SUPPLEMENT

For

Constructing Multi-country Rational Expectations Models

By

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1 Introduction

This supplement provides additional results, tables, figures and computational details for Dees, Pesaran, Smith and Smith (DPSS, 2013): Constructing Multi-country Rational Expectations Models. It should be read in conjunction with that paper.

2 GVAR data 1979Q1-2011Q2 used to construct the GVAR deviations

This version of the GVAR data set revises and extends up to 2011Q2 the last available GVAR dataset (the '2009 Vintage') which covered the period 1979Q1-2009Q4. Both vintages are available at: https://sites.google.com/site/gvarmodelling/data. This updated data set (1979Q1-2011Q2) will be referred to as the '2011 Vintage'.[1] The revisions primary affected the GDP series and to a lesser extent the inflation series.

S.1. Real GDP

In order to compile the 2011 Vintage real GDP, the International Financial Statistics (IFS) database and Inter-American Development Bank Latin Macro Watch Database (IDB LMW hereinafter) were used.[2] Countries are divided into three groups. First, those for which quarterly and seasonally adjusted data are available. Second, those for which quarterly data are available, but they are not seasonally adjusted. Third, those for which only annual data are available.

For the first group, the IFS data was used (Concept: Gross Domestic Product, Real Index, Quarterly, 2005 = 100) for Australia, Canada, France, Germany, Italy, Japan, Netherlands, New Zealand, South Africa, Spain, Switzerland, United Kingdom, and United States.[3] The 2009 Vintage real GDP was extrapolated using quarterly growth rates of the IFS series from 2004Q1 to 2010Q4.

For the Latin American countries, namely for Argentina, Brazil, Chile, Mexico, and Peru, the IDB LMW data was used (Concept: GDP, Real Index SA) and the series were updated in the same manner described for the quarterly seasonally adjusted data. For Philippines, the quarterly rate of change of the seasonal adjusted real GDP index (Source: Bloomberg. Ticker: PHNAGDPS Index) was used to extrapolate forward the 2009 Vintage real GDP from 2004Q1 to 2010Q4. For Norway, the series from IFS continued to show evidence of seasonality after seasonal adjustment. The series from OECD (Ticker: GPSA, Concept: Growth rate compared to previous quarter, seasonally adjusted) was used instead, and the 2009 Vintage real GDP was extrapolated forward using this growth rate from 1979Q1 to 2010Q4.

For the second group, the IFS data (Concept: Gross Domestic Product, Real Index, Quarterly, 2005 = 100) was used for Austria, Belgium, Finland, India, Indonesia, Korea, Malaysia, Singapore, Sweden, Thailand, and Turkey. When IFS data was not available, gaps were filled using Bloomberg data: India in 2011Q2 (Ticker: INQGGDPY Index) and Singapore in 2000Q2, 2000Q3 and 2011Q2 (Ticker: SGDPYOY Index). These series were seasonally adjusted using Eviews, applying the National Bureau's X12 program.[4] As in the first group, the data set was extended with forward

[1] The '2011 Vintage' was prepared by Gang Zhang, Ambrogio Cesa Bianchi, and Alessandro Rebucci of the Inter-American Development Bank

[2] For further information see http://www.iadb.org/Research/LatinMacroWatch/lmw.cfm

[3] All series in the IMF IFS database have been reclassified. The concepts used here correspond to the ones used for the 2009 Vintage real GDP, namely 99BVRZF, 99BVPZF and BVPZF.

[4] Seasonal adjustment was performed on the log difference of GDP using the additive option. Using the first

extrapolation of the 2009 Vintage using quarterly growth rates of the adjusted IFS series from 2004Q1 to 2010Q4.

For Saudi Arabia the annual seasonally unadjusted IFS data (Concept: Gross Domestic Product, Real index, Annually, 2005 = 100) was interpolated to obtain the quarterly values.[5] This series was then treated as the quarterly seasonally unadjusted data.

As no institution publishes a quarterly real GDP Index for China, it has to be compiled from a nominal GDP series. The National Bureau of Statistics (NBS) of China releases quarterly nominal GDP series without seasonal adjustment.[6] Accordingly, a quarterly real GDP index for China was constructed as follows. First, the nominal GDP from NBS was seasonally adjusted. Then, the following formula was used

$$log(RGDP)_1 = log(\frac{GDP_1}{CPI_1})$$

$$log(RGDP)_t = log(RGDP)_{t-1} + log(\frac{GDP_t}{GDP_{t-1}}) - log(\frac{CPI_t}{CPI_{t-1}}),\ t > 1$$

where CPI is defined in Section S.2. The series was updated in the same manner as described for the quarterly seasonally adjusted data.

The adjustment procedure for real GDP frequently failed to give sensible estimates towards the end of the sample period. For the final two quarters, the respective annual growth rate was calculated relative to the same quarters in the previous year from the original source, then divided by four, and finally the updated data was extrapolated forward from 2011Q1 to 2011Q2.

S.2. Consumer Price Indices

In order to create the 2011 Vintage CPI, IFS data (Concept: Consumer Prices, All items, Quarterly, 2005 = 100) was collected for all countries with the exception of China.[7] For the series that did not need seasonal adjustment, the quarterly growth rates were used to extrapolate forward the 2009 Vintage from 2004Q1 to 2011Q2. Consistent with the procedure in Section S.1., the CPI series for the following countries were seasonally adjusted: Austria, Belgium, Canada, Chile, Finland, France, Germany, India, Indonesia, Italy, Japan, Korea, Mexico, Netherlands, Norway, South Africa, Spain, Sweden, Switzerland, Thailand, United Kingdom, United States. [8] The quarterly rate of change of the adjusted IFS series was used to extrapolate forward the 2009 Vintage CPI from 2004Q1 to 2011Q2, in order to obtain the 2011 Vintage.

For China, Bloomberg data (Ticker: CNCPIYOY Index, quarterly rate of change of CPI index, NSA) was used. First, the quarterly rate of change was seasonally adjusted using Eviews, applying the National Bureau's X12 program. Then, the 2011 Vintage CPI for China was obtained by forward extrapolation of the 2009 Vintage using the rate of change of the adjusted Bloomberg series from 2004Q1 to 2011Q2.

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observation of the un-adjusted log GDP series, the adjusted log changes were then accumulated. Finally, the seasonally adjusted level series were obtained by taking the exponential of the log adjusted series. The seasonal adjustment window is the whole sample period, namely from 1979Q1 to 2011Q2.

[5] The interpolation procedure is described in Supplement A of Dees, di Mauro, Pesaran, and Smith (2007) and can also be found in the Appendix of the user guide of the GVAR Toolbox.

[6] For further information see: http://www.stats.gov.cn/english/statisticaldata/Quarterlydata/

[7] The series in the IMF IFS database have been reclassified. The concept used here corresponds to the IFS CPI 64zf (level) series, which is the one used in the 2009 Vintage CPI.

[8] Note that the UK inflation series has changed since August 2011. Instead of "Retail Price Index", IMF has started to publish "the Consumer Price Index", which was previously published as "the Harmonized Consumer Price Index", as the official inflation series of UK.

S.3. Exchange Rates

Exchange rate series are from Bloomberg. A quarterly average of the nominal bilateral exchange rates vis-a-vis the US dollar (units of foreign currency per US dollar) was obtained for each country.[9] The quarterly average was computed based on the closing value of the last Wednesday of each month. The 2011 Vintage exchange rate was obtained by forward extrapolation of the 2009 Vintage using the rate of change of the new series from 2004Q1 to 2011Q2.

The exchange rate series of the euro economies refer to the pre-euro exchange rate (i.e. national currency per dollar). To denominate them in euro, the quarterly average of the euro exchange rate vis-a-vis the US dollar was used (Source: Bloomberg. Ticker: EUR Currency). The 1999Q1 value of this series was then used as the base value, which was extrapolated backwards and forwards using the rate of change of the series denominated in national currency.

S.4. Short-Term Interest Rates

IFS is the main source of data for the short term interest rates. Consistent with the 2009 Vintage, IFS data is used for Argentina, Chile, China, and Turkey (Concept: Interest Rates, Deposit Rate); for New Zealand and Peru (Concept: Interest Rates, Discount Rate); for Canada, Malaysia, Mexico, Philippines, South Africa, Sweden, UK and US (Concept: Interest Rates, Treasury Bill Rate); and for Australia, Brazil, Finland, Germany, Indonesia, Italy, Japan, Korea, Norway, Singapore, Spain, Switzerland, and Thailand (Concept: Interest Rates, Money Market Rate).[10]

For Austria, Belgium, France, and the Netherlands no data is available for any of these series from 1999Q1 when the euro was introduced. The country specific IFS Money Market Rate was used from 1979Q1 to 1998Q4 and the series was completed to 2011Q2 using the corresponding data for Germany as the representative euro area interest rate.

For India, quarterly averages of daily Bloomberg data (India Treasury Bill 3-Month Yield. Ticker: GINTB3MO Index) are constructed in the same way as the quarterly exchange rate series.[11] When IFS data was not available, gaps were filled using Bloomberg data: Norway in 2007Q1 and from 2009Q4 to 2011Q2 (Ticker: NKDRC CMPN Curncy), Philippines in 2003Q4, 2005Q4, 2006Q4 and 2008Q2 (Ticker: PH91AVG Index). The 2009 Vintage short term interest rates are extended with these series from 2004Q1 to 2011Q2.

S.5. Oil Price Index

For the oil price index a Brent crude oil price from Bloomberg was used (Series: Current pipeline export quality Brent blend. Ticker: CO1 Comdty). To construct the quarterly series, the average of daily closing prices was obtained for all trading days within the quarter. The quarterly rate of change of this new series was used to extrapolate forward the 2009 Vintage oil price index from 2004Q1 to 2011Q2.

[9] The list of Bloomberg tickers is as follows: ARS JPMQ Curncy, AUD BGN Curncy, ATS CMPN Curncy, BEF CMPN Curncy, BRL BGN Curncy, CAD BGN Curncy, CNY BGN Curncy, CLP BGN Curncy, COP BGN Curncy, FIM CMPN Curncy, FRF CMPN Curncy, DEM BGN Curncy, INR CMPN Curncy, IDR BGN Curncy, ITL BGN Curncy, JPY BGN Curncy, KRW BGN Curncy, MYR BGN Curncy, MXN BGN Curncy, NLG CMPN Curncy, NOK BGN Curncy, NZD BGN Curncy, PEN BGN Curncy, PHP BGN Curncy, ZAR BGN Curncy, SAR BGN Curncy, SGD BGN Curncy, ESP CMPN Curncy, SEK BGN Curncy, CHF BGN Curncy, THB BGN Curncy, TRY BGN Curncy, GBP BGN Curncy, VEF BGN Curncy.

[10] All series in the IMF IFS database have been reclassified. The concepts used here correspond to the ones used in the 2009 Vintage for the short term interest rates, namely the 60Lzf series, the 60Czf series, the 60Bzf series, and the 60zf series.

[11] This is an indicative Treasury Bill Rate polled daily by Bloomberg from various sources. The constructed series is not exactly equal to the original DdPS series, however they are very close.

S.6. Trade Matrix

To construct the trade matrices, the IMF Direction of Trade statistics was used. For all the countries considered the matrix of Exports and Imports (c.i.f.) was downloaded at the annual frequency. The data for 2010 Exports and Imports (average) is appended to the trade matrices associated with the 2009 Vintage.

