency comovement in both variables, and are largely responsible for the lack of a clean Phillips-curve-like pattern in the observed data. Supply shocks, on the other hand, lead to a near-zero comovement. Note that this is still consistent with wage inflation equation (3) (given the forward-looking nature of the latter), for their implied responses of unemployment display a sign switch (see Figure 5), thus leaving wage inflation largely unchanged as a result.

Figure 14 displays analogous evidence for unemployment and price inflation. As in the case of wage inflation, the four demand shocks generate a clear negative comovement between price inflation and the unemployment rate, while wage markup shocks underlie a low frequency positive comovement. Contrary to traditional textbook analyses, productivity shocks are also shown to generate a negative comovement between price inflation and the unemployment rate. On the other hand, price markup shocks produce a nearly vertical Phillips curve, since their impact on the unemployment rate

is tiny, while their effect on price inflation is substantial.

6 Robustness

In this section we briefly summarize the findings based on a number of alternative specifications. First, we use hours worked rather than employment as our measure of labour input. While the benchmark model is written in terms of employment, the actual labour input that enters the production function should be total hours worked. Using employment will therefore distort the estimated productivity process. When we use hours, we leave the unemployment rate unchanged, thus making the implicit assumption that those who are unemployed want to work the same number of hours as those who are employed.¹⁹ In that alternative specification we also use wage per hour. When we leave the model unchanged but use hours worked rather than employment as our measure of labour input, the main results emphasized above are not affected. The full set of results is available on request. Two differences are worth mentioning. First, as expected, the contribution of productivity shocks to output fluctuations becomes less important. Second, the degree of wage rigidity is estimated to be higher (0.60) and as a result the slope of the Phillips curve becomes less steep, due to the greater cyclical volatility of wage per worker relative to wage per hour.

Second, we also estimate the model using only the compensation series as a wage measure. Again, the main results are unchanged. The main impact of the higher volatility in the compensation series is to increase the estimate of

¹⁹In order to address these issues, ideally we need to explicitly include the intensive margin, i.e. hours worked per employee, in the model and re-estimate it accordingly. That extension is part of our currently ongoing research.

the inverse Frisch elasticity of the labour supply to 5.6 when unemployment is added. With higher observed volatility of wages, the response of labour supply to real wages is estimated to be less. This has an additional impact on some of the other parameters, such as the degree of habit formation.

Thirdly, we have also estimated the model under KPR preferences (i.e., imposing $v = 0$) and an alternative set of Jaimovich-Rebelo preferences where the Z_t factor evolves in line with aggregate productivity instead of aggregate consumption. The model with KPR preferences leads to a significant deterioration of the empirical fit by about 15 points. As discussed above, in this case the labor force moves countercyclically in response to monetary policy and other demand shocks. However, the modified JR model leads to a significantly improved empirical fit by about 28 points. Moreover, the parameter v rises back to 0.9 (from 0.02 in the baseline model) suggesting that in response to productivity shocks the data prefer stronger short-run wealth effects on labor supply. We still need to think harder about the interpretation of these results.

Finally, we have also re-estimated our model using data up to 2010Q4, thus ignoring the potential problems raised earlier (likely mis-specification of the interest rate rule and the wage equation due to nonlinearities at work during this period). The main difference with the benchmark results is that the estimated wage stickiness rises and the overall persistence in the economy as captured by the persistence of the shocks also goes up.

7 Conclusion

In this paper we have developed a reformulated version of the Smets-Wouters (2007) framework that embeds the theory of unemployment proposed in Galí (2011a,b). We estimate the resulting model using postwar U.S. data, while treating the unemployment rate as an additional observable variable. This helps overcome the lack of identification of wage markup and labor supply shocks highlighted by Chari, Kehoe and McGrattan (2008) in their criticism of New Keynesian models. In turn, our approach allows us to estimate a "correct" measure of the output gap. In addition, the estimated model can be used to analyze the sources of unemployment fluctuations.

A number of key results emerge from our analysis. First, we show that wage markup shocks play a smaller role in driving output and employment fluctuations than previously thought. Secondly, fluctuations in our estimated output gap are shown to be the near mirror image of those experienced by the unemployment rate, and to be well approximated by conventional measures of the cyclical component of GDP. Thirdly, demand shocks are the main driver of unemployment fluctuations at business cycle frequencies, but wage markup shocks are shown to be more important at lower frequencies. Finally, our estimates point to an adverse risk-premium shock as the key force behind the initial rise in unemployment during the Great Recession. The important role uncovered for monetary policy and wage markup shocks at a later stage may be interpreted as capturing the likely effects of the zero lower bound on the nominal rate and of downward wage rigidities (as opposed to those of truly exogenous shocks).

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APPENDIX

In this appendix, we summarize the remaining log-linear equations of the model. For a more detailed presentation, we refer to the discussion in SW.

- Consumption Euler equation:

$$\widehat{c}_t = c_1 E_t [\widehat{c}_{t+1}] + (1 - c_1) \widehat{c}_{t-1} - c_2 (\widehat{R}_t - E_t [\widehat{\pi}_{t+1}] - \widehat{\varepsilon}_t^b)$$

with $c_1 = 1/(1 + h)$, $c_2 = c_1(1 - h)$ where h is the external habit parameter. $\widehat{\varepsilon}_t^b$ is the exogenous AR(1) risk premium process.

- Investment Euler equation:

$$\widehat{i}_t = i_1 \widehat{i}_{t-1} + (1 - i_1) \widehat{i}_{t+1} + i_2 \widehat{Q}_t^k + \widehat{\varepsilon}_t^q$$

with $i_1 = 1/(1 + \beta)$, $i_2 = i_1/\Psi$ where β is the discount factor, and Ψ is the elasticity of the capital adjustment cost function. $\widehat{\varepsilon}_t^q$ is the exogenous AR(1) process for the investment specific technology.

- Value of the capital stock:

$$\widehat{Q}_t^k = -(\widehat{R}_t - E_t [\widehat{\pi}_{t+1}] - \widehat{\varepsilon}_t^b) + q_1 E_t [r_{t+1}^k] + (1 - q_1) E_t [Q_{t+1}^k]$$

with $q_1 = r_*^k / (r_*^k + (1 - \delta))$ where r_*^k is the steady state rental rate to capital, and δ the depreciation rate.

- Aggregate demand equals aggregate supply:

$$\begin{aligned} \widehat{y}_t &= \frac{c_*}{y_*} \widehat{c}_t + \frac{i_*}{y_*} \widehat{i}_t + \widehat{\varepsilon}_t^g + \frac{r_*^k k_*}{y_*} \widehat{u}_t \\ &= \mathcal{M}_p (\alpha \widehat{k}_t + (1 - \alpha) \widehat{L}_t + \widehat{\varepsilon}_t^a) \end{aligned}$$

with \mathcal{M}_p reflecting the fixed costs in production which corresponds to the price markup in steady state. $\widehat{\varepsilon}_t^g$, $\widehat{\varepsilon}_t^a$ are the AR(1) processes representing exogenous demand components and the TFP process.

- Price-setting under the Calvo model with indexation:

$$\widehat{\pi}_t - \gamma_p \widehat{\pi}_{t-1} = \pi_1 (E_t [\widehat{\pi}_{t+1}] - \gamma_p \widehat{\pi}_t) - \pi_2 \widehat{\mu}_t^p + \widehat{\varepsilon}_t^p$$

with $\pi_1 = \beta$, $\pi_2 = (1 - \xi_p \beta)(1 - \xi_p) / [\xi_p(1 + (\mathcal{M}_p - 1)\varepsilon_p)]$, with θ_p and γ_p respectively the probability and indexation of the Calvo model, and ε_p the curvature of the aggregator function. The price markup $\widehat{\mu}_t^p$ is equal to the inverse of the real marginal $\widehat{m}c_t = (1 - \alpha) \widehat{w}_t + \alpha \widehat{r}_t^k - \widehat{A}_t$.

