Cost Pass-Through under Delegation

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Abstract

The rate of cost pass-through exceeds 50% under strategic delegation of decision-making to managers with sales revenue contracts—regardless of the number of firms in the industry and demand curvature. This contrasts sharply with profit-maximization, for which cost pass-through can take on any positive value. The key intuition is that firms under delegation act as if they faced more rivals than they actually do, thus pushing cost pass-through towards 100%. Cost pass-through with market share contracts is similarly bounded below, and this note also generalizes existing results on equilibrium characterization for this case.

KEYWORDS: cost pass-through, excise taxation, executive compensation, market share, strategic delegation

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1 Introduction

The rate of pass-through from changes in marginal costs onto prices is central to the analysis of many important questions in public economics and industrial organization. How large is the incidence of unit taxes on consumer goods (say, gasoline or tobacco products)? What is the retail price impact of changes in the price of commodities used as inputs for production? To what extent do firms pass savings from cost-cutting exercises on to their customers?

In practice, firms delegate decision-making in the product market to managers. These managers in turn are frequently rewarded on the basis of measures of firm size, such as sales revenue or market share, that are ‘more aggressive’ than profits. Indeed, giving such aggressive incentives to managers is in the interest of shareholders when competition is in strategic substitutes, see, e.g., the seminal papers by Fershtman and Judd (1987), Sklivas (1987) and Vickers (1985).

Under delegation, firms’ incentives and hence outcomes in the product market are thus likely to differ from those that arise under profit-maximization. This has been widely recognized and recently incorporated, for example, into the analysis of collusion (e.g., Lambertini and Trombetta, 2002), mergers (e.g., Ziss, 2001), cost control (e.g., Szymanski, 1994) and patent licensing (e.g., Saracho, 2002). It is also relevant for understanding what drives cost pass-through onto prices; however, no such treatment exists in the delegation literature thus far.

This note analyzes the economics of cost pass-through under such delegation, and compares it to that of profit-maximization. It is well-known that with profit-maximizing firms the rate of cost pass-through can take on any positive value—depending on the shape of the demand curve. The surprising main result of this note is that cost pass-through exceeds 50% when managerial incentives are based on sales revenue contracts (as in the seminal papers listed above, and almost all subsequent work)—regardless of the number of firms in the industry and for any given demand curvature. The key intuition developed is that firms under delegation act as if they faced more rivals than they actually do, which pushes cost pass-through towards 100% (and also lowers firms’ profit margins).

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1 Indeed, in an early analysis of price responses to tax changes, Musgrave (1959, p. 281) already recognized that “in a market characterized by a small number of sellers supplying a standardized product . . . the solution depends on the strategy pursued by the participating firms” (italics added).

2 The crucial role of demand curvature was first pointed out by Bulow and Pfeiderer (1983), who examined the monopoly case.
The results for the (analytically more involved) case of market share contracts are very similar, and the underlying intuition for them is the same. (Jansen et al. (2007) and Ritz (2008) have recently shown that firms may prefer to provide incentives based on market share rather than sales revenue. Given this, the present note covers both types of contracts.) Here, cost pass-through has a lower bound of \( 33\frac{1}{3}\% \), although it again always exceeds 50\% as long as there are at least three firms in the industry.\(^3\)

Topical applications for these results include the impact of environmental policies—such as ‘carbon taxes’ and emissions trading schemes—on consumer prices in different sectors,\(^4\) and the extent to which food producers pass rising costs of agricultural commodities (such as wheat, cocoa or soybeans) on to consumers.\(^5\)

The remainder of the paper is organized as follows. Section 2 sets up the model (for the case of sales revenue contracts) and derives its subgame-perfect Nash (“incentive”) equilibrium. Section 3 presents the main results on cost pass-through under delegation. Section 4 offers concluding comments.

For expositional purposes, the parallel analysis for contracts based on market share is presented in the Appendix. This analysis also generalizes the results from the existing literature on equilibrium characterization under delegation with market share incentives to allow for both non-linear demand curves and an arbitrary number of firms in the industry.\(^6\)

\(^3\)The technical condition used to derive these results is that inverse demand is assumed throughout to have an iso-elastic slope. This is a common assumption in models of imperfect competition, and is satisfied by all demands typically employed in applied economics, such as linear, log-linear and constant-elasticity demand curves. (See footnote 8 for more discussion.)

