Exponent of Cross-sectional Dependence for Residuals*

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Abstract

In this paper, we focus on estimating the degree of cross-sectional dependence in the error terms of a classical panel data regression model. For this purpose we propose an estimator of the exponent of cross-sectional dependence denoted by α, which is based on the number of non-zero pair-wise cross correlations of these errors. We prove that our estimator, $\tilde{\alpha}$, is consistent and derive the rate at which $\tilde{\alpha}$ approaches its true value. We also propose a resampling procedure for the construction of confidence bounds around the estimator of α. We evaluate the finite sample properties of the proposed estimator by use of a Monte Carlo simulation study. The numerical results are encouraging and supportive of the theoretical findings. Finally, we undertake an empirical investigation of α for the errors of the CAPM model and its Fama-French extensions using 10-year rolling samples from S&P 500 securities over the period Sept 1989 - May 2018.

Keywords: Pair-wise correlations, Cross-sectional dependence, Cross-sectional averages, Weak and strong factor models, CAPM and Fama-French Factors.

JEL Codes: C21, C32

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1 Introduction

Interest in the analysis of cross-sectional dependence applied to households, firms, markets, regional and national economies has become prominent over the past decade, especially so in the aftermath of the latest financial crisis given its effects on the global economy. Researchers in many fields have turned to network theory, spatial and factor models to obtain a better understanding of the extent and nature of such cross dependencies. There are many issues to be considered: how to test for the presence of cross-sectional dependence, how to measure the degree of cross-sectional dependence, how to model cross-sectional dependence, and how to carry out counterfactual exercises under alternative network formations or market interconnections. Many of these topics are the subject of ongoing research. In this paper we focus on measuring cross-sectional dependence.

Bailey, Kapetanios and Pesaran (2016, BKP hereafter) give a thorough account of the rationale and motivation behind the need for determining the extent of cross-sectional dependence, be it in finance, micro or macroeconomics. They focus on the asymptotic behaviour of the variance of the cross section average of the observations on a double array of random variables, say x_{it}, indexed by $i = 1, 2, \ldots, N$ and $t = 1, 2, \ldots, T$, over space and time. In particular, they analyse the rate at which this variance tends to zero and show that it depends on the degree or exponent of cross-sectional dependence which they denote by α. They explore a factor model setting as a vehicle for characterising strong and semi-strong covariance structures as defined in Chudik et al. (2011). They relate these to the degree of pervasiveness of factors in unobserved factor models often used in the literature to model cross-sectional dependence.

In this paper we build on BKP and extend the analysis in two respects. First, we consider a more generic setting which does not require a common factor representation and holds more generally for both moderate to sizable cross-sectional dependence. We achieve this by directly considering the significance of individual pair-wise correlations, and do not concern ourselves with the factors that might underlie these pair-wise correlations. Second, we consider estimating the exponent of cross-sectional dependence, α, of the residuals obtained from a panel data regression model.

We propose a new estimator of α based on the number of statistically significant pair-wise correlations of the residuals from the panel regression under consideration. To establish the statistical significance of the correlation coefficients we adopt the thresholding multiple testing (MT) estimator proposed by Bailey et al. (2018), BPS. Other thresholding estimators can also be used. See, for example, Bickel and Levina (2008) or Karoui (2008) and Cai and Liu (2011) or Fan et al. (2013). The MT testing procedure advanced by BPS has the advantage that it directly considers the statistical significance of the correlation coefficient which is invariant to scales. Other thresholding procedures focus on the sample covariances and resort to cross validation to identify the threshold. Bickel and Levina (2008) use universal thresholding, namely comparing all the sample covariances to the same threshold value, whilst Cai and Liu (2011) propose an 'adaptive' thresholding procedure that allows for differing thresholds across the different pairs of sample covariances. Other contributions to this literature include the work of Huang et al. (2006), Rothman et al. (2009), Cai and Zhou (2011) and Cai and Zhou (2012), Wang and

Zhou (2010), and Fan et al. (2011).[1] All these contributions apply the thresholding procedure to sample covariances and do not apply to the residuals from a panel regression model that concerns us in this paper. It is also important to bear in mind that when estimating α we do not assume that the underlying error covariance matrix is sparse, as is assumed in the literature on regularization of the sample covariance. Our objective is to estimate the degree of sparsity of the covariance matrix rather than assume sparsity for the purpose of consistent estimation of the covariance matrix or its inverse. What matters for estimation of α is to ensure that all non-zero entries of the correlation matrix are correctly identified.

We establish consistency of our estimator under the assumptions of exogeneity of regressors and symmetry of the error distribution. We also explain how the derivations can be extended to the case when weakly exogenous variables are present, as for example in a dynamic panel data setting. The proposed estimator is simple to compute and is shown to perform well in small samples, for a variety of correlation matrices, irrespective of whether the cross correlations are generated from a multi-factor structure or specified by a given correlation matrix with a specified degree of sparsity. This is especially the case as compared to basing the estimation of α on the largest eigenvalue of the correlation matrix, which performs particularly poorly. The rate of convergence of our preferred estimator is complex and depends on an interplay of the cross-sectional and time dimensions, N and T. The Monte Carlo results also show that the error in estimating α is smaller for values of α close to unity, which is likely to be of greater interest in practice. The problem of making inference about the value of α raises additional technical difficulties and will not be addressed in this paper. In practice, bootstrap techniques can be used to obtain confidence bounds around our proposed estimator. We provide some Monte Carlo results in support of estimating the empirical distribution of the proposed estimator of α, using cross-sectional resampling as suggested in Kapetanios (2008). Finally, we provide an empirical application investigating the degree of inter-linkages between financial variables using the Standard & Poor's 500 index. We present 10-year rolling estimates of α applied to excess returns on securities included in the S&P 500 data set as well as α estimates applied to the residuals obtained from the CAPM and its Fama-French extensions used extensively in the finance literature.

The rest of the paper is organised as follows: Section 2 discusses alternative characterisations of α, the exponent of cross-sectional dependence, and the conditions under which these measures are equivalent as $N \to \infty$. Section 3 sets up the panel data model and discusses its underlying assumptions. Section 4 proposes the estimator of α in terms of the number of statistically significant non-zero pair-wise correlations of the residuals. Section 5 presents the main theoretical results of the paper for a static panel data model with strictly exogenous regressors. Extensions to dynamic panels or panels with weakly exogenous regressors are discussed in the sub-section 5.2 while inference of α by use of bootstrap procedures is discussed in sub-section 5.3. Section 6 presents a detailed Monte Carlo simulation study. The empirical application is discussed in Section 7. Finally, Section 8 concludes. Proofs of all theoretical results are provided in the Appendix.

[1] Shrinkage procedures have also been proposed in the literature for regularization of covariance matrices. See, for example, Stein (1956), Ledoit and Wolf (2003) and Ledoit and Wolf (2004). However, the shrinkage procedure does not set any elements of the covariance matrix to zero, and is not suitable for estimation of α which builds on the support recovery properties of the estimated covariance matrix.

2 Degrees of cross-sectional dependence: alternative measures

Our analysis focuses on the covariance matrix of $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Nt})'$, where $\boldsymbol{\varepsilon}_t$ is the $N \times 1$ vector of errors from a panel data regression model. Let $\boldsymbol{\Sigma}_N = E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') = (\sigma_{ij})$, and denote its largest eigenvalue by $\lambda_{\max}(\boldsymbol{\Sigma}_N) > 0$. The errors ε_{it} are said to be strongly cross-sectionally correlated, if $\lambda_{\max}(\boldsymbol{\Sigma}_N) = \ominus(N)$, where $\ominus$ denotes exact order of magnitude, and they are said to be weakly cross-sectionally correlated, if $\lambda_{\max}(\boldsymbol{\Sigma}_N)$ is bounded in N. All intermediate cases can be parameterized in terms of the exponent α_λ, such that

$$\lambda_{\max}(\boldsymbol{\Sigma}_N) = \ominus(N^{\alpha_\lambda}). \tag{1}$$

The weak and strong cross dependence cases then relate to $\alpha_\lambda = 0$ and $\alpha_\lambda = 1$, respectively. It is important to emphasise that the exponent, α_λ, is an asymptotic concept, in the sense that α_λ can be identified only as $N \to \infty$, as the definition in (1) makes clear.

Suppose now that the cross dependence of ε_{it} is characterized by the following approximate multiple-factor error process

$$\varepsilon_{it} = \beta_i' \mathbf{f}_t + u_{it}, \tag{2}$$

where $\mathbf{f}_t$ is the $m \times 1$ vector of unobserved common factors with zero means, and $\boldsymbol{\beta}_i = (\beta_{i1}, \beta_{i2}, \ldots, \beta_{im})'$ is the associated $m \times 1$ vector of factor loadings, and u_{it} is the idiosyncratic component assumed to have mean zero and the covariance matrix $\mathbf{V} = E(\mathbf{u}_t \mathbf{u}_t')$, where $\mathbf{u}_t = (u_{1t}, u_{2t}, \ldots, u_{Nt})'$. Then

$$\boldsymbol{\Sigma}_N = E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') = \mathbf{B}\mathbf{B}' + \mathbf{V},$$

where $\mathbf{B} = (\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \ldots, \boldsymbol{\beta}_N)'$, and without loss of generality we have set $E(\mathbf{f}_t \mathbf{f}_t') = \mathbf{I}_m$. To identify the factor component from the idiosyncratic component we assume that $\lambda_{\max}(\mathbf{V}) = O(1)$, but allow the factor loadings to satisfy the condition

$$\mathbf{B}'\mathbf{B} = \sum_{i=1}^{N} \boldsymbol{\beta}_i \boldsymbol{\beta}_i' = \ominus(N^{\alpha_\beta}), \ \alpha_\beta > 0, \tag{3}$$

where α_β measures the degree to which the factors are pervasive, in the sense that they have non-zero effects on the individual errors, ε_{it}. In what follows we refer to α_β as the exponent of factor loadings. In the standard approximate factor models it is assumed that $\alpha_\beta = 1$, whilst in practice, where the possibility of weak factors can not be ruled out, α_β could be a parameter of interest to be estimated.

To see how α_β and α_λ are related note that

$$\lambda_{\max}(\boldsymbol{\Sigma}_N) \leq \|\mathbf{B}\mathbf{B}' + \mathbf{V}\|_1 \leq \|\mathbf{B}\|_1 \|\mathbf{B}\|_\infty + \|\mathbf{V}\|_1,$$

where $\|\mathbf{B}\|_1$ and $\|\mathbf{B}\|_\infty$ are column and row norms of $\mathbf{B}$, respectively. To ensure that $Var(\varepsilon_{it})$ is bounded we must have $\|\mathbf{B}\|_\infty < K$. Also to ensure that $\lambda_{\max}(\mathbf{V}) = O(1)$, we must have $\|\mathbf{V}\|_1 < K$. Therefore, the rate at which $\lambda_{\max}(\boldsymbol{\Sigma}_N)$ can rise with N is controlled by

$$\|\mathbf{B}\|_1 = \max_{1 \leq j \leq N} \sum_{i=1}^{N} |\beta_{ij}|. \tag{4}$$

Setting $\sum_{i=1}^{N}|\beta_{ij}| = \ominus(N^{\alpha_{\beta_j}})$, for $j = 1, 2, \ldots, m$, then $\|\mathbf{B}\|_1 = \ominus(N^{\alpha_\beta})$, where $\alpha_\beta = \max_j(\alpha_{\beta_j})$. Then

$$\lambda_{\max}(\mathbf{\Sigma}_N) = \ominus(N^{\alpha_\beta}) + O(1).$$

To distinguish the effects of the factor component from those of the idiosyncractic component we must have $\alpha_\beta > 0$. Comparing this result with (1) establishes that $\alpha_\lambda = \alpha_\beta > 0$, as $N \to \infty$.

The above analysis suggests two alternative ways of estimating α_λ. A direct procedure would be to base the estimate of α_λ on $\lambda_{\max}(\mathbf{\Sigma}_N)$ and set

$$\lambda_{\max}(\mathbf{\Sigma}_N) = O(N^{\alpha_\lambda}) = \kappa N^{\alpha_\lambda},$$

where κ is a constant independent of N. Then

$$\alpha_\lambda = \frac{\ln[\lambda_{\max}(\mathbf{\Sigma}_N)]}{\ln(N)} - \frac{\ln(\kappa)}{\ln(N)}. \tag{5}$$

In order to identify α_λ, as $N \to \infty$, we set $\kappa = 1$, so that (5) becomes

$$\alpha_\lambda = \frac{\ln[\lambda_{\max}(\mathbf{\Sigma}_N)]}{\ln(N)}. \tag{6}$$

In this form, the value of α_λ is susceptible to the scaling of elements in $\boldsymbol{\varepsilon}_t$. For this reason we focus our attention rather on the corresponding correlation matrix $\boldsymbol{R}_N = (\rho_{ij})$ given by

$$\boldsymbol{R}_N = \boldsymbol{D}_N^{-1/2}\mathbf{\Sigma}_N\boldsymbol{D}_N^{-1/2},$$

where

$$\mathbf{D}_N = \mathrm{diag}(\sigma_{ii}, i = 1, 2, \ldots, N). \tag{7}$$

Hence, (6) finally becomes

$$\alpha_\lambda = \frac{\ln[\lambda_{\max}(\boldsymbol{R}_N)]}{\ln(N)}, \tag{8}$$

and α_λ has fixed bounds at zero and unity, as $N \to \infty$.

Developing a theory based on the maximum eigenvalue of the correlation matrix $\boldsymbol{R}_N$ can be challenging. To avoid some of the technical problems involved in estimating $\lambda_{\max}(\boldsymbol{R}_N)$, and noting that $Var(\bar{\varepsilon}_t) \leq N^{-1}\lambda_{\max}(\mathbf{\Sigma}_N)$, BKP propose basing the estimation of α_λ on $Var(\bar{\varepsilon}_t)$, where $\bar{\varepsilon}_t = N^{-1}\sum_{i=1}^{N}\varepsilon_{it}$. In the case where ε_{it} has a factor representation, BKP show that $Var(\bar{\varepsilon}_t) = O\left[\max\left(N^{2(\alpha_\beta-1)},\ N^{-1}\right)\right]$, which reduces to $Std(\bar{\varepsilon}_t) = O\left(N^{(\alpha_\beta-1)}\right)$, if $2(\alpha_\beta - 1) > -1$, or if $\alpha_\beta > 1/2$. This means that at least $N^{1/2}$ of the factor loadings must have non-zero values for $\mathbf{\Sigma}_N$ to differ sufficiently from a diagonal $\mathbf{\Sigma}_N$.

In this paper we consider an alternative estimation strategy that does not require ε_{it} to have a factor representation. Since

$$\lambda_{\max}(\boldsymbol{R}_N) \leq \|\boldsymbol{R}_N\|_1 = \max_{1\leq j\leq N}\sum_{i=1}^{N}|\rho_{ij}|,$$

we focus directly on estimation of ρ_{ij} and distinguish between values of ρ_{ij} that are close to zero and those that are significantly different from zero, and measure the exponent of cross-sectional dependence in terms of the number of significant (non-zero) cross-correlation coefficients. Specifically, we define α such that $M_N = N^{2\alpha}$ where M is the number of non-zero

elements of $\boldsymbol{R}_N$ which can be written equivalently as $M_N = \boldsymbol{\tau}'_N \boldsymbol{\Delta}_N \boldsymbol{\tau}_N$, where $\boldsymbol{\tau}_N$ is an $N \times 1$ vector of ones and $\boldsymbol{\Delta}_N = (\delta_{ij})$ is an $N \times N$ matrix of population correlation indicators with typical elements given by

$$\delta_{ij} = I\left(\rho_{ij} \neq 0\right), \; i, j = 1, 2, \ldots, N,$$

in which $I(\mathcal{A})$ is equal to unity if $\mathcal{A}$ is true and zero otherwise. Note that by construction $\delta_{ii} = 1$. Hence,

$$\alpha = \frac{\ln\left(M_N\right)}{\ln(N^2)} = \frac{\ln\left(\boldsymbol{\tau}'_N \boldsymbol{\Delta}_N \boldsymbol{\tau}_N\right)}{\ln N^2}. \tag{9}$$

Cross-sectional independence refers to the case when $\mathbf{R}_N = \mathbf{I}_N$, and $\alpha = 1/2$, while the case of cross-sectional strong dependence corresponds to all pair-wise correlation coefficients being non-zero such that $\alpha = 1$. Note that by construction $1/2 \leq \alpha \leq 1$, with $\alpha = 1/2$ arising when $\boldsymbol{\Delta}_N = \mathbf{I}_N$, and $\alpha = 1$ if $\rho_{ij} \neq 0$ for all i and j.

Other exponents of cross-sectional dependence can be defined by focussing only on the off-diagonal elements of $\mathbf{R}_N$ and consider the following exponent of cross-sectional dependence:[2]

$$\alpha^{\circ} = \frac{\ln\left[\boldsymbol{\tau}'_N \left(\boldsymbol{\Delta}_N - \mathbf{I}_N\right) \boldsymbol{\tau}_N\right]}{\ln N\left(N-1\right)},$$

assuming that $\boldsymbol{\Delta}_N \neq \mathbf{I}_N$. Unlike α the above measure is not defined if $\mathbf{R}_N = \mathbf{I}_N$. The two measures coincide, namely $\alpha = \alpha^{\circ} = 1$, if $\rho_{ij} \neq 0$ for all i and j, as $N \to \infty$. In cases where ε_{it} have a multi-factor error representation given by (2), the largest exponent of the factor loadings is given by $\alpha_\beta > 0$. Assuming, for simplicity that $\mathbf{V}$ is diagonal, it then readily follows that $\boldsymbol{\tau}'_N \left(\boldsymbol{\Delta}_N - \mathbf{I}_N\right) \boldsymbol{\tau}_N = N^{2\alpha_\beta} - N^{\alpha_\beta}$, where $\left(N^{2\alpha_\beta} - N^{\alpha_\beta}\right) = N^{\alpha_\beta}\left(N^{\alpha_\beta} - 1\right)$ is the total number of off-diagonal non-zero pair-wise cross correlations of the errors. In such a multi-factor error set up we have

$$\alpha = \frac{\ln\left(\boldsymbol{\tau}'_N \boldsymbol{\Delta}_N \boldsymbol{\tau}_N\right)}{\ln N^2} = \frac{\ln\left(N^{2\alpha_\beta} - N^{\alpha_\beta} + N\right)}{\ln N^2},$$

and

$$\alpha^{\circ} = \frac{\ln\left[\boldsymbol{\tau}'_N \left(\boldsymbol{\Delta}_N - \mathbf{I}_N\right) \boldsymbol{\tau}_N\right]}{\ln N\left(N-1\right)} = \frac{\ln\left(N^{2\alpha_\beta} - N^{\alpha_\beta}\right)}{\ln N\left(N-1\right)}.$$

Recall that we must have $\alpha_\beta > 1/2$ for factors to be distinguishable from the idiosyncratic components. It is then easily seen that $\lim_{N\to\infty} \alpha = \lim_{N\to\infty} \alpha^{\circ} = \alpha_\beta$. However, the two measures could differ if N is not sufficiently large. In finite samples α° can be written in terms of α by first solving the quadratic equation

$$N^{2\alpha_\beta} + \left(N - N^{\alpha_\beta}\right) = N^{2\alpha}, \tag{10}$$

[2]One can also consider only the distinct off-diagonal elements of $\mathbf{R}_N$ and define α as

$$\alpha^{\circ} = \frac{\ln\left[\frac{1}{2}\boldsymbol{\tau}'_N \left(\boldsymbol{\Delta}_N - \mathbf{I}_N\right) \boldsymbol{\tau}_N\right]}{\ln\left[\frac{1}{2}N\left(N-1\right)\right]} = \frac{\ln\left[\boldsymbol{\tau}'_N \left(\boldsymbol{\Delta}_N - \mathbf{I}_N\right) \boldsymbol{\tau}_N\right] - \ln(2)}{\ln\left[N\left(N-1\right)\right] - \ln(2)}$$
$$= \frac{\alpha - \frac{\ln(2)}{\ln[N(N-1)]}}{1 - \frac{\ln(2)}{\ln[N(N-1)]}} \to \alpha, \text{ as } N \to \infty.$$

for α_β, namely

$$\alpha_\beta = \frac{\ln\left(1+\sqrt{1-4\left(N-N^{2\alpha}\right)}\right)-\ln 2}{\ln N}. \tag{11}$$

Since $[N^{\alpha_\beta}]$ can only take positive or zero values the second root of (10) is clearly redundant. In what follows, we focus on α since it is defined even if $\mathbf{R}_N = \mathbf{I}_N$, and α is suitably scaled to lie in the range $(1/2, 1]$.

3 Panel data model

Consider the panel data regression model

$$y_{it} = \gamma_i' \mathbf{x}_{it} + \varepsilon_{it}, \text{ for } i = 1, 2, \ldots, N; t = 1, 2, \ldots, T, \tag{12}$$

where $\mathbf{x}_{it}$ is a $k \times 1$ vector of observed regressors, γ_i is the associated vector of coefficients, and ε_{it} are the model's errors. We are interested in estimating the exponent of the cross-sectional dependence of the errors, ε_{it}, defined by (9). First, we obtain residuals e_{it} computed as

$$e_{it} = y_{it} - \mathbf{x}_{it}'\hat{\gamma}_i = y_{it} - \mathbf{x}_{it}'(\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i'\mathbf{y}_i, \tag{13}$$

where $\mathbf{X}_i$ is the $T \times k$ matrix of observations on the regressors for the ith unit, and $\mathbf{y}_i$ is the $T \times 1$ vector of observations on the dependent variable of the ith unit. We assume that the regressors are strictly exogenous.

We define the standardized errors, ξ_{it}, and the associated standardized residuals, z_{it}, as

$$\xi_{it} = \frac{\varepsilon_{it}}{\left(T^{-1}\boldsymbol{\varepsilon}_i'\mathbf{M}_i\boldsymbol{\varepsilon}_i\right)^{1/2}}, \tag{14}$$

$$z_{it} = \frac{e_{it}}{\left(T^{-1}\mathbf{e}_i'\mathbf{M}_i\mathbf{e}_i\right)^{1/2}} = \frac{e_{it}}{\left(T^{-1}\mathbf{e}_i'\mathbf{e}_i\right)^{1/2}}, \tag{15}$$

where $\varepsilon_i = (\varepsilon_{i1}, \varepsilon_{i2}, \ldots, \varepsilon_{iT})'$, $\mathbf{e}_i = (e_{i1}, e_{i2}, \ldots, e_{iT})'$, and $\mathbf{M}_i = \mathbf{I}_T - \mathbf{X}_i(\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i'$. Note that (since $\mathbf{e}_i = \mathbf{M}_i\boldsymbol{\varepsilon}_i$)

$$z_{it} = \frac{\varepsilon_{it} + \mathbf{x}_{it}'(\gamma_i - \hat{\gamma}_i)}{\left(T^{-1}\boldsymbol{\varepsilon}_i'\mathbf{M}_i\boldsymbol{\varepsilon}_i\right)^{1/2}} = \xi_{it} - \mathbf{a}_{it}'\boldsymbol{\xi}_i, \tag{16}$$

where

$$\mathbf{a}_{it} = \mathbf{X}_i\left(\mathbf{X}_i'\mathbf{X}_i\right)^{-1}\mathbf{x}_{it},$$

and $\boldsymbol{\xi}_i = (\xi_{i1}, \xi_{i2}, \ldots, \xi_{iT})'$.

Further, in what follows we assume that the error terms are symmetrically distributed.

Assumption 1 *Conditional on $\mathbf{X}_i$, the errors of the panel data model, (12), (a) ε_{it} are symmetrically distributed with zero means and variances $0 < c < \sigma_i^2 < K < \infty$, (b) ε_{it} are serially independent, (c) ε_{it} and ε_{jt} are distributed independently if $E(\varepsilon_{it}\varepsilon_{jt}) = 0$, for all $i \neq j$.*

Under the above assumption, and using (14) it readily follows that

$$E\left(\xi_i \,|\mathbf{X}_i\right) = \mathbf{0}, \tag{17}$$

and

$$E(\xi_i \xi_j' \,|\mathbf{X}_i, \mathbf{X}_j\,) = \rho_{ij}\mathbf{I}_T. \tag{18}$$

Our main analysis will condition on the observed regressors. Remark 2 will discuss an unconditional version of our results. For the observed regressors, we make the following assumption:

Assumption 2 *The $k\times 1$ vector of regressors $\mathbf{x}_{it}$ in (12) is bounded: $\sup_{i,t}\|\mathbf{x}_{it}\| < \infty$. Further, for some T_0, we have*

$$\inf_{T>T_0,i} \lambda_{\min}\left(\frac{\mathbf{X}_i'\mathbf{X}_i}{T}\right) > 0.$$

Under Assumption 1 it readily follows that

$$E\left(\mathbf{a}_{it}'\boldsymbol{\xi}_i \,|\mathbf{X}_i\,\right) = \mathbf{x}_{it}'(\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i' E\left(\boldsymbol{\xi}_i \,|\mathbf{X}_i\,\right) = 0,$$

where $\mathbf{x}_{it}'$ is the tth row of $\mathbf{X}_i = (\mathbf{x}_{i1}, \mathbf{x}_{i2}, \ldots, \mathbf{x}_{iT})'$. Hence,

$$E\left(z_{it}\,|\mathbf{X}_i\,\right) = 0, \tag{19}$$

which in turn implies that z_{it} is a martingale difference process with respect to the non-decreasing information set, $\Omega_{it} = (\mathbf{x}_{i1}, \mathbf{x}_{i2}, \ldots, \mathbf{x}_{it})$. By construction $\Omega_{iT} \equiv \mathbf{X}_i$. Indeed, since Ω_{it} is a subset of Ω_{iT}, then by the chain rule of conditional expectations we have

$$E\left(z_{it}\,|\Omega_{it}\,\right) = E\left[E\left(z_{it}\,|\Omega_{iT}\,\right)|\Omega_{it}\,\right].$$

But in view of (19), $E\left(z_{it}\,|\Omega_{iT}\,\right) = 0$, and it also follows that $E\left(z_{it}\,|\Omega_{it}\,\right) = 0$, for all i and t.

We also require the following assumption that sets a lower bound condition on the non-zero values of the pair-wise correlations.

Assumption 3 *Let $\rho_{\min} = \inf_{i,j}\left(|\rho_{ij}| \;\; |\rho_{ij} \neq 0\right)$. Then,*

$$\lim_{T\to\infty} \frac{\ln(T)}{\sqrt{T}\rho_{\min}} = 0. \tag{20}$$

Remark 1 *This assumption is needed for successful recovery of non-zero pair-wise correlations, and is weaker than requiring $\rho_{\min} > 0$, as it allows $\rho_{\min}$ to tend to zero with N or T or both, so long as its rate of decline is slower than $\ln(T)/\sqrt{T}$.*

4 Consistent estimation of α

Consider the sample estimate of the pair-wise correlation coefficients of the residuals from units i and j,

$$\hat{\rho}_{ij} = \frac{T^{-1}\mathbf{e}_i'\mathbf{e}_j}{\left(T^{-1}\mathbf{e}_i'\mathbf{e}_i\right)^{1/2}\left(T^{-1}\mathbf{e}_j'\mathbf{e}_j\right)^{1/2}}, \tag{21}$$

where $\mathbf{e}_i = (e_{i1}, e_{i2}, \ldots, e_{iT})'$, e_{it} is defined by (13), and by construction the sample mean of e_{it} is exactly zero. We can re-write (21) equivalently as

$$\hat{\rho}_{ij} = T^{-1} \sum_{t=1}^{T} z_{it} z_{jt}, \tag{22}$$

where z_{it} is defined by (15). In order to identify whether the pair-wise correlation coefficients $\hat{\rho}_{ij}$ are significantly different from zero we follow Bailey et al. (2018) and apply the multiple testing estimator associated with $\hat{\rho}_{ij}$. This is defined by

$$\tilde{\rho}_{ij} = \hat{\rho}_{ij} I \left(|\hat{\rho}_{ij}| > \frac{c_p(n, \delta)}{\sqrt{T}} \right), \tag{23}$$

where

$$c_p(n, \delta) = \Phi^{-1} \left(1 - \frac{p/2}{n^{\delta}} \right), n = \frac{1}{2} N(N-1), \tag{24}$$

n is the number of tests carried out, p is the nominal size of the individual test, which can be set to 1%, 5% or 10%, $\Phi^{-1}(.)$ is the inverse of the standard normal distribution function, and δ is a tuning parameter to be set *a priori.* This thresholding method is based on the notion that for each unit (i, j) pairs we carry out a total of $\frac{1}{2}N(N-1)$ individual tests of the null hypothesis that $\rho_{ij} = 0$ where $j \neq i$, $i, j = 1, 2, \ldots, N$. Such tests can result in spurious outcomes especially when N is larger than T. The critical value function, $c_p(n, \delta)$, is therefore adjusted using parameter δ to take account of the effects of the multiple testing procedure for the estimation of α. It is important to bear in mind that the multiple testing problem encountered here differs from the standard one studied in the literature by Bonferroni (1935), Holm (1979) and others. Our focus here is on identifying the range of values for δ such that α can be consistently estimated, rather than controlling the overall size of the multiple tests being carried out.

Accordingly, we propose to estimate α by

$$\tilde{\alpha} = \frac{\ln \left(\boldsymbol{\tau}' \tilde{\boldsymbol{\Delta}} \boldsymbol{\tau} \right)}{2 \ln N}, \tag{25}$$

where $\tilde{\boldsymbol{\Delta}} = (\tilde{\delta}_{ij})$, with

$$\tilde{\delta}_{ij} = \begin{cases} \tilde{\rho}_{ij}, \text{ if } |\hat{\rho}_{ij}| > \frac{c_p(n,\delta)}{\sqrt{T}}, \text{ for } i \neq j \\ 1, \quad \text{for } i = j \\ 0, \quad \text{otherwise} \end{cases}.$$

5 Theoretical Derivations

5.1 Main Results

To establish that $\tilde{\alpha}$ converges to α, in addition to Assumptions 1, 2 and 3, we also require the following additional technical sub-exponential assumption:

Assumption 4 *There exist sufficiently large positive constants C_0, C_1, $s > 0$, such that*

$$\sup_{i,t} \Pr\left(|\varepsilon_{it}| > \alpha\right) \leq C_0 \exp\left(-C_1 \alpha^s\right), \text{ for all } \alpha > 0. \tag{26}$$

This assumption is used to allow a relatively simple bounding of an infinite sum of probabilities, needed for the proof of Lemmas 2 and 3 in the Appendix. It can be relaxed to allow for fatter tails, at the expense of smaller allowable values for N.

The rate of convergence of $\tilde{\alpha}$ to α is given in Theorem 1 below:

Theorem 1 *Consider the panel data regression model (12) and suppose that Assumptions 1-4 hold. Let $\tilde{\alpha}$ and δ be defined by (25), (23), and (24). Then, conditional on the observed $\mathbf{x}_{it}$, as $N, T \to \infty$, and if, for some $d > 0$, $N = O(T^d) = o\left(\exp(T)\right)$,*

$$2\left(\ln N\right)\left(\tilde{\alpha} - \alpha\right) = O\left(N^{2(1-\alpha-\varkappa\delta)}\right) + O\left(N^{2(1-\alpha)} \exp\left(-C_0 T^{C_1}\right)\right) + O\left(N^{-\alpha}\right) + O\left(N^{1-2\alpha}\right) = \tag{27}$$
$$O\left(T^{2d(1-\alpha-\varkappa\delta)}\right) + O\left(\exp\left[2d(1-\alpha)\ln(T) - C_0 T^{C_1}\right]\right) + O\left(T^{-d\alpha}\right) + O\left(T^{d(1-2\alpha)}\right)$$

for any $0 < \varkappa < 1$, and some $C_0, C_1 > 0$.

As long as δ is set large enough ($\delta > 1 - \alpha$), the first term on the RHS of (27) can be made sufficiently small.

Remark 2 *If we do not wish to condition on the observed $\mathbf{x}_{it}$, one could obtain the result of Theorem 1, unconditionally, if it is assumed that the regressors satisfy the following sub-exponential condition for some $s > 0$,*

$$\sup_{i,t} \Pr\left(\|\mathbf{x}_{it}\| > \alpha\right) \leq C_0 \exp\left(-C_1 \alpha^s\right), \text{ for all } \alpha > 0, \tag{28}$$

and if for some T_0, $\left(\frac{\mathbf{X}_i'\mathbf{X}_i}{T}\right)^{-1}$ exists for all $T > T_0$. Under these conditions on $\mathbf{x}_{it}$ (which replace Assumption 2), we can then use Lemma A6 of Chudik et al. (2018) to establish suitable probability bounds on $\left(\frac{\boldsymbol{\xi}_i'\mathbf{X}_i}{T}\right)\left(\frac{\mathbf{X}_i'\mathbf{X}_i}{T}\right)^{-1}\left(\frac{\mathbf{X}_i'\boldsymbol{\xi}_i}{T}\right)$, and to show that, for some $C_0, C_1 > 0$, and $0 < \pi < 1$,

$$\sup_{i,j} \Pr\left(\left|\textstyle\sum_{t=1}^T z_{it} z_{jt}\right| > \sqrt{T} c_p\left(n, \delta\right)\right) \leq \sup_{i,j} \Pr\left(\left|\textstyle\sum_{t=1}^T \xi_{it}\xi_{jt}\right| > (1-\pi)\sqrt{T} c_p\left(n, \delta\right)\right) + \exp\left(-C_0 T^{C_1}\right).$$

5.2 Extension to panels with weakly exogenous regressors

In the case of panels with lagged dependent variables, the use of OLS residuals for estimation of α could still be justifiable so long as T is sufficiently large, such that the time series bias in the estimated residuals is not too large. This is supported by the Monte Carlo evidence provided for dynamic panels below.

However, the mathematical proofs provided above will not be applicable to the OLS residuals if the panel regression model, (12), contains weakly exogenous regressors, such as lagged values of y_{it}. An alternative approach which avoids some of the technical issues associated with the use

of OLS residuals would be to base the estimation of α on the recursive residuals. Specifically, one could consider the recursive residuals defined by

$$\check{e}_{it} = y_{it} - \check{\gamma}'_{i,t-1}\mathbf{x}_{it},$$

where

$$\check{\gamma}_{i,t-1} = \left(\sum_{\tau=1}^{t-1}\mathbf{x}_{i\tau}\mathbf{x}'_{i\tau}\right)^{-1}\left(\sum_{\tau=1}^{t-1}\mathbf{x}_{i\tau}y_{i\tau}\right).$$

Then the pair-wise correlations based on these recursive residuals are given by

$$\check{\rho}_{ij} = (T-h)^{-1}\sum_{t=h}^{T}\check{z}_{it}\check{z}_{jt},$$

where

$$\check{z}_{it} = \frac{\check{e}_{it}}{\tilde{\sigma}_{it}},$$

and

$$\check{\sigma}^2_{it} = \frac{1}{t}\sum_{\tau=1}^{t}\check{e}^2_{i\tau}.$$

Here h is the size of the training period, which needs to be set by the researcher. It is then easily seen that under cross-sectional independence, $\check{z}_{it}\check{z}_{jt}$ is a martingale process with respect to $\Omega_{\iota i,t-1}$, where $\Omega_{\iota i,t-1} = (y_{i\tau}, x_{i\tau}; \text{for } \tau = t-1, t-2, \ldots, 1)$. This and other related results then allow us to apply the mathematical analysis of the previous sections to the recursive residuals, after suitable adjustments. The main open question is what critical value to use when checking the significance of $\check{\rho}_{ij}$, and hence the threshold value in the determination of the indicators δ_{ij} defined above. This issue will not be pursued in this paper.

5.3 Confidence intervals for α

Quantifying the uncertainty surrounding the proposed estimator of α is clearly of interest. Given the complexity of developing asymptotic inferential theory, a fruitful avenue is to use bootstrap procedures. In the case of panel datasets a number of important data features matter. One possibility is to consider a parametric bootstrap where residuals are resampled. Again there are a number of ways to construct such a bootstrap method. The first is to resample, with replacement, from the rows of the residual matrix with the rows referring to time periods. This procedure applies if the residuals are serially uncorrelated, although block resampling methods can be considered to deal with the serial correlation. It is important to resample whole rows as otherwise the nature of the cross sectional dependence of the original sample, on which α depends, is not inherited by the bootstrap sample. Initial experimentation suggests that coverage rates for such bootstrap methods are very low. An alternative is to use some estimation method for large dimensional covariance matrices, such as thresholding, to estimate the covariance matrix of the residuals and then implement a wild bootstrap. This approach has been considered, for example, by Gonçalves and Perron (2018). However, its desirable properties depend on the true covariance being sparse and certainly sparser that the

structures we consider in this paper. Another alternative is to resample from columns (cross section units) of the residual matrix, namely to resample across units keeping all the residuals (observations) on a given together. This procedure is robust to the serial correlation problem. This resampling procedure, initially proposed by Kapetanios (2008), will be used to obtain bounds on the estimator of α in the Monte Carlo section below.

6 Monte Carlo Simulations

We investigate the small sample properties of our proposed estimator of α, defined by (25), using a number of different simulation designs, allowing for dynamics as well as non-Gaussian errors. We consider the following relatively general dynamic panel data model

$$y_{it} = a_i + \vartheta_i y_{i,t-1} + \gamma_i x_{it} + \varepsilon_{it}, \text{ for } i = 1, 2, \ldots, N; \; t = 2, 3, \ldots, T, \tag{29}$$

with exogenous, but serially correlated, regressors:

$$x_{it} = \rho_{ix} x_{i,t-1} + \left(1 - \rho_{ix}^2\right)^{1/2} \nu_{it}, \text{ for } i = 1, 2, \ldots, N; \; t = -50, -49, \ldots, 0, \ldots, T,$$

where $\rho_{ix} \sim IIDU\,(0, 0.95)$, and $\nu_{it} \sim IIDN\,(0, 1)$, with $x_{i,-50} = \nu_{i,-50}$ for $i = 1, 2, \ldots, N$. The first 50 observations are disregarded for all units to minimize the effects of the initial values on the observations used in the estimation.

We consider the following cases: (i) a static panel data model, where $\vartheta_i = 0$, $a_i \sim IIDN\,(1, 1)$, and $\gamma_i \sim IIDN\,(1, 1)$, for $i = 1, 2, \ldots, N$; and (ii) a dynamic panel data model with exogenous regressors, where $\vartheta_i \sim IIDU\,(0, 0.95)$, $\gamma_i \sim IIDN\,(1, 1)$, and $a_i \sim IIDN\,(1, 1)$, for $i = 1, 2, \ldots, N$.[3] Our estimator is robust to possible correlations between the fixed effects, α_i, and the regressors, x_{it}.

