Does competition increase pass-through?

Robert A. Ritz
Judge Business School & Faculty of Economics
Cambridge University
r.ritz@jbs.cam.ac.uk

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Abstract

In recent years, the literature has seen a surge of interest in pass-through as an economic tool. At the same time, widespread concerns have emerged about the rising market power of firms. How does competition affect pass-through? A standard intuition is that more competition makes prices more cost-reflective and hence raises the rate of cost pass-through. This paper shows this conclusion is sensitive to the routine assumption that firms’ marginal costs are constant. With modestly convex costs, market power can raise pass-through (even when it lies below 100%). These results have implications for antitrust policy, environmental regulation, and welfare analysis.

Keywords: Cost pass-through, market power, price theory, capacity constraints, cost convexity

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1 Introduction

In recent years, the literature has seen a surge of interest in using cost pass-through as a tool for economic analysis across fields including industrial organization (Weyl & Fabinger 2013), environmental economics (Fabra & Reguant 2014), development economics (Atkin & Donaldson 2015), and international trade (Mrázová & Neary 2017). At the same time, widespread concerns have emerged about the rising market power of firms, as the result of globalization, soft antitrust enforcement, ownership concentration, and other factors (Shapiro 2019; Syverson 2019).

This paper addresses a basic question that links these two themes: how does market power affect pass-through? A common intuition is that firms with market power have an incentive to absorb part of a cost increase whereas, under perfect competition, price equals marginal cost \((P = MC)\) so the rate of pass-through of a market-wide increase in marginal cost \((\partial P/\partial MC)\) is 100%. This suggests that more intense competition leads to stronger pass-through. Perhaps most prominently, this intuition holds in a textbook linear Cournot model, with a 50% pass-through rate under monopoly which rises up to 100% as the number of firms grows large.

Yet this intuition and existing theory literature on pass-through under imperfect competition (e.g., Bulow & Pfleiderer 1983; Kimmel 1992; Anderson & Renault 2003; Weyl & Fabinger 2013; Mrázová & Neary 2017) routinely maintain the assumption that firms have constant marginal costs. On one hand, this is a substantive economic assumption which may be appropriate for some markets but less so for others. On the other hand, it obscures the comparison with the benchmark of perfect competition—precisely because it restricts competitive pass-through to a “knife-edge” rate of 100%.

This paper unifies earlier results from the pass-through literature and highlights their sensitivity to the assumption of constant marginal cost. The model has two key features. First, to facilitate the comparison with perfect competition, firms sell a homogeneous product and the setup nests monopoly, oligopoly and perfect competition as special cases. Second, firms have convex cost functions, which can be justified purely on technological grounds, by frictions arising from principal-agent problems within the firm (Hart 1995), or by imperfections in labour, financial, and other input markets.

The main point is that, if firms have even modestly increasing marginal costs, the standard intuition can be overturned—and market power actually increases pass-through. Importantly, this finding applies to the “normal” case where pass-through is incomplete, i.e., lies below 100%.

The quickest way to see the result is to look at Figure 1. Market demand \((P)\) is linear and the industry marginal revenue \((MR)\) curve is twice as steep. The marginal cost of production \((MC)\) is a constant \(c\) up to the industry’s \(K\) units of capacity. A monopoly optimally produces

\[\text{footnote}{\text{Weyl & Fabinger (2013) allow for convex costs in some parts of their analysis though much of their treatment of oligopoly reverts to constant marginal cost. Adachi & Fabinger (2018) generalize many incidence results to settings with non-constant marginal costs. None of these papers focus specifically on the interplay between pass-through and market power studied in the present paper.}}\]
Figure 1. A case in which monopoly cost pass-through exceeds competitive pass-through

$Q^m < K$ units at marginal cost $c$, leading to the textbook result of a pass-through rate of

$$\frac{\partial P^m}{\partial MC} = \frac{1}{2} \left( \frac{\text{slope of } P(Q)}{\text{slope of } MR(Q)} \right).$$

A competitive industry, by contrast, produces at capacity, $Q^c = K$, so its market price does not change and so its rate of pass-through $\frac{\partial P^c}{\partial MC} = 0$ ($= \frac{\text{price elasticity of supply}}{\text{price elasticity of supply}}$). Therefore competition here reduces cost pass-through. Intuitively, a less flexible production technology, with more convex costs, always leads to lower pass-through because it makes quantities—and hence price—less responsive to the cost change. Yet this cost-convexity effect can be more pronounced in a more competitive market because it has higher industry output. While very simple, this point appears to be novel to the literature on price theory.

This paper uses two approaches to present more general versions of this basic insight. One approach extends the argument in Figure 1 to general demand and cost curves as well as richer market structures. It derives conditions on primitives to characterize when more intense competition reduces equilibrium pass-through. Sufficient conditions are that (i) the market demand curve is concave, linear or not too convex, (ii) demand is weakly more convex at higher prices, and (iii) cost convexity is pronounced enough.

The other approach compares in the cross-section two markets which may have different underlying demand and/or cost functions. For a like-for-like comparison, suppose that any such differences are controlled for—specifically, in the price elasticity of demand, the curvature of

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2This paper follows the literature on industrial organization and environmental economics in focusing on the pass-through rate $\partial P/\partial MC$ rather than the elasticity $\partial \ln P/\partial \ln MC$ that is more widely used in macroeconomics and international trade. The condition for market power to raise a pass-through elasticity is tighter due to the higher price-cost mark-up $P/MC$. However, it is immediate from Figure 1 that the main point can also apply to a pass-through elasticity—which is positive for monopoly but zero under perfect competition.
demand, and the curvature of costs. Here, too, the more competitive market always has lower pass-through if cost convexity is sufficiently pronounced. For example, this occurs if market demands are linear or convex and firms’ cost functions are at least as convex as a quadratic.

In sum, the standard intuition on pass-through and market power can be overturned in both approaches under plausible conditions on demand, costs and conduct.\(^3\)

Section 2 sets up the model, and Section 3 presents a unifying equilibrium result on cost pass-through that applies under both perfect and imperfect competition. Section 4 derives conditions under which more competition leads to weaker cost pass-through. Section 5 discusses the empirical implications of the theory in light of recent econometric work on cost pass-through. Section 6 concludes and discusses policy implications.

