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Evaluating Projects and Assessing Sustainable Development in Imperfect Economies*

by

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ABSTRACT

We are interested in three related questions: (1) How should accounting prices be estimated? (2) How should we evaluate policy change in an imperfect economy? (3) How can we check whether intergenerational well-being will be sustained along a projected economic programme? We do not presume that the economy is convex, nor do we assume that the government optimizes on behalf of its citizens. We show that the same set of accounting prices should be used both for policy evaluation and for assessing whether or not intergenerational welfare along a given economic path will be sustained. We also show that a comprehensive measure of wealth, computed in terms of the accounting prices, can be used as an index for problems (2) and (3) above. The remainder of the paper is concerned with rules for estimating the accounting prices of several specific environmental natural resources, transacted in a few well known economic institutions.

JEL Classification: D6, D9, E2, O2, O4, Q2, Q3

Contents

1 Introduction

In several recent publications, it has been shown that there is a wealth like measure that can serve as an index of intergenerational welfare. The index enables one (a) to check whether welfare will be sustained along an economic forecast, and (b) to conduct social cost-benefit analysis of policy reforms (e.g., investment projects). Excepting under special circumstances, however, the index in question is not wealth itself, but an adaptation of wealth. Interestingly, the results do not require the economy to be convex, nor do they require the assumption that the government optimizes on behalf of its citizens.[1]

An economy's wealth is the worth of its capital assets. As is widely recognised today, the list of assets should include not only manufactured capital, but also human capital (health, knowledge, and skills), and natural capital. Formally, an economy's wealth is a linear combination of its capital stocks, the weights awarded to the stocks being the latter's accounting prices.

The term accounting prices was used originally in the literature on economic planning (Tinbergen, 1954). The underlying presumption there was that governments are intent on maximizing social welfare. Public investment criteria were subsequently developed for economies enjoying good governance (Little and Mirrlees, 1968, 1974; Arrow and Kurz, 1970). In its turn the now-extensive literature exploring various concepts of sustainable development has also been directed at societies where governments choose policies so as to maximize intergenerational welfare.[2]

Sustainability is different from optimality. To ask whether collective well-being is sustained along an economic forecast is to ask, roughly speaking, whether the economy's production possibility set is growing. The concept of sustainability is useful for judging the performance of economies where the government, whether by design or incompetence, does not choose policies that maximise intergenerational welfare. One can argue, therefore, that the term "sustainable development" acquires particular bite when it is put to work in <u>imperfect economies</u>, that is, economies suffering from weak, or even bad, governance. Recently the theory of intertemporal welfare indices has been extended to such economies.[3] The theory's reach

[1] Dasgupta and Mäler (2000), Dasgupta (2001a,b), and Section 2 below.

[2] For references to the technical literature on sustainable development, see Pezzey and Toman (2002).

[3] Dasgupta and Mäler (2000), Dasgupta (2001a,b), and Section 2 below.

therefore now extends to actual economies. The theory has also been put to use in a valuable paper by Hamilton and Clemens (1999) for judging whether in the recent past countries have invested sufficiently to expand their productive bases.[4] Among the resources making up natural capital, only commercial forests, oil and minerals, and the atmosphere as a sink for carbon dioxide were included in the Hamilton-Clemens work. Not included were water resources, forests as agents of carbon sequestration, fisheries, air and water pollutants, soil, and biodiversity. Nor were discoveries of oil and mineral reserves taken into account. Moreover, there is a certain awkwardness in several of the steps Hamilton and Clemens took when estimating changes in the worth of an economy's capital assets. Our aim in this paper is to clarify a number of issues that arise in putting the theory of welfare indices to practical use. It is our hope that the findings documented here will prove useful in future empirical work.

We are interested in three related questions: (1) How should accounting prices be estimated? (2) How should we evaluate policy change in an imperfect economy? (3) How can we check whether intergenerational well-being will be sustained along a projected economic programme?

For simplicity, we confine our analysis until Section 14 to a deterministic world. In Section 2 we rehearse the basic theory.[5] We prove that the same set of accounting prices should be used both for policy evaluation and for assessing whether or not intergenerational welfare along a given economic path will be sustained. We also show that a comprehensive measure of wealth, computed in terms of the accounting prices, can be used as an index for problems (2) and (3) above. These results do not require that the economy be convex, nor do they depend on the assumption that the government optimizes on behalf of its citizens subject to constraints.

In Section 3 we use the Ramsey-Solow model of national saving in a convex economy to illustrate the theory. In Section 4 we show that the theory can be put to use in non-convex economies by studying a particular class of ecosystems, namely, shallow lakes. The remainder of the paper is concerned with rules for estimating the accounting prices of specific environmental natural resources, transacted in a few well known economic institutions.

In order to make our findings easily accessible for empirical work, we report our findings

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[4] Serageldin (1995) and Pearce, Hamilton, and Atkinson (1996) were early explorations of the practicalities of estimating a nation's comprehensive wealth.

[5] The material in Section 2 has been taken from Dasgupta and Mäler (2000) and Dasgupta (2001a,b).

as a catalogue of results. Rules for estimating accounting prices of exhaustible natural resources under both free and restricted entry are derived in Section 5. In Section 6 we show how expenditure toward the discovery of new deposits ought to be incorporated in national accounts. Section 7 develops methods for including forest depletion; and in Section 8 we show how the production of human capital could be taken into account. In Section 9 we study the valuation of global public goods.

If an economy were to face exogenous movements in certain variables, its dynamics would not be autonomous in time. Non-autonomy in time introduces additional problems for the construction of the required welfare index, in that the wealth measure requires to be augmented. Exogenous growth in factor productivities, for example, is a potential reason for non-autonomous dynamics. In Section 10 we show that by suitably redefining variables, it is often possible to transform a non-autonomous economic system into one that is autonomous. But such helpful transformations are not available in many other cases. In Section 11 we show that the required welfare index can nevertheless be constructed, by studying a small country exporting an exhaustible natural resource at a price that is time-dependent. The way defensive expenditure against pollution ought to be included in national accounts is discussed in Section 12.

The theory developed upto and including Section 12 assumes that population remains constant. In Section 13 we extend the theory to cover population change.[6] In Section 14 we show how future uncertainty in commodity transformation possibilities can be incorporated. Section 15 contains concluding remarks.

2. The Basic Model

2.1 Preliminaries

We assume that the economy is closed. Time is continuous and is denoted variously by τ and t (τ, t $\geq$ 0). The horizon is taken to be infinite. For simplicity of exposition, we aggregate consumption into a single consumption good, C, and let **R** denote a vector of resource flows (e.g., rates of extraction of natural resources, expenditure on education and health). Labour is supplied inelastically and is normalised to be unity. Intergenerational welfare (henceforth, "social welfare") at t ($\geq$ 0) is taken to be of the Ramsey-Koopmans form,

$$W_t = \int_t^{\infty} U(C_\tau)e^{-\delta(\tau-t)}d\tau, \qquad (\delta > 0), \tag{1}$$

where the utility function, U(C), is strictly concave and monotonically increasing.

[6] In a companion paper (Arrow, Dasgupta, and Mäler, 2003) we have developed criteria for identifying sustainable development under changing population size in optimizing economies.

The state of the economy is represented by the vector **K**, where **K** is a comprehensive list of capital assets. The economy under study faces not only technological and ecological constraints, but also a wide variety of institutional constraints. By the economy's "institutions" we mean market structures, property rights, tax rates, non-market arrangements for credit, insurance, and common property resources, the character of various levels of government, and so forth. We do <u>not</u> assume that the government is necessarily bent on maximizing social welfare subject to constraints. It could be that the government is predatory, or is at best neglectful, and has objectives of its own that are not congruent with social welfare. Nor do we imagine institutions to be unchanging over time. What we do assume is that institutions coevolve with the state of the economy (**K**) in ways that are understood. It is no doubt a truism that social and political institutions influence the evolution of the state of an economy, but it has also been argued by political scientists (Lipset, 1959) that the state of an economy (**K**) influences the evolution of social and political institutions. The theory we develop below accommodates this mutual influence.

Let $\{C_\tau, \mathbf{R}_\tau, \mathbf{K}_\tau\}_t^\infty$ be an economic programme from t to ∞. Given technological possibilities, resource availabilities, and the dynamics of the ecological-economic system, the decisions made by individual agents and consecutive governments from t onwards will determine C_τ, $\mathbf{R}_\tau$, and $\mathbf{K}_\tau$ - for $\tau \geq t$ - as functions of $\mathbf{K}_t$, τ, and t. Thus let $f(\mathbf{K}_t, \tau, t)$, $\mathbf{g}(\mathbf{K}_t, \tau, t)$, and $\mathbf{h}(\mathbf{K}_t, \tau, t)$, respectively, be consumption, the vector of resource flows, and the vector of capital assets at date τ ($\geq t$) if $\mathbf{K}_t$ is the vector of capital assets at t. Now write

$$(\xi_\tau)_t^\infty \equiv \{C_\tau, \mathbf{R}_\tau, \mathbf{K}_\tau\}_t^\infty, \text{ for } t \geq 0. \quad (2)$$

Let $\{t, K_t\}$ denote the set of possible t and $\mathbf{K}_t$ pairs, and $\{(\xi_\tau)_t^\infty\}$ the set of economic programmes from t to infinity.

Definition 1 A <u>resource allocation mechanism</u>, α, is a (many-one) mapping

$$\alpha: \{t, \mathbf{K}_t\} \rightarrow \{(\xi_\tau)_t^\infty\}. \quad (3)$$

It bears emphasis that we do <u>not</u> assume that α maps $\{t, \mathbf{K}_t\}$ into to optimum economic programmes (starting at t), nor even that it maps $\{t, \mathbf{K}_t\}$ into efficient programmes (starting at t). The following analysis is valid even if α is riddled with economic distortions and inequities. Nor do we assume, in defining α, that the economy's institutions are fixed. If institutions and the state of the economy were known to coevolve, that coevolution would be reflected in α. Note too that we do <u>not</u> assume commodity transformation possibility sets to be convex. This is significant, because ecological processes involve transformation possibility sets that are frequently non-convex; displaying, for example, threshold effects. The reason we are able to accommodate non-convex production structures is that we are developing welfare economics in imperfect economies: we assume that the government (rather, some honest agency in government) seeks only to institute policy reform. For an optimizing government the matter would be different. As the Second Fundamental Theorem of Welfare Economics makes clear, production structures need to be convex if the optimum allocation is to be decentralized.

Definition 2 α is time-autonomous (henceforth autonomous) if for all $\tau \geq t$, ξ_τ is a function solely of $\mathbf{K}_t$ and $(\tau\text{-}t)$.

Notice that if α is autonomous, economic variables at date τ ($\geq$ t) are functions of $\mathbf{K}_t$ and $(\tau\text{-}t)$ only. α would be non-autonomous if, for example, knowledge or the terms of trade (for a trading economy) were to change exogenously over time. In certain cases exogenous changes in population size would mean that α is not autonomous. However, by suitably redefining state variables, non-autonomous resource allocation mechanisms can sometimes be mapped into autonomous mechanisms (Sections 10 and 13).

