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*Effective Central Bank Independence and the Inflation-Output Trade-Off**

What is the impact of changing central bank independence (CBI) on the inflation-output trade-off? This paper introduces the notion of *effective* CBI and distinguishes its political from its economic effects on the trade-off by analyzing the interaction between the government, the median voter and the central bank. It is found that the two effects may work in opposite directions, with the political effect counteracting or reinforcing the economic effect. A taxonomy of possible combinations of expected inflation and inflation-output variability is built based on the equilibrium response of the median voter's expected loss to changes in effective CBI. Also, a threshold degree of CBI is obtained such that increasing effective CBI from below the threshold induces greater absolute change in inflation variability than output variability, the stylized fact underlying the claim that CBI is a free lunch.

1. Introduction

Is central bank independence (CBI) a free lunch? Following Rogoff's (1985) theoretical results of lower and more stable inflation and more variable output growth associated with higher CBI, empirical research has found mixed results: although there is strong negative correlation between the levels of CBI and average inflation and inflation variability, CBI is not significantly positively correlated with the variance of output growth or unemployment.[1] These findings have not prompted much theoretical work despite their strong policy-making implications. A notable exception is Alesina and Gatti (1995) who, in the context of a partisan business cycle model, have shown that increasing CBI can yield lower output variability if it implies less political influence in monetary policy.[2] The absence of a clear link between CBI and output variability has prompted the notion that the inflation benefits

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[1]The theoretical results have been extended by Lohmann (1992), and optimal contracts for implementing CBI have been designed by Persson and Tabellini (1994) and Walsh (1995). On the empirical results see Alesina and Summers (1993), Eijfinger and Schaling (1993, 1996) and Fischer (1994).

[2]The key contributions to rational partisan business cycle theory are by Alesina (1987, 1988).

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of establishing an independent central bank come at no apparent output cost.

This paper addresses these issues in a monetary policy model where changes in the *effective* degree of CBI introduce a political effect to the inflation-output trade-off which may counter the traditional economic effect. The key idea is as follows. The government would like the inflation-output combination to reflect its own monetary policy preferences, rather than the central bank's. However, it has a political incentive to satisfy a median voter who generally has policy preferences different than its own. If macroeconomic performance is an important electoral issue, higher CBI lowers the government's reelection probability if the median voter is less inflation averse than itself, but improves its reelection chances if the median voter is more inflation averse. The government reacts to the expected change in its reelection probability by shifting its policy preferences relatively toward, or away from, those of the median voter, as the case may be. The effective stance of monetary policy is then jointly determined by the interaction of the government's political incentive and the central bank's traditional economic influence. I show that the first-order condition of the government's optimization problem is nonlinear in the degree of effective CBI and proceed to study the relationship between an exogenous increase in effective CBI and the inflation-output trade-off.

There are three main results. First, as the political and economic effects of changing CBI may counteract each other, their net impact on average inflation and inflation-output variability is generally ambiguous. In absolute terms, the magnitude of the political effect is larger the lower the initial degree of effective CBI and the government's political incentive, and the greater the policy differences between the government and the median voter. When the political effect is strong, increasing effective CBI may yield higher average and more variable inflation and less variable output growth.

Second, the political effect's relative weight depends upon the median voter's expected gain or loss from higher effective CBI. I derive a result linking the median voter's expected reaction to her relative inflation aversion vis-á-vis the government, the degree of time inconsistency and inflation shock variability in the economy. This yields a taxonomy of equilibrium outcomes: when the median voter expects to *benefit* from higher CBI the government optimizes by *lowering* its political incentive, thus moving the effective stance of monetary policy away from the median voter's preferences. Therefore, if the median voter is *more* inflation averse than the government the political effect counteracts the economic effect and the effective policy stance loosens. Increasing effective CBI then induces smaller absolute changes in inflation-output variability and on average inflation. In contrast, if the median voter is *less* inflation averse than the government, the political effect reinforces the economic effect and the absolute impact on average inflation and inflation-output variability is stronger than otherwise. The intuition is symmetric to the first case: as the median voter expects to *lose* from higher CBI, the government optimally reacts by *increasing* its political incentive. In both cases, the net impact of the two effects is a function of the parameters affecting the median voter's expected loss.

Third, a threshold is derived for the initial degree of effective CBI below which the absolute change in inflation variability following a change in CBI exceeds that in output variability. The level of the threshold is shown to be decreasing in the government's and median voter's inflation aversion and increasing in the central bank's inflation aversion. Therefore, the "free lunch" claim whereby higher CBI induces lower average and less variable inflation with no apparent rise in output variability may be explained by a combination of low inflation aversion for the government and/or median voter and not excessively high inflation aversion for the central bank. Intuitively, if the initial degree of effective CBI is less than the threshold, inflation variability will be more sensitive than output variability to changes in CBI.

The remainder of the paper is arranged as follows. In Section 2, the model is presented, effective CBI is defined and reduced forms for equilibrium effective inflation and output growth obtained. In Section 3, I derive a key result linking the government's electoral incentive to its expected macroeconomic performance. Section 4 then analyzes the net impact of increasing effective CBI on the components of the inflation-output trade-off. The discussion focuses on the relative weight of the economic and political effects of changing CBI on average inflation and inflation-output variability. Section 5 concludes.

2. The Model

The model involves a two-period game where the government chooses the degree of CBI, the central bank implements monetary policy, and the representative agent—or median voter—sets expected inflation and votes for the government or the opposition.[3] The timing follows Rogoff (1985) and Alesina and Gatti (1995). All variables refer to representative period t: the period-t reelection probability refers to the elections for period t occurring at the end of period $t - 1$. The length of each period coincides with the incumbent political party's term in office. The government knows the central bank's and the median voter's expected loss functions and the distribution of the inflation shock. Upon being elected for period t at the end

[3]The identity of the political party in power does not matter for the results, so I focus on the policy outcomes independently of the government's partisan affiliation.

of period $t - 1$, it has to choose the degree of CBI before observing the inflation shock aiming to minimize its expected losses. Period-t nominal wage contracts and expected inflation are then set and the actual inflation-output growth combination (π_t, y_t) determined as a joint function of the degree of CBI and the inflation shock realization.

