Monetary Policy with a Nonlinear Phillips Curve and Asymmetric Loss

Demosthenes N. Tambakis
City University Business School
d.tambakis@city.ac.uk

Abstract. Recent theoretical and empirical work has cast doubt on the hypotheses of a linear Phillips curve and a symmetric quadratic loss function underlying traditional thinking on monetary policy. This paper studies the one-period optimal monetary policy problem under an asymmetric loss function corresponding to the “opportunistic approach” to disinflation and a convex Phillips curve. The policy-inaction range and its properties are derived analytically. Numerical simulations are then used to assess the implications of asymmetric loss for the distributional properties of the equilibrium levels of inflation and unemployment. For parameter values relevant to the U.S., it is found that the asymmetric loss function yields an average inflation rate in excess of the target, and that bias is larger than the standard symmetric loss function. For moderate policy-maker preferences, the asymmetric loss function also yields a smaller gap between average unemployment and the natural rate, and higher (lower) variance of inflation (unemployment) compared to the symmetric benchmark. Calibrating the model to match the observed average unemployment rate requires a high degree of inflation aversion and small asymmetry.

Keywords. monetary policy, nonlinear Phillips curve, asymmetric loss function

Acknowledgments. This research began while the author was at the Research Department of the International Monetary Fund, Washington, D.C. The author would like to thank Hamid Faruqee, Peter Isard, Thomas Krueger, Douglas Laxton, David Rose, and an anonymous referee for their helpful comments and discussions. The usual disclaimer applies.

1 Introduction

Recent empirical contributions to the monetary policy literature have argued that the short-run Phillips curve in the U.S. is moderately convex, such that at any given point on the curve, the inflation increase associated with an incremental decline in the unemployment rate exceeds the inflation decline associated with an equal rise in the unemployment rate.1 This paper analyzes Barro and Gordon’s (1983) optimal monetary policy problem under an asymmetric loss-function corresponding to the “opportunistic approach” to disinflation when the Phillips curve is convex. Extending the loss-function specification seems natural, as the advantage of

---

symmetric quadratic loss functions is that they imply linear decision rules when optimized subject to linear constraints. Indeed, this is one reason why researchers have opted for linear Phillips curves.

The key implication of a nonlinear (convex) Phillips curve is that the short-run trade-off between inflation and unemployment is path dependent: a 1% unemployment decline leads to a smaller increase in inflation at high rates of unemployment than at low rates. As a result, the unemployment rate consistent with maintaining a stable average inflation rate over time is not the same in a stochastic setting as it is in a deterministic setting. In a stochastic economy with a nonlinear Phillips curve, the nonaccelerating inflation rate of unemployment is greater than its deterministic counterpart for any distribution of inflation shocks. In contrast, under the linear model, the nonaccelerating inflation rates of unemployment with and without shocks coincide. In this paper, this distinction is emphasized by referring to the nonaccelerating inflation rate of unemployment in a stochastic economy as the NAIRU, while reserving the term “deterministic NAIRU,” or DNAIRU, for the nonaccelerating inflation rate of unemployment in a deterministic economy.2

Given a nonlinear Phillips curve, the time-series properties of inflation and unemployment are sensitive to the loss function used by the policy maker. Recently, several anecdotal and theoretical arguments have been made in support of the “opportunistic approach” to disinflation. This approach is founded upon the perception that, in contrast to the traditional quadratic loss function, social welfare is decreasing in the level of the unemployment rate, as well as on the variance of unemployment and inflation. For example, as early as 1989, President Boehne of the Federal Reserve Bank of Philadelphia stated the following during an FOMC meeting (FRB 1989, p. 19):

“...if we could bring inflation down from cycle to cycle just as we let it build up from cycle to cycle, that would be considerable progress over what we’ve done in other periods in history.”

In the same vein, former vice chairman of the Federal Reserve Board Alan Blinder has suggested that (Blinder 1994, p. 4):

“If monetary policy is used to cut our losses on the inflation front when luck runs against us, and pocket the gains when good fortune runs our way, we can continue to chip away at the already-low inflation rate.”

Blinder (1997) has argued that opportunism may arise because unemployment deviations from target induce a greater social distortion than corresponding inflation deviations. For example, unemployment at 2% above the natural rate implies that 2% of workers are fully unemployed, rather than all workers being 2% unemployed. An alternative rationale could be that the central bank’s vulnerability to political attack makes it more sensitive to positive rather than negative deviations of unemployment from the natural rate, as positive deviations could threaten its independence. A model of monetary policy with a linear Phillips curve rationalizing asymmetric preferences yielding “opportunistic” disinflation strategies was introduced by Orphanides and Wilcox (1996) and Orphanides and colleagues (1996).

In line with the above arguments, this paper departs from the quadratic specification by introducing a loss term that is linearly increasing in unemployment when the latter is above the DNAIRU, and zero when unemployment is at, or below, the DNAIRU. This asymmetry is intended to capture the underlying convex policy trade-off combined with an opportunistic monetary policy strategy. Under the latter, the central bank is guarding against any incipient rise in inflation, but waits for the next favorable inflation shock to lower inflation toward the target, rather than seeking to actively lower inflation in a manner that pushes the unemployment rate higher. The analysis proceeds in two stages. First, it is shown analytically that a nonlinear and asymmetric specification of the one-period Barro-Gordon problem maintains the key result of an “inaction range” of inflation shocks, over which the optimal monetary policy setting does not adjust and the equilibrium level of unemployment is the DNAIRU.3 Moreover, with a convex Phillips curve, the magnitude of the inaction range is increasing in the DNAIRU. The implication is that a given amount of disinflation is more difficult starting from a higher unemployment rate, because the policy maker’s one-period trade-off shifts toward stabilizing unemployment.

