Econometric Analysis of Nonlinear Dynamic Models with Applications in International Macroeconomics

End-of Award Report

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Abstract

The project "Econometric Analysis of Nonlinear Dynamic Models with Applications in International Macroeconomics" was an ESRC-funded project (reference number: R00235524) carried out in the Department of Applied Economics at the University of Cambridge and in the Department of Economics at the University of Leicester over the period 1/09/95 - 31/08/98. In what follows, we describe the aims of the project, and provide a non-technical summary, a full-report, and a list of papers produced on the project.

Summary of aims and objectives of project

1. To investigate the econometric properties of a general class of non-linear dynamic models, emphasising dynamic endogenous switching models and models incorporating rational expectations.

2. To investigate the dynamic properties of such models through the development of measures of persistence of shocks and through the impulse response analysis of dynamic models.

3. To develop testing procedures to compare non-linear models.

4. To investigate the possible asymmetric effects of macroeconomic and other shocks on macroeconomic variables.

Non-Technical Summary

In recent years, there has been considerable interest in non-linear models in economics. The continuing development of sophisticated theoretical models of economic phenomena has resulted in an increasing dissatisfaction with the approximations that are provided by linear systems. And the growth in computing power experienced in recent years has made it feasible to estimate a very wide range of non-linear models. However, it can be argued that one of the major shortcomings of this growing literature is that estimated non-linear models are not always very closely linked to economic theory. Rather, the models are designed to capture particular non-linear features of the data. Moreover, given the variety of non-linear models that are available, the need for a systematic strategy for model evaluation and selection becomes paramount, and yet this literature is still in its early stages of development.

This research project has aimed to develop new techniques for the analysis of non-linear dynamic models and to apply them in the analysis of macroeconomic and international financial markets data. The need for cohesion between a model and the underlying analytic account of how economic agents operate has meant that our research has been restricted to a specific class of models and to a particular set of topics in macroeconomics. Specifically, attention has concentrated on the class of non-linear dynamic models that accommodate the possibility of regime changes (including 'threshold' models and 'endogenous switching regression' models). We have also considered their application in the study of fluctuations of the outputs of various countries and exchange rate movements (although we have considered models of stock returns also). In terms of the development of econometric methods, attention has focused on the development of testing procedures to compare non-linear models and the development of suitable techniques for the analysis of dynamic properties of non-linear stochastic models.

The project has made significant contributions in four main areas. *First*, it has contributed to the development of the analysis of non-linear dynamic models involving rational expectations models, focusing on two classes of model: namely, models obtained as the outcome of a (finite-horizon) intertemporal optimisation problem and the limited-dependent variable models in which there exist bounds within which the variable of interest is constrained to move. Both classes of models are widely observed in economics and, because they are intrinsically dynamic in nature, the solution of the models is complicated in the presence of Rational Expectations. Efficient computational techniques and estimation methods have been developed in the project for the solution and econometric analysis of these models. *Second*, appropriate techniques have been derived with which the dynamic properties of non-linear dynamic models can be investigated. Specifically, the impulse response analysis of non-linear dynamic models has been developed in some detail, and the use of these new methods has already become established in the literature. *Third*, important progress has been made in the area of the evaluation and comparison of non-linear dynamic models, having extended the literature on non-nested testing between non-linear models, and having examined the use of various selection criteria used when choosing between different non-linear models. *Fourth*, these theoretical developments have been applied successfully in a number of areas and, specifically, in the study of non-linearities in business cycle fluctuations, in the modelling of the exchange rate subject to target zones, and in the analysis of stock returns in financial markets.

The work of the project demonstrates the complexity involved in the analysis of non-linear dynamic models and the importance of relating econometric modelling activity to the underlying economic theory. On the other hand, the work also illustrates the power of non-linear dynamic models in capturing the properties of a wide variety of economic phenomena, and suggests that non-linear dynamic models will become more popular and more extensively used in future years.

Full Report

Background

In recent years, there has been considerable interest in non-linear models in economics. The continuing development of sophisticated theoretical models of economic phenomena has resulted in an increasing dissatisfaction with the approximations that are provided by linear systems. And the growth in computing power experienced in recent years has made it feasible to estimate a very wide range of non-linear models. However, it can be argued that one of the major shortcomings of this growing literature is that estimated non-linear models are not always very closely linked to economic theory. Rather, the models are designed to capture particular non-linear features of the data. Moreover, given the variety of non-linear models which are available, the need for a systematic strategy for model evaluation and selection becomes paramount, and yet this literature is still in its early stages of development.

