# Mediating Market Power in Networks

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**Abstract**

We have shown that competitive transmission markets are not necessarily efficient in the presence of market power. By "competitive" we mean the nodal prices that occur if transmission capacity is auctioned to bidders who have perfect information about generation prices and who do not own generation supplies. In particular, welfare may be higher if a monopolist is allowed to own scarce transmission capacity (model I) or if flow gate rights are allocated in a way that differs from physical transmission constraints (model II).

## 1 INTRODUCTION

The EU Electricity Directive has as its intended goal a single European market for electricity, in order to take advantage of regionally differentiated generation sources (hydro, nuclear, gas, brown coal, etc.), differences in plant age and efficiency, non-coincidental demand peaks, reduced volatility and greater security afforded by an integrated and larger system, as well as increased competition, all of which should improve efficiency and lower prices. One major obstacle to realising the gains from increased efficiency is the presence of localised market power, resulting in inefficient dispatch, higher prices adversely affecting industrial competitiveness, and inefficient incentives for investment.

This paper addresses the question of how to mitigate market power in meshed electricity networks, where transmission constraints can fragment the network and create local pockets of enhanced market power for the generators sheltered by the transmission constraint. Two examples illustrate this. Scotland has a large potential export surplus of power, as a result of the shift away from energy-intensive heavy industry, but exports to England are constrained by the capacity of the interconnectors (and grid links further south, notably over the National Parks in North Yorkshire). Scotland remains a vertically integrated duopoly, while England and Wales have moved from a price-setting duopoly at privatisation in 1990 to a fairly fragmented market by 2001. Figure 1 shows the rapid increase in the number of different generation companies in England and Wales.

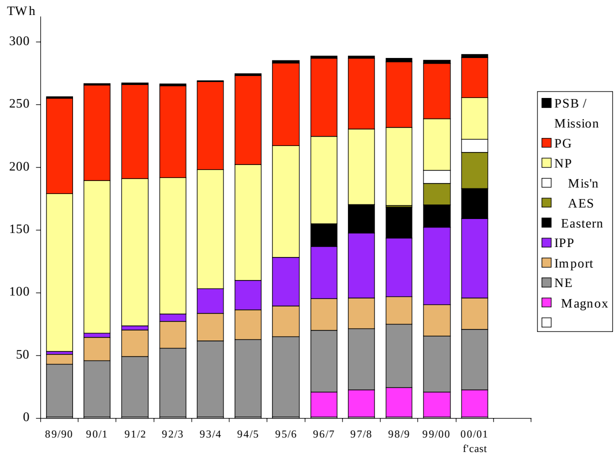


Figure 1: Generation by companies in England and Wales, 1990-2001

The consequences of the divergence of concentration in the two interconnected markets has been a reversal in the ranking of final electricity prices, as Figure 2 shows. Originally final consumer prices in Edinburgh, Scotland were about 10% lower than in London, England, but after 1997 were actually more expensive.

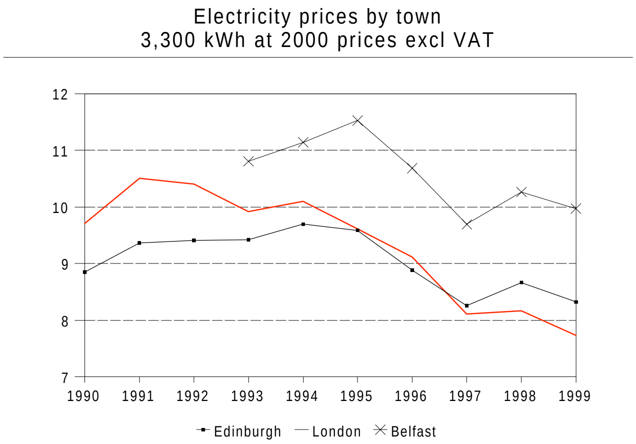


Figure 2: Domestic electricity prices in Scotland, England and N. Ireland

The second example comes from The Netherlands, which is interconnected with Germany (2 ties) and Belgium (and hence to France).[1] Again The Netherlands experiences congestion, though this time on importing, particularly from the cheaper German market, and is less competitive than the German market (at least in the period 2000-2001).

[1] The electricity network in continental Europe is highly nashed therefore too complex for an explicit analysis and illustration of the effects of market power. We therefore reduce the complex network to the simplest possible structure with network effects, a triangle.

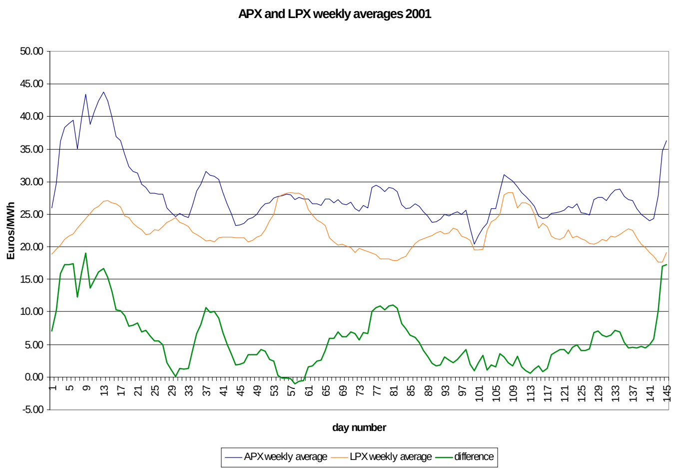


Figure 3: Weekly moving average prices in Germany and The Netherlands, Jan-June 2001

Figure 3 shows the difference in price on each side of the border, reflected in the price that clears the auction market for spot interconnection capacity to import from the Leipzig Power Exchange (LPX) and the Amsterdam Power Exchange (APX).

Previous literature showed that transmissions constraints can enhance market power [10]. Bushnell shows that generators and transmission rights owners with market power can increase profits by withholding transmission capacity [5]. We assume use-it-or lose-it conditions that can prevent withholding of capacity.

Borenstein, Bushnell and Knittel conclud from a theoretical analysis and comparison between historical and present flow patterns between the western and eastern portion of the PJM (Pennsylvania, New Jersey, Maryland) pool that transmission links have been more frequently congested after market liberalisation than under the previous centrally dispatched system, due to the exercise of local market power [3]. Borenstein, Bushnell and Soft show that market power in two oligopolistic markets can be reduced if the capacity of the interconnector is increased, allowing a larger number of generators to compete directly with each other for more of the time [4].

Joskow and Tirol show that with financial rights, a generator with market power can increase profits by manipulating output to affect the prices of transmission rights [11]. They also show that the method for allocating rights matters. We consider the effects of allowing generators to bid for transmission in model I, and while in model II we show that an Independent System Operator (ISO) can manipulate transmission rights to increase welfare in the presence of generation with market power.

Harvey and Hogan show that market power has less effect on market efficiency and social welfare in a nodal pricing regime than if prices are averaged over several nodes (zonal pricing) [8]. That is because with zonal pricing, constraint costs are socialised, thus reducing the demand elasticity, and enhancing generators' market power. In the models we analyse the nodes are treated separately.

We shall study a simple two-node model of export from a node with market power to a competitive market (analogous to the Scotland-England case), and a more complex three-node model of a meshed network corresponding to The Netherlands interconnected to Germany and Belgium.