We consider two different designs for generating the errors, ε_{it}, both with the same exponent of cross-sectional dependence, α :

Design 1 We draw $N \times 1$ vector $\mathbf{b}_N = (b_1, b_2, \ldots, b_N)'$ as $Uniform\,(0.7, 0.9)$ for the first $N_b\,(\leq N)$ elements, where $N_b = [N^{\alpha_\beta}]$ and set the remaining elements to zero. Then, we construct the correlation matrix $\mathbf{R}_N$ given by

$$\mathbf{R}_N = \mathbf{I}_N + \mathbf{b}_N \mathbf{b}_N' - \mathbf{B}_N^2, \tag{30}$$

where $\mathbf{B}_N = \text{Diag}\,(\mathbf{b}_N)$ is an $N \times N$ diagonal matrix with its ith diagonal element given by b_i. The degree of sparseness of $\mathbf{R}_N$ is determined by the choice of α_β. If $\alpha_\beta = 0$ then $\mathbf{R}_N = \mathbf{I}_N$ and $\alpha = 1/2$, while if $\alpha_\beta = 1$ then all elements of $\mathbf{R}_N$ will be non-zero and we have $\alpha = 1$. For all intermediate values of α_β, $\mathbf{R}_N$ will have a total of $[N^{\alpha_\beta}\,(N^{\alpha_\beta} - 1) + N]$ non-zero elements. The exact relationship between α and α_β is given by (11). Further, we generate the variances of ε_{it} as $\sigma_{ii} \sim IID\;0.5\,[1 + 0.5\chi^2(2)]$, for $i = 1, 2, \ldots, N$, and set $\mathbf{D}_N = \text{Diag}(\sigma_{ii}, i = 1, 2, \ldots, N)$. We now generate ε_{it} so that its correlation matrix is

[3] We also consider (iii) a dynamic panel data model with no exogenous regressors, where $\vartheta_i \sim IIDU\,(0, 0.95)$ and $\gamma_i = 0$, for $i = 1, 2, \ldots, N$. Simulation results for case (iii) are similar to those for cases (i) and (ii) and are available in the online supplement, Tables S1a-S1d.

equal to $\mathbf{R}_N$. To this end we first obtain matrix $\mathbf{P}_N$ as the Cholesky factor of $\mathbf{R}_N$, and then set $\mathbf{W}_N = \mathbf{D}_N^{1/2}\mathbf{P}_N = (w_{ij})$, where w_{ij} is the (i,j) element of $\mathbf{W}_N$. We then generate ε_{it} as

$$\varepsilon_{it} = \sum_{j=1}^{N} w_{ij} u_{jt},\ i = 1, 2, \ldots, N;\ t = 1, 2, \ldots, T, \tag{31}$$

where u_{jt} are IID draws from Gaussian or non-Gaussian distributions, to be specified below.[4]

Design 2 The second design closely follows the set up in BKP and employs the two-factor specification given by

$$\varepsilon_{it} = \beta_{i1} f_{1t} + \beta_{i2} f_{2t} + u_{it},\ \text{for } i = 1, 2, \ldots, N;\ t = 1, 2, \ldots, T, \tag{32}$$

where $f_{jt} \sim IIDN(0,1)$, $j = 1, 2$ and $u_{it} \sim IIDN(0,1)$ for $i = 1, 2, \ldots, N$. With regard to the factor loadings, we generate them as follows:

$$\begin{aligned} &\beta_{i1} = \dot{v}_{i1},\ \text{for } i = 1, 2, \ldots, [N^{\alpha_{\beta 1}}] \\ &\beta_{i1} = 0,\ \text{for } i = [N^{\alpha_{\beta 1}}] + 1, [N^{\alpha_{\beta 1}}] + 2, \ldots, N \\ &\beta_{i2} = \dot{v}_{i2},\ \text{for } i = 1, 2, \ldots, [N^{\alpha_{\beta 2}}], \\ &\beta_{i2} = 0,\ \text{for } i = [N^{\alpha_{\beta 2}}] + 1, [N^{\alpha_{\beta 2}}] + 1, \ldots, N, \end{aligned} \tag{33}$$

where β_{i2} are then randomised across i to achieve independence from β_{i1}. The loadings are generated as $\dot{v}_{ij} \sim IIDU(\mu_{v_j} - 0.2, \mu_{v_j} + 0.2)$, for $j = 1, 2$. We examine the case where $\alpha_{\beta 2} < \alpha_{\beta 1} = \alpha_\beta$ and consider values of α_β and $\alpha_{\beta 2}$ such that $\alpha_{\beta 2} = \frac{2\alpha_\beta}{3}$. We set $\mu_{v_2} = 0.71$ and $\mu_{v_1} = \sqrt{\mu_v^2 - N^{2(\alpha_{\beta 2} - \alpha_\beta)}\mu_{v_2}^2}$ such that $\mu_{v_1}^2 + \mu_{v_2}^2 = \mu_v^2 = 0.75$ - see BKP for further details. Both μ_{v_1} and μ_{v_2} are chosen such that $\mu_{v_j} \neq 0$, $j = 1, 2$, without $\mu'_{v_j}s$ being too distant from zero either. As before, the exact relationship between α and α_β is given by (11).

In both designs, we examine two cases for the innovations u_{jt}: (i) Gaussian, where $u_{jt} \sim IIDN(0,1)$ for $j = 1, 2, \ldots, N$; (ii) non-Gaussian, where u_{jt} follows a multivariate t-distribution with v degrees of freedom. This is achieved by generating u_{jt} as

$$u_{jt} = \left(\frac{\mathrm{v} - 2}{\chi_{\mathrm{v},t}^2}\right)^{1/2} \tilde{\nu}_{jt},\ \text{for } j = 1, 2, \ldots, N,$$

where $\tilde{\nu}_{jt} \sim IIDN(0,1)$ and $\chi_{\mathrm{v},t}^2$ is a chi-squared random variate with $\mathrm{v} = 8$ degrees of freedom.

For the estimation of α, in each replication $r = 1, 2, \ldots, R$, we first compute the OLS residuals

$$e_{it}^{(r)} = y_{it} - \hat{a}_i^{(r)} - \hat{\vartheta}_i^{(r)} y_{i,t-1} - \hat{\gamma}_i^{(r)} x_{it},\ i = 1, 2, \ldots, N;\ t = 2, 3, \ldots, T,$$

where $\hat{a}_i^{(r)}$, $\hat{\vartheta}_i^{(r)}$, and $\hat{\gamma}_i^{(r)}$ are the OLS estimators of the regressions of y_{it} on an intercept, $y_{i,t-1}$ and x_{it}, computed using the observations $t = 2, 3, \ldots, T$, for each i. In the case of the static

[4]Note that $\boldsymbol{\varepsilon}_t = \mathbf{W}_N \mathbf{u}_t$, and $Var(\boldsymbol{\varepsilon}_t) = \mathbf{W}_N \mathbf{W}'_N = \mathbf{D}_N^{1/2}\mathbf{P}_N\mathbf{P}'_N\mathbf{D}_N^{1/2} = \mathbf{D}_N^{1/2}\mathbf{R}_N\mathbf{D}_N^{1/2}$, as required.

regressions where $\vartheta_i = 0$, the residuals are computed from regressions of y_{it} on an intercept and x_{it} using the observations $t = 1, 2, \ldots, T$. The sample covariance matrix, $\hat{\boldsymbol{\Sigma}}_N^{(r)} = (\hat{\sigma}_{ij}^{(r)})$, is then computed as

$$\hat{\sigma}_{ij}^{(r)} = T^{-1} \sum_{t=2}^{T} e_{it}^{(r)} e_{jt}^{(r)}, \text{ for } i, j = 1, 2, \ldots, N,$$

and diagonal elements $\hat{\sigma}_{ii}^{(r)}$ collected in $\hat{\boldsymbol{D}}_N^{(r)} = \text{Diag}(\hat{\sigma}_{ii}^{(r)},\ i = 1, 2, \ldots, N)$. The corresponding sample correlation matrix is then given by

$$\hat{\boldsymbol{R}}_N^{(r)} = \hat{\boldsymbol{D}}_N^{(r)-1/2} \hat{\boldsymbol{\Sigma}}_N^{(r)} \hat{\boldsymbol{D}}_N^{(r)-1/2}.$$

We evaluate the bias and root mean squared error (RMSE) of the exponent of cross-sectional dependence $\tilde{\alpha}$ computed as in (25) with the critical value, $c_p(n, \delta)$, given by (24). For p and δ we consider the values $p = \{0.05, 0.10\}$ and $\delta = \{1/2, 1/3\}$.[5] Further, we compute the bias-corrected version of the exponent of cross-sectional dependence estimator developed in BKP, $\mathring{\alpha}$, and compare its performance with that of $\tilde{\alpha}$.[6] However, it is important to bear in mind that BKP provide theoretical justification for their estimator only in the case of demeaned observations, namely $x_{it} - \bar{x}_i$, and do not consider residuals from panel regressions as we do in this paper. As a by-product, this paper also provides Monte Carlo evidence on the properties of the estimator, $\mathring{\alpha}$, when applied to residuals from panel regressions.

Finally, for the construction of confidence intervals for $\tilde{\alpha}$, we propose the following bootstrap procedure:

Bootstrap In each replication, $r = 1, 2, \ldots, R$, we collect residuals in matrix $E^{(r)} = \left(e_{it}^{(r)}\right)_{T \times N}$ and proceed to resample (with replacement) from the columns a total number of B times. More precisely, in each bootstrap $b = 1, 2, \ldots, B$, the N cross section units are reshuffled with replacement to generate a new matrix of residuals $E^{(r),(b)} = \left(e_{(i)t}^{(r),(b)}\right)_{T \times N}$, where (i) denotes the resampled index of the N units. For each b, we then compute the correlation matrix $\hat{R}_N^{(r),(b)}$ corresponding to residuals $E^{(r),(b)}$ and estimate α, which we denote by $\tilde{\alpha}^{(r),(b)}$. The bootstrap estimates of α are collected in the vector $\tilde{\alpha}^{(r),B} = \left(\tilde{\alpha}^{(r),(1)}, \tilde{\alpha}^{(r),(2)}, \ldots, \tilde{\alpha}^{(r),(B)}\right)'$ from where we obtain the estimates of α that correspond to the 0.05 and 0.95 percentiles, and which we denote by $\tilde{\alpha}_{0.05}^{(r),B}$ and $\tilde{\alpha}_{0.95}^{(r),B}$, respectively. Finally, we evaluate the frequency at which $\tilde{\alpha}_{0.05}^{(r),B} < \tilde{\alpha}^{(r)} < \tilde{\alpha}_{0.95}^{(r),B}$ across the R replications of each MC experiment.

[5] The value of $\delta = 1/4$ was also considered. Results are in line with those for $\delta = 1/3$ and are available in the online supplement, Tables S2a-S2d.

[6] Recall that $\mathring{\alpha}$ corresponds to the most robust bias-adjusted estimator of the exponent of cross-sectional dependence considered in Bailey et al. (2016) and allows for both serial correlation in the factors and weak cross-sectional dependence in the error terms. It is given by

$$\mathring{\alpha} = \mathring{\alpha}\left(\hat{\mu}_v^2\right) = 1 + \frac{1}{2}\frac{\ln(\hat{\sigma}_{\bar{x}}^2)}{\ln(N)} - \frac{\ln\left(\hat{\mu}_v^2\right)}{2\ln(N)} - \frac{\hat{c}_N}{2\left[N\ln(N)\right]\hat{\sigma}_{\bar{x}}^2}, \tag{34}$$

where $\hat{\sigma}_{\bar{x}}^2$, $\hat{\mu}_v^2$ and $\hat{c}_N$ are consistent estimators of $\sigma_{\bar{x}}^2$, μ_v^2, and c_N - see BKP for further details. We use four principal components when estimating $\hat{c}_N$.

For all experiments we consider the values of $\alpha = 0.55, 0.60, \ldots, 0.90, 0.95, 1.00$, and the sample sizes $N, T = 100, 200, 500$, and carry out all experiments with $R = 2,000$ replications and $B = 500$ bootstraps. The values of a_i, ϑ_i and γ_i are drawn randomly in each replication.

6.1 Small sample results

First, we consider the small sample performance of our proposed estimator, $\tilde{\alpha}$, and investigate its robustness to different choices of p and δ, that govern the critical value, $c_p(n, \delta)$, used in estimating it. The results for Gaussian errors are provided in Tables A1a to A2b, and the results for non-Gaussian errors are reported in Tables A3a to A4b. Each table reports bias and RMSE of $\tilde{\alpha}$ computed using the residuals from either static or dynamic panel data regressions. Tables A1a and A1b give the results for static and dynamic panels, respectively, when the cross-sectional dependence in the errors are generated according to Design 1, whilst the same results for Design 2 are summarized in Tables A2a and A2b. Similarly, the results in Tables A3a and A3b give bias and RMSE of $\tilde{\alpha}$ for static and dynamic panels when the errors are non-Gaussian and the cross correlations are generated according to Design 1, whilst the same results for Design 2 are provided in Tables A4a and A4b. In each of these tables, the left panels give bias and RMSE for $p = 0.05$, and the right panels for $p = 0.10$, whilst the top panels give the results for $\delta = 1/2$, and the bottom panels for $\delta = 1/3$. Specifically, each Table gives four sets of results for the combinations (p, δ), with $p = 0.05, 0.10$ and $\delta = 1/3, 1/2$.

Comparing the left and right panels of the tables, it is clear that $\tilde{\alpha}$ is robust to the choice of p, irrespective of the value of δ, and for all N and T combinations. Observing that $n = N(N-1)/2$ is quite large even for moderate values of N, the effective p-value of the underlying individual tests is given by $2p/n^{\delta}$, which is likely to be dominated by the choice of δ as compared to p. Therefore, the test outcomes are more likely to be robust to the choice of p as compared to δ.

Turning to the choice of δ, comparing the results reported in top and bottom panels of the tables, we note that for all N and T combinations the choice of $\delta = 1/2$ produces smaller bias and RMSE as compared to $\delta = 1/3$ for values of α close to $1/2$ $(\alpha \leq 0.75)$. The reverse is true when considering values of α close to unity $(\alpha > 0.80)$. Again, this is consistent with the result of Theorem 1 which requires δ to be larger than $1 - \alpha$. Hence, for $\alpha \to 1/2$ setting $\delta = 1/2$ is more appropriate, while as $\alpha \to 1$ values of δ below $1/2$ are more appropriate. In cases where there is no priori information regarding the range in which the true value of α might fall, the simulation results suggest setting δ to its upper bound value of $\delta = 1/2$.

Overall, irrespective of whether we consider static or dynamic panel regressions, with Gaussian or non-Gaussian errors, the tabulated results show that the small sample performance of $\tilde{\alpha}$ improves as the true exponent of cross-sectional dependence, α, rises from 0.55 towards 1.0, uniformly over N and T combinations. This finding holds for both Designs, although $\tilde{\alpha}$ generally performs better when Design 1 is used to generate the error cross-sectional dependence. Further, both bias and RMSE of $\tilde{\alpha}$ diminish as N rises for all values of T considered. These results are in line with our main theoretical findings as set out in Theorem 1. It is also interesting to note that under Design 1, the bias and RMSE of $\tilde{\alpha}$ are particularly small for values of α in the range of $0.9 - 1$, even if we consider dynamic panels with non-Gaussian errors. For example,

for $T = 100$ and $N = 500$, $p = 0.05$, $\delta = 1/3$, the bias and RMSE of estimating $\alpha = 0.95$ by $\tilde{\alpha}$, in the case of dynamic panels with non-Gaussian errors are -0.00008 and 0.00067, respectively. (see Table A3b).

Tables A2a and A2b summarize the results for static and dynamic panel data models, respectively, when the error cross-sectional dependence is generated by Design 2 (the two-factor structure). Compared with Tables A1a and A1b, both bias and RMSE are more sizeable across the range of α when $T = 100$. However, as T increases, the performance of $\tilde{\alpha}$ improves for all values of α, especially when α approaches unity, as to be expected. Perhaps, the signal-to-noise ratio implied by (32) becomes somewhat distorted when the T dimension is short and adversely affects the accuracy of the multiple testing procedure used to identify the non-zero elements of the error correlation matrix, $\mathbf{R}_N$. To verify this conjecture, we repeated the same experiments attaching a scaling parameter of $\varsigma = \sqrt{1/2}$ to u_{it} in (32), in line with the simulation setup in BKP. Performance of $\tilde{\alpha}$ is much improved in this case and comparable to those shown in Tables A1a and A1b, even when T is small.[7] Our conclusions regarding the robustness of $\tilde{\alpha}$ to the choice of p and δ arrived at under Design 1 continue to hold for Design 2.

We now consider the small sample performance of $\tilde{\alpha}$ relative to that of $\mathring{\alpha}$, the estimator of α proposed in BKP. $\mathring{\alpha}$ is a biased-corrected estimator of α based on the standard deviation of the cross-sectional average of the residuals. As noted earlier, the asymptotic properties of $\mathring{\alpha}$ are established only for demeaned observations, but it is conjectured that these asymptotic properties are likely to hold even if $\mathring{\alpha}$ is computed using residuals from panel regressions. For comparison we consider bias and RMSE of $\tilde{\alpha}$ computed using $p = 0.05$ and $\delta = 1/2$, and note that similar results are obtained for other choices of p and δ. Table B1 compares the resulting bias and RMSE of the two α estimators when applied to residuals obtained from a static (top panel) and dynamic panel data models (bottom panel). These results refer to Design 1 with Gaussian errors. Both estimators perform well for all values of α, and irrespective of whether the panel regressions are static or dynamic. This is particularly so for values of $\alpha > 0.8$. In comparative terms, $\tilde{\alpha}$ outperforms $\mathring{\alpha}$ on average, for all values of α, and all N and T combinations. The superior performance $\tilde{\alpha}$ is more pronounced when $\alpha \leq 0.75$ uniformly over N and T. The results for Design 2 (where the cross-sectional dependence of the errors are generated using a two-factor specification) are summarized in Table B2 which has the same format as Table B1. In the case of these experiments, $\mathring{\alpha}$ (the estimator proposed by BKP) performs better than $\tilde{\alpha}$ when T is small ($T = 100$), but the bias and RMSE of $\tilde{\alpha}$ becomes more comparable to $\mathring{\alpha}$ as both N and T rise. As noted above, scaling u_{it} by ς in (32) eliminates this relative outperformance of $\mathring{\alpha}$.[8]

Further, Table C displays bias and RMSE results for estimates of the exponent of cross-sectional dependence, given by (8), and computed using the maximum eigenvalue of the correlation matrices $\hat{\boldsymbol{R}}_N$ derived from the residuals from a static or dynamic panel data model with Gaussian errors generated under Designs 1 or 2. It is clear that all eigenvalue based estimates of α perform rather poorly even for large values of N and T, and even for values of α close to 1.

[7] These results are available in the online supplement - see Table S3.

[8] Comparison of $\tilde{\alpha}$ and $\mathring{\alpha}$ when errors are non-Gaussian are provided in Tables S4a-S4b.of the online supplement.

Turning to the problem of sampling uncertainty, we compute bootstrapped confidence intervals for the estimate $\tilde{\alpha}$ in the case of Design 1 with Gaussian errors. The results are summarised in Table D and give the average coverage rates (in percent) over the R simulations of $\tilde{\alpha}$ being between the 5th and 95th percentiles of its empirical distribution. The related confidence intervals are constructed by applying the resampling procedure with replacement to the residuals obtained for the static and dynamic panel data models, with the results summarized in the top and bottom panels of Table D, respectively. We set $\delta = 1/2$ in the critical value function $c_p(n, \delta)$, and $p = 0.05$ (left panel) $p = 0.10$ (right panel), in each replication of the MC experiments and in each bootstrap. For $\alpha = 0.60$ coverage is low for small values of N, but rises steadily towards unity as N increases and for all values of T considered. For $0.65 \leq \alpha \leq 0.95$ coverage stands universally at 100% but drops significantly when $\alpha = 1.00$. Indeed, in this case for small T coverage is relatively high and improves as N rises but drops toward zero when the T dimension is increased for all values of N considered. This is to be expected since, as noted earlier, the error in estimating $\tilde{\alpha}$ becomes negligible when $\alpha = 1.00$.

Overall, we can conclude that using multiple testing for identifying non-zero elements of $\hat{\boldsymbol{R}}_N$ when computing $\tilde{\alpha}$ is computationally attractive, has sound theoretical properties, with comparable performance to the estimator of the exponent of cross-sectional dependence, $\mathring{\alpha}$, developed in BKP.

7 An Empirical Application: identifying the weak factor component of CAPM

In their paper BKP investigate the extent to which excess returns on the Standard & Poor's 500 (S&P 500) securities are interconnected through the market factor by computing rolling estimates of α, the degree of cross-sectional dependence of S&P 500 securities. According to asset pricing theories such as the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965), and arbitrage pricing (APT) of Ross (1976), such estimates of α should be close to unity at all times. This is because both CAPM and APT assume security returns have a common factor representation with at least one strong common factor, with the idiosyncratic component being weakly correlated - see also the approximate factor model due to Chamberlain (1983). Such a factor structure implies that all individual stock returns are significantly affected by the common factor(s) and in consequence they are all pair-wise correlated with varying degrees.

The subsequent analysis in BKP reveals that a disconnect between some asset returns and the market factor does occur particularly at times of stock market bubbles and crashes where these asset returns could be driven by non-fundamentals. In this paper, we focus on the exponent of cross-sectional dependence of the residuals obtained from different versions of the CAPM model, and provide rolling estimates of the exponent of the cross-sectional dependence of the errors from CAPM and related APT models. This is important since under CAPM, after allowing for the market factor, the errors can not be cross-sectionally strongly correlated. It is therefore of interest to see if this is in fact true at all times, or if there are episodes where market factors are not sufficient to capture all the significant interdependencies that might exist

across the security returns.

We update the BKP analysis and consider monthly excess returns of the securities included in the S&P 500 index over the period from September 1989 to May 2018. We obtain estimates of α using rolling samples of 120 months (10 years) to capture possible time variations in the degree of cross-sectional dependence.[9] Since the composition of the S&P 500 index changes over time, we compiled returns on all 500 securities at the end of each month and included in our analysis only those securities that had at least 10 years of data in the month under consideration. On average, we ended up with 442 securities at the end of each month for the 10-year rolling samples. The one-month US treasury bill rate was chosen as the risk-free rate (r_{ft}), and excess returns were computed as $\tilde{r}_{it} = r_{it} - r_{ft}$, where r_{it} is the monthly return on the ith security in the sample inclusive of dividend payments (if any).[10]

First, following BKP, we estimated α for excess security returns to see the extent to which securities in S&P 500 index are fully interconnected at all times. As noted above, under CAPM we would expect estimates of α to be close to unity. To this end we used our proposed estimator, $\tilde{\alpha}$, defined by (25), and computed 10-year rolling estimates $\tilde{\alpha}_t$, for $t =$ September 1989 to May 2018 - a total of 345 estimates - with $p = 0.05$. To check the robustness of the estimates to the choice of δ we computed the rolling estimates for $\delta = 1/2$ and $1/3$.[11] The resultant estimates are shown in Figure 1.[12] As expected, estimates of $\tilde{\alpha}_t$, for $\delta = 1/2$, lie below those of $\tilde{\alpha}_t$, using $\delta = 1/3$, but the series track each other very closely. Also the quantitative differences between the two estimates are not that large. Specifically, all the 345 rolling estimates $\tilde{\alpha}_t\,(\delta = 1/2)$ fall in the interval 0.82 to 0.96, whilst the corresponding estimates $\tilde{\alpha}_t\,(\delta = 1/3)$ all lie in the range $0.86 - 0.97$. These estimates show a high degree of inter-linkages across individual securities, and are very close to unity at the start and at the end of the sample, with important departures from unity in between. Considering $\tilde{\alpha}_t\,(\delta = 1/2)$, it recorded lows around 0.86 in 1998 before recovering temporarily and falling further to 0.84. This episode coincides with the Russian and the Long-Term Capital crises of 1998 which originated in bond markets but rapidly transmitted through international equity markets. The $\tilde{\alpha}$ estimates remained low, falling to $0.82 - 0.83$, around the turn of the century which saw the burst of the Dotcom bubble and 9/11 terrorist attacks in the US. During the less volatile period of $2003 - 2007$, $\tilde{\alpha}$ rose slightly to about 0.85 before a new low of 0.82 was recorded in August 2008, around the time of the sub-prime mortgage crisis in the US, and the ensuing global financial meltdown and economic recession.[13] The estimates of $\tilde{\alpha}$ gradually increase to pre-1997 levels of $0.92 - 0.93$ in 2011 and have since remained in this range to the end of our sample, May 2018.

We now turn our attention to the exponent of cross-sectional correlations of the error terms of the CAPM model, and two well known extensions using additional Fama and French factors.

[9] We also consider rolling samples of size 60 months (5 years). Results for this setting are shown in the online supplement - Figures 3 and 4.

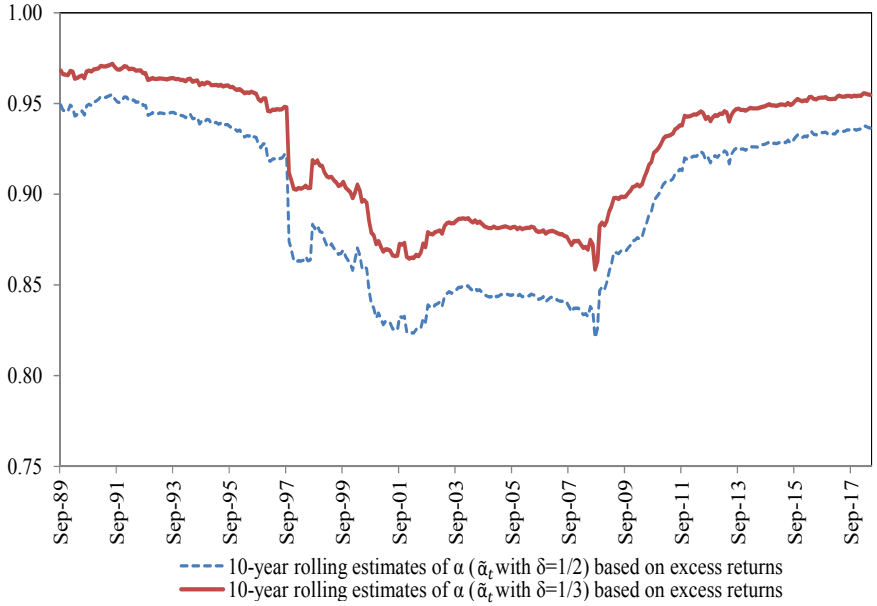
[10] For further details of data sources and definitions see Pesaran and Yamagata (2012).

[11] For the remaining parameters in (25) we set $p = 0.05$ and $n = N_t(N_t - 1)/2$, where N_t is the number of securities in a given 10-year rolling window ($t = 1, 2, \ldots, 345$).

[12] The same estimates including their 90% confidence intervals are shown in Figures 5 and 6 of the online supplement, where in critical value $c_p(n, \delta)$ we set $\delta = 1/2$ and $1/3$, respectively.

[13] The measured increase in $\tilde{\alpha}$ estimates during $2003 - 2007$ is partly attributed to the length of the rolling windows being set to 10 years (120 months). Opting for 5-year rolling windows (60 months) produces more pronounced increments in $\tilde{\alpha}$ estimates which is expected - see Figure 3 in the online supplement.

Figure 1: 10-year rolling estimates of the exponent of cross-sectional correlation ($\tilde{\alpha}_t$) of S&P 500 securities' excess returns



Specifically, the first regression is the usual CAPM one-factor representation given by

$$r_{it} - r_{ft} = a_i + \beta_i (r_{mt} - r_{ft}) + u_{1i,t}, \text{ for } i = 1, 2, \ldots, N, \tag{35}$$

where r_{mt} is the market return computed as the value-weighed returns on all NYSE, AMEX, and NASDAQ stocks. The second and third regressions assume the following extensions to (35) proposed by Fama and French (2004):

$$r_{it} - r_{ft} = a_i + \beta_{1i} (r_{mt} - r_{ft}) + \beta_{2i} smb_t + u_{2i,t}, \text{ for } i = 1, 2, \ldots, N \tag{36}$$

and

$$r_{it} - r_{ft} = a_i + \beta_i (r_{mt} - r_{ft}) + \beta_{2i} smb_t + \beta_{3i} hml_t + u_{3i,t}, \text{ for } i = 1, 2, \ldots, N, \tag{37}$$

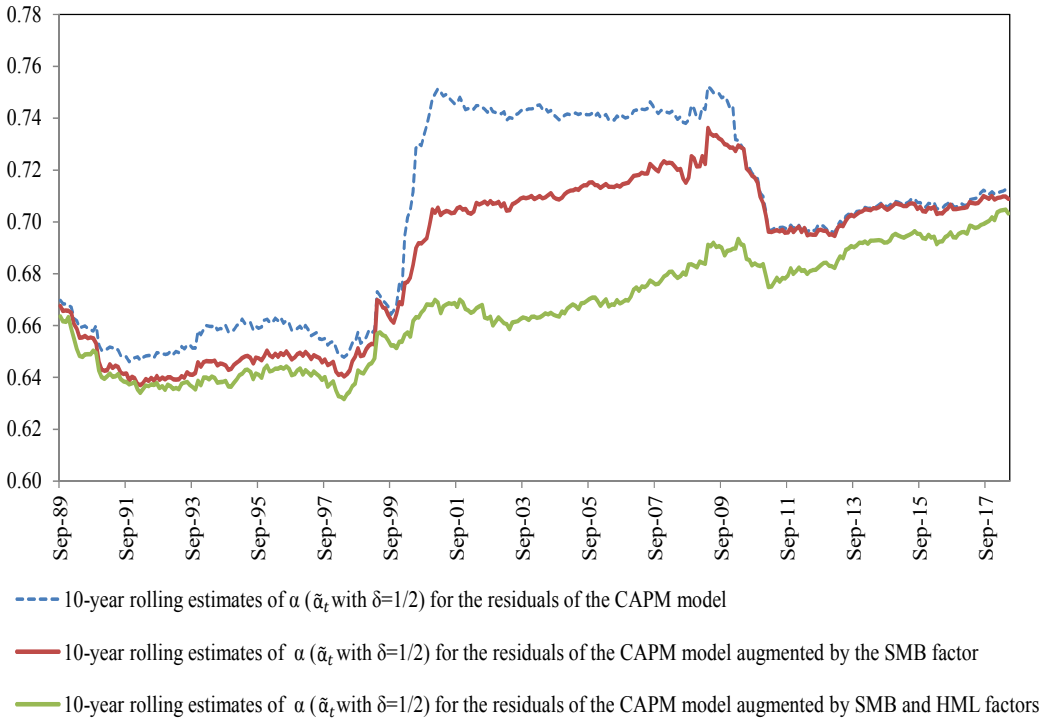
where smb_t stands for average return on the three small portfolios minus the average return on the three big portfolios formed by size, while hml_t refers to the average return on securities with high book value to market value ratio minus the average return of securities with low book value to market value ratio.[14]

As noted previously, under CAPM we would expect the errors, $u_{1i,t}$, to be cross-sectionally weakly correlated, with α_{u_1} to be close to 1/2. But this need not be the case in reality. In fact the introduction of FF factors, smb_t and hml_t, could be viewed as an attempt to ensure cross-sectionally weakly correlated errors for the augmented CAPM model. It is therefore of interest to consider the estimates of α for the errors, $u_{1i,t}$, $u_{2i,t}$, and $u_{3i,t}$, and see if they are close to 1/2 as required by the theory. To this end, we compute 10-year rolling estimates of α based on the pair-wise correlations of the OLS residuals $\hat{u}_{1i,t}$, $\hat{u}_{2i,t}$ and $\hat{u}_{3i,t}$ in the panel regressions

[14]For further details of data sources and definitions see Pesaran and Yamagata (2012).

(35), (36) and (37), respectively. These estimates denoted by $\tilde{\alpha}_{\hat{u}_j t}$, $j = 1, 2, 3$ for $t =$ September 1989 to May 2018, are shown in Figure 2.[15,16]

Figure 2: 10-year rolling estimates of the exponent of cross-sectional correlation ($\tilde{\alpha}_t$) of residuals from CAPM and its two Fama-French extensions



Notes: CAPM model includes excess market returns, CAPM model augmented by SMB includes excess market returns and small minus big (SMB) firm returns, and CAPM model augmented by SMB and HML includes excess market returns, small minus big (SMB) firm returns and high minus low (HML) firm returns as regressors in (35), (36) and (37), respectively.

As expected, estimates of α based on the residuals are smaller compared to the estimates obtained for the securities themselves (as depicted in Figure 1). It is also interesting that all the three estimates $\tilde{\alpha}_{\hat{u}_1 t}$, $\tilde{\alpha}_{\hat{u}_2 t}$ and $\tilde{\alpha}_{\hat{u}_3 t}$ are closely clustered over the two sub-periods September 1989 to September 1997, and February 2011 to May 2018, suggesting that the standard CAPM model provides an adequate characterisation of the cross-sectional correlations of securities, and the additional FF factors are not required in these sub-periods. It is also worth nothing that, over these two sub-periods, estimates of α fall in the narrow range of 0.63-0.71 which are sufficiently small and support CAPM as an adequate model for characterising cross-correlations of S&P 500 security returns. In contrast, the estimates $\tilde{\alpha}_{\hat{u}_1 t}$, $\tilde{\alpha}_{\hat{u}_2 t}$ and $\tilde{\alpha}_{\hat{u}_3 t}$ tend to diverge over the period from October 1997 to January 2011, and more importantly they all start to rise sharply, suggesting important departures from the basic CAPM model. Using only the market

[15]We set $p = 0.05$ and $\delta = 1/2$ when estimating α for the residuals, since *a priori* we would expect the true value of α for the errors of CAPM models to be close to 1/2. See the discussion in Section 6.

[16]The same estimates including their 90% confidence intervals are shown in Figures 7, 8 and 9 of the online supplement, respectively.

factor, as in (35), results in $\tilde{\alpha}_{\hat{u}_1 t}$ jumping to levels around $0.74 - 0.76$. Adding smb_t to (35) reduces the α estimates of the resulting residuals to $0.69 - 0.73$, suggesting that the size portfolio does have some influence on individual security returns during this period. Adding the second FF factor (as in (37)), further reduces the estimates of α to the range $0.66 - 0.68$. These results are also in line with the sharp drop in the estimates of α we reported for the excess returns during the period $1998 - 2010$, and provide further evidence in favour of the argument that the presence of factors other than the market factor, namely smb_t and hml_t, tend to become relevant during periods of financial crises and turmoils.

8 Conclusions

Cross-sectional dependence and the extent to which it occurs in large multivariate data sets is of great interest for a variety of economic, econometric and financial analyses. Such analyses vary widely. Examples include the effects of idiosyncratic shocks on aggregate macroeconomic variables, the extent to which financial risk can be diversified by investing in disparate assets or asset classes and the performance of standard estimators such as principal components when applied to data sets with unknown collinearity structures. A common characteristic of such analyses is the need to quantify the degree of cross-sectional dependence, especially when it is prevalent enough to materially affect the outcome of the analysis.

In this paper we generalize the work of Bailey et al. (2016) by proposing a method of measuring the extent of inter-connections in the residuals of large panel data sets in terms of a single parameter, α. We refer to this as the exponent of cross-sectional dependence of the residuals. We show that this exponent can be used to characterize the degree of sparsity of correlation matrices, or the prevalence of factors in multi-factor representations routinely used in economic and financial analysis. We propose a simple consistent estimator of the cross-sectional exponent and derive the rate at which it approaches its true value. We also propose a resampling procedure for the construction of confidence bounds around the estimator of α.

A detailed Monte Carlo study suggests that the proposed estimator has desirable small sample properties especially when $\alpha > 3/4$. We apply our measure to the widely analysed Standard & Poor's 500 data set. We find that for individual securities in S&P 500 index, the 10-year rolling estimates of cross-sectional exponents are sufficiently close to unity over the two sub-periods $1989 - 1997$ and $2011 - 2018$, but not during the intervening period $1998 - 2010$, when markets have been subject to a number of financial turmoils, starting with the LTCM crisis and the Dotcom bubble, and ending with the credit crunch of $2007 - 2008$. These results carry over when we consider the cross-sectional dependence of errors from the CAPM model and its multi-factor extensions using Fama-French factors. Estimates of α based on the residuals from the CAPM model lend support to CAPM during the sub-periods $1989 - 1997$ and $2011 - 2018$, but not when we consider the period $1998 - 2010$.

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Appendices

Appendix I: Statement of Lemmas

Lemma 1 *Consider the panel data regression model (12) and suppose that Assumptions 1-4 hold. Then,*

$$\sup_{i,j}\Pr\left(\left|\sum_{t=1}^{T} z_{it}z_{jt} - E\left(z_{it}z_{jt}|\Omega_{i,j,t}\right)\right| > \sqrt{T}c_p\left(n,\delta\right)\right) \leq \sup_{i,j}\Pr\left(\left|\sum_{t=1}^{T}\xi_{it}\xi_{jt} - E\left(\xi_{is}\xi_{js}|\Omega_{i,j,t}\right)\right| > (1-\pi)\sqrt{T}c_p\left(n,\delta\right)\right) + \exp\left(-C_0T^{C_1}\right),$$

for some $C_0, C_1 > 0$, where $z_{it} = \frac{e_{it}}{\left(T^{-1}\mathbf{e}_i'\mathbf{M}_i\mathbf{e}_i\right)^{1/2}}$.

Lemma 2 *Let*

$$W_{NT}^0 = N^{-2\alpha}\sum_{i\neq j}^{N} I\left(\left|T^{-1}\sum_{t=1}^{T} z_{it}z_{jt}\right| > \frac{c_p(n,\delta)}{\sqrt{T}}\,|\rho_{ij}=0\right)$$

where $z_{it} = \frac{e_{it}}{\left(T^{-1}\mathbf{e}_i'\mathbf{M}_i\mathbf{e}_i\right)^{1/2}}$. Under Assumptions 1-4,

$$E\left(W_{NT}^0\right) = O\left(N^{2(1-\alpha-\varkappa\delta)}\right) + O\left(N^{2(1-\alpha)}\exp\left(-C_0T^{C_1}\right)\right),$$

for any $0 < \varkappa < 1$, and some $C_0, C_1 > 0$, where δ is defined below (23).

Lemma 3 *Let*

$$W_{NT}^1 = N^{-2\alpha}\sum_{i\neq j}^{N} I\left(\left|T^{-1}\sum_{t=1}^{T} z_{it}z_{jt}\right| \leq \frac{c_p(n,\delta)}{\sqrt{T}}\,|\rho_{ij}\neq 0\right)$$

Under Assumptions 1-4, and as long as $N = o\left(\exp(T)\right)$, $\inf_{i,j}|E(\hat{\rho}_{ij})| > 0$,

$$E\left(W_{NT}^1\right) = O\left(\exp\left(-C_0T^{C_1}\right)\right),$$

for some $C_0, C_1 > 0$, where δ is defined below (23) and $z_{it} = \frac{e_{it}}{\left(T^{-1}\mathbf{e}_i'\mathbf{M}_i\mathbf{e}_i\right)^{1/2}}$.

Appendix II: Proofs of Lemmas

Proof of Lemma 1

Recall $a_{i,t} = (a_{i,t1},\ldots,a_{i,tT})' = \mathbf{X}_i(\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{x}_{it}$. Then, using (15),

$$\begin{aligned}\sum_{t=1}^{T} z_{it}z_{jt} &= \sum_{t=1}^{T}\left(\xi_{it} + \sum_{s=1}^{T} a_{i,ts}\xi_{is}\right)\left(\xi_{jt} + \sum_{s=1}^{T} a_{j,ts}\xi_{js}\right) = \\ &\sum_{t=1}^{T}\xi_{it}\xi_{jt} + \sum_{t=1}^{T}\xi_{it}\left(\sum_{s=1}^{T} a_{j,ts}\xi_{js}\right) + \\ &\sum_{t=1}^{T}\xi_{jt}\left(\sum_{s=1}^{T} a_{i,ts}\xi_{is}\right) + \\ &\sum_{t=1}^{T}\left(\sum_{s=1}^{T} a_{i,ts}\xi_{is}\right)\left(\sum_{s'=1}^{T} a_{j,ts'}\xi_{js'}\right) \\ &= \sum_{i=1}^{4} A_i.\end{aligned}$$

We focus on A_4. A_2 and A_3 can be treated similarly. We have

$$\sum_{t=1}^{T}\left(\sum_{s=1}^{T} a_{i,ts}\xi_{is}\right)\left(\sum_{s'=1}^{T} a_{j,ts'}\xi_{js'}\right) = \sum_{s=1}^{T}\sum_{s'=1}^{T}\xi_{is}\xi_{js'}\sum_{t=1}^{T} a_{i,ts}a_{j,ts'}$$

$$\sum_{s=1}^{T}\sum_{s'=1}^{T}\xi_{is}\xi_{js'} = \sum_{s=1}^{T}\xi_{is}\xi_{js} + \sum_{s=1}^{T}\xi_{is}\left(\sum_{s\neq s',s'=1}^{T}\xi_{js'}\right)$$

Further,

$$\sum_{s=1}^{T}\sum_{s'=1}^{T}\xi_{is}\xi_{js'}\sum_{t=1}^{T} a_{i,ts}a_{j,ts'} = \sum_{s=1}^{T}\xi_{is}\xi_{js}\left(\sum_{t=1}^{T} a_{i,ts}a_{j,ts'}\right) + \sum_{s=1}^{T}\xi_{is}\left(\sum_{s\neq s',s'=1}^{T}\xi_{js'}\right)\left(\sum_{t=1}^{T} a_{i,ts}a_{j,ts'}\right).$$

Define

$$a_{T,ij,ss'} = T\sum_{t=1}^{T} a_{i,ts}a_{j,ts'}.$$

By Assumption 2, $\sup_{T,i,j,s,s'} a_{T,ij,ss'} < K < \infty$. Further, define

$$\tilde{\xi}_{1,ijs} = \xi_{is}\xi_{js}a_{T,ij,ss'}$$

and

$$\tilde{\xi}_{2,ijs} = \xi_{is}\left(\frac{1}{\sqrt{T}}\sum_{s\neq s',s'=1}^{T}\xi_{js'}\right)a_{T,ij,ss'}.$$

Then,

$$\sum_{s=1}^{T}\xi_{is}\xi_{js}\left(\sum_{t=1}^{T} a_{i,ts}a_{j,ts'}\right) + \sum_{s=1}^{T}\xi_{is}\left(\sum_{s\neq s',s'=1}^{T}\xi_{js'}\right)\left(\sum_{t=1}^{T} a_{i,ts}a_{j,ts'}\right) = \frac{1}{T}\sum_{s=1}^{T}\tilde{\xi}_{1,ijs} + \frac{1}{\sqrt{T}}\sum_{s=1}^{T}\tilde{\xi}_{2,ijs}.$$

It can be easily seen that $\tilde{\xi}_{1,ijs} - E\left(\tilde{\xi}_{1,ijs}|\Omega_{i,j,t}\right)$ and $\tilde{\xi}_{2,ijs} - E\left(\tilde{\xi}_{2,ijs}|\Omega_{i,j,t}\right)$ are martingale difference series with finite variances, and that if $\rho_{ij} = 0$ then $E\left(\tilde{\xi}_{1,ijs}|\Omega_{i,j,t}\right) = E\left(\tilde{\xi}_{2,ijs}|\Omega_{i,j,t}\right) = 0$. Define $c_{ij,t} = E\left(z_{it}z_{jt}|\Omega_{i,j,t}\right)$. Then, using Lemma A3 of Chudik et al. (2018), it easily follows that

$$\sup_{i,j}\Pr\left(\left|\sum_{t=1}^{T} z_{it}z_{jt} - c_{ij,t}\right| > \sqrt{T}c_p\left(n,\delta\right)\right) \leq \sup_{i,j}\Pr\left(\left|\sum_{t=1}^{T}\xi_{it}\xi_{jt} - E\left(\xi_{is}\xi_{js}|\Omega_{i,j,t}\right)\right| > (1-\pi)\sqrt{T}c_p\left(n,\delta\right)\right) + \exp\left(-C_0T^{C_1}\right),$$

for some $C_0, C_1 > 0$.