The inflation-output trade-off is given by a stylized short-run aggregate supply function:

$$y_t = \pi_t - E_{t-1}\pi_t + \varepsilon_t , \tag{1}$$

where y_t and π_t denote levels of real output growth and inflation, $E_{t-1}\pi_t$ is expected inflation determined at the end of period $t - 1$, and ε_t is a normal iid inflation shock with mean zero and variance σ_ε^2. The slope of (1) and the natural rate of output growth are normalized to one and zero, respectively, while the inflation and output growth targets are set to zero and $y^* > 0$. The government thus has a time-inconsistent incentive to create surprise inflation and temporarily raise output growth.[4] The period-t losses of the government, the central bank and the median voter, L_t^G, L_t^{CB} and L_t^V respectively, are defined over quadratic deviations of policy outcomes from their targets:

$$L_t^G = \frac{1}{2}\,[\pi_t^2 + b^G(y_t - y^*)^2 - \gamma_t P_t] , \quad b^G > 0 ;$$

$$L_t^{CB} = \frac{1}{2}\,[\pi_t^2 + b^{CB}(y_t - y^*)^2] , \quad b^{CB} > 0 ;$$

$$L_t^V = \frac{1}{2}\,[\pi_t^2 + b^V(y_t - y^*)^2] , \quad b^V > 0 . \tag{2}$$

The loss functions differ from each other in two respects. First, it is assumed that $b^G > b^{CB}$ and $b^V > b^{CB}$: the central bank is more inflation averse than the government and the median voter. However, the latter may be either more or less inflation averse than the government, reflecting shifts in the electorate's macroeconomic policy preferences within a term in office. Second, L_t^G also involves the government's political incentive in period t. Unlike the central bank, the government is sensitive to P_t, the *actual* percentage of the vote it gets in the election. The government's losses are strictly decreasing in P_t. As the degree of CBI is set before observing the inflation

[4]Due, for example, to labor market distortions and/or inflation tax considerations: see Barro and Gordon (1983) and Cukierman (1992). As there are no explicit monetary policy instruments, the government is assumed to have direct control over inflation.

shock, the government can *ex ante* react to changes in its expected share of the vote, that is, in its reelection probability. I assume that the government can change its political incentive γ_t as follows:

$$\frac{\partial \gamma_t}{\partial E_{t-1} P_t} \equiv -1 \; . \tag{3}$$

Macroeconomic policy is more likely to be affected by political considerations if the incumbent is less likely to be reelected, and vice versa. A decline in $E_{t-1}\, P_t$ induces a rise in γ_t as the government tries to stem its falling reelection probability by shifting monetary policy more in line with the median voter's preferences. Conversely, a government whose reelection probability rises will lower its political sensitivity, other things equal.[5]

The government's actual percentage of the vote, P_t, is assumed to be a decreasing function of the median voter's loss in period t:

$$P_t = A - B\, L_t^V \; ,$$

where A, $B > 0$ are positive constants normalized to yield a well-defined probability measure. Taking expectations yields the government's reelection probability—its *expected* percentage of the vote—as a function of the median voter's expected loss:

$$E_{t-1}P_t = A - B\; E_{t-1}L_t^V \; .$$

Coefficient B measures the negative impact of the median voter's expected loss on the election outcome. Worse expected macroeconomic performance, evaluated according to the median voter's preferences, lowers the government's reelection probability by a factor B. A larger B suggests that the median voter is more sensitive to the state of the economy, ceteris paribus.

Substituting the actual election outcome P_t in the government's loss function yields

$$L_t^G = \frac{1}{2}\,[(1 + \gamma_t B)\pi_t^2 + (b^G + \gamma_t B b^V)(y_t - y^*)^2] - \gamma_t\, A \; . \tag{4}$$

Equation (4) will be referred to as the government's *effective* loss function

[5]Assuming $d\gamma_t/dE_{t-1}\, P_t = -1$ simplifies the calculations, although in general it suffices that it be negative. The results are robust to an asymmetric specification wherein γ_t rises when $E_{t-1}\, P_t$ falls, but stays unchanged when $E_{t-1}P_t$ rises.

to emphasize that, in the presence of a political incentive, the government's effective policy preferences are dependent on the median voter's. Minimizing (4) subject to the short-run supply function (1) yields the actual inflation-output combination when the government has full control over monetary policy:

$$\pi_t^G = \frac{b^G + \gamma_t B b^V}{1 + \gamma_t B} y^* - \frac{b^G + \gamma_t B b^V}{1 + b^G + \gamma_t B(1 + b^V)} \varepsilon_t ;$$

$$y_t^G = \frac{1 + \gamma_t B}{1 + b^G + \gamma_t B(1 + b^V)} \varepsilon_t . \tag{5}$$

Similarly, minimizing the central bank's loss function in (2) yields the inflation-output combination when the central bank is completely independent:

$$\pi_t^{CB} = b^{CB} y^* - \frac{b^{CB}}{1 + b^{CB}} \varepsilon_t ;$$

$$y_t^{CB} = \frac{1}{1 + b^{CB}} \varepsilon_t . \tag{6}$$

The inflation-output combinations in Equations (5) and (6) denote the actual policy outcomes, as the preferred combination of both the government and the central bank is just $\pi_t = 0$ and $y_t = y^*$.

At this point variable $g_t \in [0, 1]$ is introduced, measuring the degree of *effective* CBI in period t. The government chooses g_t at the beginning of its term in office, that is, after expected inflation has been set but before observing the inflation shock's realization, and is committed to its choice for the whole term in office:

$$\pi_t \equiv g_t \pi_t^{CB} + (1 - g_t)\pi_t^G ;$$

$$y_t \equiv g_t y_t^{CB} + (1 - g_t) y_t^G . \tag{7}$$

Equations (7) define the *effective* inflation and output growth rates as weighted averages of the inflation-output combinations when $g_t = 0$ and $g_t = 1$, where the weights are g_t and $1 - g_t$. Thus $g_t = 0$ implies that the central bank is effectively a branch of government, while when $g_t = 1$ the central bank is solely responsible for monetary policy. For intermediate values of g_t, the effective inflation-output combination captures the joint influence on monetary policy of the government's and central bank's preferences, with higher values indicating that the central bank is effectively more independent.[6]

Substituting (7) into (4) and minimizing the government's loss function yields the equilibrium effective inflation and output growth rates:

$$\pi_t = g_t \left[b^{CB} y^* - \frac{b^{CB}}{1 + b^{CB}} \varepsilon_t \right]$$

$$+ (1 - g_t) \left[\frac{b^G + \gamma_t B b^V}{1 + \gamma_t B} y^* - \frac{b^G + \gamma_t B b^V}{1 + b^G + \gamma_t B(1 + b^V)} \varepsilon_t \right]; \quad (8)$$

$$y_t = \frac{g_t}{1 + b^{CB}} \varepsilon_t + (1 - g_t) \frac{1 + \gamma_t B}{1 + b^G + \gamma_t B(1 + b^V)} \varepsilon_t . \quad (9)$$

3. Endogenous Political Incentive and Equilibrium CBI

The effects of changing g_t on the expectation and variance of inflation and output growth depend upon the impact of changing effective CBI on the government's political incentive. The latter is an implicit function of the change in the median voter's expected loss:

$$\frac{\partial \gamma_t}{\partial g_t} = \frac{\partial \gamma_t}{\partial E_{t-1} P_t} \frac{\partial E_{t-1} P_t}{\partial E_{t-1} L_t^V} \frac{\partial E_{t-1} L_t^V}{\partial g_t} . \quad (10)$$

Substituting in (10) the values of $\partial \gamma_t / \partial E_{t-1} P_t$ and $\partial E_{t-1} P_t / \partial E_{t-1} L_t^V$ implies:

$$\frac{\partial \gamma_t}{\partial g_t} = B \frac{\partial E_{t-1} L_t^V}{\partial g_t} . \quad (11)$$

[6]The effective CBI parameter g may be interpreted narrowly to encompass the legislated statutes of the central bank, specifying, among other things, the terms in office of the monetary authorities and the government's difficulty in replacing them in the event of disagreement over objectives. However, unless these statutes are written into the constitution, a government may use its discretion to change the legislation controlling the central bank's operating procedures as suggested in Blinder (1997). Such changes reflect either a change of political party in office, or simply a realization by the incumbent that economic conditions have changed. A broader interpretation of g would combine both legislative and discretionary influences on CBI, thus capturing the overall variation in the central bank's independence from the executive. Such an interpretation encompasses CBI in both instruments and goals, as defined by Debelle and Fischer (1996) and Fischer (1995).