In a two-node network the transmission operator can increase welfare if he allows oligopolists to pre-empt the market for transmission capacity. This enables the exporters to pre-commit some capacity before competing in the spot market, and reduces the demand still to be satisfied at their home-market price. Reduced demand reduces the incentive to increase prices, much as forward contracts reduce market power in conventional pools (without transmission constraints) ([7], [12], [1]). In a three-node network with market power at one node, the transmission operator can reduce prices and monopoly rent, and hence improve welfare, by applying so-called flow gate proportionality factors which differ from those that would be computed from the underlying physical network characteristics. We show that the two step approach, common in Europe, of first deciding how scarce transmission capacity is to be divided between different countries for them to allocate for transmission contracts, and then each country auctioning these transmission contracts separately, is inefficient. Market power can be better dealt with if both steps are integrated such that jointly used scarce transmission capacity is used for transmissions between countries where firms offer the highest bids.

The illustrations in this paper show that if market power is a problem, then it has to be taken into account when evaluating different transmission access designs. Looking at impacts on individual prices in the three-node network gives little indication of the size of the total welfare change - implying that market power in meshed networks is likely to be hard to address effectively with a decentralised approach.

# 2 Pre-commitment in two node case: exporters have market power

Consider two electricity nodes interconnected by a transmission line of capacity $\overline{K}$. In the first model the upstream market is a symmetric duopoly, with constant variable costs normalised to zero[2], producing an export surplus. The upstream price is $p_1$. The downstream market is perfectly competitive with a residual demand (net of local competitive supply). The downstream price is $p_2 > p_1$.

[2] The results do not depend on this assumption, and the general case of an arbitrary number of upstream generation (including monopoly case), with differing variable costs can readily be modelled with essentially unchanged results, as shown in appendix A.

### 2.1 Case A: Generators denied access to auction

The Transmission System Operator (TSO) auctions the entire available capacity to traders (and generators are prevented from acquiring capacity). The market clearing price for capacity is established either by rational expectations or re-trading in the spot market at the difference in prices $t = p_2 - p_1$. Market demands are given by:

$$y_1 = A_1 - p_1, \qquad y_2 = A_2 - bp_2, \tag{1}$$

for the upstream market (subscript 1) and downstream (subscript 2) markets respectively. If there is adequate transmission capacity, then the upstream generators will maximise profits for the market as a whole, subject to a market clearing price

$$p_1 = \frac{A_1 + A_2 - Q}{1+b}, \qquad K = A_2 - bp < \overline{K}, \tag{2}$$

where $q_m$ is output by generator $m$ and $Q = \sum q_m$. In order for capacity to be scarce and therefore fully used,[3]

$$\overline{K} < A_2 - \frac{b(A_1 + A_2)}{3(1+b)} < A_2. \tag{3}$$

We assume this condition holds in the following.

In that case, market clearing prices with capacity fully used are:

$$p_1 = A_1 + \overline{K} - Q, \qquad p_2 = \frac{A_2 - \overline{K}}{b}, \tag{4}$$

Profit for generator $m, m = 1, 2$, in the upstream market is

$$\pi_m = p_1 q_m = (A_1 + \overline{K} - Q) q_m. \tag{5}$$

The first order condition (FOC) for the choice of output $q_m$ is:

$$q_m = A_1 + \overline{K} - Q, \tag{6}$$

and so

$$q_m = q_n = \frac{1}{2}Q, \quad q_m = \frac{1}{3}\left(A_1 + \overline{K}\right), \quad p_1 = \frac{1}{3}\left(A_1 + \overline{K}\right). \tag{7}$$

[3] The first order condition for the choice of each generator's output, $q_m = \frac{1}{2}Q$ is $q_m = (A_1 + A_2)/3$, hence $p = \frac{(A_1+A_2)}{3(1+b)}$. In a competitive market capacity will be fully used if $\overline{K} < A_2$.

## 2.2 Case B: Generators allowed to pre-empt transmission

The TSO sets a price for interconnect $t$ and anyone is free to acquire capacity. The price is set to ensure that demand equals supply at full capacity. Consider first the case in which generator $m$ buys $k_m$, $\sum k_m = K \leq \overline{K}$ . Profit for generator $m$ is now

$$\begin{aligned} \pi_m &= p_1(q_m - k_m) + k_m(p_2 - t), \\ &= (A_1 + K - Q)(q_m - k_m) + k_m(\frac{A_2 - K}{b} - t). \end{aligned} \tag{8}$$

If traders buy transmission so long as $p_2 \geq p_1 + t$, then the only equilibrium will be an arbitraged one in which $p_2 - t = p_1$, $K = \overline{K}$, in which case equation 8 collapses to 5 and the equilibrium will be as before. If, however, the TSO sets a high enough transmission price, $t > p_2 - p_1$, traders will not buy any transmission and only generators will buy. In that case the FOC for $k_m$ and $q_m$ respectively are:

$$\text{FOC } k_m \ : \quad -(A_1 + K - Q) + (q_m - k_m) + \frac{A_2 - K}{b} - t - \frac{k_m}{b} = 0, \tag{9}$$

$$\text{FOC } q_m \ : \quad +(A_1 + K - Q) - (q_m - k_m) = 0. \tag{10}$$

Equation 10 gives the upstream equilibrium prices and quantities (noting that again symmetry requires $q_m = \frac{1}{2}Q$, $k_m = \frac{1}{2}K$) as

$$\begin{aligned} k_m &= \frac{1}{3}(A_2 - bt), \ q_m = \frac{1}{3}(A_1 + A_2 - bt), \qquad m = 1, 2 \\ p_1 &= \frac{1}{3}A_1, \qquad\qquad p_2 = \frac{A_2 - K}{b} \end{aligned} \tag{11}$$

Compared to 6, provided all transmission capacity is used, the downstream price $p_2$ is the same and the upstream price $p_1$ is lower, and hence welfare is higher.[4] Adding equations 9 and 10 and noting that, by symmetry, $2k_m = K$, gives the market clearing price for fully using the interconnector, $t$, and hence the no arbitrage condition $t > p_2 - p_1$:

$$t = \frac{A_2 - \frac{3}{2}\overline{K}}{b} > \frac{A_2 - \overline{K}}{b} - \frac{A_1}{3} \quad \Longrightarrow \quad A_1 > \frac{3\overline{K}}{2b}. \tag{12}$$

If this condition is satisfied, and if generators are free to bid in an auction for interconnection, they would pre-empt the entire capacity (given the use-it-or-lose-it condition), outbidding any traders or consumers, and raising possible regulatory concerns. The reason why such pre-emptive bidding improves welfare is that, first, downstream prices are purely a function of downstream imports, which are constrained by the available capacity no matter who secures it, and second, bidding for interconnect capacity precommits the generators to behave more competitively in their local market, driving down prices relative to no commitment to export.

[4] If the transmission price is set higher, demand for transmission will be less than supply, and the downstream price will be higher (though the upstream price will be unaffected). The use-it-or lose it condition ensures that capacity will be fully used, and hence rational generator demand would be zero at any transmission price above the market clearing level.