Definition 3 α is time-consistent if

$$\mathbf{h}(\mathbf{K}_{\tau'},\tau'',\tau') = \mathbf{h}(\mathbf{K}_t,\tau'',t), \text{ for all } \tau'', \tau', \text{ and t.} \quad (4)$$

Time-consistency implies a weak form of rationality. An autonomous resource allocation mechanism, however, has little to do with rationality; it has to do with the influence of external factors (e.g., whether trade prices are changing autonomously). In what follows, we assume that α is time-consistent.

Definition 4 The value function reflects social welfare (equation (1)) as a function of initial capital stocks and the resource allocation mechanism. We write this as

$$W_t = V(\mathbf{K}_t, \alpha, t). \quad (5)$$

In what follows, we will often write $V(\mathbf{K}_t, \alpha, t) = V_t$.

Let K_i be the ith capital stock. We assume that V is differentiable in $\mathbf{K}$.[7]

Definition 5 The accounting price, p_{it}, of the ith capital stock is defined as

$$p_{it} = \partial V(\mathbf{K}_t, \alpha, t)/\partial K_{it} \equiv \partial V_t/\partial K_{it}. \quad (6)$$

[7] Differentiability everywhere is a strong assumption. For practical purposes, however, it would suffice to assume that V is differentiable in K_i almost everywhere. The latter would appear to be a reasonable assumption even when production possibilities (including ecological processes) are realistically non-convex. See Section 4 below. However, if the location of these points on the space of capital stocks is uncertain and the uncertainty a smooth probability distribution, the expected value of V_t would be continuous.

Note that accounting prices are defined in terms of hypothetical perturbations to an economic forecast. Specifically, the accounting price of a capital asset is the present discounted value of the perturbations to U that would arise from a marginal increase in the quantity of the asset. Given the resource allocation mechanism, accounting prices at t are functions of $\mathbf{K}_t$, and possibly of t as well (i.e., $p_{it} = p_i(\mathbf{K}_t,t)$). The prices depend also on the extent to which various capital assets are substitutable for one another. It should be noted that accounting prices of private "goods" can be negative if property rights are dysfunctional, such as those that lead to the tragedy of the commons. Note too that if α is autonomous, accounting prices are not explicit functions of time, and so, $p_{it} = p_i(\mathbf{K}_t)$.

2.2 Marginal Rates of Substitution vs Market Observables

Using (1) and (6), it can be shown that, if α is autonomous, p_{it} satisfies the dynamical equation,

$$dp_{it}/dt = \delta p_{it} - U'(C_t)\partial C_t/\partial K_{it} - \Sigma_j p_{jt}\partial(dK_{jt}/dt)/\partial K_i. \quad (7)$$

(7) reduces to Pontryagin equations for co-state variables in the case where α is an optimum resource allocation mechanism. In any event, we show below that, in order to study the evolution of accounting prices under simple resource allocation mechanisms, it is often easier to work directly with (6).

From (6) it also follows that accounting price ratios (p_{it}/p_{jt}, $p_{i\tau}/p_{it}$, and consumption discount rates (see below)) are defined as marginal social rates of substitution between goods. In an economy where the government maximizes social welfare, marginal rates of substitution among goods and services equal their corresponding marginal rates of transformation. As the latter are observable in market economies (e.g. border prices for traded goods in an open economy), accounting prices are frequently defined in terms of marginal rates of transformation among goods and services. However, marginal rates of substitution in imperfect economies do not necessarily equal the corresponding marginal rates of transformation. A distinction therefore needs to be made between the ingredients of social welfare and "market observables". Using market observables to infer social welfare can be misleading in imperfect economies. That we may have to be explicit about welfare parameters (e.g. δ and the elasticity of $U'(C)$) in order to estimate marginal rates of substitution in imperfect economies is not an argument for pretending that the economies in question are not imperfect after all. In principle it could be hugely misleading to use the theory of optimum control to justify an exclusive interest in market observables.

2.3 Genuine Investment as a Measure of Sustainable Development

IUCN (1980) and World Commission (1987) introduced the concept of sustainable development. The latter publication defined sustainable development to be "... development that meets the needs of the present without compromising the ability of future generations to meet their own needs" (World Commission, 1987: 43). Several formulations are consistent with this phrase. But the underlying idea is straightforward enough: we seek a measure that would enable us to judge whether an economy's production possibility set is, in a loose sense, growing. Our analysis is based on an interpretation of sustainability that is based on the maintainence of social welfare, rather than on the maintainenance of the economy's productive base. We then show that the requirement that economic development be sustainable implies, and is implied by, the requirement that the economy's productive base be maintained (Theorems 1-3). These results give intellectual support for the definition of sustainability we adopt here.[8]

Definition 6 The economic programme $\{C_t, \mathbf{R}_t, \mathbf{K}_t\}_0^\infty$ corresponds to a sustainable development path at t if $dV_t/dt \geq 0$.[9]

Notice that the above criterion does not attempt to identify a unique economic programme. In principle any number of technologically and ecologically feasible economic programmes could satisfy the criterion. On the other hand, if substitution possibilities among capital assets are severely limited and technological advances are unlikely to occur, it could be that there is no sustainable economic programme open to an economy. Furthermore, even if the government were bent on optimising social welfare, the chosen programme would not correspond to a sustainable path if the utility discount rate, δ, were too high. It could also be that along an optimum path social welfare declines for a period and then increases thereafter, in which case the optimum programme does not correspond to a sustainable path locally, but does so in the long run.[10]

Optimality and sustainability are thus different notions. The concept of sustainability

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[8] It is not our purpose to review the several ways in which sustainable development can be, and has been, defined. Pezzey (1992) contains an early, but thorough, classification.

[9] For convenience we have defined sustainability only for a moment in time. One could insist on the infinitely more demanding requirement: $dV_t/dt \geq 0$ for all t. Readers can confirm that our results can be rephrased in the obvious manner to be in accordance with this stiffer condition.

[10] One of us (KJA) has produced an example of an optimum economic programme displaying the latter feature.

helps us to better understand the character of economic programmes, and is particularly useful for judging the performance of imperfect economies.

We may now state

Theorem 1 $dV_t/dt = \Sigma_i p_{it} dK_{it}/dt + \partial V_t/\partial t$. (8)

The proof follows directly from equations (5) and (6).

Definition 6 The accounting value of the rate of change in the stocks of capital assets is called genuine investment.

If α is autonomous, then $\partial V_t/\partial t = 0$, and so, from equation (8) we have,

Theorem 2 If α is autonomous, then $dV_t/dt = \Sigma_i p_{it} dK_{it}/dt$.[11] (9)

Equation (9) states that at each date the rate of change in social welfare equals genuine investment. Theorem 2 gives a local measure of sustainability. Integrating (9) yields a non-local measure:

Theorem 3 If α is autonomous, for all $T \geq 0$,

$V_T - V_0 = \Sigma_i[p_{iT}K_{iT} - p_{i0}K_{i0}] - {}_0\int^T[\Sigma_i(dp_{i\tau}/d\tau)K_{i\tau}]d\tau$. (10)

Equation (10) shows that in assessing whether or not social welfare has increased between two dates, the "capital gains" on the assets that have accrued over the interval should be deducted from the difference in wealth between the dates.

Each of Theorems 1, 2 and 3 is an equivalence result. None says whether α gives rise to an economic programme along which social welfare is sustained. For example, it can be that an economy is incapable of achieving a sustainable development path, owing to scarcity of resources, limited substitution possibilities among capital assets, or whatever. Or it can be that although the economy is in principle capable of achieving a sustainable development path, social welfare is unsustainable along the path that has been forecast because of bad government policies. Or it can be that α is optimal, but that because the chosen utility discount rate is large, social welfare is not sustained along the optimum economic programme. Or it can be that along an optimum path social welfare declines for a period and then increases thereafter.

2.4 What Else Does Genuine Investment Measure?

Genuine investment is related to changes in future consumption brought about by it. Imagine that the capital base at t is not $\mathbf{K}_t$ but $\mathbf{K}_t+\Delta\mathbf{K}_t$, where as before, Δ is an operator signifying a small difference. In the obvious notation,

[11] Pearce and Atkinson (1993) noted this result for optimizing economies.

$$V(\alpha, \mathbf{K}_t+\Delta\mathbf{K}_t) - V(\alpha, \mathbf{K}_t) \approx {}_t\int^{\infty} U'(C_\tau)\Delta(C_\tau)e^{-\delta(\tau-t)}d\tau. \tag{11}$$

Now suppose that at t there is a small change in α, but only for a brief moment, Δt, after which the resource allocation mechanism reverts back to α. We write the increment in the capital base at t+Δt consequent upon the brief increase in genuine investment as $\Delta\mathbf{K}_t$. So $\Delta\mathbf{K}_t$ is the consequence of an increase in genuine investment at t and $(\mathbf{K}_{t+\Delta t}+\Delta\mathbf{K}_t)$ is the resulting capital base at t+Δt. Let Δt tend to zero. From equation (11) we obtain

Theorem 4 Genuine investment measures the present discounted value of the changes to consumption services brought about by it.[12]

2.5 Project Evaluation Criteria

Theorem 4 provides a criterion for social cost-benefit analysis of policy reforms. Imagine that even though the government does not optimize, it can bring about small changes to the economy by altering the existing resource allocation mechanism in minor ways. The perturbation in question could be small adjustments to the prevailing structure of taxes for a short while, or it could be minor alterations to the existing set of property rights for a brief period, or it could be a small public investment project. Call any such perturbation a "policy reform".

Consider as an example an investment project. It can be viewed as a perturbation to the resource allocation mechanism α for a brief period (the lifetime of the project), after which the mechanism reverts back to its earlier form. We consider projects that are small relative to the size of the economy. How should they be evaluated?

For simplicity of exposition, we suppose there is a single manufactured capital good (K) and a single extractive natural resource (S). The rate of extraction is denoted by R. Let the project's lifetime be the period [0, T]. Denote the project's output and inputs at t by the vector $(\Delta Y_t, \Delta L_t, \Delta K_t, \Delta R_t)$. We imagine that if the project is accepted, the project manager would rent ΔK_t at t for the period t to t+Δt.[13]

The project's acceptance would perturb consumption under α. Let the perturbation at t

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[12] Theorem 4 is, of course, familiar for economies where the government maximises social welfare (see e.g., Arrow and Kurz, 1970).

[13] If the project has been designed efficiently, we would have:

$$\Delta Y_t = (\partial F/\partial K)\partial K_t + (\partial F/\partial L)\Delta L_t + (\partial F/\partial R)\Delta R_t,$$

where F is an aggregate production function (Y = F(K,L,R)). The analysis that follows in the text does not require the project to have been designed efficiently. As we are imagining that aggregate labour supply is fixed, ΔL_t used in the project would be the same amount of labour displaced from elsewhere.

(≥ 0) be $\tilde{\Delta}C_t$. It would affect U_t by the amount $U'(C_t)\tilde{\Delta}C_t$. However, because the perturbation includes all "general equilibrium effects", it would be tiresome if the project evaluator were required to estimate $\tilde{\Delta}C_t$ for every project that came up for consideration. Accounting prices are useful because they enable project evaluators to estimate $\tilde{\Delta}C_t$ indirectly, which means that they do not have to go beyond project data in order to evaluate projects. Now, it is most unlikely that consumption and investment have the same accounting price in an imperfect economy. So we divide ΔY_t into two parts: changes in consumption and in investment in manufactured capital. Denote them as ΔC_t and $\Delta(dK/dt)$, respectively.