- Capital accumulation equation:

$$\widehat{k}_t = \kappa_1 \widehat{k}_{t-1} + (1 - \kappa_1) \widehat{i}_t + \kappa_2 \widehat{\varepsilon}_t^q$$

with $\kappa_1 = 1 - (i_*/\bar{k}_*)$, $\kappa_2 = (i_*/\bar{k}_*)(1 + \beta)\Psi$. Capital services used in production is defined as: $\widehat{k}_t = \widehat{u}_t + \widehat{k}_{t-1}$

- Optimal capital utilisation condition:

$$\widehat{u}_t = (1 - \psi) / \psi \widehat{r}_t^k$$

with ψ is the elasticity of the capital utilisation cost function.

- Optimal capital/labor input condition:

$$\widehat{k}_t = \widehat{w}_t - \widehat{r}_t^k + \widehat{L}_t$$

- Monetary policy rule:

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + (1 - \rho_R)(r_\pi \widehat{\pi}_t + r_y(\widehat{ygap}_t) + r_{\Delta y} \Delta(\widehat{ygap}_t) + \widehat{\varepsilon}_t^r)$$

with $\widehat{ygap}_t = \widehat{y}_t - \widehat{y}_t^{flex}$, the difference between actual output and the output in the flexible price and wage economy in absence of distorting price and wage markup shocks.

The following parameters are not identified by the estimation procedure and therefore calibrated: $\delta = 0.025$, $\varepsilon_p = 10$.

Table 1: Posterior Estimates for the model with and without unemployment as observed variable - Complete list of parameters

	prior distribution			posterior distribution							
	type	mean	st.dev	With UR				Without-UR			
				mode	mean	5%	95%	mode	mean	5%	95%
st.dev. of the innovations ¹											
σ_a	U	2.5	1.44	0.41	0.42	0.37	0.46	0.42	0.42	0.37	0.46
σ_b	U	2.5	1.44	1.73	1.60	0.56	2.50	0.73	0.91	0.35	1.66
σ_g	U	2.5	1.44	0.47	0.48	0.43	0.52	0.47	0.48	0.43	0.52
σ_q	U	2.5	1.44	0.42	0.42	0.34	0.49	0.38	0.38	0.30	0.46
σ_r	U	2.5	1.44	0.21	0.22	0.19	0.24	0.23	0.23	0.21	0.26
σ_p	U	2.5	1.44	0.05	0.11	0.03	0.18	0.06	0.32	0.02	0.73
σ_w	U	2.5	1.44	0.04	0.06	0.01	0.13	0.07	0.10	0.03	0.20
σ_{ls}	U	2.5	1.44	1.07	1.17	0.89	1.45	-	-	-	-
σ_{wC}	U	2.5	1.44	0.45	0.46	0.41	0.50	0.45	0.45	0.40	0.50
σ_{wE}	U	2.5	1.44	0.34	0.36	0.32	0.41	0.33	0.34	0.29	0.39
persistence of the exogenous processes: $\rho = \text{AR}(1)$, $\mu = \text{MA}(1)$											
ρ_a	B	0.5	0.2	0.98	0.98	0.97	0.99	0.98	0.97	0.96	0.99
ρ_b	B	0.5	0.2	0.36	0.42	0.19	0.67	0.66	0.64	0.39	0.86
ρ_g	B	0.5	0.2	0.97	0.97	0.96	0.99	0.98	0.98	0.96	0.99
ρ_q	B	0.5	0.2	0.72	0.75	0.62	0.88	0.75	0.74	0.62	0.86
ρ_r	B	0.5	0.2	0.09	0.10	0.02	0.17	0.09	0.11	0.02	0.19
ρ_p	B	0.5	0.2	0.76	0.43	0.07	0.79	0.84	0.64	0.23	0.93
ρ_w	B	0.5	0.2	0.99	0.98	0.97	1.00	0.99	0.99	0.99	1.00
μ_p	B	0.5	0.2	0.59	0.57	0.24	0.96	0.68	0.73	0.46	0.97
μ_w	B	0.5	0.2	0.67	0.63	0.35	0.91	0.66	0.65	0.38	0.91
a_g^2	N	0.5	0.25	0.69	0.69	0.55	0.83	0.71	0.70	0.56	0.85
structural parameters											
Ψ	N	4.0	1.0	4.09	3.96	2.34	5.58	3.33	3.77	2.32	5.20
h	B	0.7	0.10	0.78	0.75	0.65	0.85	0.66	0.68	0.57	0.81
φ	N	2.0	1.0	3.99	4.35	3.37	5.32	3.32	3.46	2.27	4.66
ν	B	0.5	0.2	0.02	0.02	0.01	0.04	0.73	0.70	0.50	0.92
θ_p	B	0.5	0.15	0.58	0.62	0.53	0.71	0.60	0.71	0.56	0.84
θ_w	B	0.5	0.15	0.47	0.55	0.44	0.66	0.61	0.66	0.56	0.76
γ_p	B	0.5	0.15	0.26	0.49	0.20	0.78	0.26	0.46	0.16	0.82
γ_w	B	0.5	0.15	0.16	0.18	0.07	0.29	0.17	0.20	0.08	0.31
ψ	B	0.5	0.15	0.57	0.56	0.36	0.75	0.41	0.42	0.24	0.60
\mathcal{M}_p	N	1.25	0.12	1.74	1.74	1.61	1.88	1.71	1.73	1.59	1.86
ρ_R	B	0.75	0.10	0.85	0.86	0.82	0.89	0.83	0.84	0.79	0.89
r_π	N	1.5	0.25	1.91	1.89	1.62	2.16	2.03	1.96	1.65	2.26
r_y	N	0.12	0.05	0.15	0.16	0.11	0.22	0.07	0.07	0.04	0.10
$r_{\Delta y}$	N	0.12	0.05	0.24	0.25	0.20	0.30	0.27	0.28	0.22	0.33
$\bar{\pi}$	G	0.62	0.1	0.62	0.66	0.49	0.83	0.79	0.80	0.61	0.99
$100(\beta^{-1} - 1)$	G	0.25	0.1	0.31	0.31	0.17	0.43	0.21	0.22	0.11	0.33
\bar{l}	N	0.0	2.0	-1.65	-1.52	-3.83	0.77	3.56	3.37	1.46	5.29
τ	N	0.4	0.1	0.34	0.34	0.30	0.37	0.40	0.39	0.36	0.43
τ_{wE}	N	0.2	0.1	0.07	0.08	0.03	0.12	0.11	0.10	0.05	0.15
\mathcal{M}_w	N	1.25	0.25	1.18	1.22	1.15	1.29	1.25 ³	1.25 ³	-	-
α	N	0.3	0.05	0.17	0.17	0.14	0.20	0.16	0.16	0.13	0.19

¹ The IG-distribution is defined by the degree of freedom. ² The effect of TFP innovations on exogenous demand. ³ The steady state wage mark-up is not identified if the unemployment rate is not observed.

Table 2: Variance Decomposition

Variance Decomposition	output	inflation	real wage	employment	labor force	unemployment
10 quarter horizon						
Demand Shocks						
Risk premium	6 / 14	2 / 8	3 / 6	16 / 25	0 / 15	20 / 25
Exogenous demand	3 / 5	1 / 0	1 / 0	7 / 10	1 / 9	8 / 1
Investment spec. techn.	9 / 7	3 / 2	8 / 2	12 / 9	2 / 3	10 / 2
Monetary policy	5 / 7	8 / 8	6 / 4	11 / 12	0 / 4	11 / 10
Supply Shocks						
Productivity	59 / 46	6 / 4	40 / 32	5 / 2	3 / 4	4 / 1
Price mark-up	2 / 6	27 / 33	30 / 45	3 / 6	5 / 3	0 / 1
Labor Market shocks						
Wage mark-up	6 / 15	53 / 46	12 / 11	18 / 35	3 / 61	41 / 61
Labor supply	11 / -	0 / -	1 / -	29 / -	86 / -	5 / -
40 quarter horizon						
Demand Shocks						
Risk premium	2 / 5	1 / 6	1 / 3	6 / 8	0 / 6	7 / 7
Exogenous demand	1 / 2	1 / 0	1 / 0	3 / 5	1 / 8	3 / 0
Investment spec. techn.	5 / 3	2 / 1	6 / 3	4 / 3	1 / 2	3 / 0
Monetary policy	2 / 3	5 / 7	3 / 3	4 / 4	0 / 2	4 / 3
Supply Shocks						
Productivity	56 / 39	4 / 3	71 / 59	3 / 1	2 / 1	1 / 0
Price mark-up	1 / 2	18 / 26	13 / 26	1 / 2	2 / 1	0 / 0
Labor Market shocks						
Wage mark-up	17 / 45	67 / 57	5 / 6	39 / 77	5 / 81	80 / 89
Labor supply	17 / -	0 / -	0 / -	40 / -	89 / -	2 / -

Note: each cell reports the contributions to the forecast error variance of the corresponding variable for the models estimated with and without unemployment, respectively.