Proof of Lemma 2

Note that W_{NT}^0 can be written as

$$W_{NT}^0 = N^{-2\alpha}\sum_{i\neq j}^{N} I\left(\left|T^{-1}\sum_{t=1}^{T} z_{it}z_{jt}\right| > \frac{c_p\left(n,\delta\right)}{\sqrt{T}}\,|\rho_{ij} = 0\right)$$

Since by assumption ξ_{it} and ξ_{jt} are distributed independently when $\rho_{ij} = 0$, it also follows that

$$\begin{aligned} E\left(z_{it}z_{jt}\,|\Omega_{i,j,t}\right) &= E\left(\xi_{it}\xi_{jt}\,|\Omega_{i,j,t}\right) + E\left(\xi_{it}q_{jt}\,|\Omega_{i,j,t}\right) + E\left(\xi_{jt}q_{it}\,|\Omega_{i,j,t}\right) + E\left(q_{it}q_{jt}\,|\Omega_{i,j,t}\right) \\ &= 0, \end{aligned}$$

and hence $\{z_{it}z_{jt}, \Omega_{i,j,t}\}$, is a zero mean martingale difference sequence, where $\Omega_{i,j,t} = \Omega_{i,t} \cup \Omega_{j,t}$. Then, we note that $z_{it}z_{jt}$ is a normalised process since $z_{it} = \frac{e_{it}}{\left(T^{-1}\mathbf{e}_i'\mathbf{e}_i\right)^{1/2}}$. Then, noting Lemma 1, using Chudik et al. (2018), and, in particular, their Lemmas A3 (which provides a martingale difference exponential probability inequality under (26)), A4 (which handles exponential probability tails for products of random variables) and A9 (which handles the normalisation by $\left(T^{-1}\mathbf{e}_i'\mathbf{e}_i\right)^{1/2}$), we have for any $0 < \pi < 1$, any bounded sequence, $d_T > 0$, and some $C_0, C_1 > 0$,

$$\sup_{i,j} \Pr\left(|\textstyle\sum_{t=1}^T z_{it}z_{jt}| > \sqrt{T}c_p(n,\delta)\,|\rho_{ij} = 0\right) \leq \exp\left[-\frac{(1-\pi)^2 c_p(n,\delta)^2}{2(1+d_T)}\right] + \exp\left(-C_0 T^{C_1}\right). \tag{38}$$

Note that $\frac{(1-\pi)^2}{1+d_T} < 1$, but can be made arbitrarily close to 1, and that, by Lemma A2 of Chudik et al. (2018), $\exp\left[-bc_p^2(n,\delta)\right] = O\left(n^{-2b\delta}\right)$. Then, for any $0 < \varkappa < 1$, and some $C_0, C_1 > 0$,

$$\sup_{i.j} \Pr\left(|\textstyle\sum_{t=1}^T z_{it}z_{jt}| > \sqrt{T}c_p(n,\delta)\,|\rho_{ij} = 0\right) = O(n^{-\varkappa\delta}) + O\left(\exp\left(-C_0 T^{C_1}\right)\right) = O(N^{-2\varkappa\delta}) + O\left(\exp\left(-C_0 T^{C_1}\right)\right).$$

Then,for some some $C_0, C_1 > 0$, any $0 < \varkappa < 1$, and if $N = O\left(T^d\right)$

$$E(W_{NT}^0) = N^{-2\alpha}\sum_{i\neq j}^{N} \Pr\left(\left|T^{-1}\sum_{t=1}^{T} z_{it}z_{jt}\right| > \frac{c_p(n,\delta)}{\sqrt{T}}\,|\rho_{ij} = 0\right) = O\left(N^{2(1-\alpha-\varkappa\delta)}\right) + O\left(N^{2(1-\alpha)}\exp\left(-C_0 T^{C_1}\right)\right) =$$
$$O\left(T^{2d(1-\alpha-\varkappa\delta)}\right) + O\left(\exp\left[2d(1-\alpha)\ln(T) - C_0 T^{C_1}\right]\right).$$

Proof of Lemma 3

We need to derive $h_{N,T}$ in

$$\Pr\left[\left|\sum_{t=1}^{T} z_{it}z_{jt}\right| \leq \sqrt{T}c_p(n,\delta)\,|\rho_{ij} \neq 0\right] = O\left(h_{N,T}\right).$$

Let $c_{ij,t} = E\left(z_{it}z_{jt}|\Omega_{i,j,t}\right)$, and note the inequality

$$\Pr\left(|X+B| \leq C\right) \leq \Pr\left(|X| > |B| - C\right),$$

where X is a random variable, B and C are constants, and $|B| \geq C > 0$. Then, for some $0 < \pi < 1$

$$\Pr\left[\left|\sum_{t=1}^{T}\left(z_{it}z_{jt} - c_{ij,t}\right) + c_{ij,t} - E\left(c_{ij,t}\right) + E\left(c_{ij,t}\right)\right| \leq \sqrt{T}c_p(n,\delta)\,|\rho_{ij} \neq 0\right]$$
$$\leq \Pr\left[\left|\sum_{t=1}^{T}\left(z_{it}z_{jt} - c_{ij,t}\right) + c_{ij,t} - E\left(c_{ij,t}\right)\right| > \left|\sum_{t=1}^{T} E(c_{ij,t})\right| - \sqrt{T}c_p(n,\delta)\,|\rho_{ij} \neq 0\right] \leq$$
$$\leq \Pr\left[\left|\sum_{t=1}^{T}\left(z_{it}z_{jt} - c_{ij,t}\right)\right| > (1-\pi)\left[\left|\sum_{t=1}^{T} E(c_{ij,t})\right| - \sqrt{T}c_p(n,\delta)\right]|\rho_{ij} \neq 0\right] + \tag{39}$$
$$\Pr\left[\left|\sum_{t=1}^{T} c_{ij,t} - E\left(c_{ij,t}\right)\right| > \pi\left[\left|\sum_{t=1}^{T} E(c_{ij,t})\right| - \sqrt{T}c_p(n,\delta)\right]|\rho_{ij} \neq 0\right] \tag{40}$$

But, by (28), (40) is bounded by $\exp\left(-C_0 T^{C_1}\right)$ for some $C_0, C_1 > 0$. We consider

$$\Pr\left[\left|\sum_{t=1}^{T}\left(z_{it}z_{jt} - c_{ij,t}\right)\right| > (1-\pi)\left[\left|\sum_{t=1}^{T} E(c_{ij,t})\right| - \sqrt{T}c_p(n,\delta)\right]|\rho_{ij} \neq 0\right]$$

But, by (20) of Assumption 1, $\lim_{N,T\to\infty}\frac{\sqrt{T}c_p(n,\delta)}{\sum_{t=1}^{T} E(c_{ij,t})} = 0$. Therefore, using again Lemma 1 and (38) of Lemma 2, we have

$$\sup_{ij,} \Pr\left[\left|\sum_{t=1}^{T}\left(z_{it}z_{jt} - c_{ij,t}\right)\right| > (1-\pi)\left[\left|\sum_{t=1}^{T} E(c_{ij,t})\right| - \sqrt{T}c_p(n,\delta)\right]|\rho_{ij} \neq 0\right] \leq \exp\left(-CT\right),$$

for some $C > 0$, proving the result.

Appendix III: Proof of Theorem 1

We prove that $\tilde{\alpha}$ converges to α under our assumptions of exogenous regressors and symmetrically distributed errors. We first note that

$$\tilde{\alpha} = \frac{1}{2}\frac{\ln\left(\frac{\boldsymbol{\tau}'\tilde{\boldsymbol{\Delta}}\boldsymbol{\tau}}{N^{2\alpha}}N^{2\alpha}\right)}{\ln N} = \frac{1}{2}\frac{\ln\left(\frac{\boldsymbol{\tau}'\tilde{\boldsymbol{\Delta}}\boldsymbol{\tau}}{N^{2\alpha}}\right)}{\ln N} + \alpha,$$

$$(\ln N)(\tilde{\alpha} - \alpha) = \frac{1}{2}\ln\left(\frac{\boldsymbol{\tau}'\tilde{\boldsymbol{\Delta}}\boldsymbol{\tau}}{N^{2\alpha}} - 1 + 1\right), \tag{41}$$

and

$$2(\ln N)(\tilde{\alpha} - \alpha) = \frac{\boldsymbol{\tau}'\tilde{\boldsymbol{\Delta}}\boldsymbol{\tau}}{N^{2\alpha}} - 1 + O\left(\left[\frac{\boldsymbol{\tau}'\tilde{\boldsymbol{\Delta}}\boldsymbol{\tau}}{N^{2\alpha}} - 1\right]^2\right).$$

Further

$$N^{-2\alpha}\boldsymbol{\tau}'\tilde{\boldsymbol{\Delta}}\tau = N^{-2\alpha}\sum_{i\neq j}^{N} I\left(|\hat{\rho}_{ij}| > \frac{c_p(n,\delta)}{\sqrt{T}}\right),$$

and

$$\begin{aligned} N^{-2\alpha}\boldsymbol{\tau}'\tilde{\boldsymbol{\Delta}}\boldsymbol{\tau} - 1 &= N^{-2\alpha}\sum_{i\neq j}^{N} I\left(|\hat{\rho}_{ij}| > \frac{c_p(n,\delta)}{\sqrt{T}}\,|\rho_{ij} = 0\right) \\ &\quad + \left(N^{-2\alpha}\sum_{i\neq j}^{N} I\left(|\hat{\rho}_{ij}| > \frac{c_p(n,\delta)}{\sqrt{T}}\,|\rho_{ij} \neq 0\right) - 1\right) \\ &= W_{NT}^0 + W_{NT}^1 + O\left(N^{1-2\alpha}\right). \end{aligned} \tag{42}$$

We now have

$$\begin{aligned} &N^{-2\alpha}\sum_{i\neq j}^{N} I\left(|\hat{\rho}_{ij}| > \frac{c_p(n,\delta)}{\sqrt{T}}\,|\rho_{ij} \neq 0\right) - 1 \\ &= N^{-2\alpha}(N^{2\alpha} - N^{\alpha}) - 1 - N^{-2\alpha}\sum_{i\neq j}^{N} I\left(|\hat{\rho}_{ij}| \leq \frac{c_p(N,\delta)}{\sqrt{T}}\,|\rho_{ij} \neq 0\right) \\ &= -N^{-2\alpha}\sum_{i\neq j}^{N} I\left(|\hat{\rho}_{ij}| \leq \frac{c_p(n,\delta)}{\sqrt{T}}\,|\rho_{ij} \neq 0\right) - N^{-\alpha}. \end{aligned}$$

Hence,

$$\begin{aligned} N^{-2\alpha}\boldsymbol{\tau}'\tilde{\boldsymbol{\Delta}}\boldsymbol{\tau} - 1 = N^{-2\alpha}\sum_{i\neq j}^{N} I\left(|\hat{\rho}_{ij}| > \frac{c_p(n,\delta)}{\sqrt{T}}\,|\rho_{ij} = 0\right) - \\ N^{-2\alpha}\sum_{i\neq j}^{N} I\left(|\hat{\rho}_{ij}| \leq \frac{c_p(n,\delta)}{\sqrt{T}}\,|\rho_{ij} \neq 0\right) - N^{-\alpha}, \end{aligned}$$

which we write more compactly as

$$N^{-2\alpha}\boldsymbol{\tau}'\tilde{\boldsymbol{\Delta}}\boldsymbol{\tau} - 1 = W_{NT}^0 - \tilde{W}_{NT}^1 + O_p\left(N^{-\alpha}\right),$$

where

$$W_{NT}^0 = N^{-2\alpha}\sum_{i\neq j}^{N} I\left(|\hat{\rho}_{ij}| > \frac{c_p(n,\delta)}{\sqrt{T}}\,|\rho_{ij} = 0\right),$$

$$\tilde{W}_{NT}^1 = N^{-2\alpha}\sum_{i\neq j}^{N} I\left(|\hat{\rho}_{ij}| \leq \frac{c_p(n,\delta)}{\sqrt{T}}\,|\rho_{ij} \neq 0\right).$$

It is worth noting that $E\left|W_{NT}^{0}\right| = E\left(W_{NT}^{0}\right)$, and $E\left|\tilde{W}_{NT}^{1}\right| = E\left(\tilde{W}_{NT}^{1}\right)$, and hence,

$$E\left|N^{-2\alpha}\boldsymbol{\tau}'\tilde{\boldsymbol{\Delta}}\boldsymbol{\tau} - 1\right| \leq E\left(W_{NT}^{0}\right) + E\left(\tilde{W}_{NT}^{1}\right) + O\left(N^{-\alpha}\right) + O\left(N^{1-2\alpha}\right).$$

Lemmas 2-3 provide bounds for $E\left(W_{NT}^{0}\right)$ and $E\left(\tilde{W}_{NT}^{1}\right)$ proving the result.

Table A1a: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a static panel data model

Cross correlations are generated using Design 1 with Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.10$									
		$\delta = 1/2$												$\delta = 1/2$									
T	N	Bias										T	N	Bias									
100	100	0.204	-0.609	-0.584	-0.355	-0.289	-0.001	-0.259	-0.279	-0.094	-0.016	100	100	0.634	-0.324	-0.405	-0.245	-0.218	0.042	-0.230	-0.261	-0.083	-0.009
	200	0.066	-0.426	-0.156	-0.302	-0.254	-0.101	-0.165	-0.080	-0.058	-0.025		200	0.395	-0.219	-0.034	-0.225	-0.205	-0.070	-0.143	-0.065	-0.046	-0.015
	500	0.090	-0.120	-0.162	-0.144	-0.149	-0.035	-0.042	-0.046	-0.050	-0.041		500	0.332	0.021	-0.079	-0.093	-0.115	-0.009	-0.021	-0.029	-0.034	-0.025
200	100	0.243	-0.584	-0.557	-0.328	-0.267	0.017	-0.240	-0.262	-0.078	0.000	200	100	0.686	-0.291	-0.372	-0.211	-0.201	0.056	-0.218	-0.251	-0.074	0.000
	200	0.118	-0.380	-0.120	-0.271	-0.226	-0.074	-0.139	-0.057	-0.035	0.000		200	0.470	-0.161	0.003	-0.198	-0.184	-0.052	-0.127	-0.051	-0.033	0.000
	500	0.158	-0.064	-0.114	-0.101	-0.106	0.007	-0.001	-0.006	-0.011	0.000		500	0.418	0.081	-0.035	-0.059	-0.084	0.019	0.005	-0.004	-0.010	0.000
500	100	0.277	-0.556	-0.549	-0.325	-0.261	0.019	-0.239	-0.262	-0.078	0.000	500	100	0.718	-0.251	-0.356	-0.210	-0.191	0.059	-0.216	-0.251	-0.073	0.000
	200	0.157	-0.367	-0.110	-0.265	-0.223	-0.073	-0.138	-0.056	-0.035	0.000		200	0.519	-0.139	0.020	-0.188	-0.179	-0.049	-0.125	-0.050	-0.033	0.000
	500	0.189	-0.047	-0.105	-0.097	-0.104	0.009	0.000	-0.006	-0.011	0.000		500	0.470	0.108	-0.021	-0.053	-0.080	0.021	0.006	-0.004	-0.010	0.000
T	N	RMSE										T	N	RMSE									
100	100	0.311	0.633	0.596	0.364	0.296	0.051	0.264	0.282	0.102	0.041	100	100	0.714	0.407	0.438	0.269	0.233	0.072	0.235	0.263	0.087	0.026
	200	0.145	0.438	0.174	0.308	0.261	0.124	0.171	0.093	0.070	0.053		200	0.436	0.255	0.094	0.233	0.211	0.089	0.147	0.072	0.053	0.033
	500	0.118	0.137	0.177	0.157	0.165	0.073	0.078	0.076	0.077	0.079		500	0.345	0.068	0.098	0.104	0.124	0.045	0.049	0.050	0.052	0.052
200	100	0.347	0.611	0.571	0.338	0.272	0.042	0.242	0.263	0.079	0.000	200	100	0.769	0.381	0.410	0.243	0.217	0.081	0.222	0.252	0.075	0.000
	200	0.183	0.391	0.133	0.274	0.228	0.076	0.140	0.057	0.035	0.000		200	0.509	0.210	0.082	0.207	0.188	0.058	0.128	0.052	0.033	0.000
	500	0.173	0.075	0.116	0.102	0.107	0.010	0.004	0.007	0.011	0.000		500	0.429	0.100	0.050	0.063	0.085	0.021	0.008	0.006	0.011	0.000
500	100	0.374	0.584	0.563	0.337	0.268	0.043	0.240	0.263	0.078	0.000	500	100	0.798	0.356	0.397	0.245	0.211	0.083	0.220	0.252	0.075	0.000
	200	0.218	0.379	0.126	0.268	0.225	0.075	0.138	0.057	0.035	0.000		200	0.560	0.193	0.089	0.199	0.184	0.056	0.126	0.051	0.033	0.000
	500	0.203	0.063	0.108	0.098	0.104	0.011	0.004	0.007	0.011	0.000		500	0.480	0.124	0.042	0.057	0.082	0.023	0.009	0.005	0.010	0.000
		$\delta = 1/3$												$\delta = 1/3$									
T	N	Bias										T	N	Bias									
100	100	1.507	0.288	-0.013	-0.003	-0.072	0.127	-0.177	-0.232	-0.070	-0.005	100	100	3.071	1.418	0.712	0.452	0.212	0.289	-0.078	-0.181	-0.050	-0.002
	200	1.433	0.445	0.354	0.011	-0.066	0.010	-0.097	-0.038	-0.032	-0.006		200	2.970	1.475	0.974	0.391	0.157	0.136	-0.028	-0.004	-0.017	-0.003
	500	1.533	0.715	0.314	0.123	0.008	0.061	0.021	-0.001	-0.015	-0.009		500	3.037	1.641	0.849	0.421	0.170	0.146	0.066	0.021	-0.004	-0.005
200	100	1.585	0.330	0.018	0.030	-0.058	0.141	-0.169	-0.226	-0.065	0.000	200	100	3.159	1.461	0.751	0.491	0.218	0.307	-0.075	-0.176	-0.046	0.000
	200	1.531	0.529	0.403	0.040	-0.046	0.024	-0.087	-0.031	-0.026	0.000		200	3.089	1.572	1.030	0.423	0.176	0.148	-0.020	0.001	-0.014	0.000
	500	1.684	0.813	0.370	0.161	0.032	0.078	0.034	0.009	-0.006	0.000		500	3.217	1.764	0.918	0.464	0.196	0.163	0.076	0.028	0.001	0.000
500	100	1.627	0.386	0.041	0.035	-0.044	0.144	-0.167	-0.225	-0.064	0.000	500	100	3.192	1.517	0.778	0.495	0.241	0.310	-0.071	-0.176	-0.045	0.000
	200	1.613	0.564	0.432	0.054	-0.039	0.028	-0.083	-0.030	-0.025	0.000		200	3.175	1.616	1.067	0.441	0.185	0.154	-0.016	0.003	-0.013	0.000
	500	1.783	0.867	0.399	0.173	0.040	0.082	0.037	0.010	-0.005	0.000		500	3.327	1.827	0.957	0.483	0.207	0.168	0.080	0.030	0.001	0.000
T	N	RMSE										T	N	RMSE									
100	100	1.571	0.448	0.248	0.170	0.143	0.153	0.187	0.236	0.075	0.015	100	100	3.124	1.492	0.790	0.514	0.280	0.316	0.123	0.192	0.061	0.008
	200	1.461	0.490	0.378	0.098	0.093	0.053	0.102	0.046	0.036	0.016		200	2.991	1.498	0.993	0.416	0.184	0.153	0.055	0.032	0.026	0.009
	500	1.541	0.722	0.321	0.133	0.037	0.067	0.032	0.019	0.023	0.022		500	3.044	1.646	0.854	0.426	0.176	0.150	0.070	0.027	0.013	0.013
200	100	1.652	0.475	0.255	0.180	0.137	0.167	0.181	0.230	0.068	0.000	200	100	3.211	1.531	0.826	0.555	0.286	0.335	0.122	0.188	0.058	0.000
	200	1.559	0.570	0.426	0.109	0.082	0.052	0.092	0.037	0.028	0.000		200	3.109	1.598	1.048	0.447	0.203	0.163	0.053	0.032	0.022	0.000
	500	1.692	0.820	0.377	0.167	0.044	0.081	0.036	0.013	0.007	0.000		500	3.223	1.769	0.923	0.469	0.201	0.166	0.079	0.031	0.008	0.000
500	100	1.693	0.524	0.254	0.191	0.138	0.170	0.179	0.229	0.068	0.000	500	100	3.245	1.589	0.854	0.561	0.307	0.338	0.121	0.187	0.059	0.000
	200	1.641	0.604	0.456	0.116	0.080	0.057	0.090	0.037	0.028	0.000		200	3.196	1.640	1.085	0.465	0.211	0.171	0.053	0.034	0.023	0.000
	500	1.790	0.874	0.405	0.180	0.051	0.085	0.040	0.014	0.007	0.000		500	3.332	1.832	0.963	0.487	0.212	0.171	0.083	0.033	0.008	0.000

Notes: Parameters of the static panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i = 0$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Gaussian errors are generated as $u_{it} \sim IIDN(0,1)$ in (31). Design 1 assumes $b_i \sim U(0.7, 0.9)$ for the first $N_b\,(\leq N)$ elements of vector $\mathbf{b}_N$, $i = 1, 2, \ldots, N$, in the construction of the correlation matrix of the errors, $\mathbf{R}_N = \mathbf{I}_N + \mathbf{b}_N\mathbf{b}_N' - \mathbf{B}_N^2$ given by (30), where $\mathbf{B}_N = \text{Diag}\,(\mathbf{b}_N)$. $c_p(n,\delta)$ corresponds to the critical value used in the multiple testing procedure shown in (23). The number of replications is set to $R = 2000$.

Table A1b: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a dynamic panel data model with exogenous regressors

Cross correlations are generated using Design 1 with Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.10$									
		$\delta = 1/2$												$\delta = 1/2$									
T	N	Bias										T	N	Bias									
100	100	0.198	-0.621	-0.594	-0.363	-0.295	-0.007	-0.261	-0.284	-0.099	-0.021	100	100	0.625	-0.330	-0.412	-0.245	-0.221	0.039	-0.230	-0.264	-0.085	-0.012
	200	0.064	-0.432	-0.161	-0.309	-0.262	-0.108	-0.173	-0.088	-0.070	-0.033		200	0.398	-0.218	-0.035	-0.229	-0.211	-0.074	-0.148	-0.069	-0.053	-0.019
	500	0.084	-0.123	-0.169	-0.158	-0.158	-0.046	-0.054	-0.059	-0.066	-0.049		500	0.326	0.018	-0.084	-0.102	-0.120	-0.016	-0.029	-0.037	-0.045	-0.030
200	100	0.255	-0.573	-0.561	-0.330	-0.266	0.018	-0.240	-0.263	-0.078	0.000	200	100	0.687	-0.280	-0.376	-0.216	-0.200	0.057	-0.217	-0.252	-0.074	0.000
	200	0.123	-0.381	-0.118	-0.272	-0.225	-0.074	-0.139	-0.056	-0.035	0.000		200	0.469	-0.160	0.008	-0.198	-0.183	-0.051	-0.126	-0.050	-0.033	0.000
	500	0.156	-0.063	-0.113	-0.101	-0.106	0.007	-0.001	-0.007	-0.011	0.000		500	0.417	0.082	-0.034	-0.060	-0.084	0.019	0.005	-0.004	-0.010	0.000
500	100	0.281	-0.550	-0.550	-0.322	-0.261	0.020	-0.240	-0.261	-0.078	0.000	500	100	0.737	-0.246	-0.361	-0.205	-0.190	0.060	-0.216	-0.249	-0.073	0.000
	200	0.153	-0.366	-0.108	-0.266	-0.222	-0.072	-0.137	-0.056	-0.035	0.000		200	0.514	-0.139	0.022	-0.190	-0.179	-0.049	-0.125	-0.050	-0.033	0.000
	500	0.185	-0.049	-0.104	-0.097	-0.104	0.009	0.000	-0.006	-0.011	0.000		500	0.467	0.107	-0.020	-0.053	-0.080	0.021	0.006	-0.004	-0.010	0.000
T	N	RMSE										T	N	RMSE									
100	100	0.302	0.644	0.607	0.375	0.305	0.056	0.266	0.289	0.108	0.045	100	100	0.702	0.411	0.444	0.270	0.238	0.070	0.235	0.267	0.090	0.027
	200	0.153	0.444	0.180	0.316	0.272	0.126	0.183	0.106	0.094	0.064		200	0.439	0.255	0.096	0.237	0.219	0.088	0.154	0.080	0.068	0.041
	500	0.117	0.144	0.185	0.179	0.174	0.102	0.102	0.102	0.107	0.090		500	0.340	0.073	0.102	0.119	0.130	0.066	0.067	0.069	0.073	0.060
200	100	0.354	0.600	0.573	0.339	0.272	0.043	0.241	0.264	0.079	0.000	200	100	0.765	0.374	0.413	0.246	0.216	0.082	0.221	0.253	0.075	0.000
	200	0.190	0.392	0.131	0.274	0.226	0.076	0.139	0.057	0.035	0.000		200	0.511	0.208	0.085	0.207	0.187	0.057	0.127	0.051	0.033	0.000
	500	0.169	0.074	0.116	0.102	0.107	0.010	0.004	0.007	0.011	0.000		500	0.427	0.100	0.049	0.063	0.085	0.021	0.008	0.005	0.011	0.000
500	100	0.375	0.580	0.563	0.332	0.267	0.045	0.241	0.262	0.079	0.000	500	100	0.815	0.360	0.402	0.236	0.208	0.084	0.220	0.251	0.075	0.000
	200	0.213	0.378	0.123	0.270	0.224	0.074	0.138	0.057	0.035	0.000		200	0.553	0.195	0.089	0.199	0.183	0.056	0.127	0.051	0.033	0.000
	500	0.198	0.065	0.107	0.098	0.104	0.011	0.004	0.007	0.011	0.000		500	0.478	0.123	0.042	0.057	0.082	0.023	0.009	0.006	0.010	0.000
		$\delta = 1/3$												$\delta = 1/3$									
T	N	Bias										T	N	Bias									
100	100	1.509	0.279	-0.026	-0.005	-0.074	0.125	-0.177	-0.233	-0.071	-0.006	100	100	3.070	1.397	0.708	0.455	0.208	0.290	-0.077	-0.181	-0.049	-0.003
	200	1.431	0.450	0.359	0.009	-0.069	0.009	-0.099	-0.040	-0.035	-0.008		200	2.974	1.486	0.983	0.387	0.156	0.135	-0.029	-0.004	-0.019	-0.004
	500	1.532	0.716	0.311	0.119	0.007	0.059	0.018	-0.004	-0.019	-0.011		500	3.038	1.642	0.848	0.418	0.169	0.145	0.065	0.020	-0.007	-0.006
200	100	1.569	0.341	0.017	0.027	-0.054	0.142	-0.168	-0.227	-0.065	0.000	200	100	3.122	1.471	0.750	0.487	0.230	0.307	-0.073	-0.178	-0.047	0.000
	200	1.552	0.529	0.407	0.042	-0.045	0.026	-0.085	-0.030	-0.026	0.000		200	3.106	1.571	1.035	0.427	0.177	0.150	-0.017	0.003	-0.014	0.000
	500	1.686	0.819	0.371	0.158	0.032	0.078	0.034	0.010	-0.006	0.000		500	3.220	1.765	0.919	0.462	0.196	0.163	0.076	0.029	0.001	0.000
500	100	1.634	0.384	0.040	0.040	-0.040	0.146	-0.167	-0.223	-0.064	0.000	500	100	3.196	1.512	0.778	0.502	0.241	0.310	-0.072	-0.174	-0.045	0.000
	200	1.609	0.563	0.432	0.057	-0.038	0.029	-0.083	-0.030	-0.025	0.000		200	3.172	1.611	1.068	0.443	0.189	0.152	-0.015	0.003	-0.013	0.000
	500	1.780	0.864	0.401	0.174	0.041	0.082	0.036	0.010	-0.005	0.000		500	3.321	1.824	0.961	0.483	0.208	0.168	0.079	0.030	0.002	0.000
T	N	RMSE										T	N	RMSE									
100	100	1.576	0.441	0.242	0.165	0.144	0.150	0.187	0.238	0.076	0.015	100	100	3.123	1.466	0.787	0.514	0.274	0.315	0.122	0.192	0.062	0.008
	200	1.459	0.493	0.383	0.095	0.098	0.050	0.105	0.049	0.043	0.020		200	2.996	1.509	1.001	0.411	0.185	0.152	0.056	0.034	0.029	0.012
	500	1.541	0.724	0.319	0.130	0.036	0.069	0.037	0.029	0.033	0.026		500	3.044	1.648	0.853	0.423	0.175	0.149	0.070	0.029	0.018	0.015
200	100	1.632	0.486	0.247	0.175	0.135	0.169	0.180	0.230	0.069	0.000	200	100	3.174	1.540	0.823	0.548	0.294	0.335	0.120	0.188	0.059	0.000
	200	1.579	0.568	0.430	0.109	0.084	0.053	0.091	0.037	0.028	0.000		200	3.127	1.595	1.055	0.451	0.205	0.166	0.053	0.034	0.023	0.000
	500	1.694	0.826	0.377	0.165	0.045	0.081	0.037	0.013	0.007	0.000		500	3.226	1.771	0.925	0.467	0.201	0.166	0.079	0.032	0.007	0.000
500	100	1.700	0.529	0.262	0.182	0.134	0.171	0.179	0.228	0.068	0.000	500	100	3.246	1.581	0.856	0.564	0.304	0.337	0.121	0.187	0.058	0.000
	200	1.636	0.602	0.454	0.116	0.081	0.056	0.090	0.037	0.028	0.000		200	3.193	1.635	1.086	0.466	0.216	0.168	0.054	0.034	0.023	0.000
	500	1.787	0.871	0.407	0.180	0.052	0.085	0.039	0.014	0.007	0.000		500	3.327	1.830	0.965	0.487	0.214	0.171	0.082	0.033	0.008	0.000

Notes: Parameters of the dynamic panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i \sim U(0, 0.95)$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Gaussian errors are generated as $u_{it} \sim IIDN(0,1)$ in (31). Design 1 assumes $b_i \sim U(0.7, 0.9)$ for the first $N_b\,(\leq N)$ elements of vector $\mathbf{b}_N$, $i = 1, 2, \ldots, N$, in the construction of the correlation matrix of the errors, $\mathbf{R}_N = \mathbf{I}_N + \mathbf{b}_N\mathbf{b}_N' - \mathbf{B}_N^2$ given by (30), where $\mathbf{B}_N = \text{Diag}(\mathbf{b}_N)$. $c_p(n,\delta)$ corresponds to the critical value used in the multiple testing procedure shown in (23). The number of replications is set to $R = 2000$.

Table A2a: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a static panel data model

Cross correlations are generated using Design 2 with Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.10$									
		$\delta = 1/2$												$\delta = 1/2$									
T	N	Bias										T	N	Bias									
100	100	-0.880	-2.378	-2.922	-3.026	-3.232	-3.131	-3.509	-3.648	-3.576	-3.598	100	100	-0.092	-1.533	-2.041	-2.119	-2.328	-2.220	-2.609	-2.763	-2.699	-2.738
	200	-1.221	-2.480	-2.800	-3.267	-3.464	-3.434	-3.595	-3.591	-3.658	-3.714		200	-0.542	-1.721	-1.988	-2.435	-2.620	-2.590	-2.750	-2.754	-2.824	-2.883
	500	-1.501	-2.658	-3.266	-3.581	-3.780	-3.779	-3.859	-3.934	-4.011	-4.057		500	-0.910	-1.952	-2.492	-2.775	-2.953	-2.948	-3.026	-3.098	-3.175	-3.222
200	100	0.705	-0.090	-0.160	0.120	0.009	0.205	-0.145	-0.299	-0.242	-0.304	200	100	1.207	0.294	0.129	0.346	0.192	0.362	-0.007	-0.177	-0.131	-0.207
	200	0.375	-0.068	0.134	-0.086	-0.146	-0.063	-0.191	-0.206	-0.269	-0.326		200	0.784	0.236	0.360	0.094	0.001	0.066	-0.071	-0.097	-0.166	-0.228
	500	0.219	-0.024	-0.086	-0.158	-0.206	-0.153	-0.212	-0.267	-0.325	-0.368		500	0.541	0.209	0.094	-0.012	-0.079	-0.038	-0.104	-0.163	-0.222	-0.266
500	100	1.043	0.295	0.247	0.563	0.433	0.625	0.264	0.084	0.114	-0.001	500	100	1.463	0.581	0.426	0.672	0.502	0.666	0.289	0.098	0.121	0.000
	200	0.683	0.313	0.550	0.326	0.254	0.329	0.203	0.161	0.087	-0.001		200	1.031	0.529	0.674	0.401	0.298	0.355	0.218	0.169	0.091	0.000
	500	0.530	0.355	0.331	0.249	0.205	0.246	0.176	0.117	0.056	-0.001		500	0.799	0.504	0.412	0.294	0.229	0.259	0.184	0.121	0.058	0.000
T	N	RMSE										T	N	RMSE									
100	100	1.252	2.688	3.301	3.469	3.699	3.623	3.974	4.094	4.029	4.051	100	100	0.826	1.877	2.412	2.542	2.752	2.671	3.023	3.155	3.097	3.137
	200	1.455	2.748	3.167	3.655	3.868	3.859	4.002	4.009	4.070	4.130		200	0.894	2.002	2.351	2.795	2.987	2.976	3.112	3.125	3.189	3.251
	500	1.662	2.899	3.580	3.938	4.153	4.176	4.260	4.336	4.411	4.456		500	1.115	2.198	2.791	3.105	3.292	3.306	3.386	3.459	3.533	3.580
200	100	0.803	0.377	0.369	0.346	0.298	0.354	0.312	0.405	0.359	0.399	200	100	1.272	0.451	0.314	0.438	0.305	0.426	0.217	0.275	0.239	0.281
	200	0.457	0.291	0.307	0.274	0.296	0.251	0.315	0.318	0.367	0.415		200	0.824	0.336	0.421	0.223	0.199	0.198	0.206	0.208	0.251	0.299
	500	0.303	0.241	0.262	0.299	0.328	0.295	0.326	0.366	0.418	0.456		500	0.571	0.283	0.214	0.196	0.211	0.197	0.216	0.252	0.300	0.337
500	100	1.068	0.337	0.272	0.571	0.442	0.631	0.279	0.116	0.136	0.002	500	100	1.495	0.621	0.451	0.682	0.510	0.672	0.303	0.127	0.141	0.001
	200	0.697	0.325	0.553	0.329	0.257	0.332	0.208	0.166	0.093	0.001		200	1.048	0.542	0.678	0.405	0.301	0.357	0.223	0.174	0.097	0.001
	500	0.534	0.357	0.332	0.250	0.206	0.246	0.177	0.118	0.057	0.001		500	0.804	0.507	0.414	0.294	0.230	0.260	0.184	0.122	0.059	0.001
		$\delta = 1/3$												$\delta = 1/3$									
T	N	Bias										T	N	Bias									
100	100	1.121	-0.410	-1.023	-1.161	-1.437	-1.372	-1.789	-1.971	-1.926	-1.985	100	100	2.947	1.111	0.208	-0.136	-0.573	-0.609	-1.093	-1.323	-1.312	-1.401
	200	0.926	-0.377	-0.770	-1.305	-1.554	-1.571	-1.758	-1.790	-1.871	-1.940		200	2.750	1.070	0.352	-0.393	-0.787	-0.898	-1.140	-1.210	-1.313	-1.396
	500	0.905	-0.329	-1.008	-1.398	-1.634	-1.672	-1.775	-1.859	-1.939	-1.991		500	2.688	1.017	0.014	-0.590	-0.950	-1.065	-1.210	-1.316	-1.407	-1.466
200	100	2.120	0.962	0.590	0.671	0.425	0.538	0.134	-0.064	-0.038	-0.131	200	100	3.649	2.070	1.338	1.167	0.758	0.763	0.293	0.047	0.039	-0.080
	200	1.900	0.990	0.855	0.438	0.246	0.250	0.080	0.027	-0.056	-0.130		200	3.436	2.034	1.512	0.863	0.520	0.428	0.203	0.113	0.009	-0.082
	500	1.870	1.040	0.622	0.337	0.173	0.156	0.059	-0.017	-0.085	-0.135		500	3.394	2.001	1.200	0.682	0.384	0.290	0.150	0.052	-0.029	-0.088
500	100	2.305	1.160	0.797	0.898	0.641	0.748	0.338	0.123	0.131	0.000	500	100	3.785	2.210	1.487	1.322	0.905	0.903	0.429	0.171	0.150	0.000
	200	2.073	1.186	1.057	0.633	0.434	0.431	0.261	0.191	0.099	0.000		200	3.571	2.177	1.653	0.999	0.650	0.553	0.328	0.224	0.112	0.000
	500	2.060	1.231	0.815	0.514	0.348	0.322	0.216	0.137	0.064	0.000		500	3.556	2.152	1.346	0.811	0.510	0.406	0.259	0.157	0.071	0.000
T	N	RMSE										T	N	RMSE									
100	100	1.349	0.991	1.458	1.608	1.841	1.798	2.153	2.306	2.267	2.325	100	100	3.029	1.332	0.832	0.857	1.049	1.073	1.429	1.611	1.603	1.685
	200	1.102	0.867	1.210	1.653	1.883	1.906	2.061	2.097	2.170	2.241		200	2.798	1.230	0.792	0.864	1.132	1.222	1.410	1.477	1.566	1.648
	500	1.017	0.766	1.325	1.693	1.916	1.966	2.063	2.144	2.220	2.272		500	2.711	1.136	0.643	0.938	1.224	1.335	1.465	1.562	1.647	1.703
200	100	2.169	1.026	0.649	0.711	0.469	0.568	0.216	0.175	0.153	0.187	200	100	3.688	2.111	1.371	1.190	0.780	0.780	0.326	0.140	0.120	0.121
	200	1.922	1.014	0.873	0.463	0.284	0.281	0.155	0.125	0.135	0.181		200	3.453	2.049	1.523	0.874	0.534	0.440	0.228	0.145	0.089	0.119
	500	1.878	1.049	0.633	0.357	0.208	0.192	0.124	0.111	0.144	0.180		500	3.399	2.005	1.205	0.688	0.394	0.301	0.169	0.093	0.088	0.122
500	100	2.345	1.200	0.824	0.910	0.651	0.755	0.350	0.149	0.150	0.001	500	100	3.823	2.246	1.513	1.336	0.917	0.911	0.441	0.192	0.168	0.000
	200	2.091	1.200	1.063	0.638	0.437	0.434	0.266	0.195	0.105	0.000		200	3.586	2.190	1.661	1.005	0.655	0.556	0.332	0.228	0.117	0.000
	500	2.065	1.234	0.817	0.515	0.349	0.322	0.217	0.138	0.065	0.000		500	3.560	2.155	1.348	0.813	0.511	0.407	0.260	0.158	0.073	0.000

Notes: Parameters of the static panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i = 0$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Design 2 assumes a two-factor model with $[N_{\alpha_\beta}]$ and $[N_{\alpha_{\beta 2}}]$ non-zero loadings for the first and second factor, respectively. We set: $\alpha_{\beta 2} = 2\alpha_\beta/3$, where α_β relates to α under (11), f_{jt} and $u_{it} \sim IIDN(0,1)$, $v_{ij} \sim IIDU(\mu_{v_j} - 0.2, \mu_{v_j} + 0.2)$, $j = 1, 2$, $\mu_v = 0.87$, $\mu_{v_2} = 0.71$, $\mu_{v_1} = \sqrt{\mu_v^2 - N^{2(\alpha_{\beta 2} - \alpha_\beta)}\mu_{v_2}^2}$), in (32) and (33). The number of replications is set to $R = 2000$.