This research project has aimed to develop new techniques for the analysis of non-linear dynamic models and to apply them in the analysis of macroeconomic and international financial markets data. The need for cohesion between a model and the underlying analytic account of how economic agents operate has meant that our research has been restricted to a specific class of models and to a particular set of topics in macroeconomics. Specifically, attention has concentrated on the class of non-linear dynamic models that accommodate the possibility of regime changes (including 'threshold' models and 'endogenous switching regression' models). We have also considered their application in the study of fluctuations of the outputs of various countries and exchange rate movements (although we have considered models of stock returns also). In terms of the development of econometric methods, attention has focused on the development of testing procedures to compare non-linear models and the development of suitable techniques for the analysis of dynamic properties of non-linear stochastic models.

The literature on non-linearities in business cycles provides a useful illustration of the recent expansion in interest in non-linear modelling in economics. In the mid-1980's, papers by Nefci (1984), DeLong and Summers (1986) and Falk (1986), among others, rekindled interest in the earlier theoretical work of Goodwin, Hicks, Minsky and Smithies providing the motivation for asymmetries in the response of output to positive and negative shocks. Persistently high unemployment rates observed in many European economies at the time also focused attention on non-linearities in business cycles, especially as they were translated into movements in the natural rate of unemployment and unemployment hysteresis; see the papers in Cross (1995). Important empirical contributions followed, including: the Markov switching model of Hamilton (1989), in which US GNP switches between regimes according to a first order Markov process; the two regime logistic and exponential smooth transition autoregressive models estimated for various OECD countries in Teräsvirta and Anderson (1992) and Granger and Teräsvirta (1993); the two regime, self-exciting threshold autoregressive model of US output in Potter (1995); the model of US output of Beaudry and Koop (1993), in which the effects of negative shocks are dampened when the economy is in a recessionary regime (defined to occur when output falls below its historical maximum); and the multi-regime models of Sichel (1993) and Tiao and Tsay (1994). Non-linear analyses of a country's output

growth was expanded to consider its joint determination with inflation, unemployment, real wages, international business cycles and various other macroeconomic magnitudes (see, for example, Balke and Wynne (1996), Clements (1998), Peel and Speight (1994, 1995)). And, given the wide variety of alternative non-linear models which were developed, a number of analyses have appeared attempting to compare the performance of linear and non-linear models of output growth (eg. Boldin (1996), Clements and Krolzig (1997), Clements and Smith (1998), or Mills (1995), inter alia). Another strand in the literature has focussed on non-linear dynamic models with a closer correspondence to economic theory such as Deaton and Laroque (1992) and McGrattan (1996). Our own research covers both of these strands.

Given this background, in what follows, we provide a brief overview of the results of the research conducted on the project. A more complete description of the work is provided in the referenced papers. The overview is organised so that we describe first some of the theoretical insights obtained that are relevant to the study of non-linear threshold models involving rational expectations. We then note some of the progress made in the derivation of econometric methods relevant in the selection, evaluation and characterisation of dynamic non-linear models. And finally, we comment on some of the empirical results obtained.

Results

I. The Econometric Properties of Non-Linear Dynamic Models Involving Expectations

In a sequence of papers, [Pesaran and Samiei (1995), Pesaran and Ruge-Murcia (1996, 1998), and Binder, Pesaran and Samiei (1998)], the solution techniques and econometric properties of two classes of non-linear models involving Rational Expectations (RE) have been elaborated. The first class of model is that resulting from the optimality conditions of a finite-horizon intertemporal optimisation problem involving (possibly inequality) constraints. This has widespread applicability in economics including, for example, the finite-lifetime life-cycle model of consumption, asset pricing models and models involving non-linear adjustment costs such as Hayashi's (1982) formulation of the neoclassical model of investment. The second type of models considered include the limited-dependent variable RE (LD-RE) model which constrains the model's dependent variable to lie within a given band, although the position of the band are not necessarily fixed over time. This class of model is also widely applicable in economics in circumstances where policy intervention is used to maintain some (usually nominal) magnitude within a publicly announced band (eg. floor and ceiling restrictions on some commodity price movements, or interest rate, inflation and exchange rate targeting in monetary policy arrangements). A common feature of both classes of model, given that they are intrinsically dynamic in nature, is the complexity of their solution under the RE hypothesis. The solution methods therefore require a pragmatic approach in which analytic intractability is circumvented through the use of approximations and numeric solution algorithms, while paying appropriate attention to evaluating their validity and robustness.