Figure 1. The Wage Markup and the Unemployment Rate

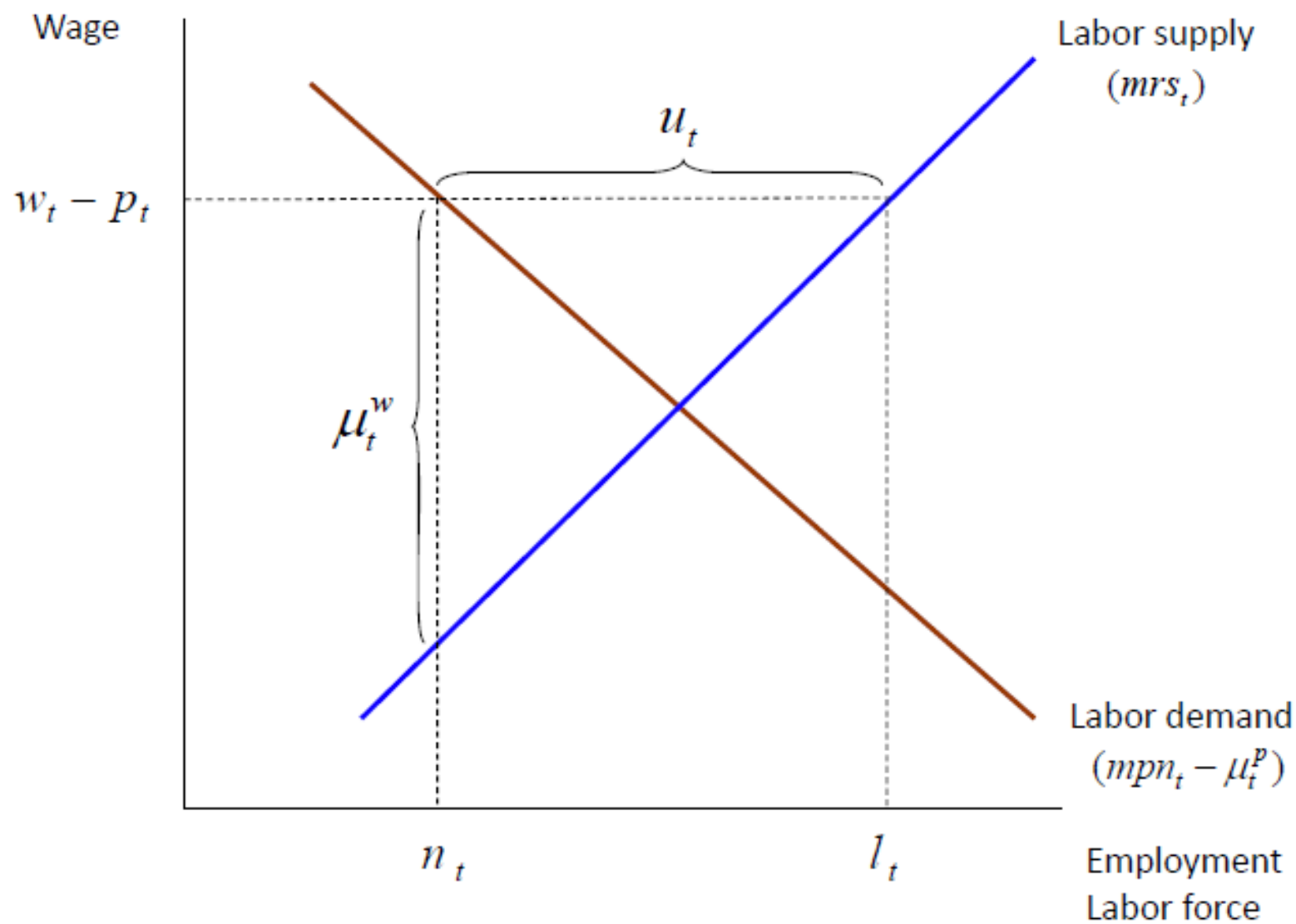


Figure 2. Two Wage Inflation Measures

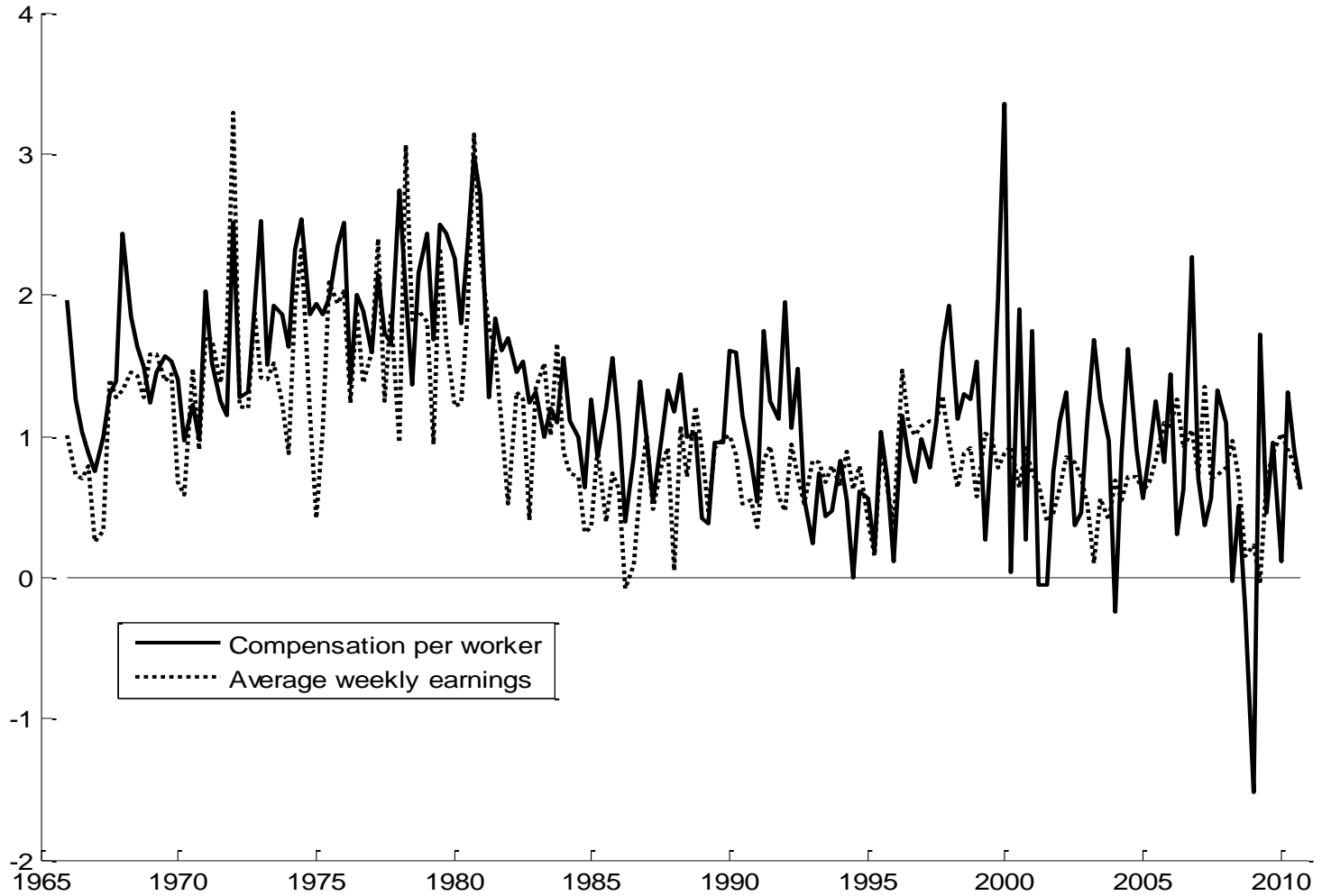


Figure 3. Dynamic Responses to Demand Shocks

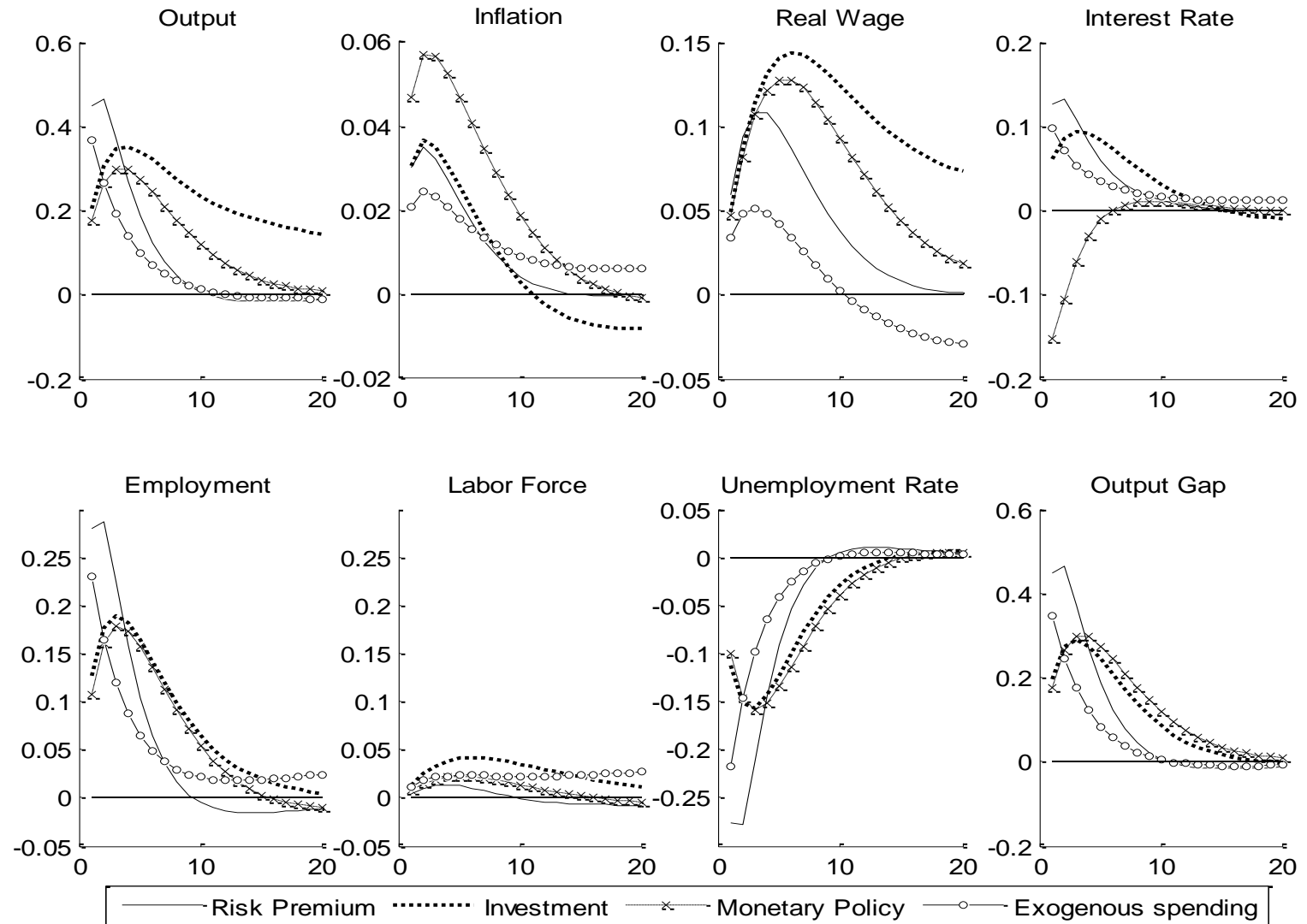