Table A2b: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a dynamic panel data model with exogenous regressors

Cross correlations are generated using Design 2 with Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.10$									
		$\delta = 1/2$												$\delta = 1/2$									
T	N	Bias										T	N	Bias									
100	100	-1.016	-2.571	-3.183	-3.319	-3.568	-3.479	-3.838	-3.998	-3.922	-3.952	100	100	-0.216	-1.691	-2.254	-2.354	-2.602	-2.509	-2.879	-3.047	-2.978	-3.024
	200	-1.320	-2.642	-3.032	-3.558	-3.745	-3.763	-3.918	-3.925	-4.003	-4.057		200	-0.627	-1.858	-2.175	-2.671	-2.846	-2.856	-3.011	-3.025	-3.104	-3.163
	500	-1.610	-2.849	-3.517	-3.890	-4.107	-4.124	-4.222	-4.304	-4.388	-4.439		500	-1.007	-2.119	-2.705	-3.035	-3.227	-3.235	-3.330	-3.409	-3.490	-3.543
200	100	0.689	-0.109	-0.179	0.088	-0.030	0.166	-0.181	-0.329	-0.272	-0.336	200	100	1.194	0.280	0.115	0.329	0.162	0.332	-0.035	-0.199	-0.153	-0.230
	200	0.357	-0.089	0.115	-0.120	-0.177	-0.095	-0.227	-0.240	-0.301	-0.360		200	0.774	0.220	0.345	0.068	-0.022	0.043	-0.099	-0.122	-0.189	-0.253
	500	0.209	-0.033	-0.095	-0.170	-0.215	-0.165	-0.225	-0.281	-0.337	-0.380		500	0.538	0.204	0.087	-0.022	-0.085	-0.047	-0.114	-0.173	-0.230	-0.275
500	100	1.040	0.294	0.248	0.565	0.436	0.627	0.261	0.081	0.115	-0.001	500	100	1.466	0.583	0.430	0.676	0.504	0.670	0.286	0.095	0.121	0.000
	200	0.685	0.313	0.548	0.324	0.253	0.330	0.202	0.162	0.086	-0.001		200	1.031	0.528	0.672	0.399	0.298	0.355	0.217	0.170	0.089	0.000
	500	0.527	0.353	0.330	0.249	0.204	0.246	0.176	0.117	0.055	-0.001		500	0.797	0.502	0.412	0.294	0.229	0.259	0.183	0.122	0.057	0.000
T	N	RMSE										T	N	RMSE									
100	100	1.360	2.876	3.549	3.762	4.031	3.981	4.301	4.455	4.378	4.411	100	100	0.871	2.032	2.621	2.783	3.038	2.976	3.296	3.453	3.383	3.430
	200	1.536	2.902	3.398	3.954	4.170	4.208	4.358	4.367	4.445	4.497		200	0.955	2.133	2.536	3.040	3.233	3.258	3.404	3.418	3.497	3.553
	500	1.769	3.101	3.849	4.273	4.512	4.554	4.659	4.740	4.822	4.870		500	1.207	2.375	3.024	3.391	3.597	3.625	3.725	3.803	3.881	3.932
200	100	0.794	0.393	0.393	0.369	0.326	0.359	0.355	0.440	0.398	0.446	200	100	1.257	0.440	0.318	0.438	0.306	0.415	0.242	0.301	0.269	0.317
	200	0.448	0.292	0.301	0.306	0.330	0.284	0.354	0.355	0.401	0.453		200	0.816	0.323	0.412	0.229	0.213	0.207	0.230	0.233	0.275	0.327
	500	0.295	0.245	0.274	0.306	0.329	0.296	0.335	0.376	0.422	0.463		500	0.567	0.281	0.217	0.197	0.209	0.194	0.221	0.258	0.301	0.342
500	100	1.065	0.332	0.272	0.573	0.443	0.634	0.276	0.118	0.135	0.002	500	100	1.501	0.622	0.457	0.686	0.513	0.676	0.300	0.129	0.141	0.001
	200	0.698	0.326	0.551	0.328	0.257	0.332	0.207	0.166	0.092	0.001		200	1.048	0.540	0.676	0.403	0.301	0.357	0.221	0.174	0.095	0.001
	500	0.531	0.355	0.331	0.250	0.205	0.246	0.176	0.118	0.057	0.001		500	0.802	0.505	0.413	0.295	0.230	0.260	0.184	0.123	0.059	0.001
		$\delta = 1/3$												$\delta = 1/3$									
T	N	Bias										T	N	Bias									
100	100	1.045	-0.523	-1.186	-1.344	-1.652	-1.597	-1.997	-2.190	-2.141	-2.207	100	100	2.885	1.029	0.091	-0.271	-0.732	-0.774	-1.247	-1.487	-1.474	-1.568
	200	0.891	-0.465	-0.900	-1.471	-1.712	-1.762	-1.947	-1.984	-2.075	-2.143		200	2.717	1.001	0.254	-0.516	-0.900	-1.042	-1.281	-1.359	-1.469	-1.550
	500	0.840	-0.446	-1.157	-1.577	-1.822	-1.868	-1.984	-2.074	-2.157	-2.213		500	2.641	0.932	-0.096	-0.725	-1.094	-1.216	-1.373	-1.485	-1.578	-1.640
200	100	2.112	0.951	0.581	0.663	0.406	0.518	0.112	-0.078	-0.052	-0.148	200	100	3.662	2.072	1.345	1.171	0.749	0.749	0.276	0.039	0.030	-0.091
	200	1.882	0.979	0.841	0.418	0.231	0.234	0.061	0.011	-0.070	-0.146		200	3.415	2.020	1.496	0.845	0.508	0.416	0.189	0.102	-0.001	-0.093
	500	1.866	1.037	0.617	0.330	0.170	0.151	0.053	-0.022	-0.088	-0.140		500	3.392	1.999	1.198	0.678	0.382	0.286	0.146	0.048	-0.031	-0.091
500	100	2.311	1.162	0.799	0.900	0.646	0.752	0.335	0.122	0.132	0.000	500	100	3.798	2.219	1.489	1.327	0.913	0.909	0.428	0.170	0.152	0.000
	200	2.077	1.187	1.057	0.632	0.433	0.431	0.260	0.191	0.098	0.000		200	3.573	2.179	1.657	0.999	0.649	0.553	0.327	0.224	0.111	0.000
	500	2.053	1.226	0.811	0.513	0.348	0.322	0.216	0.137	0.063	0.000		500	3.549	2.147	1.341	0.810	0.509	0.406	0.258	0.157	0.071	0.000
T	N	RMSE										T	N	RMSE									
100	100	1.289	1.085	1.613	1.794	2.068	2.041	2.366	2.538	2.491	2.557	100	100	2.973	1.295	0.863	0.961	1.206	1.240	1.584	1.785	1.773	1.861
	200	1.074	0.929	1.335	1.827	2.056	2.113	2.276	2.310	2.398	2.463		200	2.766	1.179	0.793	0.977	1.255	1.378	1.575	1.640	1.743	1.819
	500	0.969	0.873	1.488	1.895	2.130	2.189	2.303	2.387	2.467	2.521		500	2.667	1.086	0.715	1.086	1.390	1.509	1.654	1.756	1.843	1.904
200	100	2.161	1.015	0.645	0.708	0.457	0.555	0.216	0.188	0.171	0.213	200	100	3.700	2.111	1.380	1.196	0.774	0.768	0.316	0.140	0.127	0.139
	200	1.906	1.006	0.861	0.448	0.273	0.272	0.153	0.131	0.148	0.198		200	3.432	2.036	1.508	0.858	0.522	0.430	0.217	0.140	0.093	0.130
	500	1.874	1.046	0.629	0.351	0.204	0.186	0.121	0.112	0.142	0.183		500	3.397	2.003	1.202	0.684	0.391	0.296	0.165	0.092	0.085	0.123
500	100	2.351	1.200	0.825	0.912	0.656	0.759	0.348	0.151	0.151	0.001	500	100	3.835	2.253	1.515	1.342	0.924	0.916	0.440	0.194	0.169	0.000
	200	2.095	1.201	1.064	0.636	0.437	0.434	0.264	0.196	0.104	0.000		200	3.589	2.193	1.664	1.004	0.654	0.556	0.330	0.228	0.116	0.000
	500	2.059	1.229	0.813	0.514	0.349	0.322	0.216	0.138	0.065	0.000		500	3.553	2.150	1.343	0.811	0.510	0.406	0.259	0.158	0.072	0.000

Notes: Parameters of the dynamic panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i \sim U(0, 0.95)$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. esign 2 assumes a two-factor model with $[N_{\alpha_\beta}]$ and $[N_{\alpha_{\beta 2}}]$ non-zero loadings for the first and second factor, respectively. We set: $\alpha_{\beta 2} = 2\alpha_\beta/3$, where α_β relates to α under (11), f_{jt} and $u_{it} \sim IIDN(0,1)$, $v_{ij} \sim IIDU(\mu_{v_j} - 0.2, \mu_{v_j} + 0.2)$, $j = 1, 2$, $\mu_v = 0.87$, $\mu_{v_2} = 0.71$, $\mu_{v_1} = \sqrt{\mu_v^2 - N^{2(\alpha_{\beta 2} - \alpha_\beta)}\mu_{v_2}^2}$), in (32) and (33). The number of replications is set to $R = 2000$.

Table A3a: Bias and RMSE $\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a static panel data model

Cross correlations are generated using Design 1 with non-Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.10$									
		$\delta = 1/2$												$\delta = 1/2$									
T	N	Bias										T	N	Bias									
100	100	2.272	0.837	0.313	0.221	0.042	0.176	-0.159	-0.243	-0.103	-0.047	100	100	3.653	1.847	0.991	0.670	0.318	0.350	-0.048	-0.179	-0.068	-0.030
	200	2.184	0.899	0.639	0.179	0.027	0.020	-0.114	-0.076	-0.078	-0.062		200	3.487	1.799	1.201	0.539	0.252	0.156	-0.032	-0.026	-0.048	-0.041
	500	2.138	1.153	0.485	0.213	0.030	0.035	-0.036	-0.067	-0.090	-0.086		500	3.317	1.929	0.948	0.493	0.199	0.137	0.029	-0.023	-0.057	-0.059
200	100	2.680	1.150	0.594	0.359	0.156	0.286	-0.090	-0.188	-0.050	0.000	200	100	4.140	2.235	1.324	0.823	0.446	0.461	0.013	-0.135	-0.030	0.000
	200	2.604	1.309	0.890	0.323	0.134	0.111	-0.022	-0.004	-0.017	0.000		200	3.993	2.288	1.499	0.703	0.364	0.236	0.051	0.031	-0.005	0.000
	500	2.642	1.440	0.792	0.418	0.178	0.140	0.069	0.027	0.000	0.000		500	3.934	2.286	1.311	0.717	0.345	0.224	0.112	0.047	0.007	0.000
500	100	2.988	1.366	0.677	0.431	0.202	0.304	-0.079	-0.177	-0.047	0.000	500	100	4.527	2.507	1.450	0.925	0.513	0.489	0.028	-0.119	-0.025	0.000
	200	3.035	1.522	1.012	0.399	0.164	0.157	-0.019	0.001	-0.014	0.000		200	4.505	2.565	1.667	0.807	0.408	0.297	0.056	0.038	0.000	0.000
	500	3.170	1.719	0.910	0.449	0.203	0.162	0.079	0.029	0.003	0.000		500	4.577	2.640	1.470	0.770	0.384	0.255	0.127	0.050	0.011	0.000
T	N	RMSE										T	N	RMSE									
100	100	2.870	1.712	1.032	0.720	0.454	0.330	0.287	0.282	0.144	0.098	100	100	4.183	2.530	1.543	1.081	0.641	0.493	0.288	0.243	0.112	0.068
	200	3.018	1.642	1.161	0.731	0.586	0.219	0.214	0.139	0.120	0.112		200	4.193	2.418	1.675	1.017	0.726	0.312	0.203	0.114	0.086	0.078
	500	3.066	1.984	1.031	0.793	0.397	0.250	0.153	0.149	0.159	0.145		500	4.127	2.686	1.446	1.000	0.519	0.323	0.151	0.120	0.114	0.104
200	100	3.191	1.765	1.190	0.747	0.456	0.459	0.211	0.217	0.075	0.003	200	100	4.572	2.725	1.801	1.136	0.700	0.633	0.245	0.195	0.076	0.002
	200	3.155	1.943	1.349	0.662	0.462	0.240	0.200	0.076	0.034	0.002		200	4.465	2.817	1.909	1.000	0.649	0.358	0.246	0.103	0.040	0.001
	500	3.240	1.965	1.265	0.867	0.439	0.230	0.139	0.108	0.027	0.001		500	4.457	2.753	1.746	1.129	0.596	0.320	0.188	0.128	0.034	0.001
500	100	3.351	1.900	1.015	0.680	0.412	0.445	0.176	0.202	0.064	0.000	500	100	4.832	2.903	1.715	1.134	0.682	0.624	0.204	0.174	0.064	0.000
	200	3.541	1.946	1.278	0.616	0.347	0.294	0.138	0.066	0.032	0.000		200	4.917	2.913	1.909	0.996	0.560	0.424	0.177	0.093	0.037	0.000
	500	3.704	2.155	1.298	0.660	0.408	0.239	0.128	0.057	0.040	0.000		500	5.018	3.016	1.810	0.964	0.567	0.334	0.179	0.079	0.047	0.000
		$\delta = 1/3$												$\delta = 1/3$									
T	N	Bias										T	N	Bias									
100	100	5.750	3.447	2.105	1.411	0.785	0.634	0.127	-0.084	-0.024	-0.018	100	100	8.496	5.631	3.681	2.501	1.492	1.069	0.394	0.058	0.035	-0.010
	200	6.202	3.781	2.496	1.376	0.777	0.461	0.143	0.069	-0.002	-0.021		200	8.936	5.912	3.985	2.381	1.414	0.837	0.355	0.177	0.043	-0.013
	500	6.775	4.358	2.488	1.433	0.753	0.444	0.203	0.073	-0.003	-0.027		500	9.469	6.422	3.913	2.350	1.303	0.749	0.367	0.154	0.032	-0.017
200	100	6.304	3.897	2.493	1.587	0.929	0.757	0.189	-0.046	0.004	0.000	200	100	9.060	6.128	4.124	2.705	1.661	1.210	0.462	0.094	0.059	0.000
	200	6.795	4.372	2.873	1.591	0.909	0.543	0.228	0.116	0.027	0.000		200	9.532	6.541	4.406	2.637	1.571	0.929	0.450	0.225	0.067	0.000
	500	7.524	4.846	2.983	1.731	0.931	0.527	0.269	0.119	0.033	0.000		500	10.206	6.939	4.468	2.694	1.512	0.841	0.434	0.196	0.061	0.000
500	100	6.747	4.228	2.664	1.727	1.029	0.796	0.210	-0.022	0.012	0.000	500	100	9.532	6.485	4.335	2.877	1.787	1.264	0.492	0.128	0.069	0.000
	200	7.390	4.731	3.114	1.747	0.980	0.633	0.239	0.128	0.033	0.000		200	10.157	6.940	4.695	2.827	1.669	1.043	0.467	0.241	0.075	0.000
	500	8.304	5.326	3.234	1.844	1.007	0.585	0.297	0.127	0.039	0.000		500	10.989	7.458	4.764	2.843	1.617	0.920	0.472	0.209	0.068	0.000
T	N	RMSE										T	N	RMSE									
100	100	6.192	3.978	2.536	1.756	1.054	0.778	0.375	0.220	0.099	0.045	100	100	8.836	6.029	4.018	2.794	1.721	1.212	0.588	0.264	0.122	0.029
	200	6.722	4.248	2.892	1.766	1.156	0.603	0.301	0.152	0.066	0.045		200	9.320	6.270	4.313	2.708	1.732	0.974	0.488	0.248	0.090	0.030
	500	7.321	4.922	2.890	1.828	1.028	0.621	0.291	0.154	0.065	0.053		500	9.867	6.859	4.250	2.686	1.555	0.922	0.455	0.225	0.070	0.035
200	100	6.652	4.277	2.875	1.848	1.149	0.926	0.366	0.189	0.088	0.002	200	100	9.321	6.410	4.425	2.920	1.852	1.369	0.607	0.254	0.128	0.001
	200	7.143	4.765	3.207	1.839	1.146	0.657	0.389	0.181	0.064	0.001		200	9.790	6.839	4.677	2.848	1.776	1.038	0.592	0.286	0.101	0.001
	500	7.877	5.184	3.325	2.062	1.156	0.631	0.351	0.196	0.060	0.001		500	10.464	7.200	4.749	2.975	1.717	0.946	0.516	0.268	0.088	0.001
500	100	6.988	4.518	2.877	1.903	1.176	0.924	0.334	0.163	0.080	0.000	500	100	9.708	6.698	4.503	3.028	1.916	1.383	0.593	0.243	0.125	0.000
	200	7.676	4.985	3.316	1.905	1.105	0.746	0.328	0.175	0.063	0.000		200	10.362	7.130	4.861	2.963	1.780	1.146	0.543	0.285	0.103	0.000
	500	8.581	5.585	3.483	2.007	1.155	0.663	0.352	0.158	0.072	0.000		500	11.189	7.655	4.962	2.985	1.748	0.996	0.527	0.239	0.099	0.000

Notes: Parameters of the static panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i = 0$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Non-Gaussian errors are generated as $u_{it} = \left(\frac{\mathrm{v}-2}{\chi^2_{\mathrm{v},t}}\right)^{1/2} \tilde{\nu}_{it}$, for $i = 1, 2, \ldots, N$, $\tilde{\nu}_{it} \sim IIDN(0,1)$ and $\chi^2_{\mathrm{v},t}$ is a chi-squared random variate with v= 8 degrees of freedom, in (31). Design 1 assumes $b_i \sim U(0.7, 0.9)$ for the first $N_b\,(\leq N)$ elements of vector $\mathbf{b}_N$, $i = 1, 2, \ldots, N$, in the construction of the correlation matrix of the errors, $\mathbf{R}_N = \mathbf{I}_N + \mathbf{b}_N\mathbf{b}_N' - \mathbf{B}_N^2$ given by (30), where $\mathbf{B}_N = \mathrm{Diag}\,(\mathbf{b}_N)$. $c_p(n,\delta)$ corresponds to the critical value used in the multiple testing procedure shown in (23). The number of replications is set to $R = 2000$.

Table A3b: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a dynamic panel data model with exogenous regressors

Cross correlations are generated using Design 1 with non-Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.10$									
		$\delta = 1/2$												$\delta = 1/2$									
T	N	Bias										T	N	Bias									
100	100	2.224	0.770	0.293	0.190	0.027	0.171	-0.178	-0.256	-0.110	-0.054	100	100	3.581	1.765	0.966	0.620	0.305	0.347	-0.067	-0.188	-0.073	-0.035
	200	2.032	0.821	0.589	0.140	-0.032	0.005	-0.128	-0.090	-0.096	-0.077		200	3.308	1.699	1.143	0.495	0.187	0.141	-0.042	-0.037	-0.059	-0.051
	500	2.043	0.966	0.483	0.171	0.015	0.017	-0.051	-0.083	-0.105	-0.105		500	3.212	1.713	0.954	0.446	0.187	0.119	0.015	-0.035	-0.067	-0.072
200	100	2.719	1.159	0.531	0.338	0.149	0.269	-0.085	-0.183	-0.051	0.000	200	100	4.192	2.239	1.248	0.796	0.437	0.440	0.017	-0.129	-0.032	0.000
	200	2.632	1.280	0.894	0.334	0.116	0.122	-0.025	-0.005	-0.016	0.000		200	4.013	2.247	1.499	0.713	0.340	0.249	0.046	0.029	-0.003	0.000
	500	2.622	1.420	0.791	0.370	0.164	0.136	0.069	0.026	0.001	0.000		500	3.915	2.254	1.300	0.659	0.326	0.218	0.112	0.045	0.008	0.000
500	100	2.974	1.350	0.694	0.405	0.203	0.303	-0.076	-0.179	-0.048	0.000	500	100	4.489	2.487	1.462	0.891	0.511	0.486	0.033	-0.123	-0.027	0.000
	200	2.913	1.474	1.005	0.405	0.170	0.146	-0.013	-0.002	-0.014	0.000		200	4.383	2.502	1.652	0.813	0.415	0.284	0.062	0.034	0.000	0.000
	500	3.138	1.674	0.918	0.478	0.198	0.176	0.078	0.029	0.001	0.000		500	4.541	2.592	1.482	0.804	0.377	0.273	0.125	0.050	0.008	0.000
T	N	RMSE										T	N	RMSE									
100	100	2.818	1.527	0.981	0.768	0.444	0.457	0.257	0.301	0.156	0.110	100	100	4.116	2.368	1.496	1.097	0.632	0.593	0.231	0.253	0.122	0.076
	200	2.730	1.507	1.096	0.686	0.416	0.262	0.213	0.159	0.156	0.136		200	3.917	2.278	1.591	0.959	0.532	0.341	0.194	0.133	0.114	0.097
	500	2.939	1.547	1.055	0.623	0.549	0.314	0.165	0.161	0.170	0.175		500	3.997	2.246	1.483	0.848	0.652	0.370	0.153	0.121	0.121	0.127
200	100	3.256	1.787	1.042	0.722	0.414	0.404	0.331	0.244	0.071	0.002	200	100	4.645	2.741	1.654	1.111	0.656	0.580	0.361	0.231	0.071	0.001
	200	3.299	1.992	1.490	0.790	0.423	0.291	0.205	0.099	0.054	0.006		200	4.572	2.834	2.004	1.101	0.599	0.412	0.247	0.123	0.062	0.005
	500	3.195	1.991	1.386	0.626	0.403	0.229	0.155	0.075	0.040	0.002		500	4.414	2.754	1.828	0.908	0.559	0.316	0.201	0.097	0.047	0.001
500	100	3.363	1.821	1.095	0.623	0.436	0.403	0.189	0.210	0.065	0.000	500	100	4.813	2.851	1.771	1.073	0.696	0.588	0.220	0.185	0.064	0.000
	200	3.257	1.957	1.356	0.704	0.371	0.286	0.237	0.060	0.035	0.000		200	4.672	2.885	1.952	1.060	0.580	0.410	0.274	0.085	0.041	0.000
	500	3.651	1.969	1.287	0.764	0.370	0.331	0.143	0.053	0.015	0.000		500	4.971	2.860	1.810	1.064	0.536	0.420	0.190	0.075	0.021	0.000
		$\delta = 1/3$												$\delta = 1/3$									
T	N	Bias										T	N	Bias									
100	100	5.659	3.338	2.058	1.326	0.770	0.632	0.109	-0.090	-0.028	-0.021	100	100	8.377	5.505	3.633	2.386	1.472	1.067	0.373	0.053	0.031	-0.012
	200	6.008	3.638	2.421	1.324	0.690	0.444	0.138	0.058	-0.008	-0.028		200	8.743	5.747	3.890	2.309	1.308	0.816	0.352	0.165	0.040	-0.017
	500	6.645	4.087	2.499	1.366	0.740	0.422	0.185	0.065	-0.008	-0.034		500	9.332	6.129	3.922	2.265	1.285	0.719	0.344	0.147	0.029	-0.021
200	100	6.349	3.883	2.405	1.544	0.921	0.730	0.191	-0.037	0.002	0.000	200	100	9.102	6.090	4.032	2.643	1.653	1.178	0.460	0.106	0.056	0.000
	200	6.815	4.303	2.874	1.600	0.879	0.558	0.220	0.112	0.028	0.000		200	9.559	6.462	4.407	2.641	1.534	0.944	0.438	0.218	0.068	0.000
	500	7.510	4.793	2.961	1.652	0.900	0.517	0.269	0.117	0.034	0.000		500	10.197	6.880	4.442	2.604	1.471	0.828	0.432	0.194	0.062	0.000
500	100	6.704	4.202	2.673	1.683	1.021	0.796	0.216	-0.027	0.009	0.000	500	100	9.483	6.464	4.330	2.827	1.775	1.265	0.497	0.119	0.065	0.000
	200	7.271	4.659	3.089	1.752	0.993	0.612	0.245	0.122	0.033	0.000		200	10.035	6.857	4.661	2.829	1.683	1.016	0.472	0.233	0.075	0.000
	500	8.259	5.286	3.253	1.885	0.993	0.609	0.292	0.128	0.035	0.000		500	10.942	7.418	4.784	2.888	1.598	0.948	0.466	0.209	0.064	0.000
T	N	RMSE										T	N	RMSE									
100	100	6.104	3.821	2.481	1.718	1.041	0.848	0.305	0.223	0.106	0.049	100	100	8.724	5.874	3.968	2.707	1.704	1.259	0.524	0.261	0.125	0.032
	200	6.466	4.085	2.794	1.699	0.954	0.613	0.292	0.165	0.084	0.058		200	9.086	6.094	4.201	2.624	1.535	0.974	0.482	0.253	0.100	0.039
	500	7.177	4.504	2.934	1.700	1.105	0.629	0.273	0.141	0.067	0.067		500	9.722	6.462	4.287	2.559	1.600	0.909	0.428	0.212	0.071	0.046
200	100	6.712	4.274	2.732	1.811	1.111	0.871	0.458	0.236	0.085	0.001	200	100	9.375	6.387	4.288	2.868	1.823	1.315	0.669	0.301	0.124	0.001
	200	7.212	4.731	3.259	1.907	1.090	0.709	0.382	0.192	0.085	0.004		200	9.848	6.783	4.707	2.894	1.717	1.083	0.578	0.291	0.119	0.004
	500	7.847	5.147	3.348	1.876	1.112	0.620	0.360	0.171	0.072	0.001		500	10.444	7.151	4.750	2.805	1.666	0.930	0.523	0.248	0.098	0.001
500	100	6.956	4.478	2.913	1.839	1.174	0.897	0.351	0.177	0.078	0.000	500	100	9.666	6.666	4.515	2.959	1.908	1.363	0.605	0.250	0.121	0.000
	200	7.483	4.928	3.320	1.944	1.128	0.722	0.402	0.165	0.067	0.000		200	10.192	7.055	4.845	2.987	1.802	1.116	0.598	0.274	0.106	0.000
	500	8.531	5.484	3.498	2.093	1.130	0.737	0.356	0.155	0.047	0.000		500	11.137	7.573	4.980	3.065	1.722	1.064	0.527	0.237	0.076	0.000

Notes: Parameters of the dynamic panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0,0.95)$, $\vartheta_i \sim U(0,0.95)$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1,2,\ldots,N$. Non-Gaussian errors are generated as $u_{it} = \left(\frac{\mathrm{v}-2}{\chi^2_{\mathrm{v},t}}\right)^{1/2} \tilde{\nu}_{it}$, for $i = 1,2,\ldots,N$, $\tilde{\nu}_{it} \sim IIDN(0,1)$ and $\chi^2_{\mathrm{v},t}$ is a chi-squared random variate with v= 8 degrees of freedom, in (31). Design 1 assumes $b_i \sim U(0.7,0.9)$ for the first $N_b\,(\leq N)$ elements of vector $\mathbf{b}_N$, $i = 1,2,\ldots,N$, in the construction of the correlation matrix of the errors, $\mathbf{R}_N = \mathbf{I}_N + \mathbf{b}_N\mathbf{b}_N' - \mathbf{B}_N^2$ given by (30), where $\mathbf{B}_N = \mathrm{Diag}\,(\mathbf{b}_N)$. $c_p(n,\delta)$ corresponds to the critical value used in the multiple testing procedure shown in (23). $c_p(n,\delta)$ corresponds to the critical value used in the multiple testing procedure shown in (23). The number of replications is set to $R = 2000$.

Table A4a: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a static panel data model

Cross correlations are generated using Design 2 with non-Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.10$									
		$\delta = 1/2$												$\delta = 1/2$									
T	N	Bias										T	N	Bias									
100	100	1.116	-0.949	-2.057	-2.527	-3.024	-3.083	-3.551	-3.745	-3.713	-3.754	100	100	2.735	0.497	-0.797	-1.407	-2.004	-2.134	-2.645	-2.879	-2.869	-2.931
	200	0.933	-1.004	-1.930	-2.775	-3.210	-3.357	-3.631	-3.693	-3.777	-3.844		200	2.503	0.356	-0.758	-1.738	-2.272	-2.485	-2.792	-2.879	-2.979	-3.055
	500	0.898	-1.048	-2.261	-3.008	-3.456	-3.615	-3.782	-3.901	-3.998	-4.063		500	2.404	0.238	-1.163	-2.048	-2.582	-2.795	-2.993	-3.126	-3.229	-3.296
200	100	2.764	1.265	0.642	0.535	0.221	0.270	-0.159	-0.346	-0.318	-0.394	200	100	4.186	2.322	1.376	1.039	0.575	0.530	0.042	-0.187	-0.184	-0.281
	200	2.655	1.332	0.886	0.307	0.020	-0.036	-0.226	-0.283	-0.361	-0.430		200	4.035	2.311	1.535	0.751	0.324	0.185	-0.053	-0.141	-0.235	-0.315
	500	2.623	1.359	0.641	0.190	-0.076	-0.138	-0.258	-0.342	-0.412	-0.460		500	3.925	2.236	1.209	0.558	0.175	0.043	-0.112	-0.215	-0.294	-0.346
500	100	3.337	1.824	1.180	1.096	0.741	0.794	0.349	0.121	0.125	-0.002	500	100	4.715	2.806	1.821	1.481	0.974	0.926	0.424	0.159	0.140	-0.001
	200	3.163	1.851	1.417	0.822	0.525	0.467	0.270	0.187	0.094	-0.002		200	4.521	2.764	1.968	1.155	0.717	0.569	0.324	0.213	0.104	-0.001
	500	3.186	1.896	1.174	0.697	0.431	0.353	0.223	0.135	0.060	-0.002		500	4.510	2.739	1.667	0.971	0.576	0.426	0.258	0.151	0.065	-0.001
T	N	RMSE										T	N	RMSE									
100	100	2.165	1.928	2.631	3.035	3.499	3.576	3.999	4.181	4.162	4.206	100	100	3.413	1.803	1.733	2.023	2.483	2.605	3.053	3.266	3.271	3.336
	200	2.179	1.962	2.499	3.204	3.624	3.796	4.066	4.140	4.227	4.292		200	3.325	1.811	1.675	2.227	2.675	2.893	3.184	3.281	3.384	3.460
	500	2.489	2.230	2.889	3.493	3.905	4.068	4.237	4.357	4.452	4.516		500	3.482	2.058	2.078	2.589	3.020	3.217	3.409	3.540	3.642	3.709
200	100	3.213	1.794	1.131	0.851	0.541	0.478	0.383	0.473	0.451	0.508	200	100	4.560	2.717	1.716	1.255	0.759	0.637	0.289	0.313	0.307	0.373
	200	3.216	1.863	1.231	0.650	0.388	0.317	0.372	0.413	0.472	0.532		200	4.514	2.751	1.828	0.996	0.521	0.334	0.240	0.272	0.334	0.399
	500	3.279	1.932	1.102	0.598	0.394	0.343	0.400	0.467	0.529	0.571		500	4.492	2.735	1.597	0.857	0.434	0.274	0.265	0.330	0.394	0.440
500	100	3.569	2.057	1.360	1.190	0.806	0.821	0.377	0.156	0.144	0.010	500	100	4.926	3.012	1.980	1.573	1.039	0.955	0.452	0.190	0.158	0.009
	200	3.466	2.107	1.556	0.915	0.578	0.486	0.280	0.193	0.100	0.004		200	4.792	3.000	2.114	1.256	0.777	0.594	0.337	0.219	0.109	0.003
	500	3.605	2.250	1.432	0.871	0.530	0.390	0.238	0.140	0.062	0.004		500	4.867	3.055	1.911	1.144	0.680	0.469	0.277	0.157	0.068	0.003
		$\delta = 1/3$												$\delta = 1/3$									
T	N	Bias										T	N	Bias									
100	100	5.029	2.438	0.805	-0.099	-0.895	-1.169	-1.767	-2.070	-2.101	-2.197	100	100	7.902	4.856	2.736	1.406	0.297	-0.215	-0.957	-1.367	-1.467	-1.611
	200	5.517	2.858	1.224	-0.128	-0.937	-1.333	-1.741	-1.900	-2.040	-2.139		200	8.402	5.243	3.068	1.291	0.150	-0.479	-1.025	-1.275	-1.466	-1.596
	500	6.307	3.401	1.320	-0.089	-0.974	-1.409	-1.729	-1.926	-2.060	-2.141		500	9.147	5.730	3.112	1.241	0.017	-0.644	-1.095	-1.363	-1.534	-1.633
200	100	6.270	3.901	2.479	1.776	1.070	0.858	0.276	-0.022	-0.062	-0.190	200	100	8.911	5.987	3.991	2.793	1.746	1.286	0.558	0.157	0.049	-0.125
	200	6.774	4.318	2.872	1.642	0.902	0.562	0.206	0.044	-0.090	-0.194		200	9.459	6.401	4.326	2.628	1.534	0.950	0.446	0.192	0.005	-0.131
	500	7.486	4.767	2.869	1.599	0.819	0.447	0.161	-0.009	-0.122	-0.192		500	10.131	6.809	4.298	2.520	1.378	0.770	0.351	0.108	-0.043	-0.133
500	100	6.772	4.331	2.855	2.129	1.378	1.155	0.555	0.225	0.164	-0.001	500	100	9.412	6.412	4.345	3.101	2.002	1.517	0.765	0.332	0.204	0.000
	200	7.252	4.723	3.222	1.942	1.179	0.822	0.457	0.274	0.126	0.000		200	9.926	6.788	4.642	2.883	1.753	1.145	0.628	0.355	0.155	0.000
	500	8.098	5.256	3.260	1.907	1.089	0.687	0.385	0.206	0.084	0.000		500	10.730	7.297	4.680	2.806	1.607	0.960	0.519	0.265	0.104	0.000
T	N	RMSE										T	N	RMSE									
100	100	5.489	3.043	1.690	1.275	1.493	1.667	2.140	2.409	2.454	2.551	100	100	8.224	5.216	3.119	1.827	1.034	0.949	1.335	1.668	1.775	1.915
	200	6.025	3.453	1.919	1.205	1.439	1.730	2.085	2.242	2.381	2.480		200	8.758	5.623	3.433	1.738	0.952	0.995	1.359	1.581	1.763	1.890
	500	6.886	4.072	2.178	1.368	1.530	1.809	2.082	2.266	2.396	2.478		500	9.551	6.175	3.594	1.836	1.054	1.131	1.433	1.665	1.827	1.926
200	100	6.571	4.206	2.736	1.945	1.198	0.928	0.375	0.198	0.192	0.263	200	100	9.141	6.221	4.193	2.934	1.851	1.344	0.614	0.229	0.145	0.182
	200	7.134	4.656	3.121	1.838	1.035	0.635	0.279	0.164	0.183	0.257		200	9.728	6.666	4.539	2.801	1.653	1.015	0.489	0.229	0.115	0.180
	500	7.867	5.131	3.173	1.828	0.974	0.530	0.234	0.151	0.203	0.258		500	10.412	7.093	4.552	2.724	1.518	0.844	0.392	0.157	0.125	0.186
500	100	6.952	4.506	2.996	2.219	1.442	1.186	0.583	0.252	0.181	0.008	500	100	9.550	6.551	4.462	3.182	2.062	1.551	0.793	0.357	0.220	0.007
	200	7.458	4.915	3.361	2.045	1.245	0.853	0.473	0.282	0.132	0.002		200	10.081	6.940	4.764	2.980	1.820	1.181	0.648	0.365	0.160	0.001
	500	8.333	5.484	3.457	2.060	1.192	0.739	0.409	0.215	0.087	0.001		500	10.902	7.473	4.843	2.941	1.704	1.014	0.547	0.276	0.107	0.001

Notes: Parameters of the static panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i = 0$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Design 2 assumes a two-factor model with $[N_{\alpha_\beta}]$ and $[N_{\alpha_{\beta 2}}]$ non-zero loadings for the first and second factor, respectively. We set: $\alpha_{\beta 2} = 2\alpha_\beta/3$, where α_β relates to α under (11) and $f_{jt} \sim IIDN(0,1)$. Non-Gaussian errors are generated as: $u_{it} = \left(\frac{\mathrm{v}-2}{\chi^2_{\mathrm{v},t}}\right)^{1/2} \tilde{\nu}_{it}$, for $i = 1, 2, \ldots, N$, $\tilde{\nu}_{it} \sim IIDN(0,1)$ and $\chi^2_{\mathrm{v},t}$ is a chi-squared random variate with v= 8 degrees of freedom, in (31). $v_{ij} \sim IIDU(\mu_{v_j} - 0.2, \mu_{v_j} + 0.2)$, $j = 1, 2$, $\mu_v = 0.87$, $\mu_{v_2} = 0.71$, $\mu_{v_1} = \sqrt{\mu_v^2 - N^{2(\alpha_{\beta 2} - \alpha_\beta)}\mu_{v_2}^2}$), in (32) and (33). The number of replications is set to $R = 2000$.