More generally, the effects of the asymmetry on the equilibrium levels of unemployment and inflation depend on the inflation-shock realization and its underlying distribution. This leads to the second, numerical part of the paper. Although no metric or discounting rule can be unambiguously specified for comparing the two loss functions, they can be evaluated by examining their implications for the time-series behavior of the

---

2The NAIRU is thus consistent with Friedman’s (1968) definition of the natural rate of unemployment as the average unemployment rate in a stochastic setting.

3This result was established by Orphanides and Wilcox (1996) for a linear Phillips curve and nonquadratic but symmetric preferences.
target variables. Using a simple iterative algorithm to derive model-consistent inflation expectations, Monte Carlo simulations confirm that equilibrium-expected inflation exceeds the inflation target under both symmetric and asymmetric loss functions. Moreover, for a moderate degree of nonlinearity, the bias is always larger when the loss function is asymmetric. We then report the mean, variance, skewness, and kurtosis of the simulated distributions of the equilibrium levels of inflation and unemployment for three different parameterizations, based on the empirical methodology for the U.S. developed by Laxton, Rose, and Tambakis (forthcoming). It is found that, for both loss-function specifications, there are significant departures from the linear and symmetric Barro-Gordon framework. However, a high degree of inflation aversion and small asymmetry are required in order to reproduce the observed average unemployment rate.

The remainder of the paper is arranged as follows. Section 2 reviews the properties of the linear-symmetric monetary policy model. Section 3 presents a model with a convex Phillips curve and an asymmetric loss function. Section 4 develops the model’s equilibrium properties, and compares them to those of a model with a symmetric loss function. The quartic polynomial equations for the equilibrium first-order conditions for inflation and unemployment are used to derive the comparative static properties of the range of inflation shocks over which the optimal policy setting and equilibrium level of unemployment do not adjust. Section 5 reports the results of numerical simulations on the equilibrium distributions of inflation and unemployment levels for parameter values estimated for the U.S. and moderate and “extreme” policy maker preferences. Section 6 concludes.

2 A Review of the Linear and Symmetric Model

The symmetric policy-loss function of Barro and Gordon (1983) is quadratic in deviations of inflation and unemployment from their target values:

\[ L_s = (\pi_t - \pi^*)^2 + \alpha (\pi_t - \pi) + \alpha U_t + \frac{\alpha}{2} U_t^2 \]  

where the inflation target is \( \pi^* > 0 \) and the unemployment target equals \( U^* \), the nonaccelerating inflation rate of unemployment in a deterministic setting. In general, assuming that \( \pi_t = \pi^* \) implies no time inconsistency, and thus no expected inflation bias. Assuming that the target unemployment rate were less than \( U^* \) would simply yield the result of positive equilibrium-inflation bias with no expected decline in unemployment.4 The policy maker’s preferences over inflation and unemployment stabilization are given by the normalized inflation-aversion parameter \( \alpha > 0 \), assumed to coincide with social preferences. The linear short-run Phillips curve has constant slope \( \gamma > 0 \):

\[ \pi_t = E_{t-\gamma} \pi_t + \gamma (U_t - U^*) + \varepsilon_t \]

where \( \varepsilon_t \) is an iid \( N(0, \sigma^2) \) normally distributed inflation (supply) shock that is realized after expected inflation has been set. In the Barro-Gordon framework, one-period optimal monetary policy is determined by minimizing loss function as in Equation (1) subject to the linear Phillips curve constraint of Equation (2). The monetary policy maker is assumed to have direct control over the inflation rate. The inflation rate is chosen after both the inflation shock and the expected inflation have been observed, so there is stabilization policy in the model. The equilibrium first-order condition is such that the sum of the policy maker’s marginal losses from inflation and unemployment is zero:

\[ L_s^\pi + L_s^U \frac{\partial U}{\partial \pi} = 0 \Rightarrow -L_s^U \frac{\partial U}{\partial \pi} = L_s^\pi \]

Substituting Equation (2) into Equation (3), the one-period equilibrium levels of inflation and unemployment become

\[ \pi_t = \pi^* + \frac{1}{1 + \alpha \gamma} \varepsilon_t \]

\[ U_t = U^* + \frac{\alpha \gamma}{1 + \alpha \gamma} \varepsilon_t \]

---

4See the work of Cukierman (1992) for a discussion of related issues.
Substituting Equations (4) into Equation (1) and taking expectations yields the expected loss to be increasing in inflation-shock variability:

\[
E_{t-1} \ell_t^3 = \frac{\alpha}{1 + \alpha \gamma^2} \sigma_e^2
\]  
(5)

Taking expectations in Equation (4) yields

\[
E_{t-1} \pi_t = \bar{\pi} \\
E_{t-1} U_t = U^*
\]  
(6)

Therefore, under the linear-symmetric model, and in the absence of a time-inconsistent incentive on the part of the policy maker, average unemployment equals the target rate \(U^*\) and there is no equilibrium inflation bias.

3 The Nonlinear-Asymmetric Model

3.1 A nonlinear Phillips curve

The nonlinear Phillips curve follows the convex functional form as described by Debelle and Laxton (1997) and Laxton, Rose, and Tambakis (forthcoming). The stylized one-period specification abstracts from leads and lags in inflation expectations, so the time subscript may be dropped:

\[
\pi = E\pi + \gamma \frac{U^* - U}{U - \varphi(U^*)} + \varepsilon, \quad 0 < \varphi(U^*) < U^*
\]  
(7)

Figure 1 shows a general form of the convex Phillips curve.\(^5\) The parameter \(\gamma\) is a horizontal asymptote: it is the maximum rate of deflation that would be generated as excess supply became unbounded. The parameter \(\varphi(U^*)\) is assumed to be a lower bound for the unemployment rate—i.e., a vertical asymptote—reflecting short-run constraints on how far aggregate demand can lower unemployment before capacity constraints become binding and inflationary pressure becomes unbounded. Although \(\varphi\) has to be less than \(U^*\), in principle it can be defined either as a fixed constant or as increasing in \(U^*\). In this paper, the latter definition is adopted:

\[
\varphi(U^*) = \min(0, U^* - \bar{\delta})
\]  
(8)