S.7. GDP, PPP

The main source for construction of the country specific PPP-GDP weights is the World Development Indicator database of the World Bank. The GDP in Purchasing Power Parity terms in current international dollars (Ticker: NY.GDP.MKTP.PP.CD) was downloaded for all countries from 2009 to 2010.[12]

3 Country-specific and GVAR results

In accordance with the theory, all variables in DPSS (2013) are measured as deviations from their steady states, which are estimated as long-horizon forecasts from a reduced-form cointegrating global vector autoregression. The version of the GVAR model used for this purpose has 131 endogenous variables, 88 stochastic trends and 43 cointegrating relations. All the roots of the global VAR model in the 33 countries either lie on or inside the unit circle. The moduli of the largest non unit eigenvalue is 0.902. It has fewer cointegrating relations than the GVAR model reported in Dees, di Mauro, Pesaran and Smith (2007, DdPS), primarily because excluding the long interest rate removes the term structure relationship, which is likely to be I(0). The lag orders for the domestic variables, p_i, and foreign variables, q_i, are selected based on the Akaike criterion with $p_{\max i} = 4$ and $q_{\max i} = 2$. The country-specific models are estimated subject to reduced rank restrictions as described in DdPS and the cointegrating relations obtained are based on the trace statistic at the 95% critical value.[13] For estimation, $\mathbf{x}_{it}^*$ are treated as "long-run forcing" or $I(1)$ weakly exogenous with respect to the parameters of the conditional model. This assumption can be tested by regressing $\mathbf{x}_{it}^*$ on the error correction terms for country i and testing whether these terms are statistically significant. Tables S1-S4 and Figures S1-S4 below provide results related to this version of the GVAR and the associated country specific VARX* models.

The table below shows the VARX* order and number of cointegrating relationships for the individual country models.

[12] WDI data was not available for Australia in 2010. Bloomberg data (Ticker: IGDVAUS Index) was used to fill the gap.

[13] The number of cointegrating relations is determined for the case where unrestricted constants and restricted trend coefficients are included in the individual country error correction models. Using these numbers, the country models are re-estimated with co-trending restrictions imposed (i.e. under the case of unrestricted constants and no trend coefficients) and all subsequent results are obtained under this case. The impulse responses of the GVAR model are computed using the sample covariance matrix with no shrinkage imposed.

Table S1: VARX* order and number of cointegrating relationships

Country	VARX*(p_i, q_i) p_i	q_i	# Cointegrating Relationships
US	3	1	1
CHINA	4	1	1
JAPAN	4	2	2
UK	3	2	1
AUSTRIA	1	1	2
BELGIUM	1	2	2
FINLAND	2	1	1
FRANCE	4	1	1
GERMANY	2	1	1
ITALY	4	2	1
NETHERLANDS	2	1	2
SPAIN	2	1	1
NORWAY	2	1	1
SWEDEN	3	1	2
SWITZERLAND	2	1	1
AUSTRALIA	3	1	1
CANADA	1	2	2
NEW ZEALAND	3	2	0
ARGENTINA	4	2	1
BRAZIL	2	2	1
CHILE	2	2	2
MEXICO	3	2	1
PERU	4	2	0
INDONESIA	3	1	3
KOREA	3	1	2
MALAYSIA	3	1	1
PHILIPPINES	3	1	2
SINGAPORE	1	1	1
THAILAND	4	2	2
INDIA	2	2	1
SOUTH AFRICA	4	1	1
SAUDI ARABIA	3	1	1
TURKEY	2	1	1

Note: The lag orders for the domestic variables, p_i, and foreign variables, q_i, are selected based on the Akaike criterion with $p_{\max i} = 4$ and $q_{\max i} = 2$. The individual country models are estimated subject to reduced rank restrictions as described in DdPS and the cointegrating relations obtained are based on the trace statistic at the 95% critical value. The number of cointegrating relations for Argentina, China, Korea, Peru and Saudi Arabia was reduced by one based on the performance of the persistence profiles of the GVAR.

Figure S1: Persistence profiles

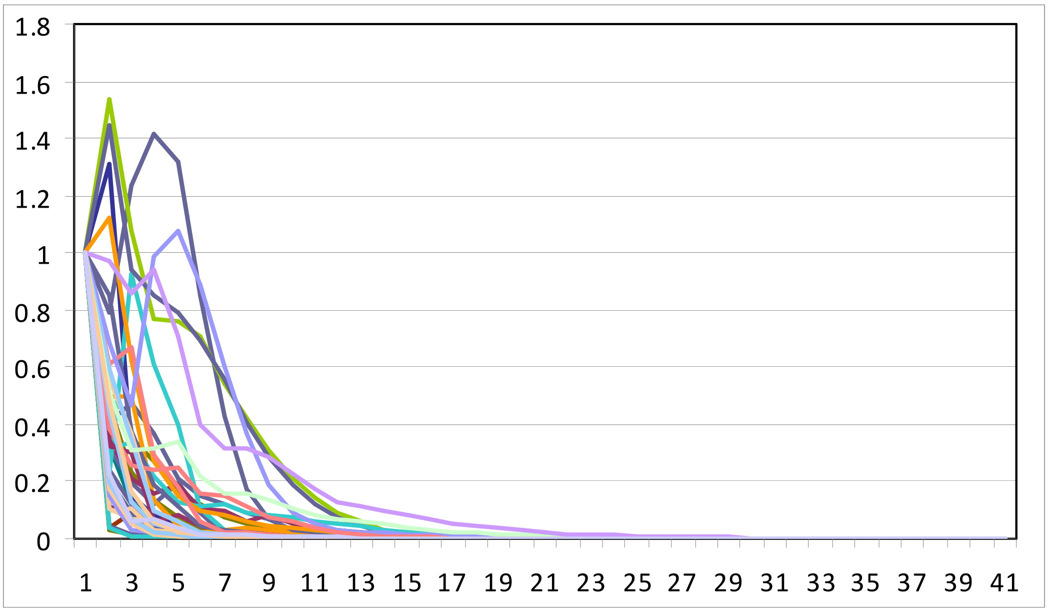


Table S2: Number of rejections of the null of parameter constancy per variable across the country specific models at the 1 percent level

Alternative Test Statistics	Domestic Variables y	π	r	ep	Numbers(%)
$PK_{\sup}$	0(0.0)	0(0.0)	0(0.0)	0(0.0)	0(0.0)
PK_{msq}	0(0.0)	0(0.0)	0(0.0)	0(0.0)	0(0.0)
N	1(3.1)	2(6.3)	2(6.5)	1(3.2)	6(4.7)
robust-N	2(6.3)	0(0.0)	0(0.0)	0(0.0)	2(1.6)
QLR	5(15.6)	8(25.0)	14(45.2)	3(9.7)	30(23.6)
robust-QLR	0(0.0)	2(6.3)	2(6.5)	0(0.0)	4(3.1)
MW	3(9.4)	4(12.5)	7(22.6)	3(9.7)	17(13.4)
robust- MW	2(6.3)	1(3.1)	2(6.5)	1(3.2)	6(4.7)
APW	4(12.5)	9(28.1)	13(41.9)	3(9.7)	29(22.8)
robust- APW	1(3.1)	1(3.1)	2(6.5)	0(0.0)	4(3.1)

Note: The test statistics $PK_{\sup}$ and PK_{msq} are based on the cumulative sums of OLS residuals, N is the Nyblom test for time-varying parameters and QLR, MW and APW are the sequential Wald statistics for a single break at an unknown change point. Statistics with the prefix robust denote the heteroskedasticity robust version of the tests. All tests are implemented at the 1% significance level.

Table S3. F-statistics for testing the serial correlation of the residuals of the country-specific VECMX* equations

Counrty	y	π	r	ep	p^o
US	0.399	3.384†	2.568†	n/a	2.076
CHINA	2.415	1.232	1.690	0.712	n/a
JAPAN	0.943	1.647	7.990†	2.465†	n/a
UK	2.080	0.926	0.375	2.272	n/a
AUSTRIA	1.716	0.963	1.258	2.035	n/a
BELGIUM	0.903	0.321	1.931	1.510	n/a
FINLAND	4.002†	1.819	0.726	0.823	n/a
FRANCE	2.491†	1.016	0.707	2.402	n/a
GERMANY	4.074†	0.449	0.977	0.968	n/a
ITALY	0.585	2.056	0.712	0.320	n/a
NETHERLANDS	1.851	2.844†	3.093†	1.319	n/a
SPAIN	1.755	4.969†	2.007	0.630	n/a
NORWAY	0.423	1.512	1.309	0.647	n/a
SWEDEN	0.191	2.050	1.830	0.703	n/a
SWITZERLAND	1.683	1.039	3.873†	0.588	n/a
AUSTRALIA	1.783	1.574	1.338	2.174	n/a
CANADA	1.766	0.392	2.047	1.544	n/a
NEW ZEALAND	0.683	1.721	2.959†	1.232	n/a
ARGENTINA	2.838†	2.369	1.598	0.334	n/a
BRAZIL	1.950	0.847	0.651	0.603	n/a
CHILE	0.635	2.497†	0.198	3.777†	n/a
MEXICO	1.625	1.130	0.546	0.594	n/a
PERU	1.164	2.165	4.421	3.718†	n/a
INDONESIA	0.613	2.242	1.022†	0.623	n/a
KOREA	0.202	0.301	0.797	1.123	n/a
MALAYSIA	0.363	1.504	2.112	0.241	n/a
PHILIPPINES	1.981	2.096	0.669	0.160	n/a
SINGAPORE	1.577	2.366	4.289†	0.280	n/a
THAILAND	2.066	2.366	1.098	3.382†	n/a
INDIA	1.106	0.781	1.022	1.072	n/a
SOUTH AFRICA	1.341	2.134	0.323	0.126	n/a
SAUDI ARABIA	20.965†	1.280	n/a	1.511	n/a
TURKEY	2.287	4.098†	2.122	1.264	n/a

Note: † denotes statistical significance at the 5% level.

Table S4: F Statistics for testing the weak exogeneity of the country-specific foreign variables and oil prices

Country	y^*	π^*	r^*	ep^*	p^o
US	3.832	2.345	n/a	0.000	n/a
CHINA	0.536	0.585	1.079	n/a	0.422
JAPAN	3.360†	1.124	1.202	n/a	0.755
UK	0.000	0.279	0.286	n/a	0.710
AUSTRIA	0.679	0.854	0.247	n/a	2.441
BELGIUM	0.194	1.237	1.105	n/a	0.619
FINLAND	3.637	4.077†	2.585	n/a	0.980
FRANCE	0.324	0.004	1.562	n/a	0.115
GERMANY	5.443†	0.007	3.711	n/a	0.124
ITALY	1.231	1.146	0.210	n/a	0.140
NETHERLANDS	1.999	1.304	4.615†	n/a	0.420
SPAIN	0.023	1.331	0.274	n/a	0.757
NORWAY	3.423	1.621	2.375	n/a	2.137
SWEDEN	0.665	0.822	0.917	n/a	0.233
SWITZERLAND	0.001	0.006	0.585	n/a	0.234
AUSTRALIA	1.052	0.226	0.747	n/a	0.800
CANADA	2.516	1.602	2.002	n/a	0.192
NEW ZEALAND	n/a	n/a	n/a	n/a	n/a
ARGENTINA	0.106	5.137†	3.624	n/a	5.150†
BRAZIL	0.828	0.370	0.054	n/a	0.001
CHILE	0.312	1.462	0.582	n/a	0.835
MEXICO	0.116	0.210	0.311	n/a	0.016
PERU	n/a	n/a	n/a	n/a	n/a
INDONESIA	1.200	1.090	0.695	n/a	2.661
KOREA	3.722†	4.299†	1.442	n/a	1.922
MALAYSIA	0.167	0.370	0.040	n/a	0.121
PHILIPPINES	0.787	0.597	0.227	n/a	0.280
SINGAPORE	2.024	3.105	0.092	n/a	0.140
THAILAND	0.688	0.053	0.064	n/a	0.650
INDIA	0.737	1.062	0.000	n/a	0.857
SOUTH AFRICA	3.595	0.005	1.226	n/a	0.005
SAUDI ARABIA	3.985†	0.444	1.733	n/a	0.700
TURKEY	0.329	0.652	0.191	n/a	0.524

Note: The lag orders for the domestic, pex_i, and foreign variables, qex_i, used in the weak exogeneity tests were set to $pex_i = p_i$ and $qex_i = q_i$ for all countries, where p_i and q_i are the lag orders of the domestic and foreign variables respectively, used in the country specificVARX* models. For those countries where $pex_i = 4$, this was reduced to 3 to enable the computation of the F-statistic which was infeasible for $pex_i > 3$ due to the large number of regressors. † denotes statistical significance at the 5% level. Increasing the lag order, qex_i, further reduced the number of statistically significant outcomes.