Table A4b: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a dynamic panel data model with exogenous regressors

Cross correlations are generated using Design 2 with non-Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.10$									
		$\delta = 1/2$												$\delta = 1/2$									
T	N	Bias										T	N	Bias									
100	100	1.016	-1.103	-2.241	-2.771	-3.270	-3.357	-3.838	-4.045	-4.008	-4.036	100	100	2.662	0.365	-0.939	-1.597	-2.202	-2.358	-2.883	-3.124	-3.111	-3.162
	200	0.813	-1.192	-2.165	-3.069	-3.534	-3.708	-3.969	-4.029	-4.118	-4.184		200	2.380	0.191	-0.959	-1.989	-2.541	-2.774	-3.072	-3.159	-3.261	-3.337
	500	0.669	-1.297	-2.534	-3.294	-3.752	-3.926	-4.100	-4.216	-4.312	-4.380		500	2.163	-0.004	-1.413	-2.297	-2.832	-3.055	-3.256	-3.387	-3.488	-3.560
200	100	2.749	1.249	0.619	0.506	0.180	0.235	-0.194	-0.387	-0.359	-0.431	200	100	4.150	2.294	1.352	1.013	0.542	0.501	0.014	-0.217	-0.216	-0.307
	200	2.578	1.267	0.834	0.256	-0.018	-0.070	-0.268	-0.316	-0.397	-0.464		200	3.950	2.241	1.482	0.701	0.292	0.157	-0.086	-0.166	-0.263	-0.340
	500	2.568	1.311	0.610	0.161	-0.098	-0.161	-0.281	-0.366	-0.436	-0.485		500	3.869	2.188	1.178	0.531	0.155	0.025	-0.130	-0.233	-0.312	-0.365
500	100	3.419	1.888	1.222	1.113	0.749	0.792	0.349	0.125	0.124	-0.002	500	100	4.812	2.886	1.868	1.505	0.989	0.927	0.426	0.164	0.139	-0.001
	200	3.202	1.882	1.439	0.839	0.535	0.471	0.272	0.191	0.092	-0.002		200	4.557	2.796	1.992	1.173	0.727	0.575	0.326	0.216	0.102	-0.001
	500	3.166	1.879	1.158	0.685	0.422	0.348	0.221	0.133	0.059	-0.002		500	4.489	2.722	1.651	0.959	0.567	0.420	0.256	0.149	0.065	-0.001
T	N	RMSE										T	N	RMSE									
100	100	2.117	2.014	2.782	3.266	3.749	3.863	4.306	4.507	4.488	4.515	100	100	3.358	1.765	1.797	2.176	2.676	2.840	3.307	3.537	3.540	3.591
	200	2.219	2.155	2.744	3.521	3.969	4.155	4.406	4.474	4.559	4.626		200	3.285	1.877	1.861	2.502	2.967	3.189	3.469	3.562	3.661	3.737
	500	2.424	2.367	3.104	3.732	4.151	4.331	4.506	4.622	4.717	4.785		500	3.321	2.061	2.232	2.785	3.223	3.432	3.628	3.757	3.858	3.930
200	100	3.191	1.751	1.070	0.792	0.492	0.453	0.407	0.521	0.502	0.555	200	100	4.533	2.687	1.675	1.210	0.707	0.603	0.287	0.347	0.347	0.408
	200	3.135	1.816	1.210	0.658	0.431	0.357	0.424	0.456	0.524	0.579		200	4.421	2.684	1.786	0.970	0.526	0.343	0.275	0.304	0.374	0.436
	500	3.270	1.965	1.181	0.708	0.486	0.394	0.425	0.483	0.545	0.591		500	4.458	2.730	1.632	0.920	0.512	0.328	0.291	0.341	0.406	0.455
500	100	3.674	2.127	1.386	1.183	0.791	0.808	0.366	0.154	0.143	0.006	500	100	5.051	3.105	2.027	1.583	1.037	0.947	0.444	0.189	0.157	0.004
	200	3.618	2.254	1.661	1.001	0.631	0.509	0.291	0.198	0.099	0.006		200	4.914	3.122	2.207	1.335	0.828	0.619	0.349	0.225	0.109	0.005
	500	3.502	2.134	1.319	0.778	0.468	0.362	0.226	0.135	0.061	0.003		500	4.792	2.967	1.819	1.063	0.621	0.439	0.263	0.151	0.067	0.002
		$\delta = 1/3$												$\delta = 1/3$									
T	N	Bias										T	N	Bias									
100	100	4.967	2.337	0.691	-0.247	-1.047	-1.341	-1.960	-2.264	-2.293	-2.378	100	100	7.868	4.785	2.655	1.301	0.186	-0.344	-1.112	-1.516	-1.617	-1.751
	200	5.386	2.703	1.059	-0.320	-1.144	-1.548	-1.952	-2.111	-2.251	-2.349		200	8.272	5.102	2.925	1.133	-0.017	-0.647	-1.190	-1.440	-1.632	-1.760
	500	6.060	3.176	1.110	-0.281	-1.155	-1.589	-1.910	-2.103	-2.234	-2.319		500	8.905	5.513	2.917	1.072	-0.136	-0.790	-1.239	-1.502	-1.669	-1.771
200	100	6.216	3.864	2.456	1.755	1.046	0.840	0.256	-0.043	-0.086	-0.208	200	100	8.879	5.963	3.979	2.784	1.732	1.277	0.545	0.142	0.031	-0.136
	200	6.674	4.238	2.815	1.590	0.872	0.538	0.182	0.028	-0.108	-0.211		200	9.350	6.310	4.262	2.570	1.498	0.925	0.425	0.179	-0.009	-0.142
	500	7.411	4.705	2.828	1.567	0.799	0.433	0.150	-0.020	-0.132	-0.203		500	10.062	6.749	4.255	2.487	1.356	0.756	0.342	0.100	-0.050	-0.141
500	100	6.882	4.427	2.919	2.162	1.399	1.163	0.559	0.232	0.165	-0.001	500	100	9.532	6.514	4.416	3.144	2.029	1.530	0.772	0.340	0.205	0.000
	200	7.284	4.756	3.250	1.961	1.193	0.830	0.460	0.279	0.125	-0.001		200	9.941	6.805	4.663	2.897	1.765	1.152	0.632	0.360	0.154	0.000
	500	8.073	5.235	3.243	1.895	1.079	0.680	0.381	0.203	0.084	0.000		500	10.705	7.276	4.661	2.793	1.598	0.953	0.515	0.262	0.103	0.000
T	N	RMSE										T	N	RMSE									
100	100	5.440	2.965	1.638	1.304	1.618	1.840	2.349	2.627	2.668	2.753	100	100	8.200	5.156	3.058	1.759	1.041	1.045	1.500	1.839	1.944	2.073
	200	5.923	3.366	1.885	1.347	1.660	1.950	2.301	2.456	2.590	2.688		200	8.643	5.511	3.336	1.690	1.036	1.148	1.527	1.750	1.927	2.055
	500	6.655	3.889	2.065	1.402	1.648	1.947	2.225	2.407	2.536	2.622		500	9.316	5.974	3.429	1.730	1.062	1.215	1.540	1.773	1.933	2.036
200	100	6.534	4.180	2.714	1.916	1.163	0.902	0.352	0.211	0.219	0.287	200	100	9.121	6.208	4.186	2.924	1.834	1.330	0.594	0.222	0.153	0.198
	200	7.025	4.571	3.063	1.791	1.016	0.622	0.276	0.175	0.210	0.281		200	9.613	6.569	4.472	2.744	1.622	0.994	0.475	0.225	0.132	0.198
	500	7.799	5.083	3.155	1.831	0.996	0.554	0.255	0.156	0.207	0.266		500	10.346	7.040	4.522	2.710	1.521	0.855	0.404	0.160	0.126	0.191
500	100	7.080	4.614	3.065	2.246	1.454	1.188	0.579	0.255	0.181	0.003	500	100	9.685	6.665	4.542	3.226	2.085	1.559	0.795	0.361	0.220	0.002
	200	7.544	5.006	3.439	2.111	1.294	0.881	0.488	0.290	0.132	0.003		200	10.131	6.996	4.821	3.029	1.860	1.206	0.662	0.373	0.161	0.003
	500	8.285	5.432	3.400	2.006	1.146	0.710	0.394	0.208	0.086	0.001		500	10.863	7.433	4.799	2.899	1.667	0.988	0.531	0.268	0.105	0.001

Notes: Parameters of the dynamic panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i \sim U(0, 0.95)$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Design 2 assumes a two-factor model with $[N_{\alpha_\beta}]$ and $[N_{\alpha_{\beta 2}}]$ non-zero loadings for the first and second factor, respectively. We set: $\alpha_{\beta 2} = 2\alpha_\beta/3$, where α_β relates to α under (11) and $f_{jt} \sim IIDN(0,1)$. Non-Gaussian errors are generated as: $u_{it} = \left(\frac{\mathrm{v}-2}{\chi^2_{\mathrm{v},t}}\right)^{1/2} \tilde{\nu}_{it}$, for $i = 1, 2, \ldots, N$, $\tilde{\nu}_{it} \sim IIDN(0,1)$ and $\chi^2_{\mathrm{v},t}$ is a chi-squared random variate with v= 8 degrees of freedom, in (31). $v_{ij} \sim IIDU(\mu_{v_j} - 0.2, \mu_{v_j} + 0.2)$, $j = 1, 2$, $\mu_v = 0.87$, $\mu_{v_2} = 0.71$, $\mu_{v_1} = \sqrt{\mu_v^2 - N^{2(\alpha_{\beta 2} - \alpha_\beta)} \mu_{v_2}^2}$), in (32) and (33). The number of replications is set to $R = 2000$.

Table B1: Comparison of Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ and $\mathring{\alpha}$ estimates of the cross-sectional exponent of the errors from a static and dynamic panel data model with exogenous regressors

Cross correlations are generated using Design 1 with Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		Bias												RMSE									
		Static model: $\vartheta_i = 0$ for $i = 1, 2, \ldots, N$																					
T	N	$\tilde{\alpha}$										T	N	$\tilde{\alpha}$									
100	100	0.204	-0.609	-0.584	-0.355	-0.289	-0.001	-0.259	-0.279	-0.094	-0.016	100	100	0.311	0.633	0.596	0.364	0.296	0.051	0.264	0.282	0.102	0.041
	200	0.066	-0.426	-0.156	-0.302	-0.254	-0.101	-0.165	-0.080	-0.058	-0.025		200	0.145	0.438	0.174	0.308	0.261	0.124	0.171	0.093	0.070	0.053
	500	0.090	-0.120	-0.162	-0.144	-0.149	-0.035	-0.042	-0.046	-0.050	-0.041		500	0.118	0.137	0.177	0.157	0.165	0.073	0.078	0.076	0.077	0.079
200	100	0.243	-0.584	-0.557	-0.328	-0.267	0.017	-0.240	-0.262	-0.078	0.000	200	100	0.347	0.611	0.571	0.338	0.272	0.042	0.242	0.263	0.079	0.000
	200	0.118	-0.380	-0.120	-0.271	-0.226	-0.074	-0.139	-0.057	-0.035	0.000		200	0.183	0.391	0.133	0.274	0.228	0.076	0.140	0.057	0.035	0.000
	500	0.158	-0.064	-0.114	-0.101	-0.106	0.007	-0.001	-0.006	-0.011	0.000		500	0.173	0.075	0.116	0.102	0.107	0.010	0.004	0.007	0.011	0.000
500	100	0.277	-0.556	-0.549	-0.325	-0.261	0.019	-0.239	-0.262	-0.078	0.000	500	100	0.374	0.584	0.563	0.337	0.268	0.043	0.240	0.263	0.078	0.000
	200	0.157	-0.367	-0.110	-0.265	-0.223	-0.073	-0.138	-0.056	-0.035	0.000		200	0.218	0.379	0.126	0.268	0.225	0.075	0.138	0.057	0.035	0.000
	500	0.189	-0.047	-0.105	-0.097	-0.104	0.009	0.000	-0.006	-0.011	0.000		500	0.203	0.063	0.108	0.098	0.104	0.011	0.004	0.007	0.011	0.000
T	N	$\mathring{\alpha}$										T	N	$\mathring{\alpha}$									
100	100	1.220	-0.126	-0.287	-0.090	-0.030	0.216	-0.096	-0.115	0.051	0.105	100	100	2.208	1.607	1.435	1.046	0.786	0.610	0.438	0.324	0.188	0.105
	200	0.463	-0.305	-0.046	-0.191	-0.091	0.040	-0.022	0.056	0.070	0.093		200	1.392	1.292	1.006	0.826	0.589	0.416	0.291	0.206	0.136	0.093
	500	0.162	-0.182	-0.116	-0.010	-0.016	0.091	0.080	0.080	0.070	0.080		500	1.057	0.996	0.762	0.575	0.386	0.295	0.203	0.142	0.097	0.080
200	100	1.916	0.155	-0.208	-0.091	-0.093	0.147	-0.134	-0.168	-0.001	0.050	200	100	2.576	1.473	1.152	0.842	0.605	0.452	0.346	0.275	0.139	0.050
	200	0.530	-0.375	-0.143	-0.221	-0.176	0.003	-0.075	-0.002	0.022	0.045		200	1.123	1.036	0.767	0.615	0.454	0.298	0.222	0.139	0.086	0.045
	500	0.144	-0.244	-0.188	-0.086	-0.081	0.052	0.044	0.036	0.031	0.040		500	0.742	0.759	0.575	0.425	0.291	0.202	0.141	0.094	0.056	0.040
500	100	6.301	2.825	0.928	0.325	0.045	0.186	-0.133	-0.190	-0.030	0.018	500	100	6.675	3.383	1.622	0.932	0.565	0.415	0.291	0.256	0.111	0.018
	200	1.495	0.021	0.033	-0.221	-0.177	-0.037	-0.104	-0.027	-0.012	0.017		200	1.869	0.881	0.607	0.477	0.343	0.212	0.178	0.098	0.057	0.017
	500	0.148	-0.269	-0.216	-0.118	-0.099	0.022	0.017	0.011	0.007	0.015		500	0.547	0.544	0.427	0.280	0.205	0.123	0.089	0.055	0.031	0.015
		Dynamic model with exogenous regressors: $\vartheta_i \sim U(0, 0.95)$ for $i = 1, 2, \ldots, N$																					
T	N	$\tilde{\alpha}$										T	N	$\tilde{\alpha}$									
100	100	0.198	-0.621	-0.594	-0.363	-0.295	-0.007	-0.261	-0.284	-0.099	-0.021	100	100	0.302	0.644	0.607	0.375	0.305	0.056	0.266	0.289	0.108	0.045
	200	0.064	-0.432	-0.161	-0.309	-0.262	-0.108	-0.173	-0.088	-0.070	-0.033		200	0.153	0.444	0.180	0.316	0.272	0.126	0.183	0.106	0.094	0.064
	500	0.084	-0.123	-0.169	-0.158	-0.158	-0.046	-0.054	-0.059	-0.066	-0.049		500	0.117	0.144	0.185	0.179	0.174	0.102	0.102	0.102	0.107	0.090
200	100	0.255	-0.573	-0.561	-0.330	-0.266	0.018	-0.240	-0.263	-0.078	0.000	200	100	0.354	0.600	0.573	0.339	0.272	0.043	0.241	0.264	0.079	0.000
	200	0.123	-0.381	-0.118	-0.272	-0.225	-0.074	-0.139	-0.056	-0.035	0.000		200	0.190	0.392	0.131	0.274	0.226	0.076	0.139	0.057	0.035	0.000
	500	0.156	-0.063	-0.113	-0.101	-0.106	0.007	-0.001	-0.007	-0.011	0.000		500	0.169	0.074	0.116	0.102	0.107	0.010	0.004	0.007	0.011	0.000
500	100	0.281	-0.550	-0.550	-0.322	-0.261	0.020	-0.240	-0.261	-0.078	0.000	500	100	0.375	0.580	0.563	0.332	0.267	0.045	0.241	0.262	0.079	0.000
	200	0.153	-0.366	-0.108	-0.266	-0.222	-0.072	-0.137	-0.056	-0.035	0.000		200	0.213	0.378	0.123	0.270	0.224	0.074	0.138	0.057	0.035	0.000
	500	0.185	-0.049	-0.104	-0.097	-0.104	0.009	0.000	-0.006	-0.011	0.000		500	0.198	0.065	0.107	0.098	0.104	0.011	0.004	0.007	0.011	0.000
T	N	$\mathring{\alpha}$										T	N	$\mathring{\alpha}$									
100	100	1.219	-0.011	-0.292	-0.137	-0.033	0.188	-0.085	-0.123	0.056	0.106	100	100	2.160	1.722	1.410	1.091	0.809	0.619	0.446	0.317	0.192	0.106
	200	0.472	-0.353	-0.080	-0.172	-0.102	0.043	-0.031	0.057	0.069	0.094		200	1.480	1.319	1.044	0.812	0.608	0.421	0.307	0.205	0.136	0.094
	500	0.152	-0.178	-0.128	-0.053	-0.031	0.092	0.091	0.082	0.073	0.081		500	1.059	0.986	0.810	0.564	0.407	0.294	0.210	0.146	0.099	0.081
200	100	1.938	0.167	-0.158	-0.096	-0.083	0.155	-0.119	-0.170	-0.003	0.051	200	100	2.598	1.444	1.152	0.863	0.601	0.476	0.346	0.276	0.135	0.051
	200	0.547	-0.363	-0.101	-0.227	-0.149	-0.011	-0.081	0.001	0.018	0.046		200	1.166	1.023	0.765	0.612	0.439	0.294	0.221	0.139	0.085	0.046
	500	0.134	-0.238	-0.200	-0.100	-0.069	0.048	0.042	0.034	0.032	0.040		500	0.741	0.746	0.589	0.403	0.296	0.205	0.138	0.093	0.058	0.040
500	100	6.145	2.677	0.952	0.313	0.043	0.184	-0.142	-0.187	-0.031	0.018	500	100	6.494	3.260	1.658	0.900	0.554	0.410	0.292	0.256	0.111	0.018
	200	1.489	0.038	0.024	-0.205	-0.191	-0.031	-0.104	-0.028	-0.010	0.017		200	1.896	0.881	0.606	0.464	0.354	0.204	0.178	0.097	0.059	0.017
	500	0.165	-0.264	-0.212	-0.124	-0.103	0.020	0.017	0.009	0.005	0.015		500	0.571	0.561	0.404	0.284	0.206	0.130	0.086	0.056	0.031	0.015

Notes: Remaining parameters of the panel data model, (29), are generated as: $a_i \sim IIDN\,(1,1)$, $\rho_{ix} \sim U(0, 0.95)$ and $\gamma_i \sim IIDN\,(1,1)$, for $i = 1, 2, \ldots, N$. Gaussian errors are generated as $u_{it} \sim IIDN(0,1)$ in (31). Design 1 assumes $b_i \sim U(0.7, 0.9)$ for the first $N_b\,(\leq N)$ elements of vector $\mathbf{b}_N$, $i = 1, 2, \ldots, N$, in the construction of the correlation matrix of the errors, $\mathbf{R}_N = \mathbf{I}_N + \mathbf{b}_N\mathbf{b}_N' - \mathbf{B}_N^2$ given by (30), where $\mathbf{B}_N = \mathrm{Diag}\,(\mathbf{b}_N)$. $\tilde{\alpha}$ is computed using $c_p(n, \delta)$ with $n = N(N-1)/2$, $\delta = 1/2$ and $p = 0.05$ in the multiple testing procedure shown in (23). $\mathring{\alpha}$ corresponds to the most robust estimator of the exponent of cross-sectional dependence considered in Bailey et al. (2016) and allows for both serial correlation in the factors and weak cross-sectional dependence in the error terms. We use four principal components when estimating $\hat{c}_N$ in the expression for $\mathring{\alpha}$. The number of replications is set to $R = 2000$.

Table B2: Comparison of Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ and $\mathring{\alpha}$ of the cross-sectional exponent of the errors of a static and dynamic panel data model with exogenous regressors

Cross correlations are generated using Design 2 with Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		Bias												RMSE									
		Static model: $\vartheta_i = 0$, for $i = 1, 2, \ldots, N$																					
T	N	$\tilde{\alpha}$										T	N	$\tilde{\alpha}$									
100	100	-0.880	-2.378	-2.922	-3.026	-3.232	-3.131	-3.509	-3.648	-3.576	-3.598	100	100	1.252	2.688	3.301	3.469	3.699	3.623	3.974	4.094	4.029	4.051
	200	-1.221	-2.480	-2.800	-3.267	-3.464	-3.434	-3.595	-3.591	-3.658	-3.714		200	1.455	2.748	3.167	3.655	3.868	3.859	4.002	4.009	4.070	4.130
	500	-1.501	-2.658	-3.266	-3.581	-3.780	-3.779	-3.859	-3.934	-4.011	-4.057		500	1.662	2.899	3.580	3.938	4.153	4.176	4.260	4.336	4.411	4.456
200	100	0.705	-0.090	-0.160	0.120	0.009	0.205	-0.145	-0.299	-0.242	-0.304	200	100	0.803	0.377	0.369	0.346	0.298	0.354	0.312	0.405	0.359	0.399
	200	0.375	-0.068	0.134	-0.086	-0.146	-0.063	-0.191	-0.206	-0.269	-0.326		200	0.457	0.291	0.307	0.274	0.296	0.251	0.315	0.318	0.367	0.415
	500	0.219	-0.024	-0.086	-0.158	-0.206	-0.153	-0.212	-0.267	-0.325	-0.368		500	0.303	0.241	0.262	0.299	0.328	0.295	0.326	0.366	0.418	0.456
500	100	1.043	0.295	0.247	0.563	0.433	0.625	0.264	0.084	0.114	-0.001	500	100	1.068	0.337	0.272	0.571	0.442	0.631	0.279	0.116	0.136	0.002
	200	0.683	0.313	0.550	0.326	0.254	0.329	0.203	0.161	0.087	-0.001		200	0.697	0.325	0.553	0.329	0.257	0.332	0.208	0.166	0.093	0.001
	500	0.530	0.355	0.331	0.249	0.205	0.246	0.176	0.117	0.056	-0.001		500	0.534	0.357	0.332	0.250	0.206	0.246	0.177	0.118	0.057	0.001
T	N	$\mathring{\alpha}$										T	N	$\mathring{\alpha}$									
100	100	1.172	0.453	0.383	0.727	0.531	0.700	0.310	0.144	0.198	0.104	100	100	2.349	1.859	1.543	1.430	1.073	1.002	0.614	0.393	0.301	0.104
	200	0.250	0.045	0.467	0.320	0.272	0.358	0.224	0.207	0.152	0.093		200	1.453	1.314	1.144	0.878	0.672	0.590	0.412	0.312	0.205	0.093
	500	-0.148	-0.003	0.183	0.210	0.204	0.269	0.213	0.164	0.120	0.080		500	1.066	0.940	0.755	0.583	0.457	0.400	0.301	0.218	0.144	0.080
200	100	2.806	1.587	1.108	1.329	0.844	0.894	0.428	0.181	0.181	0.051	200	100	3.438	2.401	1.862	1.879	1.277	1.157	0.699	0.414	0.288	0.051
	200	0.906	0.459	0.687	0.431	0.301	0.357	0.218	0.178	0.117	0.046		200	1.478	1.251	1.179	0.869	0.612	0.540	0.373	0.262	0.164	0.046
	500	0.246	0.109	0.219	0.197	0.172	0.229	0.174	0.127	0.081	0.040		500	0.781	0.714	0.614	0.468	0.363	0.325	0.239	0.166	0.100	0.040
500	100	7.425	4.897	3.572	3.447	2.355	1.976	1.184	0.587	0.374	0.019	500	100	7.723	5.218	3.881	3.685	2.633	2.223	1.451	0.825	0.522	0.019
	200	2.637	2.154	2.102	1.495	0.940	0.763	0.490	0.275	0.156	0.018		200	2.956	2.505	2.397	1.811	1.231	0.984	0.670	0.383	0.223	0.018
	500	0.623	0.529	0.652	0.399	0.305	0.281	0.185	0.121	0.064	0.016		500	0.948	0.917	0.953	0.625	0.479	0.370	0.242	0.156	0.085	0.016
		Dynamic model with exogenous regressors: $\vartheta_i \sim U(0, 0.95)$ for $i = 1, 2, \ldots, N$																					
T	N	$\tilde{\alpha}$										T	N	$\tilde{\alpha}$									
100	100	-1.016	-2.571	-3.183	-3.319	-3.568	-3.479	-3.838	-3.998	-3.922	-3.952	100	100	1.360	2.876	3.549	3.762	4.031	3.981	4.301	4.455	4.378	4.411
	200	-1.320	-2.642	-3.032	-3.558	-3.745	-3.763	-3.918	-3.925	-4.003	-4.057		200	1.536	2.902	3.398	3.954	4.170	4.208	4.358	4.367	4.445	4.497
	500	-1.610	-2.849	-3.517	-3.890	-4.107	-4.124	-4.222	-4.304	-4.388	-4.439		500	1.769	3.101	3.849	4.273	4.512	4.554	4.659	4.740	4.822	4.870
200	100	0.689	-0.109	-0.179	0.088	-0.030	0.166	-0.181	-0.329	-0.272	-0.336	200	100	0.794	0.393	0.393	0.369	0.326	0.359	0.355	0.440	0.398	0.446
	200	0.357	-0.089	0.115	-0.120	-0.177	-0.095	-0.227	-0.240	-0.301	-0.360		200	0.448	0.292	0.301	0.306	0.330	0.284	0.354	0.355	0.401	0.453
	500	0.209	-0.033	-0.095	-0.170	-0.215	-0.165	-0.225	-0.281	-0.337	-0.380		500	0.295	0.245	0.274	0.306	0.329	0.296	0.335	0.376	0.422	0.463
500	100	1.040	0.294	0.248	0.565	0.436	0.627	0.261	0.081	0.115	-0.001	500	100	1.065	0.332	0.272	0.573	0.443	0.634	0.276	0.118	0.135	0.002
	200	0.685	0.313	0.548	0.324	0.253	0.330	0.202	0.162	0.086	-0.001		200	0.698	0.326	0.551	0.328	0.257	0.332	0.207	0.166	0.092	0.001
	500	0.527	0.353	0.330	0.249	0.204	0.246	0.176	0.117	0.055	-0.001		500	0.531	0.355	0.331	0.250	0.205	0.246	0.176	0.118	0.057	0.001
T	N	$\mathring{\alpha}$										T	N	$\mathring{\alpha}$									
100	100	1.092	0.325	0.312	0.674	0.496	0.664	0.291	0.130	0.187	0.105	100	100	2.274	1.834	1.492	1.393	1.047	0.958	0.597	0.386	0.289	0.105
	200	0.157	-0.008	0.432	0.286	0.251	0.341	0.222	0.201	0.148	0.094		200	1.470	1.363	1.163	0.888	0.687	0.586	0.418	0.308	0.201	0.094
	500	-0.160	-0.033	0.162	0.193	0.188	0.257	0.204	0.157	0.116	0.081		500	1.110	0.978	0.772	0.587	0.457	0.399	0.298	0.215	0.142	0.081
200	100	2.791	1.573	1.132	1.355	0.832	0.878	0.414	0.168	0.187	0.051	200	100	3.395	2.377	1.890	1.911	1.280	1.161	0.708	0.414	0.305	0.051
	200	0.881	0.432	0.679	0.416	0.290	0.345	0.211	0.175	0.114	0.046		200	1.463	1.254	1.197	0.864	0.623	0.542	0.372	0.261	0.162	0.046
	500	0.261	0.128	0.240	0.206	0.181	0.237	0.180	0.133	0.081	0.040		500	0.801	0.733	0.630	0.473	0.369	0.332	0.244	0.174	0.101	0.040
500	100	7.388	4.904	3.529	3.455	2.334	1.980	1.188	0.579	0.377	0.019	500	100	7.714	5.231	3.849	3.696	2.620	2.222	1.448	0.829	0.527	0.019
	200	2.603	2.107	2.046	1.437	0.883	0.723	0.459	0.266	0.143	0.018		200	2.932	2.460	2.343	1.753	1.176	0.939	0.642	0.370	0.209	0.018
	500	0.636	0.531	0.644	0.397	0.292	0.274	0.183	0.121	0.063	0.016		500	0.955	0.904	0.935	0.620	0.472	0.363	0.240	0.156	0.084	0.016

Notes: Remaining parameters of the panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Design 2 assumes a two-factor model with $[N_{\alpha_\beta}]$ and $[N_{\alpha_{\beta 2}}]$ non-zero loadings for the first and second factor, respectively. We set: $\alpha_{\beta 2} = 2\alpha_\beta/3$, where α_β relates to α under (11), f_{jt} and $u_{it} \sim IIDN(0,1)$, $v_{ij} \sim IIDU(\mu_{v_j} - 0.2, \mu_{v_j} + 0.2)$, $j = 1, 2$, $\mu_v = 0.87$, $\mu_{v_2} = 0.71$, $\mu_{v_1} = \sqrt{\mu_v^2 - N^{2(\alpha_{\beta 2} - \alpha_\beta)}\mu_{v_2}^2}$), in (32) and (33). $\tilde{\alpha}$ is computed using $c_p(n, \delta)$ with $n = N(N-1)/2$, $\delta = 1/2$ and $p = 0.05$ in the multiple testing procedure shown in (23). $\mathring{\alpha}$ corresponds to the most robust estimator of the exponent of cross-sectional dependence considered in Bailey et al. (2016) and allows for both serial correlation in the factors and weak cross-sectional dependence in the error terms. We use four principal components when estimating $\hat{c}_N$ in the expression for $\mathring{\alpha}$. The number of replications is set to $R = 2000$.

Table C: Bias and RMSE ($\times 100$) of α estimates of the cross-sectional exponent of the errors using the maximum eigenvalue of their sample correlation matrix in a static and dynamic panel data model with exogenous regressors

Cross correlations are generated using Designs 1 and 2 with Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		Bias												RMSE									
		Design 1: Static model - $\vartheta_i = 0$, for $i = 1, 2, \ldots, N$																					
T	N	α estimate using eigenvalue approach										T	N	α estimate using eigenvalue approach									
100	100	-14.080	-12.332	-10.881	-10.088	-9.817	-9.406	-9.705	-9.782	-9.700	-9.706	100	100	14.131	12.391	10.942	10.152	9.889	9.471	9.771	9.845	9.763	9.772
	200	-11.253	-9.674	-8.509	-8.338	-8.267	-8.189	-8.364	-8.352	-8.443	-8.534		200	11.291	9.723	8.564	8.393	8.323	8.245	8.418	8.406	8.496	8.588
	500	-7.719	-6.671	-6.427	-6.469	-6.682	-6.750	-6.931	-7.069	-7.197	-7.283		500	7.751	6.710	6.471	6.515	6.728	6.796	6.977	7.114	7.239	7.328
200	100	-15.987	-13.479	-11.634	-10.520	-10.076	-9.572	-9.806	-9.839	-9.650	-9.641	200	100	16.024	13.519	11.674	10.563	10.117	9.614	9.843	9.874	9.685	9.676
	200	-13.357	-10.916	-9.294	-8.883	-8.609	-8.368	-8.450	-8.371	-8.395	-8.375		200	13.384	10.946	9.327	8.915	8.640	8.400	8.479	8.400	8.423	8.402
	500	-10.018	-8.102	-7.384	-7.110	-7.032	-6.977	-7.013	-7.123	-7.159	-7.177		500	10.037	8.125	7.408	7.135	7.055	7.000	7.038	7.148	7.182	7.199
500	100	-17.221	-14.208	-12.095	-10.830	-10.230	-9.670	-9.826	-9.780	-9.577	-9.513	500	100	17.248	14.236	12.122	10.858	10.256	9.694	9.846	9.800	9.596	9.529
	200	-14.655	-11.707	-9.762	-9.195	-8.799	-8.474	-8.501	-8.360	-8.337	-8.320		200	14.675	11.728	9.782	9.213	8.816	8.490	8.515	8.373	8.350	8.332
	500	-11.613	-9.002	-7.948	-7.476	-7.302	-7.085	-7.074	-7.095	-7.106	-7.114		500	11.626	9.015	7.961	7.489	7.314	7.096	7.085	7.105	7.115	7.124
		Design 1: Dynamic model with exogenous regressors - $\vartheta_i \sim U(0, 0.95)$ for $i = 1, 2, \ldots, N$																					
T	N	α estimate using eigenvalue approach										T	N	α estimate using eigenvalue approach									
100	100	-14.273	-12.513	-11.143	-10.300	-10.022	-9.608	-9.905	-10.049	-9.907	-9.935	100	100	14.323	12.572	11.210	10.368	10.094	9.678	9.967	10.112	9.971	9.997
	200	-11.387	-9.744	-8.615	-8.508	-8.457	-8.329	-8.544	-8.536	-8.669	-8.723		200	11.425	9.792	8.669	8.561	8.512	8.384	8.596	8.590	8.725	8.776
	500	-7.782	-6.713	-6.555	-6.633	-6.792	-6.889	-7.059	-7.231	-7.380	-7.403		500	7.813	6.753	6.599	6.680	6.836	6.934	7.108	7.277	7.426	7.446
200	100	-16.109	-13.524	-11.717	-10.657	-10.169	-9.691	-9.870	-9.912	-9.764	-9.682	200	100	16.144	13.564	11.760	10.701	10.210	9.733	9.907	9.949	9.798	9.718
	200	-13.395	-10.991	-9.355	-8.997	-8.683	-8.451	-8.496	-8.453	-8.470	-8.438		200	13.420	11.022	9.388	9.028	8.712	8.482	8.527	8.481	8.500	8.464
	500	-10.067	-8.154	-7.470	-7.169	-7.125	-7.015	-7.111	-7.189	-7.205	-7.269		500	10.087	8.176	7.493	7.193	7.149	7.039	7.134	7.212	7.227	7.292
500	100	-17.237	-14.217	-12.116	-10.862	-10.276	-9.736	-9.876	-9.808	-9.632	-9.554	500	100	17.266	14.245	12.143	10.889	10.302	9.759	9.896	9.826	9.650	9.570
	200	-14.701	-11.724	-9.774	-9.242	-8.828	-8.512	-8.508	-8.399	-8.383	-8.344		200	14.721	11.744	9.794	9.260	8.844	8.528	8.522	8.412	8.395	8.357
	500	-11.622	-9.037	-7.975	-7.525	-7.308	-7.110	-7.118	-7.131	-7.143	-7.145		500	11.635	9.051	7.989	7.537	7.320	7.121	7.129	7.140	7.152	7.154
		Design 2: Static model - $\vartheta_i = 0$, for $i = 1, 2, \ldots, N$																					
T	N	α estimate using eigenvalue approach										T	N	α estimate using eigenvalue approach									
100	100	-20.441	-19.904	-19.231	-18.859	-18.828	-18.720	-19.151	-19.367	-19.377	-19.458	100	100	20.494	19.971	19.306	18.939	18.911	18.804	19.235	19.451	19.461	19.542
	200	-16.689	-16.128	-15.570	-15.791	-15.961	-16.051	-16.357	-16.489	-16.672	-16.813		200	16.736	16.185	15.636	15.859	16.030	16.122	16.427	16.560	16.743	16.885
	500	-12.052	-11.948	-12.233	-12.612	-13.038	-13.298	-13.613	-13.879	-14.103	-14.260		500	12.083	11.990	12.282	12.664	13.092	13.355	13.670	13.938	14.162	14.320
200	100	-23.448	-21.789	-20.444	-19.630	-19.337	-19.024	-19.310	-19.402	-19.314	-19.324	200	100	23.488	21.833	20.491	19.678	19.384	19.072	19.356	19.447	19.359	19.369
	200	-19.823	-18.132	-16.909	-16.718	-16.586	-16.469	-16.617	-16.628	-16.714	-16.780		200	19.853	18.167	16.947	16.756	16.624	16.507	16.654	16.665	16.751	16.817
	500	-15.427	-14.141	-13.740	-13.655	-13.743	-13.750	-13.888	-14.025	-14.149	-14.240		500	15.450	14.168	13.770	13.686	13.774	13.781	13.919	14.056	14.180	14.271
500	100	-25.468	-23.006	-21.249	-20.175	-19.708	-19.258	-19.452	-19.476	-19.322	-19.289	500	100	25.496	23.034	21.277	20.201	19.732	19.281	19.475	19.498	19.342	19.309
	200	-21.987	-19.443	-17.732	-17.272	-16.946	-16.678	-16.734	-16.667	-16.696	-16.715		200	22.008	19.465	17.753	17.292	16.964	16.695	16.751	16.683	16.712	16.731
	500	-17.787	-15.539	-14.644	-14.250	-14.124	-13.980	-14.009	-14.065	-14.127	-14.177		500	17.801	15.553	14.657	14.263	14.137	13.993	14.022	14.078	14.140	14.189
		Design 2: Dynamic model with exogenous regressors - $\vartheta_i \sim U(0, 0.95)$ for $i = 1, 2, \ldots, N$																					
T	N	α estimate using eigenvalue approach										T	N	α estimate using eigenvalue approach									
100	100	-20.516	-20.025	-19.395	-19.051	-19.063	-18.960	-19.387	-19.614	-19.633	-19.726	100	100	20.568	20.092	19.469	19.130	19.144	19.043	19.469	19.696	19.715	19.809
	200	-16.722	-16.214	-15.701	-15.947	-16.112	-16.236	-16.543	-16.686	-16.880	-17.026		200	16.765	16.269	15.765	16.015	16.182	16.307	16.616	16.759	16.955	17.100
	500	-12.067	-12.003	-12.324	-12.736	-13.174	-13.452	-13.782	-14.057	-14.287	-14.454		500	12.099	12.047	12.375	12.791	13.231	13.511	13.842	14.119	14.349	14.516
200	100	-23.502	-21.871	-20.547	-19.757	-19.473	-19.155	-19.429	-19.532	-19.454	-19.459	200	100	23.542	21.917	20.596	19.807	19.522	19.204	19.477	19.578	19.500	19.505
	200	-19.885	-18.203	-16.982	-16.815	-16.671	-16.562	-16.722	-16.742	-16.825	-16.896		200	19.915	18.238	17.019	16.853	16.710	16.600	16.761	16.780	16.863	16.934
	500	-15.445	-14.156	-13.756	-13.679	-13.765	-13.786	-13.930	-14.068	-14.191	-14.277		500	15.466	14.182	13.785	13.708	13.795	13.816	13.960	14.098	14.221	14.307
500	100	-25.472	-23.054	-21.306	-20.220	-19.759	-19.304	-19.518	-19.538	-19.386	-19.356	500	100	25.501	23.085	21.336	20.249	19.786	19.329	19.541	19.559	19.407	19.376
	200	-21.991	-19.433	-17.726	-17.280	-16.950	-16.703	-16.762	-16.700	-16.729	-16.755		200	22.012	19.455	17.748	17.300	16.968	16.721	16.779	16.716	16.745	16.770
	500	-17.821	-15.585	-14.692	-14.280	-14.152	-14.010	-14.044	-14.095	-14.160	-14.209		500	17.835	15.600	14.707	14.295	14.166	14.024	14.057	14.108	14.173	14.222

Notes: Remaining parameters of the panel data model, (29), are generated as: $a_i \sim IIDN\,(1,1)$, $\rho_{ix} \sim U(0, 0.95)$ and $\gamma_i \sim IIDN\,(1,1)$, for $i = 1, 2, \ldots, N$. Gaussian errors are generated as $u_{it} \sim IIDN(0,1)$ in (31). For details on Designs 1 and 2 refer to footnotes of Tables A1a and A2a. The number of replications is set to $R = 2000$.

Table D: Coverage ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a static and dynamic panel data model with exogenous regressors

Cross correlations are generated using Design 1 with Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$, $\delta = 1/2$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$, $\delta = 1/2$ and $p = 0.10$									
		Static model: $\vartheta_i = 0$ for $i = 1, 2, \ldots, N$																					
T	N											T	N										
100	100	0.00	20.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	65.95	100	100	0.00	27.35	100.00	100.00	100.00	100.00	100.00	100.00	100.00	53.70
	200	0.00	89.30	100.00	100.00	100.00	100.00	100.00	100.00	100.00	92.30		200	0.00	90.15	100.00	100.00	100.00	100.00	100.00	100.00	100.00	85.30
	500	0.00	99.70	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.60		500	0.00	99.70	100.00	100.00	100.00	100.00	100.00	100.00	100.00	98.90
200	100	0.00	19.80	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.15	200	100	0.00	25.05	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.00
	200	0.00	88.40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.65		200	0.00	88.80	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.25
	500	0.00	99.90	100.00	100.00	100.00	100.00	100.00	100.00	100.00	6.35		500	0.00	99.90	100.00	100.00	100.00	100.00	100.00	100.00	100.00	2.85
500	100	0.00	20.95	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.00	500	100	0.00	28.20	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.00
	200	0.00	88.90	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.00		200	0.00	90.20	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.00
	500	0.00	99.75	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.00		500	0.00	99.90	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.00
		Dynamic model with exogenous regressors: $\vartheta_i \sim U(0, 0.95)$ for $i = 1, 2, \ldots, N$																					
T	N											T	N										
100	100	0.00	18.30	100.00	100.00	100.00	100.00	100.00	100.00	100.00	69.95	100	100	0.00	24.35	100.00	100.00	100.00	100.00	100.00	100.00	100.00	58.20
	200	0.00	86.75	100.00	100.00	100.00	100.00	100.00	100.00	100.00	94.60		200	0.00	88.20	100.00	100.00	100.00	100.00	100.00	100.00	100.00	89.60
	500	0.00	99.65	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.85		500	0.00	99.70	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.65
200	100	0.00	20.70	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.10	200	100	0.00	26.90	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.10
	200	0.00	88.95	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.70		200	0.00	90.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.30
	500	0.00	99.75	100.00	100.00	100.00	100.00	100.00	100.00	100.00	7.60		500	0.00	99.80	100.00	100.00	100.00	100.00	100.00	100.00	100.00	4.25
500	100	0.00	21.50	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.00	500	100	0.00	28.05	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.00
	200	0.00	88.20	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.00		200	0.00	89.40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.00
	500	0.00	99.70	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.00		500	0.00	99.55	100.00	100.00	100.00	100.00	100.00	100.00	100.00	0.00

Notes: Remaining parameters of the panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Gaussian errors are generated as $u_{it} \sim IIDN(0,1)$ in (31). Design 1 assumes $b_i \sim U(0.7, 0.9)$ for the first $N_b (\leq N)$ elements of vector $\mathbf{b}_N$, $i = 1, 2, \ldots, N$, in the construction of the correlation matrix of the errors, $\mathbf{R}_N = \mathbf{I}_N + \mathbf{b}_N \mathbf{b}_N' - \mathbf{B}_N^2$ given by (30), where $\mathbf{B}_N = \text{Diag}(\mathbf{b}_N)$. $\tilde{\alpha}$ and its empirical distribution are computed using $c_p(n,\delta)$ with $n = N(N-1)/2$, $\delta = 1/2$ and $p = 0.05, 0.10$ in the multiple testing procedure shown in (23). Coverage refers to the frequency at which $\tilde{\alpha}_{0.05}^{(r),B} < \tilde{\alpha}^{(r)} < \tilde{\alpha}_{0.95}^{(r),B}$, where $r = 1, 2, \ldots, R$ and $b = 1, 2, \ldots, B$. The number of replications is set to $R = 2000$ and the number of bootstraps is set to $B = 500$.