\(^4\)Such environmental policies typically lead to increases in firms’ marginal costs as modelled here. See also footnote 19 on the recent introduction of a ‘carbon tax’ in Quebec (the Canadian province) and cost pass-through of allowance prices in the European emissions trading scheme (EU ETS).

\(^5\)See, e.g., “Few crumbs of comfort as food costs climb,” Financial Times, 10 July 2007 for some evidence suggesting that these costs are indeed passed through. Similar considerations apply to changes in packaging costs (e.g., due to aluminium prices) or to changes in transport costs (e.g., due to oil prices), and all of these may ultimately have a broader impact on the overall rate of price inflation.

\(^6\)Jansen et al. (2007) restrict attention to linear demand while Ritz (2008) allows for non-linear demand (for the same class of demand curves employed in this note) but considers only the duopoly case.
2 Model

2.1 Setup

Consider a homogeneous product market with \( n \geq 2 \) quantity-setting firms that all have constant marginal cost \( c > 0 \). The firms face a downward-sloping inverse demand curve \( P(Q) \), where \( Q = \sum_{i=1}^{n} q_i \) denotes total industry output.

For tractability, we assume that inverse demand has an iso-elastic slope, such that demand curvature \( E = -QP''(Q)/P'(Q) \) does not vary with \( Q \). This parameterizes a rich class of demand curves of the general form

\[
P(Q) = \alpha - \frac{\beta Q^{(1-E)}}{(1 - E)}, \tag{1}
\]

for which the ratio of the slope of industry marginal revenue to the slope of demand is constant for all \( Q \). Note that inverse demand is convex (concave) when \( E \geq 0 \) \((E \leq 0)\). Many familiar demands, including linear \((E = 0)\), log-linear \((E = 1)\), using l'Hôpital’s rule) and constant-elasticity \((E = 1 + 1/\eta)\), with industry price elasticity of demand \( \eta(Q) = |P'(Q)/P(Q)Q| > 0 \), are nested as special cases.\(^7\)

Decision-making in the product market is delegated to managers whose compensation contracts depend on a combination of firm profits \((\Pi_i)\) and sales revenue \((S_i)\) as in

\[
\Omega_i = \theta_i \Pi_i + (1 - \theta_i) S_i \quad \text{for} \; i = 1, 2, \ldots, n, \tag{2}
\]

where \( \Pi_i = (P(Q) - c) q_i \) and \( S_i = P(Q) q_i \).\(^8\) This objective function was suggested by Fershtman and Judd (1987) and Sklivas (1987) and has been employed widely since. Note that sales revenue is ‘more aggressive’ than profits in the sense that it has a higher marginal return, \( \partial S_i / \partial q_i - \partial \Pi_i / \partial q_i = c > 0 \).\(^9\)

\(^7\)This can be thought of as an index of demand curvature akin to constant relative risk aversion in utility theory. (Indeed, if consumer utility exhibits this property, then the associated market demand will also have constant curvature.) Robinson (1933, pp. 40–41) refers to \(-E\) as the “adjusted concavity” of demand.

\(^8\)Comparative statics (and comparisons of equilibria more generally) in delegation models unfortunately appear to be intractable beyond this class of demand curves (see also, e.g., Ziss (2001) who compares pre-merger and post-merger market structures). This assumption is however also often employed for similar reasons in “non-delegation” analyses, both theoretical and empirical. Recent examples include Corchón (2008), Cowan and Vickers (2007) and Genesove and Mullin (1998).

\(^9\)To guarantee their commitment value, these contracts are assumed to be publicly observable. See Gal-Or (1997) for a survey containing further discussion of this assumption.

\(^10\)Contracts based on profits and sales revenue are analytically convenient since the strate-
The game has two stages. In the first stage, each firm’s shareholders choose the incentive weight $\theta_i$ to maximize their firm’s profits. In the second stage, each firm’s manager chooses output $q_i$ to maximize his compensation.

The shareholders may thereby set $\theta_i \neq 1$ as a strategic commitment to non-profit-maximizing behaviour. Managers are held to their outside option (which is normalized to zero) and all rents accrue to shareholders.