Figure 4. Dynamic Responses to Labor Market Shocks

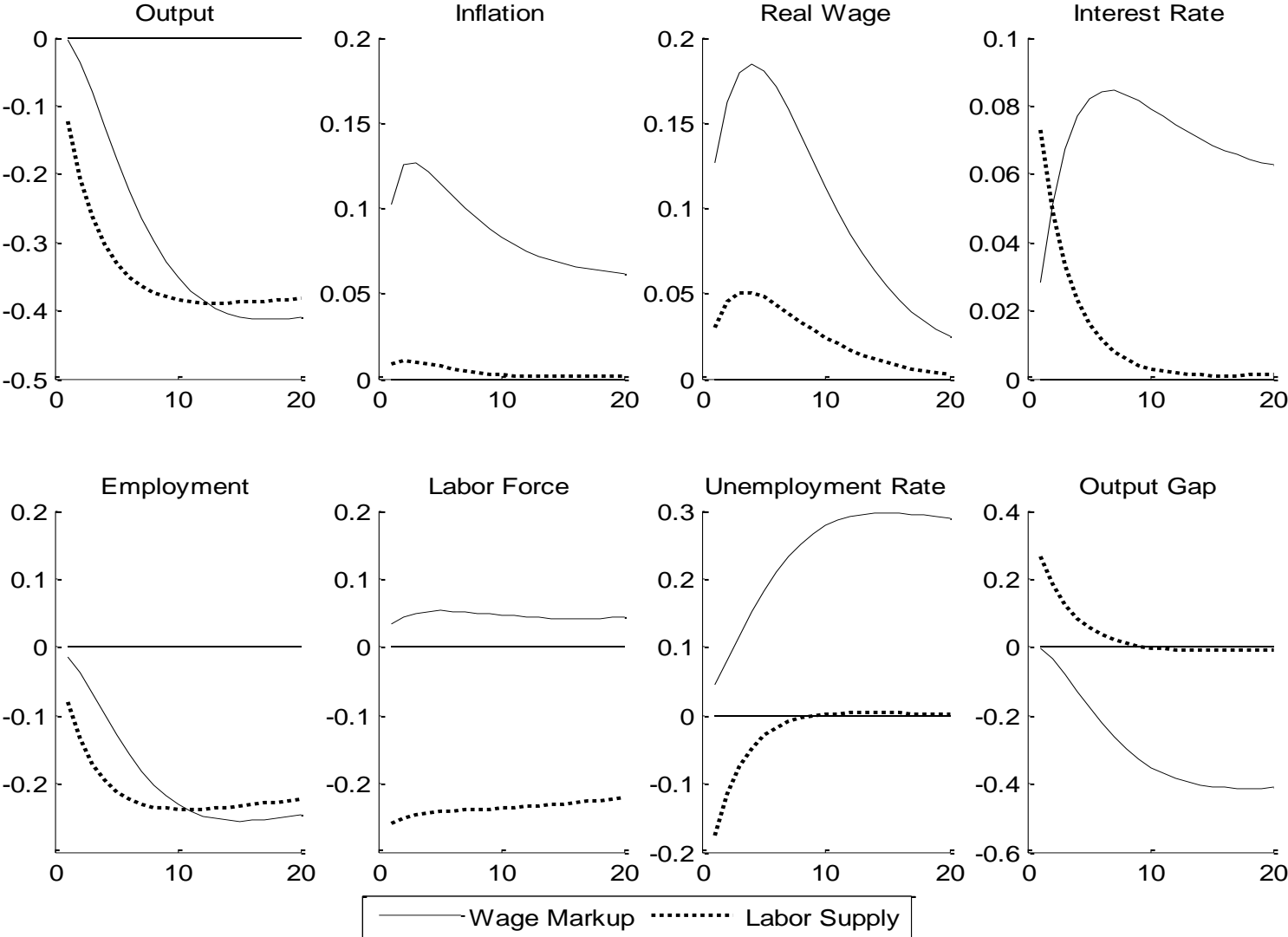


Figure 5. Dynamic Responses to Supply Shocks

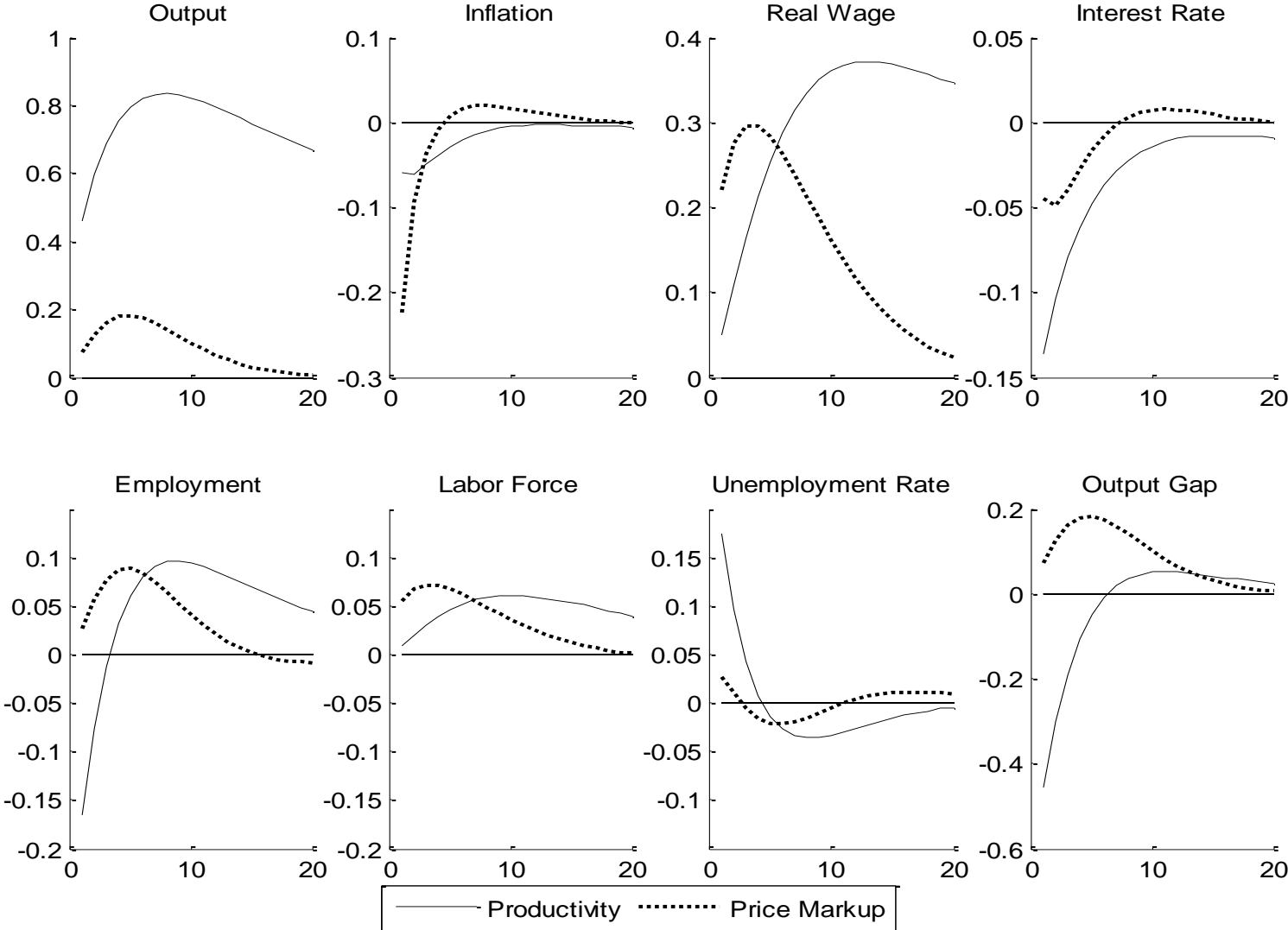


Figure 6. Monetary Policy Shocks and the Role of Wealth Effects