### 2 The model

Consider a simple model of imperfect competition between \(n\) symmetric firms that nests perfect competition and monopoly as special cases. Direct demand is \(D(p)\) and the corresponding inverse demand curve is \(p(X)\), where \(p\) is the market price, \(X\) is industry output and \(p'(\cdot) < 0\). Let \(\varepsilon^D \equiv \frac{-p(X)}{XP'(X)} > 0\) be the price elasticity of demand and let \(\xi^D \equiv \frac{-Xp''(X)}{p'(X)}\) be a measure of demand curvature. Demand is concave if \(\xi^D \leq 0\) and convex otherwise; it is log-concave (i.e., the log of direct demand \(\ln D(p)\) is concave in \(p\)) if \(\xi^D \leq 1\) and log-convex otherwise. Demand curvature can also be expressed as \(\xi^D = 1 + \frac{1 - \psi^D}{\varepsilon^D}\), where \(\psi^D \equiv \frac{d\varepsilon^D(p)/dp}{\varepsilon^D(p)/p}\) is the superelasticity of demand, i.e., the elasticity of the elasticity (Kimball 1995). So demand is log-concave \(\xi^D \leq 1\) if and only if it is unit-superelastic \(\psi^D \geq 1\).\(^4\)

Firm \(i\) has a cost function \(\hat{C}(x_i) \equiv [C(x_i) + \tau x_i]\) where \(x_i\) is its output (so \(X \equiv_i x_i\)), \(\tau\) is a market-wide cost shifter such as a tax or common cost factor, and which satisfies \(C'(\cdot) > 0, C''(\cdot) \geq 0\) (where \(\hat{C}''(x_i) = C''(x_i)\)). The cost shifter raises marginal cost according to \(\frac{\partial \hat{C}(x_i)}{\partial \tau} = 1\). Let \(\eta^S_i \equiv x_i \hat{C}''(x_i)/\hat{C}'(x_i) \geq 0\) be the elasticity of \(i\)'s marginal cost which, given symmetry, will be identical across firms with \(\eta^S_i = \eta^S\). This can be seen as a measure of the inflexibility of the production technology. (The assumption of firm symmetry is made for simplicity and is not crucial to the main results.\(^5\))

**Remark 1.** The model defines the elasticity of firm \(i\)'s marginal cost \(\hat{C}'(x_i)\) including the cost shifter \(\tau\). Many papers on pass-through focus on the case in which the initial value of the cost

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\(^3\)The present paper does not address dynamic considerations related to the speed and frequency of price adjustments in response to cost shocks.

\(^4\)Mrázová & Neary (2017) use the term “subconvex” for demands with positive superelasticity \(\psi^D \geq 0\); this condition is sometimes also referred to as Marshall’s “second law of demand.”

\(^5\)For example, the analysis would extend to marginal-cost asymmetry of the form \(C_i(x_i) = c_i + \mu(x_i)\) (given a fixed number of firms \(n\)). Similarly, the analysis would extend to a simple model of vertical product differentiation in which firm \(i\)'s price \(p_i(X) = \sigma_i + p(X)\) reflects its product quality \(\sigma_i\)—even if this would complicate the comparison with the benchmark of perfect competition.
shifter is zero, \( \tau = 0 \), and, for example, a small new unit tax is introduced. Then marginal cost is (locally) identical including and excluding the cost shifter \( \hat{C}'(x_i) = C'(x_i) \), and so the cost elasticity \( \eta^S_i = x_i C''(x_i)/C'(x_i) \) can equivalently be written without the cost shifter.\(^6\) This paper does not restrict attention to \( \tau \to 0 \), though its findings also apply to this case.

Firm \( i \)'s profits are \( \Pi_i = p(X)x_i - C(x_i) - \tau x_i \). Each firm chooses its output \( x_i \) in a generalized version of quantity competition. The industry’s conduct parameter \( \theta \in [0, 1] \) measures the intensity of competition. Formally, firms’ equilibrium outputs \( (x_i^*)_{i=1..n} \) satisfy:

\[
x_i^* = \arg \max_{x_i \geq 0} \{p(\theta(x_i - x_i^*) + X^*)x_i - C(x_i) - \tau x_i\}.
\]

Firm \( i \), in deviating its output by \( (x_i - x_i^*) \), conjectures that industry output will change by \( \theta(x_i - x_i^*) \) as a result. In this “conduct equilibrium”, lower values of \( \theta \) correspond to more intense competition. This setup can be viewed as a reduced-form representation of a dynamic game (Cabral 1995). The Cournot-Nash equilibrium, where each firm takes its rivals’ output as given, occurs where \( \theta = 1 \), and perfect competition with price-taking firms where \( \theta = 0 \).

Two further conditions will ensure a well-behaved interior equilibrium. First, a sufficient condition for an interior equilibrium is that \( p(0) > \hat{C}(0) = C'(0) + \tau \). Second, the condition \( \xi^D < 2 \), such that the industry’s marginal revenue is downward-sloping, will ensure a well-behaved equilibrium, regardless of the intensity of competition.

The first-order condition for firm \( i \) is:

\[
p(X) + \theta x_ip'(X) - \hat{C}'(x_i) = 0 \text{ at } x_i = x_i^*.
\]  \( \text{(2)} \)

This says that a generalized version of firm \( i \)'s marginal revenue equals its marginal cost.\(^7\) In symmetric equilibrium, \( x_i^* = x^* \), and so the first-order condition becomes:

\[
p(nx^*) + \theta x^*p'(nx^*) - \hat{C}'(x^*) = 0.
\]  \( \text{(3)} \)

Let \( \theta^S \equiv (\theta/n) \in [0, 1] \) be an index of market power which is higher with a higher conduct parameter or fewer firms. Writing \( p(\tau, \theta^S) = p(nx^*; \tau, \theta^S) \) for the equilibrium price (and dropping asterisks again for notational simplicity), the role of this index is made precise as follows:

**Lemma 1** The equilibrium elasticity-adjusted Lerner index \( L \equiv \varepsilon^D[p(X) - \hat{C}'(x)]/p(X) = \theta^S \in [0, 1] \), where the equilibrium market price \( p(\tau, \theta^S) \) increases with market power \( \theta^S \).

As expected, less intense competition, as measured by a higher \( \theta^S \), leads to a higher market price (and lower industry output). The setup therefore facilitates comparative statics on market

\(^6\)More generally, they are related according to \( \eta^S_i = [x_i C''(x_i)/C'(x_i)]/[1 + \tau/C'(x_i)] \).

\(^7\)The second-order condition for firm \( i \) is: \( (1 + \theta)p'(X) + \theta p''(X)x_i - C''(x_i) < 0 \Leftrightarrow (1 + \theta) - (x_i/X)\theta \xi + C''(x_i)/[-p'(X)] > 0 \), which is always satisfied given the assumptions \( \theta \in [0, 1] \), \( \xi^D < 2 \), \( C''(x_i) \geq 0 \).
power via a change in the value of $\theta^S$, due to a change in competitive conduct $\theta$ and/or in market structure $n$. Note also that, at equilibrium, the price elasticity of demand cannot be too low, with $\varepsilon^D > \theta^S$ (and so $\varepsilon^D > 1$ for monopoly).

3 Equilibrium cost pass-through

This section derives a unifying expression for pass-through that holds under both perfect and imperfect competition. The rate of cost pass-through is defined as the change in the equilibrium market price arising from a small market-wide shift in marginal cost, $\rho \equiv \partial p(\tau, \theta^S)/\partial \tau$.