### 2.2 Equilibrium

The game is solved backwards to find the subgame-perfect Nash equilibrium.\(^{11}\) The representative first-order condition for a manager’s choice of output in the second stage (taking $\theta_i$ as given) is

\[
\frac{\partial \Omega_i}{\partial q_i} = \theta_i \frac{\partial \Pi_i}{\partial q_i} + (1 - \theta_i) \frac{\partial S_i}{\partial q_i} = 0. \tag{3}
\]

This condition defines a manager’s best response in the product market. The Nash equilibrium of the stage occurs where all managers are simultaneously playing their respective best responses (for given incentive parameters). Thus let $q_i^*(\theta_1, \theta_2, \ldots, \theta_n)$ denote the equilibrium output of firm $i$ as a function of all managers’ incentive weights.

These first-order conditions also determine the aggregate (best) response of all other firms to a change in firm $i$’s output. We denote this by $R'_{-i} = dQ_{-i}/dq_i$, where $Q_{-i} = \sum_{j \neq i} q_j$ is also a function of the incentive weights.

Given the Nash equilibrium in the second stage, each firm’s shareholders strategically choose their manager’s incentives. The representative first-order condition for the first stage can be written as

\[
\frac{d\Pi_i^*}{d\theta_i} = [P(Q) - c + P'(Q)q_i^*(1 + R'_{-i})] \frac{dq_i^*}{d\theta_i} = 0, \tag{4}
\]

where a higher incentive weight on profits (lower weight on sales revenue) decreases output, $dq_i^*/d\theta_i < 0$.\(^{12}\) The $R'_{-i}$ term captures the strategic effect that

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\(^{11}\)The equilibrium conditions are derived using the approach introduced by Ritz (2008). This allows us to perform the analysis without explicitly computing the equilibrium incentive weights.

\(^{12}\)This intuitive result follows formally by applying the implicit function theorem to (3).
may induce deviations from profit-maximization in compensation contracts, and is exploited here by way of strategic delegation to managers.

Combining the two previous first-order conditions for firm $i$ (which must hold in the subgame-perfect Nash equilibrium) shows that the incentive equilibrium can be characterized by

$$ (1 - \theta^*_i) \left[ \frac{\partial S_i}{\partial q_i} - \frac{\partial \Pi_i}{\partial q_i} \right] = P'(Q)q_i^* R'_{-i}. \quad (5) $$

Whenever $R'_{-i} < 0$, such that outputs are (locally) strategic substitutes, shareholders find it optimal to give managers aggressive incentives by setting $\theta^*_i < 1$, thus placing positive weight on sales revenue. This induces managers to expand output beyond the profit-maximizing level, thereby reducing profit margins.

We assume throughout that the Hahn condition $E < n$ (such that inverse demand is not too convex) is satisfied. This is the necessary condition to ensure that firms indeed compete in strategic substitutes and it also guarantees stability and uniqueness of equilibrium.

### 3 Results

Summing the left-hand component of the $n$ first-order conditions for the first stage from (4) and noting that in symmetric equilibrium $R'_{-i} = R'$ for all $i$ yields

$$ n \left( P(Q^*) - c \right) + P'(Q^*)Q^*(1 + R') = 0. \quad (6) $$

This expression implicitly defines total industry output, and hence the equilibrium market price. It has the convenient feature that the case of profit-maximization is nested when $R' = 0$ and any strategic effects are not exploited.\(^\text{13}\) Making use of this yields the following benchmark result.

\(^\text{13}\)The key observation here is that the incentive equilibrium under delegation coincides with the profit-maximization equilibrium when $R' = 0$ (see the first-order conditions in (4), which are used to derive the rate of cost pass-through). Of course, this does not mean that the slope of the (aggregate) best response curve is actually zero under Cournot competition (recall also footnote 10). However, since any such opportunity for strategic commitment is not exploited in this case (by construction, as it is a one-stage setting), for the purposes of analysis, the case of profit-maximization is nested when $R' = 0$. 

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Lemma 1. Under profit-maximization, the rate of cost pass-through is
\[
\frac{dP^*}{dc} \bigg|_{R' = 0} = \frac{n}{(n + 1 - E)} > 0.
\]

Proof. Setting \( R' = 0 \) in (6), implicitly differentiating and recalling that \( E = -QP''(Q)/P'(Q) \) gives the result. Note that \( [dP^*/dc]_{R' = 0} > 0 \) since \( E < n \).