I.1 Non-linear RE Models based on Intertemporal Optimisation

In Binder, Pesaran and Samiei (1998), a backward recursive procedure is used to characterise and solve the dynamic non-linear RE models obtained as the optimality conditions derived

from a finite-horizon intertemporal optimisation problem. These models are distinct from those which are more frequently considered in the literature on numerical solution methods for non-linear RE models (eg Fair and Taylor (1983, 1990), Rust (1996), Judd (1996) or Miranda and Rui (1997)) as they generally do not have time-invariant decision rules. The backward recursion provides the time-varying decision rules, although solution of these will typically require numerical methods. For example, the paper considers a 'minimum weighted residuals' approach to achieve this solution in the context of an analysis of consumption behaviour. The solution method is used to derive decision rules in a form which can be investigated econometrically. Although no formal econometric modelling is presented in the paper, this provides a considerable advance, compared to previous calibration exercises or exercises in which certainty equivalence is assumed, towards integrating the relevant theory with an empirical analysis of households' optimal consumption and savings decisions, under general non-quadratic utility specifications and/or liquidity constraints.

I.2 Limited-Dependent Variable RE Models

The analysis and solution of LD-RE models also involves considerable complexity. Early analyses of such models found in Chanda and Maddala (1983), Shonkweiler and Maddala (1985) and Donald and Maddala (1992) and Pesaran and Samiei (1992a,b), for example, focused on LD-RE models with current expectations of the dependent variable subject to fixed or known bounds. In this project, we have extended this analysis in two directions to improve their relevance to real-world examples: by incorporating *future* expectations in LD-RE models in Pesaran and Samiei (1995); and by allowing for bounds which are subject to unpredictable movements in Pesaran and Ruge-Murcia (1996, 1998).

In Pesaran and Samiei (1995), an analysis of the LD-RE model with future expectations is considered. The inclusion of future expectations represents an important extension to the previous literature given the role played in the determination of current asset prices by expectations of their future price (because of intertemporal arbitrage) and given that it is precisely these markets in which policy intervention often renders LD-RE models appropriate. Analytic solutions are derived in the paper, again based on backward recursion, for those models in which such solutions exist. Approximations and numerical methods are suggested as a means for obtaining solutions for models that do not have analytic solutions, and Monte Carlo experiments are described to illustrate the robustness of these solutions.

In Pesaran and Ruge-Murcia (1996, 1998), the analysis of LD-RE models is extended to the situation where the bounds within which the endogenous variable moves are themselves time varying. In the first paper, the thresholds of the bounds are assumed to vary randomly every period (so that the bounds evolve over time stochastically). Using Monte Carlo experiments, the authors show that the assumption that the band is perfectly predictable by agents (when inappropriate) can seriously bias the estimates of the model parameters and the inferences based o them. The solution of the LD-RE model with stochastic thresholds is derived, a Full Information Maximum Likelihood (FIML) procedure for their estimation is developed, and Monte Carlo experiments are described illustrating the practical feasibility of the estimation methods and illustrating some of its small sample properties.

Despite the advances made in Pesaran and Ruge-Murcia (1996), it was recognised that in