Fixing \(U^* - \varphi\) to a constant implies that \(\varphi\) increases linearly with the trend unemployment rate \(U^*\) when it rises above 4\%, while if \(U^*\) is at or below 4\%, then \(\varphi\) is constrained to zero. The empirical fit of this model for U.S. data is examined in Section 5. The slope of the convex Phillips curve is a joint function of the parameter \(\gamma\) and the levels of \(U^*\) and \(\varphi\). It is absolutely decreasing in \(U^*\):

\[
\frac{\partial \pi}{\partial U} = -\gamma \frac{U^* - \varphi(U^*)}{(U - \varphi(U^*))^2} < 0, \quad \frac{\partial^2 \pi}{\partial U^2} > 0
\]  
(9)

Evaluating \(\partial \pi / \partial U\) at \(U^*\) yields

\[
\frac{\partial \pi}{\partial U}(U = U^*) = -\frac{\gamma}{U^* - \varphi(U^*)}
\]  
(10)

Note that, provided \(\frac{\partial \varphi}{\partial U^*} < 1\), a higher \(U^*\) implies a locally flatter Phillips curve at \(U^*\), and hence higher unemployment costs for a given amount of disinflation. Unlike the linear Phillips curve, \(U^*\) is not a feasible stable-inflation equilibrium in a stochastic economy with nonlinearity. Referring to Figure 1, for any

---

\(^5\) For a review of sources of nonlinearity in the Phillips curve, see the work of Clark, Laxton, and Rose (1996). This particular functional form is based on a labor market model described by Layard, Nickell, and Jackman (1991). In general, nonlinearity also includes concave alternatives. Examples of the latter include works by Eisner (1997) and Stiglitz (1997). The term “asymmetric” is reserved for the loss function.
distribution of inflation shocks, the stochastic NAIRU is the linear combination of points on the Phillips curve yielding a zero-mean inflation-forecast error: $E(\pi - E\pi) = 0$. Without loss of generality, Figure 1 shows two points on the Phillips curve, $(U^1, \pi^1 - E\pi)$ and $(U^2, \pi^2 - E\pi)$, corresponding to a pair of inflation shocks $U^1 < U^*$ and $U^2 > U^*$ distributed symmetrically about $U^*$. Clearly, nonlinearity implies that the intersection with the unemployment axis of the one-dimensional simplex of any two such points will always be to the right of $U^*$. The average unemployment rate $EU$ consistent with stable inflation in a stochastic equilibrium is subsequently referred to as the NAIRU. The term deterministic NAIRU (DNAIRU) is adopted for $U^*$, the nonaccelerating inflation rate of unemployment in the absence of shocks. Stabilization policy with a convex Phillips curve thus has first-order welfare effects, because success in reducing unemployment variability will also lower its mean. Policy makers who are more successful in stabilizing the business cycle will be inducing a lower average unemployment rate, as the gap between the NAIRU and the DNAIRU is increasing in the variance of the shock distribution, as well as in the degree of nonlinearity of the Phillips curve. Ceteris paribus, the NAIRU also rises if the shock distribution becomes more positively skewed; i.e., as inflation shocks are more positive, and vice versa if the inflation-shock distribution is skewed to the left.

### 3.2 An asymmetric loss function

Following the earlier discussion on the opportunistic approach to monetary policy, the key assumption is made that when unemployment gets above the DNAIRU, the policy maker cares about the level, as well as the variance, of the unemployment rate. In contrast, when the unemployment rate is to the left of the DNAIRU, the quadratic loss function in Equation (1) applies. A convenient way of modeling this asymmetry in macroeconomic policy preferences is by introducing a breakpoint in the loss function at $U^*$. This takes the form of a term linear in unemployment deviations from the DNAIRU, which is zero if the economy is in the expansion range ($U < U^*$), and positive if the economy enters in the recession range ($U > U^*$). The

---

6This result is robust to both continuously differentiable and piecewise-linear functional forms for the Phillips curve. The point was made in Mankiw's (1988) comment on De Long and Summers' work (1988). See also the work of Debelle and Laxton (1997).

7For more on this point refer to Bean's (1996) discussion.
asymmetric loss function $L^A$ then is

$$L^A = (U - U^*)^2 + \alpha(\pi - \pi)^2 + 2\psi \max(0, U - U^*), \psi > 0$$

Equation (11) differs from the nonlinear but symmetric loss function used by Orphanides and Wilcox (1996) and the symmetric policy rule by Orphanides and colleagues (1996) in two respects. First, by explicitly introducing a differential valuation of deviations from the inflation and unemployment targets, both specifications induce an “inaction range” of inflation shocks, over which the optimal policy setting and equilibrium unemployment does not adjust. However, under the Orphanides and Wilcox specification, the inaction range is only a function of policy maker preferences ($\alpha, \psi$) and the degree of convexity of the Phillips curve ($\gamma$). In particular, it is independent of the level of the DNAIRU, as the Phillips curve is linear. In the next section, it is shown that the magnitude of the inaction range induced by either the asymmetric loss function in Equation (11) or the Orphanides-Wilcox formulation is increasing in $U^*$. A necessary condition for this is that the Phillips curve be convex. An exogenous increase in the DNAIRU increases the marginal cost of disinflation, so the policy maker is willing to forgo pursuing the long-run inflation objective for a wider range of inflation shocks.