Figure S2a: Generalised impulse responses of a one standard error global inflation shock on inflation (per cent per quarter)

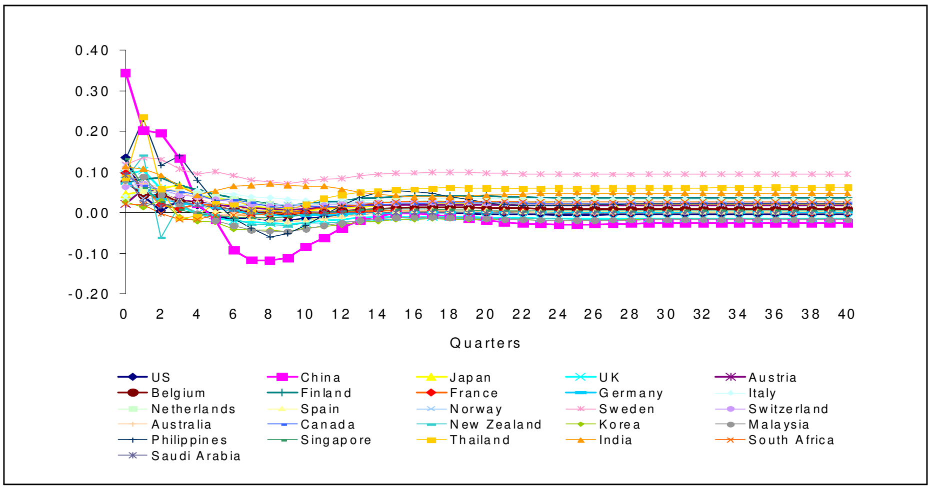


Figure S2b: Generalised impulse responses of a one standard error global inflation shock on output (per cent per quarter)

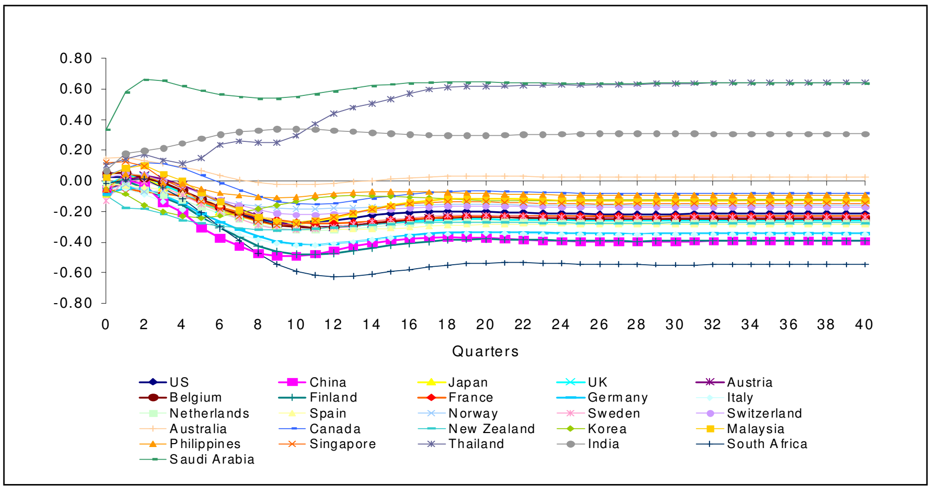


Figure S2c: Generalised impulse responses of a one standard error global inflation shock on interest rates (per cent per quarter)

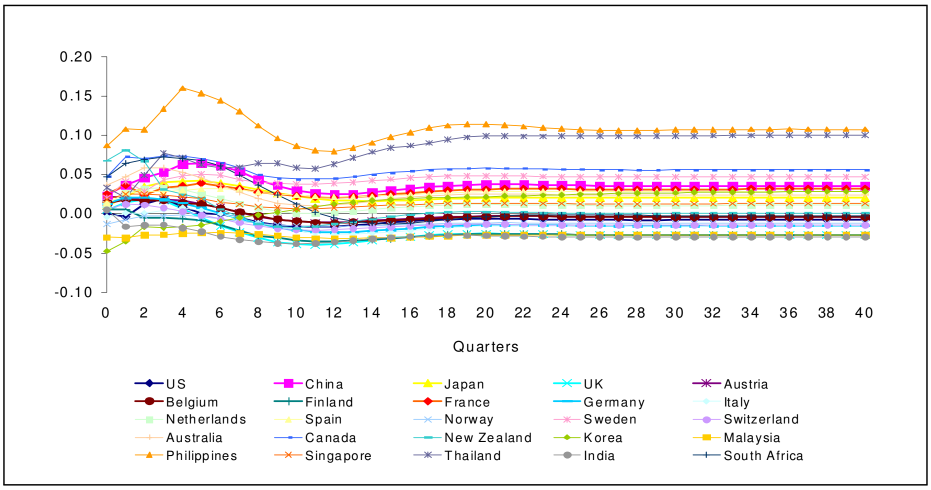


Figure S3a: Generalised impulse responses of a one standard error global output shock on output (per cent per quarter)

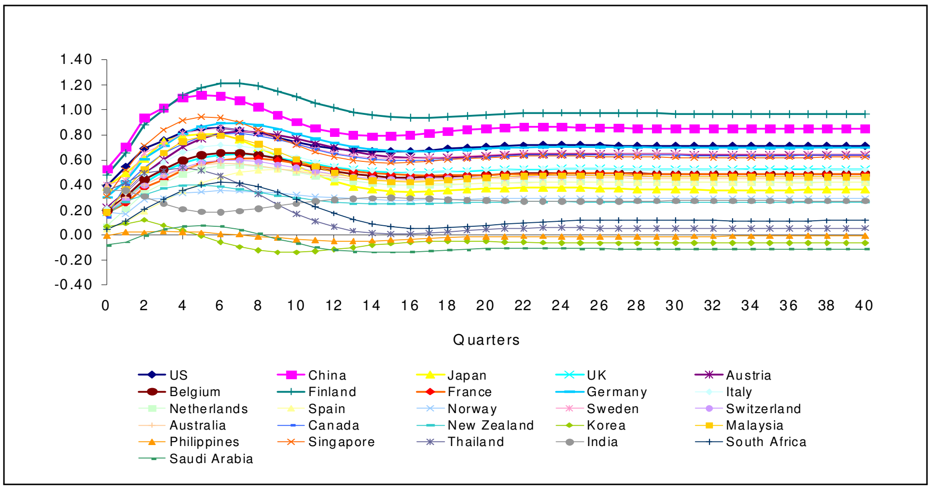


Figure S3b: Generalised impulse responses of a one standard error global output shock on inflation (per cent per quarter)

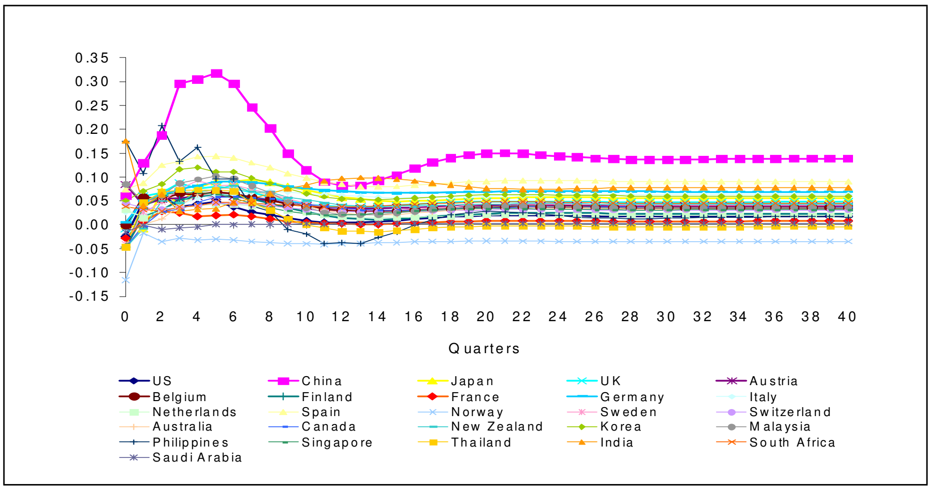


Figure S3c: Generalised impulse responses of a one standard error global output shock on interest rates (per cent per quarter)

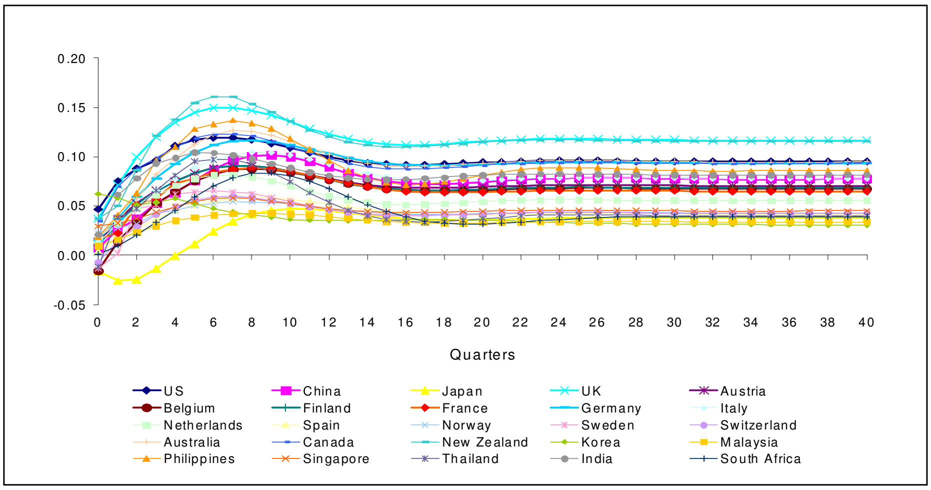


Figure S4a: Generalised impulse responses of a one standard error US interest rate shock on interest rates (per cent per quarter)