This expression for cost pass-through in Cournot markets is well-known and appears, for example, in Bulow and Klemperer (1998) and Kimmel (1992). It shows that pass-through depends on the number of firms in the industry and the shape of the demand curve. It also shows that cost pass-through can take on any positive value under profit-maximization. In particular, when demand becomes very concave (\( E \) becomes very negative), the rate of cost pass-through approaches zero.

We now turn to the corresponding expression for when firms strategically delegate decision-making to managers.

Proposition 1. Under delegation with sales revenue contracts, the rate of cost-pass through is
\[
\frac{dP^*}{dc} = \frac{n(n - E) + E}{n(n - E) + 1} > 0.
\]

Proof. The strategic effect \( R'_{-i} = dQ_{-i}/dq_i \) is endogenously determined by the \( n - 1 \) other managers’ first-order conditions from the second stage, see (3). Implicit differentiation shows that it satisfies
\[
R'_{-i} = \frac{\partial \left[ \sum_{j \neq i} \partial \Omega_j / \partial q_j \right] / \partial q_i}{-\partial \left[ \sum_{j \neq i} \partial \Omega_j / \partial q_j \right] / \partial Q_{-i}}. \tag{7}
\]
Performing these calculations, recalling that \( E = -QP''(Q)/P'(Q) \) and imposing symmetry with \( q_i^* = q^* = Q^*/n \) and \( R'_i = R' \) for all \( i \) yields that
\[
R'(E, n) = \frac{-(n - 1) + [(n - 1)/n] E}{n - [(n - 1)/n] E} < 0. \tag{8}
\]
Note that \(-1 < R'(E, n) < 0 \) since \( E < n \). Now differentiating (6) shows that
the rate of cost pass-through satisfies
\[
n \left( \frac{dP^*}{dc} - 1 \right) + \frac{dP^*}{dc} (1 - E) [1 + R'(E, n)] = 0. \tag{9}
\]
Combining (8) and (9) and some further rearranging gives the desired result. Again, \( dP^*/dc > 0 \) since \( E < n \).

Proposition 1 shows that the rate of cost pass-through under delegation is also driven by the number of firms and the shape of the demand curve. Its key properties are now discussed in conjunction with four corollaries.

The Appendix shows that the results for delegation with contracts based on market share are very similar. Corollaries 1-3 are identical and there is also a lower bound on pass-through (albeit a slightly weaker one), just as in Corollary 4. The underlying intuition for the results is also the same as that now developed for the case of sales revenue contracts.

**Corollary 1.** Under delegation with sales revenue contracts, the rate of cost pass-through satisfies \( dP^*/dc \geq 1 \) according to \( E \geq 1 \).

Price changes by more than the change in marginal costs and ‘overshifting’ occurs whenever demand is log-convex (\( E > 1 \), e.g., constant-elasticity).\(^{14}\) Undershifting occurs when demand is log-concave (\( E < 1 \), e.g., linear) and there is 100% pass-through to prices when demand is log-linear (\( E = 1 \)).

These findings extend the results of Anderson et al. (2001) who show that these three above regimes of pass-through—well-known from homogeneous product Cournot oligopoly (see Lemma 1)—also apply in a differentiated products Bertrand setting. Corollary 1 demonstrates that this conclusion also goes through in Cournot markets with delegation.\(^{15}\)

A second similarity arises in the limiting behaviour of cost pass-through as the number of firms grows large.

**Corollary 2.** Under delegation with sales revenue contracts, the rate of cost pass-through satisfies \( \lim_{n \to \infty} dP^*/dc = 1 \).

\(^{14}\)Auerbach and Hines (2003, p. 129) note that “overshifting has intrigued public finance economists at least since the time of Edgeworth.” Perhaps the most obvious instance of overshifting occurs when firms find it optimal for price to be a constant percentage mark-up on costs (‘cost-plus pricing’).

\(^{15}\)There is empirical evidence consistent with all three of these regimes, although rates of pass-through are often found to be clustered fairly closely around 100%, see, e.g., the discussion in Poterba (1996).
This result is the flip side of the one that price converges to marginal cost as the number of firms grows large. To see this, note that the equilibrium condition from (6) can be rewritten in terms of the equilibrium elasticity-adjusted Lerner index \( L^* = \eta(Q^*) (P(Q^*) - c) / P(Q^*) \) as

\[
L^*_\eta = \frac{1 + R'(E, n)}{n} = \frac{1}{n^2 - (n - 1)E} < \frac{1}{n},
\]

where the second equality makes use of (8). It is now immediate that \( P(Q^*) - c \) approaches zero and hence \( dP^*/dc \) approaches unity as the number of firms grows large, as is the case under profit-maximization.