The second difference also relates to convexity in the Phillips curve. As discussed in Section 1, convexity essentially amounts to higher unemployment per unit of disinflation at the margin as the unemployment rate goes up—equivalently, it amounts to higher inflation at the margin as unemployment declines. Equation (11), specifying that marginal losses from unemployment deviations only increase to the right of the DNAIRU, is thus consistent with the underlying structure of the economy, which symmetric preferences fail to take into account. Therefore, setting the breakpoint at $U^*$ is an arbitrary, albeit convenient, choice. In principle, provided the Phillips curve is globally convex, the asymmetric term could be triggered at any point. Of course, other things being equal, a higher value of $\psi$ reflects stronger asymmetry in the policy maker’s preferences. To summarize, modeling an asymmetric loss function, as opposed to a nonlinear-symmetric specification, is a reasonable way of combining the underlying convex policy trade-off with the important characteristic of the opportunistic approach to disinflation.

4 Equilibrium

4.1 First-order conditions

Although the policy maker’s choice variable is the inflation rate, for computational purposes, the first-order condition for the nonlinear optimization problem may be expressed as a function of either target-policy variable by applying the chain rule. First, differentiating Equation (11) and substituting in the Phillips curve of Equation (7) yields the first-order condition in terms of the inflation rate:

$$-L^A_{\pi}(\pi) \frac{\partial U}{\partial \pi} = L^A_{\pi}$$

The asymmetry implies that the first-order condition consists of two segments, each corresponding to expansion and recession regions of the economy. The one-period equilibrium level of inflation, $\pi$, is the solution to the quartic polynomial equations:

$$U < U^* \Rightarrow \alpha(\pi - \pi)(\pi - E\pi - \varepsilon + \gamma)^3 - \gamma^2(U^* - \varphi)^2 + \gamma(U^* - \varphi)^3(\pi - E\pi - \varepsilon + \gamma) = 0$$

$$U > U^* \Rightarrow \alpha(\pi - \pi)(\pi - E\pi - \varepsilon + \gamma)^3 - \gamma^2(U^* - \varphi)^2 + \gamma(U^* - \varphi)(U^* - \varphi - \psi)(\pi - E\pi - \varepsilon + \gamma) = 0$$

Equation (13) corresponds to the expansion region to the left of the DNAIRU. Taking expectations, the second and third terms always sum to zero, regardless of the magnitude of expected inflation. However, the higher moments in the first term imply that imposing zero inflation bias ($E\pi = \pi$) does not necessarily satisfy

---

8This is clearly also the first-order condition for the equilibrium inflation rate of a symmetric loss function under a nonlinear Phillips curve.
the first-order condition. Thus we cannot a priori rule out nonzero equilibrium expected inflation bias in expected minimization of a symmetric loss function under a convex Phillips curve. Equation (14) corresponds to the economy’s recession region where welfare is also negatively related to the level of the unemployment rate. Taking expectations, the sum of the second and third terms is strictly negative for all \( \psi > 0 \). Therefore, the first term has to be strictly positive. Thus, compared to the symmetric region, a larger positive expected inflation bias (\( \bar{\pi} > \bar{\pi} \)) may be required in equilibrium.

To gain more intuition, the equilibrium first-order condition as a function of the unemployment rate can be derived from Equation (12):

\[
-L^A_\pi(U) \frac{\partial \pi}{\partial U} = L^A_U
\]

The one-period equilibrium unemployment level is the solution to the following quartic polynomial equations in \( U \):

\[
U < U^* \Rightarrow \alpha \gamma^2(U^* - \varphi)^2 + \frac{\alpha \gamma(U^* - \varphi)}{(U - \varphi)^2} (\bar{\pi} - \pi + \psi)
= \ U - U^*
\]

\[
U > U^* \Rightarrow \alpha \gamma^2(U^* - \varphi)^2 + \frac{\alpha \gamma(U^* - \varphi)}{(U - \varphi)^2} (\bar{\pi} - \pi + \psi)
= \ U - U^* + \psi
\]

The marginal loss function of unemployment is linear with unit slope, and discontinuous and nondifferentiable at \( U = U^* \) because of the asymmetry. The difference between the two segments lies in the right side of Equation (17), representing the marginal loss function of unemployment in the recession region. The left side, representing the marginal loss function of inflation expressed in terms of the unemployment rate, is identical in both segments and may be written as

\[
-L^A_\pi \frac{\partial \pi}{\partial U} = \frac{A_1}{(U - \varphi)^3} + \frac{A_2}{(U - \varphi)^2}
\]

where \( A_1 \) and \( A_2 \) are as follows:

\[
A_1 = \alpha \gamma^2(U^* - \varphi)^2
A_2 = \alpha \gamma(U^* - \varphi)(\bar{\pi} - \pi + \psi)
\]

Whereas coefficient \( A_1 \) is always positive, the sign of \( A_2 \) is ambiguous, as it is a function of expected inflation, the inflation target, the Phillips-curve slope coefficient, and the realization of the shock. Consequently, constraining \( A_1 \) to be positive is neither necessary nor sufficient for Equation (18) to be positive. Figure 2 graphs the two marginal loss functions \(-L^A_\pi \frac{\partial \pi}{\partial U}\) and \(L^A_U\) against the unemployment rate, with the range of \(-L^A_\pi \frac{\partial \pi}{\partial U}\) corresponding to the feasible range of unemployment (\( \varphi, 1 \)). There are only two possible solutions to the equation \(-L^A_\pi \frac{\partial \pi}{\partial U} = 0\), depending on the sign of coefficient \( A_2 \). Either the equation has no real root, in which case it has a general hyperbolic shape, or it has one real positive root and a pair of complex conjugate roots, in which case it has one stationary point and one inflexion point.