As such, it has the same competitive effect that offering contracts does in the domestic market, as noted by Allaz and Vila [1]. An alternative and equivalent explanation is that by selling export capacity at a fixed price, the marginal revenue from exports is $p_2 - t$, whereas the marginal revenue from selling in the upstream market is equal to marginal cost, which is zero.

# 3 Pre-commitment in two node case: importers have market power

Now consider the same configuration with two electricity nodes interconnected by a transmission line of capacity $\overline{K}$, but this time the upstream exporting market is competitive, and the downstream importers are an $n$-firm oligopoly. For this configuration to result in imports, the importing firms would normally have higher costs, for simplicity taken as constant per unit at $c$. Again for simplicity, only the net exports are modelled, not the underlying demand and supply:

$$Q_1 - y_1 = K \ = \ bp_1, \qquad y_2 = A_2 - p_2, \tag{13}$$

where total exports are $K \leq \overline{K}$. The prices will then be

$$p_1 = \frac{K}{b}, \qquad p_2 = A_2 - (Q \ + K), \quad Q = \sum q_m, \quad K \leq \overline{K}. \tag{14}$$

Again we consider the two cases in which generators are either denied or granted access to the interconnector auction. In the first case, generator profits are

$$\pi_m = q_m(p_2 - c) = q_m(A_2 - Q - K - c) \tag{15}$$

and the FOC when the capacity constraint binds for $q_m$ gives

$$p_2 = \frac{A_2 - \overline{K} + nc}{n+1}, \quad p_1 = \frac{\overline{K}}{b}, \qquad \overline{K} < \frac{b(A_2 + nc)}{n+1+b}, \tag{16}$$

where the last term is the condition for the capacity constraint to bind.

In the case in which the generators are allowed to bid $t$ for interconnect capacity, proft for generator $m$ will be

$$\begin{aligned} \pi_m &= q_m(p_2 - c) + k_m(p_2 - t - p_1), \\ &= (A_2 - K - Q)(q_m + k_m) - q_m c - k_m(\frac{K}{b} + t). \end{aligned} \tag{17}$$

Generators will only be successful in acquiring import capacity if they are willing to bid more than the equilibrium price difference, i.e. provided $t > p_2 - p_1$. We shall see that this is impossible, so there is no harm (but no benefit) in allowing generators to bid. Suppose they face no competition from traders, their choice of output and transmission capacity in this symmetric case will be given by the FOCs:

$$\text{FOC } k_m \quad : \quad (A_2 - K - Q) - (q_m + k_m) - t - \frac{K}{b} - \frac{k_m}{b} = 0, \tag{18}$$

$$\text{FOC } q_m \quad : \quad (A_2 - K - Q) - (q_m + k_m) - c = 0. \tag{19}$$

Solving for prices gives

$$p_2 = \frac{A_2 + nc}{n+1}, \quad t = c - \frac{(n+1)K}{nb} = c - \frac{n+1}{n} p_1. \tag{20}$$

The no arbitrage condition requires

$$p_2 - t < p_1 \to \frac{A_2 + nc}{n+1} - c + \frac{n+1}{n} p_1 < p_1 \tag{21}$$

which is impossible given that $A_2 > c$.

The conclusion is that while there appears to be no harm in allowing importers with market power to bid for interconnect capacity, as their objective is to increase the margin between the price in the export market and in the import market, provided all the interconnection is fully used (as required under "use-it-or-lose-it") they will merely encourage traders to outbid them.

## 4 Mitigating Market Power in Meshed Networks

In a simple two-node network with a single link there is no ambiguity about the concept of transmission capacity, as all power from one node must flow along the single link to the other. In a meshed network with more than one possible path from one node to another, electricity will flow over all links, distributed according to Kirchoff's Laws [2]. A generator at one node may sign a contract to deliver power to a consumer at another node, and then seek to sign a contract with the transmission operator of the most direct link, but only some of the power will actually flow along this link, with the balance creating 'loop flows' along all other paths connecting the source (the generator) to the sink (final consumer). Dealing with these loop flows bedevils the management of federal transmission systems, in which various sub-grids of the interconnected system are under the jurisdiction of separate Transmission System Operators (TSOs). One direct consequence of these loop flows is that a transmission constraint on one link impacts on the flows that are possible on every electricity transmission link in the network.

Some systems (i.e. England and Wales under pool and NETA) allow market participants to insert and withdraw energy up to the contracted connection capacity. The system operator is subsequently required to resolve transmission constraints in the balancing market whilst the incurred costs are socialised. Such a policy can be profitable abused by market participants by first creating congestion and susbequently contracting in the balancing market to resolve it. Furthermore wrong locational signals for investment can result. Such a policy is infeasible in networks with significant transmission constraints (EU, Scottland-England) or networks within which costs can not be socialised (EU).

Two different approaches have been proposed to explicitly manage transmission constraints in a liberalised electricity market, nodal prices and property rights. In the nodal pricing approach, the system operator finds the feasible market clearing prices at each node that respect the transmission and other constraints. Generators receive the nodal price of their injection point, and consumers pay the nodal price at the off-take point. Financial transmission contracts then allow market participants to hedge against the risk that the nodal prices may differ substantially over space. The system operator de facto simulates a market of property rights for scarce transmission assets to determine the nodal prices. Nodal pricing can therefore be interpreted as an interface to simplify the underlying market structure and reduce transaction costs. We can restrict our analysis to physical transmission rights, because any design of physical transmission rights can subsequently be implemented by the system operator when simulating property rights to determine nodal prices.

In the second approach property rights are created and auctioned. Flow gate rights are the fundamental entity corresponding to individual links. The system operator calculates proportionality factors $\gamma_{ij}^k$ to determine what proportion of energy flow between injection node $i$ and offtake node $j$ will pass over link $k$. The proportionality factor $\gamma_{ij}^k$ is negative if the energy flow goes in the opposite direction to the defined orientation of the link.[5] Subsequently market participants $m$ multiply the power volume $q_{ij}$ (positive amount of MW)[6] they want to transmit between two nodes with the corresponding proportionality factor. This determines how many flow gate rights they have to obtain for each link. The system operator can issue a net amount of flow gates $F_k$ up to the capacity $\overline{K}_k$ of the link. Transmission capacity can be used for net flows in either direction of the transmission line.[7]

$$-\overline{K}_k \leq F_k = \sum_m \sum_{i,j} \gamma_{ij}^k q_{ij}^m \leq \overline{K}_k, \qquad \forall k. \tag{22}$$

Coase argued that if all externalities can be handled by costless bargaining then the result would be efficient[6], so in the absence of any market power flow gate rights (if accurately defined) should lead to the same efficient equilibrium as competitive nodal pricing.

Implementing a full set of flow gate rights is complex and creates difficulties because dynamic changes of the network, i.e. due to maintenance of lines, imply that physical proportionality

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[5] The orientations are determined arbitrarily, and these will determine the signs of the factors $\gamma_{ij}^k$ and hence the consistency of the flow analysis.

[6] Energy is measured in MWh, while capacity is measured in MW. We define a unit of time during which flows are constant, and the energy is then MW multipled by the time interval, taken here as 1 unit.