practice bands tend to remain fixed for extended periods of time and are subject to random occasional jumps. The analysis was therefore further extended in Pesaran and Ruge-Murcia (1998) to accommodate this possibility. Here, both the timing of the adjustment and the magnitude of the adjustment (when it occurs) are modelled. The occurrence of a jump is modelled using a discrete state variable that is postulated to follow an ergodic Markov-chain with (possibly) time-varying transition probabilities. Conditional on a jump in the band occurring, the size of the adjustment is dependent on the model's forcing variables. The model retains the property common to all LD-RE models that the endogenous variable is censored by the presence of the band, but in such a model the effects of government intervention is confined not only to the direct influence of changes in the fundamentals but also to their effects on the stochastic nature of the band. This adds further complexity to the solution methods required in such models, and the paper again provides a pragmatic approach to circumventing analytic intractabilities that arise. Hence, for example, while the model of the paper accommodates the possibility of the presence of future expectations, it demonstrates that a simplifying assumption, obtained by reformulating the model in terms of current expectations only, introduces an approximation error which is negligible in the examples considered. The authors then show that a solution to this model exists, that it is unique with plausible parameter values, and that it encompasses the cases of perfectly predictable bands and stochastically varying bands examined previously. These results hold even when the jump probability is itself varying stochastically and when the error terms in the model are conditionally heteroscedastic. The usefulness of this modelling framework, as in the other work on LD-RE models described above, was investigated in the context of exchange rate determination and this is described in the Section III below.

I.3 Stochastic Models of Growth

A related modelling exercise conducted under the project aimed to investigate the dynamic processes underlying economic growth. The non-linear dynamic models of output growth typically considered in the literature, such as the threshold autoregressive models, are ad hoc and lack a proper theoretical rationale. Binder and Pesaran (1999) derive a non-linear vector autoregressive (NVAR) model in output growth, employment and capital output ratio explicitly from a stochastic version of the Solow-Swan growth model. Two important cross-sectional implications of the model are also derived. It is established that the mean of the capital-output ratio depends in a precise way not only on the saving rate and the growth rate of labour input (the standard variables in the existing empirical growth literature), but also on the variance and all higher-order cumulants of the capital-output ratio, which in turn depend on the parameters of the technology and labour input processes. It is also shown that the mean of the steady state distribution of the capital-output ratio under the stochastic model exceeds the value of the steady state capital-output ratio under the corresponding deterministic model. Neither of these implications follow from a deterministic or a log-linearized version of the Solow-Swan growth model. Using the Summers-Heston data for 72 countries from 1960 to 1992, strong support is found for the predictions of the stochastic Solow-Swan model as compared to those of its deterministic counterpart (as well as those of the `AK' model), including a significant negative cross-sectional relationship between the mean and the variance of the capital-output ratio. The system estimation of the NVAR model and its evaluation will be addressed in a future work.

II. Model Selection and Evaluation

Under the project, we have also made a contribution to the development of model selection and evaluation for the analysis of non-linear models. In this, we have considered both the uses of model selection criteria in the choice between models, as well as the application of nonnested hypothesis testing. Of course, hypothesis testing relates to model features and does not, in itself, constitute a model selection strategy (which must rely on the specification of a loss function relevant to the task in hand in choosing between a set of models). Nonnested testing is relevant, however, where the selection of the model can be formulated as a choice between two fully specified competing models. In terms of model diagnostics, our attention has focused on the development of the Generalised Impulse Response analysis as a means of chacterising the dynamic properties of an estimated model.

II.1 Nonnested Hypothesis Testing

The general theory for testing nonnested hypotheses was developed by Cox (1961, 1962), and introduced to linear and non-linear regression models in econometrics by Pesaran (1974) and Pesaran and Deaton (1978). Pesaran and Pesaran (1993) suggested the use of simulation methods in the computation of certain key statistics in the implementation of the Cox procedure, providing a relatively straightforward means of implementing the test in a wide variety of nonnested models. In Pesaran and Pesaran (1995), their stochastic simulation technique was applied to the problem of testing linear versus log-linear models and level-differenced versus log-differenced stationary models.

The application of the Pesaran & Pesaran (PP) simulated Cox testing procedure to linear versus threshold autoregressive models is considered in Kapetanios (1998c). He notes that the test tends to over-reject the null hypothesis, and following Coulibaly and Brorsen (1997) and Lee and Brorsen (1994), argues in favour of bootstrap techniques in order to overcome the over-rejection problem of the PP procedure. A Monte Carlo experiment is set up in which a range of bootstrap procedures are employed in a comparison of a two regime and a three regime Self-exciting Threshold Autoregression (SETAR) model and then in a comparison of a SETAR and a Endogenous Delay Threshold Autoregression (EDTAR) model. It was found that, in the context of nonnested tests between threshold models, simple bootstrap based tests perform well relative to analytical or more complicated bootstrap procedures. Given that many of the asymptotic arguments used in the derivation of nonnested test statistics do not appear to be valid in threshold models, even in large samples, bootstrap techniques offer a useful and practical alternative.