U is the unit of account.[14] Let w_t denote the accounting wage rate. Next, let q_t be the accounting price of the extractive resource input of the project and λ_t the social cost of borrowing capital (i.e., $\lambda_t = \delta - [dp_t/dt]/p_t$).[15]

From the definition of accounting prices, it follows that:

$$\int_0^\infty U'(C_\tau)\tilde{\Delta}C_\tau e^{-\delta\tau}d\tau =$$

$$\int_0^T (U'(C_\tau)\Delta C_\tau + p_\tau\Delta(dK_\tau/d\tau) - w_\tau\Delta L_\tau - \lambda_\tau p_\tau\Delta K_\tau - q_\tau\Delta R_\tau)e^{-\delta\tau}d\tau. \qquad (12)$$

But the RHS of (12) is the present discounted value of social profits from the project (in utility numeraire). Moreover, $\int_0^\infty U'(C_\tau)\tilde{\Delta}C_\tau e^{-\delta\tau}d\tau = \Delta V_0$, the latter being the change in social welfare if the project were accepted. We may therefore write (12) as,

$$\Delta V_0 = \int_0^T (U'(C_\tau)\Delta C_\tau + p_\tau\Delta(dK_\tau/d\tau) - w_\tau\Delta L_\tau - \lambda_\tau p_\tau\Delta K_\tau - \hat{q}_\tau\Delta R_\tau)e^{-\delta\tau}d\tau. \qquad (13)$$

Equation (13) leads to the well-known criterion for project evaluation:

Theorem 5 A project should be accepted if and only if the present discounted value of its social profits is positive.

2.6 Numeraire

So far we have taken utility to be the unit of account. In applied welfare economics,

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[14] Dasgupta, Marglin, and Sen (1972) and Little and Mirrlees (1974), respectively, developed their accounts of social cost-benefit analysis with consumption and government income as numeraire. Which numeraire one chooses is, ultimately, not a matter of principle, but one of practical convenience.

[15] Thus

$$q_t = \int_t^\infty U'(C_\tau)\partial C_\tau/\partial R_\tau e^{-\delta(\tau-t)}d\tau.$$

Notice that if manufactured capital were to depreciate at a constant rate, say γ, the social cost of borrowing capital would be $\lambda_t = \delta + \gamma - (dp_t/dt)/p_t$.

Let $\hat{q}_t$ be the accounting price of the resource in situ. At a full-optimum, $p_t\partial F/\partial R_t = q_t = \hat{q}_t$, and $U'(C_t) = p_t$.

however, it has been found useful to express benefits and costs in terms of current consumption. It will pay to review the way the theory being developed here can be recast in consumption numeraire. For simplicity of exposition, assume that there is a single commodity, that is, an all-purpose durable good that can be consumed or reinvested for its own accumulation. Assume too that the elasticity of marginal utility is a constant, η. Define $\overline{p}_t$ to be the accounting price of the asset at t in terms of consumption at t; that is,

$$\overline{p}_t = p_t/U'(C_t). \tag{14}$$

It follows from (14) that,

$$(d\overline{p}_t/dt)/\overline{p}_t = (dp_t/dt)/p_t + \eta(dC_t/dt)/C_t. \tag{15}$$

Let ρ_t be the social rate of discount in consumption numeraire. ρ_t is sometimes referred to as the consumption rate of interest (Little and Mirrlees, 1974). From (1),

$$\rho_t = \delta + \eta(dC_t/dt)/C_t.\text{[16]} \tag{16}$$

Using (16) in (15) we obtain the relationship between the asset's prices in the two units of account:

$$(d\overline{p}_t/dt)/\overline{p}_t = (dp_t/dt)/p_t + \rho_t - \delta.\text{[17]} \tag{17}$$

2.7 Intragenerational distribution

The distribution of well-being within a generation has been ignored so far. Theoretically it is not difficult to include this. If there are N people in each generation and person j consumes C_j, her welfare would be $U(C_j)$.[18] A simple way to express intragenerational welfare would be to "concavify" U. Let G be a strictly concave, increasing function of real numbers. We may then express intragenerational welfare as $\Sigma_j(G(U(C_j)))$. Some people would be well-off, others badly-off. The formulation ensures that at the margin, the well-being of someone who is badly off is awarded greater weight than that of someone well-off.

The social worth of consumption services (C) depends on who gets what. To accommodate this idea, we have to enlarge the set of commodities so as to distinguish, at the

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[16] To prove (16) notice that, by definition, ρ_t satisfies the equation

$$U'(C_t)\exp(-\delta t) = U'(C_0)\exp(-{}_0\int^t \rho_\tau d\tau).$$

If we differentiate both sides of the above equation with respect to t, (16) follows.

[17] Notice that in imperfect economies δ and η may be unobservable. See Section 2.2.

[18] Person-specific factors (e.g., age, health status, gender) can be included in the welfare function. This is routinely done in applied economics.

margin, a good consumed or supplied by one person from that same good consumed or supplied by another. Thus, a piece of clothing worn by a poor person should be regarded as a different commodity from that same type of clothing worn by someone who is rich. With this re-interpretation of goods and services, the results we have obtained continue to hold.

Relatedly, we should note that the connection between rural poverty in the world's poorest regions and the state of the local ecosystems is a close one. When wetlands, inland and coastal fisheries, woodlands, forests, ponds and lakes, and grazing fields are damaged (owing, say, to agricultural encroachment, or urban extensions, or the construction of large dams, or organizational failure at the village level), traditional dwellers suffer. For them - and they are among the poorest in society - there are frequently no alternative source of livelihood. In contrast, for rich eco-tourists or importers of primary products, there is something else, often somewhere else, which means that there are alternatives. Whether or not there are substitutes for a particular resource is therefore not only a technological matter, nor a mere matter of consumer taste: among poor people location can matter too. The poorest of the poor experience non-convexities in a way the rich do not. Even the range between a need and a luxury is context-ridden. Macroeconomic reasoning glosses over the heterogeneity of Earth's resources and the diverse uses to which they are put - by people residing at the site and by those elsewhere.[19]

3 Illustration, 1: a convex production economy

It will prove useful to illustrate the theory by means of a simple example, based on Ramsey (1928) and Solow (1956). As in Section 2.6, imagine that there is an all-purpose durable good, whose stock at t is K_t (≥ 0). The good can be consumed or reinvested for its own accumulation. There are no other assets. Write output (GNP) as Y. Technology is linear. So $Y = \mu K$, where $\mu > 0$. μ is the output-wealth ratio. GNP at t is $Y_t = \mu K_t$.

Imagine that a constant proportion of GNP is saved at each moment. There is no presumption though that the saving rate is optimum; rather, it is a behavioural characteristic of consumers, reflecting their response to an imperfect credit market. Other than this imperfection, the economy is assumed to function well. At each moment expectations are fulfilled and all markets other than the credit market clear. This defines the resource allocation mechanism, α. Clearly, α is autonomous in time. We now characterise α explicitly.

[19] See the interchange between Johnson (2001) and Dasgupta (2001c) on this. For a more detailed analysis of the connection between environmental and resource economics and the economics of poverty, see Dasgupta (1982, 1993, 2000, 2003).

Let the saving ratio be s $(0 < s < 1)$. Write aggregate consumption as C_t. Therefore,

$$C_t = (1\text{-}s)Y_t = (1\text{-}s)\mu K_t. \tag{18}$$

Capital is assumed to depreciate at a constant rate γ (> 0). Genuine investment is therefore,

$$dK_t/dt = (s\mu\text{-}\gamma)K_t. \tag{19}$$

K_0 is the initial capital stock. The economy grows if $s\mu > \gamma$, and shrinks if $s\mu < \gamma$. To obtain a feel for orders of magnitude, suppose $\gamma = 0.05$ and $\mu = 0.25$. The economy grows if $s > 0.2$, and shrinks if $s < 0.2$.

Integrating (19), we obtain,

$$K_\tau = K_t e^{(s\mu-\gamma)(\tau-t)}, \qquad \tau \geq t \geq 0, \tag{20}$$

from which it follows that,

$$C_\tau = (1\text{-}s)\mu K_\tau = (1\text{-}s)\mu K_t e^{(s\mu-\gamma)(\tau-t)}, \qquad \tau \geq t \geq 0. \tag{21}$$

If the capital stock was chosen as numeraire, wealth would be K_t, and NNP would be $(\mu\text{-}\gamma)K_t$. Each of wealth, GNP, NNP, consumption and genuine investment expands at the exponential rate $(s\mu - \gamma)$ if $s\mu > \gamma$; they all contract at the exponential rate $(\gamma - s\mu)$ if $s\mu < \gamma$. We have introduced capital depreciation into the example so as to provide a whiff (albeit an artificial whiff) of a key idea, that even if consumption is less than GNP, wealth declines when genuine investment is negative. Wealth declines when consumption exceeds NNP.

Current utility is $U(C_t)$. Consider the form

$$U(C) = -C^{-(\eta-1)}, \qquad \text{where } \eta > 1.\text{[20]} \tag{22}$$

η is the elasticity of marginal utility and δ is the social rate of discount if utility is numeraire. Let ρ_t be the social rate of discount if consumption is the unit of account. It follows that

$$\rho_t = \delta + \eta(dC_t/dt)/C_t = \delta + \eta(s\mu - \gamma). \tag{23}$$

The sign of ρ_t depends upon the resource allocation mechanism α. In particular, ρ_t can be negative. To see why, suppose the unit of time is a year, $\delta = 0.03$, $\gamma = 0.04$, $s = 0.10$, $\eta = 2$, and $\mu = 0.20$. Then $\eta(dC_t/dt)/C_t = -0.04$ per year, and (23) says that $\rho_t = -0.01$ per year.[21]

Social welfare at t is,

[20] Estimates of the elasticity of marginal utility obtained from consumer behaviour, or, alternatively, from consumer responses to questions, have typically been in the range 1.5-2.5. The evidence thus acquired does not of course reflect what we mean by η here, but it is close enough.

[21] These are not fanciful figures. Per capita consumption in a number of countries in sub-Saharan Africa declined over the past three decades at as high a rate as 1 percent per year, implying that for small values of δ, the consumption rate of interest would have been negative.

$$V_t = {}_t\!\int^{\infty} U(C_\tau)e^{-\delta(\tau-t)}d\tau. \tag{24}$$

Using (21) and (22) in (24), we have:

$$V_t = -[(1-s)\mu K_t]^{-(\eta-1)}{}_t\!\int^{\infty} e^{-[(\eta-1)(s\mu-\gamma)+\delta](\tau-t)}d\tau,$$

or, assuming that $[(\eta-1)(s\mu-\gamma)+\delta] > 0$,

$$V_t = -[(1-s)\mu K_t]^{-(\eta-1)}/[(\eta-1)(s\mu-\gamma)+\delta]. \tag{25}$$

V is differentiable in K everywhere. Moreover, $\partial V_t/\partial t = 0$. Equations (20) and (25) confirm Theorem 1.[22]

We turn now to accounting prices.