An Online Supplement for

Exponent of Cross-sectional Dependence for Residuals

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April 4, 2019

This online supplement provides additional Monte Carlo and empirical results.

Appendix A

Additional Monte Carlo results

The Monte Carlo results provided in the tables below are based on the designs set out in Section 6 of the paper.

Table S1a: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a dynamic panel data model with no exogenous regressors

Cross correlations are generated using Design 1 with Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.10$									
		$\delta = 1/2$												$\delta = 1/2$									
T	N	Bias										T	N	Bias									
100	100	0.211	-0.618	-0.595	-0.357	-0.287	-0.002	-0.259	-0.281	-0.094	-0.017	100	100	0.639	-0.327	-0.415	-0.246	-0.216	0.043	-0.230	-0.262	-0.083	-0.009
	200	0.074	-0.424	-0.155	-0.302	-0.257	-0.102	-0.166	-0.081	-0.064	-0.029		200	0.406	-0.214	-0.030	-0.225	-0.207	-0.070	-0.144	-0.066	-0.050	-0.017
	500	0.087	-0.121	-0.164	-0.149	-0.152	-0.036	-0.043	-0.053	-0.051	-0.042		500	0.331	0.020	-0.081	-0.096	-0.117	-0.010	-0.022	-0.034	-0.035	-0.025
200	100	0.267	-0.579	-0.553	-0.329	-0.270	0.017	-0.241	-0.263	-0.078	0.000	200	100	0.709	-0.285	-0.366	-0.220	-0.204	0.057	-0.218	-0.251	-0.074	0.000
	200	0.124	-0.385	-0.117	-0.271	-0.226	-0.074	-0.139	-0.056	-0.035	0.000		200	0.471	-0.163	0.006	-0.197	-0.183	-0.051	-0.127	-0.050	-0.033	0.000
	500	0.153	-0.064	-0.112	-0.101	-0.106	0.007	-0.001	-0.007	-0.011	0.000		500	0.416	0.081	-0.033	-0.059	-0.084	0.018	0.005	-0.004	-0.010	0.000
500	100	0.282	-0.555	-0.549	-0.323	-0.264	0.020	-0.239	-0.262	-0.078	0.000	500	100	0.734	-0.245	-0.358	-0.208	-0.196	0.060	-0.216	-0.249	-0.073	0.000
	200	0.154	-0.363	-0.107	-0.266	-0.224	-0.072	-0.138	-0.056	-0.035	0.000		200	0.520	-0.135	0.023	-0.191	-0.181	-0.048	-0.125	-0.050	-0.032	0.000
	500	0.188	-0.048	-0.104	-0.097	-0.103	0.009	0.000	-0.006	-0.011	0.000		500	0.468	0.109	-0.020	-0.052	-0.079	0.021	0.006	-0.004	-0.010	0.000
T	N	RMSE										T	N	RMSE									
100	100	0.323	0.640	0.608	0.367	0.296	0.055	0.263	0.284	0.101	0.041	100	100	0.720	0.403	0.447	0.271	0.232	0.074	0.235	0.264	0.087	0.025
	200	0.156	0.436	0.174	0.308	0.264	0.117	0.175	0.096	0.088	0.061		200	0.449	0.251	0.094	0.234	0.214	0.083	0.149	0.074	0.065	0.038
	500	0.117	0.140	0.180	0.165	0.168	0.085	0.085	0.099	0.084	0.078		500	0.344	0.070	0.100	0.109	0.127	0.054	0.055	0.067	0.057	0.051
200	100	0.359	0.602	0.566	0.339	0.275	0.043	0.242	0.263	0.079	0.000	200	100	0.786	0.379	0.407	0.252	0.218	0.082	0.222	0.252	0.075	0.000
	200	0.187	0.396	0.131	0.274	0.227	0.076	0.140	0.057	0.035	0.000		200	0.510	0.209	0.088	0.206	0.188	0.058	0.128	0.052	0.033	0.000
	500	0.168	0.075	0.115	0.102	0.106	0.009	0.004	0.007	0.011	0.000		500	0.427	0.098	0.049	0.063	0.085	0.021	0.008	0.006	0.011	0.000
500	100	0.379	0.586	0.562	0.333	0.270	0.044	0.241	0.262	0.078	0.000	500	100	0.813	0.360	0.399	0.240	0.213	0.084	0.221	0.251	0.075	0.000
	200	0.210	0.376	0.124	0.269	0.225	0.075	0.138	0.057	0.035	0.000		200	0.556	0.192	0.092	0.200	0.185	0.056	0.126	0.051	0.033	0.000
	500	0.202	0.063	0.108	0.098	0.104	0.011	0.004	0.007	0.011	0.000		500	0.479	0.124	0.042	0.056	0.081	0.023	0.009	0.006	0.010	0.000
		$\delta = 1/3$												$\delta = 1/3$									
T	N	Bias										T	N	Bias									
100	100	1.515	0.274	-0.030	-0.008	-0.068	0.129	-0.177	-0.232	-0.069	-0.005	100	100	3.099	1.393	0.699	0.444	0.215	0.296	-0.079	-0.180	-0.049	-0.002
	200	1.453	0.459	0.363	0.014	-0.065	0.010	-0.098	-0.039	-0.034	-0.007		200	3.005	1.495	0.988	0.398	0.157	0.135	-0.030	-0.004	-0.019	-0.004
	500	1.544	0.723	0.315	0.123	0.007	0.061	0.021	-0.003	-0.015	-0.009		500	3.052	1.650	0.853	0.422	0.169	0.147	0.066	0.020	-0.004	-0.005
200	100	1.604	0.335	0.024	0.022	-0.061	0.142	-0.169	-0.225	-0.065	0.000	200	100	3.163	1.461	0.752	0.480	0.220	0.306	-0.073	-0.176	-0.046	0.000
	200	1.552	0.528	0.408	0.042	-0.045	0.024	-0.086	-0.031	-0.026	0.000		200	3.103	1.570	1.037	0.425	0.180	0.149	-0.019	0.002	-0.014	0.000
	500	1.684	0.816	0.373	0.160	0.032	0.077	0.035	0.009	-0.006	0.000		500	3.217	1.765	0.923	0.464	0.196	0.162	0.077	0.028	0.001	0.000
500	100	1.633	0.378	0.036	0.033	-0.050	0.146	-0.167	-0.224	-0.063	0.000	500	100	3.211	1.498	0.775	0.491	0.234	0.313	-0.071	-0.173	-0.044	0.000
	200	1.621	0.574	0.433	0.054	-0.041	0.029	-0.084	-0.029	-0.025	0.000		200	3.183	1.620	1.068	0.440	0.182	0.155	-0.016	0.004	-0.013	0.000
	500	1.775	0.869	0.400	0.175	0.042	0.082	0.037	0.010	-0.005	0.000		500	3.322	1.833	0.961	0.485	0.210	0.168	0.080	0.030	0.002	0.000
T	N	RMSE										T	N	RMSE									
100	100	1.579	0.437	0.239	0.168	0.140	0.156	0.188	0.236	0.074	0.014	100	100	3.148	1.464	0.775	0.509	0.282	0.323	0.122	0.191	0.061	0.008
	200	1.482	0.503	0.387	0.097	0.095	0.049	0.104	0.047	0.042	0.019		200	3.027	1.518	1.006	0.422	0.186	0.151	0.056	0.033	0.028	0.011
	500	1.552	0.730	0.322	0.132	0.037	0.068	0.034	0.029	0.025	0.022		500	3.058	1.656	0.858	0.427	0.175	0.150	0.070	0.029	0.014	0.013
200	100	1.670	0.487	0.255	0.181	0.134	0.168	0.181	0.230	0.069	0.000	200	100	3.214	1.531	0.824	0.543	0.283	0.334	0.122	0.188	0.059	0.000
	200	1.580	0.568	0.431	0.108	0.084	0.053	0.092	0.037	0.029	0.000		200	3.123	1.592	1.055	0.449	0.208	0.166	0.054	0.032	0.024	0.000
	500	1.691	0.822	0.379	0.167	0.044	0.080	0.038	0.013	0.007	0.000		500	3.222	1.770	0.928	0.470	0.201	0.165	0.080	0.031	0.008	0.000
500	100	1.701	0.520	0.253	0.180	0.134	0.171	0.180	0.228	0.068	0.000	500	100	3.261	1.568	0.846	0.553	0.298	0.341	0.122	0.186	0.059	0.000
	200	1.647	0.612	0.458	0.114	0.080	0.057	0.090	0.037	0.028	0.000		200	3.203	1.643	1.086	0.464	0.208	0.172	0.053	0.036	0.023	0.000
	500	1.783	0.876	0.407	0.181	0.054	0.085	0.040	0.014	0.007	0.000		500	3.328	1.838	0.966	0.490	0.216	0.171	0.083	0.033	0.008	0.000

Notes: Parameters of the dynamic panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i \sim U(0, 0.95)$ and $\gamma_i = 0$, for $i = 1, 2, \ldots, N$. Gaussian errors are generated as $u_{it} \sim IIDN(0,1)$ in (31). Design 1 assumes $b_i \sim U(0.7, 0.9)$ for the first $N_b\,(\leq N)$ elements of vector $\mathbf{b}_N$, $i = 1, 2, \ldots, N$, in the construction of the correlation matrix of the errors, $\mathbf{R}_N = \mathbf{I}_N + \mathbf{b}_N\mathbf{b}_N' - \mathbf{B}_N^2$ given by (30), where $\mathbf{B}_N = \text{Diag}\,(\mathbf{b}_N)$. $c_p(n,\delta)$ corresponds to the critical value used in the multiple testing procedure shown in (23). The number of replications is set to $R = 2000$.

Table S1b: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a dynamic panel data model with no exogenous regressors

Cross correlations are generated using Design 2 with Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.10$									
		$\delta = 1/2$												$\delta = 1/2$									
T	N	Bias												Bias									
100	100	-0.928	-2.437	-3.044	-3.137	-3.332	-3.247	-3.622	-3.752	-3.669	-3.693	100	100	-0.125	-1.582	-2.137	-2.206	-2.403	-2.321	-2.710	-2.851	-2.777	-2.816
	200	-1.276	-2.535	-2.883	-3.350	-3.548	-3.538	-3.722	-3.728	-3.791	-3.837		200	-0.595	-1.763	-2.057	-2.499	-2.683	-2.670	-2.854	-2.866	-2.932	-2.983
	500	-1.531	-2.737	-3.388	-3.723	-3.923	-3.937	-4.023	-4.094	-4.174	-4.239		500	-0.941	-2.029	-2.601	-2.897	-3.078	-3.084	-3.167	-3.237	-3.316	-3.380
200	100	0.716	-0.078	-0.151	0.112	0.008	0.201	-0.158	-0.307	-0.249	-0.312	200	100	1.211	0.298	0.129	0.339	0.188	0.356	-0.018	-0.183	-0.136	-0.212
	200	0.368	-0.073	0.136	-0.088	-0.145	-0.067	-0.198	-0.212	-0.273	-0.331		200	0.787	0.235	0.362	0.093	0.004	0.063	-0.077	-0.102	-0.168	-0.231
	500	0.219	-0.026	-0.082	-0.154	-0.199	-0.148	-0.210	-0.265	-0.321	-0.365		500	0.546	0.209	0.097	-0.009	-0.073	-0.033	-0.102	-0.161	-0.218	-0.263
500	100	1.044	0.298	0.250	0.566	0.432	0.629	0.260	0.086	0.113	0.000	500	100	1.464	0.581	0.431	0.676	0.501	0.669	0.284	0.100	0.119	0.000
	200	0.674	0.310	0.547	0.325	0.254	0.329	0.203	0.162	0.085	-0.001		200	1.022	0.524	0.672	0.399	0.298	0.355	0.218	0.171	0.089	0.000
	500	0.529	0.355	0.331	0.249	0.205	0.247	0.176	0.117	0.056	-0.001		500	0.797	0.504	0.412	0.294	0.230	0.260	0.184	0.121	0.058	0.000
T	N	RMSE										T	N	RMSE									
100	100	1.302	2.747	3.430	3.585	3.792	3.754	4.092	4.217	4.140	4.162	100	100	0.850	1.930	2.518	2.637	2.829	2.789	3.130	3.258	3.190	3.230
	200	1.494	2.797	3.252	3.742	3.962	3.974	4.152	4.164	4.221	4.265		200	0.928	2.042	2.423	2.862	3.058	3.064	3.236	3.253	3.313	3.363
	500	1.704	3.009	3.738	4.119	4.340	4.378	4.467	4.532	4.613	4.675		500	1.160	2.306	2.936	3.264	3.456	3.484	3.567	3.630	3.710	3.772
200	100	0.810	0.360	0.357	0.337	0.308	0.355	0.327	0.409	0.365	0.405	200	100	1.277	0.449	0.305	0.427	0.310	0.425	0.222	0.277	0.244	0.286
	200	0.459	0.300	0.313	0.276	0.295	0.260	0.319	0.325	0.372	0.419		200	0.831	0.341	0.427	0.226	0.198	0.203	0.207	0.212	0.254	0.301
	500	0.301	0.237	0.258	0.288	0.310	0.280	0.317	0.357	0.404	0.446		500	0.574	0.280	0.214	0.188	0.196	0.184	0.208	0.243	0.287	0.329
500	100	1.069	0.339	0.274	0.575	0.440	0.635	0.275	0.120	0.134	0.002	500	100	1.500	0.623	0.457	0.686	0.509	0.675	0.298	0.130	0.139	0.001
	200	0.687	0.322	0.550	0.328	0.257	0.332	0.208	0.167	0.092	0.001		200	1.039	0.538	0.676	0.403	0.301	0.357	0.222	0.175	0.095	0.001
	500	0.534	0.357	0.332	0.250	0.206	0.247	0.177	0.118	0.057	0.002		500	0.803	0.507	0.414	0.295	0.230	0.261	0.184	0.122	0.059	0.001
		$\delta = 1/3$												$\delta = 1/3$									
T	N	Bias										T	N	Bias									
100	100	1.115	-0.445	-1.089	-1.228	-1.493	-1.453	-1.871	-2.042	-1.989	-2.049	100	100	2.942	1.087	0.161	-0.183	-0.608	-0.668	-1.153	-1.376	-1.363	-1.451
	200	0.913	-0.396	-0.812	-1.348	-1.594	-1.626	-1.832	-1.870	-1.949	-2.012		200	2.747	1.058	0.325	-0.425	-0.813	-0.936	-1.194	-1.272	-1.371	-1.451
	500	0.891	-0.383	-1.088	-1.487	-1.725	-1.771	-1.877	-1.958	-2.042	-2.106		500	2.692	0.986	-0.041	-0.655	-1.021	-1.144	-1.291	-1.395	-1.490	-1.559
200	100	2.123	0.960	0.587	0.663	0.425	0.534	0.124	-0.068	-0.040	-0.134	200	100	3.671	2.077	1.349	1.164	0.762	0.760	0.286	0.045	0.038	-0.083
	200	1.892	0.989	0.855	0.434	0.248	0.248	0.076	0.025	-0.057	-0.132		200	3.425	2.029	1.508	0.860	0.520	0.426	0.200	0.112	0.008	-0.083
	500	1.870	1.040	0.625	0.340	0.177	0.159	0.060	-0.015	-0.081	-0.133		500	3.396	2.001	1.203	0.685	0.387	0.292	0.151	0.052	-0.026	-0.086
500	100	2.305	1.161	0.801	0.900	0.641	0.752	0.333	0.126	0.129	0.000	500	100	3.780	2.216	1.487	1.324	0.908	0.908	0.425	0.174	0.148	0.000
	200	2.065	1.182	1.058	0.633	0.434	0.431	0.262	0.192	0.098	0.000		200	3.569	2.180	1.657	1.001	0.650	0.553	0.329	0.225	0.110	0.000
	500	2.062	1.231	0.815	0.514	0.349	0.323	0.216	0.137	0.064	0.000		500	3.564	2.157	1.349	0.813	0.512	0.408	0.259	0.157	0.071	0.000
T	N	RMSE												RMSE									
100	100	1.355	1.026	1.527	1.678	1.902	1.893	2.238	2.390	2.344	2.402	100	100	3.027	1.330	0.855	0.895	1.086	1.142	1.487	1.674	1.666	1.745
	200	1.090	0.883	1.253	1.696	1.924	1.967	2.151	2.189	2.262	2.323		200	2.794	1.222	0.791	0.889	1.158	1.266	1.476	1.548	1.636	1.711
	500	1.017	0.848	1.439	1.816	2.041	2.099	2.196	2.269	2.351	2.414		500	2.718	1.134	0.712	1.035	1.329	1.447	1.573	1.663	1.754	1.821
200	100	2.173	1.024	0.645	0.700	0.470	0.566	0.214	0.173	0.155	0.189	200	100	3.711	2.118	1.382	1.187	0.785	0.777	0.320	0.135	0.122	0.122
	200	1.915	1.016	0.875	0.460	0.285	0.281	0.151	0.126	0.137	0.181		200	3.442	2.045	1.519	0.872	0.533	0.439	0.224	0.144	0.090	0.119
	500	1.878	1.048	0.637	0.358	0.206	0.190	0.119	0.105	0.135	0.175		500	3.401	2.006	1.208	0.690	0.395	0.301	0.168	0.090	0.080	0.118
500	100	2.342	1.199	0.826	0.911	0.651	0.758	0.346	0.152	0.149	0.001	500	100	3.815	2.250	1.513	1.339	0.919	0.915	0.437	0.196	0.166	0.000
	200	2.082	1.196	1.064	0.638	0.438	0.434	0.265	0.197	0.103	0.000		200	3.584	2.193	1.664	1.007	0.655	0.556	0.333	0.230	0.116	0.000
	500	2.068	1.234	0.817	0.515	0.350	0.323	0.217	0.138	0.065	0.000		500	3.568	2.160	1.351	0.815	0.513	0.408	0.260	0.158	0.072	0.000

Notes: Parameters of the dynamic panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i \sim U(0, 0.95)$ and $\gamma_i = 0$, for $i = 1, 2, \ldots, N$. Design 2 assumes a two-factor model with $[N_{\alpha_\beta}]$ and $[N_{\alpha_{\beta 2}}]$ non-zero loadings for the first and second factor, respectively. We set: $\alpha_{\beta 2} = 2\alpha_\beta/3$, where α_β relates to α under (11), f_{jt} and $u_{it} \sim IIDN(0,1)$, $v_{ij} \sim IIDU(\mu_{v_j} - 0.2, \mu_{v_j} + 0.2)$, $j = 1, 2$, $\mu_v = 0.87$, $\mu_{v_2} = 0.71$, $\mu_{v_1} = \sqrt{\mu_v^2 - N^{2(\alpha_{\beta 2} - \alpha_\beta)}\mu_{v_2}^2}$), in (32) and (33). The number of replications is set to $R = 2000$.

Table S1c: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a dynamic panel data model with no exogenous regressors

Cross correlations are generated using Design 1 with non-Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.10$									
		$\delta = 1/2$												$\delta = 1/2$									
T	N	Bias										T	N	Bias									
100	100	2.253	0.812	0.371	0.201	0.044	0.188	-0.164	-0.250	-0.103	-0.050	100	100	3.623	1.816	1.052	0.642	0.327	0.364	-0.053	-0.184	-0.067	-0.032
	200	2.107	0.944	0.664	0.162	-0.008	0.030	-0.108	-0.080	-0.085	-0.066		200	3.396	1.840	1.242	0.523	0.214	0.166	-0.021	-0.026	-0.052	-0.044
	500	2.050	1.043	0.488	0.204	0.012	0.034	-0.040	-0.072	-0.096	-0.092		500	3.226	1.802	0.952	0.483	0.183	0.140	0.026	-0.027	-0.061	-0.064
200	100	2.761	1.164	0.575	0.385	0.144	0.269	-0.096	-0.183	-0.049	0.000	200	100	4.226	2.262	1.302	0.854	0.432	0.441	0.005	-0.130	-0.028	0.000
	200	2.628	1.268	0.853	0.337	0.136	0.109	-0.028	-0.007	-0.016	0.000		200	4.016	2.242	1.454	0.718	0.363	0.235	0.042	0.027	-0.003	0.000
	500	2.659	1.468	0.747	0.396	0.170	0.159	0.071	0.028	0.000	0.000		500	3.961	2.317	1.259	0.692	0.334	0.248	0.114	0.048	0.007	0.000
500	100	3.035	1.321	0.722	0.444	0.218	0.294	-0.071	-0.174	-0.043	0.000	500	100	4.588	2.462	1.491	0.942	0.531	0.475	0.038	-0.117	-0.021	0.000
	200	2.930	1.529	1.049	0.411	0.180	0.146	-0.019	-0.003	-0.013	0.000		200	4.403	2.567	1.713	0.824	0.427	0.286	0.057	0.032	0.001	0.000
	500	3.062	1.717	0.888	0.492	0.215	0.168	0.076	0.028	0.001	0.000		500	4.461	2.648	1.446	0.820	0.396	0.261	0.123	0.050	0.008	0.000
T	N	RMSE										T	N	RMSE									
100	100	2.888	1.621	1.236	0.625	0.561	0.382	0.274	0.283	0.140	0.100	100	100	4.183	2.465	1.734	0.993	0.735	0.551	0.272	0.235	0.107	0.069
	200	2.851	1.818	1.221	0.717	0.401	0.401	0.283	0.147	0.132	0.117		200	4.041	2.563	1.736	0.981	0.541	0.472	0.292	0.125	0.093	0.082
	500	2.841	1.783	1.050	0.679	0.331	0.282	0.192	0.166	0.164	0.163		500	3.922	2.464	1.469	0.917	0.449	0.351	0.189	0.131	0.118	0.120
200	100	3.399	1.838	1.156	0.836	0.375	0.398	0.206	0.226	0.067	0.001	200	100	4.762	2.789	1.759	1.229	0.623	0.574	0.231	0.207	0.068	0.001
	200	3.173	1.903	1.249	0.742	0.573	0.215	0.245	0.069	0.044	0.001		200	4.487	2.766	1.810	1.066	0.733	0.335	0.279	0.094	0.051	0.000
	500	3.212	2.004	1.101	0.750	0.463	0.339	0.177	0.105	0.028	0.003		500	4.453	2.797	1.594	1.028	0.609	0.428	0.217	0.127	0.035	0.002
500	100	3.411	1.715	1.186	0.728	0.427	0.380	0.218	0.210	0.087	0.000	500	100	4.904	2.765	1.847	1.171	0.702	0.565	0.252	0.187	0.092	0.000
	200	3.312	2.098	1.432	0.749	0.423	0.264	0.108	0.056	0.033	0.000		200	4.717	3.015	2.048	1.102	0.629	0.394	0.148	0.079	0.040	0.000
	500	3.468	2.134	1.182	0.818	0.454	0.249	0.115	0.052	0.015	0.000		500	4.807	3.004	1.711	1.117	0.616	0.346	0.165	0.075	0.022	0.000
		$\delta = 1/3$												$\delta = 1/3$									
T	N	Bias										T	N	Bias									
100	100	5.722	3.405	2.160	1.368	0.793	0.652	0.120	-0.086	-0.023	-0.019	100	100	8.467	5.599	3.744	2.451	1.502	1.085	0.384	0.059	0.037	-0.011
	200	6.099	3.823	2.554	1.360	0.726	0.473	0.160	0.072	-0.003	-0.023		200	8.831	5.954	4.051	2.362	1.354	0.849	0.378	0.182	0.044	-0.014
	500	6.683	4.203	2.494	1.419	0.738	0.455	0.200	0.070	-0.003	-0.030		500	9.373	6.256	3.921	2.329	1.289	0.763	0.363	0.151	0.033	-0.019
200	100	6.361	3.929	2.460	1.621	0.916	0.731	0.177	-0.038	0.008	0.000	200	100	9.129	6.167	4.082	2.744	1.642	1.179	0.444	0.106	0.064	0.000
	200	6.818	4.315	2.816	1.609	0.906	0.542	0.213	0.111	0.028	0.000		200	9.564	6.471	4.342	2.652	1.565	0.926	0.431	0.219	0.068	0.000
	500	7.569	4.881	2.922	1.698	0.911	0.561	0.272	0.121	0.033	0.000		500	10.253	6.974	4.408	2.656	1.488	0.881	0.438	0.198	0.061	0.000
500	100	6.819	4.181	2.700	1.743	1.045	0.780	0.223	-0.020	0.017	0.000	500	100	9.611	6.456	4.367	2.898	1.812	1.239	0.507	0.129	0.076	0.000
	200	7.295	4.739	3.165	1.761	1.007	0.616	0.243	0.119	0.034	0.000		200	10.076	6.945	4.752	2.843	1.701	1.021	0.473	0.229	0.076	0.000
	500	8.174	5.343	3.207	1.906	1.021	0.593	0.291	0.127	0.035	0.000		500	10.861	7.477	4.736	2.913	1.630	0.929	0.466	0.208	0.064	0.000
T	N	RMSE										T	N	RMSE									
100	100	6.180	3.909	2.689	1.670	1.119	0.835	0.358	0.202	0.093	0.045	100	100	8.816	5.980	4.152	2.713	1.766	1.264	0.571	0.247	0.118	0.030
	200	6.581	4.355	2.962	1.722	0.988	0.713	0.395	0.169	0.069	0.048		200	9.190	6.355	4.388	2.661	1.583	1.056	0.575	0.266	0.091	0.031
	500	7.160	4.694	2.915	1.783	0.971	0.647	0.314	0.153	0.069	0.065		500	9.726	6.639	4.271	2.646	1.508	0.950	0.469	0.222	0.076	0.046
200	100	6.786	4.339	2.819	1.929	1.081	0.866	0.347	0.206	0.085	0.001	200	100	9.439	6.473	4.362	2.994	1.789	1.310	0.580	0.273	0.128	0.000
	200	7.170	4.700	3.107	1.889	1.191	0.640	0.399	0.170	0.074	0.000		200	9.824	6.760	4.580	2.885	1.800	1.023	0.584	0.275	0.110	0.000
	500	7.908	5.236	3.200	1.981	1.145	0.732	0.368	0.198	0.060	0.001		500	10.504	7.252	4.640	2.901	1.696	1.043	0.529	0.271	0.088	0.001
500	100	7.069	4.413	2.974	1.931	1.194	0.873	0.380	0.185	0.112	0.000	500	100	9.797	6.629	4.575	3.057	1.941	1.330	0.631	0.263	0.155	0.000
	200	7.522	5.049	3.427	1.974	1.171	0.713	0.312	0.158	0.066	0.000		200	10.241	7.171	4.960	3.014	1.841	1.112	0.535	0.266	0.106	0.000
	500	8.401	5.589	3.413	2.142	1.200	0.681	0.337	0.154	0.049	0.000		500	11.026	7.665	4.905	3.110	1.787	1.015	0.513	0.236	0.078	0.000

Notes: Parameters of the dynamic panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i \sim U(0, 0.95)$ and $\gamma_i = 0$, for $i = 1, 2, \ldots, N$. Non-Gaussian errors are generated as $u_{it} = \left(\frac{\mathrm{v}-2}{\chi^2_{\mathrm{v},t}}\right)^{1/2} \tilde{\nu}_{it}$, for $i = 1, 2, \ldots, N$, $\tilde{\nu}_{it} \sim IIDN(0,1)$ and $\chi^2_{\mathrm{v},t}$ is a chi-squared random variate with v= 8 degrees of freedom, in (31). Design 1 assumes $b_i \sim U(0.7, 0.9)$ for the first $N_b\,(\leq N)$ elements of vector $\mathbf{b}_N$, $i = 1, 2, \ldots, N$, in the construction of the correlation matrix of the errors, $\mathbf{R}_N = \mathbf{I}_N + \mathbf{b}_N\mathbf{b}'_N - \mathbf{B}^2_N$ given by (30), where $\mathbf{B}_N = \mathrm{Diag}\,(\mathbf{b}_N)$. $c_p(n,\delta)$ corresponds to the critical value used in the multiple testing procedure shown in (23). The number of replications is set to $R = 2000$.

Table S1d: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a dynamic panel data model with no exogenous regressors

Cross correlations are generated using Design 2 with non-Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.10$									
		$\delta = 1/2$												$\delta = 1/2$									
T	N	Bias												Bias									
100	100	1.186	-0.898	-2.047	-2.570	-3.052	-3.125	-3.605	-3.814	-3.780	-3.820	100	100	2.840	0.564	-0.764	-1.430	-2.018	-2.163	-2.693	-2.935	-2.926	-2.989
	200	0.953	-1.004	-1.983	-2.865	-3.320	-3.493	-3.754	-3.805	-3.909	-3.970		200	2.530	0.368	-0.797	-1.811	-2.361	-2.595	-2.894	-2.973	-3.087	-3.160
	500	0.712	-1.198	-2.406	-3.140	-3.569	-3.722	-3.886	-4.008	-4.101	-4.159		500	2.201	0.073	-1.310	-2.173	-2.686	-2.889	-3.081	-3.215	-3.314	-3.375
200	100	2.816	1.279	0.648	0.531	0.198	0.241	-0.187	-0.370	-0.345	-0.424	200	100	4.235	2.349	1.391	1.043	0.563	0.508	0.021	-0.205	-0.205	-0.304
	200	2.495	1.215	0.809	0.255	-0.007	-0.049	-0.242	-0.291	-0.368	-0.434		200	3.846	2.171	1.441	0.686	0.290	0.168	-0.070	-0.149	-0.241	-0.317
	500	2.592	1.330	0.625	0.174	-0.084	-0.147	-0.266	-0.350	-0.419	-0.469		500	3.906	2.212	1.194	0.544	0.168	0.036	-0.118	-0.220	-0.299	-0.352
500	100	3.405	1.873	1.221	1.125	0.752	0.798	0.355	0.124	0.127	-0.002	500	100	4.805	2.872	1.872	1.520	0.993	0.933	0.432	0.163	0.142	-0.001
	200	3.125	1.824	1.398	0.813	0.519	0.463	0.266	0.186	0.094	-0.002		200	4.487	2.736	1.948	1.143	0.708	0.565	0.320	0.212	0.104	-0.001
	500	3.153	1.876	1.159	0.686	0.425	0.350	0.222	0.134	0.059	-0.002		500	4.471	2.713	1.649	0.958	0.569	0.422	0.256	0.149	0.065	-0.001
T	N	RMSE										T	N	RMSE									
100	100	2.409	2.073	2.718	3.131	3.556	3.628	4.059	4.266	4.243	4.287	100	100	3.644	2.010	1.860	2.119	2.539	2.647	3.107	3.338	3.341	3.408
	200	2.287	2.057	2.599	3.322	3.753	3.940	4.197	4.254	4.358	4.418		200	3.404	1.901	1.771	2.336	2.786	3.011	3.296	3.377	3.492	3.564
	500	2.169	2.100	2.899	3.554	3.972	4.144	4.319	4.442	4.538	4.595		500	3.160	1.817	2.021	2.622	3.071	3.278	3.476	3.611	3.712	3.773
200	100	3.281	1.792	1.110	0.820	0.509	0.451	0.401	0.507	0.487	0.548	200	100	4.637	2.751	1.728	1.249	0.737	0.609	0.289	0.339	0.337	0.406
	200	2.867	1.563	1.026	0.508	0.349	0.321	0.404	0.434	0.493	0.549		200	4.179	2.465	1.623	0.842	0.441	0.309	0.262	0.291	0.352	0.413
	500	3.184	1.857	1.057	0.575	0.378	0.334	0.392	0.459	0.520	0.566		500	4.412	2.662	1.548	0.824	0.415	0.263	0.257	0.320	0.385	0.434
500	100	3.764	2.240	1.498	1.264	0.840	0.829	0.385	0.159	0.146	0.009	500	100	5.119	3.185	2.118	1.660	1.084	0.970	0.465	0.196	0.161	0.008
	200	3.402	2.053	1.517	0.892	0.561	0.477	0.275	0.192	0.100	0.004		200	4.736	2.948	2.075	1.231	0.757	0.584	0.330	0.217	0.110	0.003
	500	3.518	2.167	1.357	0.815	0.500	0.378	0.233	0.138	0.061	0.005		500	4.792	2.983	1.844	1.089	0.648	0.454	0.270	0.154	0.067	0.004
		$\delta = 1/3$												$\delta = 1/3$									
T	N	Bias										T	N	Bias									
100	100	5.161	2.533	0.852	-0.095	-0.901	-1.189	-1.810	-2.115	-2.149	-2.246	100	100	8.034	4.962	2.799	1.420	0.299	-0.228	-0.993	-1.402	-1.505	-1.651
	200	5.538	2.858	1.192	-0.184	-1.004	-1.417	-1.820	-1.972	-2.122	-2.218		200	8.408	5.235	3.039	1.248	0.099	-0.543	-1.085	-1.330	-1.531	-1.658
	500	6.093	3.222	1.169	-0.209	-1.067	-1.483	-1.794	-1.989	-2.117	-2.193		500	8.928	5.545	2.958	1.121	-0.073	-0.712	-1.151	-1.415	-1.578	-1.672
200	100	6.311	3.936	2.505	1.789	1.068	0.846	0.263	-0.033	-0.078	-0.206	200	100	8.966	6.036	4.030	2.820	1.755	1.287	0.550	0.151	0.037	-0.136
	200	6.569	4.157	2.760	1.563	0.856	0.535	0.187	0.036	-0.095	-0.195		200	9.248	6.224	4.195	2.534	1.474	0.914	0.424	0.182	0.000	-0.131
	500	7.481	4.754	2.859	1.586	0.812	0.442	0.159	-0.012	-0.124	-0.195		500	10.137	6.805	4.295	2.513	1.372	0.766	0.350	0.107	-0.043	-0.135
500	100	6.848	4.402	2.908	2.167	1.397	1.164	0.563	0.229	0.167	-0.001	500	100	9.485	6.474	4.392	3.140	2.019	1.528	0.775	0.337	0.207	0.000
	200	7.225	4.701	3.206	1.933	1.170	0.818	0.453	0.274	0.126	0.000		200	9.885	6.757	4.617	2.867	1.741	1.140	0.625	0.355	0.156	0.000
	500	8.051	5.221	3.235	1.890	1.079	0.681	0.381	0.204	0.084	-0.001		500	10.685	7.261	4.654	2.787	1.596	0.953	0.515	0.262	0.103	0.000
T	N	RMSE										T	N	RMSE									
100	100	5.711	3.248	1.868	1.395	1.569	1.708	2.192	2.470	2.513	2.614	100	100	8.417	5.394	3.265	1.932	1.145	1.011	1.387	1.721	1.824	1.967
	200	6.071	3.497	1.959	1.290	1.534	1.825	2.174	2.317	2.464	2.558		200	8.785	5.643	3.441	1.759	1.010	1.066	1.431	1.640	1.828	1.950
	500	6.602	3.810	1.944	1.247	1.526	1.840	2.125	2.313	2.441	2.517		500	9.285	5.935	3.376	1.651	0.956	1.127	1.463	1.702	1.861	1.955
200	100	6.638	4.256	2.771	1.960	1.195	0.911	0.363	0.211	0.214	0.286	200	100	9.217	6.285	4.244	2.967	1.863	1.342	0.605	0.231	0.155	0.197
	200	6.831	4.394	2.923	1.685	0.941	0.585	0.264	0.173	0.198	0.266		200	9.449	6.415	4.341	2.647	1.551	0.954	0.459	0.223	0.127	0.186
	500	7.822	5.079	3.131	1.793	0.953	0.521	0.229	0.139	0.193	0.252		500	10.389	7.060	4.522	2.695	1.499	0.836	0.390	0.150	0.115	0.179
500	100	7.100	4.652	3.115	2.299	1.489	1.206	0.598	0.260	0.185	0.007	500	100	9.675	6.665	4.560	3.259	2.107	1.574	0.812	0.366	0.224	0.006
	200	7.419	4.878	3.331	2.026	1.227	0.844	0.467	0.281	0.132	0.001		200	10.032	6.900	4.730	2.956	1.800	1.171	0.642	0.363	0.161	0.001
	500	8.267	5.427	3.405	2.015	1.160	0.721	0.400	0.211	0.086	0.002		500	10.845	7.423	4.798	2.902	1.675	0.996	0.537	0.271	0.107	0.002

Notes: Parameters of the dynamic panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i \sim U(0, 0.95)$ and $\gamma_i = 0$, for $i = 1, 2, \ldots, N$. Design 2 assumes a two-factor model with $[N_{\alpha_\beta}]$ and $[N_{\alpha_{\beta 2}}]$ non-zero loadings for the first and second factor, respectively. We set: $\alpha_{\beta 2} = 2\alpha_\beta/3$, where α_β relates to α under (11) and $f_{jt} \sim IIDN(0,1)$. Non-Gaussian errors are generated as: $u_{it} = \left(\frac{\mathrm{v}-2}{\chi^2_{\mathrm{v},t}}\right)^{1/2} \tilde{\nu}_{it}$, for $i = 1, 2, \ldots, N$, $\tilde{\nu}_{it} \sim IIDN(0,1)$ and $\chi^2_{\mathrm{v},t}$ is a chi-squared random variate with v= 8 degrees of freedom, in (31). $v_{ij} \sim IIDU(\mu_{v_j} - 0.2, \mu_{v_j} + 0.2)$, $j = 1, 2$, $\mu_v = 0.87$, $\mu_{v_2} = 0.71$, $\mu_{v_1} = \sqrt{\mu_v^2 - N^{2(\alpha_{\beta 2} - \alpha_\beta)}\mu_{v_2}^2}$), in (32) and (33). The number of replications is set to $R = 2000$.