[7] This is a common assumption in the direct current approximation of networks. Security constraints, voltage constraints and reactive power flow usually implies asymmetric constraints on individual flow gates.

factors change over time. Therefore either node-to-node transmission contracts or entry/exit rights might be used to define suitable property rights. Both designs are for our purposes identical. We focus on entry/exit rights because they facilitate subsequent calculations. One unit of exit right is the negative of one unit of entry right, therefore only entry rights have to be defined.[8]

We assume contracts are obligations (with penalties for non-fulfillment); therefore market participants obtain the net amount of entry rights to match net demand $Y_i$ =Demand−Generation.[9] As electricity cannot be stored net demand equals the sum of imports minus exports:

$$Y_i = \text{Import} - \text{Export} = \sum_m \sum_{j \neq i} \left( q_{ji}^m - q_{ij}^m \right), \tag{23}$$

Market participants are required to match net-demand at any node with entry rights such that $Y_i = K_i$. The system operator has to decide how many entry rights $K_i$ for node $i$ to issue such that transmission constraints are not violated by the resulting flows.[10] He can make use of two network properties: first, the total net energy inserted to the network is zero. Therefore rights are only required for all but one node, which is referred to as reference node $r$. Second, energy flows superimpose additively on the network. This allows us to pretend that at any nodes $i$ the issued entry rights $K_i$ are used to satisfy net demand $Y_i$ by only trading with reference node $r$. This means we obtain the same flows as in reality if we pretend that all entry rights $K_i$ will be used to schedule net flows $q_{ri}$ from the reference node $r$ to node $i$. If node $i$ is a net exporter $q_{ri}$ is negative. All other $q_{ij}$ with $i \neq r$ are 0 as we assumed that net demand is only traded with the reference node. Substituting $K_i$ for $q_{ri}$ in equation 22 and setting all other $q_{ij} = 0$ we determine the feasible amount of entry rights to be issued.

$$-\overline{K}_k \leq \sum_{i \neq r} \gamma_{ri}^k K_i \leq \overline{K}_k, \qquad \forall k. \tag{24}$$

In a perfect world the system operator can determine the optimal $K_i$ and equation 22 and equation 24 result in the same market equilibrium.[11] Uncertainty about future demand

[8] It is undecided whether demand can be predicted sufficiently accurate and demand side management is sufficiently irrelevant to allow for a further simplification: Only generators not demand has to obtain access rights.

[9] If flow gate rights have a non-negative value they are frequently defined as options. Non execution of the option reduces load on a constrained link but might increase loads on other links, to the point that they become constrained and reduce available (and already allocated) capacity. If flow gate rights are aggregated to entry/exit rights then a right with a positive value might entitle the holder to increase congestion on some links while relieving congestion on other links. Therefore even entry/exit rights with positive values should be treated as obligations to ensure that the anticipated congestion-relieving flows actually take place and release the required capacity on those links.

[10] The consequence of common notation for flow directions, bus load and net demand requires that one unit of entry rights corresponds to $K = -1$.

[11] Linear superposition of flows implies for the matrix $\gamma$ that $\gamma_{ij} + \gamma_{jk} = \gamma_{ik}$, $\forall i, j, k$.

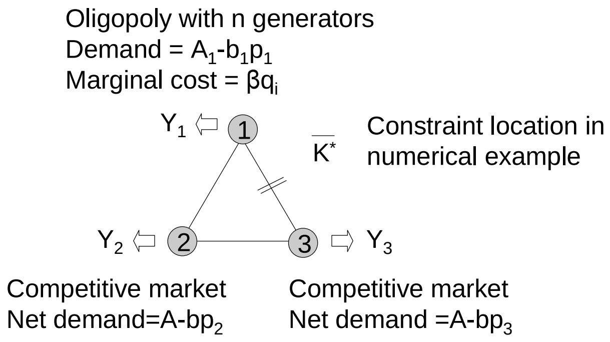


Figure 4: Three node network with one capacity constraint transmission line

or market power require that the system operator adjusts $K_i$ in order to use transmission capacity where it is most effective. But transaction costs, illiquid markets and time constraints can prevent the system operator from adjusting $K_i$ and break the identity between flow gates and entry rights.[12]

The entire subsequent analysis is performed on entry rights. If the $K_i$s can be readjusted continuously we will refer to a swapping system operator exchanging transmission contracts. If the $K_i$s are fixed we will describe the system operator as restrained (from actively trading), as he is currently in all designs of physical property rights based on point to point contracts or entry rights.

The section is structured as follows. The first subsection explains entry rights in the three node model and describes the assumptions about demand and supply. Subsection two contains the analytic solutions for equilibria with different transmission access designs. Subsection three describes two scenarios of an importing monopolist followed in subsection four by an exporting duopoly.

## 4.1 The model

The effects of market power are analysed in a three node network, illustrated in Figure 4.[13]Constraints on feasible entry rights are given in equation 24. As contracts are defined as obligations the amount of entry rights issued $K_i$ matches net demand at node $Y_i$. Whilst the algebra will be kept at this general level numerical examples are based on a network with equal resistance on all links and one binding constraint as indicated in figure 4. We first assess the distribution of flows from node one to node two. Ohm's law implies that flows are

[12]Transmission access designs are simplified in order to focus on the effect of market power in networks. Designs that seem similar in the simplified model may differ in efficiency, welfare distribution and treatment of intermittent and small generators if we consider uncertainty of future demand, dynamical aspects of electricity networks, transaction costs and illiquidity of markets.

[13]Losses, voltage limits, security constraints and reactive power are ignored in the analysis following similar approaches by i.e. [9].

divided proportionally to the inverse of the resistance on the chosen route. The route from node one via node three to node two is twice as long as the direct route, therefore resistance is twice as high. The split factor 2 : 1 implies that one third of energy will be transmitted via the constrained link between node one and node three. The same argument shows that two thirds of energy from node one to node three will pass the constrained link. The system operator has to restrict the net amount of entry rights $K_i$ to node two and three such that $-\overline{K}^* \leq \frac{1}{3}K_2 + \frac{2}{3}K_3 \leq \overline{K}^*$. The factor in front of $K_2$ can always be set to one by rescaling $\overline{K}$. Equation 25 gives a general expression for one transmission constraint in a three node network.

$$-\overline{K} \leq K_2 + \gamma_P * K_3 \leq \overline{K}. \tag{25}$$

In the symmetric network with one constraint as indicated in Figure 4, $\overline{K}$ equals three times the transmission capacity on the constrained link and physical proportionality factor $\gamma_P$ equals 2.

In the subsequent analysis of market power we will use three different designs of entry rights. The first approach corresponds to the traditional usage of flow gate rights with $\gamma_F = \gamma_P$. The system operator continuously offers to swap access rights to node two and access rights to node three at the ratio $\gamma_P : 1$. In the second approach the system operator can chose the financial proportionality factor $\gamma_F$ at which he will subsequently swap entry rights. The design has the disadvantage that constraint 25 is no longer automatically satisfied. If market participants swap rights the system operator has to change the total amount of outstanding rights to keep 25 binding. A third approach resembles the typical entry rights or point-to-point contracts. The system operator initially determines the amount of entry rights for each node. He chooses $K_2$ and $K_3$ to maximise welfare.[14]

For the subsequent illustration we assume an oligopoly with $N$ generators in node one. Each oligopolist incurs marginal costs $c_m$ linear increasing with output $q_m$:

$$c_m = \beta q_m. \tag{26}$$

Generators at node one face a linear demand schedule.

$$D = A_1 - b_1 p_1. \tag{27}$$

At node two and three competitive generation and demand is located, represented by a linear net demand function

$$Y_i = A - b p_i, \qquad i = 2, 3\ . \tag{28}$$

[14] Arguments in favor of such a restrained approach are reduced transaction costs and the difficulties of trading entry/exit rights in a potentially illiquid market.