Lemma 2 The equilibrium rate of cost pass-through equals:

$$\rho(\varepsilon^D, \xi^D, \eta^S; \theta^S) = \frac{1}{1 + (\varepsilon^D - \theta^S)\eta^S + \theta^S(1 - \xi^D)} > 0.$$ 

Lemma 2 gives a simple expression for pass-through in terms of familiar elasticity and curvature metrics that encompasses various results from prior literature. First, under perfect competition ($\theta^S = 0$), the first-order condition (2) defines firm $i$’s supply curve; letting $\varepsilon_i^S \equiv px'_i(p)/x_i(p) > 0$ be firm $i$’s price elasticity of supply, at symmetric equilibrium, $\varepsilon_i^S = \varepsilon^S$ and $\eta^S = 1/\varepsilon^S$. This leads to the textbook result that competitive pass-through $\rho = \varepsilon^S/(\varepsilon^S + \varepsilon^D)$ is driven by the ratio of demand and supply elasticities—and is never greater than 100%.

Second, under monopoly (Bulow & Pfleiderer 1983) or monopolistic competition (Mrázová & Neary 2017) with constant marginal cost ($n = 1, \theta = 1, \eta^S = 0$), pass-through $\rho = 1/(2 - \xi^D)$ is determined solely by demand curvature $\xi^D$—with no distinct role for the price elasticity of demand $\varepsilon^D$.

Third, under Cournot-Nash competition (Kimmel 1992; Atkin & Donaldson 2015) with constant marginal cost ($\theta = 1, \eta^S = 0$), pass-through $\rho = 1/[1 + \theta^S(1 - \xi^D)]$ is additionally determined by market structure—as then given by $\theta^S \equiv (1/n)$.

Lemma 2 shows that, more generally, pass-through is determined by four factors: the price elasticity of demand $\varepsilon^D$, demand curvature $\xi^D$, the elasticity of marginal cost $\eta^S$, and the intensity of competition $\theta^S$. The role of the demand elasticity $\varepsilon^D$ is predicated on the presence of the cost elasticity, $\eta^S > 0$, which is often assumed away in prior literature.

It is easy to see that, all else equal, pass-through is always lower for a less flexible production technology, that is, $\partial \rho/\partial \eta^S < 0$. In this sense, a basic insight from perfect competition extends to settings with market power. In the limiting case, pass-through tends to zero, $\rho \to 0$.

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8For constant-elasticity demand, $\xi^D = 1 + 1/\varepsilon^D$, so the two parameters directly imply one another.

9To the best of my knowledge, the particular way of writing equilibrium cost pass-through in Lemma 2 is a new result. Weyl & Fabinger (2013) obtain the same underlying characterization of pass-through, instead written in terms of the elasticity of marginal consumer surplus.
as technology becomes entirely inflexible, \( \eta^S \to \infty \), for example, because firms face binding capacity constraints (as in Figure 1). In such a situation, the change in marginal cost induces no change in output—and hence also no price change.

As is well-known, it is possible for pass-through under imperfect competition to exceed 100%. Lemma 2 makes precise that this occurs whenever \( \theta^S(\xi^D - 1) \geq \eta^S(\varepsilon^D - \theta^S) \). Several things are needed: (i) there is market power \( \theta^S > 0 \); (ii) demand is log-convex \( \xi^D > 1 \) (equivalently, unit-superinelastic \( \psi^D < 1 \)); and (iii) the elasticity of marginal cost \( \eta^S \) cannot be too large (for example, if \( \eta^S \geq \max\{0, (\varepsilon^D - 1)^{-1}\} \equiv \eta^S \) then \( \rho \leq 1 \) for any \( \theta^S \in [0, 1] \) and \( \xi^D < 2 \)).

A sufficient condition for the “normal” case with pass-through below 100% to obtain, for any competitive conduct and cost conditions, is that the demand curve is log-concave, with \( \xi^D \leq 1 \). This is a common assumption across different fields of economic theory (Bagnoli & Bergstrom 2005) and is met by any demand curve that is concave, linear or not too convex.

4 Pass-through and market power

What is the equilibrium impact of more competition on cost pass-through? Answering this question requires some care because varying the intensity of competition via \( \theta^S \) can, in general, also affect the (equilibrium) values of the demand and cost parameters \( (\varepsilon^D, \xi^D, \eta^S) \) as none of these are necessarily constants.

Two approaches are presented. First, the “between markets” approach compares pass-through in two different markets on a like-for-like basis in the cross section, where one market is more competitive than the other but identical in terms of their values of \( (\varepsilon^D, \xi^D, \eta^S) \). This approach may be of particular interest to work in international trade and macroeconomics that spans a wide range of markets. Second, as in Figure 1, the “within market” approach compares pass-through in the same market following an exogenous increase in its intensity of competition, taking into account any knock-on effects on \( (\varepsilon^D, \xi^D, \eta^S) \). This approach may lend itself, for example, to work in industrial organization that tracks the competitiveness of individual markets.

Under both approaches, it will turn out that cost convexity makes the standard intuition—more competition raises pass-through—quite fragile.

4.1 Varying competition between markets

Consider two markets, 1 and 2, with different values of the intensity of competition, \( \theta_1^S \) and \( \theta_2^S \), where \( \theta_1^S < \theta_2^S \). Firm conduct is more competitive in market 1 because there are more firms or because rivalry is more intense for the same number of firms.

The markets may differ in terms of their demand and cost functions. Lemma 2 makes clear that the relevant demand and cost conditions for pass-through are given by \( (\varepsilon^D, \xi^D, \eta^S) \). The
idea here is that an econometric analysis will control for any differences between the markets in terms of their values of \((\varepsilon^D, \xi^D, \eta^S)\).

Direct comparison of the pass-through rates using Lemma 2 yields the following result:

**Proposition 1** Consider two markets 1 and 2 with the same demand conditions (as given by the demand elasticity \(\varepsilon^D\) and demand curvature \(\xi^D\)) and cost conditions (as given by cost elasticity \(\eta^S\)) where market 1 is more competitive than market 2 with \(\theta^S_1 < \theta^S_2\). Equilibrium cost pass-through is lower in the more competitive market, \(\rho(\theta^S_1) \leq \rho(\theta^S_2)\), if and only if demand and cost conditions satisfy:

\[ \eta^S + \xi^D \geq 1, \]

which always holds for a sufficiently large elasticity of marginal cost \(\eta^S\).

Proposition 1 yields the opposite of the standard intuition: whenever costs are sufficiently convex, pass-through is lower in the market with more intense competition. If demand is linear, with \(\xi^D = 0\), the condition boils down to whether the elasticity of marginal cost is at least unity, \(\eta^S \geq 1\). Roughly put, cost convexity dampens pass-through and a more competitive market is more exposed. Importantly, this finding obtains even in the “normal” case where pass-through always lies below 100%.