Table S2a: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a static panel data model

Cross correlations are generated using Design 1 with Gaussian and non-Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$, $\delta = 1/4$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$, $\delta = 1/4$ and $p = 0.10$									
		Gaussian errors												Gaussian errors									
T	N	Bias												Bias									
100	100	3.117	1.451	0.736	0.466	0.220	0.294	-0.075	-0.179	-0.049	-0.002	100	100	5.785	3.470	2.112	1.356	0.779	0.627	0.124	-0.076	-0.010	-0.001
	200	3.367	1.750	1.145	0.496	0.219	0.171	-0.009	0.006	-0.013	-0.003		200	6.148	3.757	2.441	1.321	0.716	0.453	0.145	0.082	0.016	-0.001
	500	3.913	2.214	1.191	0.613	0.276	0.202	0.095	0.035	0.001	-0.004		500	6.774	4.221	2.470	1.365	0.698	0.425	0.208	0.088	0.021	-0.002
200	100	3.205	1.495	0.774	0.505	0.227	0.312	-0.072	-0.175	-0.045	0.000	200	100	5.846	3.486	2.139	1.398	0.788	0.644	0.123	-0.075	-0.006	0.000
	200	3.491	1.851	1.203	0.528	0.239	0.183	-0.001	0.010	-0.010	0.000		200	6.231	3.845	2.499	1.352	0.734	0.465	0.153	0.085	0.018	0.000
	500	4.100	2.339	1.264	0.661	0.304	0.218	0.104	0.041	0.005	0.000		500	6.942	4.338	2.539	1.416	0.728	0.443	0.217	0.093	0.024	0.000
500	100	3.238	1.551	0.802	0.509	0.250	0.315	-0.068	-0.174	-0.044	0.000	500	100	5.863	3.546	2.162	1.391	0.806	0.644	0.125	-0.075	-0.006	0.000
	200	3.566	1.893	1.238	0.548	0.247	0.189	0.003	0.012	-0.009	0.000		200	6.299	3.884	2.533	1.368	0.741	0.471	0.157	0.086	0.018	0.000
	500	4.208	2.406	1.307	0.681	0.315	0.225	0.108	0.043	0.006	0.000		500	7.028	4.406	2.587	1.439	0.742	0.450	0.222	0.095	0.024	0.000
T	N	RMSE												RMSE									
100	100	3.169	1.525	0.812	0.527	0.287	0.322	0.121	0.190	0.061	0.008	100	100	5.822	3.515	2.160	1.398	0.823	0.657	0.190	0.125	0.056	0.004
	200	3.386	1.771	1.163	0.518	0.243	0.187	0.051	0.035	0.025	0.008		200	6.160	3.772	2.456	1.338	0.733	0.466	0.164	0.097	0.036	0.004
	500	3.918	2.220	1.196	0.619	0.281	0.205	0.098	0.039	0.012	0.011		500	6.778	4.224	2.473	1.369	0.703	0.429	0.211	0.091	0.026	0.006
200	100	3.258	1.564	0.848	0.568	0.293	0.340	0.121	0.187	0.058	0.000	200	100	5.882	3.530	2.185	1.442	0.831	0.675	0.191	0.124	0.056	0.000
	200	3.510	1.875	1.221	0.550	0.262	0.198	0.053	0.036	0.021	0.000		200	6.244	3.861	2.514	1.368	0.751	0.478	0.173	0.100	0.035	0.000
	500	4.105	2.344	1.269	0.665	0.308	0.222	0.107	0.044	0.010	0.000		500	6.945	4.341	2.542	1.420	0.732	0.446	0.220	0.096	0.027	0.000
500	100	3.291	1.622	0.876	0.575	0.314	0.344	0.120	0.186	0.059	0.000	500	100	5.902	3.594	2.210	1.438	0.851	0.675	0.193	0.125	0.059	0.000
	200	3.585	1.916	1.255	0.570	0.270	0.206	0.055	0.039	0.022	0.000		200	6.312	3.899	2.547	1.385	0.757	0.486	0.177	0.103	0.036	0.000
	500	4.212	2.411	1.312	0.685	0.320	0.228	0.112	0.046	0.011	0.000		500	7.031	4.410	2.591	1.443	0.746	0.454	0.226	0.098	0.028	0.000
		Non-Gaussian errors												Non-Gaussian errors									
T	N	Bias										T	N	Bias									
100	100	8.568	5.689	3.724	2.532	1.512	1.081	0.402	0.062	0.036	-0.010	100	100	12.070	8.641	5.993	4.153	2.608	1.779	0.831	0.291	0.127	-0.006
	200	9.530	6.389	4.333	2.622	1.569	0.929	0.408	0.203	0.054	-0.012		200	12.998	9.290	6.524	4.199	2.621	1.572	0.777	0.390	0.127	-0.007
	500	10.724	7.428	4.649	2.841	1.607	0.920	0.459	0.198	0.050	-0.014		500	14.084	10.250	6.836	4.378	2.595	1.492	0.770	0.348	0.107	-0.008
200	100	9.130	6.188	4.168	2.735	1.680	1.223	0.470	0.098	0.061	0.000	200	100	12.583	9.125	6.437	4.381	2.801	1.935	0.909	0.331	0.152	0.000
	200	10.120	7.023	4.759	2.884	1.731	1.024	0.505	0.252	0.077	0.000		200	13.532	9.905	6.968	4.496	2.807	1.679	0.890	0.444	0.150	0.000
	500	11.439	7.948	5.220	3.202	1.829	1.017	0.528	0.239	0.077	0.000		500	14.702	10.742	7.413	4.766	2.848	1.606	0.847	0.390	0.132	0.000
500	100	9.603	6.544	4.379	2.909	1.809	1.277	0.500	0.132	0.071	0.000	500	100	13.054	9.478	6.676	4.564	2.959	2.007	0.948	0.375	0.166	0.000
	200	10.739	7.425	5.053	3.082	1.833	1.144	0.523	0.269	0.086	0.000		200	14.108	10.304	7.279	4.715	2.934	1.827	0.914	0.468	0.161	0.000
	500	12.206	8.470	5.529	3.365	1.947	1.106	0.570	0.255	0.085	0.000		500	15.404	11.243	7.734	4.956	2.993	1.719	0.902	0.412	0.142	0.000
T	N	RMSE												RMSE									
100	100	8.905	6.084	4.058	2.823	1.740	1.224	0.595	0.266	0.123	0.029	100	100	12.305	8.914	6.238	4.387	2.799	1.917	0.991	0.436	0.197	0.018
	200	9.889	6.726	4.647	2.937	1.875	1.066	0.537	0.273	0.098	0.027		200	13.240	9.528	6.764	4.449	2.867	1.701	0.891	0.457	0.166	0.018
	500	11.067	7.813	4.955	3.152	1.849	1.089	0.548	0.268	0.082	0.029		500	14.313	10.519	7.066	4.624	2.805	1.649	0.860	0.414	0.134	0.018
200	100	9.390	6.467	4.466	2.949	1.871	1.381	0.614	0.257	0.129	0.001	200	100	12.760	9.319	6.658	4.552	2.961	2.076	1.033	0.447	0.210	0.001
	200	10.361	7.303	5.017	3.088	1.930	1.132	0.644	0.313	0.111	0.001		200	13.694	10.100	7.162	4.658	2.970	1.778	1.010	0.503	0.183	0.000
	500	11.662	8.179	5.474	3.460	2.025	1.121	0.610	0.309	0.103	0.001		500	14.852	10.903	7.602	4.968	3.015	1.705	0.926	0.454	0.158	0.000
500	100	9.778	6.756	4.547	3.059	1.937	1.396	0.600	0.246	0.127	0.000	500	100	13.174	9.623	6.800	4.685	3.068	2.114	1.035	0.458	0.215	0.000
	200	10.931	7.603	5.212	3.213	1.941	1.245	0.598	0.313	0.113	0.000		200	14.235	10.427	7.399	4.820	3.026	1.917	0.980	0.510	0.188	0.000
	500	12.378	8.643	5.706	3.498	2.070	1.180	0.625	0.286	0.114	0.000		500	15.519	11.362	7.862	5.062	3.096	1.787	0.955	0.443	0.169	0.000

Notes: Parameters of the static panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i = 0$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Gaussian errors are generated as $u_{it} \sim IIDN(0,1)$ in (31). Non-Gaussian errors are generated as $u_{it} = \left(\frac{\mathrm{v}-2}{\chi^2_{\mathrm{v},t}}\right)^{1/2} \tilde{\nu}_{it}$, for $i = 1, 2, \ldots, N$, $\tilde{\nu}_{it} \sim IIDN(0,1)$ and $\chi^2_{\mathrm{v},t}$ is a chi-squared random variate with v= 8 degrees of freedom, in (31). Design 1 assumes $b_i \sim U(0.7, 0.9)$ for the first $N_b\,(\leq N)$ elements of vector $\mathbf{b}_N$, $i = 1, 2, \ldots, N$, in the construction of the correlation matrix of the errors, $\mathbf{R}_N = \mathbf{I}_N + \mathbf{b}_N\mathbf{b}_N' - \mathbf{B}_N^2$ given by (30), where $\mathbf{B}_N = \mathrm{Diag}\,(\mathbf{b}_N)$. $c_p(n,\delta)$ corresponds to the critical value used in the multiple testing procedure shown in (23). The number of replications is set to $R = 2000$.

Table S2b: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a static panel data model

Cross correlations are generated using Design 2 with Gaussian and non-Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$, $\delta = 1/4$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$, $\delta = 1/4$ and $p = 0.10$									
		Gaussian errors												Gaussian errors									
T	N	Bias										T	N	Bias									
100	100	3.000	1.154	0.241	-0.109	-0.553	-0.591	-1.078	-1.309	-1.300	-1.389	100	100	5.848	3.447	1.981	1.212	0.465	0.211	-0.407	-0.735	-0.792	-0.931
	200	3.189	1.408	0.603	-0.200	-0.634	-0.771	-1.028	-1.109	-1.218	-1.305		200	6.118	3.689	2.255	1.020	0.278	-0.067	-0.450	-0.615	-0.773	-0.890
	500	3.662	1.735	0.529	-0.212	-0.655	-0.822	-0.996	-1.117	-1.215	-1.279		500	6.695	4.020	2.156	0.921	0.159	-0.206	-0.493	-0.677	-0.808	-0.890
200	100	3.693	2.102	1.360	1.182	0.768	0.769	0.297	0.049	0.041	-0.079	200	100	6.255	4.032	2.697	2.056	1.346	1.132	0.530	0.191	0.120	-0.046
	200	3.826	2.305	1.683	0.973	0.589	0.471	0.231	0.131	0.020	-0.075		200	6.517	4.250	2.952	1.801	1.105	0.781	0.420	0.241	0.083	-0.044
	500	4.262	2.571	1.549	0.889	0.506	0.361	0.194	0.080	-0.009	-0.073		500	7.065	4.539	2.811	1.654	0.951	0.612	0.337	0.163	0.040	-0.044
500	100	3.829	2.241	1.509	1.335	0.914	0.908	0.432	0.172	0.150	0.000	500	100	6.320	4.105	2.786	2.152	1.435	1.216	0.615	0.269	0.188	0.000
	200	3.954	2.440	1.816	1.100	0.711	0.587	0.347	0.233	0.116	0.000		200	6.589	4.330	3.033	1.880	1.183	0.856	0.495	0.306	0.143	0.000
	500	4.410	2.707	1.679	1.002	0.614	0.461	0.287	0.170	0.076	0.000		500	7.169	4.634	2.902	1.731	1.025	0.679	0.398	0.221	0.094	0.000
T	N	RMSE										T	N	RMSE									
100	100	3.080	1.365	0.835	0.847	1.033	1.059	1.414	1.596	1.590	1.672	100	100	5.886	3.501	2.076	1.363	0.789	0.675	0.786	0.996	1.040	1.158
	200	3.227	1.519	0.899	0.752	0.997	1.100	1.294	1.370	1.463	1.548		200	6.133	3.716	2.305	1.144	0.619	0.572	0.730	0.855	0.978	1.086
	500	3.675	1.789	0.771	0.676	0.948	1.092	1.240	1.348	1.439	1.499		500	6.700	4.031	2.188	1.022	0.514	0.564	0.736	0.881	0.993	1.068
200	100	3.732	2.143	1.393	1.205	0.790	0.785	0.329	0.140	0.120	0.120	200	100	6.285	4.061	2.721	2.075	1.362	1.144	0.548	0.226	0.153	0.074
	200	3.841	2.319	1.693	0.983	0.601	0.481	0.251	0.157	0.086	0.110		200	6.528	4.260	2.961	1.808	1.112	0.787	0.429	0.251	0.103	0.069
	500	4.266	2.575	1.552	0.893	0.511	0.368	0.205	0.105	0.072	0.102		500	7.068	4.541	2.813	1.656	0.954	0.615	0.340	0.170	0.063	0.065
500	100	3.867	2.277	1.535	1.349	0.925	0.916	0.444	0.194	0.168	0.000	500	100	6.350	4.134	2.809	2.168	1.448	1.225	0.627	0.286	0.204	0.000
	200	3.968	2.452	1.824	1.106	0.716	0.590	0.351	0.237	0.121	0.000		200	6.599	4.339	3.041	1.886	1.188	0.860	0.500	0.310	0.148	0.000
	500	4.414	2.710	1.681	1.003	0.615	0.462	0.287	0.171	0.077	0.000		500	7.172	4.636	2.904	1.732	1.026	0.680	0.399	0.222	0.095	0.000
		Non-Gaussian errors												Non-Gaussian errors									
T	N	Bias										T	N	Bias									
100	100	7.973	4.917	2.785	1.443	0.326	-0.192	-0.939	-1.352	-1.454	-1.599	100	100	11.565	7.997	5.259	3.336	1.756	0.878	-0.101	-0.684	-0.901	-1.123
	200	9.015	5.756	3.468	1.595	0.376	-0.308	-0.889	-1.162	-1.366	-1.504		200	12.573	8.804	5.879	3.441	1.737	0.678	-0.143	-0.580	-0.880	-1.073
	500	10.440	6.818	3.962	1.870	0.471	-0.313	-0.837	-1.146	-1.338	-1.448		500	13.866	9.790	6.357	3.666	1.752	0.582	-0.186	-0.638	-0.907	-1.056
200	100	8.979	6.043	4.031	2.821	1.765	1.298	0.565	0.161	0.052	-0.123	200	100	12.354	8.843	6.165	4.314	2.785	1.939	0.970	0.395	0.172	-0.077
	200	10.036	6.865	4.660	2.858	1.683	1.041	0.501	0.224	0.023	-0.121		200	13.395	9.656	6.761	4.363	2.676	1.648	0.860	0.421	0.124	-0.077
	500	11.352	7.797	5.022	3.004	1.676	0.941	0.447	0.162	-0.011	-0.113		500	14.596	10.544	7.146	4.495	2.625	1.491	0.749	0.320	0.069	-0.074
500	100	9.483	6.468	4.386	3.128	2.019	1.528	0.771	0.335	0.205	0.000	500	100	12.825	9.246	6.494	4.590	3.001	2.122	1.124	0.517	0.273	0.000
	200	10.496	7.246	4.970	3.107	1.894	1.226	0.672	0.376	0.162	0.000		200	13.810	10.006	7.045	4.582	2.855	1.790	0.983	0.525	0.216	0.000
	500	11.933	8.278	5.400	3.284	1.893	1.115	0.597	0.299	0.115	0.000		500	15.114	10.992	7.509	4.766	2.826	1.640	0.867	0.420	0.156	0.000
T	N	RMSE										T	N	RMSE									
100	100	8.292	5.272	3.162	1.856	1.039	0.938	1.318	1.652	1.761	1.901	100	100	11.779	8.225	5.481	3.535	1.970	1.143	0.699	0.982	1.173	1.375
	200	9.347	6.109	3.800	1.975	0.995	0.889	1.228	1.463	1.655	1.789		200	12.794	9.037	6.097	3.649	1.949	0.962	0.653	0.882	1.132	1.310
	500	10.786	7.197	4.361	2.315	1.128	0.915	1.184	1.437	1.615	1.723		500	14.095	10.043	6.620	3.929	2.023	0.944	0.676	0.918	1.146	1.288
200	100	9.208	6.276	4.232	2.961	1.870	1.356	0.621	0.232	0.146	0.181	200	100	12.515	9.010	6.317	4.430	2.875	1.993	1.014	0.430	0.205	0.121
	200	10.289	7.114	4.865	3.027	1.800	1.105	0.541	0.255	0.111	0.168		200	13.569	9.836	6.923	4.504	2.781	1.712	0.896	0.439	0.147	0.112
	500	11.595	8.048	5.255	3.196	1.811	1.014	0.485	0.193	0.103	0.161		500	14.760	10.721	7.323	4.653	2.746	1.563	0.787	0.338	0.099	0.111
500	100	9.621	6.606	4.503	3.210	2.080	1.561	0.799	0.360	0.221	0.007	500	100	12.922	9.346	6.586	4.660	3.056	2.155	1.151	0.539	0.287	0.006
	200	10.642	7.390	5.088	3.202	1.961	1.262	0.692	0.385	0.168	0.001		200	13.910	10.110	7.139	4.664	2.916	1.828	1.005	0.536	0.222	0.001
	500	12.082	8.434	5.548	3.410	1.986	1.169	0.625	0.311	0.119	0.001		500	15.213	11.101	7.619	4.867	2.906	1.691	0.896	0.434	0.161	0.001

Notes: Parameters of the static panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i = 0$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Design 2 assumes a two-factor model with $[N_{\alpha_\beta}]$ and $[N_{\alpha_{\beta 2}}]$ non-zero loadings for the first and second factor, respectively. We set: $\alpha_{\beta 2} = 2\alpha_\beta/3$, where α_β relates to α under (11) and $f_{jt} \sim IIDN(0,1)$. Gaussian errors are given by: $u_{it} \sim IIDN(0,1)$ and non Gaussian errors are generated as: $u_{it} = \left(\frac{\mathrm{v}-2}{\chi^2_{\mathrm{v},t}}\right)^{1/2} \tilde{\nu}_{it}$, for $i = 1, 2, \ldots, N$, $\tilde{\nu}_{it} \sim IIDN(0,1)$ and $\chi^2_{\mathrm{v},t}$ is a chi-squared random variate with v= 8 degrees of freedom, in (31). $v_{ij} \sim IIDU(\mu_{v_j} - 0.2, \mu_{v_j} + 0.2)$, $j = 1, 2$, $\mu_v = 0.87$, $\mu_{v_2} = 0.71$, $\mu_{v_1} = \sqrt{\mu_v^2 - N^{2(\alpha_{\beta 2} - \alpha_\beta)} \mu_{v_2}^2}$), in (32) and (33). The number of replications is set to $R = 2000$.

Table S2c: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a dynamic panel data model with exogenous regressors

Cross correlations are generated using Design 1 with Gaussian and non-Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$, $\delta = 1/4$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$, $\delta = 1/4$ and $p = 0.10$									
		Gaussian errors												Gaussian errors									
T	N	Bias										T	N	Bias									
100	100	3.118	1.431	0.731	0.469	0.217	0.295	-0.074	-0.179	-0.048	-0.003	100	100	5.795	3.451	2.105	1.357	0.780	0.631	0.123	-0.075	-0.008	-0.001
	200	3.378	1.759	1.154	0.494	0.218	0.170	-0.010	0.005	-0.015	-0.004		200	6.150	3.770	2.457	1.321	0.717	0.453	0.146	0.082	0.014	-0.002
	500	3.916	2.212	1.190	0.611	0.275	0.201	0.093	0.034	-0.001	-0.005		500	6.783	4.218	2.466	1.363	0.697	0.424	0.208	0.087	0.020	-0.002
200	100	3.169	1.506	0.773	0.502	0.238	0.312	-0.069	-0.176	-0.046	0.000	200	100	5.829	3.494	2.147	1.388	0.795	0.640	0.125	-0.075	-0.008	0.000
	200	3.510	1.847	1.206	0.533	0.240	0.186	0.002	0.012	-0.010	0.000		200	6.257	3.845	2.503	1.364	0.736	0.468	0.157	0.087	0.018	0.000
	500	4.097	2.342	1.265	0.658	0.303	0.219	0.104	0.042	0.005	0.000		500	6.937	4.342	2.540	1.413	0.727	0.444	0.217	0.094	0.024	0.000
500	100	3.242	1.546	0.801	0.516	0.250	0.315	-0.069	-0.172	-0.045	0.000	500	100	5.851	3.528	2.170	1.398	0.806	0.642	0.125	-0.072	-0.007	0.000
	200	3.568	1.889	1.240	0.549	0.252	0.187	0.005	0.012	-0.010	0.000		200	6.308	3.885	2.529	1.375	0.747	0.468	0.159	0.086	0.017	0.000
	500	4.201	2.401	1.309	0.681	0.317	0.224	0.107	0.043	0.006	0.000		500	7.024	4.399	2.589	1.440	0.745	0.449	0.221	0.096	0.025	0.000
T	N	RMSE										T	N	RMSE									
100	100	3.171	1.500	0.809	0.528	0.282	0.321	0.121	0.191	0.062	0.008	100	100	5.833	3.494	2.152	1.399	0.825	0.659	0.188	0.124	0.058	0.004
	200	3.399	1.781	1.171	0.515	0.243	0.186	0.052	0.036	0.027	0.011		200	6.163	3.785	2.471	1.337	0.733	0.467	0.165	0.098	0.033	0.005
	500	3.921	2.217	1.195	0.616	0.280	0.205	0.098	0.040	0.015	0.012		500	6.787	4.221	2.470	1.367	0.701	0.428	0.212	0.091	0.025	0.006
200	100	3.220	1.574	0.845	0.562	0.302	0.340	0.119	0.187	0.059	0.000	200	100	5.865	3.538	2.192	1.429	0.839	0.670	0.190	0.122	0.056	0.000
	200	3.529	1.869	1.224	0.555	0.264	0.201	0.055	0.038	0.022	0.000		200	6.270	3.860	2.517	1.380	0.753	0.482	0.178	0.103	0.035	0.000
	500	4.102	2.347	1.270	0.663	0.308	0.222	0.107	0.045	0.010	0.000		500	6.940	4.345	2.545	1.417	0.731	0.447	0.221	0.097	0.027	0.000
500	100	3.291	1.615	0.877	0.578	0.313	0.343	0.120	0.186	0.058	0.000	500	100	5.884	3.573	2.219	1.444	0.850	0.671	0.194	0.126	0.057	0.000
	200	3.588	1.911	1.257	0.570	0.276	0.202	0.056	0.039	0.023	0.000		200	6.321	3.901	2.543	1.391	0.764	0.481	0.180	0.103	0.035	0.000
	500	4.207	2.406	1.313	0.686	0.322	0.228	0.111	0.046	0.011	0.000		500	7.027	4.403	2.593	1.444	0.749	0.452	0.225	0.099	0.028	0.000
		Non-Gaussian errors												Non-Gaussian errors									
T	N	Bias										T	N	Bias									
100	100	8.447	5.562	3.677	2.417	1.491	1.079	0.381	0.057	0.032	-0.012	100	100	11.973	8.498	5.932	4.013	2.585	1.768	0.805	0.286	0.124	-0.007
	200	9.340	6.220	4.231	2.548	1.460	0.907	0.405	0.192	0.051	-0.015		200	12.809	9.116	6.417	4.116	2.496	1.543	0.778	0.375	0.126	-0.009
	500	10.588	7.135	4.657	2.749	1.587	0.886	0.433	0.191	0.047	-0.017		500	13.950	9.966	6.836	4.270	2.569	1.448	0.736	0.340	0.105	-0.010
200	100	9.172	6.148	4.076	2.673	1.674	1.191	0.468	0.110	0.058	0.000	200	100	12.636	9.066	6.349	4.291	2.795	1.901	0.900	0.343	0.146	0.000
	200	10.146	6.941	4.761	2.886	1.692	1.040	0.493	0.245	0.078	0.000		200	13.554	9.818	6.965	4.486	2.761	1.695	0.874	0.435	0.151	0.000
	500	11.429	7.889	5.192	3.109	1.784	1.002	0.526	0.238	0.078	0.000		500	14.698	10.680	7.381	4.666	2.792	1.586	0.843	0.389	0.133	0.000
500	100	9.551	6.522	4.375	2.858	1.797	1.278	0.505	0.124	0.067	0.000	500	100	13.003	9.468	6.681	4.519	2.939	2.009	0.950	0.362	0.160	0.000
	200	10.626	7.339	5.016	3.082	1.849	1.115	0.529	0.261	0.086	0.000		200	14.002	10.220	7.242	4.716	2.952	1.790	0.921	0.457	0.161	0.000
	500	12.160	8.434	5.549	3.413	1.926	1.135	0.564	0.255	0.080	0.000		500	15.355	11.209	7.757	5.006	2.970	1.751	0.894	0.413	0.137	0.000
T	N	RMSE										T	N	RMSE									
100	100	8.793	5.928	4.009	2.737	1.723	1.271	0.531	0.263	0.126	0.031	100	100	12.213	8.756	6.182	4.264	2.780	1.936	0.932	0.431	0.198	0.019
	200	9.661	6.547	4.530	2.851	1.680	1.063	0.532	0.277	0.107	0.037		200	13.029	9.348	6.646	4.355	2.680	1.687	0.891	0.453	0.171	0.024
	500	10.925	7.431	4.990	3.023	1.881	1.068	0.516	0.255	0.083	0.039		500	14.176	10.176	7.087	4.492	2.811	1.609	0.818	0.402	0.134	0.026
200	100	9.442	6.443	4.330	2.897	1.843	1.328	0.676	0.304	0.125	0.001	200	100	12.822	9.271	6.537	4.469	2.941	2.028	1.063	0.486	0.206	0.001
	200	10.416	7.244	5.047	3.129	1.870	1.177	0.629	0.316	0.128	0.003		200	13.735	10.029	7.173	4.675	2.910	1.817	0.991	0.499	0.196	0.003
	500	11.643	8.128	5.467	3.298	1.970	1.104	0.615	0.292	0.114	0.001		500	14.842	10.846	7.582	4.820	2.952	1.681	0.928	0.442	0.167	0.000
500	100	9.733	6.723	4.559	2.989	1.929	1.377	0.612	0.253	0.122	0.000	500	100	13.125	9.607	6.815	4.624	3.049	2.099	1.042	0.455	0.209	0.000
	200	10.773	7.524	5.191	3.233	1.965	1.212	0.650	0.301	0.116	0.000		200	14.101	10.346	7.371	4.834	3.051	1.875	1.018	0.495	0.189	0.000
	500	12.328	8.570	5.726	3.576	2.044	1.246	0.624	0.284	0.093	0.000		500	15.468	11.304	7.885	5.133	3.070	1.847	0.951	0.442	0.150	0.000

Notes: Parameters of the dynamic panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i \sim U(0, 0.95)$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Gaussian errors are generated as $u_{it} \sim IIDN(0,1)$ in (31). Non-Gaussian errors are generated as $u_{it} = \left(\frac{\mathrm{v}-2}{\chi^2_{\mathrm{v},t}}\right)^{1/2} \tilde{\nu}_{it}$, for $i = 1, 2, \ldots, N$, $\tilde{\nu}_{it} \sim IIDN(0,1)$ and $\chi^2_{\mathrm{v},t}$ is a chi-squared random variate with v= 8 degrees of freedom, in (31). Design 1 assumes $b_i \sim U(0.7, 0.9)$ for the first $N_b\,(\leq N)$ elements of vector $\mathbf{b}_N$, $i = 1, 2, \ldots, N$, in the construction of the correlation matrix of the errors, $\mathbf{R}_N = \mathbf{I}_N + \mathbf{b}_N\mathbf{b}_N' - \mathbf{B}_N^2$ given by (30), where $\mathbf{B}_N = \mathrm{Diag}\,(\mathbf{b}_N)$. $c_p(n,\delta)$ corresponds to the critical value used in the multiple testing procedure shown in (23). The number of replications is set to $R = 2000$.

Table S2d: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a dynamic panel data model with exogenous regressors

Cross correlations are generated using Design 2 with Gaussian and non-Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$, $\delta = 1/4$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$, $\delta = 1/4$ and $p = 0.10$									
		Gaussian errors												Gaussian errors									
T	N	Bias										T	N	Bias									
100	100	2.937	1.072	0.123	-0.244	-0.710	-0.755	-1.231	-1.471	-1.460	-1.555	100	100	5.779	3.382	1.900	1.113	0.353	0.095	-0.515	-0.850	-0.907	-1.050
	200	3.160	1.347	0.512	-0.315	-0.740	-0.908	-1.162	-1.250	-1.366	-1.452		200	6.113	3.655	2.200	0.947	0.210	-0.163	-0.545	-0.716	-0.880	-0.997
	500	3.620	1.662	0.436	-0.331	-0.784	-0.957	-1.142	-1.268	-1.368	-1.435		500	6.666	3.972	2.092	0.837	0.066	-0.306	-0.603	-0.790	-0.922	-1.008
200	100	3.709	2.103	1.368	1.185	0.759	0.755	0.280	0.042	0.032	-0.090	200	100	6.276	4.043	2.708	2.063	1.341	1.124	0.518	0.188	0.115	-0.052
	200	3.807	2.290	1.669	0.956	0.577	0.459	0.217	0.120	0.011	-0.084		200	6.494	4.229	2.939	1.783	1.094	0.771	0.409	0.232	0.077	-0.050
	500	4.264	2.572	1.549	0.886	0.504	0.358	0.190	0.077	-0.010	-0.075		500	7.057	4.533	2.808	1.649	0.948	0.609	0.334	0.161	0.039	-0.046
500	100	3.840	2.250	1.510	1.340	0.921	0.914	0.431	0.172	0.152	0.000	500	100	6.358	4.135	2.802	2.163	1.447	1.226	0.615	0.270	0.191	0.000
	200	3.955	2.440	1.818	1.099	0.709	0.587	0.345	0.233	0.114	0.000		200	6.603	4.333	3.042	1.881	1.183	0.856	0.494	0.306	0.142	0.000
	500	4.407	2.704	1.675	1.001	0.614	0.461	0.287	0.170	0.075	0.000		500	7.166	4.630	2.898	1.729	1.024	0.679	0.398	0.222	0.094	0.000
T	N	RMSE										T	N	RMSE									
100	100	3.023	1.327	0.863	0.947	1.187	1.223	1.567	1.769	1.758	1.847	100	100	5.820	3.447	2.011	1.305	0.777	0.712	0.880	1.119	1.162	1.287
	200	3.200	1.471	0.874	0.844	1.110	1.247	1.450	1.523	1.632	1.711		200	6.129	3.683	2.259	1.100	0.629	0.644	0.841	0.964	1.103	1.205
	500	3.635	1.730	0.757	0.787	1.094	1.248	1.412	1.524	1.616	1.681		500	6.671	3.984	2.133	0.974	0.552	0.663	0.864	1.015	1.130	1.209
200	100	3.747	2.142	1.403	1.210	0.783	0.774	0.320	0.141	0.126	0.138	200	100	6.305	4.072	2.733	2.082	1.358	1.137	0.539	0.222	0.152	0.085
	200	3.823	2.305	1.679	0.967	0.589	0.471	0.240	0.150	0.088	0.120		200	6.505	4.239	2.946	1.791	1.101	0.778	0.418	0.243	0.100	0.074
	500	4.268	2.575	1.552	0.890	0.510	0.364	0.202	0.103	0.068	0.103		500	7.059	4.536	2.810	1.651	0.951	0.612	0.338	0.168	0.060	0.065
500	100	3.877	2.283	1.535	1.354	0.932	0.921	0.443	0.196	0.170	0.000	500	100	6.386	4.162	2.825	2.178	1.460	1.235	0.627	0.289	0.206	0.000
	200	3.969	2.453	1.826	1.105	0.714	0.590	0.349	0.237	0.120	0.000		200	6.613	4.343	3.049	1.887	1.188	0.860	0.498	0.310	0.147	0.000
	500	4.411	2.707	1.677	1.002	0.615	0.462	0.287	0.171	0.077	0.000		500	7.168	4.632	2.900	1.731	1.025	0.680	0.399	0.223	0.095	0.000
		Non-Gaussian errors												Non-Gaussian errors									
T	N	Bias										T	N	Bias									
100	100	7.943	4.850	2.707	1.341	0.217	-0.321	-1.092	-1.499	-1.603	-1.738	100	100	11.542	7.952	5.212	3.270	1.685	0.788	-0.211	-0.790	-1.015	-1.228
	200	8.884	5.619	3.330	1.443	0.217	-0.469	-1.046	-1.320	-1.524	-1.660		200	12.449	8.683	5.761	3.318	1.609	0.552	-0.265	-0.700	-0.998	-1.191
	500	10.209	6.610	3.777	1.710	0.329	-0.446	-0.967	-1.272	-1.459	-1.571		500	13.651	9.595	6.184	3.520	1.628	0.472	-0.288	-0.733	-0.995	-1.147
200	100	8.946	6.018	4.020	2.811	1.750	1.289	0.553	0.147	0.033	-0.135	200	100	12.328	8.823	6.154	4.304	2.769	1.932	0.961	0.386	0.157	-0.085
	200	9.923	6.769	4.592	2.799	1.646	1.015	0.479	0.211	0.010	-0.131		200	13.282	9.557	6.685	4.302	2.635	1.622	0.840	0.410	0.114	-0.084
	500	11.279	7.734	4.975	2.969	1.652	0.926	0.437	0.154	-0.018	-0.120		500	14.528	10.481	7.097	4.456	2.599	1.475	0.740	0.314	0.065	-0.079
500	100	9.601	6.570	4.457	3.173	2.047	1.540	0.778	0.343	0.206	0.000	500	100	12.946	9.356	6.579	4.646	3.034	2.138	1.133	0.527	0.275	0.000
	200	10.511	7.264	4.991	3.122	1.907	1.233	0.676	0.381	0.162	0.000		200	13.820	10.018	7.060	4.596	2.864	1.799	0.986	0.531	0.217	0.000
	500	11.910	8.257	5.382	3.272	1.884	1.108	0.593	0.296	0.115	0.000		500	15.094	10.975	7.494	4.755	2.816	1.632	0.862	0.417	0.156	0.000
T	N	RMSE										T	N	RMSE									
100	100	8.273	5.217	3.104	1.788	1.044	1.032	1.482	1.822	1.928	2.059	100	100	11.763	8.187	5.442	3.474	1.917	1.104	0.780	1.106	1.301	1.495
	200	9.230	5.995	3.701	1.910	1.037	1.023	1.387	1.625	1.812	1.946		200	12.676	8.927	5.993	3.555	1.869	0.927	0.741	1.002	1.249	1.432
	500	10.559	7.000	4.194	2.191	1.077	0.965	1.273	1.531	1.709	1.821		500	13.881	9.852	6.453	3.794	1.917	0.878	0.706	0.981	1.212	1.358
200	100	9.187	6.261	4.226	2.951	1.852	1.342	0.602	0.225	0.152	0.196	200	100	12.495	8.996	6.310	4.422	2.858	1.983	0.999	0.421	0.196	0.130
	200	10.170	7.014	4.794	2.967	1.767	1.083	0.526	0.249	0.124	0.184		200	13.450	9.732	6.842	4.440	2.741	1.686	0.880	0.432	0.146	0.124
	500	11.526	7.991	5.217	3.175	1.807	1.020	0.493	0.195	0.101	0.166		500	14.694	10.662	7.278	4.621	2.730	1.559	0.789	0.337	0.095	0.114
500	100	9.753	6.720	4.583	3.254	2.104	1.569	0.801	0.364	0.221	0.002	500	100	13.053	9.466	6.678	4.719	3.088	2.170	1.158	0.547	0.290	0.002
	200	10.689	7.444	5.143	3.251	2.000	1.287	0.706	0.395	0.169	0.003		200	13.940	10.145	7.177	4.700	2.946	1.851	1.018	0.546	0.224	0.002
	500	12.047	8.398	5.510	3.374	1.954	1.145	0.610	0.303	0.117	0.001		500	15.187	11.076	7.592	4.842	2.881	1.670	0.882	0.426	0.159	0.000

Notes: Parameters of the dynamic panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i \sim U(0, 0.95)$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Design 2 assumes a two-factor model with $[N_{\alpha_\beta}]$ and $[N_{\alpha_{\beta 2}}]$ non-zero loadings for the first and second factor, respectively. We set: $\alpha_{\beta 2} = 2\alpha_\beta/3$, where α_β relates to α under (11) and $f_{jt} \sim IIDN(0,1)$. Gaussian errors are given by: $u_{it} \sim IIDN(0,1)$ and non Gaussian errors are generated as: $u_{it} = \left(\frac{\mathrm{v}-2}{\chi^2_{\mathrm{v},t}}\right)^{1/2} \tilde{\nu}_{it}$, for $i = 1, 2, \ldots, N$, $\tilde{\nu}_{it} \sim IIDN(0,1)$ and $\chi^2_{\mathrm{v},t}$ is a chi-squared random variate with v= 8 degrees of freedom, in (31). $v_{ij} \sim IIDU(\mu_{v_j} - 0.2, \mu_{v_j} + 0.2)$, $j = 1, 2$, $\mu_v = 0.87$, $\mu_{v_2} = 0.71$, $\mu_{v_1} = \sqrt{\mu_v^2 - N^{2(\alpha_{\beta 2} - \alpha_\beta)} \mu_{v_2}^2}$), in (32) and (33). The number of replications is set to $R = 2000$.

Table S3: Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ estimate of the cross-sectional exponent of the errors from a static panel data model

Cross correlations are generated using Design 2 with Gaussian errors scaled by $\varsigma = \sqrt{1/2}$

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.05$												$c_p(n,\delta)$ with $n = N(N-1)/2$ and $p = 0.10$									
		$\delta = 1/2$												$\delta = 1/2$									
T	N	Bias										T	N	Bias									
100	100	0.812	0.027	-0.037	0.234	0.107	0.295	-0.080	-0.244	-0.199	-0.275	100	100	1.253	0.359	0.203	0.423	0.256	0.424	0.037	-0.137	-0.100	-0.184
	200	0.446	0.046	0.268	0.039	-0.039	0.032	-0.112	-0.138	-0.213	-0.279		200	0.802	0.298	0.449	0.179	0.076	0.133	-0.015	-0.049	-0.125	-0.193
	500	0.303	0.114	0.070	-0.003	-0.061	-0.023	-0.096	-0.164	-0.234	-0.288		500	0.575	0.297	0.205	0.101	0.029	0.060	-0.016	-0.084	-0.152	-0.204
200	100	1.013	0.278	0.233	0.550	0.416	0.616	0.252	0.073	0.105	-0.003	200	100	1.426	0.558	0.406	0.660	0.487	0.659	0.280	0.091	0.114	-0.002
	200	0.649	0.289	0.530	0.311	0.241	0.319	0.192	0.153	0.077	-0.004		200	0.982	0.496	0.650	0.384	0.286	0.346	0.210	0.163	0.083	-0.002
	500	0.497	0.335	0.317	0.239	0.195	0.236	0.168	0.109	0.048	-0.004		500	0.748	0.474	0.393	0.281	0.219	0.251	0.177	0.115	0.052	-0.003
500	100	1.046	0.301	0.254	0.574	0.444	0.636	0.275	0.092	0.120	0.000	500	100	1.465	0.587	0.432	0.682	0.509	0.674	0.296	0.103	0.124	0.000
	200	0.688	0.321	0.558	0.335	0.262	0.338	0.213	0.168	0.092	0.000		200	1.035	0.535	0.680	0.408	0.304	0.362	0.225	0.175	0.094	0.000
	500	0.534	0.361	0.339	0.256	0.213	0.253	0.183	0.123	0.060	0.000		500	0.801	0.508	0.418	0.299	0.235	0.265	0.189	0.126	0.061	0.000
T	N	RMSE										T	N	RMSE									
100	100	0.875	0.315	0.302	0.388	0.319	0.417	0.325	0.390	0.372	0.428	100	100	1.304	0.470	0.331	0.495	0.353	0.486	0.254	0.275	0.261	0.309
	200	0.496	0.235	0.358	0.238	0.243	0.242	0.270	0.292	0.343	0.405		200	0.832	0.362	0.489	0.259	0.201	0.228	0.190	0.204	0.241	0.295
	500	0.341	0.210	0.202	0.197	0.209	0.214	0.240	0.286	0.346	0.397		500	0.591	0.329	0.251	0.179	0.152	0.171	0.167	0.196	0.247	0.293
200	100	1.038	0.325	0.263	0.561	0.427	0.623	0.269	0.115	0.127	0.013	200	100	1.461	0.604	0.437	0.672	0.498	0.666	0.296	0.127	0.135	0.009
	200	0.663	0.303	0.534	0.316	0.246	0.323	0.198	0.158	0.085	0.012		200	0.999	0.510	0.655	0.390	0.291	0.350	0.215	0.168	0.090	0.008
	500	0.501	0.338	0.318	0.240	0.196	0.237	0.169	0.110	0.051	0.010		500	0.753	0.477	0.395	0.283	0.221	0.252	0.178	0.116	0.054	0.007
500	100	1.071	0.343	0.279	0.582	0.451	0.642	0.289	0.123	0.141	0.000	500	100	1.498	0.630	0.460	0.693	0.518	0.680	0.310	0.133	0.145	0.000
	200	0.702	0.333	0.561	0.339	0.266	0.341	0.217	0.173	0.098	0.000		200	1.053	0.550	0.685	0.413	0.308	0.365	0.230	0.179	0.100	0.000
	500	0.538	0.363	0.340	0.257	0.213	0.254	0.184	0.124	0.061	0.000		500	0.807	0.511	0.419	0.300	0.236	0.266	0.190	0.127	0.062	0.000
		$\delta = 1/3$												$\delta = 1/3$									
T	N	Bias										T	N	Bias									
100	100	2.104	0.973	0.617	0.712	0.461	0.578	0.159	-0.038	-0.016	-0.116	100	100	3.603	2.048	1.338	1.180	0.776	0.785	0.305	0.063	0.054	-0.071
	200	1.826	0.977	0.880	0.472	0.280	0.284	0.110	0.055	-0.032	-0.109		200	3.338	1.987	1.502	0.869	0.530	0.445	0.219	0.130	0.023	-0.069
	500	1.784	1.026	0.650	0.381	0.224	0.205	0.104	0.025	-0.045	-0.101		500	3.255	1.936	1.186	0.692	0.408	0.317	0.178	0.079	-0.003	-0.065
200	100	2.261	1.131	0.774	0.886	0.628	0.743	0.331	0.119	0.126	-0.001	200	100	3.735	2.174	1.457	1.308	0.897	0.902	0.425	0.169	0.146	-0.001
	200	2.010	1.144	1.030	0.617	0.423	0.424	0.255	0.187	0.094	-0.001		200	3.504	2.131	1.627	0.983	0.640	0.547	0.323	0.221	0.108	-0.001
	500	1.969	1.175	0.782	0.495	0.336	0.314	0.211	0.133	0.061	-0.001		500	3.450	2.084	1.304	0.787	0.495	0.398	0.254	0.153	0.069	-0.001
500	100	2.304	1.161	0.798	0.903	0.644	0.752	0.341	0.126	0.133	0.000	500	100	3.780	2.208	1.489	1.325	0.906	0.905	0.431	0.173	0.151	0.000
	200	2.073	1.190	1.060	0.637	0.437	0.436	0.265	0.194	0.101	0.000		200	3.575	2.182	1.657	1.004	0.653	0.556	0.331	0.227	0.113	0.000
	500	2.062	1.233	0.817	0.516	0.351	0.325	0.219	0.139	0.066	0.000		500	3.555	2.152	1.348	0.813	0.512	0.408	0.261	0.159	0.072	0.000
T	N	RMSE										T	N	RMSE									
100	100	2.148	1.029	0.671	0.750	0.505	0.610	0.255	0.186	0.180	0.212	100	100	3.643	2.090	1.373	1.206	0.801	0.805	0.349	0.161	0.145	0.143
	200	1.848	1.002	0.897	0.496	0.312	0.314	0.173	0.145	0.142	0.182		200	3.355	2.004	1.514	0.882	0.545	0.459	0.244	0.165	0.104	0.123
	500	1.791	1.033	0.657	0.391	0.240	0.225	0.141	0.105	0.121	0.158		500	3.260	1.940	1.190	0.697	0.414	0.325	0.191	0.108	0.080	0.108
200	100	2.300	1.173	0.805	0.901	0.641	0.752	0.347	0.152	0.146	0.006	200	100	3.774	2.214	1.488	1.327	0.913	0.913	0.441	0.198	0.166	0.004
	200	2.029	1.159	1.038	0.624	0.429	0.428	0.261	0.192	0.101	0.004		200	3.521	2.147	1.637	0.992	0.647	0.551	0.329	0.226	0.114	0.003
	500	1.975	1.179	0.784	0.497	0.337	0.315	0.212	0.134	0.062	0.003		500	3.454	2.088	1.307	0.790	0.497	0.399	0.255	0.154	0.070	0.002
500	100	2.346	1.205	0.830	0.919	0.656	0.759	0.356	0.154	0.153	0.000	500	100	3.819	2.250	1.521	1.344	0.922	0.916	0.447	0.199	0.170	0.000
	200	2.093	1.206	1.069	0.644	0.443	0.439	0.270	0.199	0.107	0.000		200	3.592	2.197	1.667	1.013	0.661	0.561	0.337	0.231	0.119	0.000
	500	2.067	1.237	0.820	0.518	0.353	0.326	0.220	0.140	0.067	0.000		500	3.559	2.156	1.351	0.815	0.513	0.409	0.262	0.160	0.074	0.000

Notes: Parameters of the static panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$, $\vartheta_i = 0$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Design 2 assumes a two-factor model with $[N_{\alpha_\beta}]$ and $[N_{\alpha_{\beta 2}}]$ non-zero loadings for the first and second factor, respectively. We set: $\alpha_{\beta 2} = 2\alpha_\beta/3$, where α_β relates to α under (11), f_{jt} and $u_{it} \sim IIDN(0,1)$ scaled by $\varsigma = \sqrt{1/2}$, $v_{ij} \sim IIDU(\mu_{v_j} - 0.2, \mu_{v_j} + 0.2)$, $j = 1, 2$, $\mu_v = 0.87$, $\mu_{v_2} = 0.71$, $\mu_{v_1} = \sqrt{\mu_v^2 - N^{2(\alpha_{\beta 2} - \alpha_\beta)}\mu_{v_2}^2}$), in (32) and (33). The number of replications is set to $R = 2000$.