4.2 The inflation-shock inaction range

The geometry of Figure 2 yields an important equilibrium property. I focus on first-order conditions of Equations (16) and (17) for unemployment, without loss of generality. The target variables’ one-period equilibrium levels are at the intersection of the marginal loss schedules. This is a function of the inflation-shock realization, expected inflation, and parameter values. However, a real-valued solution is not guaranteed, because of the discontinuity of the unemployment marginal loss schedule at \( U^* \). Existence of a real solution depends on the value of the inflation marginal loss function at the DNAIRU. In particular, \(-L^A_\pi \frac{\partial \pi}{\partial U}(U^*)\) has to lie outside \((0, \psi)\), the discontinuity range of \( L^A_U\).
The unemployment marginal loss at $U^*$ equals $\psi$, while Equations (18) and (19) yield the inflation marginal loss at $U^*$ to be

$$- L^A \frac{\partial \pi}{\partial U} (U^*) = \frac{\alpha \gamma}{U^* - \varphi} (E\pi - \bar{\pi} + \varepsilon)$$  \hspace{1cm} (20)$$

This implies the following three cases for equilibrium unemployment:

1. The inflation-shock realization is such that Equation (20) is positive and greater than $\psi$, implying $E\pi - \bar{\pi} + \varepsilon > \psi(U^* - \psi) \frac{U^*}{U^* - \varphi}$. The inflation marginal loss function intersects $L^A_U$ in the recession region to the right of the DNAIRU, reflecting a large positive inflation-shock realization such as an oil-price shock. The equilibrium level of unemployment is then above $U^*$, and is given by the real positive root of the quartic polynomial in Equation (16). Intuitively, such shock realizations induce tighter monetary policy, so the one-period equilibrium unemployment rate exceeds the DNAIRU.

2. The shock realization is such that Equation (20) is negative; i.e., $E\pi - \bar{\pi} + \varepsilon < 0$: the inflation marginal loss function then intersects $L^A_U$ in the economy’s expansion region. The equilibrium level of unemployment lies below $U^*$, and is given by the real positive root of the quartic polynomial in Equation (17). The intuition is symmetric to case 1: a large negative inflation-shock realization induces looser monetary policy, hence the equilibrium unemployment rate is less than the DNAIRU.

3. Finally, if $- L^A \frac{\partial \pi}{\partial U} (U^*)$ is within the range of discontinuity $(0, \psi)$ of $L^A_U$, the two marginal loss functions do not intersect, and all roots are complex-valued. From Equation (20), the relevant range of inflation shocks is

$$0 \leq \frac{\alpha \gamma^2}{U^* - \varphi} + \frac{\alpha \gamma}{U^* - \varphi} (E\pi - \bar{\pi} + \varepsilon - \gamma) \leq \psi$$
This can be written as:

\[ 0 \leq E\pi - \pi + \varepsilon \leq \frac{\psi(U^*-\varphi)}{\alpha\gamma} \]

\[ \Rightarrow \pi - E\pi \leq \varepsilon \leq \frac{\psi(U^*-\varphi)}{\alpha\gamma} + \pi - E\pi \]  

(21)

The asymmetric loss function then implies that the one-period equilibrium unemployment rate is \( U^* \). Inequalities (21) thus define an inaction range of inflation shocks, over which the optimal policy setting and equilibrium unemployment does not adjust. Note that this is the case for either a linear or a nonlinear Phillips curve, as it is the asymmetry in the loss function that underlies the nonintersection of the marginal loss schedules.

The comparative static properties of the inaction range are as follows. First, as shown by Orphanides and Wilcox (1996), increasing \( \psi \) widens the inaction range, as unemployment rates above \( U^* \) become relatively more costly. In contrast, higher values of the inflation-aversion coefficient \( \alpha \) and/or the Phillips-curve coefficient \( \gamma \) induce a smaller inaction range. The intuition for \( \alpha \) is that a more inflation-averse policy maker pursues a tighter monetary policy, hence is more likely to disinflate in any given period, other things being equal. In the case of a larger \( \gamma \), a steeper Phillips curve implies that a 1% decline in the inflation rate involves smaller unemployment costs; i.e., the sacrifice ratio is smaller. This creates a higher incentive to disinflate, so the range of inflation shocks for which \( U^* \) is the equilibrium level of unemployment is narrower. Finally, an exogenous increase in \( U^* \) widens the inaction range. Importantly, the intuition centers on the value of the slope of the Phillips curve at \( U^* \). From Equation (10), a larger \( U^* \) implies a locally flatter Phillips curve, and thus a higher sacrifice ratio, so there is a wider range of inflation shocks for which \( U^* \) is the equilibrium unemployment rate. This property is in contrast to the linear Phillips curve, in which the size of the inaction range is independent of \( U^* \). Therefore, an asymmetric loss function implies that the DNAIRU has an effect on the inaction range only under a convex Phillips curve.9

5 Equilibrium Simulations for the U.S. Output-Inflation Trade-Off

5.1 Parameter values

Reduced-form models of monetary policy involve shocks to the DNAIRU, unemployment (aggregate-demand shocks), inflation expectations, and inflation (aggregate-supply shocks). The focus of this paper is on the last type only. In particular, there is no discussion of the desirability of modest experimentation—“cautious probing”—with different policy rules in the face of uncertainty about the DNAIRU. In general, however, asymmetric preferences combined with a linear Phillips curve—such as described by Orphanides and Wilcox (1996)—imply policy makers have an incentive to accommodate negative (recessionary) aggregate-demand shocks less than when the Phillips curve is nonlinear (convex). In the former case, the policy maker wishes to stabilize fast because of the asymmetric cost of unemployment above the natural rate. In the latter case, there would also be a strong initial response, as there would be little risk of inflation and the effects on unemployment would be relatively strong. However, in this case, the policy response would become more measured as unemployment fell in order to avoid significant overshooting.10

The inflation target \( \pi \) is set at 2%, and the inflation-aversion parameter and the asymmetric coefficient in the loss function are initially set at \( \alpha = 1 \) and \( \psi = 1 \). Following the empirical methodology of Debelle and Laxton (1997) and Laxton, Rose, and Tambakis (forthcoming), the unobserved time-varying DNAIRU and the parameters of the nonlinear Phillips curve can be jointly estimated using a random-walk specification with boundary conditions \( (U^*_t = U^*_{t-1} + \varepsilon_t^{U^*}) \) for the trend unemployment rate and an extension of Equation (7). The resulting nonlinear estimation problem can be solved using a Kalman filter.11

The resulting value of the Phillips-curve slope parameter for U.S. quarterly data from 1955–1996 is \( \gamma = 4.71 \). The estimated standard error of the inflation-shock distribution is \( \hat{\sigma}_\varepsilon = 0.0147 \), which is somewhat

---

9An asymmetric loss function is necessary for the existence of the inaction range, but it is convexity in the Phillips curve that underlies the result of the positive effect of changes in \( U^* \) on the inaction range. Each of the two conditions on its own is necessary, but not sufficient for this result.