However, it is also clear that convergence is much faster under delegation since the strategic effect \( R'(E, n) \) tends to \(-1\) as \( n \to \infty \). (Intuitively, there is more scope for strategic manipulation with more firms.) Formally, the rate of convergence to 100% cost pass-through (zero profit margins) under profit-maximization is of order \( 1/n \) (since then \( R'(E, n) = 0 \)), while the convergence rate under delegation is of order \( 1/n^2 \).

The next result is closely related.

**Corollary 3.** Under delegation with sales revenue contracts, the rate of cost pass-through is closer to 100% than under profit-maximization, \( |1 - dP^*/dc| \leq |1 - [dP^*/dc]_{R'=0}| \) for any \( E < n \).

Corollary 3 is perhaps best understood by noting (e.g., from (10) and (11)) that firms under delegation act as if they faced more rivals than they actually do. For example, if demand is linear \( (E = 0) \), \( dP^*/dc = n^2 / (n^2 + 1) \) and \( n \) ‘sales-maximizing’ firms act like \( n^2 \) profit-maximizing firms. This pushes the rate of cost pass-through closer to 100% (see Corollary 2), which increases (reduces) pass-through if \( E \leq 1 \) \((E \geq 1)\) (see Corollary 1).

Applied tax incidence studies frequently employ an a priori assumption of full pass-through to consumers (see, e.g., Fullerton and Metcalf (2002) and the references cited therein) which corresponds either to a perfectly competitive market (where price equals marginal cost) or to log-linear demand (for which price-cost margins are constant). Although still restrictive, Corollary 3 reveals

\[\text{16}\]

However, it is also worth recalling that, even though pass-through can be higher or lower under delegation, consumers always prefer decision-making to be delegated to managers since this leads to a lower equilibrium price than under profit-maximization, and thus higher consumer surplus (and higher total welfare). (See, e.g., Fershtman and Judd (1987); this is also clear from (10) and (11) and consistent with the ‘more firms’ intuition developed here.)
that such an assumption is somewhat less unsatisfactory under delegation than it is under profit-maximization.

The following corollary states our main result.

**Corollary 4.** Under delegation with sales revenue contracts, the rate of cost pass-through satisfies $dP^*/dc \geq \frac{1}{2}$.

This follows directly from Proposition 1 since $dP^*/dc$ is increasing in $E$, so $dP^*/dc \geq \{\lim_{E \to -\infty} dP^*/dc = (n - 1)/n\} \geq \frac{1}{2}$ since $n \geq 2$.

Corollary 4 contrasts sharply with profit-maximization, for which the rate of cost pass-through can take on *any* positive value—depending on the shape of the demand curve (recalling Lemma 1). However, under delegation with sales revenue contracts a wide range of pass-through rates can be ruled out, namely anything below 50%.

To understand this result, note that as demand becomes more concave ($E$ becomes more negative), firms’ marginal revenue curves become steeper relative to the (inverse) demand curve. This has two opposing effects:

First, firms need to adjust their output by less in response to a cost change to ensure optimality at the margin. This effect pushes cost pass-through towards zero and is present both under profit-maximization and delegation.

Second, the scope for strategic manipulation increases (since $R'(E, n)$ also becomes more negative), which amplifies the ‘more rivals’ intuition discussed above. This effect pushes cost pass-through towards unity and is present only under delegation.

Formally, these two forces can be seen by rearranging (9) to show that pass-through under delegation satisfies

$$
\left(1 - E\right) \frac{dP^*}{dc} + \frac{n}{[1 + R'(E, n)]} \left(\frac{dP^*}{dc} - 1\right) = 0.
$$

(12)

Term I captures the first effect, which clearly pushes pass-through towards zero as $E$ becomes more negative.

Term II captures the second effect, which under profit-maximization does not vary with $E$ since $R'(E, n) = 0$. However, under delegation with sales revenue contracts recall that $n/[1 + R'(E, n)] = n^2 - (n - 1)E > n$ (‘more rivals’), so it pushes pass-through towards unity as $E$ becomes more negative.