Thus γ changes in the same direction as the median voter's expected loss, and by a factor $B > 0$. Consider an exogenous rise in g_t. If $\partial E_{t-1}L_t^V/\partial g_t > 0$ the median voter is worse off in expectation. The government then reacts by increasing its political incentive γ_t, thus shifting the effective stance of monetary policy toward the median voter's policy preferences. In so doing, it partially mitigates the adverse consequences to its reelection probability. Conversely, if $\partial EL^V/\partial g < 0$ the median voter is better off in expectation, so the government lowers γ_t in its effective loss function. The effective stance of monetary policy then shifts away from the median voter's preferences.

Therefore, the equilibrium change in the political incentive following an exogenous change in g_t is a function of the corresponding change in the expected loss of the median voter. As expected inflation is set before the government's choice of g_t, the median voter takes γ_t as given in minimizing $E_{t-1}L_t^V$. In Appendix A the following is shown:

LEMMA.

$$\frac{\partial E_{t-1}L_t^V}{\partial g_t} = (A_1 + g_t A_2)\sigma_\epsilon^2 + A_3(g_t)y^{*2} ;$$

where

$$A_1 = (b^V - b^G)(1 + b^{CB})(b^G + \gamma_t B b^V - (1 + \gamma_t B)b^{CB}) ;$$

$$\begin{aligned} A_2 = {} & \frac{(b^V + b^{CB^2})(1 + b^G + \gamma_t B(1 + b^V))^2}{1 + b^{CB}} \\ & + (1 + b^{CB})^2[b^V(1 + \gamma_t B)^2 + (b^G + \gamma_t B b^V)^2] \\ & - 2(1 + b^{CB})(1 + b^G + \gamma_t B(1 + b^V)) \\ & \cdot[b^V(1 + \gamma_t B) + (b^G + \gamma_t B b^V)] > 0 ; \end{aligned}$$

$$\begin{aligned} A_3(g_t) = {} & \frac{g_t b^{CB}(1 + \gamma_t B) + (1 - g_t)(b^G + \gamma_t B b^V)}{(1 + \gamma_t B)^2} \\ & \cdot[(1 + \gamma_t B)b^{CB} - b^G - \gamma_t B b^V] < 0 . \end{aligned}$$

From Equation (11) we get:

COROLLARY.

$$\frac{\partial \gamma_t}{\partial g_t} = B[(A_1 + g_t A_2)\sigma_\varepsilon^2 + A_3(g_t)y^{*2}] .$$

The Lemma decomposes the response of the median voter's expected loss to a change in g in two additively separable terms.[7] The term in σ_ϵ^2 captures the influence on $\partial EL^V/\partial g$ of the inflation shock variance—hence of the variability of inflation and output growth—while the term in y^{*2} captures the influence of the output target. Separability of the terms in σ_ε^2 and y^* implies that the effects of changing g on the expectation (A_3) and variances (A_1 and A_2) of inflation and output growth are evaluated independently. I presently analyze an exogenous increase in effective CBI by evaluating its effect on the median voter's expected loss.

In Appendix A, it is shown that $A_1 > (<)0$ as $b^V > (<)b^G$, $A_2 > 0$ and $A_3 < 0$. Moreover, coefficients A_1 and A_2 are independent of g whereas A_3 is not. The intuition for the sign of A_1 is as follows. Consider, without loss of generality, an increase in the volatility of the inflation shock. A median voter who is less inflation averse than the government ($b^V > b^G$) prefers *lower* effective CBI in order to accommodate the impact of higher volatility on output growth, so increasing g makes this median voter worse off: $\partial EL^V/\partial g > 0$. Conversely, a median voter who is more inflation averse than the government ($b^V < b^G$) prefers *higher* effective CBI in order to accommodate the effects of higher volatility on the inflation rate, that is, $\partial EL^V/\partial g < 0$.

The second term ($g\ A_2\sigma_\varepsilon^2 > 0$) relates $\partial EL^V/\partial g$ to the *initial* degree of effective CBI. When g increases, the median voter is always worse off in expectation as $A_2 > 0$ regardless of the initial value of g. This is because relatively *more* of the increase in inflation shock volatility will affect output growth. In other words, the median voter is always worse off as she is less inflation averse than the central bank, and raising g shifts the effective stance of monetary policy toward the central bank's preferences. This is consistent with $gA_2\sigma_\varepsilon^2$ increasing in the initial value of g: higher inflation volatility is worse for the median voter the higher the initial degree of effective CBI.

The last term ($A_3(g)y^* < 0$) captures the link between the time-inconsistent output growth target and expected inflation. If y^* rises but g is constant, the median voter is worse off regardless of her policy preferences, as raising g would mitigate the rise in expected inflation at no cost to expected output growth. Increasing effective CBI partially alleviates the greater time inconsistency problem, so the expected benefit to the median voter is always positive. A median voter facing higher y^* thus always favors more effective CBI. This is consistent with coefficient $A_3(g)$ absolutely decreasing in g: $\partial A_3/\partial g < 0$. Ceteris paribus, any degree of time inconsistency harms the median voter less if the central bank is more independent.

[7]The time subscripts will be subsequently dropped except when necessary, as in a two-period framework g_t only gets determined once. In a multi-period model g_t would be a state variable.

4. The Impact of CBI on the Inflation-Output Trade-Off

Effective CBI and Expected Inflation

In Appendix B it is shown that the first-order condition determining the *optimal* degree of effective CBI is nonlinear in g and the underlying parameters. In this section attention is restricted to analyzing the simpler case of an *exogenous* increase in effective CBI and its impact on the inflation-output trade-off. The effects of an exogenous decrease in effective CBI are symmetric.

Taking expectations in (9) implies that expected output growth is zero regardless of the degree of effective CBI. Expected inflation involves taking expectations in (8):

$$E\pi = \left[\frac{b^G + \gamma B b^V}{1 + \gamma B} + g\left(b^{CB} - \frac{b^G + \gamma B b^V}{1 + \gamma B}\right)\right] y^* . \qquad (12)$$

Differentiating (12) with respect to g and using $\partial\gamma/\partial g = B\ \partial EL^V/\partial g$ yields:

PROPOSITION 1.

$$\frac{\partial E\pi}{\partial g} = \left(b^{CB} - \frac{b^G + \gamma B b^V}{1 + \gamma B}\right) y^* + (1 - g)\frac{B^2 y^*}{(1 + \gamma B)^2}(b^V - b^G)\frac{\partial EL^V}{\partial g} .$$

The two additive terms capture the *economic* and *political* effects of increasing g on expected inflation. The first term, which is negative for all $y^* > 0$, is the traditional economic effect whereby higher effective CBI lowers average inflation because the central bank is more inflation averse than the government and the median voter.[8] The second term captures the political effect of changing g: if this were zero, expected inflation would be strictly decreasing in g. In general, however, the political effect is nonzero so the net impact of higher g on $\partial E\pi/\partial g$ is ambiguous.