It is useful to consider a couple of examples in which pass-through is less than 100% and the condition of Proposition 1 is met. Suppose that demand is convex but log-concave, \(\xi^D \in [0, 1]\), and that costs are at least as convex as a quadratic cost function, \(C(x_i) \propto x_i^2\); in such cases, Proposition 1 always holds for pass-through \(\rho|_{\tau \to 0}\) of a small new tax, as then \(\eta^S \geq 1\). The required degree of cost convexity can be given a microfoundation based on a standard Cobb-Douglas technology. Let \(x_i = Ak_i^\alpha l_i^\beta\) be firm \(i\)’s production technology for output, where \(k_i\) is factor of production, say capital, that is fixed (e.g., in the short run) while \(l_i\) is a flexible factor, say labour, and \(A, \alpha, \beta > 0\) are parameters. Taking factor prices as given, firm \(i\)’s optimal cost function \(C_i(x_i; k_i)\) is at least as convex as a quadratic whenever \(\beta \leq 1/2\). A larger endowment of the fixed factor \(k_i\) reduces marginal cost but leaves the cost elasticity \(\eta^S = (1 - \beta)/\beta\) unchanged. These kinds of quadratic cost functions (with \(\alpha = \beta = 1/2\)) are frequently used in the literature on merger analysis (McAfee & Williams 1992). Of course, they also underlie textbook expositions of perfect competition with linear demand and a linearly upward-sloping supply curve.

To see another example, consider the marginal-cost function with a “soft” capacity constraint given by \(C''(x_i) = c + \max\{0, \lambda(x_i - K)\}\) where \(\lambda > 0\) is a parameter and \(K\) is firm \(i\)’s installed capacity. While it is possible for production to exceed capacity, \(x_i > K\), this becomes increasingly costly as a strain on resources. If so, for a small new tax \(\tau \to 0\), cost convexity is \(\eta^S = \lambda(x_i/K)/(c + \lambda(x_i/K - 1))\) and so \(\eta^S \geq 1\) holds whenever the cost-convexity effect dominates with \(\lambda \geq c\). More generally, the condition from Proposition 1 always holds for a sufficiently large (but finite) \(\eta^S\), regardless of demand conditions and competitive intensity.
By contrast, existing literature on pass-through under imperfect competition typically assumes that marginal costs are constant, \( \eta^S = 0 \). Then the standard intuition obtains whenever demand is log-concave, \( \xi^D \leq 1 \). Conversely, if the condition from Proposition 1 holds with \( \xi^D \geq 1 \), both markets feature pass-through in excess of 100%—but it is closer to 100% in the more competitive market, \( \rho(\theta^2_S) \geq \rho(\theta^1_S) \geq 1 \). A familiar example occurs with Cournot competition and a highly convex demand curve with constant elasticity (\( \xi^D = 1 + 1/\varepsilon^D \)) for which the equilibrium price-cost margin \( (p - \hat{C})/p = \theta^S/\varepsilon^D \) is constant—so that cost pass-through \( \rho = \varepsilon^D/(\varepsilon^D - \theta^S) \) always exceeds 100%. This paper instead focuses on the “normal” case of pass-through that lies below 100%. The main point is that, with non-constant marginal cost, \( \eta^S > 0 \), more competition can yield lower pass-through even when it always lies below 100%.

To get a sense of numbers on the demand side, the range \( \xi^D \in [0,1] \) is satisfied by three of the four demand specifications in the influential study of oligopolistic competition by Genesove & Mullin (1998): linear (\( \xi^D = 0 \)), quadratic (\( \xi^D = \frac{1}{2} \)) and exponential (\( \xi^D = 1 \)) demand.\(^{10}\) In the macroeconomics literature, Gopinath & Itskhoki (2011) calibrate a model of monopolistic competition and, as baseline parameters, use a demand elasticity \( \varepsilon^D = 5 \) and with a superelasticity \( \psi^D = 6 \); taken together, these imply that demand curvature \( \xi^D = 0 \), that is, demand is (locally) exactly linear. On the empirical side, Beck & Lein (2019) estimate a discrete choice model based on a homescanner dataset of a large number of consumer goods. They find that the demand elasticity \( \varepsilon^D \) ranges between 3 to 5 while the superelasticity \( \psi^D \) ranges between 1 to 2. This, in turn, converts into a surprisingly tight range of demand curvatures: \( \xi^D \in \left[\frac{2}{3}, 1\right] \).

Taken together, and acknowledging the differences in methodologies, this combination of theoretical and empirical considerations suggests that demand being convex but log-concave, \( \xi^D \in [0,1] \) will, in many cases, be a plausible baseline assumption on demand curvature. This has three implications for whether market power can increase pass-through: (i) cost convexity is a (weakly) necessary condition, (ii) a modest degree of cost convexity can be sufficient, and (iii) a (grossly) sufficient condition is that cost convexity satisfies \( \eta^S \geq 1 \)—which is met by cost curves that are at least quadratic.

What is driving the result from Proposition 1? Recall from Lemma 2 that a less flexible production technology always means lower pass-through, \( \partial \rho/\partial \eta^S < 0 \). A key observation is that this effect is mitigated by market power in the following sense:

**Lemma 3.** Equilibrium cost pass-through satisfies

\[ \frac{\partial}{\partial \theta^S} \left[ \frac{\partial}{\partial \eta^S} \rho(\varepsilon^D, \xi^D, \eta^S, \theta^S) \right] \geq 0 \]

if and only if the elasticity of marginal cost satisfies \( \eta^S \leq [1 + (1 - \xi^D)(2\varepsilon^D - \theta^S)]/(\varepsilon^D - \theta^S) \),

\(^{10}\)Their fourth specification is constant-elasticity demand for which the condition from Proposition 1 is always met but pass-through exceeds 100%.
for which $\eta^S \leq 1 - \xi^D$ is a sufficient condition.

Lemma 3 shows that, for modest values of $\eta^S$, the pass-through function is supermodular in the cost elasticity and market power. A less flexible production technology means lower pass-through—and then more strongly so for a more competitive market. This helps explain why, in markets with a fairly inflexible production technology, more competition can be associated with less pass-through.\textsuperscript{11}

Through the lens of incidence analysis, these results highlight the role of the producer surplus generated by a competitive market. In the “normal” case with pass-through below 100%, cost convexity is necessary for Proposition 1 to apply; this, in turn, means that the producer surplus associated with a competitive market is non-zero. By contrast, recent literature that employs pass-through as a tool for incidence analysis often makes assumptions—specifically constant marginal cost and firm symmetry—that imply that a competitive industry always makes zero profits.

4.2 Varying competition within a market

Now consider the second approach: in the same market, with given demand and cost functions, competition (exogenously) intensifies as measured by a lower value of $\theta^S$. How does more competition affect pass-through $\rho(\theta^S)$?

Write the equilibrium price in terms of the conduct parameter $p(\theta^S)$, and think of the (equilibrium) values of the demand and cost metrics as $(\varepsilon^D(p(\theta^S)), \xi^D(p(\theta^S), \eta^S(p(\theta^S)))$. Also let $\phi^S_i \equiv x_iC''(x_i)/C''(x_i)$ be the elasticity of the slope of $i$’s marginal cost which, given symmetry, will again be identical across firms with $\phi^S_i = \phi^S$ (also recalling that $\hat{C}''(\cdot) = C''(\cdot)$ and $\hat{C}'''(\cdot) = C'''(\cdot)$).

The next result provides necessary and sufficient conditions under which—contrary to standard intuition—a small increase in competition, $d\theta^S < 0$, leads to weaker pass-through in the same market.