A piece of work in a related area carried out on the project is described in Coe (1998b). Here, it is noted that testing a single state model against the alternative of Hamilton's (1989) Markov switching model is problematic due to the presence of nuisance parameters. A Monte Carlo experiment is used to explore the issue of how power varies with sample size with respect to the likelihood ratio test statistic under these non-standard conditions. It is found that power is low for sample sizes of under 400. Of course, this is approximately twice the number of quarterly observations available to the empirical macroeconomist.

II.2 Model Selection Criteria

Kapetanios (1998c) also considers the use of model selection criteria for threshold models. He focuses attention on a range of alternative information criteria (including the familiar Akaike, Schwarz, and Hannan-Quinn Information Criteria). The conditions for the consistency of the use of the criteria in the choice of lag order in different threshold models are elaborated. And the use of the criteria in lag selection, in the estimation of threshold parameters and in selection between alternative threshold models is discussed and examined through Monte Carlo experiments. The evidence suggests that the desirable asymptotic properties of the less familiar Generalised Information Criterion and Informational Complexity Criterion appear to have relatively little relevance in small samples, and the more familiar Information Criteria of Akaike and Schwarz perform relatively well in threshold models.

The comparison of linear and loglinear models arises also in the paper by van Garderen et al (1999). Here, choice criteria are derived for choosing between the use of an aggregate model, based only on aggregate data, and a disaggregate model based on sectoral or individual, when the models are to be judged purely in terms of their ability to predict an aggregate of interest. The aggregate of interest may be a non-linear function of the variable modeled in the aggregate model and a complicated non-linear function of the disaggregate variables explained by the disaggregate model. (The most obvious example, and one which is described in detail in the paper, is the comparison of a loglinear aggregate model, explaining a variable $\log(Y_t)$ and a disaggregate model consisting of loglinear relationships explaining $\log(Y_{it})$ for sector i,i=1,2,..,m say, where the aggregate of interest is $Y_t = \sum_{i=1} Y_{it}$. The selection criteria proposed in the paper are shown to be consistent in the sense that, in large samples, the disaggregate model would be chosen in preference to the aggregate on average if it was the correct model. Bootstrap techniques are described for implementing the selection criteria in small samples and the ideas are illustrated through a general discussion of log-linear models and a specific empirical study of log-linear production functions using data for eight industrial sectors of the UK.

II.3 Impulse Response Analysis

The final contribution to the development of strategies for model selection and evaluation in the context of non-linear models has been through the development of the Generalised Impulse Response function. Impulse response analysis provides extremely useful information with which to characterise the dynamics of a model by illustrating the evolution over time of the effects of shocks on variables and, importantly, on the persistence of the effects of the shocks at long horizons. The interpretation of the impulse responses of multivariate models requires considerable care, however: even in linear models, there might be important interactions between shocks to different variables which should be taken into account (see Lee and Pesaran (1993), and Pesaran and Shin (1998)). But in non-linear models there is the additional complexity that impulse responses are both history and shock dependent. That means that the ultimate effect of a shock can vary depending on the state of the system at the time of the impact of the shock, and on the sign and magnitude of the shock. The development of the Generalised Impulse Response function in Koop, Pesaran & Potter (1996) provides a unified approach to impulse response analysis that can be used in linear and non-linear, univariate and multivariate models and which fully takes into account the historical patterns of the

correlations observed among different shocks, and the history and shock dependence nature of the non-linear dynamic model.

In the Generalised Impulse Response function, the constructed response is an average of what might happen in the future given the present and the past. This is compared with a baseline response defined with respect to the past only. By writing the response in terms of an arbitrary current shock, and then recognising that the shock is a realisation of a random variable, we can consider the impulse response function itself to be a random variable. Various conditional versions of the generalised impulse response function can be considered, conditioning on a particular shock, or a particular history. In Pesaran and Potter (1997), for example, the authors consider a three-regime threshold model in which the impulse responses are presented conditional on the regime observed; this helps highlight the differences in response that arise because of the non-linearity of the system. It is worth noting that, in the case of linear models, the Generalised Impulse responses are history or shock dependent, and are unique in the sense that they are invariant to the ordering of the variables in the model (in contrast to the more familiar 'orthogonalised' impulse responses). The Generalised Impulse Response function can also be helpful in characterising the extent to which shocks are persistent since, the density of the response should narrow to a spike as the horizon gets large in the absence of persistence. Most importantly, however, the responses provide a clear illustration of the role of history and shock dependence in the evolution of a variable over time and provide a reliable and informative diagnostic tool with which to quantify the importance of non-linear effects in the model under consideration.