(i) Utility Numeraire

Begin by taking utility to be numeraire. Let p_t be the accounting price of capital. Now

$$p_t \equiv \partial V_t/\partial K_t = {}_t\!\int^{\infty} U'(C_\tau)[\partial C_\tau/\partial K_t]e^{-\delta(\tau-t)}d\tau. \tag{26}$$

Using (25) in (26) we have,

$$p_t = (\eta-1)[(1-s)\mu]^{-(\eta-1)}K_t^{-\eta}/[(\eta-1)(s\mu-\gamma)+\delta]. \tag{27}$$

Using equations (20), (21), (25), and (27) it is simple to check that $p_t \neq U'(C_t)$, except when $s = (\mu+(\eta-1)\gamma-\delta)/\mu\eta$. Let s* be the optimum saving rate. From equation (25) we have,

$$s^* = (\mu+(\eta-1)\gamma-\delta)/\mu\eta. \tag{28}$$

Note that $p_t < U'(C_t)$ if $s > s^*$, which means there is excessive saving. Conversely, $p_t > U'(C_t)$ if $s < s^*$, which means there is excessive consumption.

(ii) Consumption Numeraire

Write $\overline{p}_t = p_t/U'(C_t)$. (29)

Using (26) in (29) yields

$$\overline{p}_t = {}_t\!\int^{\infty}[U'(C_\tau)/U'(C_t)][\partial C_\tau/\partial K_t]e^{-\delta(\tau-t)}d\tau. \tag{30}$$

Now use (21), (22) and (30) to obtain

$$\overline{p}_t = {}_t\!\int^{\infty}(1-s)\mu e^{(-\rho+(s\mu-\gamma))(\tau-t)}d\tau, \tag{31}$$

where $\rho = \delta + \eta(s\mu-\gamma)$.

From (31) we have

$$\overline{p}_t = (1-s)\mu/(\rho-(s\mu-\gamma)). \tag{32}$$

Observe that $\overline{p}_t > 1$ (resp. < 1) if $s < s^*$ (resp. $> s^*$).[23]

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[22] As the economy has a single asset, Theorem 3 is trivially true.

[23] A special case of formula (32) appears in Dasgupta, Marglin, and Sen (1972). However, unlike our present work, the earlier publication did not provide a rigorous welfare economic theory for imperfect economies.

In order to obtain a sense of orders of magnitude, suppose $\eta = 2$, $\mu = 0.20$, $\gamma = 0.05$, and $\delta = 0$. From (28) we have $s^* = 0.625$. Now imagine that $s = 0.40$ (by Ramsey's criterion, this is undersaving!). Using (23) we have $\rho = 0.06$ per unit of time. So (32) reduces to $\overline{p}_t = 4$. In other words, a saving rate that is approximately 30 percent short of the optimum corresponds to a high figure for the accounting price of investment: investment should be valued four times consumption.

Although intergenerational equity is nearly always discussed in terms of the rate at which future well-being is discounted (see, e.g., Portney and Bryant, 1998), equity would be more appropriately discussed in terms of the curvature of U. Let the unit of time be a year. Suppose $\gamma = 0$, $\delta = 0.02$, and $\mu = 0.32$. Consider two alternative values of η: 25 and 50. It is simple to confirm that $s^* = 0.038$ if $\eta = 25$ and $s^* = 0.019$ if $\eta = 50$. Intergenerational equity in both consumption and welfare (the latter is a concave function of the former) can be increased indefinitely by making η larger and larger: C_t becomes "flatter" as η is increased. In the limit, as η goes to infinity, s^* tends to γ (equation (28)), which reflects the Rawlsian maxi-min consumption as applied to the intergenerational context.[24]

4 Illustration, 2: a non-convex ecosystem

The Ramsey-Solow economy discussed above is convex. In this section we confirm that the theory presented in Section 2 can be applied to non-convex economies. We do this by studying a model of shallow lakes.[25]

A key determinant of the overall state of a shallow lake is phosphorus, which is a necessary nutrient for such ecological services in the lake as those that provide a habitat for fish populations. But at high levels of concentration phosphorus is a pollutant, causing as it does increased plant growth, algae blooms, decrease in water transparency, bad odour, oxygen depletion, and fish kills. Thus, the state of a lake can be taken to be the quantity of phosphorus in the water column, which we denote by a scalar, S.

The rate of phosphorus inflow into a lake is a byproduct of agriculture in the watershed (e.g., as fertilizer runoff from farms). We bring these considerations together and postulate that current utility is a strictly concave and twice differential function U(C,S), where U is an increasing function of phosphorus inflow, C. Imagine next that phosphorus has a deletarious

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[24] Solow (1974) and Hartwick (1977) are the key articles on this limiting case.

[25] For the ecology of shallow lakes, see Scheffer (1997) and Carpenter, Ludwig, and Brock (1999).

effect on the lake at all levels of concentration (and not just at high levels of concentration); which is to say that U is a decreasing function of S for all S. This assumption brings into sharp relief those economic problems where a produced good has positive social worth as a flow, even though it is a pollutant as a stock.

Social welfare at t is

$V(S_t) = \int_t^{\infty} U(C_\tau, S_\tau)e^{-\delta(\tau-t)}d\tau$, where $U_S < 0$ and $U_C > 0$.

4.1 Constant Phosphorus Inflow

For simplicity of exposition, we suppose in what follows that

$$U(C,S) = \log C - hS^2, \qquad h > 0. \tag{33}$$

Consider the case where the resource allocation mechanism for phosphorus inflow is such that C_t is a constant, say $\bar{C}$. Studies have confirmed that there is a feedback of phosphorus from bottom sediments when the density of algae in the lake is large. This feedback is reflected in the form of recycling - from sediment to the water column. Experiments suggest that the recycling rate, R, is a sigmoid function of S. A simple form of the relationship is,

$$R_t = bS_t^2/(1+S_t^2), \qquad \text{where } b > 0. \tag{34}$$

The rate of input of phosphorus into the water column is therefore $[\bar{C}+bS_t^2/(1+S_t^2)]$.

However, phosphorus is depleted from the water column owing to sedimentation and water outflow. Assuming that the rate of loss is proportional to S, say γS ($\gamma > 0$), the phosphorus content in the lake's water column is governed by the equation,

$$dS_t/dt = \bar{C} + bS_t^2/(1+S_t^2) - \gamma S_t. \tag{35}$$

For a range of parameter values $\bar{C}$, b, and γ, the curves $[\bar{C}+bS^2/(1+S^2)]$ and γS intersect at three points. This is shown in Fig. 1. The upper and lower intersects, S_3 and S_1, are stable stationary points of (35), whereas the intermediate intersect, S_2, is unstable. Thus, S_2 is the unique separatrix of the dynamical system. S_3 and S_1 should be thought of as eutrophic and oligotrophic states, respectively. Thus, given S_t, the resource allocation mechanism, α, governing the lake's quality can be expressed as,

$$dS_\tau/d\tau = \bar{C} + bS_\tau^2/(1+S_\tau^2) - \gamma S_\tau, \qquad \tau \geq t. \tag{36}$$

Clearly, α is autonomous and time consistent. It is simple to confirm that V(S) is differentiable in S everywhere, excepting S_2. It is simple to confirm as well that, although V(S) is discontinuous at S_2, it possesses both right- and left-hand derivatives there. We can therefore define the accounting price of the lake's quality to be $p(S) = \partial V/\partial S$ at all $S \neq S_2$ and apply the theory locally for the purposes of project evaluation and sustainability assessment. It should be noted that because phosphorus is a pollutant in the lake, $p(S) < 0$.[26]

4.2 Optimum Phosphorus Inflow

The resource allocation mechanism defined by (36) reflects an imperfect economy. Brock and Starrett (2003) have studied the optimum resource allocation mechanism. To review their work, we generalize (36). If C_t is the inflow of phosphorus, the lake's dynamics are given by the equation,

$$dS_t/dt = C_t + bS_t^2/(1+S_t^2) - \gamma S_t, \qquad \text{for } t \geq 0, \tag{37}$$

where S_0 is given as an initial condition.

The problem is to choose $\{C_t\}_0^\infty$ so as to maximize (33), subject to (37).

Clearly, the optimum resource allocation mechanism is both autonomous and time consistent. In what follows, we restrict ourselves to the case where the optimum is an interior one (i.e. $C_t > 0$). Let p_t be the accounting price of phosphorus in the lake. Brock and Starrett confirmed that, for $\{C_t\}_0^\infty$ to be an optimum, it is necessary that C_t and S_t satisfy not only (37), but also the Pontryagin conditions,

$$p_t = -U_C \ (< 0), \text{ for all } t, \tag{38}$$

and $$(dp_t/dt)/p_t = \delta + \gamma - U_S/U_C - 2bS_t/(1+S_t^2)^2, \text{ for all } t. \tag{39}$$

The point therefore is to select p_0 (equivalently, C_0) optimally and allow the dynamical system to evolve in accordance with equations (37)-(39). The authors showed that, in the (p, S) space, equations (37)-(39) can have at most a countable number of stationary points. They studied in detail the class of parameter values for which the number of stationary points is three. They found that two of them (call them S_1 and S_3, with $S_1 < S_3$, corresponding to what could be interpreted to be the oligotrophic and eutrophic state, respectively) are saddle points, while the intermediate point (call it S_2) is a spiral source (i.e., it is unstable).[27] The authors showed that there exists a value of phosphorus stock, $\bar{S}$, such that if $S_0 > \bar{S}$, the optimum programme asymptotes to S_3; but if $S_0 < \bar{S}$, it asymptotes to S_1. In short, history matters.[28] It is easy to

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[26] Note too that because the resource allocation mechanism is imperfect, $-U_C \neq \partial V/\partial S$ (see Section 4.2 below).

[27] Although, for ease of exposition, we are using the same notation, the points S_1, S_2, and S_3 here are not the same as the points S_1, S_2, and S_3 in the previous sub-section.

[28] To the best of our knowledge, Kurz (1968) was the first to note that if utility depends directly on capital stocks, the optimality conditions may possess multiple stationary points even in a convex world. Skiba (1978) showed that in non-convex economies the optimality conditions may possess multiple stationary points even if the utility function is independent of stocks. The model of Brock and Starrett

confirm that if, by fluke, $S_0 = \bar{S}$, there are two equally desirable optimal programmes, one that asymptotes to S_1, another that asymptotes to S_3. This last property can be shown to imply that V(S), although not differentiable at $\bar{S}$, is continuous at $\bar{S}$ and possesses both left- and right-derivatives. $\bar{S}$ is an endogenously determined separatrix.[29]

Since the optimum resource allocation mechanism is autonomous, we may write by p(S) the optimum policy function. Phosphorus being a pollutant in the lake, we have $p(S) < 0$. It can be shown that V(S) is differentiable everywhere excepting at $\bar{S}$. It can also be demonstrated that p(S) is discontinuous at $\bar{S}$, but is left- and right-differentiable there. Moreover,

$$p(S) = \partial V/\partial S \ (< 0), \quad \text{for all } S \neq \bar{S}. \tag{40}$$

Writing by $[p(S)]_{\bar{S}-0}$ (resp., $[p(S)]_{\bar{S}+0}$) the limit of p(S) as S tends to $\bar{S}$ from the left (resp., right), and similarly for $[\partial V/\partial S]_{\bar{S}-0}$ and $[\partial V/\partial S]_{\bar{S}+0}$, it can be shown too that $[p(S)]_{\bar{S}-0} = [\partial V/\partial S]_{\bar{S}-0}$ and $[p(S)]_{\bar{S}+0} = [\partial V/\partial S]_{\bar{S}+0}$. The theory we have outlined in Section 2 is thus applicable to the optimum resource allocation mechanism of this particular non-convex economy.