Proposition 1 implies the following necessary and sufficient condition for the economic effect to outweigh the political effect:

$$\frac{\partial E\pi}{\partial g} < 0 \iff$$

$$(1 - g)\frac{B^2}{1 + \gamma B}(b^V - b^G)\frac{\partial EL^V}{\partial g} < b^G - b^{CB} + \gamma B(b^V - b^{CB}) . \qquad (13)$$

[8]This is one part of Rogoff's (1985) insight on the benefits of instituting a conservative independent central bank, the other being lower inflation variability.

As the economic effect on the right side is positive, the inequality will always hold if the political effect is negative: $(b^V - b^G)\partial EL^V/\partial g < 0$. If, however, the political effect is *positive* then the net impact of raising g on expected inflation depends on the two effects' relative magnitude. On the one hand, the magnitude of the economic effect increases in $b^G - b^{CB}$ and $b^V - b^{CB}$, reflecting the inflationary gains of delegating monetary policy to a conservative central bank. On the other hand, the political effect's magnitude is increasing in $|b^V - b^G|$ and B but decreasing in γ and the initial g value, ceteris paribus. A lower initial g value increases the government's control over monetary policy, while changing γ in either direction from a low initial value has a larger impact on the effective stance of monetary policy. In contrast, if $|b^V - b^G| \to 0$ and/or $B \to 0$ the economic effect dominates: the median voter's preferences are very close to the government's, and/or macroeconomic performance is not an important political issue.[9] To summarize, if the political effect is positive and absolutely smaller than the economic effect, their net impact is negative. However, if the political effect is absolutely greater than the economic effect, their net impact is positive so that increasing g will raise expected inflation.

Determining the political effect's relative magnitude requires signing $(b^V - b^G)\partial EL^V/\partial g$. From Section 3, if $\partial EL^V/\partial g < 0$ then the government reacts to higher g by lowering γ. If $b^V < b^G$ the political effect is positive and counters the economic effect: the effective monetary policy stance becomes less inflation averse. Higher effective CBI then leads to a smaller decline in expected inflation than in the absence of the political incentive. In contrast, if $b^V > b^G$ monetary policy effectively tightens as the negative political effect reinforces the economic effect. Increasing effective CBI then leads to a bigger decline in expected inflation. Conversely, if $\partial EL^V/\partial g > 0$ the government responds to higher g by raising γ. If $b^V < b^G$, then the political effect is negative and the decline in expected inflation greater than otherwise. In contrast, if $b^V > b^G$ the political effect is positive, so that increasing g induces a smaller absolute decline in expected inflation. The four cases are summarized in Table 1.

The Lemma of Section 3 evaluates the political effect by focusing on $\partial EL^V/\partial g$. Its components include coefficients A_1 and A_2 in σ_ϵ^2 and coefficient A_3 in y^*. First, A_1 is positive (negative) if $b^V > (<)\ b^G$. Second, the term in A_2 is always positive, inducing $\partial EL^V/\partial g > 0$. Finally, coefficient A_3 is always negative. The impact of A_3, and hence y^*, on $\partial EL^V/\partial g$ increases in absolute if the central bank becomes more inflation averse given b^V and b^G ($\downarrow b^{CB}$) and/or if G and V become relatively less inflation averse ($\uparrow b^V, b^G$).

[9]The economic effect also dominates if initially effective CBI is very high ($g \to 1$). The government has then effectively delegated control of monetary policy to the central bank.

TABLE 1. *The Political Effect on* $\frac{\partial E\pi}{\partial g}$

	$\frac{\partial EL^V}{\partial g} < 0$	$\frac{\partial EL^V}{\partial g} > 0$
$b^V < b^G$	+	−
$b^V > b^G$	−	+

Referring to Table 1, $b^V < b^G$ implies $\partial EL^V/\partial g$ is negative on account of A_1 and A_3 and positive on account of A_2. If the initial value of g is small, the negative impact of A_1 and A_3 is likely to dominate $gA_2\sigma_\varepsilon^2$, so overall the political effect is positive and counters the economic effect. This is consistent with the political effect absolutely decreasing in g. Conversely, $b^V > b^G$ implies $\partial EL^V/\partial g$ is positive on account of A_1 and A_2 and negative on account of A_3. Provided g is small and the output growth target y^* is significantly positive, the negative impact of A_3 is likely to outweigh A_1 and $gA_2\sigma_\varepsilon^2$, so that in net $\partial EL^V/\partial g < 0$. This is also the likely outcome if σ_ε is relatively small. The political effect is then negative and reinforces the economic effect.

Note that in both cases parameters were such that $\partial EL^V/\partial g < 0$: the median voter is better off with higher g. Intuitively, a small initial g value implies that a median voter who is more inflation averse than the government is always better off with increased effective CBI. In contrast, when $b^V > b^G$ the median voter is worse off because of the smaller degree of inflation shock accommodation (the A_1 and A_2 terms), but better off because of the lower time inconsistency (the A_3 term). Overall, provided b^V is not much greater than b^G, and σ_ε^2 and/or the initial value of g are relatively small, the median voter is likely to be better off with higher g. As a result, the government lowers γ, which in this case implies a greater decline in expected inflation.[10] Nevertheless, it should be stressed that there is no a priori reason for selecting a particular outcome. A full taxonomy requires a case-by-case eval-

[10]Note that in the extreme case that y^* is very large compared to σ_ε then $A_3(g)y^{*2} \ll 0$ outweighs any positive effect from the volatility terms in A_1 and A_2. This results in $\partial EL^V/\partial g < 0$ and the following two possibilities. If $b^V > b^G$ then $(b^V - b^G)\partial EL^V/\partial g < 0$ and the political effect is negative, reinforcing the economic effect. Conversely, if $b^V < b^G$ then $(b^V - b^G)\partial EL^V/\partial g > 0$ and the political effect is positive, so the sign of $\partial E\pi/\partial g$ is ambiguous without further information on parameter values. Equivalently, if the inflation rate is almost constant ($\sigma_\varepsilon \to 0$) then $\partial EL^V/\partial g < 0$ regardless of the degree of time inconsistency implicit in y^*. Once more, the political effect is negative if $b^V > b^G$ and positive if $b^V < b^G$.

uation of the net impact of increasing g on $\partial E\pi/\partial g$, which is beyond the scope of this paper.