Table S4a: Comparison of Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ and $\mathring{\alpha}$ estimates of the cross-sectional exponent of the errors from a static and dynamic panel data model with exogenous regressors

Cross correlations are generated using Design 1 with non-Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		Bias												RMSE									
		Static model: $\vartheta_i = 0$, for $i = 1, 2, \ldots, N$																					
T	N	$\tilde{\alpha}$										T	N	$\tilde{\alpha}$									
100	100	2.272	0.837	0.313	0.221	0.042	0.176	-0.159	-0.243	-0.103	-0.047	100	100	2.870	1.712	1.032	0.720	0.454	0.330	0.287	0.282	0.144	0.098
	200	2.184	0.899	0.639	0.179	0.027	0.020	-0.114	-0.076	-0.078	-0.062		200	3.018	1.642	1.161	0.731	0.586	0.219	0.214	0.139	0.120	0.112
	500	2.138	1.153	0.485	0.213	0.030	0.035	-0.036	-0.067	-0.090	-0.086		500	3.066	1.984	1.031	0.793	0.397	0.250	0.153	0.149	0.159	0.145
200	100	2.680	1.150	0.594	0.359	0.156	0.286	-0.090	-0.188	-0.050	0.000	200	100	3.191	1.765	1.190	0.747	0.456	0.459	0.211	0.217	0.075	0.003
	200	2.604	1.309	0.890	0.323	0.134	0.111	-0.022	-0.004	-0.017	0.000		200	3.155	1.943	1.349	0.662	0.462	0.240	0.200	0.076	0.034	0.002
	500	2.642	1.440	0.792	0.418	0.178	0.140	0.069	0.027	0.000	0.000		500	3.240	1.965	1.265	0.867	0.439	0.230	0.139	0.108	0.027	0.001
500	100	2.988	1.366	0.677	0.431	0.202	0.304	-0.079	-0.177	-0.047	0.000	500	100	3.351	1.900	1.015	0.680	0.412	0.445	0.176	0.202	0.064	0.000
	200	3.035	1.522	1.012	0.399	0.164	0.157	-0.019	0.001	-0.014	0.000		200	3.541	1.946	1.278	0.616	0.347	0.294	0.138	0.066	0.032	0.000
	500	3.170	1.719	0.910	0.449	0.203	0.162	0.079	0.029	0.003	0.000		500	3.704	2.155	1.298	0.660	0.408	0.239	0.128	0.057	0.040	0.000
T	N	$\mathring{\alpha}$										T	N	$\mathring{\alpha}$									
100	100	2.181	0.661	0.217	0.273	0.178	0.399	0.058	-0.009	0.098	0.104	100	100	3.497	2.601	2.036	1.541	1.166	0.901	0.639	0.451	0.272	0.104
	200	1.145	0.159	0.306	0.023	0.100	0.174	0.068	0.105	0.108	0.093		200	2.499	1.945	1.624	1.164	0.946	0.615	0.419	0.286	0.190	0.093
	500	0.485	0.174	0.038	0.073	0.092	0.156	0.143	0.116	0.094	0.080		500	1.862	1.757	1.117	0.943	0.648	0.468	0.311	0.223	0.134	0.080
200	100	3.293	1.318	0.615	0.402	0.221	0.372	0.016	-0.075	0.050	0.050	200	100	4.217	2.571	2.022	1.392	0.988	0.825	0.514	0.342	0.216	0.050
	200	1.446	0.283	0.274	0.061	0.014	0.117	0.010	0.066	0.057	0.045		200	2.318	1.734	1.220	0.906	0.655	0.474	0.381	0.233	0.139	0.045
	500	0.601	0.104	0.071	0.075	0.042	0.121	0.097	0.074	0.052	0.040		500	1.528	1.103	1.040	0.757	0.464	0.330	0.241	0.210	0.094	0.040
500	100	8.330	4.834	2.300	1.191	0.594	0.484	0.051	-0.078	0.033	0.018	500	100	8.775	5.465	2.960	1.806	1.127	0.815	0.447	0.300	0.194	0.018
	200	3.326	1.257	0.733	0.201	0.066	0.137	-0.004	0.040	0.027	0.017		200	3.905	1.988	1.372	0.775	0.529	0.408	0.254	0.179	0.104	0.017
	500	0.979	0.238	0.111	0.061	0.026	0.097	0.067	0.043	0.029	0.015		500	1.525	0.917	0.810	0.485	0.410	0.250	0.158	0.104	0.085	0.015
		Dynamic model with exogenous regressors: $\vartheta_i \sim U(0, 0.95)$ for $i = 1, 2, \ldots, N$																					
T	N	$\tilde{\alpha}$										T	N	$\tilde{\alpha}$									
100	100	2.224	0.770	0.293	0.190	0.027	0.171	-0.178	-0.256	-0.110	-0.054	100	100	2.818	1.527	0.981	0.768	0.444	0.457	0.257	0.301	0.156	0.110
	200	2.032	0.821	0.589	0.140	-0.032	0.005	-0.128	-0.090	-0.096	-0.077		200	2.730	1.507	1.096	0.686	0.416	0.262	0.213	0.159	0.156	0.136
	500	2.043	0.966	0.483	0.171	0.015	0.017	-0.051	-0.083	-0.105	-0.105		500	2.939	1.547	1.055	0.623	0.549	0.314	0.165	0.161	0.170	0.175
200	100	2.719	1.159	0.531	0.338	0.149	0.269	-0.085	-0.183	-0.051	0.000	200	100	3.256	1.787	1.042	0.722	0.414	0.404	0.331	0.244	0.071	0.002
	200	2.632	1.280	0.894	0.334	0.116	0.122	-0.025	-0.005	-0.016	0.000		200	3.299	1.992	1.490	0.790	0.423	0.291	0.205	0.099	0.054	0.006
	500	2.622	1.420	0.791	0.370	0.164	0.136	0.069	0.026	0.001	0.000		500	3.195	1.991	1.386	0.626	0.403	0.229	0.155	0.075	0.040	0.002
500	100	2.974	1.350	0.694	0.405	0.203	0.303	-0.076	-0.179	-0.048	0.000	500	100	3.363	1.821	1.095	0.623	0.436	0.403	0.189	0.210	0.065	0.000
	200	2.913	1.474	1.005	0.405	0.170	0.146	-0.013	-0.002	-0.014	0.000		200	3.257	1.957	1.356	0.704	0.371	0.286	0.237	0.060	0.035	0.000
	500	3.138	1.674	0.918	0.478	0.198	0.176	0.078	0.029	0.001	0.000		500	3.651	1.969	1.287	0.764	0.370	0.331	0.143	0.053	0.015	0.000
T	N	$\mathring{\alpha}$										T	N	$\mathring{\alpha}$									
100	100	2.104	0.586	0.255	0.282	0.173	0.365	0.063	-0.034	0.105	0.105	100	100	3.368	2.626	1.986	1.530	1.176	0.983	0.610	0.434	0.273	0.105
	200	1.096	0.120	0.255	0.033	0.094	0.165	0.063	0.123	0.109	0.093		200	2.415	1.888	1.551	1.145	0.847	0.638	0.429	0.309	0.204	0.093
	500	0.461	0.050	0.070	0.138	0.120	0.163	0.137	0.118	0.093	0.081		500	1.878	1.414	1.235	0.914	0.688	0.487	0.302	0.218	0.134	0.081
200	100	3.371	1.310	0.534	0.356	0.214	0.347	0.016	-0.068	0.055	0.051	200	100	4.322	2.617	1.822	1.382	0.955	0.765	0.575	0.388	0.226	0.051
	200	1.451	0.234	0.336	0.056	0.011	0.121	0.025	0.067	0.059	0.046		200	2.459	1.627	1.459	0.955	0.694	0.499	0.372	0.252	0.162	0.046
	500	0.603	0.097	0.049	0.061	0.044	0.125	0.096	0.071	0.057	0.040		500	1.724	1.229	1.126	0.660	0.466	0.318	0.272	0.155	0.107	0.040
500	100	8.194	4.656	2.335	1.190	0.557	0.499	0.051	-0.072	0.035	0.018	500	100	8.624	5.271	3.003	1.796	1.088	0.819	0.457	0.335	0.183	0.018
	200	3.275	1.278	0.706	0.195	0.069	0.117	0.006	0.036	0.028	0.017		200	3.852	2.096	1.352	0.771	0.552	0.407	0.332	0.173	0.116	0.017
	500	0.976	0.245	0.122	0.076	0.031	0.113	0.069	0.045	0.025	0.015		500	1.630	0.927	0.827	0.572	0.380	0.368	0.204	0.104	0.058	0.015

Notes: Remaining parameters of the panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Non-Gaussian errors are generated as $u_{it} = \left(\frac{\mathrm{v}-2}{\chi^2_{\mathrm{v},t}}\right)^{1/2} \tilde{\nu}_{it}$, for $i = 1, 2, \ldots, N$, $\tilde{\nu}_{it} \sim IIDN(0,1)$ and $\chi^2_{\mathrm{v},t}$ is a chi-squared random variate with v= 8 degrees of freedom, in (31). Design 1 assumes $b_i \sim U(0.7, 0.9)$ for the first $N_b\,(\leq N)$ elements of vector $\mathbf{b}_N$, $i = 1, 2, \ldots, N$, in the construction of the correlation matrix of the errors, $\mathbf{R}_N = \mathbf{I}_N + \mathbf{b}_N\mathbf{b}_N' - \mathbf{B}_N^2$ given by (30), where $\mathbf{B}_N = \mathrm{Diag}\,(\mathbf{b}_N)$. $\tilde{\alpha}$ is computed using $c_p(n, \delta)$ with $n = N(N-1)/2$, $\delta = 1/2$ and $p = 0.05$ in the multiple testing procedure shown in (23). $\mathring{\alpha}$ corresponds to the most robust estimator of the exponent of cross-sectional dependence considered in Bailey et al. (2016) and allows for both serial correlation in the factors and weak cross-sectional dependence in the error terms. We use four principal components when estimating $\hat{c}_N$ in the expression for $\mathring{\alpha}$. The number of replications is set to $R = 2000$.

Table S4b: Comparison of Bias and RMSE ($\times 100$) for the $\tilde{\alpha}$ and $\mathring{\alpha}$ of the cross-sectional exponent of the errors of a static and dynamic panel data model with exogenous regressors

Cross correlations are generated using Design 2 with non-Gaussian errors

	α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00		α	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
		Bias												Bias									
		Static model: $\vartheta_i = 0$, for $i = 1, 2, \ldots, N$																					
T	N	$\tilde{\alpha}$												$\tilde{\alpha}$									
100	100	1.116	-0.949	-2.057	-2.527	-3.024	-3.083	-3.551	-3.745	-3.713	-3.754	100	100	2.165	1.928	2.631	3.035	3.499	3.576	3.999	4.181	4.162	4.206
	200	0.933	-1.004	-1.930	-2.775	-3.210	-3.357	-3.631	-3.693	-3.777	-3.844		200	2.179	1.962	2.499	3.204	3.624	3.796	4.066	4.140	4.227	4.292
	500	0.898	-1.048	-2.261	-3.008	-3.456	-3.615	-3.782	-3.901	-3.998	-4.063		500	2.489	2.230	2.889	3.493	3.905	4.068	4.237	4.357	4.452	4.516
200	100	2.764	1.265	0.642	0.535	0.221	0.270	-0.159	-0.346	-0.318	-0.394	200	100	3.213	1.794	1.131	0.851	0.541	0.478	0.383	0.473	0.451	0.508
	200	2.655	1.332	0.886	0.307	0.020	-0.036	-0.226	-0.283	-0.361	-0.430		200	3.216	1.863	1.231	0.650	0.388	0.317	0.372	0.413	0.472	0.532
	500	2.623	1.359	0.641	0.190	-0.076	-0.138	-0.258	-0.342	-0.412	-0.460		500	3.279	1.932	1.102	0.598	0.394	0.343	0.400	0.467	0.529	0.571
500	100	3.337	1.824	1.180	1.096	0.741	0.794	0.349	0.121	0.125	-0.002	500	100	3.569	2.057	1.360	1.190	0.806	0.821	0.377	0.156	0.144	0.010
	200	3.163	1.851	1.417	0.822	0.525	0.467	0.270	0.187	0.094	-0.002		200	3.466	2.107	1.556	0.915	0.578	0.486	0.280	0.193	0.100	0.004
	500	3.186	1.896	1.174	0.697	0.431	0.353	0.223	0.135	0.060	-0.002		500	3.605	2.250	1.432	0.871	0.530	0.390	0.238	0.140	0.062	0.004
T	N	$\mathring{\alpha}$										T	N	$\mathring{\alpha}$									
100	100	2.007	0.792	0.564	0.800	0.580	0.729	0.327	0.146	0.199	0.104	100	100	3.303	2.266	1.724	1.516	1.114	1.016	0.625	0.395	0.301	0.104
	200	0.645	0.087	0.437	0.268	0.241	0.330	0.214	0.199	0.149	0.092		200	2.034	1.611	1.259	0.928	0.700	0.580	0.417	0.308	0.202	0.092
	500	0.006	-0.107	0.057	0.124	0.151	0.237	0.195	0.154	0.110	0.080		500	1.649	1.307	0.946	0.667	0.480	0.400	0.298	0.216	0.137	0.080
200	100	3.804	2.191	1.416	1.521	0.963	0.939	0.459	0.196	0.197	0.051	200	100	4.589	3.065	2.155	2.028	1.400	1.209	0.731	0.422	0.305	0.051
	200	1.493	0.719	0.816	0.501	0.342	0.382	0.232	0.190	0.121	0.046		200	2.260	1.570	1.335	0.929	0.651	0.560	0.386	0.271	0.169	0.046
	500	0.447	0.118	0.214	0.199	0.183	0.238	0.178	0.129	0.081	0.040		500	1.258	0.876	0.678	0.493	0.375	0.337	0.243	0.170	0.101	0.040
500	100	9.139	5.921	3.981	3.658	2.478	2.109	1.245	0.613	0.393	0.019	500	100	9.496	6.288	4.312	3.896	2.762	2.355	1.493	0.836	0.535	0.019
	200	3.786	2.622	2.230	1.557	0.963	0.765	0.487	0.280	0.153	0.018		200	4.224	3.014	2.535	1.877	1.248	0.976	0.672	0.394	0.221	0.018
	500	0.993	0.664	0.720	0.436	0.315	0.290	0.185	0.122	0.066	0.016		500	1.412	1.062	1.019	0.667	0.492	0.382	0.242	0.158	0.086	0.016
		Dynamic model with exogenous regressors: $\vartheta_i \sim U(0, 0.95)$ for $i = 1, 2, \ldots, N$																					
T	N	$\tilde{\alpha}$										T	N	$\tilde{\alpha}$									
100	100	1.016	-1.103	-2.241	-2.771	-3.270	-3.357	-3.838	-4.045	-4.008	-4.036	100	100	2.117	2.014	2.782	3.266	3.749	3.863	4.306	4.507	4.488	4.515
	200	0.813	-1.192	-2.165	-3.069	-3.534	-3.708	-3.969	-4.029	-4.118	-4.184		200	2.219	2.155	2.744	3.521	3.969	4.155	4.406	4.474	4.559	4.626
	500	0.669	-1.297	-2.534	-3.294	-3.752	-3.926	-4.100	-4.216	-4.312	-4.380		500	2.424	2.367	3.104	3.732	4.151	4.331	4.506	4.622	4.717	4.785
200	100	2.749	1.249	0.619	0.506	0.180	0.235	-0.194	-0.387	-0.359	-0.431	200	100	3.191	1.751	1.070	0.792	0.492	0.453	0.407	0.521	0.502	0.555
	200	2.578	1.267	0.834	0.256	-0.018	-0.070	-0.268	-0.316	-0.397	-0.464		200	3.135	1.816	1.210	0.658	0.431	0.357	0.424	0.456	0.524	0.579
	500	2.568	1.311	0.610	0.161	-0.098	-0.161	-0.281	-0.366	-0.436	-0.485		500	3.270	1.965	1.181	0.708	0.486	0.394	0.425	0.483	0.545	0.591
500	100	3.419	1.888	1.222	1.113	0.749	0.792	0.349	0.125	0.124	-0.002	500	100	3.674	2.127	1.386	1.183	0.791	0.808	0.366	0.154	0.143	0.006
	200	3.202	1.882	1.439	0.839	0.535	0.471	0.272	0.191	0.092	-0.002		200	3.618	2.254	1.661	1.001	0.631	0.509	0.291	0.198	0.099	0.006
	500	3.166	1.879	1.158	0.685	0.422	0.348	0.221	0.133	0.059	-0.002		500	3.502	2.134	1.319	0.778	0.468	0.362	0.226	0.135	0.061	0.003
T	N	$\mathring{\alpha}$										T	N	$\mathring{\alpha}$									
100	100	1.854	0.704	0.481	0.745	0.542	0.707	0.305	0.139	0.186	0.105	100	100	3.177	2.262	1.725	1.472	1.089	1.006	0.614	0.388	0.287	0.105
	200	0.609	0.031	0.385	0.237	0.215	0.316	0.202	0.192	0.144	0.093		200	2.042	1.618	1.244	0.914	0.690	0.574	0.412	0.303	0.199	0.093
	500	0.010	-0.106	0.060	0.124	0.151	0.242	0.197	0.156	0.112	0.080		500	1.522	1.155	0.862	0.636	0.476	0.402	0.297	0.215	0.139	0.080
200	100	3.816	2.133	1.371	1.412	0.901	0.891	0.424	0.181	0.182	0.051	200	100	4.636	3.039	2.152	1.945	1.344	1.159	0.697	0.422	0.294	0.051
	200	1.464	0.701	0.811	0.496	0.352	0.379	0.228	0.185	0.118	0.046		200	2.273	1.579	1.351	0.952	0.682	0.575	0.386	0.274	0.169	0.046
	500	0.441	0.101	0.204	0.190	0.178	0.237	0.178	0.128	0.081	0.040		500	1.404	1.045	0.813	0.565	0.410	0.343	0.245	0.169	0.102	0.040
500	100	9.059	5.866	3.980	3.556	2.417	1.994	1.202	0.591	0.370	0.019	500	100	9.450	6.268	4.333	3.831	2.719	2.248	1.465	0.843	0.515	0.019
	200	3.747	2.521	2.193	1.494	0.925	0.732	0.478	0.272	0.148	0.018		200	4.201	2.926	2.502	1.816	1.233	0.955	0.669	0.387	0.218	0.018
	500	1.030	0.646	0.705	0.433	0.321	0.286	0.191	0.125	0.066	0.016		500	1.456	1.044	1.006	0.661	0.496	0.375	0.250	0.161	0.086	0.016

Notes: Remaining parameters of the panel data model, (29), are generated as: $a_i \sim IIDN(1,1)$, $\rho_{ix} \sim U(0, 0.95)$ and $\gamma_i \sim IIDN(1,1)$, for $i = 1, 2, \ldots, N$. Design 2 assumes a two-factor model with $[N_{\alpha_\beta}]$ and $[N_{\alpha_{\beta 2}}]$ non-zero loadings for the first and second factor, respectively. We set: $\alpha_{\beta 2} = 2\alpha_\beta/3$, where α_β relates to α under (11) and $f_{jt} \sim IIDN(0,1)$. Gaussian errors are given by: $u_{it} \sim IIDN(0,1)$ and non Gaussian errors are generated as: $u_{it} = \left(\frac{\mathrm{v}-2}{\chi^2_{\mathrm{v},t}}\right)^{1/2} \tilde{\nu}_{it}$, for $i = 1, 2, \ldots, N$, $\tilde{\nu}_{it} \sim IIDN(0,1)$ and $\chi^2_{\mathrm{v},t}$ is a chi-squared random variate with v= 8 degrees of freedom, in (31). $v_{ij} \sim IIDU(\mu_{v_j} - 0.2, \mu_{v_j} + 0.2)$, $j = 1, 2$, $\mu_v = 0.87$, $\mu_{v_2} = 0.71$, $\mu_{v_1} = \sqrt{\mu_v^2 - N^{2(\alpha_{\beta 2} - \alpha_\beta)} \mu_{v_2}^2}$), in (32) and (33). The number of replications is set to $R = 2000$.

Appendix B

Additional Empirical results

The figures below provide the estimates displayed in Figures 1 and 2 but using the shorter 5-year rolling samples.

Figure 3: 5-year rolling estimates of the exponent of cross-sectional correlation ($\tilde{\alpha}_t$) of S&P 500 securities' excess returns

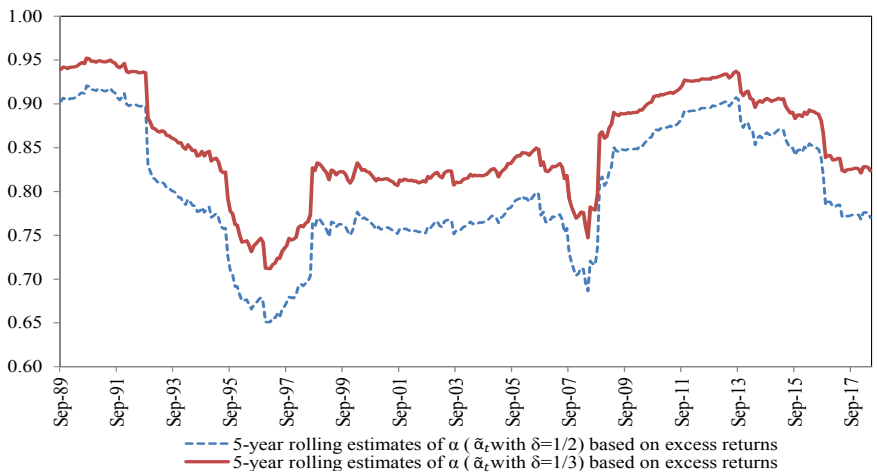
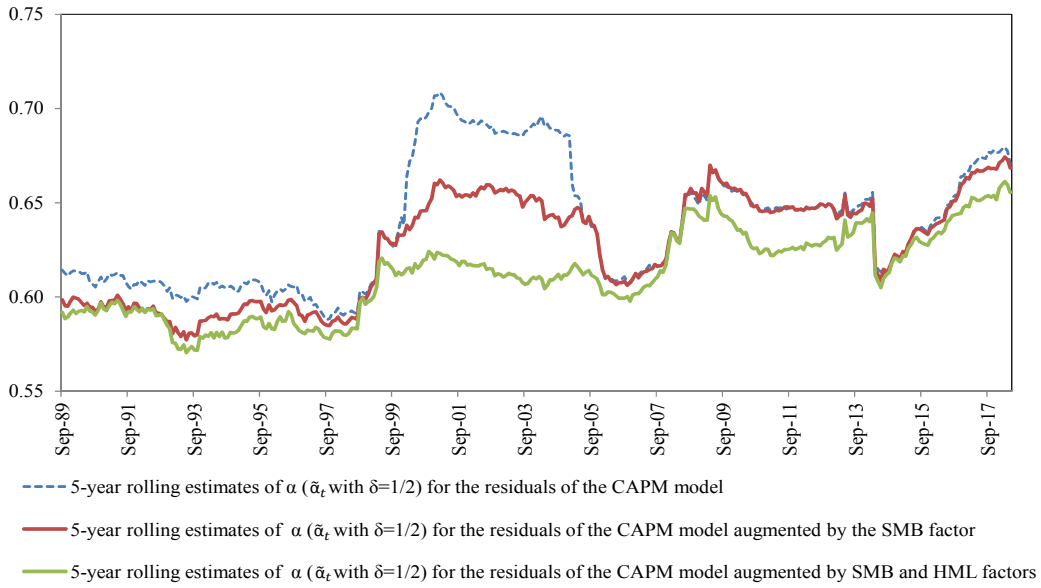


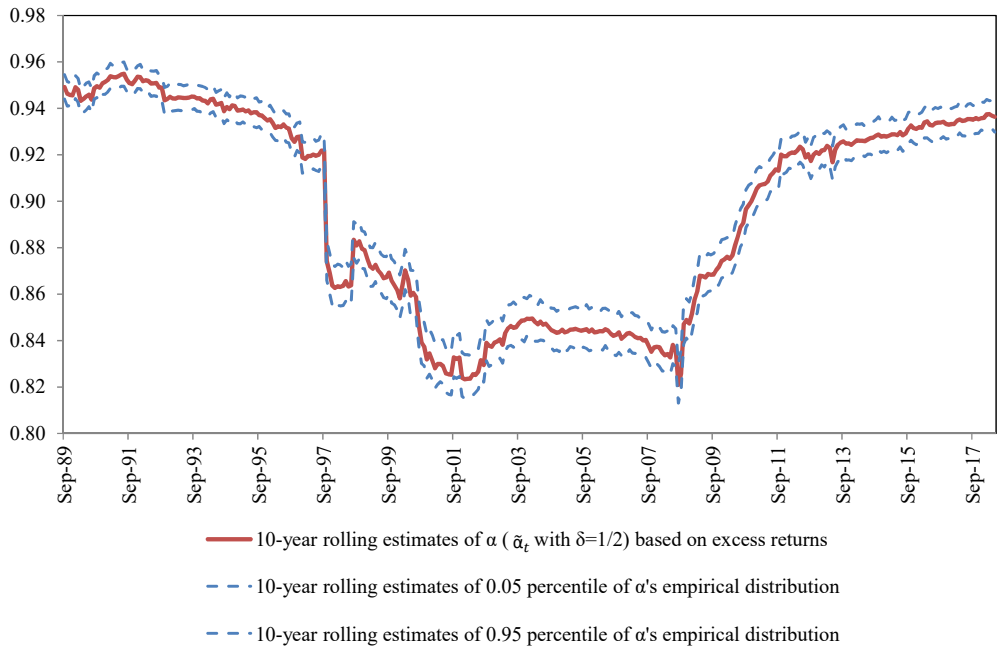
Figure 4: 5-year rolling estimates of the exponent of cross-sectional correlation ($\tilde{\alpha}_t$) of residuals from CAPM and its two Fama-French extensions



Notes: CAPM model includes excess market returns, CAPM model augmented by SMB includes excess market returns and small minus big (SMB) firm returns, and CAPM model augmented by SMB and HML includes excess market returns, small minus big (SMB) firm returns and high minus low (HML) firm returns as regressors in (35), (36) and (37), respectively.

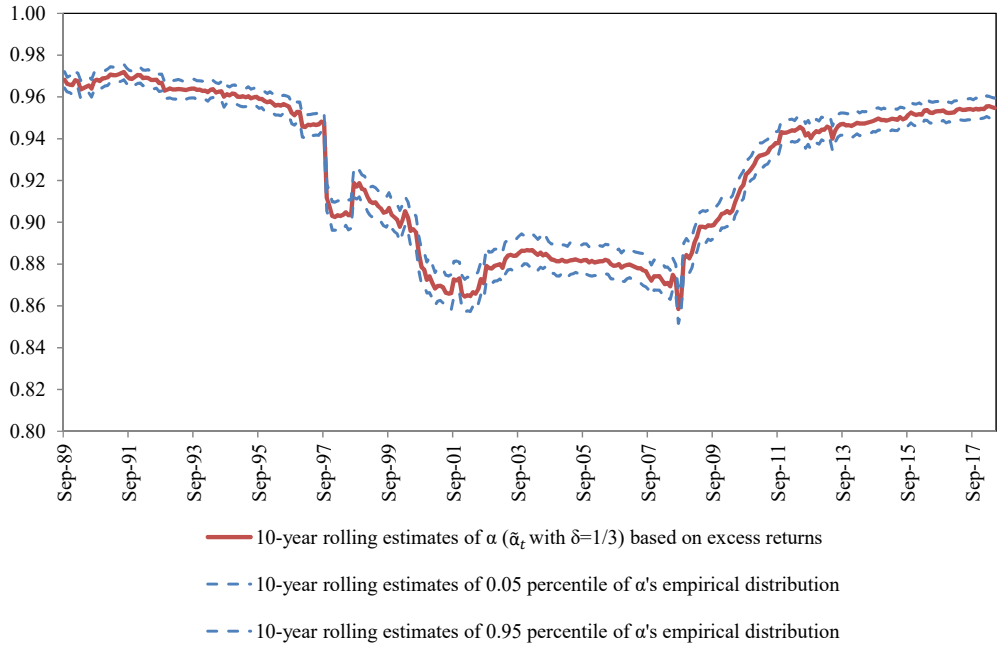
The figures below provide the estimates displayed in Figures 1 and 2 including their 95% confidence bands.

Figure 5: 10-year rolling estimates of the exponent of cross-sectional correlation ($\tilde{\alpha}_t$) of S&P 500 securities' excess returns, with 90% confidence intervals



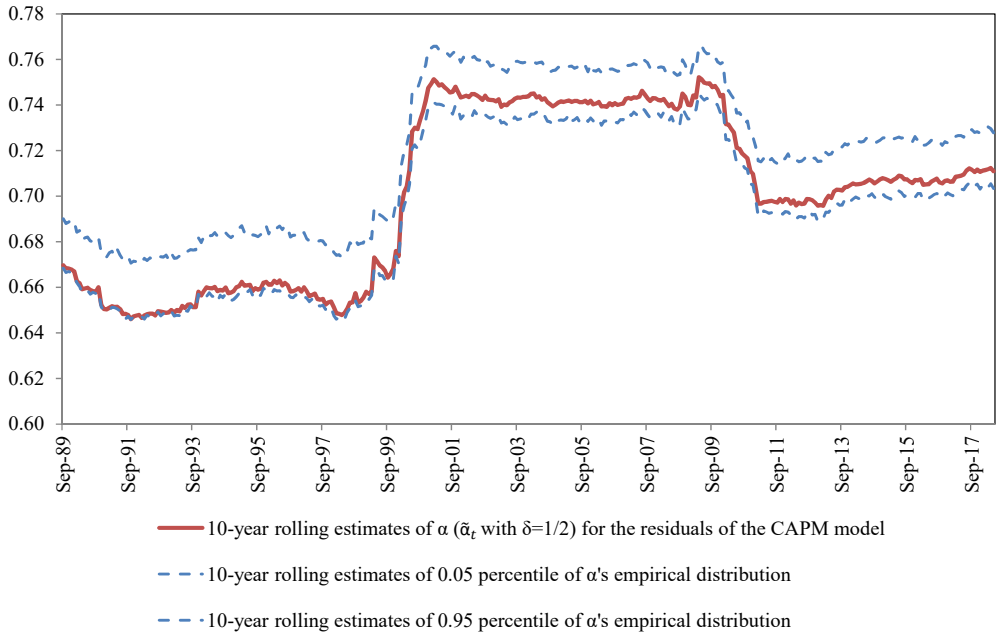
Notes: In the critical value, $c_p(n, \delta)$ we set $p = 0.05$ and $\delta = 1/2$.

Figure 6: 10-year rolling estimates of the exponent of cross-sectional correlation ($\tilde{\alpha}_t$) of S&P 500 securities' excess returns, with 90% confidence intervals



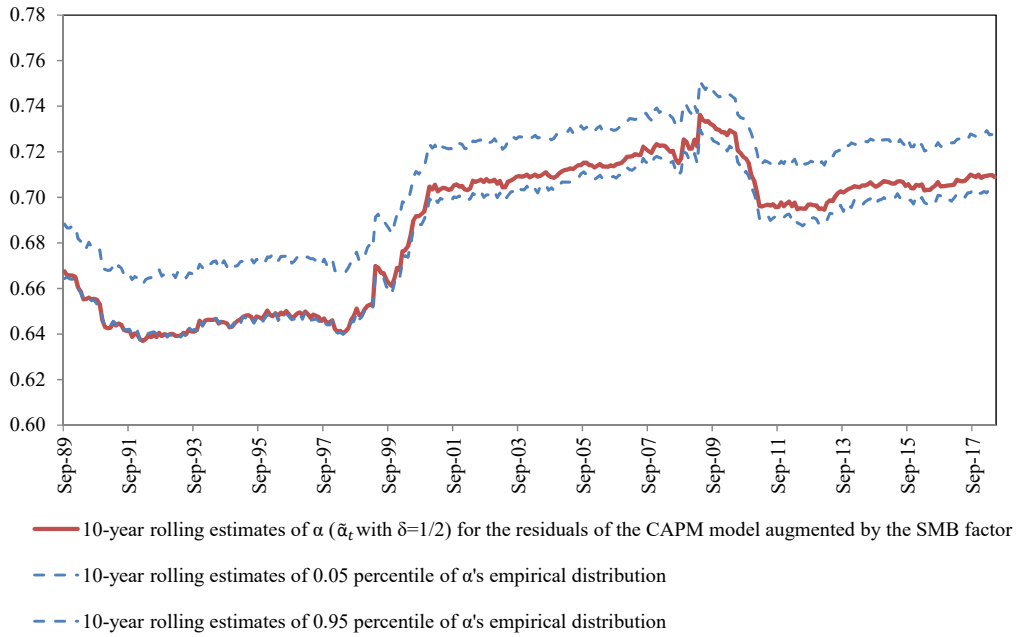
Notes: In the critical value, $c_p(n, \delta)$ we set $p = 0.05$ and $\delta = 1/3$.

Figure 7: 10-year rolling estimates of the exponent of cross-sectional correlation ($\tilde{\alpha}_t$) of residuals from CAPM model, with 90% confidence intervals



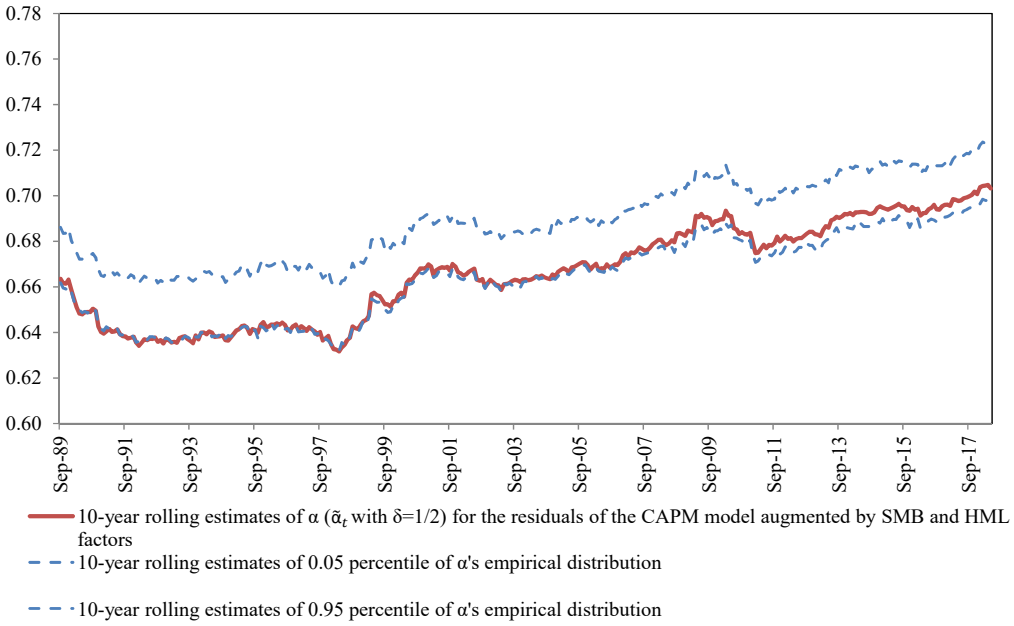
Notes: CAPM model includes excess market returns as regressor in (35). In the critical value, $c_p(n, \delta)$ we set $p = 0.05$ and $\delta = 1/2$. Confidence intervals are computed using the boostrap procedure described in Section 6.

Figure 8: 10-year rolling estimates of the exponent of cross-sectional correlation ($\tilde{\alpha}_t$) of residuals from CAPM model augmented by SMB, with 90% confidence intervals



Notes: CAPM model augmented by SMB includes excess market returns and small minus big (SMB) firm returns as regressors in (36). In the critical value, $c_p(n, \delta)$ we set $p = 0.05$ and $\delta = 1/2$. Confidence intervals are computed using the boostrap procedure described in Section 6.

Figure 9: 10-year rolling estimates of the exponent of cross-sectional correlation ($\tilde{\alpha}_t$) of residuals from CAPM model augmented by SMB and HML, with 90% confidence intervals



Notes: CAPM model augmented by SMB and HML includes excess market returns, small minus big (SMB) firm returns and high minus low (HML) firm returns as regressors in (37). In the critical value, $c_p(n, \delta)$ we set $p = 0.05$ and $\delta = 1/2$. Confidence intervals are computed using the boostrap procedure described in Section 6.