**Proposition 2.** (a) Equilibrium cost pass-through is lower with more competition, that is, $d\rho(\theta^S)/d\theta^S \geq 0$, if and only if demand and cost conditions and firm conduct satisfy:

\[
\frac{(\varepsilon^D - \theta^S)\eta^S}{[1 + \theta^S(1 - \xi^D) + (\varepsilon^D - \theta^S)\eta^S]}(\phi^S + \xi^D) \geq \frac{d}{d\theta^S} [\theta^S(1 - \xi^D)];
\]

which always holds if the elasticities of marginal cost $\eta^S$ and of the slope of marginal cost $\phi^S$ are sufficiently large;

\textsuperscript{11}Note that the supermodularity property cannot hold globally because pass-through $\rho \to 0$ as inflexibility $\eta^S \to \infty$ regardless of intensity of competition.
(b) Equilibrium cost pass-through lies below 100%, \(\rho(\theta^S) \leq 1\), and is lower with more competition \(d\rho(\theta^S)/d\theta^S \geq 0\) if:

- **Demand** is log-concave \(\xi^D \leq 1\) and its curvature is non-decreasing \(d\xi^D(p)/dp \geq 0\);

- **Costs** are sufficiently convex with \(\eta^S \geq \hat{\eta}^S\) and satisfy \(d\eta^S(p)/dp \leq 0\) where \(\hat{\eta}^S \geq 0\) solves \([\eta^S - 2(1 - \xi^D)](\varepsilon^D - \theta^S)\eta^S = (1 - \xi^D)(1 + \theta^S(1 - \xi^D))\) (for which \(\eta^S \geq 2(1 - \xi^D) \geq 0\) is necessary).

Proposition 2 delivers a similar conclusion to Proposition 1: Under plausible conditions on the degree of cost convexity, it is possible for more competition to reduce “within market” pass-through—and the standard intuition is overturned.

Part (a) of the result provides a condition that is necessary and sufficient. To get a sense for the required degree of cost convexity, consider the case in which demand is linear so that the condition simplifies to \([d\rho(\theta^S)/d\theta^S]_{\theta^S=0} \geq 0 \iff \eta^S(\phi^S - 1) \geq (1 + \theta^S)/(\varepsilon^D - \theta^S)\). If five firms initially play Cournot-Nash and the initial price elasticity of demand is two \((n = 5, \theta = 1, \varepsilon^D = 2)\), then greater competitive intensity reduces pass-through as long as production costs satisfy \(\eta^S(\phi^S - 1) \geq 2/3\). More generally, the condition in part (a) is more likely to be met if demand is more elastic (higher \(\varepsilon^D\)) and more convex (higher \(\xi^D\)) and marginal cost is more convex (higher \(\eta^S\)) and its slope is more convex (higher \(\phi^S\)).

Part (b) gives a simple set of sufficient conditions that apply to the “normal” case where pass-through is less than 100%. First, demand is log-concave \(\xi^D \leq 1\) and is more convex at a higher price \(d\xi^D(p)/dp \geq 0\), which applies, for example, to any demand curve of the family \(p(X) = \alpha - \beta X^\gamma\), with constant curvature \(\xi^D = 1 - \gamma\) (with parameters \(\alpha > 0, \beta > 0, \gamma \geq 0\)). This demand family corresponds to consumer valuations being drawn from a generalized Pareto distribution (Bulow & Klemperer 2012); it also corresponds to the direct demand function \(D(p)\) being “\(\rho\)-linear” (Anderson & Renault 2003). Second, firms’ costs are sufficiently convex, that is, \(\eta^S > 0 \iff C''(\cdot) > 0\) is sufficiently large and convexity is less pronounced at higher prices \(d\eta^S(p)/dp \leq 0\)—that is, it is more pronounced at higher output, \(d\eta^S(x)/dx \geq 0\). The latter condition is consistent with the notion that production inflexibility becomes more acute at higher output levels.

Proposition 1’s “between markets” condition \(\eta^S + \xi^D \geq 1\) is, in general, neither necessary nor sufficient for the “within market” result of Proposition 2 to hold. The reason is that Proposition 2 additionally relies on third-order properties of demand and cost functions, specifically the change in demand curvature \(d\xi^D(p)/dp\) and the elasticity of the slope of marginal cost \(\phi^S\). For example, if \(\phi^S\) is sufficiently large, then the condition from Proposition 2(a) can be met even if that of Proposition 1 is violated. Conversely, even if the condition from Proposition 1 holds, that of Proposition 2(a) can be violated if \(d\xi^D(p)/dp\) is strongly negative.
For a discrete increase in market power from, say, $\theta_1^S$ to $\theta_2^S$ (where $\theta_2^S > \theta_1^S$), if either of the conditions from Proposition 2(a) or 2(b) holds over the range $\theta^S \in [\theta_1^S, \theta_2^S]$, then this is sufficient (but not necessary) to conclude that $\rho(\theta_2^S) = \rho(\theta_1^S) + \int_{\theta_1^S}^{\theta_2^S} \rho'(\theta^S) d\theta^S > \rho(\theta_1^S)$.

To illustrate such a discrete change in competitive intensity, consider a market with a single firm and linear demand curve with slope $p' = -\beta$ ($n = 1$, $\xi^D = 0$). Suppose that the firm is initially a price-taker ($\theta^S = 0$) and then becomes a monopolist ($\theta^S = 1$). Let $x^c \equiv x(0)$ denote the competitive output and $x^m \equiv x(1)$ the monopoly output, where $x^m < x^c$. Cost pass-through under monopoly $\rho^m$ is higher than with perfect competition $\rho^c$ whenever:

$$\rho^m = \frac{1}{2 + \frac{C''(x^m)}{\beta}} \geq \frac{1}{1 + \frac{C''(x^c)}{\beta}} = \rho^c \quad (4)$$

which holds if and only if $\int_{x^c}^{x^m} C''(y) dy = [C''(x^c) - C''(x^m)] \geq \beta$.\(^{12}\) So competition reduces pass-through if $C''(\cdot) > 0$, and $C''(\cdot)$ is large enough. The result of Figure 1, with $\rho^m = \frac{1}{2} > \rho^c = 0$, is nested where $C''(x^m) = 0 \iff \eta^S = 0$ (as marginal cost is constant at $c$ around the monopoly output) and $C''(x^c) \to \infty \iff \eta^S \to \infty$ (as the competitive industry produces at capacity $K$).\(^{13}\)

This basic insight also speaks to recent policy concerns about rising market power—and its welfare implications. The standard intuition, based on constant marginal costs, suggests an appealing test: any empirical evidence of declining rates of cost pass-through would indeed be indicative of weaker competition. However, in the setting of Figure 1, with a capacity constraint, it is clear that perfect competition produces higher social welfare than monopoly, $W^c < W^m$, even though it features lower pass-through $\rho^c < \rho^m$. This shows that the relationship between pass-through and social welfare is complex and that any inference on the extent of market power needs to take in account the details of firms’ cost conditions and capacity constraints.

4.3 Example: “Between markets” vs “within market”

A simple example illustrates how (i) the “between markets” and “within market” approaches can yield the same conclusion, (ii) the degree of cost convexity needed to overturn the standard intuition may be modest in both cases.