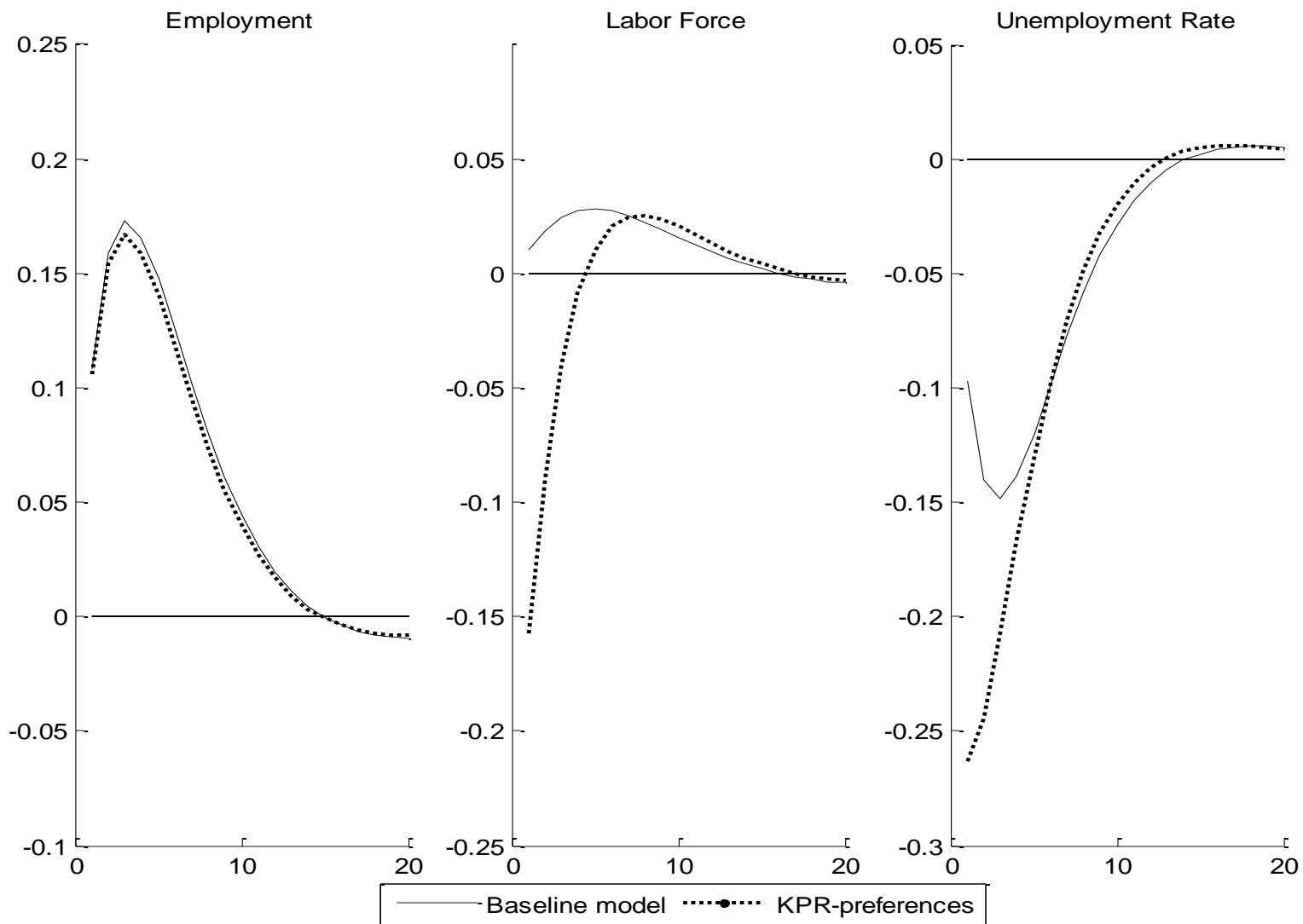


Figure 7. Two Measures of the Output Gap

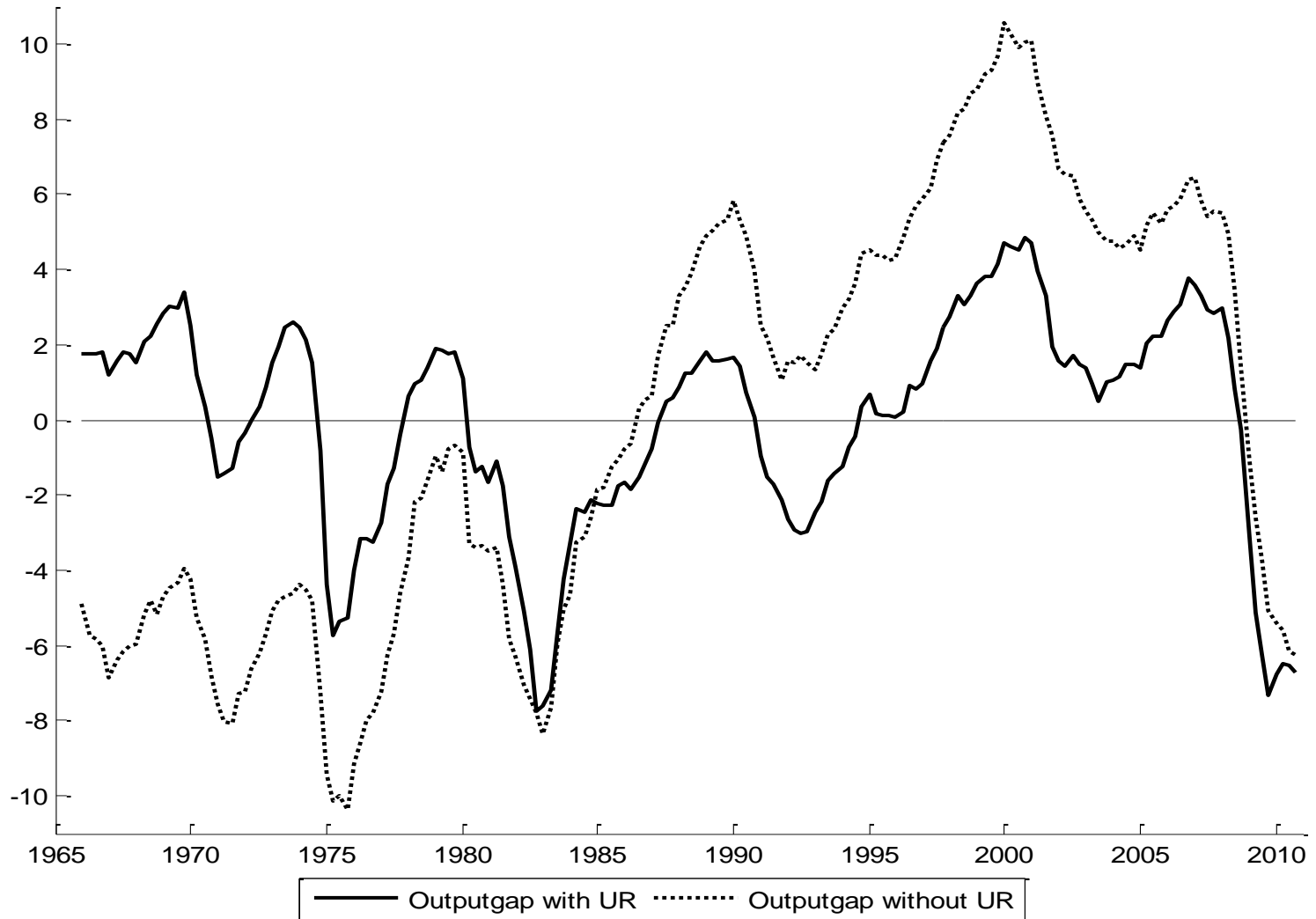


Figure 8. The Output Gap and the Unemployment Rate

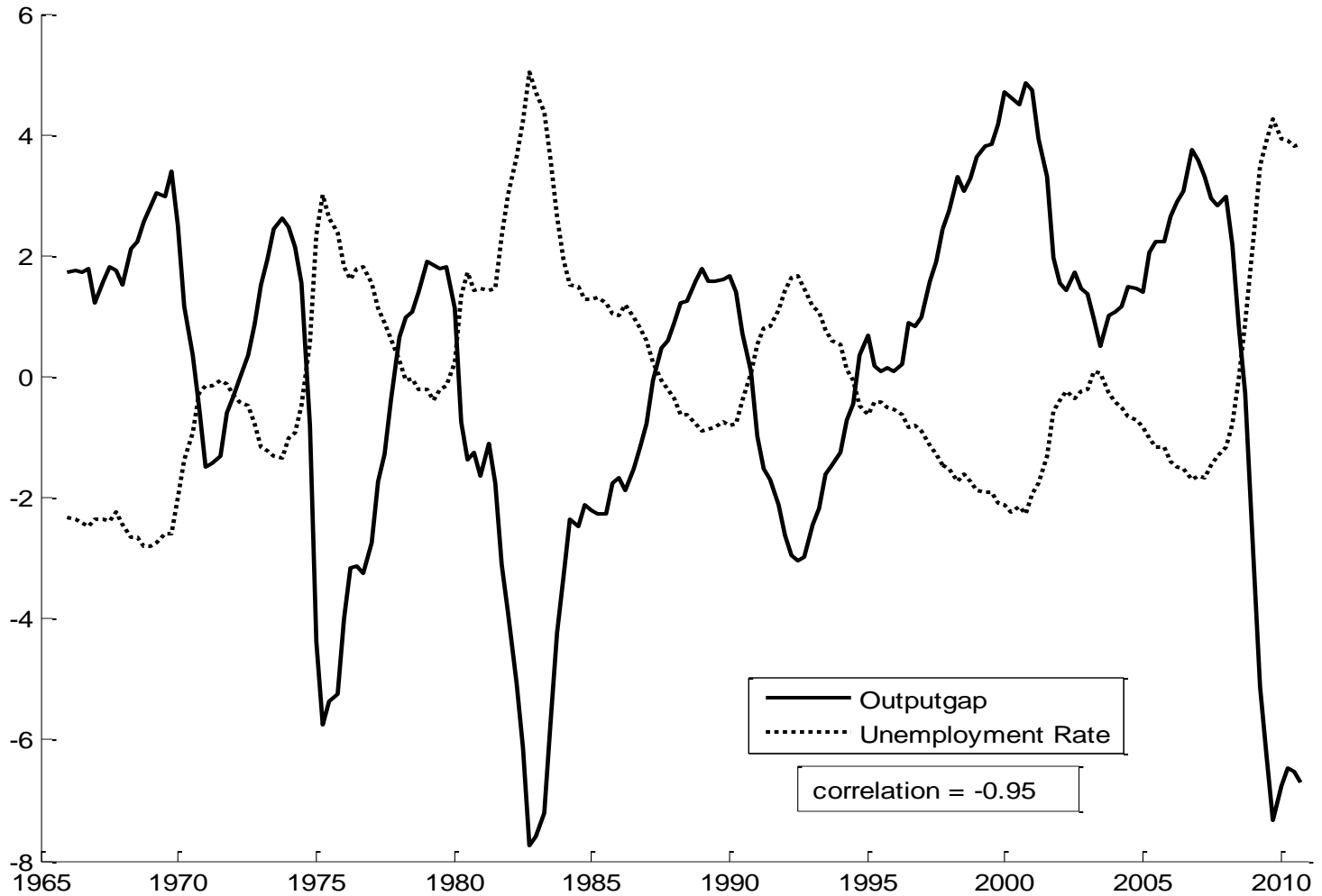


Figure 9. The Output Gap vs. Detrended GDP