### 4.2 Algebra

Equilibrium prices evolve in a three-stage game. In the first stage the swapping system operator sets the financial proportionality factor $\gamma_F$ whilst a restrained system operator chooses the volume of different entry rights $K_2$ and $K_3$. In the second stage electricity generators with market power chose their optimal output level. In the third stage competitive generators choose output such that marginal costs equal price.

In our analysis we proceed in the inverse order because complete information allows decision makers in earlier stages to anticipate and incorporate the implications of their choices. Starting with stage three we calculate the net imports to node one from competitive markets at node two and three (Section 4.2.1). In stage two the equilibrium price at node one is calculated. In section 4.2.2 we assume perfect competition at node one followed by the assumption of oligopolistic competition at node one in section 4.2.3. Finally at stage one we calculate the optimal choice of parameters for the system operator. In section 4.2.4 we calculate total welfare gains as performance indicator for the system operator. Whereas the previous calculation was focused on a swapping system, section 4.2.5 summarises the analysis for a restrained system operator determining the amount of entry rights ex ante.

#### 4.2.1 Net imports to node one

Assuming that the physical transmission constraint is binding, the market clearing condition for entry rights follows from equation 25. If the oligopoly is exporting then $\overline{K}$ is positive, otherwise negative. The amount of entry rights $K_2$ and $K_3$ for nodes two and three are

$$K_2 + \gamma_P * K_3 = \overline{K}. \tag{29}$$

If the system operator allocates transmission capacity ex ante and the transmission constraint is subsequently binding then the conservation of enery implies that net imports in node one are fixed $Y_1 = -Y_2 - Y_3 = -K_2 - K_3$.

If the system operator is willing to swap entry rights to nodes two and three, then imports to node one can change. The system operator announces a financial proportionality factor $\gamma_F$ and will exchange one unit of entry rights for node three for $\gamma_F$ units of entry rights for node two. Market participants will swap entry rights as long as arbitrage is possible. The value of entry rights to node two equals the price difference between node two and node one, $p_1 - p_2$. The value of entry rights for node 3 is $p_1 - p_3$. The no arbitrage condition therefore requires:

$$p_1 - p_3 = \gamma_F (p_1 - p_2). \tag{30}$$

We assume full utilisation of capacity $Y_i = K_i$ in equation 29 and combine it with the net-demand equations 28 for nodes two and three to obtain a relation between $p_2$ and $p_3$:

$$p_2 + \gamma_P * p_3 = \frac{A(1+\gamma_P) - \overline{K}}{b}. \tag{31}$$

Equation 31 and 30 can be combined to find $p_2$ and $p_3$ as function of $p_1$:

$$p_2 = \frac{\frac{A(1+\gamma_P)-\overline{K}}{b} + \gamma_P\left(\gamma_F - 1\right)p_1}{1+\gamma_F\gamma_P}, \qquad p_3 = \frac{\gamma_F\frac{A(1+\gamma_P)-\overline{K}}{b} - \left(\gamma_F - 1\right)p_1}{1+\gamma_F\gamma_P}. \tag{32}$$

Inserting these prices in net demand function 28 gives the aggregate net imports at node one as function of $p_1$:

$$Y_1 = -Y_2 - Y_3 = \frac{-(\gamma_F+1)\overline{K} + \left(\gamma_P - 1\right)\left(\gamma_F - 1\right)\left(bp_1 - A\right)}{1+\gamma_F\gamma_P}. \tag{33}$$

For further use we define slope $\Omega$ and intercept $\Theta$ of the previous equation as $Y_1 = \Omega p_1 + \Theta$:

$$\Theta = \frac{-(\gamma_F+1)\overline{K} - \left(\gamma_P - 1\right)\left(\gamma_F - 1\right)A}{1+\gamma_F\gamma_P}, \qquad \Omega = b\frac{\left(\gamma_P - 1\right)\left(\gamma_F - 1\right)}{1+\gamma_F\gamma_P}. \tag{34}$$

### 4.2.2 Swapping system operator and perfect competition at node one

Assuming the generators at node one act as price takers, they choose output such that marginal costs equal the price (equation 26):

$$\beta\frac{A_1 - Y_1 - b_1 p_1}{N} = p_1. \tag{35}$$

Substituting the net import quantity $Y_1$ from equation 33 we obtain an equation for the price at node one as a function of all parameters:

$$p_1 = \frac{A_1 - \Theta(\gamma_F, \gamma_P, \overline{K}, A)}{b_1 + N/\beta + \Omega(\gamma_F, \gamma_P, b)}. \tag{36}$$

Equation 32 can then be used to calculate the corresponding prices at nodes two and three.

### 4.2.3 Swapping system operator and market power at node one

The oligopolists at node one determine output $q_i$ to maximise profit taking the output of all other firms $q_j$ as given. We assume $N$ symmetrical firms. A one-to-one mapping exists between the output an oligopolist $i$ sets and the resulting equilibrium price at node one. Therefore we first calculate the price $p_1$ which maximises firm $i$'s profit and then determine the corresponding $q_i$, always taking output of the other firms $q_j$ for $j \neq i$ as fixed. Profit of firm $i$ is given by quantity sold times revenue minus generation costs, where total costs are $\frac{\beta}{2}q_i^2$:

$$\pi_m = (A_1 - b_1p_1 - Y_1 - (N-1)q_j)\left(p_1 - \frac{\beta}{2}(A_1 - b_1p_1 - Y_1 - (N-1)q_j)\right). \tag{37}$$

The first order condition of equation 37 gives the profit maximising $p_1$ as a function of output $q_j$ of any of the other symmetric firms. As all firms are identical total output $Nq_j$ equals demand minus imports, $Nq_j = A_1 - b_1p_1 - Y_1(p_1)$. Combining this market clearing condition with the first order condition we obtain the oligopoly price at node one:

$$p_1 = \frac{\left(A_1 - \Theta(\gamma_F, \gamma_P, \overline{K}, A)\right)\left(1 + \beta(b_1 + \Omega(\gamma_F, \gamma_P, b))\right)}{(b_1 + \Omega(\gamma_F, \gamma_P, b))(N + 1 + \beta(b_1 + \Omega(\gamma_F, \gamma_P, b)))}. \tag{38}$$

The corresponding prices at nodes two and three follow again from equations 32.

#### 4.2.4 Welfare analysis

Market power can contribute to welfare losses by two routes. First, prices above marginal cost at node one imply a deadweight loss. Second, distorted prices result in inefficient scheduling of generation. We calculate welfare change by adding up welfare changes at the nodes and changes in transmission revenues.