The ‘weights’ on both of these effects thus vary linearly with $E$. It follows that, in the limit as $E \to -\infty$, there is a (relative) weight of 1 on $dP/dc = 0$ (first effect; direct curvature) and a weight of $n - 1$ on $dP/dc = 1$ (second
effect; indirect strategic). Therefore, equilibrium cost pass-through satisfies $dP^*/dc \geq (n - 1)/n$.

The existence of a lower bound clearly is the most distinguishing feature of cost pass-through under delegation. The Appendix shows that this feature also applies with market share contracts, as recently examined by Jansen et al. (2007) and Ritz (2008). In particular, cost pass-through with strategic incentives for market share has a lower bound of $33\frac{1}{3}\%$, although it again always exceeds 50% whenever there are at least three firms in the industry.

4 Conclusion

It is well-known that with profit-maximizing firms, the rate of cost pass-through can take on any positive value. This has the unfortunate consequence that theory effectively predicts that ‘anything could happen.’

The surprising main result of this note is that the rate of cost pass-through when decision-making is strategically delegated to managers is bounded below for a fairly general class of demand curves (which includes all those typically employed in applied economics). With sales revenue contracts, pass-through always exceeds 50%—regardless of the number of firms and demand curvature. Similarly, with market share contracts, cost pass-through also always exceeds 50% as long as there are at least three firms in the industry. More generally, cost pass-through under delegation is closer to 100% (for any given demand curvature) than it is under profit-maximization.

The analysis with delegation thus yields a sharper theoretical prediction that is consistent with the stylized facts, both on cost pass-through and executive compensation. Indeed, empirical studies often find rates of pass-through that are clustered fairly closely around 100%, and there is scant evidence for the very low rates (substantial undershifting) possible under profit-maximization.\(^{17,18}\)

\(^{17}\)For example, two recent papers—Delipalla and O’Donnell (2001) on cigarette taxes in various European countries and Kenkel (2005) on alcohol excise taxes for various products—find no pass-through rates below 67% and 81% respectively. (Both of these studies also find evidence for substantial tax overshifting.)

\(^{18}\)A vivid example of a recent policy debate on cost pass-through is provided by the introduction of a ‘carbon tax’ (on CO\(_2\) emissions from the energy sector) in Quebec (the Canadian province) as of October 2007. A few months before, it was reported that Quebec’s minister for natural resources “expects that the companies will absorb the higher costs, though he ‘can’t guarantee’ that producers and refiners won’t pass them on to consumers.” However, consistent with the results reported here, it appears that a very substantial portion (probably all) of the tax has been passed through to consumers. (See, e.g., “Quebec approves carbon tax to cut greenhouse gases,” Bloomberg.com, 7 July 2007 and “Carbon tax bill in Quebec introduces new fees to cut greenhouse gases,” Bloomberg.com, 7 July 2007)
The key intuition for understanding the results in this note is that firms under delegation act as if they faced more rivals than they actually do, which pushes cost pass-through towards 100% (and lowers profit margins). Surprisingly, strategic considerations therefore imply that passing on to consumers the majority of a change in costs is almost always in managers’ (and firms’) best interest.

Appendix: Cost pass-through with market share incentives

The analysis for the case when decision-making in the product market is delegated to managers whose compensation contracts depend on a combination of firm profits and market share (see Jansen et al., 2007 and Ritz, 2008) parallels that of the main text, to which the reader is referred for more detailed discussion.

Setup

The notation and assumptions used here are the same as those for the case of sales revenue contracts in the main text. In particular, inverse demand is assumed to have an iso-elastic slope, such that demand curvature \( E = -QP''(Q)/P'(Q) \) is constant, and the Hahn condition \( E < n \) is assumed to be satisfied, such that competition is in strategic substitutes (in equilibrium).

With market share contracts, the objective function becomes

\[
\Omega_i = (1 - \lambda_i)\Pi_i + \lambda_i \sigma_i \quad \text{for} \quad i = 1, 2, ..., n, \tag{13}
\]

where \( \sigma_i = q_i/Q \) is firm \( i \)'s market share and \( \lambda_i \) is the incentive weight on market share.