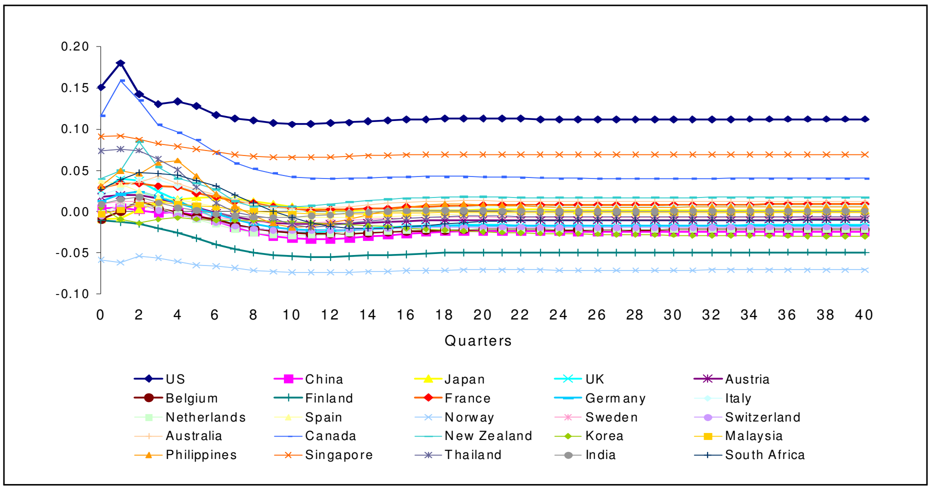


Figure S4b: Generalised impulse responses of a one standard error US interest rate shock on inflation (per cent per quarter)

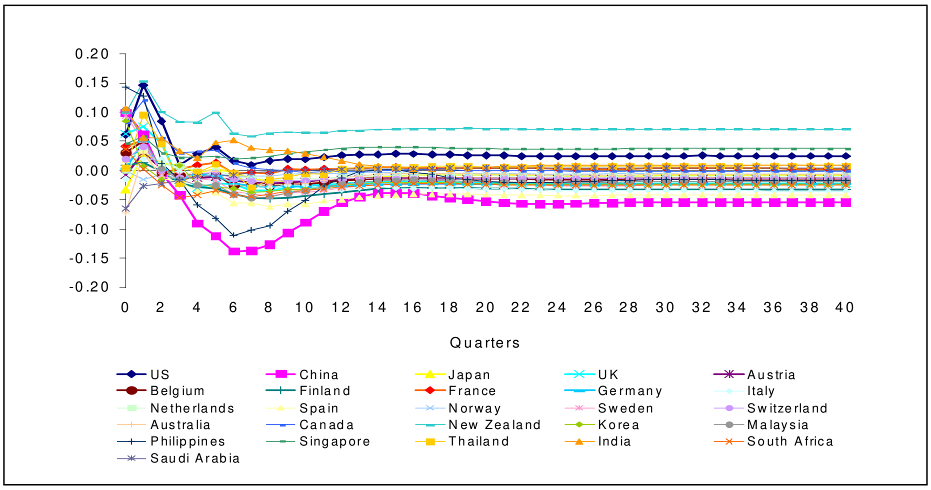
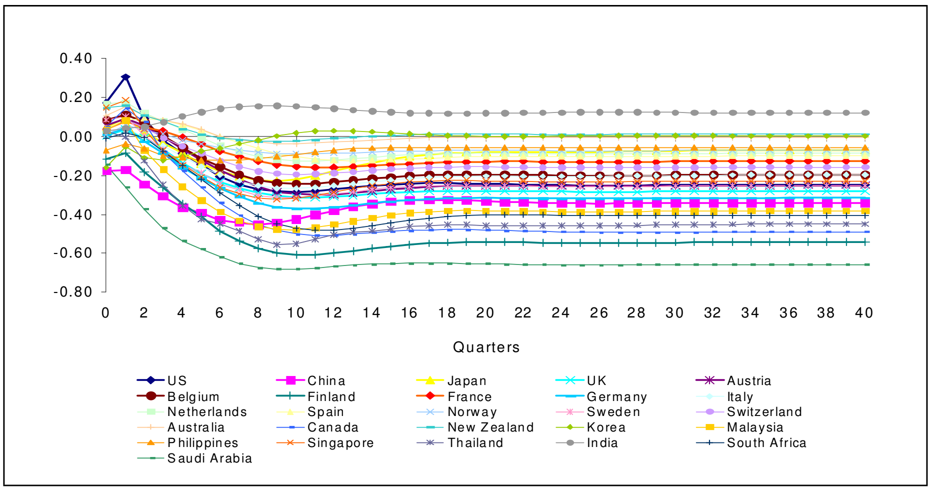


Figure S4c: Generalised impulse responses of a one standard error US interest rate shock on output (per cent per quarter)



4 Additional Results on the MCNK Model

The tables and figures that follow provide additional results associated with the MCNK model, which are not reported in the paper.

Table S6a: Inequality-constrained IV estimates using GVAR estimates of deviations from steady states for all countries in the model with no real effective exchange rate and foreign output in the IS equation of the US

	PC			IS				TR			REER
Country	β_{ib}	β_{if}	β_{iy}	α_{ib}	α_{ir}	α_{ie}	α_{iy*}	γ_{ib}	$\gamma_{i\pi}$	γ_{iy}	ρ_i
US	0.11	0.88	0.07	0.76	-0.11	n/a	n/a	0.90	0.22	0.00	n/a
China	0.26	0.64	0.12	0.53	-0.49	-0.05	0.26	0.82	0.02	0.03	0.79
Japan	0.02	0.97	0.01	0.71	-0.14	-0.06	0.24	0.92	0.08	0.00	0.74
UK	0.22	0.77	0.04	0.36	-0.40	0.05	0.92	0.84	0.23	0.00	0.62
Austria	0.09	0.83	0.02	0.30	-0.19	-0.10	0.95	0.93	0.00	0.02	0.50
Belgium	0.06	0.93	0.04	0.15	-0.11	-0.12	1.04	0.89	0.20	0.00	0.63
Finland	0.12	0.87	0.03	0.03	0.00	0.15	1.31	0.91	0.11	0.02	0.60
France	0.17	0.82	0.04	0.31	0.00	-0.07	0.67	0.86	0.20	0.02	0.57
Germany	0.10	0.59	0.06	0.07	0.00	-0.03	1.22	0.81	0.16	0.03	0.57
Italy	0.42	0.57	0.03	0.16	0.00	-0.20	0.44	0.95	0.06	0.04	0.51
Netherlands	0.00	0.99	0.00	0.03	0.00	-0.01	0.83	0.94	0.01	0.04	0.75
Spain	0.00	0.99	0.04	0.27	0.00	-0.14	0.67	0.87	0.25	0.00	0.85
Norway	0.00	0.99	0.07	-0.10	-0.14	-0.01	0.49	0.85	0.12	0.02	0.52
Sweden	0.00	0.99	0.09	0.00	-0.22	0.08	1.23	0.91	0.10	0.00	0.74
Switzerland	0.23	0.76	0.05	-0.02	-0.54	-0.05	0.81	0.07	0.00	0.11	0.59
Australia	0.00	0.99	0.08	0.29	-0.33	0.01	0.50	0.43	0.12	0.12	0.65
Canada	0.12	0.87	0.03	0.48	0.00	-0.01	0.85	0.76	0.33	0.00	0.94
New Zealand	0.00	0.99	0.13	0.00	0.00	-0.04	0.54	0.48	0.37	0.15	0.52
Argentina	0.00	0.99	0.00	0.49	0.00	0.00	0.59	-0.21	0.33	0.00	0.58
Brazil	0.21	0.78	0.08	0.08	0.00	-0.03	1.22	-0.49	1.36	0.00	0.36
Chile	0.07	0.88	0.00	0.39	0.00	-0.20	1.44	0.56	0.53	0.13	0.68
Mexico	0.30	0.69	0.00	0.37	-0.09	-0.04	0.73	0.07	0.36	0.07	0.76
Peru	0.29	0.51	0.00	0.59	0.00	-0.03	0.34	-0.21	0.38	0.22	0.59
Indonesia	0.38	0.61	0.00	0.57	0.00	-0.09	1.16	0.68	0.21	0.10	0.52
Korea	0.00	0.99	0.12	0.51	0.00	-0.01	0.15	0.79	0.17	0.09	0.87
Malaysia	0.07	0.92	0.04	0.16	0.00	0.00	1.46	0.55	0.00	0.01	0.69
Philippines	0.00	0.77	0.19	0.82	0.00	0.07	0.32	0.80	0.16	0.00	0.69
Singapore	0.23	0.69	0.04	0.02	-0.16	0.05	1.46	0.98	0.07	0.01	0.61
Thailand	0.06	0.89	0.02	0.76	-0.47	-0.03	1.16	0.83	0.19	0.00	0.64
India	0.25	0.49	0.00	0.49	0.00	-0.07	0.00	0.79	0.04	0.00	0.54
South Africa	0.16	0.83	0.01	0.73	-0.73	-0.10	0.48	0.65	0.13	0.06	0.79
Saudi Arabia	0.23	0.63	0.01	0.51	n/a	-0.15	0.48	n/a	n/a	n/a	0.84
Turkey	0.01	0.83	0.20	0.08	-0.16	-0.36	1.28	0.78	0.05	0.06	0.26

Note: The estimation sample is 1980Q3-2011Q1 for the PC and IS equations, except for the PC equation of Argentina where it is 1990Q1-2011Q1. For the TR and exchange rate equations the sample is 1980Q3-2011Q2. For Saudi Arabia interest rate data are not available. The parameters for each country are estimated separately by instrumental variables (IV) subject to the theory restrictions referred to in DPSS (2013). The instruments used are an intercept, the lagged values of the country-specific endogenous variables $\widetilde{y}_{i,t-1}$, $\widetilde{\pi}_{i,t-1}, \widetilde{r}_{i,t-1}, \widetilde{re}_{i,t-1}$, the current values of the country-specific foreign variables $\widetilde{y}^*_{it}, \widetilde{\pi}^*_{it}, \widetilde{r}^*_{it}$, and the log oil price deviation, $\widetilde{p}^o_t$.

Table S6b: Long-run estimates based on inequality-constrained IV procedure applied to GVAR deviations in the model with no real effective exchange rate and foreign output in the IS equation of the US

	IS			TR	
Country	κ_{ir}	κ_{ie}	κ_{iy^*}	$\mu_{i\pi}$	μ_{iy}
US	-0.45	0.00	0.00	2.23	0.00
China	-1.05	-0.11	0.55	0.12	0.15
Japan	-0.48	-0.19	0.83	0.98	0.00
UK	-0.62	0.07	1.44	1.44	0.00
Austria	-0.28	-0.14	1.36	0.00	0.34
Belgium	-0.13	-0.14	1.22	1.80	0.04
Finland	0.00	0.16	1.36	1.19	0.22
France	0.00	-0.10	0.98	1.46	0.17
Germany	0.00	-0.03	1.31	0.81	0.17
Italy	0.00	-0.24	0.52	1.20	0.82
Netherlands	0.00	-0.01	0.86	0.23	0.72
Spain	0.00	-0.19	0.92	1.90	0.00
Norway	-0.13	-0.01	0.44	0.78	0.14
Sweden	-0.22	0.08	1.23	1.09	0.04
Switzerland	-0.53	-0.05	0.80	0.00	0.12
Australia	-0.47	0.01	0.70	0.22	0.21
Canada	0.00	-0.02	1.62	1.40	0.00
New Zealand	0.00	-0.04	0.54	0.71	0.29
Argentina	0.00	0.01	1.16	0.27	0.00
Brazil	0.00	-0.03	1.33	0.91	0.00
Chile	0.00	-0.33	2.34	1.20	0.30
Mexico	-0.15	-0.06	1.17	0.38	0.07
Peru	0.00	-0.08	0.83	0.31	0.18
Indonesia	0.00	-0.20	2.67	0.64	0.30
Korea	0.00	-0.02	0.31	0.81	0.45
Malaysia	0.00	0.00	1.74	0.00	0.02
Philippines	0.00	0.40	1.76	0.82	0.00
Singapore	-0.16	0.05	1.50	3.31	0.64
Thailand	-1.91	-0.14	4.73	1.10	0.00
India	0.00	-0.14	0.00	0.18	0.00
South Africa	-2.69	-0.36	1.76	0.38	0.17
Saudi Arabia	n/a	-0.31	1.00	n/a	n/a
Turkey	-0.18	-0.39	1.39	0.21	0.28