Having illustrated the theory by means of a three examples, we now proceed to obtain rules for estimating accounting prices. We do this by focussing on specific categories of capital assets and several well known institutional imperfections.

5 Exhaustible Resources: the closed economy

Accounting prices of exhaustible resources when depletion rates are optimal have been much studied (e.g., Dasgupta and Heal, 1979; see below). What is the structure of their accounting prices when resources are instead common pools?

Two property-rights regimes suggest themselves: open access and restricted entry. They in turn need to be compared to an optimum regime. It is simplest if we avoid a complete capital model. So we resort to a partial equilibrium world: income effects are assumed to be negligible. Let R_t be the quantity extracted at t. Income is the numeraire. Let U(R) be the area under the demand curve below R. So U′(R) is taken to be the market demand function. U is assumed to be an increasing and strictly concave function of R for positive values of R. In order to have a notation that is consistent with the one in the foregoing example, we take the social rate of interest to be an exogenously given constant, ρ. Let S_t be the stock. Then,

$$dS_t/dt = -R_t. \tag{41}$$

(2003) combines the two features.

[29] Brock and Starrett (2003) refer to $\bar{S}$ as a Skiba point, the reference being to Skiba (1978).

5.1 The Optimum Regime

In order to construct a benchmark against which imperfect economies can be evaluated, we first study an optimizing economy. Assume that extraction is costless (constant unit extraction cost can be introduced easily). Social welfare at t is,

$$V_t = {}_t\int^\infty U(R_\tau)e^{-\rho(\tau-t)}d\tau. \tag{42}$$

Let p_t^* denote the accounting price of the resource underground (equivalently, the Hotelling rent, or the optimum depletion charge per unit extracted). We know that

$$dp_t^*/dt = \rho p_t^*. \tag{43}$$

This is the Hotelling Rule. Moreover, optimum extraction, R_t^*, must satisfy the condition,

$$U'(R_t) = p_t^*. \tag{44}$$

Assume that

$$U(R) = -R^{-(\eta-1)}, \qquad \text{where } \eta > 1. \tag{45}$$

Then

$$R_t^* = (\rho/\eta)S_0e^{-\rho t/\eta}. \tag{46}$$

We next consider the two imperfect regimes.

5.2 Restricted Entry

For vividness, assume that there are N identical farmers (i, j = 1,2,...,N), drawing from an unrechargeable aquifer. Extraction is costless. We model the situation in the following way:[30]

At t, farmer i owns a pool of size S_{it}. Each pool is separated from every other pool by a porous barrier. Water percolates from the pool which is larger to the one which is smaller. Let λ_{ij} (> 0), be the rate at which water diffuses from pool i to pool j. We assume that $\lambda_{ij} = \lambda_{ji}$. Denote by R_{it} the rate at which i draws from his pool. There are then N depletion equations:

$$dS_{it}/dt = \Sigma_{N-i}[\lambda_{ji}(S_{jt} - S_{it})] - R_{it}, \tag{47}$$

where "Σ_{N-i}" denotes summation over all j other than i.

The payoff function for farmer i at time t is

$${}_t\int^\infty U(R_{i\tau})e^{-\rho(\tau-t)}d\tau. \tag{48}$$

Farmers play non-cooperatively. For tractablity, we study an open loop solution: Farmers are assumed to be naive (when computing his own optimum extraction rates, each takes the others' extraction rates as given).

[30] McKelvey (1980) has studied a special case of the model of diffusion developed below.

Let p_{it} be the (spot) personal accounting price of a unit of i's own resource pool. The present value Hamiltonian for *i*'s optimization problem would then be,

$$H_0 = U(R_{it})e^{-\rho t} + [\Sigma_{N\text{-}i}\lambda_{ji}(S_{jt} - S_{it}) - R_{it}]p_{it}e^{-\rho t}. \qquad (49)$$

It follows from (49) that p_{it} obeys the equation,

$$dp_{it}/dt = (\rho + \Sigma_{N\text{-}i}\lambda_{ji})p_{it}. \qquad (50)$$

For notational simplicity, assume that $\lambda_{ij} = \lambda$ for all i, j. Then (50) reduces to

$$dp_{it}/dt = (\rho + (N\text{-}1)\lambda)p_{it}. \qquad (51)$$

Write $(\rho + (N\text{-}1)\lambda) = \beta$. We conclude that the rush to extract because of insecure property rights amounts to each extractor using an implicit discount rate, β, which is in excess of the social discount rate ρ.[31] Assume now that the elasticity of demand is a constant, η (> 1). Using (46) and (51), we conclude that the extraction rate from the common pool is

$$R_\tau = (\beta/\eta)S_t e^{-\beta(\tau-t)/\eta}, \text{ for all } \tau \geq t. \qquad (52)$$

In order to have a meaningful problem, we take it that $\beta/\eta > \beta\text{-}\rho$ (see below).

Let p_t be the resource's (social) accounting price. We know that $p_t = \partial V_t/\partial S_t$. Using (46), it follows that,

$$p_t = {}_t\!\int^{\infty} U'(R_\tau)[\partial R_\tau/\partial S_t]e^{-\rho(\tau-t)}d\tau. \qquad (53)$$

Write $\overline{p}_t = p_t/U'(R_t)$. Then (51) and (53) imply

$$\overline{p}_t = \beta/(\beta\text{-}\eta(\beta\text{-}\rho)) > 1. \qquad (54)$$

(Notice that $\overline{p}_t = 1$ if $\beta = \rho$.)

As a numerical illustration, consider the case where $\rho = 0.06$, $\beta = 0.10$, and $\eta = 2$. In this case, $\overline{p}_t = 5$, which reflects a considerable imperfection in the resource allocation mechanism in question: the resource's accounting price is five times its market price.

5.3 Open Access

We next study an open-access pool. To have a meaningful problem, we now assume that extraction is costly. For simplicity, let the unit extraction cost be a constant k (> 0). Under open access, Hotelling rents are dissipated completely. Therefore, the equilibrium extraction rate, R_t, is the solution of the equation,

$$U'(R_t) = k. \qquad (55)$$

Equation (55) confirms that, for any given level of reserves, there is excessive extraction. Let $\overline{R}$ be the solution of (55). We then have,

[31] In the limit, as λ tends to infinity, β tends to infinity, implying that depletion is instantaneous.

$dS_t/dt = -\bar{R}$.

Reserves remain positive for a period $T = S/\bar{R}$. Let us normalize utility by setting $U(0) = 0$. It follows that,

$$V_t = \int_t^{(t+S(t)/\bar{R})}(U(\bar{R})-k\bar{R})e^{-\rho(\tau-t)}d\tau. \tag{56}$$

Let p_t be the accounting price of the unextracted resource. Then,

$$p_t = \partial V_t/\partial S_t = [(U(\bar{R})-k\bar{R})/\bar{R}]e^{-\rho St/\bar{R}} > 0. \tag{57}$$

Write $\bar{p}_t = p_t/U'(\bar{R})$, which is the ratio of the resource's shadow price to its unit extraction cost. Then, from (55) and (57),

$$\bar{p}_t = [(U(\bar{R})-k\bar{R})/k\bar{R}]e^{-\rho St/\bar{R}} > 0. \tag{58}$$

(58) resembles a formula proposed by El Serafy (1989) for estimating depletion charges.[32] The charge is positive because an extra unit of water in the aquifer would extend the period of extraction. Notice that $\bar{p}_t$ is bounded above by the ratio of the Marshallian consumer surplus to total extraction cost; furthermore, it increases as the aquifer is depleted and attains its upper bound at the date at which the pool is exhausted. If reserves are large, $\bar{p}_t$ is small, and free access involves no great loss - a familiar result.

What are plausible orders of magnitude? Consider the linear demand function. Assume therefore that

$$U(R) = aR - bR^2, \qquad \text{where } a > k \text{ and } b > 0. \tag{59}$$

From (55) and (59),

$$\bar{R} = (a-k)/2b. \tag{60}$$

Substituting (59) and (60) in (58),

$$\bar{p}_t = ((a-k)/2k)e^{-2b\rho St/(a-k)}. \tag{61}$$

Equation (61) says that

$$\bar{p} \geq 1 \text{ iff } \rho S \leq ((a-k)/2b)\ln((a-k)/2k).$$

(61) expresses the magnitude of $\bar{p}$ in terms of the parameters of the model. Suppose, for example, that $\rho = 0.02$ per year, $S/\bar{R} = 100$ years (i.e. at the current rate of extraction, the aquifer will be exhausted in 100 years), $(a-k)/2k = 20$ (e.g., $k = \$0.50$ and $(a-k) = \$20$). Then

$$\bar{p} = 20\exp(-2) \approx 7. \tag{62}$$

We should conclude that the value to be attributed to water at the margin is high (about 7 times extraction cost). As the date of exhaustion gets nearer, the accounting price rises to its upper

[32] See also Hartwick and Hageman (1993) for a fine discussion that links El Serafy's formula to Hicks' formulation of the concept of national income (Hicks, 1942).

bound, 20.

6 Exploration and Discoveries

How should one account for expenditure on explorations of new deposits of exhaustible resources? We imagine that the rate at which new reserves are discovered (N) is an increasing function of (1) current expenditure on explorations (E) and (2) the accumulated expenditure on explorations (M), but is a declining function of (3) accumulated extraction (Z). Denote the discovery function be $N(E_t,M_t,Z_t)$, where

$$dM_t/dt = E_t, \tag{63}$$

and $$dZ_t/dt = R_t. \tag{64}$$

We revert to the model containing one manufactured capital good, K, and an exhaustible natural resource, S. In the familiar notation, $Y = F(K, R)$ is taken to be the aggregate production function. The remaining equations of motion are,

$$dK_t/dt = F(K_t,R_t) - C_t - E_t. \tag{65}$$

$$dS_t/dt = N(E_t,M_t,Z_t) - R_t. \tag{66}$$

The model has four capital assets K, S, M, and Z. Their accounting prices are denoted by p_K, p_S, p_M, and p_Z, respectively. Social welfare is given by (1). From Theorem 1, we have

$$dV_t/dt = p_K[F(K_t,R_t)-C_t-E_t] + p_S[N(E_t,M_t,Z_t)-R_t] + p_M E_t + p_Z R_t. \tag{67}$$

There are two cases to consider:

(A) Assume that $\partial N/\partial M = 0$ (implying that p_M=0) and $\partial N/\partial Z < 0$ (implying that $p_{Zt} < 0$). Even in this case genuine investment is not the sum of investment in manufactured capital and changes in proven reserves (N_t-R_t). This is because new reserves are valued differently from existing reserves. Note too that exploration costs should not be regarded as investment.