A marginal price increase at competitive nodes two and three increases generators profits by the amount of energy produced and reduces consumers welfare by the amount of energy consumed. We ignore substitution and income effects because few substitutes for electric energy are available and electric energy - if not used for heating - constitutes only a small proportion of most budgets. Total marginal welfare change at a node is therefore the difference between generation and consumption. This difference equals the negative of net demand $Y_i$:

$$\frac{\partial W_i}{\partial p_i} = -Y_i = -A + bp_i, \qquad i = 2, 3. \tag{39}$$

At node one we have explicit cost and demand functions and can therefore calculate total welfare. The first component of equation 40 gives the area of the triangle under the demand curve for consumer surplus. The second component is the triangle between aggregate supply curve and marginal costs, an indicator of generators' profit in a competitive setting. The third component gives the monopoly surplus, produced volume times the price difference between market price and cost of marginal unit generated.

$$\begin{aligned} W_1 \quad = \quad & \frac{(A_1 - b_1 p_1)^2}{2b_1} + \frac{\beta}{2N}(A_1 - b_1 p_1 + Y_2 + Y_3)^2 \\ & + (p_1 - \beta \frac{A_1 - b_1 p_1 + Y_2 + Y_3}{N})(A_1 - b_1 p_1 + Y_2 + Y_3). \end{aligned} \tag{40}$$

Transmission revenues can be interpreted as welfare gains as far as they are not required to cover transmission losses (which we ignore).[15] We assume the system operator has a fixed regulated income and so changes in transmission revenue fall on consumers. Transmission revenue is given by the price difference between the nodes times transmitted energy volume. It can be rewritten as the net imported energy quantity $Y_i$ at each node times the energy price at the node $p_i$.

$$W_t = \sum_i p_i Y_i. \tag{41}$$

Equation 42 gives a general expression for the marginal change of total welfare due to a change of an arbitrary parameter $\lambda$. It combines the welfare change at node two and three given by equation 39 with changes in revenue for transmission entry rights (equation 41) and welfare changes at node one (equation 40). Equations 41 and 40 have been differentiated

[15] Grid investment represents sunk and hence fixed costs, therefore changes in the entire revenue give changes in welfare surplus.

with respect to $\lambda$ to obtain changes rather than levels of welfare.

$$\begin{aligned}\frac{\partial W}{\partial \lambda} &= -b_1 p_1 \frac{\partial p_1}{\partial \lambda} - b p_2 \frac{\partial p_2}{\partial \lambda} - b p_3 \frac{\partial p_3}{\partial \lambda} \\ &\quad + \frac{\beta}{N}(A_1 + 2A - b_1 p_1 - b p_2 - b p_3)(b_1 \frac{\partial p_1}{\partial \lambda} + b \frac{\partial p_2}{\partial \lambda} + b \frac{\partial p_3}{\partial \lambda}).\end{aligned} \tag{42}$$

Equation 42 will be used three-fold. The restrained system operator substitutes $\lambda = K_3$ to decide on the welfare maximising allocation of transmission rights. The swapping system operator sets $\lambda = \gamma_F$ to determine the optimal proportionality factor. Investment decisions are guided by the marginal utility of additional grid capacity, obtained by setting $\lambda = \overline{K}$.

#### 4.2.5 Restrained system operator and market power at node one

The system operator cannot swap entry rights. Therefore he has to decide on the welfare maximising combination of entry rights for node two and three ex ante. We assume the system operator chooses the amount $K_3$ of entry rights at node three. We only analyse scenarios with binding transmission constraints. Therefore equation 29 gives the corresponding amount $K_2$ of entry rights for node two. As constraints are binding market participants obtain the entire available capacity: $Y_i = K_i$ for $i = 2, 3$. Net demand equation 28 gives the corresponding prices $p_2$ and $p_3$. The price at node one follows, as in section 4.2.3, from Cournot competition in quantities. Firms maximise profit function 37. As long as the transmission line is constrained the quantity of electricity imported $Y_1 = -K_2 - K_3$ is in this scenario independent of prices. It is only influenced by the ex ante assignment of entry rights. Using symmetry and market clearing condition we obtain the oligopoly price in node one:

$$p_1 = \frac{\left(A_1 + (1 - \gamma_P) K_3 + \overline{K}\right)(1 + b_1 \beta)}{(N+1) b_1 + b_1^2 \beta}, \tag{43}$$

so $p_1$is a linear function of $K_3$. To simplify further notation slope $M$ and intercept $Z$ of the previous equation are given by:

$$M = \frac{(1 - \gamma_P)(1 + b_1 \beta)}{(N+1) b_1 + b_1^2 \beta}, \qquad Z = \frac{\left(A_1 + \overline{K}\right)(1 + b_1 \beta)}{(N+1) b_1 + b_1^2 \beta}. \tag{44}$$

Based on this analysis the system operator can calculate resulting prices and welfare as function of his choice of $K_3$. Equation 42 gives the first derivative of total welfare with respect to $\lambda = K_3$, and can be solved to obtain the optimal $K_3$:

$$K_3 = \frac{-b_1 M Z - (A - \overline{K})\gamma_P / b + A/b + \beta(A_1 - b_1 Z + \overline{K})(b_1 M + \gamma_P - 1)/N}{b_1 M^2 + \gamma_P^2 / b + 1/b + \beta(b_1 M + \gamma_P - 1)^2 / N}. \tag{45}$$

### 4.3 Comparison of designs for importing oligopoly

The importing oligopoly qualitatively relates to our highly simplified representation of The Netherlands: An oligopoly faces imports from Germany and Belgium. We assume parameter values $\gamma_P = 2$, $\overline{K} = -3$, $b = 1$, $A = 0$, $b_1 = 1.5$, $A_1 = 5$, $N = 1$, and $\beta = 1$.

Table 1 summarises the calculated prices for optimal and reference choices of the decision variables.[16] Market power increases prices at node one. Access design options are sorted such that the design resulting in maximum welfare is at the top. This sorting does not correlate with prices at node two and three, illustrating that mediation of market power in meshed networks needs to be assessed based on total welfare change rather than on local prices.

| | $p_1$ | $p_2$ | $p_3$ |
|---|---|---|---|
| Competitive | 1.1852 | 1.0741 | 0.9630 |
| Swapping SO $\gamma_F = 2.29$ | 1.3661 | 1.1695 | 0.9153 |
| Swapping SO $\gamma_F = 2$ | 1.3736 | 1.1494 | 0.9253 |
| Restrained SO $K_3 = -0.95$ | 1.4053 | 1.0979 | 0.9511 |

Table 1: Prices for different transmission access designs with importing monopoly

Figure 5 illustrates the effect of market power with a swapping system operator. Welfare losses relative to a competitive scenario are depicted for a changing financial proportionality factor $\gamma_F$. The categories total welfare loss and change in transmission revenue are self explanatory. Change in consumer welfare combines consumer4 losses and lost profits of generators assuming they are selling at competitive level. Change in oligopoly profits shows profits of oligopolies from selling above marginal costs.

Consumer loss is mainly divided between monopolies rent and additional transmission revenue. The remaining loss is total welfare loss which is minimal for $\gamma_F = 2.29$. The traditional flow gate approach setting $\gamma_F = \gamma_P = 2$ therefore does not result in a welfare optimum. The system operator might choose an even higher financial proportionality factor $\gamma_F$ as figure 5 show that total welfare would hardly be affected, but more of monopoly surplus could be returned to consumers.