Figure S5a. Impulse responses of a one standard error US monetary policy shock on interest rates (per cent per quarter)

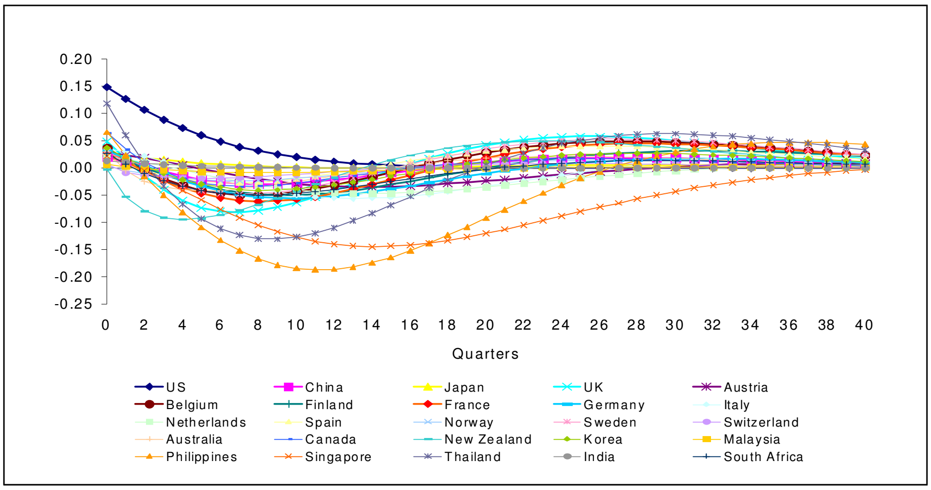


Figure S5b. Impulse responses of a one standard error US monetary policy shock on inflation (per cent per quarter)

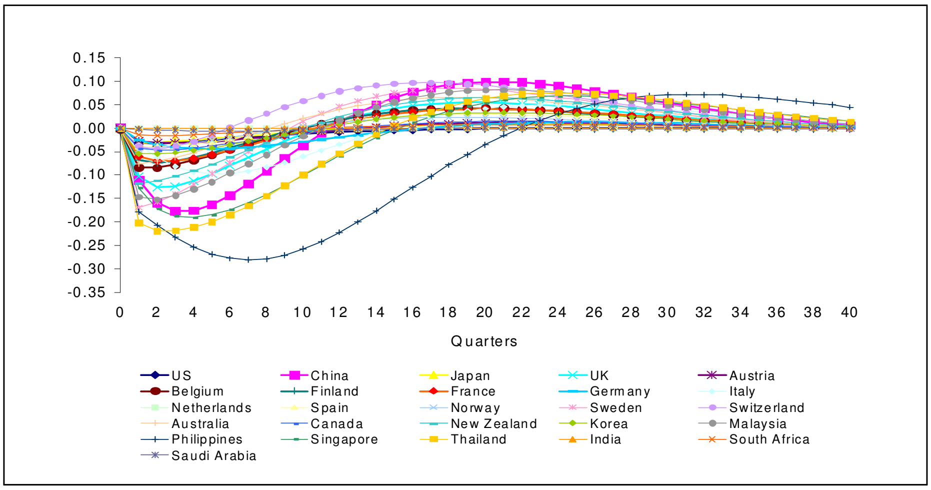


Figure S5c. Impulse responses of a one standard error US monetary policy shock on output (per cent per quarter)

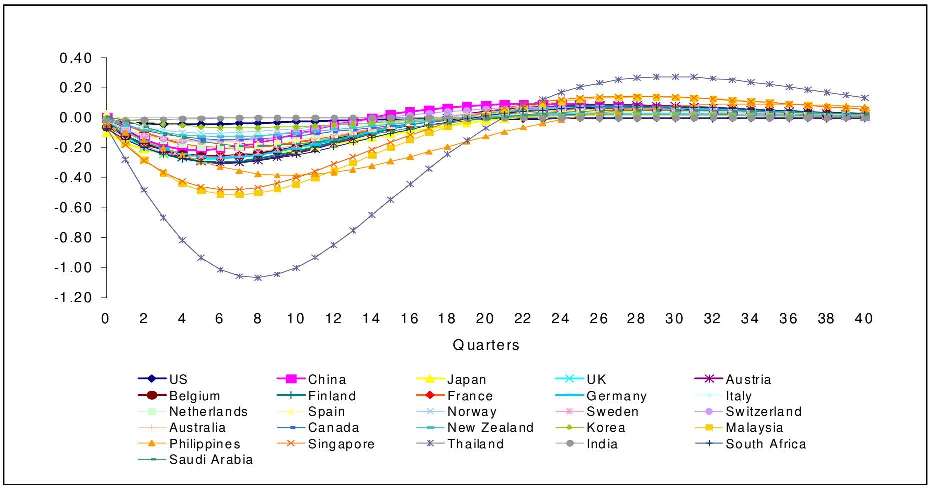


Figure S6a: Impulse responses of a one standard error global demand shock on US and euro area output, inflation and interest rates under alternative covariance matrices (per cent per quarter)

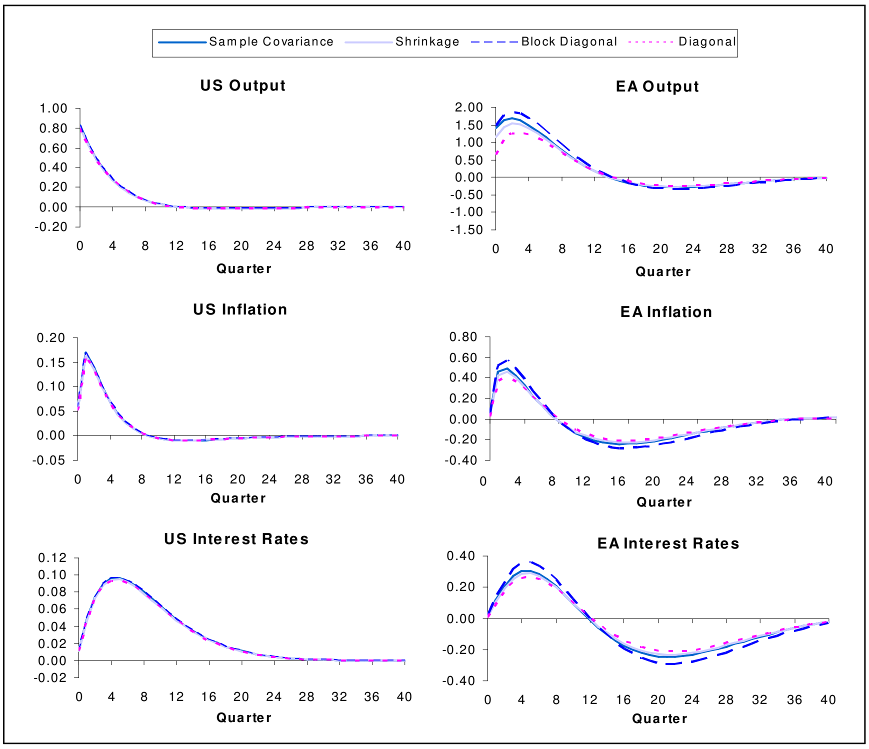


Figure S6b: Impulse responses of a one standard error US monetary policy shock on US and euro area interest rates, inflation and output under alternative covariance matrices (per cent per quarter)

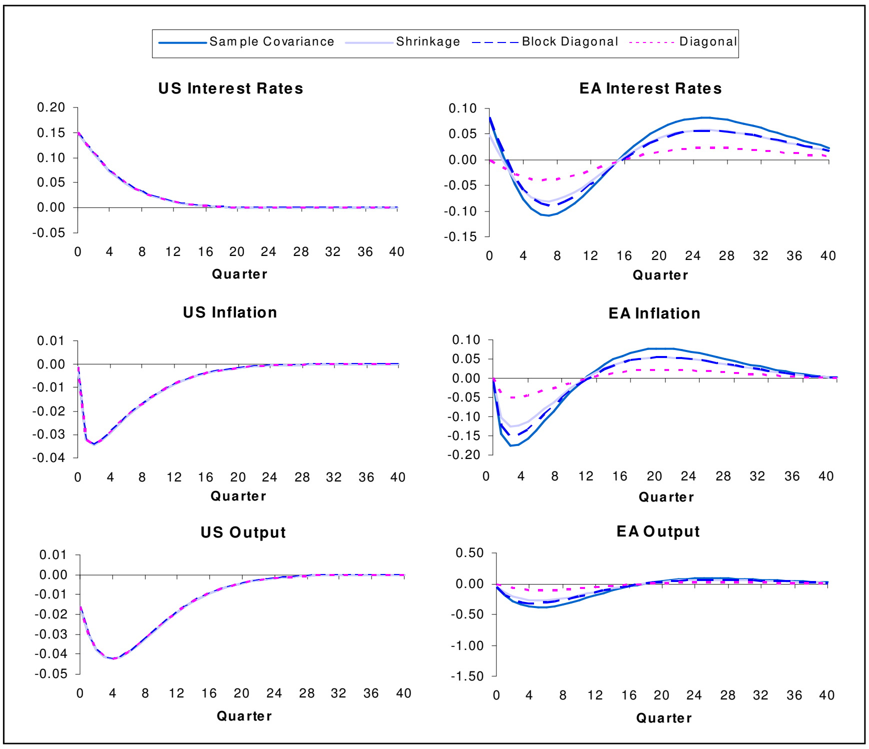


Figure S7. Impulse responses of a one standard error US monetary policy shock on US and euro area interest rates, inflation and output (per cent per quarter, bootstrap median estimates together with 90% bootstrap bands)

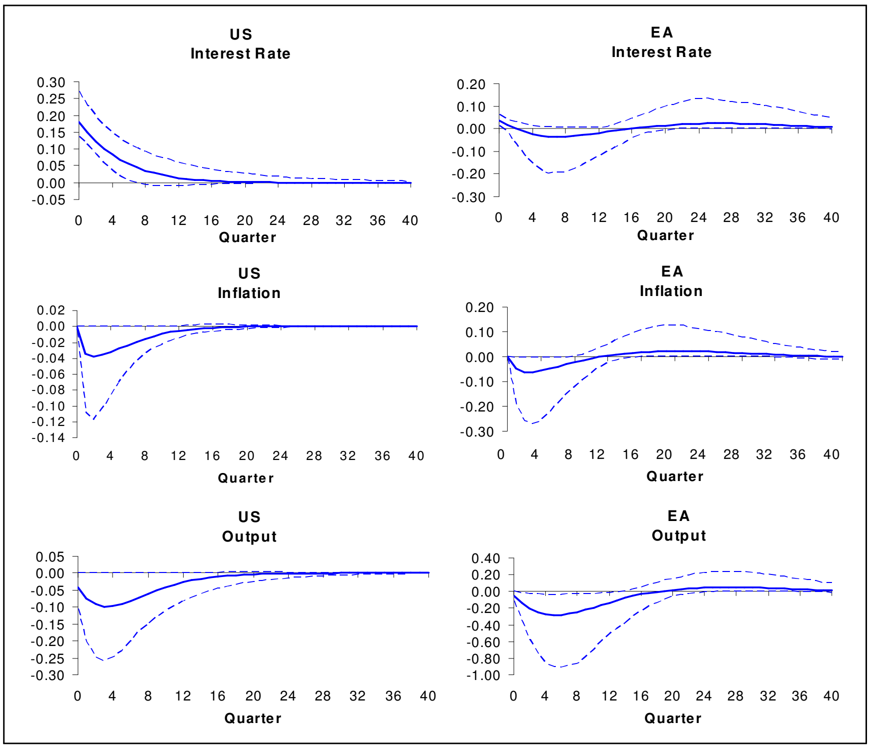


Table S7a: Inequality-constrained IV estimates using Hodrick-Prescott (HP) filtered output deviations with smoothing parameter $\lambda = 1600$ and constant steady states for other variables in the model with no real effective exchange rate and foreign output ($\widetilde{y}^{HP*}$) in the IS equation of the US