Consider now the special case where the mining industry optimizes.[33] Then $p_K = p_S\partial N/\partial E$. If, in addition, $p_S N_t$ can be approximated by $p_K E_t$, one could exclude discoveries of new reserves from genuine investment, but regard instead exploration costs as part of that investment.

(B) Suppose $\partial N/\partial M > 0$. If the industry optimizes, we have

$$p_K = p_M + p_S\partial N/\partial E, \tag{68}$$

and so $p_K > p_M$. It follows that genuine investment should now include not only new discoveries and investment in manufactured capital (as in Case A), but also exploration costs, using an accounting price that is less than that of manufactured capital.

[33] That the industry optimizes does not mean that the economy is following an optimum programme.

7 Forests and Trees

As stocks, forests offer a multitude of services. Here we focus on forests as a source of timber. Hamilton and Clemens (1999) regard the accounting value of forest depletion to be the stumpage value (price minus logging costs) of the quantity of commercial timber and fuelwood harvested in excess of natural regeneration rates. This is an awkward move, since the authors do not say what is intended to happen to the land being deforested. For example, if the deforested land is converted into an urban sprawl, the new investment in the sprawl would be recorded in conventional accounting statistics.[34] But if it is intended to be transformed into farmland, matters would be different: the social worth of the land as a farm should be included as an addition to the economy's stock of capital assets. In what follows, we consider the simple case where the area is predicted to remain a forest.

Let the price of timber, in consumption numeraire, be unity and let ρ (assumed constant) be the social rate of discount. Holding all other assets constant, if B_t is aggregate forest land at, we may express social welfare as $V(B_t)$. The accounting price of forest land is then $\partial V_t/\partial B_t$, which we write as p_t.

Consider a unit of land capable of supporting a single tree and its possible successors. If the land is virgin, if a seed is planted at t=0, if F(T) is the timber yield of a tree aged T, and if T is the rotation cycle, then the present discounted value of the land as a tree-bearer is,

$$p_0 = F(T)e^{-\rho T}/(1-e^{-\rho T}). \tag{69}$$

Suppose instead that at t=0 the piece of land in question houses a tree aged τ. What is the value of the land?

If the cycle is expected to be maintained, we have

$$p_0 = F(T)e^{-\rho(T-\tau)}/(1-e^{-\rho(T-\tau)}). \tag{70}$$

If instead the tree is logged now, but the cycle is expected to be maintained, the value of the land, after the tree has been felled, is given by (69). Depreciation of the forest, as a capital asset, is the difference between (70) and (69).

8 Human Capital

To develop an accounting framework for knowledge acquisition and skill formation,

[34] It should be noted though that the value of urban land would be more than just the new investment: there is a contribution to the value (which could be of either sign) arising from changes in population density - both in the newly developed property and in places of origin of those who migrate to the property.

consider a modified version of the basic model of Section 2. In particular, the underlying resource allocation mechanism is assumed to be autonomous. Labour hours are assumed to be supplied inelastically and population is constant, we may as well then normalize by regarding the labour-hours supplied to be unity.

Production of the consumption good involves physical capital, K_{1t}, and human capital, H_{1t}. Here, H_{1t} is to be interpreted to be the human capital embodied in those who work in the sector producing the consumption good. Thus, if Y_t is output of the consumption good,

$$Y_t = F(K_{1t}, H_{1t}), \tag{71}$$

where F is an increasing function of its arguments.

Assume that human capital is produced with the help of physical capital, K_{2t}, and human capital, H_{2t}, and that, owing to mortality, it depreciates at a constant rate, γ. Output of human capital is given by the technology

$$G(K_{2t}, H_{2t}), \tag{72}$$

where G is an increasing function of its arguments and strictly concave, representing that the input of students is given.

By assumption, all individuals at a given moment of time have the same amount of human capital. Therefore, $H_{1t}/(H_{1t}+H_{2t})$ is the proportion of people employed in the sector producing the consumption good. Let the total stock of human capital be H. It follows that

$$H_{1t} + H_{2t} = H_t. \tag{73}$$

Write

$$K_{1t} + K_{2t} = K_t. \tag{74}$$

For simplicity of exposition, we assume that physical capital does not depreciate. Accumulation of physical capital can be expressed as

$$dK_t/dt = F(K_{1t},H_{1t}) - C_t, \tag{75}$$

and the accumulation of human capital as

$$dH_t/dt = G(K_{2t},H_{2t}) - \gamma H_t. \tag{76}$$

Since the resource allocation mechanism, α, is assumed to be autonomous, we have

$$V_t = V(\alpha,K_{1t},K_{2t},H_{1t},H_{2t}). \tag{77}$$

Let p_{1t} and p_{2t} be the accounting prices of physical capital and q_{1t} and q_{2t} the accounting prices of human capital, in the two sectors, respectively (i.e., $p_{1t} = \partial V_t/\partial K_{1t}$, $q_{2t} = \partial V_t/\partial H_{2t}$, and so forth). Therefore, wealth can be expressed as,

$$Z_t = p_{1t}K_{1t} + p_{2t}K_{2t} + q_{1t}H_{1t} + q_{2t}H_{2t},$$

and genuine investment by

$$I_t = p_{1t}dK_{1t}/dt + p_{2t}dK_{2t}/dt + q_{1t}dH_{1t}/dt + q_{2t}dH_{2t}/dt. \quad (78)$$

Estimating q_{1t} and q_{2t} poses difficult problems in practice. It has been customary to identify human capital with education and to estimate its accounting price in terms of the market return on education (i.e., salaries over and above raw labour). But this supposes, as we have assumed in the above model, that education offers no direct utility. If education does offer direct utility (and it is widely acknowledged to do so), the market return on education is an underestimate of what we should ideally be after. Furthermore, human capital includes health, which too is both a durable consumption good and capital good.

An alternative is to use estimates of expenditures on health and education for the purpose in hand. Such a procedure may be be a reasonable approximation for poor societies, but it is in all probability far off the mark for rich societies.

If α were an optimum resource allocation mechanism, we would have $p_{1t} = p_{2t} = p_t$, say, and $q_{1t} = q_{2t} = q_t$, say. These prices would be related by the optimality conditions

$$U'(C_t) = p_t; \qquad p_t\partial F/\partial K_1 = q_t\partial G/\partial K_2;$$

and $\quad p_t\partial F/\partial H_1 = q_t\partial G/\partial H_2$.

9 Global Public Goods

Countries interact with one another not only through trade in international markets, but also via transnational externalities. Hamilton and Clemens (1999) include carbon dioxide in the atmosphere in their list of assets and regard the accounting price (a negative number) of a country's emission to be the amount it would be required to pay the rest of the world if carbon emissions were the outcome of a fully cooperative agreement. Their procedure is, consequently, valid only if each country is engaged in maximising global welfare, an unusual scenario. In what follows, we develop the required analysis.

Let G_t be the stock of a global common at t. We imagine that G is measured in terms of a "quality" index which, to fix ideas, we shall regard as carbon dioxide concentration in the atmosphere. Being a global common, G is an argument in the value function V of every country. For simplicity of notation, we assume that there is a single private capital good. Let K_{jt} be the stock of the private asset owned by citizens of country j and let α_j be j's (autonomous) resource allocation mechanism and α the vector of resource allocation mechanisms. If V_j is j's value function, we have

$$V_{jt} = V_j(\alpha, K_{jt}, G_t). \quad (79)$$

Let $p_{jt} = \partial V_{jt}/\partial K_{jt}$ and $g_{jt} = \partial V_{jt}/\partial G_t$. It may be that G is an economic "good" for some countries, while it is an economic "bad" for others. For the former, $g_j > 0$; for the latter, $g_j < 0$. Let E_{kt} be the emission rate from country k and let γ be the rate at which carbon in the atmosphere is sequestered. It follows that

$$dG_t/dt = \Sigma_k E_{kt} - \gamma G_t. \tag{80}$$

Genuine investment in j is,

$$I_t = dV_{jt}/dt = p_{jt}dK_{jt}/dt + g_{jt}dG_t/dt,$$

which, on using (80), can be expressed as

$$I_t = p_{jt}dK_{jt}/dt + g_{jt}(\Sigma_k E_{kt} - \gamma G_t). \tag{81}$$

Notice that the expression on the RHS of (81) is the same whether or not α is based on international cooperation. On the other hand, dK_{jt}/dt and dG_t/dt do depend on how the international resource allocation mechanisms are arrived at (e.g., whether they are cooperative or non-cooperative); and they affect the accounting prices, p_{jt} and g_{jt}.[35]

10 Exogenous Productivity Growth

To assume exogenous growth in total factor productivity (the residual) over the indefinite future is imprudent. It is hard to believe that serendipity, unbacked by R&D effort and investment, can be a continual source of productivity growth. Moreover, many environmental resources go unrecorded in growth accounting. If the use of natural capital in an economy has in fact been increasing, estimates of the residual could be presumed to be biased upward. On the other hand, if a poor country were able to make free use of the R&D successes of rich countries, it would enjoy a positive residual.

The residual can have short bursts in imperfect economies. Imagine that a government reduces economic inefficiencies by improving the enforcement of property rights, or reducing centralized regulations (import quotas, price controls, and so forth). We would expect the factors of production to find better uses. As factors realign in a more productive fashion, total factor productivity would increase.

In the opposite vein, the residual could become negative for a period. Increased government corruption could be a cause; the cause could also be civil strife, which destroys capital assets and damages a country's institutions. When institutions deteriorate, assets are used

[35] Social cost-benefit analysis, as sketched in Section 2.4, would enable a country to estimate whether it ought to alter its emissions. Nordhaus and Yang (1996) have studied international carbon emissions as the outcome of a non-cooperative equilibrium game among nations.

even more inefficiently than before and the residual declines. This would appear to have happened in sub-Saharan Africa during the past forty years (Collins and Bosworth, 1996).

We now study sustainability in the context of two models of exogenous productivity growth.