10For experimentation issues see the works of Laxton, Rose, and Tambakis (forthcoming), Stiglitz (1997), and Wieland (1996).

11This methodology has been applied by Kuttner (1992, 1994) for measuring potential output.
larger than the standard error of 0.0129 reported by Gordon (1997). Bearing in mind the likely omitted influences and measurement errors in reduced-form models, the first estimate is used as a benchmark in the subsequent simulations. The average unemployment rate is $EU = 0.064$, while the average of the estimated model-consistent DNAIRU series is $U^* = 0.061$. Therefore, the average gap between the DNAIRU and the NAIRU in the U.S. has been about 0.3%. Finally, recall from Equation (8) that the minimum unemployment rate $\varphi(U^*)$ was specified as linearly increasing in the DNAIRU. Therefore, a hypothetical doubling of the DNAIRU estimate for the U.S. from 5% to 10% would raise $\varphi$ from 1% to 6%.12

5.2 Stochastic properties of inflation and output under alternative loss functions

In contrast to the linear model, the nonlinearity of the first-order conditions implies that $\varpi$ cannot be used for the equilibrium expected inflation rate. This is so regardless of the particular loss function under consideration: as argued earlier, drawing a large number of inflation shocks and solving first-order condition (12) using the inflation target as an estimate of expected inflation can yield an average inflation rate different from $\varpi$, so expected inflation would not be model-consistent. This problem is resolved by means of a numerical algorithm converging to a fixed point for expected inflation. An arbitrary initial estimate of the excess of expected inflation over the target is specified; call it $b_0 = E\pi - \varpi > 0$. Given this value, the one-period equilibrium level of inflation is computed for a large number of shocks, yielding an inflation sample mean of $\hat{\pi}(b_0)$. This yields the excess of expected inflation over the target corresponding to $b_1$: $b_0 = \hat{\pi}(b_0) - \varpi$. The value of $b_0$ is then compared against $b_0$. If the latter is not consistent with $b_0$ (i.e., if $b_0 \neq b_0$), then a second iteration is carried out using $b_1 = b_0$ as the updated estimate of inflation bias. Repeated iteration of the choice of expected inflation bias eventually converges to a value $b^*$ such that $b^* = \hat{\pi}(b^*) - \varpi$. Then $b^*$ is a fixed point of the inflation bias for the particular inflation-shock distribution. The resulting inflation sample mean $\hat{\pi}(b^*)$ is therefore model-consistent and can be used in the first-order conditions for deriving the equilibrium levels of inflation and unemployment.13

The stochastic properties of inflation and output under the symmetric and asymmetric loss functions are analyzed by means of numerical simulations of first-order conditions (13), (14), (16), and (17) for a large number of inflation shocks. Tables 1–3 present convergent values of the first four moments of the distributions of one-period equilibrium levels of inflation and unemployment.14 Each table corresponds to a different level for the variance of the inflation shock. The reported numerical simulations involve the benchmark set of parameter values and two alternatives, for a total of nine parameter combinations. In each case, the moments of the inflation and unemployment distributions are obtained for both the symmetric and the asymmetric loss functions.

Table 1 reports the moments of the equilibrium inflation and unemployment distributions in the benchmark case where the variance of the underlying inflation-shock distribution takes on the estimated standard error for the U.S. ($\hat{\sigma}_\pi = 0.0147$). The simulations refer to the following parameter combinations: (1) a benchmark case in which $U^* = 0.06$, $\varpi = 0.02$, $\varphi = 0.02$, and $\psi = 1$; (2) a first alternative in which the asymmetry in the loss function is reduced to $\psi = 0.1$, indicating lower losses than the benchmark when unemployment is in the recession region; and (3) a second alternative where the DNAIRU increases to $U^* = 0.10$ with $\varphi$ rising to 0.06.15

The symmetric outcomes in the benchmark case and the first alternative are identical because they are independent of the asymmetry in the loss function. There are two observations on the behavior of the expectations of the target variables. First, for all three parameter combinations, the gap between the average unemployment rate and the DNAIRU is very small—generally less than 0.1%. This confirms the findings of Laxton, Rose, and Tambakis (forthcoming) that nonlinearity in the Phillips curve is difficult to identify when policy makers are broadly successful in stabilizing the business cycle; i.e., when the estimated shock variances are small. Second, in each case there is positive expected inflation bias ranging from 0.08% in case 2 ($\psi = 0.1$) to 0.44% in case 3 ($U^* = 0.10$). Moreover, the bias is always larger for the asymmetric loss function.

---

12The simple rule for $\varphi$ with unit slope and the trigger point set at 4% was chosen as it provided the highest likelihood values among the alternatives considered. For more details on the estimation process and results, interested readers are referred to Laxton, Rose, and Tambakis’s work (forthcoming).

13It is assumed that the inflation distribution is ergodic, so that convergence occurs for finite sample sizes after a finite number of iterations.

14The number of inflation-shock draws is 1,000, determined by the model-consistency requirement that the realized inflation sample mean should confirm the ex-ante choice of expected inflation.