Market demands are exponential $D(p) = \exp \left( \frac{\alpha -\beta p}{\beta} \right) \Leftrightarrow p(X) = \alpha - \beta \ln X$, where $\alpha, \beta > 0$ are parameters, and so demand curvature $\xi^D = 1$. Firms’ cost functions are given by $C(x_i) = \delta x_i^\mu$, with parameters $\delta > 0$ and $\mu > 1$. Assuming the the cost shifter is initially zero, $\tau = 0$,

\(^{12}\) The relationship $\eta^S(\varepsilon^D - \theta^S) = C''(\cdot)/[-p'(\cdot)]$ from the proof of Lemma 2 here presentationally simplifies the comparison.

\(^{13}\) Strictly speaking, this involves a non-differentiability of the cost function around the capacity constraint $K$. However, Figure 1 can be closely approximated using a marginal cost function $C'(x) = c - \omega \ln(1 - x/K)$, where $\omega > 0$ is a (small) parameter and $x/K \in [0,1]$ is the rate of capacity utilization, and for which $\eta^S \to \infty$ as $x/K \to 1$. 
this pins down the marginal-cost elasticity as $\eta^S = \mu - 1 > 0$ and the elasticity of the slope of marginal cost as $\phi^S = \mu - 2 \geq 0$. Using Lemma 2, equilibrium cost pass-through $\rho|_{\tau=0} = [1 + (\varepsilon^D - \theta^S)(\mu - 1)]^{-1} < 1$ is always incomplete.\footnote{With exponential demand $\xi^D = 1$ and constant marginal cost $\eta^S = 0$, cost pass-through $\rho(\theta^S) = 1$ is invariant to market power $\theta^S$.}

Between markets (Proposition 1): Conditional on the equilibrium values of the demand elasticity ($\varepsilon^D$) and the cost elasticity (via $\mu$), it is immediate from the expression for $\rho|_{\tau=0}$ that the more competitive market 1 always has strictly lower pass-through in the cross section, i.e., $\rho(\theta^S_1) < \rho(\theta^S_2)$ for $\theta^S_1 < \theta^S_2$ and for any value of the cost-elasticity parameter $\mu$. This is the opposite of the standard intuition.

Within market (Proposition 2): By inspection of Lemma 2, under these assumptions, pass-through declines with more competition, $d\rho(\theta^S)/d\theta^S > 0$, whenever $\frac{d}{d\theta^S}[\theta^S - \varepsilon^D(p(\theta^S))] > 0$. It is easy to check that $d\varepsilon^D(p(\theta^S))/d\theta^S = \rho < 1$ (as the superelasticity $\psi^D = 1$ whenever demand curvature $\xi^D = 1$, see the proof of Lemma 1), so the condition of Proposition 2(a) also always holds for any value of the cost-elasticity parameter $\mu$. This is again the opposite of the standard intuition.

So the standard intuition here fails even for an arbitrarily small degree of cost convexity $\eta^S$, and can fail even if the elasticity of marginal cost $\phi^S$ is zero or mildly negative.

5 Empirical implications

This section discusses the empirical implications and testability of the theory. An emerging literature at the intersection of industrial organization and public economics has begun to explore the empirical relationship between pass-through and market power (Miller, Osborne & Sheu 2017; Stolper 2018; Genakos & Pagliero 2019). These papers consider a single industry with multiple regional markets and—akin to Proposition 1—focus on cross-sectional differences in competition. However, while this literature also highlights the importance for pass-through of finer details on market conditions, it has so far engaged only little with the role of cost convexity and capacity constraints.

Genakos & Pagliero (2019) study the relationship between competition and pass-through using 2010 daily retail prices for gasoline in isolated markets on the Greek islands. They argue that the firm-level marginal cost of gasoline stations is approximately constant in their short-run setting, i.e., $\eta^S \approx 0$. In line with standard intuition, they find pass-through is just below 50% in monopoly markets and quickly rises towards 100% in markets with at least four firms. This is also remarkably consistent with a textbook Cournot model with linear demand and constant marginal cost, for which $\rho = 1/(1 + n^{-1})$ (Lemma 2 with $\theta = 1$, $\xi^D = 0$, $\eta^S = 0$). Put differently, taking together (i) the initial argument that costs satisfy $\eta^S \approx 0$, (ii) the cross-sectional estimates of cost pass-through, and (iii) a structural model of pass-through
(that delivers Lemma 2), suggests that demand curves for gasoline on the Greek islands are approximately linear, $\xi^D \approx 0$.

Miller, Osborne & Sheu (2017) estimate pass-through rates around 130–180% using 30 years of annual data on the US Portland cement industry over the period 1980–2010. Their discussion of the institutional context also suggests that $\eta^S \approx 0$ is likely. By Lemma 2, pass-through above 100% means that demand must be log-convex ($\xi^D > 1$); by Proposition 1, cross-sectional pass-through is then unambiguously lower with greater competition. Using several measures of rivalry, Miller et al. (2017) find evidence across different regional markets that is consistent with this theoretical prediction. This is the reverse of the standard intuition—albeit driven by demand conditions rather than cost conditions.

Stolper (2018) estimates pass-through using 2007 daily firm- and market-level price data for 10,000 gasoline retail stations in Spain, with a primary interest in the distributional implications of fuel cost shocks. He finds an average cost pass-through rate of around 90%, with the large majority of station-specific rates between 70–115%. Moreover, greater market power—as proxied by a lower spatial density of competition and greater product branding—is strongly associated with higher pass-through. This finding appears to be potentially consistent with the condition of Proposition 1. However, as his analysis also assumes constant marginal costs, $\eta^S \approx 0$, the economic mechanism by which competition reduces pass-through may be more subtle. In principle, the theory with cost constraints can explain a “regime switch” : if demand is log-convex ($\xi^D > 1$), then changes in the value of $\eta^S$—across markets and/or over time—can generate pass-through that lies above or below 100%.

Looking ahead, it would be valuable for future research to use longer periods of high-frequency data to study the role of varying cost constraints—and test this paper’s Propositions 1 and 2. It seems clear that, in practice, many industries do at times experience “soft” capacity constraints and that industrial sectors can face “hard” constraints in the short-term. In terms of Proposition 1’s between-markets approach, as motivated by the Cobb-Douglas example discussed earlier, this suggests the capital-labour ratio as a proxy for soft capacity constraints. This could be used to control for cross-industry variation in $\eta^S$ and thus help isolate the impact of market power on pass-through. In terms of Proposition 2’s within-market approach, an em-

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15Ganapati, Shapiro & Walker (2019) use estimate fuel cost pass-through in a panel of six homogenous-product US industries (boxes, bread, cement, concrete, gasoline, and plywood) and find large inter-industry heterogeneity. In their sample, cement is the most concentrated industry and has the highest pass-through rate (181%) but they do not attempt like-for-like comparisons between industries so their results do not speak directly to Proposition 1.