In the current discussion revenue from auctioning of scarce transmission access is proposed to be used for additional transmission investment (as is the intention in the Netherlands). If the

[16] As a reference scenario we calculate the situation of perfect competition at node one with equation 36. The oligopoly equilibrium price at node one for traditional nodal pricing or flow gate rights can be calculated with equation 38 by setting the financial proportionality factor $\gamma_F = \gamma_P = 2$.

A system operator combating market power chooses the welfare maximising proportionality factor $\gamma_F$. We calculate $\gamma_F$ by setting the first derivative of total welfare change with respect to $\gamma_F$ to be zero (equation 42 with $\lambda = \gamma_F$). The system operator restrained from swapping entry/exit rights determines the welfare maximising allocation of entry/exit rights by setting the first derivative of total welfare change with respect to $K_3$ to be zero (equation 42 with $\lambda = K_3$).

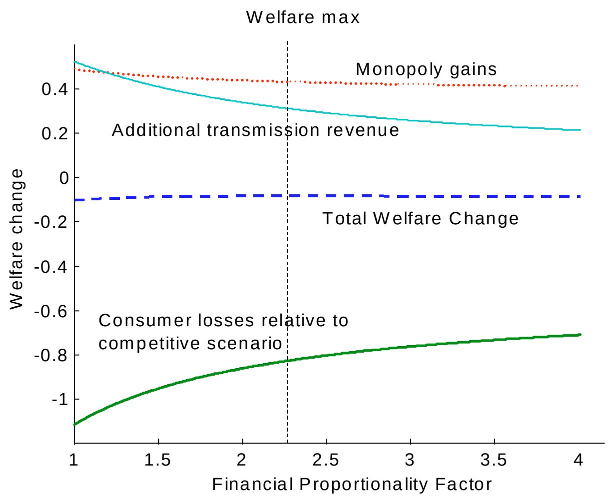


Figure 5: Change in Welfare relative to competitive scenario in the case of an importing monopolist at node one ($b_1 = 1.5$, $A_1 = 5$, $b = 1$, $A = 0$, $\beta = 1$).

price differences are due to market power then the revenue represents part of the monopoly rent from consumers. Divesture might be used to mitigate market power without incurring costs of grid investment, but if this is not feasible, grid investment may be the second best solution to market power.

Table 2 gives numerical results of the welfare changes and allows a comparison with a restrained system operator. A restrained system operator is the least efficient design to mitigate market power and incurs the highest total welfare losses and highest consumer welfare losses relative to a competitive scenario. Fixed assignment of transmission capacity eliminates the price elasticity of imports which usually serve to mitigates market power at node one.

We calculated the marginal benefit of one additional unit of transmission capacity on the constrained link by setting $\lambda = K_3$ in equation 42. When calculating marginal utility of additional capacity in the second scenario it has to be taken into account that a change of transmission capacity changes the optimal financial proportionality factor $\gamma_F$. Additional transmission capacity is obviously only of value whilst the constraint is binding. Therefore the interpretation of the results are for infinitesimal changes in capacity. The numerical example supports the initial intuition that additional transmission capacity is most effective in the presence of market power. But results for another parametrisation in Table 3 proves that intuition can be wrong, as shown below.

| | $\triangle W$ | $\triangle W_{mon}$ | $\triangle W_{trans}$ | $\triangle W_{cons}$ | $\frac{\partial W}{\partial \overline{K}}$ |
|---|---|---|---|---|---|
| Competitive | 0 | 0 | 0 | 0 | 0.333 |
| Swapping SO $\gamma_F = 2.29$ | -0.0812 | +0.4332 | 0.3094 | -0.8238 | 0.426 |
| Swapping SO $\gamma_F = 2$ | -0.0815 | +0.4400 | 0.3392 | -0.8606 | 0.426 |
| Restrained SO $K_3 = -0.95$ | -0.0952 | +0.4739 | 0.4361 | -1.0053 | 0.441 |

Table 2: Welfare losses incured in different transmission access designs with importing monopoly

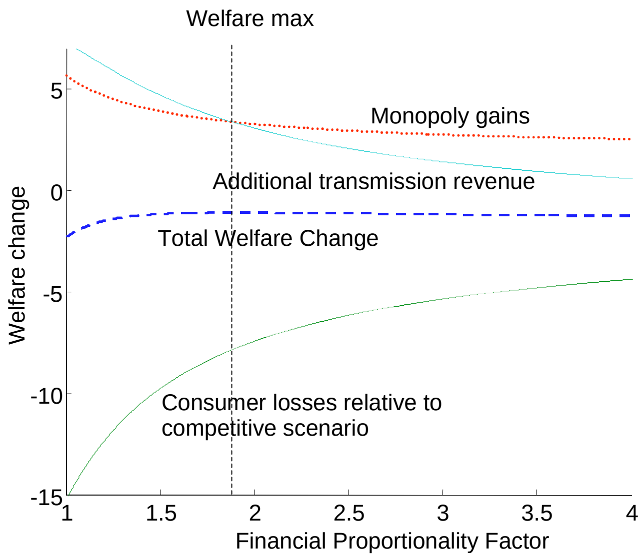


Figure 6: Change demand slope in zone one to $b_1 = 0.3$ changes welfare maximising financial proportionality factor $\gamma_F = 1.92$. Other parameters stay constant ($A_1 = 5$, $b = 1$, $A = 0$, $\beta = 1$)

Figure 6 illustrates the effect of reducing the demand slope at node one from $b_1 = 1.5$ to 0.3. The optimal financial proportionality factor $\gamma_F = 1.92$ is now below the physical proportionality factor $\gamma_P = 2$. Monopoly rent increases with lower $\gamma_F$. The welfare maximising choice of financial proportionality factor increases monopoly rent compared to setting the financial proportionality factors equal to the physical proportionality factors. But whether a system operator should implement such a proportionality factor is questionable as monopoly rents cannot easily be transferred to consumers.

The results show that the current approach in the Netherlands of separately allocating transmission capacity to different flow gate rights is inefficient. Welfare losses due to oligopoly power can be reduced if the system operator can swap transmission rights. The model shows that the system operator can increase total welfare if he chooses a proportionality factor different from the physical proportionality factor. The differences between the two parametrisations shows that the optimal choice depends on the precise representation of network, consumers and generators.

## 4.4 Comparison of designs for exporting oligopoly

This scenario is related to the Scottish situation of an exporting duopoly linked towards the English market with two interconnectors along the West and East coast. A current proposal by Ofgem suggests that physical entry rights are defined for nodes in England and Wales. The system operator cannot swap these rights. We chose the following parameter values to obtain qualitative but not quantitative approximations: $N = 2$, $A_1 = 30$, $b_1 = 1$, $\beta = 4$, $A = 30$, $b = 1$, $\gamma_P = 2$, $\overline{K} = 4$.

Table 3 shows equilibrium prices in the different designs. The swapping system operator with $\gamma_F = 2.9 > \gamma_P$ offers the best way to deal with market power, followed by the system operator with a financial proportionality factor equalling the physical proportionality factor. The restrained system operator, as currently proposed, cannot create the elasticity of export demand and therefore creates the highest prices at node one and the highest welfare loss relative to the competitive scenario.