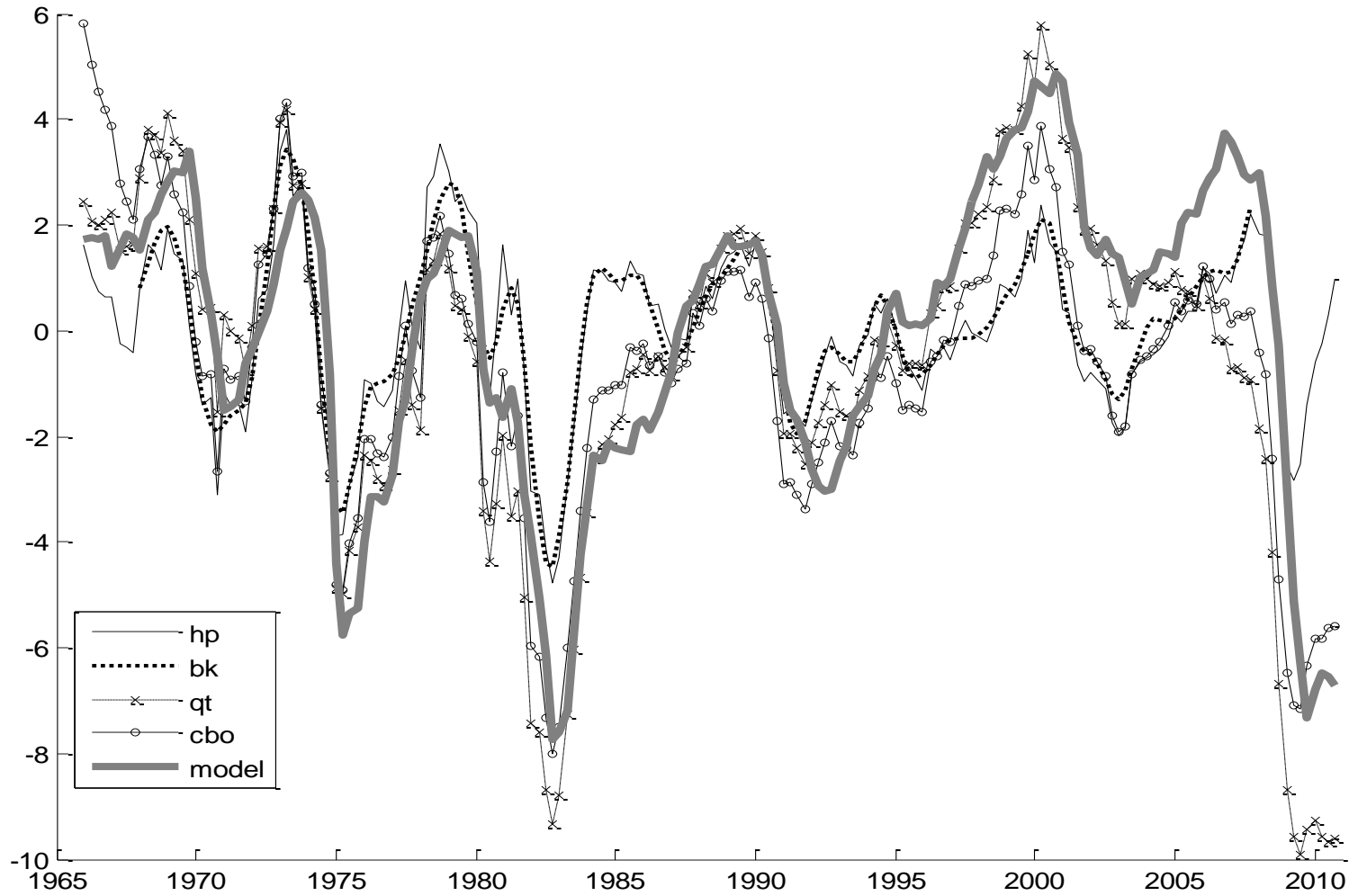


Figure 10. The Natural Rate of Unemployment

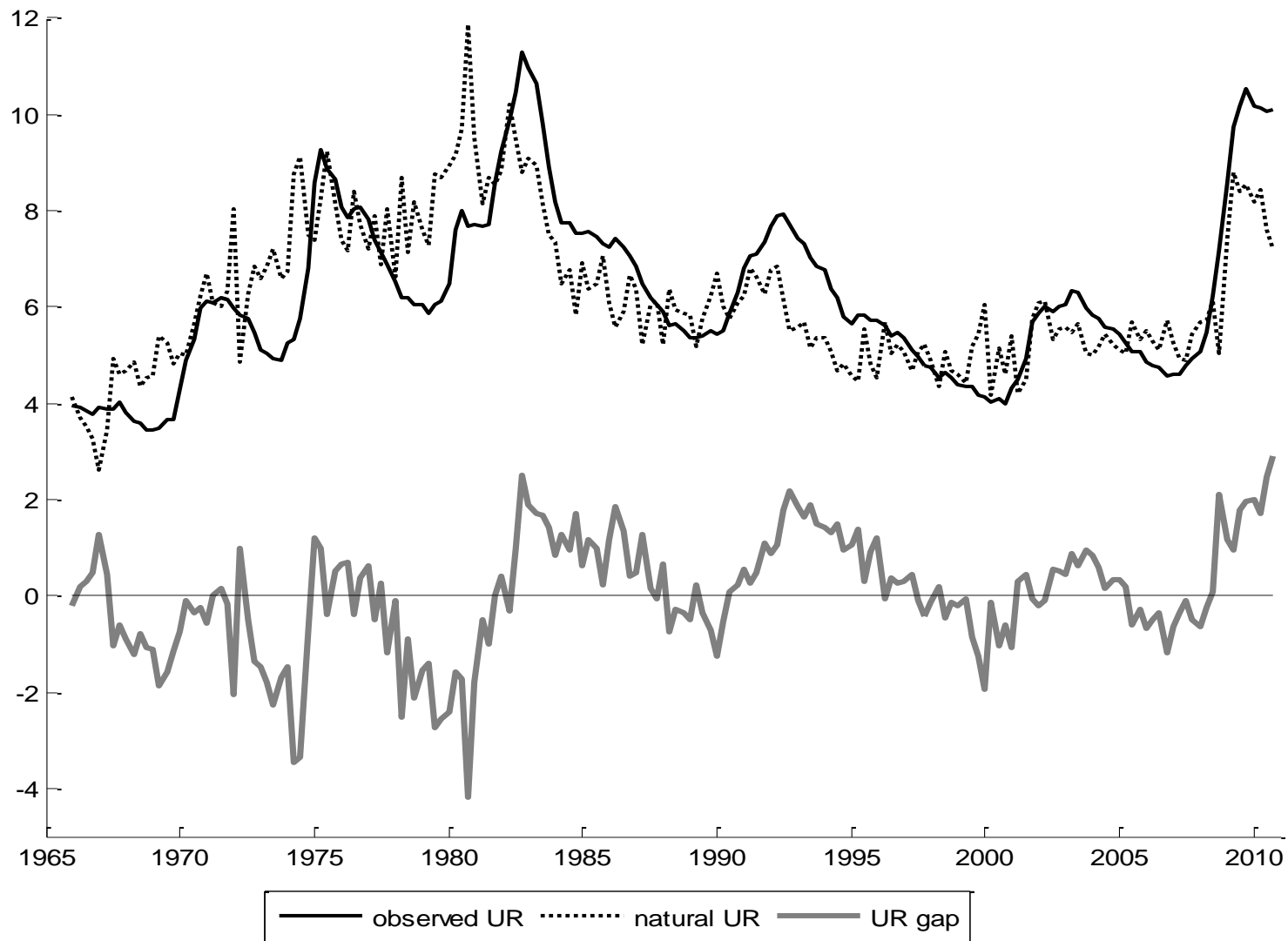


Figure 11. Sources of Unemployment Rate Fluctuations

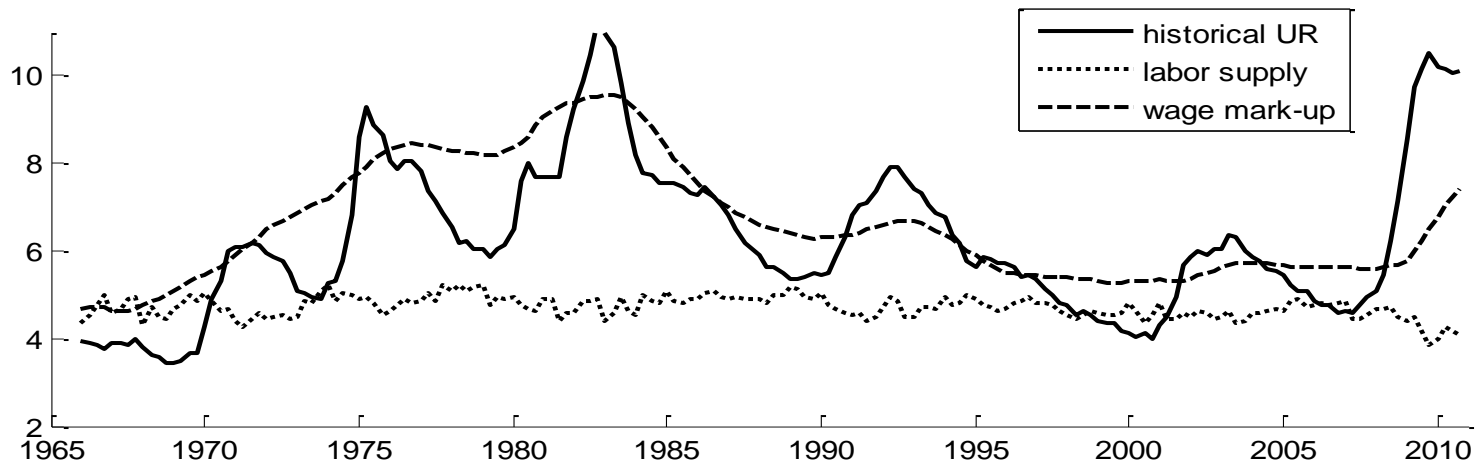
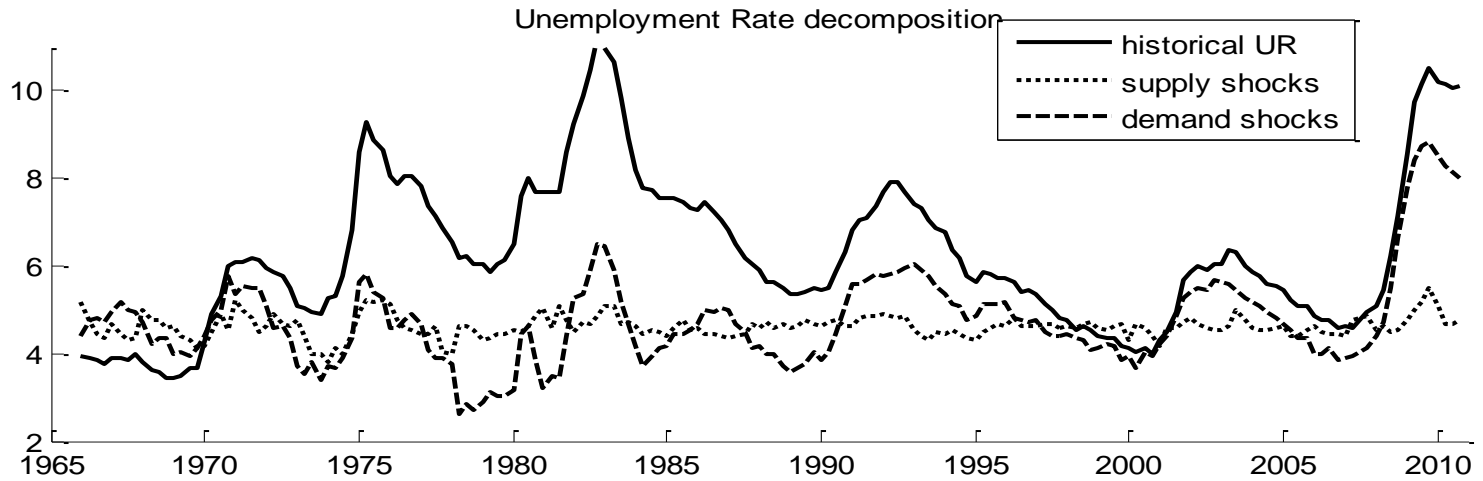