Effective CBI and Inflation-Output Variability

Turning to the second moments of the equilibrium policy outcomes, the variance of the effective inflation-output combination from Equations (8)–(9) is just

$$\sigma_\pi^2 = \left(\frac{gb^{CB}}{1 + b^{CB}} + (1 - g)\frac{b^G + \gamma Bb^V}{1 + b^G + \gamma B(1 + b^V)}\right)^2 \sigma_\varepsilon^2 \tag{14}$$

$$\sigma_y^2 = \left(\frac{g}{1 + b^{CB}} + (1 - g)\frac{1 + \gamma B}{1 + b^G + \gamma B(1 + b^V)}\right)^2 \sigma_\varepsilon^2 \tag{15}$$

Differentiating with respect to g and applying Equation (11) yields

PROPOSITION 2.

$$\frac{\partial\sigma_\pi^2}{\partial g} = 2\left(g\frac{b^{CB}}{1 + b^{CB}} + (1 - g)\frac{b^G + \gamma Bb^V}{1 + b^G + \gamma B(1 + b^V)}\right)\sigma_\varepsilon^2 \cdot \left[\frac{b^{CB}}{1 + b^{CB}} - \frac{b^G + \gamma Bb^V}{1 + b^G + \gamma B(1 + b^V)} + \frac{(1 - g)B^2(b^V - b^G)}{(1 + b^G + \gamma B(1 + b^V))^2}\frac{\partial EL^V}{\partial g}\right];$$

$$\frac{\partial\sigma_y^2}{\partial g} = 2\left(g\frac{1}{1 + b^{CB}} + (1 - g)\frac{1 + \gamma B}{1 + b^G + \gamma B(1 + b^V)}\right)\sigma_\varepsilon^2 \cdot \left[\frac{1}{1 + b^{CB}} - \frac{1 + \gamma B}{1 + b^G + \gamma B(1 + b^V)} + \frac{(1 - g)B^2(b^G - b^V)}{(1 + b^G + \gamma B(1 + b^V))^2}\frac{\partial EL^V}{\partial g}\right].$$

Both $\partial\sigma_\pi^2/\partial g$ and $\partial\sigma_y^2/\partial g$ are absolutely increasing in the variance of the inflation shock. However, whereas the first multiplicative terms are always positive and less than 1, the sign of the second terms is ambiguous. The difference of the first two components in square brackets is unambiguously negative for $\partial\sigma_\pi^2/\partial g$ and positive for $\partial\sigma_y^2/\partial g$. This captures the traditional economic effect of increasing effective CBI, involving less variable inflation and more variable output growth. The last components in square brackets capture the political effect of increasing g. The political effects are absolutely decreasing in the initial value of g, as in the expression for expected inflation in Proposition 1. They are equal and opposite but not necessarily of the same sign as the economic effects. There are 3 general cases:

(i) A negative political effect of higher g on $\partial\sigma_\pi^2/\partial g$ reinforces the economic

TABLE 2. *The Political Effect on* $\dfrac{\partial\sigma_\pi^2}{\partial g}$ and $\dfrac{\partial\sigma_y^2}{\partial g}$

	$\dfrac{\partial EL^V}{\partial g} < 0$	$\dfrac{\partial EL^V}{\partial g} > 0$
$b^V < b^G$	+ (−)	− (+)
$b^V > b^G$	− (+)	+ (−)

effect on inflation variability. Similarly, a positive political effect on $\partial\sigma_y^2/\partial g$ reinforces the respective economic effect. An increase in g then induces less variable inflation and more variable output growth than in the absence of the political incentive.

(ii) A positive political effect on $\partial\sigma_\pi^2/\partial g$ counters the economic effect. If the political effect is absolutely smaller than the economic effect, their net impact is dominated by the economic effect and $\partial\sigma_\pi^2/\partial g$ will still be negative. Similarly, if the negative political effect of higher g on $\partial\sigma_y^2/\partial g$ is absolutely smaller than the respective positive economic effect, $\partial\sigma_y^2/\partial g$ will be positive but smaller than otherwise.

(iii) If the political effect on $\partial\sigma_\pi^2/\partial g$ is positive and absolutely greater than the negative economic effect, inflation variability *increases* with g. For $\partial\sigma_y^2/\partial g$, the net impact is negative when the negative political effect is bigger in absolute than the economic effect. Higher g then implies less variable output growth.

The possible outcomes for the influence of the political effect on $\partial\sigma_\pi^2/\partial g$ and $\partial\sigma_y^2/\partial g$ are summarized in Table 2 (signs for $\partial\sigma_y^2/\partial g$ are in parentheses).

The net impact of changing g on the variability of inflation and output growth is determined by the response of the median voter's expected loss, as was the case for expected inflation. In signing $\partial EL^V/\partial g$, recall from the Lemma that if either $b^V \ll b^G$, σ_ε^2 and/or g are small or y^* is large, the median voter is likely to be better off in expectation with higher g. Table 2 then indicates that for $b^V < b^G$ the political effect is positive on $\partial\sigma_\pi^2/\partial g$ and negative on $\partial\sigma_y^2/\partial g$, hence the absolute changes in inflation and output variability are smaller than otherwise. Conversely, if either $b^V \gg b^G$, σ_ε^2 and/or g are large or y^* is small, the median voter is likely to be worse off with higher g. The political effect is then positive on $\partial\sigma_\pi^2/\partial g$ and negative on $\partial\sigma_y^2/\partial g$ so once more effective inflation and output growth become less variable. For intermediate values of the parameters, a numerical exploration of the Lemma is required before referring to Table 2.

Relative Variability of Inflation and Output Growth

This section compares the relative magnitudes of the changes in inflation and output growth variability following an exogenous rise in g. The economic and political effects of higher g on σ_π^2 and σ_y^2 are equal and opposite, so any difference in the absolute values of $\partial\sigma_\pi^2/\partial g$ and $\partial\sigma_y^2/\partial g$ is attributed to their respective first multiplicative term. Comparing the expressions in Proposition 2 implies

PROPOSITION 3.

$$g < (>)\ \bar{g} \Leftrightarrow \left|\frac{\partial\sigma_\pi^2}{\partial g}\right| > (<) \left|\frac{\partial\sigma_y^2}{\partial g}\right| ,$$

where the *threshold* degree of effective CBI, $\bar{g}$, is given by

$$\bar{g} = \frac{(1 + b^{CB})(b^G + \gamma B b^V - 1 - \gamma B)}{2(b^G + \gamma B b^V - (1 + \gamma B) b^{CB})} .$$

As the denominator of $\bar{g}$ is always positive, requiring $\bar{g} \in [0, 1]$ implies the following conditions on the inflation aversion coefficients:

$$\begin{aligned} &\bar{g} \geq 0 \Leftrightarrow b^G + \gamma B b^V > 1 + \gamma B ; \\ &\bar{g} \leq 1 \Leftrightarrow 0 < b^{CB} < 1 . \end{aligned}$$

These restrictions are in line with the ordering of monetary policy preferences: at least one of (b^V, b^G) has to exceed 1 and b^{CB} has to be less than 1, so b^V can lie on either side of b^G.