	PC			IS				TR			REER
Country	β_{ib}	β_{if}	β_{iy}	α_{ib}	α_{ir}	α_{ie}	α_{iy*}	γ_{ib}	$\gamma_{i\pi}$	γ_{iy}	ρ_i
US	0.12	0.87	0.05	0.89	-0.17	n/a	n/a	0.93	0.12	0.00	n/a
China	0.37	0.62	0.03	0.80	-0.21	0.00	0.13	0.92	0.06	0.00	0.96
Japan	0.09	0.77	0.04	0.70	-0.05	0.00	0.33	0.91	0.12	0.00	0.96
UK	0.30	0.69	0.00	0.82	-0.25	0.01	0.23	0.94	0.06	0.03	0.95
Austria	0.00	0.99	0.04	0.28	-0.09	0.02	0.77	0.90	0.14	0.03	0.98
Belgium	0.00	0.99	0.05	0.39	-0.16	0.02	0.61	0.94	0.03	0.06	0.95
Finland	0.17	0.82	0.02	0.61	-0.57	-0.04	0.75	0.96	0.05	0.02	0.97
France	0.10	0.89	0.03	0.64	0.00	0.03	0.30	0.94	0.06	0.05	0.92
Germany	0.05	0.94	0.02	0.57	-0.02	0.01	0.60	0.93	0.07	0.03	0.96
Italy	0.38	0.61	0.01	0.37	-0.10	0.00	0.66	0.93	0.09	0.06	0.95
Netherlands	0.00	0.99	0.01	0.62	-0.13	0.02	0.51	0.95	0.01	0.04	0.96
Spain	0.09	0.90	0.03	0.78	-0.01	0.00	0.30	0.91	0.12	0.02	0.98
Norway	0.00	0.99	0.00	0.57	-0.04	0.04	0.25	0.97	0.00	0.10	0.87
Sweden	0.05	0.94	0.02	0.34	-0.57	-0.02	0.91	0.89	0.13	0.02	0.95
Switzerland	0.17	0.82	0.02	0.63	-0.16	0.02	0.44	0.90	0.12	0.00	0.96
Australia	0.00	0.99	0.00	0.81	-0.14	0.01	0.19	0.90	0.09	0.05	0.95
Canada	0.20	0.79	0.02	0.62	-0.02	0.01	0.51	0.92	0.07	0.06	0.98
New Zealand	0.09	0.90	0.08	0.73	-0.04	0.00	0.26	0.78	0.23	0.06	0.96
Argentina	0.22	0.77	0.00	0.90	0.00	0.00	0.00	-0.17	0.69	0.00	0.93
Brazil	0.33	0.66	0.00	0.71	-0.07	0.00	0.38	-0.26	1.25	0.00	0.93
Chile	0.12	0.85	0.01	0.83	-0.45	-0.02	0.32	0.61	0.41	0.15	0.95
Mexico	0.34	0.65	0.00	0.66	-0.04	-0.03	0.47	0.78	0.15	0.07	0.93
Peru	0.34	0.64	0.00	0.81	0.00	0.01	0.40	0.52	0.25	0.23	0.97
Indonesia	0.37	0.62	0.00	0.50	-0.64	-0.02	0.42	0.73	0.30	0.12	0.95
Korea	0.18	0.80	0.00	0.69	-0.34	-0.04	0.00	0.95	0.05	0.09	0.92
Malaysia	0.18	0.81	0.01	0.61	-0.23	-0.01	0.72	0.92	0.05	0.01	0.99
Philippines	0.25	0.62	0.00	0.77	-0.19	-0.01	0.07	0.85	0.18	0.01	0.91
Singapore	0.28	0.68	0.02	0.67	-0.47	0.03	0.61	0.91	0.09	0.01	0.97
Thailand	0.07	0.92	0.00	0.78	-0.98	-0.09	0.23	0.92	0.02	0.08	0.94
India	0.27	0.45	0.11	0.72	-0.05	0.00	0.04	0.91	0.01	0.01	0.98
South Africa	0.28	0.71	0.02	0.80	-0.10	-0.01	0.40	0.90	0.06	0.07	0.94
Saudi Arabia	0.20	0.74	0.01	0.84	n/a	0.00	0.36	n/a	n/a	n/a	0.98
Turkey	0.00	0.99	0.00	0.66	-0.19	-0.02	0.31	0.86	0.09	0.14	0.97

Note: The estimation sample is 1980Q3-2011Q1 for the PC and IS equations, except for the PC equation of Argentina where it is 1990Q1-2011Q1. For the TR and exchange rate equations the sample is 1980Q3-2011Q2. For Saudi Arabia interest rate data are not available. The HP filter of log real output is computed using the smoothing parameter of 1600 for all countries. The output deviations based on the HP filter are then computed, which we denote by $\widetilde{y}_{it}^{HP}$, for $i = 0, 1, ..., N$. The country-specific NK models are then estimated by the IV procedure subject to the theoretical restrictions referred to in DPSS (2013), with an intercept included to allow for the assumed constant steady state values. The instruments used are an intercept, the lagged values of the country specific endogenous variables, $\widetilde{y}_{i,t-1}^{HP}$, $\pi_{i,t-1}, r_{i,t-1}, re_{i,t-1}$, the current values of the foreign variables $\widetilde{y}_{it}^{HP*}, \pi_{it}^*, r_{it}^*$, and the first difference of the oil price variable, Δp_{it}^o. The foreign output variable based on HP steady state values are computed as $\tilde{y}_{it}^{*HP} = \Sigma_{j=0}^{N} w_{ij} \tilde{y}_{jt}^{HP}$.

Table S7b: Long-run estimates based on the inequality-constrained IV procedure applied to Hodrick-Prescott (HP) filtered output deviations in the model with no real effective exchange rate and foreign output ($\widetilde{y}^{HP*}$) in the IS equation of the US

	IS			TR	
Country	κ_{ir}	κ_{ie}	κ_{iy^*}	$\mu_{i\pi}$	μ_{iy}
US	-1.57	n/a	n/a	1.71	0.00
China	-1.03	0.01	0.63	0.69	0.00
Japan	-0.15	0.01	1.11	1.33	0.00
UK	-1.40	0.04	1.29	0.95	0.43
Austria	-0.12	0.02	1.08	1.50	0.35
Belgium	-0.26	0.03	1.01	0.44	0.96
Finland	-1.47	-0.10	1.92	1.35	0.57
France	0.00	0.08	0.84	0.98	0.77
Germany	-0.04	0.03	1.40	0.98	0.45
Italy	-0.16	0.00	1.05	1.29	0.84
Netherlands	-0.34	0.04	1.33	0.15	0.78
Spain	-0.04	-0.01	1.40	1.32	0.18
Norway	-0.08	0.09	0.58	0.03	3.28
Sweden	-0.86	-0.03	1.38	1.16	0.22
Switzerland	-0.42	0.04	1.17	1.20	0.02
Australia	-0.76	0.03	1.00	0.91	0.55
Canada	-0.04	0.02	1.34	0.90	0.79
New Zealand	-0.17	0.00	0.95	1.06	0.28
Argentina	0.00	0.03	0.00	0.59	0.00
Brazil	-0.23	-0.01	1.32	0.99	0.00
Chile	-2.64	-0.12	1.89	1.04	0.38
Mexico	-0.12	-0.10	1.37	0.70	0.31
Peru	0.00	0.03	2.08	0.53	0.49
Indonesia	-1.27	-0.03	0.84	1.13	0.44
Korea	-1.11	-0.13	0.00	0.99	1.70
Malaysia	-0.58	-0.03	1.85	0.56	0.14
Philippines	-0.82	-0.06	0.28	1.23	0.07
Singapore	-1.42	0.08	1.84	1.02	0.08
Thailand	-4.48	-0.41	1.08	0.27	0.93
India	-0.17	0.00	0.16	0.14	0.15
South Africa	-0.48	-0.03	1.96	0.62	0.71
Saudi Arabia	n/a	-0.01	2.18	n/a	n/a
Turkey	-0.56	-0.05	0.91	0.62	0.99

Table S8a: Inequality-constrained IV estimates using GVAR estimates of deviations from steady states for all countries inthe model with no real effective exchange rate and foreign output in the IS equation of the US, and no foreign output included in the IS equation for all other countries

	PC			IS			TR			REER
Country	β_{ib}	β_{if}	β_{iy}	α_{ib}	α_{ir}	α_{ie}	γ_{ib}	$\gamma_{i\pi}$	γ_{iy}	ρ_i
US	0.11	0.88	0.07	0.76	-0.11	n/a	0.90	0.22	0.00	n/a
China	0.26	0.64	0.12	0.69	-0.42	0.00	0.82	0.02	0.03	0.79
Japan	0.02	0.97	0.01	0.76	-0.29	-0.04	0.92	0.08	0.00	0.74
UK	0.22	0.77	0.04	0.76	-0.24	-0.14	0.84	0.23	0.00	0.62
Austria	0.09	0.83	0.02	0.73	-0.20	0.37	0.93	0.00	0.02	0.50
Belgium	0.06	0.93	0.04	0.84	-0.45	0.13	0.89	0.20	0.00	0.63
Finland	0.12	0.87	0.03	0.66	-0.55	0.23	0.91	0.11	0.02	0.60
France	0.17	0.82	0.04	0.49	0.00	0.23	0.86	0.20	0.02	0.57
Germany	0.10	0.59	0.06	0.58	0.00	0.37	0.81	0.16	0.03	0.57
Italy	0.42	0.57	0.03	0.24	-0.02	-0.40	0.95	0.06	0.04	0.51
Netherlands	0.00	0.99	0.00	0.66	0.00	-0.03	0.94	0.01	0.04	0.75
Spain	0.00	0.99	0.04	0.23	0.00	-0.31	0.87	0.25	0.00	0.85
Norway	0.00	0.99	0.07	0.38	-0.49	0.23	0.85	0.12	0.02	0.52
Sweden	0.00	0.99	0.09	0.48	0.00	-0.17	0.91	0.10	0.00	0.74
Switzerland	0.23	0.76	0.05	0.85	-0.20	0.12	0.07	0.00	0.11	0.59
Australia	0.00	0.99	0.08	0.55	0.00	-0.08	0.43	0.12	0.12	0.65
Canada	0.12	0.87	0.03	0.85	-0.23	-0.02	0.76	0.33	0.00	0.94
New Zealand	0.00	0.99	0.13	0.65	-0.54	-0.13	0.48	0.37	0.15	0.52
Argentina	0.00	0.99	0.00	0.60	0.00	-0.01	-0.21	0.33	0.00	0.58
Brazil	0.21	0.78	0.08	0.69	-0.02	0.07	-0.49	1.36	0.00	0.36
Chile	0.07	0.88	0.00	0.66	-0.24	-0.17	0.56	0.53	0.13	0.68
Mexico	0.30	0.69	0.00	0.42	-0.09	-0.18	0.07	0.36	0.07	0.76
Peru	0.29	0.51	0.00	0.62	0.00	-0.01	-0.21	0.38	0.22	0.59
Indonesia	0.38	0.61	0.00	0.74	-0.14	-0.12	0.68	0.21	0.10	0.52
Korea	0.00	0.99	0.12	0.51	0.00	-0.01	0.79	0.17	0.09	0.87
Malaysia	0.07	0.92	0.04	0.69	-1.36	-0.12	0.55	0.00	0.01	0.69
Philippines	0.00	0.77	0.19	0.75	-0.17	-0.02	0.80	0.16	0.00	0.69
Singapore	0.23	0.69	0.04	0.66	-0.59	-0.15	0.98	0.07	0.01	0.61
Thailand	0.06	0.89	0.02	0.80	-0.26	0.07	0.83	0.19	0.00	0.64
India	0.25	0.49	0.00	0.49	0.00	-0.07	0.79	0.04	0.00	0.54
South Africa	0.16	0.83	0.01	1.23	-2.72	-0.04	0.65	0.13	0.06	0.79
Saudi Arabia	0.23	0.63	0.01	0.55	n/a	-0.03	n/a	n/a	n/a	0.84
Turkey	0.01	0.83	0.20	0.55	-0.61	-0.45	0.78	0.05	0.06	0.26