Again we calculated the marginal benefit of one additional unit of transmission capacity on the constrained link by setting $\lambda = K_3$ in equation 42. In contrast to Table 2 marginal welfare gain due to additional transmission capacity is highest in the competitive scenario.

| | $p_1$ | $p_2$ | $p_3$ | $\triangle W$ | $\frac{\partial W}{\partial \overline{K}}$ |
|---|---|---|---|---|---|
| Competitive | 22.58 | 26.23 | 29.88 | 0 | 10.94 |
| Swapping SO $\gamma_F = 2.99$ | 23.97 | 25.99 | 30.01 | -2.58 | 10.84 |
| Swapping SO $\gamma_F = 2$ | 23.79 | 26.72 | 29.64 | -2.97 | 10.66 |
| Restraint SO $K_3 = 0.158$ | 24.17 | 26.32 | 29.84 | -3.90 | 10.57 |

Table 3: Equilibrium prices for different transmission access designs with exporting duopoly

## 4.5 Conclusion

Welfare can be improved and simultaneously market power mediated in a design with a swapping system operator, if he offers exchanges entry rights at a rate $\gamma_F$ which differs from the rate $\gamma_P$ which would be implied by the underlying physical structure. If the regulator puts higher wait on consumer surplus than on monopoly profits, then he will set $\gamma_F > \gamma_P$ in all scenarious analysed here.

These result directly translates to nodal pricing. In the presence of market power it is no longer optimal to determine nodal prices simply based on the constraints implied by the physical network but rather to represent a flow gate approach with different physical and financial proportionality factors. The system operator has to announce the changes such that oligopolists incorporate the information into their profit function and adapt their bidding behaviour.

The approach of assigning fixed proportions of transmission capacity as common in entry rights and point to point contracts lacks the positive effect of increasing net demand elasticity and is therefore dominated by traditional as well as extended swapping system operator in total welfare as well as in price levels at node one.The results indicate that the separate allocation of transmission rights towards the Netherlands is inefficient as is the Ofgem proposal for entry rights in the England/Wales which does not allow the system operator to swap entry rights.

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## A The n-firm oligopoly case with variable costs

In this model, the upstream market has $N$ firms with variable costs $m_i$ . Consider the case in which generator $i$ buys $k_i$ , $\sum k_i = K \leq \overline{K}$ . Profit for generator $i$ is now

$$\begin{aligned} \pi_i &= (p_1 - c)(q_i - k_i) + k_i(p_2 - t - m_i) \\ &= (A_1 + \overline{K} - Q - m_i)(q_i - k_i) + k_i(\frac{A_2 - K}{b} - t - m_i) \end{aligned} \tag{46}$$

If traders buy transmission so long as $p_2 \geq p_1 + t$, then the only equilibrium will be an arbitraged one in which $p_2 - t = p_1$, in which case equation 46 collapses to

$$\pi_i = qi(p_1 - m_i) = q_i(A_1 + \overline{K} - Q - m_i) \tag{47}$$

In this case, summing over $q_i$ in the FOC for 47 gives

$$Q = \frac{N(A_1 + \overline{K} - \sum m_j)}{N+1} \tag{48}$$

and hence from 4 and 47

$$q_i = \frac{A_1 + \overline{K} - \sum m_j}{N+1} - m_i; \qquad p_1 = \frac{A_1 + \overline{K} + \sum m_j}{N+1}. \tag{49}$$

If the TSO sets a high enough transmission price, $t > p_2 - p_1$, traders will not buy any transmission and only generators will buy. In that case the FOC for $k_i$ and $q_i$ respectively are:

$$\text{FOC } k_i \quad : \quad -(A + K - Q - m_i) + (q_i - k_i) + \frac{A_2 - K}{b} - t - m_i - \frac{k_i}{b} = 0; \tag{50}$$

$$\text{FOC } q_m \quad : \quad (A_1 + K - Q - m_i) - (q_i - k_i) = 0. \tag{51}$$

Summing equation 51 over $m$ gives the upstream equilibrium prices and quantities:

$$Q = \overline{K} + \frac{N(A_1 - m)}{N+1}, \qquad p_1 = \frac{A_1 + \sum m_j}{N+1}, \tag{52}$$

where $m = \frac{1}{N}\sum m_j$ is the average marginal cost. Compared to 49, prices are lower and hence welfare higher. Adding equations 50 and 51 gives the market clearing price for the interconnector, $t$, and hence the no arbitrage condition $t > p_2 - p_1$:

$$t = \frac{A_2 - \frac{N+1}{N}\overline{K}}{b} - m > \frac{A_2 - K}{b} - \frac{A_1 + Nm}{N+1} \quad \Longrightarrow \quad A_1 > \frac{N+1}{N}\frac{\overline{K}}{b} + m \tag{53}$$

When it comes to calibration, allowing for variable costs is critical, for the demand elasticities upstream and downstream (as positive numbers) are given by

$$\varepsilon_1 = \frac{A_1 + Nm}{N(A_1 - m)} > \frac{1}{N}; \qquad \varepsilon_2 = \frac{A_2 - \overline{K}}{\overline{K}} > \frac{1}{N}(1 + \frac{Nbm}{\overline{K}}). \tag{54}$$

The first inequality is required for a positive value of $A_1$, and the last inequality follows from the requirement that $t > 0$ in 53. Putting these two inequalities together with the condition on $A_1$ in 53 gives

$$\frac{A_1}{Nm} = \frac{1 + \varepsilon_1}{N\varepsilon_1 - 1} > \frac{1 + \varepsilon_2}{N\varepsilon_2 - 1} \quad \Longrightarrow \quad \varepsilon_2 > \varepsilon_1 > \frac{1}{N}. \tag{55}$$

The condition that the net demand elasticity downstream is higher than that upstream is neither strong nor surprising, but the condition that the demand elasticity upstream must exceed $1/N$ is strong for electricity and would require either many upstream firms or a fringe of price-taking suppliers, making equation 1 also a net demand function.

The following parameters illustrate a possible equilibrium. If generators are allowed to pre-empt interconnector capacity, and if $A_1 = 25$, $N = 5$, $m = 1, \overline{K} = 12$, $A_2 = 18, b = 1$, $\varepsilon_1 = 1/4$, $\varepsilon_2 = 1/2$, $p_1 = 5$, $p_2 = 6$, $t = 3\frac{3}{5}$ .then $Q = 32$ and profits would be $96\frac{4}{5}$ . If generators are prevented from buying, then there is no equilibrium with export capacity fully used, and arbitrage would equate the expected prices downstream an upstream. Given the actual import demand, $K < \overline{K}$ , generators would maximise profits to give the solution 49 but with $K$ instead of $\overline{K}$. In this case $K = 11$ , $p_1 = p_2 = 6\frac{6}{7}$ , and the interconnect price would fall to zero. In this case $Q = 29\frac{2}{7}$ , and profits would be 171.53.

Comparing the pre-emptive case with that in which generators are forbidden to buy capacity, profits are lower by 74.73, interconnector revenues are higher by $43\frac{1}{5}$ , and consumer welfare by 31.42 upstream and 9.92 downstream, so the sum of the surpluses is 9.81 higher in the pre-emption case. While it is individually rational for the oligopolists to bid for interconnection (or buy at the elastic price), collectively they are substantially worse off as a result, as the domestic price falls appreciably and they have to pay to export.