Equilibrium

The representative first-order condition for the second stage (taking \( \lambda_i \) as given) is

\[
\frac{\partial \Omega_i}{\partial q_i} = (1 - \lambda_i)\frac{\partial \Pi_i}{\partial q_i} + \lambda_i \frac{\partial \sigma_i}{\partial q_i} = 0. \tag{14}
\]
Let \( q_i^*(\lambda_1, \lambda_2, \ldots, \lambda_n) \) denote the equilibrium output of firm \( i \) as a function of all managers’ incentive weights.

Given this Nash equilibrium in the second stage, the representative first-order condition for the first stage can be written as

\[
\frac{d\Pi_i^*}{d\lambda_i} = [P(Q) - c + P'(Q)q_i^*(1 + R_{-i}')] \frac{dq_i^*}{d\lambda_i} = 0, \tag{15}
\]

where a higher incentive weight on market share (lower weight on profits) increases output, \( dq_i^*/d\lambda_i > 0 \), applying the implicit function theorem to (14).

Combining the two previous first-order conditions for firm \( i \) shows that the incentive equilibrium can be characterized by

\[
\frac{\lambda_i^*}{1 - \lambda_i^*} \frac{\partial \sigma_i}{\partial q_i} = P'(Q)q_i^* R_{-i}'. \tag{16}
\]

Again, whenever \( R_{-i}' < 0 \), such that outputs are (locally) strategic substitutes, shareholders find it optimal to give managers aggressive incentives by setting \( \lambda_i^* \in (0, 1) \), thus inducing them to expand output beyond the profit-maximizing level.\(^{19}\)

**Results**

Summing the left-hand component of the \( n \) first-order conditions for the first stage from (15) and noting that in symmetric equilibrium \( R_{-i}' = R' \) for all \( i \) yields

\[
n (P(Q^*) - c) + P'(Q^*)Q^*(1 + R') = 0. \tag{17}
\]

This expression implicitly defines total industry output, and hence the equilibrium market price, just as in the case with sales revenue contracts from the main text.

**Proposition 1A.** Under delegation with market share contracts, the rate of cost-pass through satisfies

\[
\frac{dP^*}{dc} = \frac{n}{n + (1 - E) [1 + R'(E, n)]} > 0,
\]

\(^{19}\)Note also that \( \lambda_i^* = 1 \) cannot be an equilibrium, since then the first-order condition (14) cannot be satisfied as \( \partial \sigma_i/\partial q_i > 0 \) in any interior solution.
where \( R'(E, n) \in (-1, 0) \) solves

\[
R' = \frac{-(n-1) + \left[(n-1)/n\right]E + \left[(n-2)/n\right]R'}{n - \left[(n-1)/n\right]E - \left[2(n-1)/n\right]R'}.
\]

**Proof.** The strategic effect \( R'_{-i} = dQ_{-i}/dq_i \) is endogenously determined by the \( n-1 \) other managers’ first-order conditions from the second stage, see (14). Implicit differentiation shows that it satisfies

\[
R'_{-i} = \frac{\partial \left[ \sum_{j \neq i} \partial \Omega_j / \partial q_j \right] / \partial q_i}{-\partial \left[ \sum_{j \neq i} \partial \Omega_j / \partial q_j \right] / \partial Q_{-i}}. \tag{18}
\]

Performing these calculations, recalling that \( E = -QP''(Q)/P'(Q) \) and imposing symmetry with \( q^*_i = q^* = Q^*/n \) and \( \lambda^*_i = \lambda^* \) for all \( i \) yields

\[
\frac{\partial}{\partial q_i} \left[ \sum_{j \neq i} \frac{\partial \Omega_j}{\partial q_j} \right] = (1 - \lambda^*) P'(Q) \left[ \frac{(n-1) - \frac{(n-1)E}{(1-\lambda^*)}}{\frac{n}{P'(Q)Q^2}} \right] \tag{19}
\]

and

\[
\frac{\partial}{\partial Q_{-i}} \left[ \sum_{j \neq i} \frac{\partial \Omega_j}{\partial q_j} \right] = (1 - \lambda^*) P'(Q) \left[ \frac{n - \frac{(n-1)E}{(1-\lambda^*)}}{\frac{n}{2(n-1)^2 P'(Q)Q^2}} \right]. \tag{20}
\]