Note: The estimation sample is 1980Q3-2011Q1 for the PC and IS equations, except for the PC equation of Argentina where it is 1990Q1-2011Q1. For the TR and exchange rate equations the sample is 1980Q3-2011Q2. For Saudi Arabia interest rate data are not available. The parameters for each country are estimated separately by instrumental variables (IV) subject to the theory restrictions referred to in DPSS (2010) with no foreign output included in the IS equation for all countries. The instruments used are an intercept, the lagged values of the country-specific endogenous variables $\widetilde{y}_{i,t-1}$, $\widetilde{\pi}_{i,t-1}, \widetilde{r}_{i,t-1}, \widetilde{re}_{i,t-1}$, the current values of the country-specific foreign variables $\widetilde{y}^*_{it}, \widetilde{\pi}^*_{it}, \widetilde{r}^*_{it}$, and the log oil price deviation, $\widetilde{p}^o_t$.

Table S8b: Long-run estimates based on the inequality-constrained IV procedure applied to GVAR deviations in the model with no real effective exchange rate and foreign output in the IS equation of the US, and no foreign output included in the IS equation for all other countries

	IS		TR	
Country	κ_{ir}	κ_{ie}	$\mu_{i\pi}$	μ_{iy}
US	-0.45	n/a	2.23	0.00
China	-1.39	-0.01	0.12	0.15
Japan	-1.18	-0.17	0.98	0.00
UK	-0.98	-0.57	1.44	0.00
Austria	-0.75	1.38	0.00	0.34
Belgium	-2.78	0.81	1.80	0.04
Finland	-1.61	0.67	1.19	0.22
France	0.00	0.44	1.46	0.17
Germany	0.00	0.88	0.81	0.17
Italy	-0.03	-0.53	1.20	0.82
Netherlands	0.00	-0.10	0.23	0.72
Spain	0.00	-0.40	1.90	0.00
Norway	-0.78	0.36	0.78	0.14
Sweden	0.00	-0.32	1.09	0.04
Switzerland	-1.38	0.85	0.00	0.12
Australia	0.00	-0.17	0.22	0.21
Canada	-1.60	-0.16	1.40	0.00
New Zealand	-1.54	-0.37	0.71	0.29
Argentina	0.00	-0.01	0.27	0.00
Brazil	-0.08	0.24	0.91	0.00
Chile	-0.70	-0.48	1.20	0.30
Mexico	-0.15	-0.31	0.38	0.07
Peru	0.00	-0.02	0.31	0.18
Indonesia	-0.53	-0.45	0.64	0.30
Korea	0.00	-0.03	0.81	0.45
Malaysia	-4.44	-0.39	0.00	0.02
Philippines	-0.68	-0.08	0.82	0.00
Singapore	-1.76	-0.46	3.31	0.64
Thailand	-1.33	0.35	1.10	0.00
India	0.00	-0.14	0.18	0.00
South Africa	11.77	0.18	0.38	0.17
Saudi Arabia	n/a	-0.06	n/a	n/a
Turkey	-1.37	-0.99	0.21	0.28

5 Inequality constrained instrumental variable estimation

The country-specific equations are estimated by the inequality constrained instrumental variables (ICIV) method both to obtain the estimates based on historical time series observations and for the bootstrap replications. When the constraints are not satisfied, the parameters are set to their boundary values and the choice between any alternative estimates that satisfy the constraints is based on the sum of squares of in-sample prediction errors. The ICIV is also applied at the bootstrap stage but for the (possibly) constrained specification estimated using the historical observations.

More specifically, ICIV estimation is a non-linear programming problem. In general, if there are n inequality restrictions there are 2^n possible unconstrained and constrained models to consider. In our application the maximum number of constraints is 4, in the PC equation, and the constrained optimization problem can be carried out by searching over all the specifications and then selecting the specification that satisfies all the constraints and has the lowest in-sample mean square prediction errors. Note that since some of the regressors are endogenous, the in-sample prediction errors and the IV residuals would not be the same. See Pesaran and Smith (1994). Consider the regression equation

$$\mathbf{y} = \mathbf{X}\beta + \varepsilon$$

where $\mathbf{y}$ and ε are $T \times 1$ vectors for the dependent variable and unobserved disturbance, $\mathbf{X}$ a $T \times k$ matrix of potentially endogenous regressors and there is also a $T \times s$ matrix of instruments $\mathbf{Z}$, $s \geq k$. Define $\mathbf{P}_z = \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$, $\mathbf{M}_z = (\mathbf{I} - \mathbf{P}_z)$ and $\hat{\mathbf{X}} = \mathbf{P}_z\mathbf{X}$. The IV estimator is $\hat{\beta} = (\mathbf{X}'\mathbf{P}_z\mathbf{X})^{-1}\mathbf{X}'\mathbf{P}_z\mathbf{y}$. The prediction errors are: $\hat{\mathbf{e}} = \mathbf{y} - \hat{\mathbf{X}}\hat{\beta}$; the IV residuals $\mathbf{e} = \mathbf{y} - \mathbf{X}\hat{\beta}$. Then as shown in Pesaran and Smith (1994), the sum of squared prediction errors equals the IV minimand plus a constant which depends only on the data, not the estimates:

$$\hat{\mathbf{e}}'\hat{\mathbf{e}} = \mathbf{e}'\mathbf{P}_z\mathbf{e} + \mathbf{y}'\mathbf{M_z}\mathbf{y}.$$

Thus minimising the sum of squares of prediction errors, $\hat{\mathbf{e}}'\hat{\mathbf{e}}$, with respect to $\hat{\beta}$ is equivalent to minimising the IV minimand, $\mathbf{e}'\mathbf{P}_z\mathbf{e}$. In some of the constrained cases, there are no endogenous variables so $\mathbf{X} = \mathbf{Z}$, $\hat{\mathbf{X}} = \mathbf{X}$ and IV reduces to least squares; in other cases $\hat{\beta}$ is fully specified by the constraints, so no parameters are estimated. The sum of squared prediction errors remains well defined in all such cases.

For illustration consider the ICIV estimation of the PC equation. There are four inequality constraints:

$$\begin{aligned} &A : \beta_{ib} + \beta_{if} \leq 0.99 \\ &B : \beta_{ib} \geq 0 \\ &C : \beta_{if} \geq 0 \\ &D : \beta_{iy} \geq 0. \end{aligned}$$

The set of possible binding constraints gives 14 cases: two cases are redundant since imposing the boundary conditions under C and D (namely $\beta_{ib} = 0$ and $\beta_{if} = 0$) implies $\beta_{ib} + \beta_{if} \leq 0.99$. All the specifications associated with the 14 cases are estimated by IV. From the set of cases whose coefficients satisfy the inequality constraints the estimates with the lowest sum of squared prediction errors are chosen. The equations for the 14 possible cases are set out in the table below, a cross in the column corresponding to the constraint indicates that the boundary condition in that constraint is imposed.

Case	A	B	C	D	Equation
1					$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}_{i,t-1}+\beta_{if}E_{t-1}\left(\widetilde{\pi}_{i,t+1}\right)+\beta_{iy}\widetilde{y}_{it}+\varepsilon_{i,st}$
2	X				$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}_{i,t-1}+(0.99-\beta_{ib})E_{t-1}\left(\widetilde{\pi}_{i,t+1}\right)+\beta_{iy}\widetilde{y}_{it}+\varepsilon_{i,st}$
3		X			$\widetilde{\pi}_{it} = \beta_{if}E_{t-1}\left(\widetilde{\pi}_{i,t+1}\right)+\beta_{iy}\widetilde{y}_{it}+\varepsilon_{i,st}$
4			X		$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}_{i,t-1}+\beta_{iy}\widetilde{y}_{it}+\varepsilon_{i,st}$
5				X	$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}_{i,t-1}+\beta_{if}E_{t-1}\left(\widetilde{\pi}_{i,t+1}\right)+\varepsilon_{i,st}$
6	X	X			$\widetilde{\pi}_{it} = 0.99E_{t-1}\left(\widetilde{\pi}_{i,t+1}\right)+\beta_{iy}\widetilde{y}_{it}+\varepsilon_{i,st}$
7	X		X		$\widetilde{\pi}_{it} = 0.99\widetilde{\pi}_{i,t-1}+\beta_{iy}\widetilde{y}_{it}+\varepsilon_{i,st}$
8	X			X	$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}_{i,t-1}+(0.99-\beta_{ib})E_{t-1}\left(\widetilde{\pi}_{i,t+1}\right)+\varepsilon_{i,st}$
9		X	X		$\widetilde{\pi}_{it} = \beta_{iy}\widetilde{y}_{it}+\varepsilon_{i,st}$
10		X		X	$\widetilde{\pi}_{it} = \beta_{if}E_{t-1}\left(\widetilde{\pi}_{i,t+1}\right)+\varepsilon_{i,st}$
11			X	X	$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}+\varepsilon_{i,st}$
12	X	X		X	$\widetilde{\pi}_{it} = 0.99E_{t-1}\left(\widetilde{\pi}_{i,t+1}\right)+\varepsilon_{i,st}$
13	X		X	X	$\widetilde{\pi}_{it} = 0.99\widetilde{\pi}_{i,t-1}+\varepsilon_{i,st}$
14		X	X	X	$\widetilde{\pi}_{it} = \varepsilon_{i,st}$

6 Solving the quadratic equation

The method used to solve the quadtratic equation

$$\mathbf{B}\mathbf{\Phi}^2 - \mathbf{\Phi} + \mathbf{A}= \mathbf{0} \tag{1}$$

in DPSS (2013) is based on an iterative back-substitution procedure which involves iterating on an initial arbitrary choice of $\mathbf{\Phi}$ and $\mathbf{\Psi}$, say $\mathbf{\Phi}_0$ and $\mathbf{\Psi}_0$, using the recursive relations

$$\mathbf{\Phi}_r = (\mathbf{I}_k - \mathbf{B}\mathbf{\Phi}_{r-1})^{-1}\mathbf{A}, \;\; \mathbf{\Psi}_r = (\mathbf{I}_k - \mathbf{B}\mathbf{\Phi}_{r-1})^{-1}\mathbf{B},$$

where $\mathbf{\Phi}_r$ and $\mathbf{\Psi}_r$ are the values of $\mathbf{\Phi}$ and $\mathbf{\Psi}$, respectively, at the r^{th} iteration ($r = 1, 2, ...$) and $\mathbf{\Psi}$ is the coefficient matrix in the forward equation

$$\mathbf{z}_t = \mathbf{\Psi}E_{t-1}(\mathbf{z}_{t+1}) + \mathbf{v}_t,$$

with

$$\mathbf{z}_t = \chi_t - \mathbf{\Phi}\chi_{t-1}, \;\;\; \mathbf{v}_t = (\mathbf{I}_k - \mathbf{B}\mathbf{\Phi})^{-1}\eta_t.$$

See Binder and Pesaran (1995, 1997) for further details.