15The values of expected shock used to solve the first-order conditions were derived using the fixed-point procedure described above. The realized inflation sample mean has to converge arbitrarily close to the ex-ante choice of expected inflation.
Table 1
Simulated inflation and unemployment distributions: $\sigma^2 = 0.022$

1. **Benchmark:** $U^* = 0.06, \phi = 0.02, \pi = 0.02, \alpha = 1.0, \gamma = 4.71, \psi = 1.0$

<table>
<thead>
<tr>
<th>Inflation</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>Symmetric</td>
<td>2.30</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>2.77</td>
</tr>
</tbody>
</table>

2. **Small Asymmetry:** $\psi = 0.1$

<table>
<thead>
<tr>
<th>Inflation</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>Symmetric</td>
<td>2.30</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>2.38</td>
</tr>
</tbody>
</table>

3. **Large DNAIRU:** $U^* = 0.10, \phi = 0.06$

<table>
<thead>
<tr>
<th>Inflation</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>Symmetric</td>
<td>3.61</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>3.94</td>
</tr>
</tbody>
</table>

Table 2
Simulated inflation and unemployment distributions: $\sigma^2 = 0.16$

1. **Benchmark:** $U^* = 0.06, \phi = 0.02, \pi = 0.02, \alpha = 1.0, \gamma = 4.71, \psi = 1.0$

<table>
<thead>
<tr>
<th>Inflation</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>Symmetric</td>
<td>3.59</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>3.94</td>
</tr>
</tbody>
</table>

2. **Small Asymmetry:** $\psi = 0.1$

<table>
<thead>
<tr>
<th>Inflation</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>Symmetric</td>
<td>3.60</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>3.65</td>
</tr>
</tbody>
</table>

3. **Large DNAIRU:** $U^* = 0.10, \phi = 0.06$

<table>
<thead>
<tr>
<th>Inflation</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>Symmetric</td>
<td>3.61</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>3.98</td>
</tr>
</tbody>
</table>
Table 3
Simulated inflation and unemployment distributions: $\sigma^2 = 0.001$

1. Benchmark: $U^* = 0.06, \varphi = 0.02, \Pi = 0.02, \alpha = 1.0, \gamma = 4.71, \psi = 1.0$

<table>
<thead>
<tr>
<th>Inflation</th>
<th>Unemployment</th>
<th>Inflation</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Symmetric</td>
<td>Asymmetric</td>
<td>Symmetric</td>
</tr>
<tr>
<td>Mean</td>
<td>2.02</td>
<td>2.71</td>
<td>6.01</td>
</tr>
<tr>
<td>Variance</td>
<td>0.02</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.43</td>
<td>-0.50</td>
<td>-0.04</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.32</td>
<td>2.88</td>
<td>3.02</td>
</tr>
</tbody>
</table>

2. Small Asymmetry: $\psi = 0.1$

<table>
<thead>
<tr>
<th>Inflation</th>
<th>Unemployment</th>
<th>Inflation</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Symmetric</td>
<td>Asymmetric</td>
<td>Symmetric</td>
</tr>
<tr>
<td>Mean</td>
<td>2.02</td>
<td>2.10</td>
<td>6.01</td>
</tr>
<tr>
<td>Variance</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.29</td>
<td>0.08</td>
<td>-0.15</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.01</td>
<td>2.57</td>
<td>2.96</td>
</tr>
</tbody>
</table>

3. Large DNAIRU: $U^* = 0.10, \varphi = 0.06$

<table>
<thead>
<tr>
<th>Inflation</th>
<th>Unemployment</th>
<th>Inflation</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Symmetric</td>
<td>Asymmetric</td>
<td>Symmetric</td>
</tr>
<tr>
<td>Mean</td>
<td>2.02</td>
<td>2.75</td>
<td>10.01</td>
</tr>
<tr>
<td>Variance</td>
<td>0.02</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.35</td>
<td>-0.58</td>
<td>-0.07</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.99</td>
<td>2.89</td>
<td>2.85</td>
</tr>
</tbody>
</table>

The intuition relates to the underlying difference in policy-making behavior: under the asymmetric loss function, monetary policy does not tighten unless the realized shock exceeds the upper bound of the inaction range. The variance of the equilibrium level of inflation (unemployment) is always greater under the asymmetric (symmetric) loss function. This is a straightforward implication of the asymmetric welfare costs of a rise in unemployment on either side of $U^*$. The equilibrium level of the variance of unemployment is greater under the asymmetric loss function, because it penalizes unemployment deviations from target more. In this respect, when the asymmetry is small ($\psi = 0.1$), the two variances are nearly the same.

Regarding the higher moments of the equilibrium distributions, inflation skewness and kurtosis seldom approach their standard normal values of 0 and 3, respectively. The inflation distribution is strongly positively skewed, reflecting a thicker right tail than a normal distribution’s. Equivalently, the mean of inflation is above the median, a finding consistent with expected inflation exceeding the inflation target. In contrast, under both loss functions, the skewness of the unemployment distribution is slightly negative, indicating a small negative gap between the expected unemployment rate—the NAIRU—and median unemployment. Ceteris paribus, this appears consistent with the small gap between the NAIRU and the DNAIRU for the chosen parameterization. Meanwhile, the kurtosis of the inflation (unemployment) distribution is always greater under the symmetric (asymmetric) loss function. This is consistent with the relation between the variances of unemployment and inflation. The fact that the unemployment distribution has less variance under the asymmetric loss function implies it has relatively more probability mass in the tails. Its kurtosis is therefore greater than the respective distribution under symmetric loss, other things being equal. Conversely, the inflation distribution has more variance under the asymmetric loss function, so it has less probability mass in the tails and is less leptokurtic than under the symmetric loss function.