16Proposition 1 makes precise that an ideal empirical test would have to control for differences in demand and cost conditions, specifically in $(\varepsilon^D, \xi^D, \eta^S)$. A typical empirical pass-through study includes demand controls that may plausibly account for variation in the demand elasticity $\varepsilon^D$. It is more challenging, however, to control for demand curvature $\xi^D$—which reflects the degree of heterogeneity in consumer valuations. This leads to possibility that empirical estimates could confound the impact of competition (differences in $\theta^S$) on pass-through with those due to differences in higher-order demand conditions (differences in $\xi^D$). Further complications would be introduced by the presence of horizontal product differentiation.
pirical test could build on the work by Marion & Muehlegger (2011) who estimate pass-through in regional US gasoline markets and use metrics such as national refinery capacity utilization as proxies for supply-side constraints. In line with Lemma 2, they find that cost-pass-through is markedly lower during times when the industry is close to its (hard) capacity constraint—but do not test for the interaction with competition that underlies Proposition 2. A strong empirical research design could combine (i) exogenous variation that shifts market-wide marginal costs (leading to changes $\tau$) with (ii) shocks to operational industry capacity such as capacity shutdowns due to regulatory interventions or safety events (leading to changes in $\eta^S$).

6 Conclusion

Theoretical and empirical literature based on imperfect competition routinely assumes that firms have constant marginal costs. As a result, studies of pass-through and recent applications across fields including industrial organization, environmental economics and international trade, have focused on demand-side properties. More competition then raises pass-through as long as it lies below 100%. This suggests that any evidence of declining rates of cost pass-through would be supportive of concerns about rising market power.

This paper has shown that this logic is perhaps surprisingly fragile. If firms have even modestly convex costs, then more competition may reduce pass-through. A rough intuition is that a more competitive industry has higher output, and is therefore more exposed to this cost-convexity effect. An immediate corollary is that the rate of pass-through of a demand shock, that is, a uniform upward shift in consumers’ willingness-to-pay, may—again perhaps counterintuitively—be more pronounced in a more competitive market.17

The interplay between pass-through and market power plays an important role across several policy areas. The pass-through of fuel costs to retail electricity prices and gasoline prices regularly attracts the attention of competition policymakers (Federal Trade Commission 2011; Competition and Markets Authority 2015). A related antitrust issue is the “passing-on defense” by which the damages from an upstream cartel may be limited by downstream firms passing the overcharge onto their own customers (Verboven & Van Dijk 2009). Another application is the design and effectiveness of market-based regulation towards climate change for which the pass-through of a carbon price imposed on emissions-intensive industries (such as cement, electricity and steel) has central importance (Fabra & Reguant 2014; Miller, Osborne & Sheu 2017). The results in this paper suggest that the role of cost constraints may deserve more attention in economic analysis related to these policy areas.

In short, prices will be more reflective of marginal cost in a more competitive market but it does not follow that price changes will necessarily be more reflective of cost changes.

17 This follows from the relationship: rate of cost pass-through + rate of demand pass-through = 1.
References


Appendix

Proof of Lemma 1. The expression for $L \equiv \varepsilon^D[p(X) - \hat{C}'(x)]/p(X)$ follows by rearranging (3) and using the definitions of $\varepsilon^D$ and $\theta^S$. Differentiating (3) shows that:

$$\frac{dp(\theta^S)}{d\theta^S} = p'(X)n\frac{dx}{d\theta^S} = p'(X)n - \frac{p'(X)X}{[n + \theta + \theta \varepsilon^D - C''(x)/p'(X)]}$$

where the denominator of this expression is positive because $(n + \theta) > \theta \varepsilon^D$ given that $n \geq 1$, $\theta \in [0, 1]$ and $\varepsilon^D < 2$ as well as $C''(x) \geq 0$ and $p'(X) < 0$.

Proof of Lemma 2. By construction, the rate of pass-through satisfies $\rho \equiv \frac{dp}{dx} = p'(X)n\frac{dx}{d\theta^S}$. Hence differentiating (3) yields:

$$\rho = \frac{p'(X)n}{[p'(X)n + \theta \varepsilon^D - C''(x)/p'(X)]} > 0, \quad \text{using the definition } \xi^D \equiv -Xp''(x)/p'(X) \text{ and where the denominator is again positive. Now rewrite the last term as follows:}$$

$$\frac{C''(x)}{-p'(X)} = \frac{x\hat{C}''(x)\hat{C}''(x)}{\hat{C}''(x)} \frac{p(X)}{p(x)} = \frac{x \varepsilon^D - \theta/n}{\varepsilon^D} \eta^S = \eta^S(\varepsilon^D - \theta^S)n,$$

which uses Lemma 1 and the definitions $\varepsilon^D \equiv -p(X)/p'(X)$, $\eta^S \equiv x\hat{C}''(x)/\hat{C}''(x)$ (at symmetric equilibrium, where $\hat{C}''(x_i) = C''(x_i)$), and $\theta^S \equiv (\theta/n)$. Combining (6) and (7) and some rearranging yields the expression for $\rho(\varepsilon^D, \xi^D, \eta^S, \theta^S)$, as claimed.

Proof of Proposition 1. Using the expression from Lemma 2, equilibrium cost pass-through in the two markets equals

$$\rho(\theta^S_j) = \frac{1}{1 + (\varepsilon^D - \theta^S_j)\eta^S + \theta^S_j(1 - \varepsilon^D)}$$

where $\theta^S_j$ is the competitive intensity in market $j = 1, 2$ and $(\varepsilon^D, \xi^D, \eta^S)$ are, by assumption, identical in both markets. Hence $\rho(\theta^S_1) \leq \rho(\theta^S_2)$ holds if and only if $(\theta^S_2 - \theta^S_1)(\eta^S - 1 + \xi^D) \geq 0$ which boils down to $\eta^S + \xi^D \geq 1$, as claimed, since $\theta^S_2 > 1$ by assumption.

Proof of Lemma 3. Differentiating the expression for equilibrium cost pass-through from Lemma 2 gives:

$$\frac{\partial}{\partial \eta^S} \rho(\varepsilon^D, \xi^D, \eta^S, \theta^S) = -\frac{(\varepsilon^D - \theta^S)}{[1 + (\varepsilon^D - \theta^S)\eta^S + \theta^S(1 - \varepsilon^D)]^2} < 0.$$
Differentiating again for the cross-partial effect gives:
\[
\frac{\partial}{\partial S} \left[ \frac{\partial}{\partial \eta^S} \partial \rho(\varepsilon^D, \xi^D, \eta^S, \theta^S) \right] = \frac{1 + (\varepsilon^D - \theta^S)\eta^S + \theta^S(1 - \varepsilon^D)}{1 + (\varepsilon^D - \theta^S)\eta^S + \theta^S(1 - \varepsilon^D)}.
\]

It is immediate that \(\frac{\partial}{\partial \eta^S} \left( \frac{\partial}{\partial \theta^S} \rho \right)\) > 0 if \(\eta^S \leq 1 - \varepsilon^D\) and some further rearranging shows that \(\frac{\partial}{\partial \eta^S} \left( \frac{\partial}{\partial \theta^S} \rho \right)\) > 0 if and only if \([1 + (1 - \varepsilon^D)(2\varepsilon^D - \theta^S)]/[\varepsilon^D - \theta^S]\), as claimed.