Proposition 3 relates the absolute values of $\partial\sigma_\pi^2/\partial g$ and $\partial\sigma_y^2/\partial g$ to the initial value of g. If the initial degree of effective CBI is less than $\bar{g}$ then $\partial\sigma_\pi^2/\partial g$ exceeds $\partial\sigma_y^2/\partial g$ in absolute. Conversely, if the initial degree of effective CBI exceeds $\bar{g}$ then $\partial\sigma_y^2/\partial g$ is absolutely greater. The initial degree of CBI may therefore be classified as "high" or "low" depending on its relation to the threshold $\bar{g}$. Intuitively, at low initial g values the government exercises more influence over monetary policy, so a given increase in effective CBI has a first-order effect on inflation variability but only a second-order effect on output growth variability. Therefore, inflation variability changes absolutely more than output variability. Conversely, at high initial g values the central bank has the larger influence on monetary policy. Increasing g further then has a first-order effect on output variability and a second-order effect on inflation variability.

Assume, without loss of generality, that the initial value of g is uniformly distributed on [0, 1], so its expected value of 0.5 reflects equal influence on monetary policy by the central bank and the government. Proposition 3 then suggests that if $\bar{g}$ exceeds the expected initial value of 0.5 inflation variability is likely to exceed output growth variability in absolute terms, and *vice versa* if $\bar{g}$ is less than 0.5. This implies the following condition on the parameters:

COROLLARY.

$$\bar{g} \geq (<) \frac{1}{2} \Leftrightarrow b^G + \gamma B b^V \geq (<) \frac{1 + \gamma B}{b^{CB}} .$$

Thus for $\bar{g}$ to exceed 0.5, b^G and/or b^V have to be greater than b^{CB}, which cannot be much smaller than 1. If the initial value of g is uniformly distributed, Proposition 3 implies a higher probability that it is less than the threshold. Increasing g will then induce a larger absolute change to inflation variability than output growth variability.

To summarize, the "free lunch" empirical observation whereby increasing effective CBI induces lower average and less variable inflation with no apparent rise in output growth variability may be explained in this framework by a high threshold value $\bar{g}$. In turn, this requires a low degree of inflation aversion for the government and/or median voter, and not excessively high inflation aversion for the central bank. If b^{CB} takes on small values then $\bar{g}$ will be less than 0.5. The initial g is then more likely to exceed the threshold, so that increasing g has a larger absolute effect on output growth variability than inflation variability.

5. Conclusion

This paper developed a political economy model for studying the effects of higher central bank independence on the inflation-output trade-off. A distinction was introduced between the economic and political effects of changing the degree of CBI through the notion of effective CBI. The resulting taxonomy of the effects of increasing CBI on the inflation-output combination allows for a variety of equilibrium outcomes.

It was shown that the political effect may either counter or reinforce the economic effect as a function of parameter values. The absolute magnitude of the political effect is greater the lower the initial degree of effective CBI and the policy differences between the government and median voter, and the smaller the initial value of the government's political incentive. The net impact of increasing CBI on the inflation-output trade-off is a function of the expected reaction of the median voter. Other things equal, more time inconsistency and higher shock variability imply the median voter is worse off in expectation, while when the median voter is more inflation averse than the government she is more likely to be better off with higher CBI. It is thus possible that increasing CBI may yield higher expected inflation and inflation variability and less variable output growth, depending on the median voter's relative inflation aversion vis-à-vis the government.

A threshold degree of CBI was also obtained such that, if effective CBI increases while below the threshold, then the absolute change in inflation variability exceeds the change in output variability, that is, the stylized fact underlying the suggestion that CBI is a free lunch. It was shown that a combination of recession averse governments and median voters and a not too inflation averse central bank is required in order for the model to rationalize this stylized fact. More precise statements would require solving numerically for the optimal degree of effective CBI derived in Appendix B. On the theoretical front, these results could be verified against alternative distributional assumptions for the initial g value. More generally, a theoretical extension to the paper would involve modeling a dynamic game in which the median voter's policy preferences evolve as a function of past and expected future macroeconomic and election outcomes. Such extensions seem important in studying the link between monetary policy and central bank independence.

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Appendix A.

Proof of the Lemma (Section 3)

The median voter's expected loss function is

$$EL^V = \frac{1}{2} E[\pi^2 + b^V (y - y^*)^2] .$$

Substituting the effective inflation-output combination from (8)–(9) and taking expectations yields

$$EL^V = \frac{1}{2}\left\{g^2\left[(b^{CB}y^*)^2 + \left(\frac{b^{CB}}{1 + b^{CB}}\right)^2 \sigma_\varepsilon^2\right]\right.$$

$$+ (1 - g)^2\left[\left(\frac{b^G + \gamma B b^V}{1 + \gamma B}\right) y^{*2} + \left(\frac{b^G + \gamma B b^V}{1 + b^G + \gamma B(1 + b^V)}\right)^2 \sigma_\varepsilon^2\right]$$

$$+ 2g(1 - g)\left[\frac{b^{CB}(b^G + \gamma B b^V)}{1 + \gamma B} y^{*2}\right.$$

$$\left. + \frac{b^{CB}\ (b^G + \gamma B b^V)}{(1 + b^{CB})(1 + b^G + \gamma B(1 + b^V))}\right] \sigma_\varepsilon^2$$

$$+ b^V\left[y^{*2} + \left(\frac{g^2}{(1 + b^{CB})^2} + \frac{(1 - g)^2(1 + \gamma B)^2}{(1 + b^G + \gamma B(1 + b^V))^2}\right.\right.$$

$$\left.\left.\left. + \frac{2g(1 - g)(1 + \gamma B)}{(1 + b^{CB})(1 + b^G + \gamma B(1 + b^V))}\right) \sigma_\varepsilon^2\right]\right\}. \tag{A1}$$

Differentiating (A1) with respect to g

$$\frac{\partial EL^V}{\partial g} = (1 - 2g)b^{CB}(b^G + \gamma B b^V)$$

$$\left[\frac{y^{*2}}{1 + \gamma B} + \frac{\sigma_\varepsilon^2}{(1 + b^{CB})(1 + b^G + \gamma B(1 + b^V))}\right]$$

$$+ g b^{CB^2}\left[y^{*2} + \frac{\sigma_\varepsilon^2}{(1 + b^{CB})^2}\right]$$

$$- (1 - g)(b^G + \gamma B b^V)^2\left[\frac{y^{*2}}{(1 + \gamma B)^2} + \frac{\sigma_\varepsilon^2}{(1 + b^G + \gamma B(1 + b^V))^2}\right]$$

$$+ b^V \sigma_\varepsilon^2\left[\frac{g}{(1 + b^{CB})^2} - \frac{(1 - g)(1 + \gamma B)^2}{(1 + b^G + \gamma B(1 + b^V))^2}\right.$$

$$\left. + \frac{(1 - 2g)(1 + \gamma B)}{(1 + b^{CB})(1 + b^G + \gamma B(1 + b^V))}\right]. \tag{A2}$$