III. Investigating financial and international macroeconomic dynamics with non-linear models

III.1 Output fluctuations

The empirical work carried out in the project to investigate the importance of non-linearities in business cycle fluctuations is documented in Pesaran and Potter (1997), Coe (1998a), Hernandez and Lee (1998), Kapetanios (1998a) and Lee and Shields (1998b). The recurrent theme of this work is the desire to estimate models with a relatively close correspondence to economic theory. This is achieved by generalising earlier models to accommodate more sophisticated feedbacks and the possibility of including an increased number of, and more economically meaningful, regimes and through a closer consideration of the de-trending transformations employed in the construction of feedback variables.

In Pesaran and Potter (1997), a new class of threshold models is introduced which provides a flexible framework for modelling multiple regimes. Their Endogenous Delay Threshold Autoregressive (EDTAR) model is based on the construction of feedback variables which allow past realisations of the variable of interest (and other variables) to influence current dynamics in a flexible and intuitive way. In the context of their study of post-Korean war quarterly US growth, an EDTAR model is formulated which allows for floor and ceiling effects to alter the dynamics of output growth, so that the model allows for three regimes corresponding to low, normal and high output growth rate regimes. The results suggest that turning points of the business cycle provide new initial conditions for the ensuing growth

process and that there are important asymmetries in the response of output to positive and negative shocks. It is noted that the observed history and shock dependence property is not present in the linear or approximately linear models of the type that arise in standard implementations of Real Business Cycle theory; rather, the estimated model displays features similar to the non-linear trade cycle models of the 1940s and 1950s developed by Hicks and Goodwin.

Kapetanios (1998b) provides a comprehensive analysis of SETAR and EDTAR models estimated using US GDP. The selection of the lag order of the models and in the estimation of threshold parameters is examined using a broad range of Information Criteria and the in-sample and out-of-sample predictive ability of the models is analysed and non-nested tests of the models compared to alternative threshold models are described. Generalised impulse response functions are used to investigate and compare the dynamic properties of the models. And experiments are conducted to investigate the robustness of the results to the application of alternative transformations and detrending techniques to the GDP data. As in Pesaran and Potter (1997), Kapetanios (1998b) finds statistically significant non-linear effects in the US output process.

Coe (1998a) also considers US output growth, but concentrates on the inter-war years and considers growth to differ in two regimes according to whether there was a 'financial crisis' occurring at each point in time. The paper argues that there was a shift to the financial crisis regime following the first banking crisis of 1930 which lasted until early 1934. This supports the view that it was the introduction the Federal Deposit Insurance Corporation in January 1934 which ended the financial crisis. The time series of probabilities over the state of the financial sector is also shown to contain predictive power for interwar output fluctuations, supporting the view that the financial crisis played a role in the Great Depression and that the end of the financial crisis was important for recovery from the Depression.

The work of Hernandez and Lee (1998) focuses on the interaction between economies in influencing business cycle fluctuations. In the paper, two hypotheses are suggested for the presence of inter-country interactions. The first, elaborated in Lee (1998), relates to the aversion of agents in the home country to the accumulation of liabilities or claims on the assets of other countries which are out of line with wealth and income levels. This provides for a balance of payments constraint on output growth in which a sequence of balance of payments surpluses or deficits could induce a response in agents' savings behaviour and hence growth itself. Alternatively, the ease with which technology can transfer from one country to another suggests that output growth in the home country is unlikely to systematically diverge from output levels abroad as best practice techniques ultimately pass across national borders (see the related discussion, and the application of the impulse response analysis, provided in Fabiani, 1997). The construction of appropriate feedback variables allows these potential influences on a country's output growth to be captured in a set of threshold models estimated for the output series of the G7 economies.