10.1 Labour-augmenting Technical Progress

Consider an adaptation of the model explored in Section 3. Physical capital and a constant labour force together produce a non-deteriorating all purpose commodity. The economy enjoys labour augmenting technological progress at a constant rate n. If K is capital and A is knowledge, we have in the usual notation,

$$Y_t = F(K_t,A_t), \tag{82}$$

$$dK_t/dt = F(K_t,A_t) - C_t, \tag{83}$$

and $$dA_t/dt = nA_t. \tag{84}$$

There are two capital goods, K and A. Let p_K and p_A, respectively, be their accounting prices in utility numeraire. The sustainability criterion is then $p_K dK_t/dt + p_A dA_t)/dt \geq 0$, or, equivalently,

$$dK_t/dt + q_t dA_t/dt \geq 0, \text{ where } q_t \equiv p_A/p_K. \tag{85}$$

It is instructive to study the case where the resource allocation mechanism is optimal. The equations of motion for p_K and p_A are,

$$dp_K/dt = \delta p_K - p_K \partial F/\partial K, \tag{86}$$

and $$dp_A/dt = \delta p_A - p_K \partial F/\partial A - np_A. \tag{87}$$

Using (85)-(87) yields,

$$dq_t/dt = (\partial F/\partial K - n)q_t - \partial F/\partial A. \tag{88}$$

Suppose F displays constant returns to scale. Define k = K/A and c = C/A. Write $f(k) \equiv F(k, 1)$. From (83) and (84) we have

$$dk_t/dt = f(k_t) - nk_t - c_t,$$

or $$dk_t/dt = (\partial F/\partial K)k_t + \partial F/\partial A - nk_t - c_t. \tag{89}$$

Adding (88) and (89) yields

$$d(q_t+k_t)/dt = (\partial F/\partial K-n)(q_t+k_t) - c_t. \tag{90}$$

It is simple to confirm that q+k is the present value of future consumption (discounted at the rate $\partial F/\partial K$) divided by A (the current state of knowledge). It follows that the sustainability criterion at t (condition (85)), divided by A_t, is

$$dk_t/dt + n(k_t+q_t) \geq 0. \tag{91}$$

10.2 Resource Augmenting Technical Progress

Consider an alternative world, where output, Y, is a function of manufactured capital (K) and the flow of an exhaustible natural resource (R). Let A_tR_t be the effective supply of the resource in production at t and S_t the resource stock at t. Then we may write,

$$Y_t = F(K_t,A_tR_t), \tag{92}$$

$$dK_t/dt = F(K_t,A_tR_t) - C_t, \tag{93}$$

$$dA_t/dt = n, \tag{94}$$

$$dS_t/dt = -R_t. \tag{95}$$

There are three state variables. But we can reduce the model to one with two state variables. Thus, write $Q_t \equiv A_tR_t$ and $X_t = A_tS_t$. Then (93) and (94) become,

$$dK_t/dt = F(K_t,Q_t) - C_t, \tag{96}$$

and $$dX_t/dt = nX_t - Q_t. \tag{97}$$

This is equivalent to a renewable resource problem, and the steady state is the Green Golden Rule, with

$$nX = Q. \tag{98}$$

Let p_K and p_X be the accounting prices of K_t and X_t, respectively. Then the sustainability condition is,

$$p_K dK_t/dt + p_X dX_t/dt \geq 0. \tag{99}$$

It is instructive to study the case where the resource allocation mechanism is optimal. Suppose also that F displays constant returns to scale. Following the approach of the previous example, let $q_t = p_X/p_K$. Then it is easy to confirm that

$$(dq_t/dt)/q_t = \partial F/\partial K - n. \tag{100}$$

Moreover, the optimal use of the productivity adjusted natural resource, Q_t, is determined by the condition,

$$\partial F/\partial Q = q_t. \tag{101}$$

Along the optimal programme, the sustainability condition (99) is,

$$F(K_t,Q_t) - C_t + q_t(nX_t - Q_t) \geq 0, \tag{102}$$

or $$(\partial F/\partial K)K_t + (\partial F/\partial Q)Q_t - C_t + q_t(nX_t - Q_t) \geq 0, \tag{103}$$

or $$(\partial F/\partial K)K_t - C_t + nq_tX_t \geq 0. \tag{104}$$

Inequality (104) says that consumption must not exceed the sum of capital income and the sustainable yield.

11 Exhaustible Resources: the exporting economy

The export of natural resources at given world prices raises issues similar to those we have just encountered in our analysis of exogenous productivity change. The exogenous "drift" term, $\partial V_t/\partial t$, in equation (8) has to be estimated.

Assume that extraction is costless. Suppose that at time τ the world market price of an exhaustible resource is q_τ. If R_τ is the volume of export, revenue is $q_\tau R_\tau$.

Write $C_\tau = q_\tau R_\tau$. (105)

The country's export policy, being governed by the underlying α, can be expressed as $R(\tau,S_t,t)$ for $\tau \geq t$. From equation (105) it follows that

$$dC_\tau/dt = q_\tau dR_\tau/dt = (\partial C_\tau/\partial S_t)dS_t/dt + q_\tau \partial R_\tau/\partial t, \tag{106}$$

As before, we assume that social welfare at t is,

$$V_t = {}_t\!\int^\infty U(C_\tau)e^{-\rho(\tau-t)}d\tau. \tag{107}$$

Let p_t denote the resource's accounting price. Since the criterion for sustainable well-being is dV_t/dt, we differentiate both sides of equation (107) with respect to t to obtain,

$$dV_t/dt = -U(C_t) + \rho V_t + {}_t\!\int^\infty U'(C_\tau)[(\partial C_\tau/\partial S_t)dS_t/dt + q_t\partial R_\tau/\partial t]e^{-\rho(\tau-t)}d\tau. \tag{108}$$

But

$$dS_t/dt = -R_t.$$

Therefore, equation (108) reduces to

$$dV_t/dt = -U(C_t) + \rho V_t + p_t dS_t/dt + {}_t\!\int^\infty U'(C_\tau)e^{-\rho(\tau-t)}(\partial C_\tau/\partial t)d\tau. \tag{109}$$

Define $\mu(\tau,t) = \partial C_\tau/\partial \tau + \partial C_\tau/\partial t$. (110)

$\mu(\tau,t)$ can be regarded as an index of the extent to which the resource allocation mechanism is non-autonomous. Using equations (105)-(107) and (110), equation (109) can be reexpressed as,

$$dV_t/dt = -U(C_t) + \rho V_t + p_t dS_t/dt + {}_t\!\int^\infty U'(C_\tau)e^{-\rho(\tau-t)}\mu(\tau,t)d\tau - {}_t\!\int^\infty U'(C_\tau)e^{-\rho(\tau-t)}(\partial C\tau/\partial \tau)d\tau \tag{111}$$

On partially integrating the last term on the RHS of equation (111) and cancelling terms, we obtain,

$$dV_t/dt = p_t dS_t/dt + {}_t\!\int^\infty U'(C_\tau)e^{-\delta(\tau-t)}\mu(\tau,t)d\tau. \tag{112}$$

The integral on the RHS of (112) is the "drift" term. As (112) shows, the index of sustainable welfare is the algebraic sum of genuine investment and the drift term. We now proceed to obtain simple rules for estimating the index in the case of two special non-optimum resource allocation mechanisms.[36]

[36] Asheim (1996), Sefton and Weale (1996), Vincent, Panayotou, and Hartwick (1997), Aronsson and Löfgren (1998), and Cairns (2002) have published related findings, but in the context of optimising economies.

Suppose C is constant.[37] In this case,

$\partial C_\tau/\partial\tau = \partial C_\tau/\partial t = 0,$

and $\mu(\tau,t) = 0$ in (112) is zero, and genuine investment measures changes in social welfare.

Suppose instead R is constant. It follows that

$$\partial R_\tau/\partial\tau + \partial R_\tau/\partial t = 0, \tag{113}$$

and $$\mu(\tau,t) = R_\tau \partial q_\tau/\partial\tau = q_\tau R_\tau(\partial q_\tau/\partial\tau)/q_\tau. \tag{114}$$

Using (113) and (114), we may write,

$$\int_t^\infty U'(C_\tau)e^{-\delta(\tau-t)}\mu(\tau,t)d\tau = \bar{\mu}_t/\delta, \tag{115}$$

where $\bar{\mu}_t$ can be interpreted as the average capital gains on the world market, as viewed from time t. Formally, (112) can be re-written as,

$$dV_t/dt = p_t dS_t/dt + \bar{\mu}_t/\delta. \tag{116}$$

12. Defensive Expenditure

How should defensive expenditure toward pollution control appear in national accounts? Denote by Q_t the stock of defensive capital and X_t investment in its accumulation. Let P_t be the stock of pollutants and Y_t aggregate output. We may then write,

$$dP_t/dt = G(Y_t, Q_t) - \pi P_t, \text{ where } G(Y_t, Q_t) \geq 0, \partial G/\partial Y > 0 \text{ and } \partial G/\partial Q < 0. \tag{117}$$

Moreover, if defensive capital depreciates at the rate γ (> 0), then

$$dQ_t/dt = X_t - \gamma Q_t. \tag{118}$$

In the usual notation, the accumulation equation is expressed as,

$$dK_t/dt = F(K_t) - C_t - X_t. \tag{119}$$

Denote by p_t the accounting price of K, m_t that of defensive capital, and r_t (< 0) the accounting price of the pollutant. Wealth can then be expressed as,

$p_tK_t + m_tQ_t + r_tP_t,$

and genuine investment at t as,

$$I_t = p_t dK_t/dt + m_t dQ_t/dt + r_t dP_t/dt. \tag{120}$$

Equation (120) says that defensive expenditure against pollution ought to be included in the estimation of genuine investment ($m_t dQ_t/dt$), but, then, so should changes in the quality of the environment be included ($r_t dP_t/dt$). To include the former, but not the latter, would be a mistake.

13. Population Change and Sustainable Development

How does demographic change affect the index of sustainable development? There are

[37] In this case the resource will be exhausted in finite time. For notational simplicity, we continue to present matters as though the horizon is infinite.

a number of conceptual problems inherent in the welfare economics of reproductive behaviour that still remain usettled. Such problems have typically been bypassed in growth accounting; instead, it has been customary there to regard changes in population to be exogenously given. We follow that practice here.[38]

We seek to determine how population change influences the drift term ($\partial V_t/\partial t$) on the RHS of equation (8). An equivalent way of casting the problem is to regard population as a capital asset. Once we do that, what could appear to be a non-autonomous model reduces to an autonomous one. To illustrate, we adopt a natural extension of Harsanyi (1955) by regarding social welfare to be the average utility of all who are ever born. We formalize this 'dynamic average utilitarianism' as follows:

Let N_t be population size at t and $n(N_t)$ the percentage rate of change of N_t.[39] For notational simplicity, we ignore intragenerational inequality and changes in the age composition of the population. Let c_t denote per capita consumption at t. If C_t is aggregate consumption, $c_t = C_t/N_t$. Assume as before that labour is supplied inelastically in each period. Current utility of the representative person is $U(c_t)$ and social wefare is,

$$V_t = {}_t\int^{\infty} N_\tau U(c_\tau)e^{-\delta(\tau-t)}d\tau / {}_t\int^{\infty} N_\tau e^{-\delta(\tau-t)}d\tau.^{40} \qquad (121)$$

If V_t is to be well-defined, we need to suppose that there exists $\varepsilon > 0$, such that $(\delta - \varepsilon)t > {}_0\int^t n(N_\tau)d\tau$ for large enough t. Notice though that, once we are given the population forecast, the denominator in (121) is independent of the policies that could be chosen at t. This means that a policy deemed to be <u>optimal</u> if (121) were used as the criterion of choice would also be judged to be optimal if instead social welfare V_t were taken to be of the form,

$$V_t = {}_t\int^{\infty} N_\tau U(c_\tau)e^{-\delta(\tau-t)}d\tau. \qquad (122)$$

But for assessing whether or not a pattern of development sustains V_t, it matters whether V_t is taken to be (121) or (122).