Grouping together terms in terms in $\sigma_\varepsilon^\varepsilon$ and y^* yields

$$\frac{\partial EL^V}{\partial g} = \left[\frac{gb^V}{(1 + b^{CB})^2} - \frac{(1 - g)b^V(1 + \gamma B)^2}{(1 + b^G + \gamma B(1 + b^V))^2}\right.$$

$$+ \frac{(1 - 2g)b^V(1 + \gamma B)}{(1 + b^{CB})(1 + b^G + \gamma B(1 + b^V))}$$

$$+ \frac{(1 - 2g)b^{CB}(b^G + \gamma Bb^V)}{(1 + b^{CB})(1 + b^G + \gamma B(1 + b^V))}$$

$$\left. + \frac{gb^{CB^2}}{(1 + b^{CB})^2} - \frac{(1 - g)(b^G + \gamma Bb^V)^2}{(1 + b^G + \gamma B(1 + b^V))^2}\right] \cdot \sigma_\varepsilon^2$$

$$+ \left[gb^{CB^2} + \frac{(1 - 2g)b^{CB}(b^G + \gamma Bb^V)}{1 + \gamma B}\right.$$

$$\left. - \frac{(1 - g)(b^G + \gamma Bb^V)^2}{(1 + \gamma B)^2}\right] \cdot y^{*2} . \tag{A3}$$

Equation (A3) is of the form $\partial EL^V/\partial g = A(g, \cdot)\sigma_\varepsilon^2 + A_3(g, \cdot)y^{*2}$. Rearranging the terms in $A(g, \cdot)$ yields

$$A(g, \cdot) = A_1(\cdot) + gA_2(\cdot)$$

$$= (1 + b^{CB})[1 + b^G + \gamma B(1 + b^V)][b^V(1 + \gamma B)$$

$$+ b^{CB}((b^G + \gamma Bb^V)]$$

$$- (1 + b^{CB})^2[b^V(1 + \gamma B)^2 + (b^G + \gamma Bb^V)^2]$$

$$+ g\left[\frac{(b^V + b^{CB^2})(1 + b^G + \gamma B(1 + b^V))^2}{1 + b^{CB}}\right.$$

$$+ (1 + b^{CB})^2[b^V(1 + \gamma B)^2 + (b^G + \gamma Bb^V)^2]$$

$$- 2(1 + b^{CB})(1 + b^G + \gamma B(1 + b^V))$$

$$\cdot [b^V(1 + \gamma B) + b^{CB}(b^G + \gamma Bb^V)]] . \tag{A4}$$

Substituting (A4) in (A3) yields $\partial EL^V/\partial g$. Signing the coefficients involves straightforward algebra. First, dividing all A_1 terms by $1 + b^{CB}$ and expanding yields

$$[1 + b^G + \gamma B(1 + b^V)][b^V (1 + \gamma B) + b^{CB}(b^G + \gamma Bb^V)] \\ -(1 + b^{CB})[b^V(1 + \gamma B)^2 + (b^G + \gamma Bb^V)^2] .$$

After manipulation this becomes

$$A_1/(1 + b^{CB}) = (b^V - b^G)(b^G + \gamma Bb^V - (1 + \gamma B)b^{CB}) ,$$

which is clearly positive (negative) iff $b^V > (<) b^G$. Turning to coefficient A_2 in (A4) and multiplying throughout by $1 + b^{CB}$ yields

$$\begin{aligned} A_2 = {} & (b^V + b^{CB^2})(1 + b^G + \gamma B(1 + b^V))^2 \\ & + (1 + b^{CB})^2 [b^V (1 + \gamma B)^2 + (b^G + \gamma Bb^V)^2] \\ & -2(1 + b^{CB})(1 + b^G + \gamma B(1 + b^V))[b^V (1 + \gamma B) \\ & \quad + b^{CB} (b^G + \gamma Bb^V)] . \end{aligned} \tag{A5}$$

The second term in (A5) is clearly positive. Subtracting one half of the last term from the first term yields

$$(b^V + b^{CB^2})(1 + b^G + \gamma B(1 + b^V))^2 \\ - (1 + b^{CB})(1 + b^G + \gamma B(1 + b^V))[b^V(1 + \gamma B) + b^{CB}(b^G + \gamma Bb^V)] .$$

Conjecturing that this expression is positive yields

$$b^{CB^2} (1 + \gamma B) + b^V b^G + \gamma Bb^{V^2} > b^V b^{CB} (1 + \gamma B) + b^{CB} b^G + \gamma Bb^{CB} b^V , \tag{A6}$$

which has ambiguous sign. Subtracting the other half of the last term in (A5) from the second term and conjecturing that their difference is positive yields

$$b^{G^2} (1 + \gamma B) + b^V b^G \gamma B + b^V b^{CB} (1 + \gamma B) > b^V b^G \\ + \gamma Bb^{V^2} + b^{CB} b^G (1 + \gamma B) . \tag{A7}$$

Adding inequalities (A6) and (A7) yields

$$(b^V - b^{CB})(b^G + \gamma Bb^V - b^{CB} b^V (1 + \gamma B)) > (b^V - b^G)(b^G \\ + \gamma Bb^V - b^{CB} b^V(1 + \gamma B)) ,$$

which implies

$$b^{CB} < b^G . \tag{A8}$$

Inequality (A8) is always true, thus confirming our earlier conjectures. Therefore coefficient A_2 is always positive.

Finally, expanding coefficient A_3 in (A3) and collecting common terms yields

$$A_3 = \frac{b^{CB}(1 + \gamma B) - b^G - \gamma B b^V}{(1 + \gamma B)^2} [g b^{CB}(1 + \gamma B) + (1 - g)(b^G + \gamma B b^V)] ,$$

which is always negative.

Appendix B.

Derivation of the Optimal Degree of Effective CBI

The government's optimization problem takes into account both the economic and the political effects of changing g. Its choice is the solution to the expected minimization of the effective loss function:

$$\min_{g,\gamma(g)} \frac{1}{2} E[(1 + \gamma B)\pi^2 + (b^G + \gamma B b^V)(y - y^*)^2] - \gamma A . \quad \text{(B1)}$$

Equivalently

$$g^* = \arg\min_g \frac{1}{2} E[(1 + \gamma(g)B)\pi(g)^2 + (b^G + \gamma(g)Bb^V)(y(g) - y^*)^2] - \gamma A . \quad \text{(B2)}$$

Substituting inflation and output growth from Equations (9)–(10):

$$g^* = \arg\min_g \frac{1}{2}\Bigg\{(1 + \gamma B)\Bigg\{\left[g b^{CB} + \frac{(1 - g)(b^G + \gamma B b^V)}{1 + \gamma B}\right]^2 y^{*2} + \left[g\frac{b^{CB}}{1 + b^{CB}} + \frac{(1 - g)(b^G + \gamma B b^V)}{1 + b^G + \gamma B(1 + b^V)}\right]^2 \sigma_\varepsilon^2\Bigg\} + (b^G + \gamma B b^V)\left(y^{*2} + \left[\frac{g}{1 + b^{CB}} + \frac{(1 - g)(1 + \gamma B)}{1 + b^G + \gamma B(1 + b^V)}\right]^2 \sigma_\varepsilon^2\right)\Bigg\} - \gamma A . \quad \text{(B3)}$$