Now noting that the symmetric equilibrium condition from (16) becomes

\[
\frac{\lambda^*}{(1 - \lambda^*)} \frac{(n-1)}{P'(Q)Q^2} = R' \tag{21}
\]

and combining this with the previous two expressions yields that

\[
R' = \frac{-(n-1) + \left[(n-1)/n\right]E + \left[(n-2)/n\right]R'}{n - \left[(n-1)/n\right]E - \left[2(n-1)/n\right]R'} \tag{22}
\]

as claimed. Note that the relevant solution of this quadratic in \( R' \) is the one where \( R'(E, n) \in (-1, 0) \) (since \( E < n \)). (The other root violates the stability condition \(|R'(E, n)| < 1\).) Differentiating (17) shows that the rate of cost pass-through satisfies

\[
n \left( \frac{dP^*}{dc} - 1 \right) + \frac{dP^*}{dc} (1 - E) \left[ 1 + R'(E, n) \right] = 0. \tag{23}
\]
Some further rearranging gives the desired result. ■

Proposition 1A shows that cost pass-through with market share incentives is also driven by the number of firms and the shape of the demand curve, albeit in a more complicated way.

Corollaries 1A-3A (stated below) follow easily from the proposition, and are identical to those (Corollaries 1-3 on sales revenue contracts) in the main text.

**Corollary 1A.** Under delegation with market share contracts, the rate of cost pass-through satisfies
\[ dP^*/dc \geq 1 \] according to \( E \geq 1 \).

**Corollary 2A.** Under delegation with market share contracts, the rate of cost pass-through satisfies
\[ \lim_{n \to \infty} dP^*/dc = 1. \]

**Corollary 3A.** Under delegation with market share contracts, the rate of cost pass-through is closer to 100% than under profit-maximization, \( |1 - dP^*/dc| \leq |1 - [dP^*/dc]_{R'=0}| \) for any \( E < n \).

Corollary 4 on the lower bound of cost-pass through under delegation is now somewhat more involved. As in the sales revenue case, pass-through rates are lower with more concave demand curves (more negative \( E \)), that is, \( dP^*/dc \) is increasing in \( E \). It can be shown that
\[ \lim_{E \to -\infty}(1 - E)[1 + R'(E, n)] = 2n/(n - 1), \]
and so
\[ dP^*/dc \geq \left\{ \lim_{E \to -\infty} dP^*/dc = (n - 1)/(n + 1) \right\} \geq \frac{1}{3}, \tag{24} \]
since \( n \geq 2 \). The two underlying drivers are the direct curvature effect and the indirect strategic effect, as discussed in the main text.

**Corollary 4A.** Under delegation with market share contracts, the rate of cost pass-through satisfies
\[ dP^*/dc \geq \frac{1}{3} \] (and \( dP^*/dc \geq \frac{1}{2} \) whenever \( n \geq 3 \)).

\(^{20}\)To see why, note from (22) that
\[ (1 - E)(1 + R') = \frac{(1 - E)(1 - R')}{n - [(n - 1)/n] E - [2(n - 1)/n] R'}, \]
where both sides have been multiplied by \( (1 - E) \). As demand becomes very concave (\( E \) becomes very negative), the right-hand side tends to \( n(1 - R')/(n - 1) \). But since \( \lim_{E \to -\infty} R' = -1 \) (again from (22), as firms’ marginal revenue curves become increasingly steep relative to inverse demand), it is clear that \( \lim_{E \to -\infty}(1 - E)(1 + R') = 2n/(n - 1) \).
Finally, it is worth noting that this analysis also generalizes existing results in the literature on equilibrium characterization with market share incentives.

In particular, note that the symmetric incentive equilibrium can be characterized by the equilibrium elasticity-adjusted Lerner index, which using (17) can be written as

\[ L^*_n = \frac{1 + R'(E, n)}{n}. \]  

The slope of the managers’ (aggregate) best response curve \( R'(E, n) \in (-1, 0) \) is given in Proposition 1A above.

These two equations, (22) and (25), provide an equilibrium characterization for market share incentives with both non-linear demand curves and an arbitrary number of firms in the industry. For a given demand curve, the equilibrium output and price can be backed out, and the equilibrium incentive weight on market share is then also easily calculated using (21).

These arguments nest Proposition 1 of Ritz (2008) as a special case when the industry is a duopoly (\( n = 2 \)) and the result of Jansen et al. (2007) as a special case when demand is linear (\( E = 0 \)).

References


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