**Proof of Proposition 2.** For the necessary and sufficient condition in part (a), differentiation of the expression from Lemma 2 shows that a small increase in market power intensifies pass-through if and only if:
\[
\frac{d\rho}{d\theta^S} \geq 0 \iff \frac{d}{d\theta^S} \left[ (\varepsilon^D - \theta^S)\eta^S + \theta^S(1 - \varepsilon^D) \right] \leq 0
\]
\[
\iff \left( \frac{d\varepsilon^D}{d\theta^S} - 1 \right) \eta^S + (\varepsilon^D - \theta^S) \frac{d\eta^S}{d\theta^S} \leq - \frac{d}{d\theta^S} \left[ \theta^S(1 - \varepsilon^D) \right]. \quad (8)
\]

The initial two steps are to derive formulae for the terms \(d\varepsilon^D/d\theta^S\) and \(d\eta^S/d\theta^S\). For the first step, begin by expanding the term as follows:
\[
\frac{d\varepsilon^D(p(\theta^S))}{d\theta^S} = \frac{d\varepsilon^D}{dX} \frac{1}{p'(X)} \frac{dp}{d\theta^S} = -\rho \frac{d\varepsilon^D}{dX},
\]
which combines the expression for \(dp/d\theta^S\) from (5) in the proof of Lemma 1 with the expression for pass-through from Lemma 2. Performing the differentiation \(d\varepsilon^D/dX\) and simplifying then gives:
\[
\frac{d\varepsilon^D}{d\theta^S} = -\rho \left[ p'(X) \frac{1}{p'(X)} \frac{dp}{d\theta^S} - \frac{p'(X) - p''(X)p(X)}{[p'(X)]^2} \right]
\]
\[
= -\rho \left[ \frac{p'(X) - (1 - \varepsilon^D)p(X)}{[p'(X)]^2} \right]
\]
\[
= \rho \left[ 1 + (1 - \varepsilon^D)p(X) \right] = \rho \varepsilon^D, \quad (9)
\]
where the second line uses the definition \(\xi^D \equiv -\varepsilon^Dp''(X)/p'(X)\) and the third line uses the relationship with the superelasticity as given by \(\xi^D = 1 + (1 - \psi^D)/\varepsilon^D\). For the second step, similarly, begin by expanding the term as follows:
\[
\frac{d\eta^S(p(\theta^S))}{d\theta^S} = \frac{d\eta^S}{dp} \frac{dp}{d\theta^S} = \frac{d}{dp} \left[ \eta^S(x(p)) \right]_{p=p(\theta^S)} \frac{dp}{d\theta^S}
\]
\[
= \frac{d\eta^S(x)}{dx} \frac{dx}{dp} \frac{dp}{d\theta^S} = - \left( \frac{d\eta^S(x)}{dx} \frac{x}{\eta^S} \right) \varepsilon^D \frac{dp}{d\theta^S} \frac{\eta^S}{p}
\]
\[
= - \left( \frac{d\eta^S(x)}{dx} \frac{x}{\eta^S} \right) \varepsilon^D \left[ p'(X) \frac{1}{p'(X)} \frac{dp}{d\theta^S} - \frac{p'(X) - p''(X)p(X)}{[p'(X)]^2} \right] \rho \eta^S, \quad (10)
\]
where the third line uses the same expression for \( dp/d\theta^S \) as the first step. Performing the differentiation of the elasticity of marginal cost \( \eta^S(x) = x\hat{C}''(x)/\hat{C}'(x) \) gives:

\[
\frac{d\eta^S(x)}{dx} = \frac{\hat{C}''(x) + x\hat{C}'''(x)}{\hat{C}'(x)} - \frac{x\hat{C}''(x)\hat{C}''(x)}{\hat{C}'(x)},
\]

where the second line uses that \( \hat{C}''(\cdot) = C''(\cdot) \) and \( \hat{C}'''(\cdot) = C'''(\cdot) \) and the third line uses the definition \( \phi^S \equiv xC'''(x)/C''(x). \) Using (11) in (10) yields:

\[
\frac{d\eta^S(\theta^S)}{d\theta^S} = -\left(1 + \phi^S - \eta^S\right) \rho \eta^S.
\]

Now using the results in (9) and (12) from these two steps in the condition for \( dp/d\theta^S \geq 0 \) from (8) gives:

\[
\frac{dp}{d\theta^S} \geq 0 \iff -\eta^S \left[ 1 - \left[ 1 + (1 - \xi^D)\varepsilon^D - (\varepsilon^D - \theta^S)(1 + \phi^S - \eta^S) \right] \rho \right] \leq -\frac{d}{d\theta^S} [\theta^S(1 - \xi^D)]
\]

\[
\iff -\eta^S \left[ 1 - \frac{\left[ 1 + (1 - \xi^D)\varepsilon^D - (\varepsilon^D - \theta^S)(1 + \phi^S - \eta^S) \right]}{[1 + (\varepsilon^D - \theta^S)\eta^S + \theta^S(1 - \xi^D)]} \right] \leq -\frac{d}{d\theta^S} [\theta^S(1 - \xi^D)]
\]

\[
\iff -\eta^S \frac{(\varepsilon^D - \theta^S)(\phi^S + \xi^D)}{[1 + (\varepsilon^D - \theta^S)\eta^S + \theta^S(1 - \xi^D)]} \leq -\frac{d}{d\theta^S} [\theta^S(1 - \xi^D)],
\]

as claimed, where the third line uses the pass-through expression from Lemma 2. For the sufficient condition in part (b), under the assumptions that, for demand, \( \xi^D \leq 1 \) and \( d\xi^D/dp \geq 0 \) (so \( d\xi^D/d\theta^S \geq 0 \)), and for costs, \( d\eta^S/d\theta^S \geq 0 \) (so \( d\eta^S/d\theta^S \leq 0 \)), it follows from (8) that:

\[-\left(\frac{d\xi^D}{d\theta^S} - 1\right) \eta^S \geq (1 - \xi^D) \Rightarrow \frac{dp}{d\theta^S} \geq 0.
\]

Using the result for \( d\xi^D/d\theta^S \) from (9) combined with Lemma 2 then yields:

\[-\left(\frac{d\xi^D}{d\theta^S} - 1\right) \eta^S \geq (1 - \xi^D) \iff -\rho \left[ 1 + (1 - \xi^D)\varepsilon^D \right] \eta^S \geq (1 - \xi^D) \iff \frac{(\varepsilon^D - \theta^S)\eta^S}{[1 + (\varepsilon^D - \theta^S)\eta^S + \theta^S(1 - \xi^D)]} (\eta^S + \xi^D - 1) \geq (1 - \xi^D).
\]

Some further rearranging yields \( [\eta^S - 2(1 - \xi^D)](\varepsilon^D - \theta^S)\eta^S \geq (1 - \xi^D)[1 + \theta^S(1 - \xi^D)] \) as a sufficient condition for \( dp/d\theta^S \geq 0 \), as claimed.