Figure 13. Unemployment and Wage Inflation

Conditional Phillips Curves: price inflation

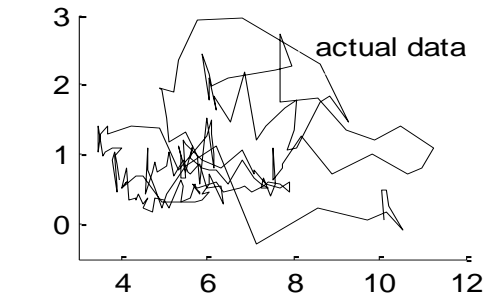
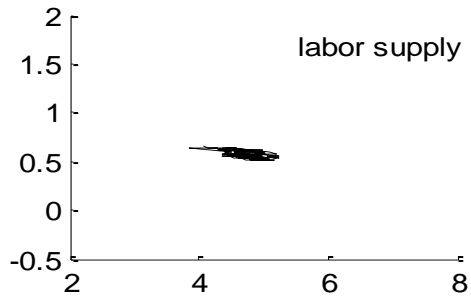
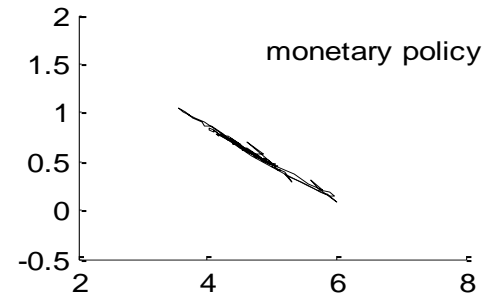
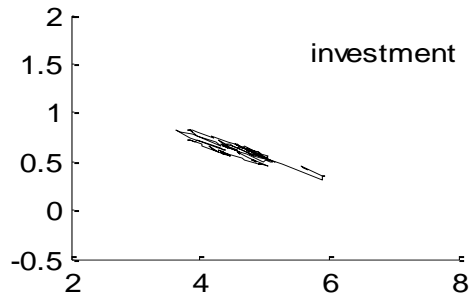