We set $\mathbf{\Phi}_0$ and $\mathbf{\Psi}_0$ to the identity matrix. As a check against multiple solutions, we also started the iterations with an initial value of $\mathbf{\Phi}$ and $\mathbf{\Psi}$ that had units along the diagonal and the off diagonal terms were drawn from a uniform distribution over the range -0.5 to +0.5. Both initial values resulted in the same solution. This iterative procedure is continued until one of the following convergence criteria is met

$$\|\mathbf{\Phi}_r - \mathbf{\Phi}_{r-1}\|_{\max} \leq 10^{-6} \text{ or } \|\mathbf{\Psi}_r - \mathbf{\Psi}_{r-1}\|_{\max} \leq 10^{-6},$$

where the max norm of a matrix $\mathbf{A} = \{a_{ij}\}$ is defined as $\|\mathbf{A}\|_{\max} = \max_{i,j}\{|a_{ij}|\}$.[14] In the numerical calculations all unknown parameters are replaced by the restricted IV estimates.

[14] Matlab and Gauss code for this procedure is available at http://ideas.repec.org/c/dge/qmrbcd/73.html.

7 Forecast error variance decomposition

Under the assumption that within country supply, demand and monetary policy shocks are orthogonal, the forecast error variance decomposition corresponding to the expression for the forecast errors in DPSS (2013), is given by

$$\begin{aligned}
Var\left(\tilde{\upsilon}_{t+n}\left|\Im_{t-1}\right.\right) &= \sum_{\ell=0}^{n}\mathbf{B}_{s,n-\ell}\boldsymbol{\Sigma}_{ss}\mathbf{B}_{s,n-\ell}' + \sum_{\ell=0}^{n}\mathbf{B}_{d,n-\ell}\boldsymbol{\Sigma}_{dd}\mathbf{B}_{d,n-\ell}' \\
&+ \sum_{\ell=0}^{n}\mathbf{B}_{m,n-\ell}\boldsymbol{\Sigma}_{mm}\mathbf{B}_{m,n-\ell}' + \sum_{\ell=0}^{n}\mathbf{B}_{e,n-\ell}\boldsymbol{\Sigma}_{ee}\mathbf{B}_{e,n-\ell}' \\
&+ \sum_{\ell=0}^{n}\mathbf{B}_{s,n-\ell}\boldsymbol{\Sigma}_{se}\mathbf{B}_{e,n-\ell}' + \sum_{\ell=0}^{n}\mathbf{B}_{e,n-\ell}\boldsymbol{\Sigma}_{es}\mathbf{B}_{s,n-\ell}' \\
&+ \sum_{\ell=0}^{n}\mathbf{B}_{d,n-\ell}\boldsymbol{\Sigma}_{de}\mathbf{B}_{e,n-\ell}' + \sum_{\ell=0}^{n}\mathbf{B}_{e,n-\ell}\boldsymbol{\Sigma}_{ed}\mathbf{B}_{d,n-\ell}' \\
&+ \sum_{\ell=0}^{n}\mathbf{B}_{m,n-\ell}\boldsymbol{\Sigma}_{me}\mathbf{B}_{e,n-\ell}' + \sum_{\ell=0}^{n}\mathbf{B}_{e,n-\ell}\boldsymbol{\Sigma}_{em}\mathbf{B}_{m,n-\ell}'.
\end{aligned}$$

The first four terms give the contributions to the variance from each of the four shocks; the remaining six terms arise from the covariances between the exchange rate shocks and the three structural shocks.

8 Computation of bootstrap error bands

To get the bootstrap errors bands for the MCNK B bootstrap samples are generated denoted by $\widetilde{\mathring{\mathbf{x}}}_t^{(b)}$, $b = 1, 2, ..., B$ from the process

$$\widetilde{\mathring{\mathbf{x}}}_t^{(b)} = \hat{\boldsymbol{\Phi}}_{11}\widetilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} + \hat{\boldsymbol{\Phi}}_{12}\widetilde{\mathring{\mathbf{x}}}_{t-2}^{(b)} + \hat{\mathbf{H}}_0^{-1}\hat{\varepsilon}_t^{(b)},\ t = 1, 2, ..., T, \tag{2}$$

by resampling the structural residuals, $\hat{\varepsilon}_t$, and setting $\widetilde{\mathring{\mathbf{x}}}_0^{(b)} = \widetilde{\mathring{\mathbf{x}}}_0$ and $\widetilde{\mathring{\mathbf{x}}}_{-1}^{(b)} = \widetilde{\mathring{\mathbf{x}}}_{-1}$, where $\widetilde{\mathring{\mathbf{x}}}_0$ and $\widetilde{\mathring{\mathbf{x}}}_{-1}$ are the observed initial data vectors that include the US real exchange rate (or equivalently the US price level). Recall that the multi-country rational expectations model is solved in terms of the US price level rather than the US inflation.

The structural shocks, $\hat{\varepsilon}_t$, are initially orthogonalised by using the inverse of the Cholesky factor, $\widetilde{\mathbf{P}}$, associated with the Cholesky decomposition of the shrinkage covariance matrix, $\hat{\Sigma}_\varepsilon(0.6)$. This way we obtain the $k \times 1$ orthogonal vector $\hat{\upsilon}_t = \widetilde{\mathbf{P}}^{-1}\hat{\varepsilon}_t$ where its j^{th} element $\hat{\upsilon}_{jt}$, $j = 1, 2, ..., k$, has unit variance. The bootstrap error vector is then obtained as $\varepsilon_t^{(b)} = \widetilde{\mathbf{P}}\hat{\upsilon}_t^{(b)}$, where $\hat{\upsilon}_t^{(b)}$ is the $k \times 1$ vector of re-sampled values from $\{\hat{\upsilon}_{jt}\}_{j=1,2,...,k;t=1,2,...,T}$. Prior to any resampling the structural residuals are recentered to ensure that their bootstrap population mean is zero.

Once a series $\widetilde{\mathring{\mathbf{x}}}_t^{(b)}$, $b = 1, 2, ..., B$ is generated, US inflation is computed from the US price level so that $\widetilde{\mathbf{x}}_{it}^{(b)}$ is constructed, with the corresponding foreign variables, $\widetilde{\mathbf{x}}_{it}^{*(b)}$, computed using the trade weights. For each bootstrap replication the individual country models are then estimated by the inequality constrained IV procedure, ensuring that any constraint which binds for the estimates based on historical realisations are also imposed on the bootstrap estimates.

The country specific models in terms of $\widetilde{\mathbf{x}}_{it}^{(b)}$ are given by

$$\hat{\mathbf{A}}_{i0}^{(b)}\widetilde{\mathbf{x}}_{it}^{(b)} = \hat{\mathbf{A}}_{i1}^{(b)}\widetilde{\mathbf{x}}_{i,t-1}^{(b)} + \hat{\mathbf{A}}_{i2}^{(b)}E_{t-1}(\widetilde{\mathbf{x}}_{i,t+1}^{(b)}) + \hat{\mathbf{A}}_{i3}^{(b)}\widetilde{\mathbf{x}}_{it}^{*(b)} + \hat{\mathbf{A}}_{i4}^{(b)}\widetilde{\mathbf{x}}_{i,t-1}^{*(b)} + \varepsilon_t^{(b)},$$

and are subsequently combined yielding the MCNK model

$$\widetilde{\mathring{\mathbf{x}}}_t^{(b)} = \hat{\mathbf{F}}_1^{(b)}\widetilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} + \hat{\mathbf{F}}_2^{(b)}\widetilde{\mathring{\mathbf{x}}}_{t-2}^{(b)} + \hat{\mathbf{F}}_3^{(b)}E_{t-1}(\widetilde{\mathring{\mathbf{x}}}_{t+1}^{(b)}) + \hat{\mathbf{F}}_4^{(b)}E_{t-1}(\widetilde{\mathring{\mathbf{x}}}_t^{(b)}) + \mathbf{u}_t^{(b)}. \quad (3)$$

Solving the quadratic matrix as described earlier, the reduced form solution of (3) follows as

$$\widetilde{\mathring{\mathbf{x}}}_t^{(b)} = \hat{\mathbf{\Phi}}_{11}^{(b)}\widetilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} + \hat{\mathbf{\Phi}}_{12}^{(b)}\widetilde{\mathring{\mathbf{x}}}_{t-2}^{(b)} + \mathbf{u}_t^{(b)},$$

with

$$\hat{\mathbf{u}}_t^{(b)} = \widetilde{\mathring{\mathbf{x}}}_t^{(b)} - \hat{\mathbf{\Phi}}_{11}^{(b)}\widetilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} - \hat{\mathbf{\Phi}}_{12}^{(b)}\widetilde{\mathring{\mathbf{x}}}_{t-2}^{(b)}$$

and

$$\hat{\varepsilon}_t^{(b)} = \hat{\mathbf{H}}_0^{(b)}\hat{\mathbf{u}}_t^{(b)}.$$

For the first bootstrap replication we begin the iterative back-substitution procedure, using the estimated $\hat{\Phi}$ from the actual data as an initial value to compute (2) and (3), so that for $b = 1$, $\Phi_0^{(1)} = \hat{\Phi}$. For each subsequent bootstrap replication, b, the initial value is set to the solution of (1) obtained under the preceding replication, $b-1$, so that $\Phi_0^{(b)} = \hat{\Phi}^{(b-1)}$ and $\Psi_0^{(b)} = (I_k - \hat{B}^{(b)}\hat{\Phi}^{(b-1)})^{-1}\hat{B}^{(b)}$. If for a particular bootstrap replication the iterative back-substitution procedure fails to converge after 500 iterations, the initial values for $\Phi_0^{(b)}$ and $\Psi_0^{(b)}$ are set to the identity matrix.

For each bootstrap replication $b = 1, 2, ..., B$, having estimated the individual country NK models using the simulated data $\widetilde{\mathring{\mathbf{x}}}_t^{(b)}$, the MCNK model is reconstructed as described above and the impulse responses are calculated $g^{(b)}(n)$, for $n = 0, 1, 2, ...$ These statistics are then sorted in ascending order, and the $(1-\alpha)100\%$ confidence interval is calculated by using the $\alpha/2$ and $(1-\alpha/2)$ quantiles, say $q_{\alpha/2}$ and $q_{(1-\alpha/2)}$, respectively of the bootstrap distribution of $g(n)$.

To compute the upper and lower confidence bounds we use 2000 convergent and stationary bootstrap replications. A convergent replication is defined as one where for the corresponding bootstrap sample, the iterative back-substitution procedure described above converges within 500 iterations, whether the initial values for $\Phi_0^{(b)}$ and $\Psi_0^{(b)}$ are set to the identity matrix or otherwise. Having achieved convergence, a bootstrap replication is checked to make sure that it yields a stationary solution. If any of the above two conditions is violated, a new bootstrap sample is computed. For our bootstrap results we had to carry out a total of 3475 bootstrap replications, of which 1475 where due to non-convergence of the iterative back-substitution procedure. No bootstrap replications were found to be non-stationary.

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