Let K_{it} denote the stock of the ith type of capital good and write $k_{it} = K_{it}/N_t$. We now express by $\mathbf{k}_t$ the vector of capital stocks per head. The state variables are therefore $\mathbf{k}_t$ and N_t. We take it that α is autonomous. Then equation (121) implies that

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[38] For a discussion of such problems and possible resolutions to the paradoxes that normative population theory has given rise to, see Dasgupta (2001b).

[39] If N_t is a logistic function, $n(N_t) = A(N^*-N_t)$, where A and N* are positive constants.

[40] See Dasgupta (2001b) for a justification of this form of intergenerational welfare.

$V_t = V(\mathbf{k}_t, N_t)$. (123)

Let the numeraire be utility. Define $\nu_t = \partial V_t/\partial N_t$. It is the contribution of an additional person at t to social well-being. ν_t is the accounting price of a person (as distinct from the accounting price of a person's human capital). Note that ν_t can be negative, depending on initial conditions at t and on the resource allocation mechanism.

Let p_{it} denote the accounting price of k_{it}. Equation (123) then implies

$$dV_t/dt = \Sigma_i p_{it} dk_{it}/dt + \nu_t dN_t/dt. \quad (124)$$

The RHS of equation (124) is genuine investment, inclusive of the change in the size of the population. It generalizes equation (8). We conclude that Proposition 1 remains valid so long as wealth comparisons mean comparisons of wealth per capita, adjusted for demographic changes.

In Arrow, Dasgupta, and Mäler (2003), we have studied optimal economies in which the adjustment term ($\nu_t dN_t/dt$) is not negligible, but nevertheless can be estimated in a simple way. Dasgupta (2001b) identified a set of circumstances where the term vanishes even in an imperfect economy. Suppose (i) $n(N_t)$ is independent of N_t; (ii) all the production processes are linear; and (iii) $c_t = c(\mathbf{k}_t)$, meaning that under the resource allocation mechanism α, per capita consumption is not a function of population size. In such circumstances V_t is independent of N_t (i.e. $\nu_t = 0$) and, so, equation (124) reduces to

$$dV_t/dt = \Sigma_i p_{it} dk_{it}/dt. \quad (125)$$

This finding can be summarised as

Theorem 6 If (i) $n(N_t)$ is independent of N_t, (ii) all the production processes are linear, and (iii) $c_t = c(\mathbf{k}_t)$, then social welfare is sustained at a point in time if and only if the value of the changes in per capita capital assets at that instant is non-negative.

The conditions underlying Theorem 6 are overly strong. It is tempting nevertheless to regard the value of changes in the per capita stocks of capital assets as a first approximation of dV_t/dt and then to estimate correction terms that reflect departures from the conditions underlying the theorem. That investigation is left for future work.[41]

14. Uncertain Productivity

How does future uncertainty in the productivity of capital assets influence accounting prices? In order to study this question in the simplest possible way, we revert to the Ramsey-Solow model of Section 3 and assume that the productivity of the single asset is uncertain.

[41] In Dasgupta (2001b) Theorem 6 was invoked to assess whether the world's poorest regions have experienced sustainable development in the recent past.

Analytically it is easiest to imagine that the underlying stochastic process generates a return on investment that is independently and identically distributed (iid) in each period. For convenience we now suppose that time is discrete ($t = 0,1,2,...$). In what follows we indicate that a variable is random by placing a tilde over it. Let us denote the uncertain productivity of investment at date t by $\tilde{\mu}_t$. We assume that $\tilde{\mu}_t$ is non-negative and that the distribution of $\tilde{\mu}_t$ is atomless.

Population is assumed to be a constant and aggregate saving is taken to be a constant proportion, s, of wealth, where $0 < s < 1$. At each t the size of the capital stock that has been inherited from the previous period is a known quantity. Consumption is a fixed proportion (1-s) of that inherited stock. Therefore, assuming that capital does not deteriorate, the discrete time, stochastic counterpart of the accumulation equation (19) is,

$$\tilde{K}_{t+1} = (K_t - C_t)\tilde{\mu}_t,$$

from which we conclude that

$$\tilde{K}_{t+1} = s\tilde{\mu}_t K_t, \qquad t \geq 0,$$

and thus,

$$\tilde{C}_\tau = (1-s)K_t[{}_t\Pi^{(\tau-1)}(s\tilde{\mu}_k)], \qquad \tau > t \geq 0. \tag{126}$$

Writing by U(C) the utility of consumption, we take it that social welfare (V) is the expected value of the sum of discounted utilities over time. Letting E denote the expectation operator, this means that

$$V_t = E[{}_t\Sigma^\infty U(\tilde{C}_\tau)\beta^{(\tau-t)}], \text{ where } \beta \equiv 1/(1+\delta) \text{ and } \delta > 0. \tag{127}$$

Suppose utility is iso-elastic. Let η be the elasticity of marginal utility. We consider the empirically interesting case, $\eta > 1$. We write U as:

$$U(C) = C^{1-\eta}/(1-\eta), \qquad \text{where } \eta > 1. \tag{128}$$

In (128), U is bounded above, but is unbounded below.

Write $E(\tilde{\mu}_t^{(1-\eta)}) = E(\tilde{\mu}^{(1-\eta)})$. If V_t is to be well-defined, we must now suppose that

$$\beta s^{(1-\eta)}E(\tilde{\mu}^{(1-\eta)}) < 1. \tag{129}$$

Using (126) and (128), and noting that the series in (127) is absolutely convergent, we can rewrite (127) as

$$V_t = -(1-s)^{(1-\eta)}K_t^{(1-\eta)}/(\eta-1)[1-\beta s^{(1-\eta)}E(\tilde{\mu}^{(1-\eta)})],$$

and, so, deduce that the asset's accounting price is

$$p_t = \partial V_t/\partial K_t = (1-s)^{(1-\eta)}K_t^{-\eta}/[1-\beta s^{(1-\eta)}E(\tilde{\mu}^{(1-\eta)})]. \tag{130}$$

How would changes in the distribution of $\tilde{\mu}_\tau$ ($\tau \geq t$) affect p_t? To study this, imagine that $\log(\tilde{\mu}_t)$ is normally distributed with mean m and variance σ^2. Denote the mean of $\tilde{\mu}_t$ by $\bar{\mu}$. In that case, we know that

$$\bar{\mu} = \exp(m + \sigma^2/2), \tag{131}$$

$$E(\tilde{\mu}^{(1-\eta)}) = \bar{\mu}^{(1-\eta)}\exp(-\eta(1-\eta)\sigma^2/2), \tag{132}$$

and $$\mathrm{var}(\tilde{\mu}) = \bar{\mu}^2[\exp(\sigma^2) - 1]. \tag{133}$$

From (130)-(133) we confirm that, holding $\mathrm{var}(\tilde{\mu})$ constant, $dp_t/d\bar{\mu} < 0$. To study the effect of an increase in $\mathrm{var}(\tilde{\mu})$ on p_t, while keeping $\bar{\mu}$ constant, we must allow σ to increase in such a way that $(m+\sigma^2/2)$ remains unchanged. It is now a simple matter to confirm that $\partial p_t/\partial(\sigma^2) > 0$. And so, we have

Theorem 7 <u>Other things the same, (i) if the expected return on investment were to increase, the assets' accounting price would decrease, and (ii) if the underlying risk in the asset's productivity were to increase, so would its accounting price increase</u>.

Part (i) of Theorem 7 says that an increase in the expected rate of return on investment would lead to a decrease in the asset's accounting price, other things the same. But Part (ii) is also consistent with intuition. From (128) we know that utility, while bounded above, is unbounded below. We would then expect V_t to be particularly sensitive to the downside risk in $\tilde{\mu}$. Part (ii) of Theorem 7 says that if the risk in $\tilde{\mu}$ were to increase, the asset (at the margin) would become more valuable - other things the same. The Theorem's message should be expected to be even stronger if the underlying transformation possibilities among goods and services were to display thresholds, or, more generally, ecological non-convexities of the kind that is present in the model of the shallow lake (Section 4).[42]

Of course, consumers could be expected to respond to an increase in the mean return on investment, or to an increase in uncertainty in the return. What would be their response? We cannot tell unless we model the economic environment in which various parties make their saving decisions. The simplest place to look is an environment where the saving rate is optimal. There, people's response to a change in risk is also optimal. Levhari and Srinivasan (1969) have shown that in the model economy being studied here, if U is homogeneous of degree $(1-\eta)$ in C, the optimal saving ratio (s^*) is the solution of the equation,

$$s^\eta = \beta E(\tilde{\mu}^{(1-\eta)}). \tag{134}$$

Let us continue to assume that $\eta > 1$. From (130) and (134) we conclude that if the saving rate

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[42] The reader can confirm that if $0 < \eta < 1$ in (128), then $dp_t/d\bar{\mu} > 0$ and $dp_t/d\sigma^2 < 0$; and if $\eta = 1$ (i.e., $U(C) = \log C$), then $dp_t/d\bar{\mu} = dp_t/d\sigma^2 = 0$. See the following footnote for an intuitive explanation for these results.

is optimal, then, other things the same, an increase in the expected return on investment leads to a decline in the accounting price of capital (i.e., $dp_t/d\bar{\mu} < 0$), and an increase in the riskyness of return leads to an increase in the accounting price (i.e., $dp_t/d\sigma^2 > 0$).[43]

Accounting prices of capital assets (as opposed to their market prices) are rarely estimated; but when they are, the estimates are mostly made on the basis of economic models that eschew uncertainty. The general moral of our finding here is that such studies underestimate the social worth of those assets.

15 Concluding Remarks

In this paper we have explored the way welfare analysis can be conducted in imperfect economies. In Sections 2-3 it was confirmed that the same set of accounting prices should be used both for the evaluation of policy reforms (e.g., project evaluation) and for assessing whether the economic programme being pursued sustains intergenerational welfare. In Sections 5-14 we studied the properties of accounting prices of environmental natural resources under a variety of institutional arrangements. We showed that for a number of cases it is possible to derive simple formulae for accounting prices. It was found that under plausible values of the relevant parameters, accounting prices of goods and services can be substantially different from their market prices.

A large empirical literature in ecology and epidemiology offers evidence that ecological processes are driven by non-convex transformation possibilities.[44] We note here in passing that metabolic processes also involve non-convex functional relationships between nutrition intake and nutritional status.[45] It was confirmed that accounting prices can be used in non-convex environments (Section 4). Our hope is that the methods developed here will be of use not only in environmental and resource economics (our focus of concern here), but also in nutrition and epidemiological studies.

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[43] The reader can confirm that if $0 < \eta < 1$ in (128) and (134), then $dp_t/d\bar{\mu} > 0$ and $dp_t/d\sigma^2 < 0$. To understand the result, note that if $0 < \eta < 1$, then U is unbounded above, but bounded below.

$\eta = 1$ corresponds to the case where $U(C) = \log C$. In this case s^* is independent of both $\bar{\mu}$ and σ^2, and so $dp_t/d\bar{\mu} = dp_t/d\sigma^2 = 0$. The opposite pulls arising from the unboundedness of U at both ends cancel each other. See Hahn (1970) for an intuitive explanation for the way η influences the relationship between σ and s^*.

[44] See, for example, Murray (1993).

[45] On this see Dasgupta (1993).

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