The work of Lee and Shields (1998b) is also concerned with international output fluctuations, and makes use of output data in various EU economies. Here, however, the focus is on the appropriate measure of the trend output level employed in transforming the data and the

sensitivity of the non-linear dynamic analysis to alternative detrending techniques. The preferred method of detrending is carried out in a first stage of the analysis, in which Survey data on expected output movements is used in conjunction with the actual output data to derive measures of potential output. The Survey data is found to provide considerable information for use in the derivation of the potential output measures, and the use of the resultant series, compared to potential output measures derived on the basis of the actual data only, has a considerable effect on the corresponding dynamic non-linear specifications.

III.2 Exchange rate determination and financial markets

The sequence of papers analysing ever-more sophisticated variants of the LD-RE model described in Section I.2 above provided the basis for a comprehensive study of the determination of the French Franc/German Mark exchange rate over the period July 1979-April 1993 in Pesaran and Ruge-Murcia (1998). During this period, the exchange rate was administered to fluctuate within a target zone, although in fact the central parity of the target zone was changed seven times. Three alternative models of exchange rate determination were estimated in addition to the non-linear LD-RE model with jumps derived in the paper: a linear model; a non-linear LD-RE model with fixed thresholds; and a non-linear LD-RE model with a constant but non-zero probability of realignment. The proposed model outperformed all three in that the estimated parameters of the model of the underlying process of the fundamentals of exchange rate determination took their expected signs only in the proposed model, and that the estimated system suggested either an explosive or a near-explosive process for the exchange rate in the other three models. Inspection of the realignment probabilities derived from the preferred model of the paper indicated that (i) there are important asymmetries in the realignment probabilities when the exchange rate is close to the upper and lower bounds (undermining the symmetry assumption made by some researchers in the field); and (ii) provides important evidence to suggest that, while the target zone has generally been a credible instrument of exchange rate management, announcements by governments about the stability of the system have not been believed in the periods preceding parity realignments and that many agents have correctly anticipated many of the parity adjustments.

The non-linear LD-RE model with time-varying realignment probabilities is important in a practical context because it allows the analysis of the target zone credibility in terms of agents' exchange rate forecasts and perceived realignment probabilities and because it addresses the 'peso problem' (in which agents' expectations of realignments affect the current value of the exchange rate regardless of whether the event takes place ex-post or not). The empirical success of the model in the context of examining exchange rate movements suggests that the modelling framework might also be successfully applied to the analysis of inflation rate and interest rate targeting in monetary policy implementation.

Although the focus of the applied work of the project has been on the study of output growth and exchange rates, we have also expanded our horizons to consider other models of financial markets, and in particular of stock returns in Pesaran and Timmermann (1995, 1998). We conclude the Results section with a brief comment on this work therefore. In these papers, we exploit a new recursive modelling approach to investigate the predictability of stock returns over time. In this approach, in order to predict stock returns at any time, the modeller searches

for a suitable model specification *among the set of models* believed a prior to be capable of predicting stock returns. Standard information criteria, and less familiar selection criteria of arguably more relevance to investors' decision rules, are used in the specification search and prediction made on the basis of the preferred model. As more information becomes available, the search is repeated so that the model selection is confirmed or overturned and parameter estimates are updated. The evolution of forecasting models might reflect the learning by the investor or the changing nature of the underlying model. The analysis of US stock returns, in Pesaran and Timmermann (1995), and UK returns, in Pesaran and Timmermann (1998) highlight the extent to which stock returns are predictable and identified factors such as dividend yields, interest rates and inflation, in generating the observed pattern of predictability. The recursive modelling approach allows complicated non-linear, or regime switching effects to be incorporated in an otherwise linear structure, and seems to provide a promising technique for identification of regime switches.

The work of the project demonstrates the complexity involved in the analysis of non-linear dynamic models and the importance of relating the econometric modelling activity to the underlying economic theory. On the other hand, the work also illustrates the power of non-linear dynamic models in capturing the properties of a wide variety of economic phenomena, and suggests that non-linear dynamic models will become more popular and more extensively used in future years.

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Project output

In what follows, we provide details of the dissemination of the work undertaken under this project. This has primarily taken the form of published, forthcoming and working. In what follows, we list the project output under the separate headings: I. Papers directly related to the project; and II. Papers written by project members but related to the project only indirectly.

I. Papers directly related to the project

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II. Papers indirectly related to the project

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