Tables 2 and 3 report the simulated moments of the inflation and unemployment distributions under two hypothetical scenarios where the variance of the inflation shock drops to 0.1%, or grows to 16%. In each case, the three parameter combinations are as before. The qualitative features of these results are similar to the benchmark of the estimated shock variance in Table 1. Quantitatively, the gap between the NAIRU and the DNAIRU increases with the shock variance: the difference is close to 0.2% in the alternative parameter.
Table 4
Benchmark inflation-shock distribution: $\tilde{\sigma}_\nu^2 = 0.022^a$

<table>
<thead>
<tr>
<th></th>
<th>Inflation</th>
<th>Unemployment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Symmetric</td>
<td>Asymmetric</td>
</tr>
<tr>
<td>A. Low Inflation Aversion: $\alpha = 0.1$, $\psi = 10$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>2.65</td>
<td>3.12</td>
</tr>
<tr>
<td>Variance</td>
<td>1.65</td>
<td>2.07</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.22</td>
<td>0.20</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.51</td>
<td>2.48</td>
</tr>
</tbody>
</table>

| B. High Inflation Aversion: $\alpha = 10$, $\psi = 0.1$ |            |            |            |            |
| Mean                  | 2.13      | 2.12       | 6.32      | 6.32       |
| Variance              | 0.08      | 0.07       | 1.51      | 1.50       |
| Skewness              | 2.73      | 2.93       | 0.75      | 0.74       |
| Kurtosis              | 11.07     | 13.59      | 3.30      | 3.29       |

$^a$The structural parameter values are as in Tables 1–3: $U^*/\bar{\pi} = 0.06$, $\tau = 0.02$, $\gamma = 4.71$, and $\varphi = 0.02$.

combinations in Table 2. In contrast, the gap is close to zero when $\sigma^2 = 0.1$ in Table 3. These findings are consistent with the model in Section 3: the gap is increasing in inflation-shock variability and the degree of asymmetry in the loss function. Note also that the variances of inflation and unemployment increase sharply with the shock’s variability across parameterizations, but the variance of unemployment rises relatively less under asymmetric loss because of the influence of the inaction range. The relative properties of skewness and kurtosis from the benchmark standard error in Table 1 are broadly preserved.

Overall, for the range of parameter values under consideration, the gap between the NAIRU and the DNAIRU is lower than its estimated value of 0.3% for the U.S., with the exception of large inflation-shock variability (Table 2). This could be because the choice of parameter values for the policy maker’s preferences is inappropriate. Table 4 therefore presents the simulated moments of equilibrium inflation and unemployment for the following “extreme” preference specifications: (1) very low inflation aversion and large asymmetry in the loss function (panel A: $\alpha = 0.1$, $\psi = 10$), and (2) high inflation aversion and small asymmetry (panel B: $\alpha = 10$, $\psi = 0.1$). Interestingly, in the case of high inflation aversion, the unemployment gap is 0.32%, approximately its estimated value. This is combined with an excess of average inflation over the target of just over 0.1%. The results also indicate a disproportionate impact of “extreme” policy-maker preferences on the second moments of the target variables: the variance of inflation (unemployment) increases sharply as the policy maker becomes relatively less (more) inflation averse. The fact that the model closely matches the data only when the asymmetry is effectively negligible is prima facie evidence against asymmetric preferences. Arguably, however, a strong rejection cannot be warranted on the basis of the present highly stylized, reduced-form framework. Further calibration of the preference parameters is a subject for future research.

6 Conclusion

This paper studied the one-period optimal monetary policy problem when the Phillips curve is convex and the policy maker’s loss function is asymmetric, and examined the implications for the equilibrium levels of inflation and unemployment. Following recent arguments favoring an “opportunistic approach” to disinflation, an asymmetric loss function was introduced specifying that, at high unemployment rates, social welfare is a negative function of the level of unemployment, as well as the variances of unemployment and inflation. The asymmetric specification gives rise to an inaction region of inflation shocks for which the optimal policy

---

$^16$Combining a high (low) degree of inflation aversion with small (large) asymmetry seems natural in the context of the asymmetric loss function.
setting and equilibrium unemployment do not adjust. It was shown that, under a convex Phillips curve, the
inaction region’s magnitude is increasing in the DNAIRU. Monte Carlo simulations were used to solve the
nonlinear first-order conditions and evaluate the moments of the equilibrium inflation and unemployment
distributions. Calibrating the model to parameter values relevant to the U.S. and moderate policy preferences,
it was found that the asymmetric loss function yields an average inflation rate in excess of the target—and that
bias is larger than the standard symmetric loss function. The asymmetric loss function also yielded higher
(lower) variance of inflation (unemployment), and a small positive gap between average unemployment and
the natural rate. However, a very high degree of inflation aversion was required to reproduce the observed
average unemployment rate. Exploring the implications of these results for the expected social welfare
properties of alternative monetary policy loss functions is an important direction of future research.

References


Advisory Panel

Jess Benhabib, New York University
William A. Brock, University of Wisconsin-Madison
Jean-Michel Grandmont, CREST-CNRS—France
Jose Scheinkman, University of Chicago
Halbert White, University of California-San Diego

Editorial Board

Bruce Mizrach (editor), Rutgers University
Michele Boldrin, University of Carlos III
Tim Bollerslev, University of Virginia
Carl Chiarella, University of Technology-Sydney
W. Davis Dechert, University of Houston
Paul De Grauwe, KU Leuven
David A. Hsieh, Duke University
Kenneth F. Kroner, BZW Barclays Global Investors
Blake LeBaron, University of Wisconsin-Madison
Stefan Mittnik, University of Kiel
Luigi Montrucchio, University of Turin
Kazuo Nishimura, Kyoto University
James Ramsey, New York University
Pietro Reichlin, Rome University
Timo Terasvirta, Stockholm School of Economics
Ruey Tsay, University of Chicago
Stanley E. Zin, Carnegie-Mellon University

Editorial Policy

The SNDE is formed in recognition that advances in statistics and dynamical systems theory may increase our understanding of economic and financial markets. The journal will seek both theoretical and applied papers that characterize and motivate nonlinear phenomena. Researchers will be encouraged to assist replication of empirical results by providing copies of data and programs online. Algorithms and rapid communications will also be published.

ISSN 1081-1826