Differentiating (B3) with respect to g, the derivative of the last term is just $A\partial\gamma/\partial g = AB\ \partial EL^V/\partial g$. Applying the Lemma to the right side of the first-order condition implies

$$AB\partial EL^V/\partial g = AB[(A_1 + gA_2)\sigma_\varepsilon^2 - A_3 y^{*2}] \,. \tag{B4}$$

Coefficients A_1 and A_2 are independent of g and A_3 is linear in g, so (B4) is also linear in g. Differentiating all remaining terms in the first-order condition (B3) with respect to g and separating terms in σ_ε^2 and y^* yields

$$\begin{aligned}
&\sigma_\varepsilon^2\Bigg\{(1 + \gamma B)\Bigg[\Bigg(\Bigg(g\Bigg[\frac{b^{CB}}{1 + b^{CB}} - \frac{b^G + \gamma B b^V}{1 + b^G + \gamma B(1 + b^V)}\Bigg] \\
&+\frac{b^G + \gamma B b^V}{1 + b^G + \gamma B(1 + b^V)} \\
&\times\Bigg(\frac{b^{CB}}{1 + b^{CB}} - \frac{b^G + \gamma B b^V}{1 + b^G + \gamma B(1 + b^V)} + (1 - g)\,\frac{B\frac{\partial\gamma}{\partial g}(b^V - b^G)}{(1 + b^G + \gamma B(1 + b^V))^2}\Bigg)\Bigg] \\
&+\frac{b^G + \gamma B b^V}{1 + b^G + \gamma B(1 + b^V)}\Bigg[g\Bigg(\frac{1}{1 + b^{CB}} - \frac{1 + \gamma B}{1 + b^G + \gamma B(1 + b^V)}\Bigg) \\
&+\frac{1 + \gamma B}{1 + b^G + \gamma B(1 + b^V)}\Bigg] \\
&\times\Bigg[\frac{b^G + \gamma B b^V - (1 + \gamma B)b^{CB}}{1 + b^G + \gamma B(1 + b^V)} - (1 - g)\,\frac{B\frac{\partial\gamma}{\partial g}(b^V - b^G)}{1 + b^G + \gamma B(1 + b^V)}\Bigg]\Bigg\} \\
&+y^{*2}(1 + \gamma B)\Bigg\{\Bigg[g\Bigg(b^{CB} - \frac{b^G + \gamma B b^V}{1 + \gamma B}\Bigg) + \frac{b^G + \gamma B b^V}{1 + \gamma B}\Bigg] \\
&\times\Bigg(b^{CB} - \frac{b^G + \gamma B b^V}{1 + \gamma B} + (1 - g)\,\frac{B\frac{\partial\gamma}{\partial g}(b^V - b^G)}{(1 + \gamma B)^2}\Bigg)\Bigg\}\,.
\end{aligned} \tag{B5}$$

After manipulation this becomes

$$\begin{aligned}
&\Bigg[\frac{b^G + \gamma B b^V}{1 + b^G + \gamma B(1 + b^V)}D_2E_3 - GI\,\frac{y^*(1 + \gamma B)}{\sigma_\varepsilon^2} - E_1D_1(1 + \gamma B)\Bigg]\,g^2 \\
&+\Bigg[(1 + \gamma B)(E_1^2 + E_1D_1 - E_2D_1) \\
&+\frac{b^G + \gamma B b^V}{1 + b^G + \gamma B(1 + b^V)}(E_3F_2 - D_2 + F_1D_2) \\
&+(G(G + I) - IH)\frac{y^*(1 + \gamma B)}{\sigma_\varepsilon^2}\Bigg]g + (1 + \gamma B)E_2(E_1 + D_1) \\
&+\frac{b^G + \gamma B b^V}{1 + b^G + \gamma B(1 + b^V)}(F_1F_2 - F_1D_2) + H(I + H)\frac{y^*(1 + \gamma B)}{\sigma_\varepsilon^2}\,.
\end{aligned} \tag{B6}$$

Expression (B6) is therefore a quadratic polynomial in g whose coefficients are given by

$$D_1 = B\frac{\partial\gamma}{\partial g}\frac{b^V - b^G}{(1 + b^G + \gamma B(1 + b^V))^2};$$

$$D_2 = B\frac{\partial\gamma}{\partial g}\frac{b^V - b^G}{1 + b^G + \gamma B(1 + b^V)};$$

$$E_1 = \frac{b^{CB}}{1 + b^{CB}} - \frac{b^G + \gamma B b^V}{1 + b^G + \gamma B(1 + b^V)};$$

$$E_2 = \frac{b^G + \gamma B b^V}{1 + b^G + \gamma B(1 + b^V)};$$

$$E_3 = \frac{1}{1 + b^{CB}} - \frac{1 + \gamma B}{1 + b^G + \gamma B(1 + b^V)};$$

and

$$F_1 = \frac{1 + \gamma B}{1 + b^G + \gamma B(1 + b^V)};$$

$$F_2 = \frac{b^G + \gamma B b^V - (1 + \gamma B)b^{CB}}{1 + b^{CB}};$$

$$G = b^{CB} - \frac{b^G + \gamma B b^V}{1 + \gamma B};$$

$$H = \frac{b^G + \gamma B b^V}{1 + \gamma B};$$

$$I = B\frac{\partial\gamma}{\partial g}\frac{b^V - b^G}{(1 + \gamma B)^2}.$$

Therefore, assuming that the second-order conditions hold, the first-order condition for the government's optimization problem (B1) is of the general form

$$C_2 g^2 + C_1 g + C_0 = A\frac{\partial\gamma}{\partial g}. \tag{B7}$$

As coefficients D_1, D_2 and I entering in C_2 are linear in $\partial\gamma/\partial g$, the remaining terms in the first-order condition form a cubic polynomial in g. This implies the existence of either one or three real roots g^*. Note that if either $B = 0$, $b^V - b^G = 0$ or $\partial\gamma/\partial g = 0$, coefficients D_1, D_2 and I become zero, implying that coefficient C_2 becomes independent of g. Thus in the absence of a political incentive the first-order condition reduces to a quadratic polynomial in g. In the presence of a political incentive, substituting $\partial\gamma/\partial g = B\ \partial EL^V/\partial g$ and Equation (B4) the first-order condition becomes

$$C_2 g^2 + C_1 g + C_0 = AB[(A_1 + gA_2)\sigma_\varepsilon^2 - A_3 y^{*2}]\ . \tag{B8}$$

Appendix C.

Variable Definitions

$g_t \in [0, 1]$	= effective central bank independence.
π_t	= period-t inflation rate.
y_t	= period-t real output growth rate.
ε_t	= period-t $N(0, 1)$ iid inflation shock.
P_t	= the government's actual share of the vote in the elections for period t.
$\gamma_t > 0$	= the government's political incentive.
$A, B > 0$	= fixed coefficients in the government's share of the vote.
$E_{t-1}P_t$	= the government's expected share of the vote (reelection probability).
$L^G, b^G > 0$	= the government's loss function and inflation aversion coefficient.
$L^V, b^V > 0$	= the median voter's loss function and inflation aversion coefficient.
$L^{CB}, b^{CB} > 0$	= the central bank's loss function and inflation aversion coefficient.