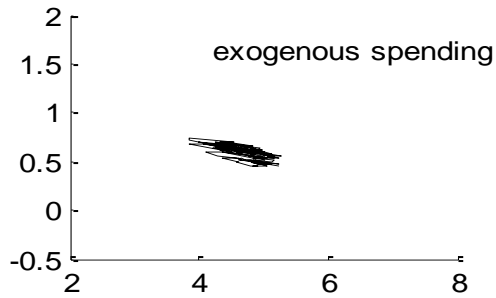
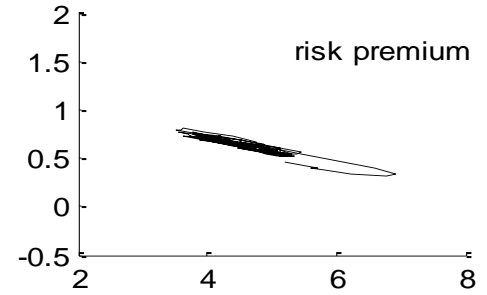
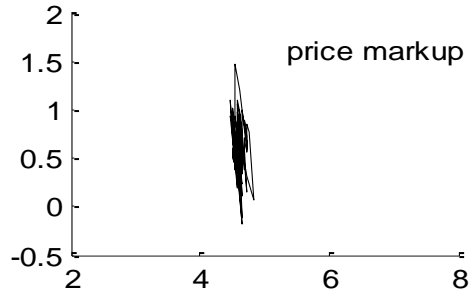
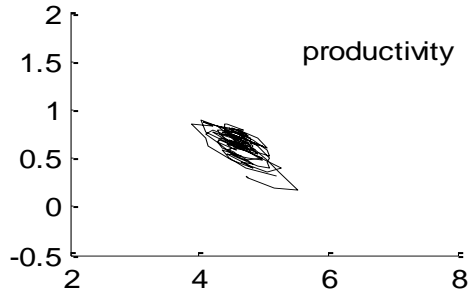


Figure 14. Unemployment and Price Inflation

Conditional Phillips Curves: wage inflation

