Commitment, Transparency and Monetary Policy*

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Abstract

Conventional wisdom says that commitment eliminates the inflationary bias of monetary policy. However, this paper shows that the inflation bias can persist even when the central bank commits. A simple model is presented in which the central bank precommits by setting the policy instrument, and the subsequent adjustment of inflation expectations is part of the transmission mechanism. Generally there is still an inflation bias, despite the absence of a time-inconsistency problem. It is caused by uncertainty about the economic disturbances to which the central bank responds. Only perfect transparency about economic information completely eliminates the inflation bias.
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1 Introduction

Following the seminal work three decades ago by Kydland and Prescott (1977) on rules versus discretion, the literature on monetary policy has devoted much attention to the inflationary bias of discretionary policy. As is well known, it arises when monetary policymakers are unable to commit themselves and aim to stimulate output beyond the natural rate. The literature suggests that the inflation bias vanishes when the central bank commits because people are able to incorporate its policy into their expectations. This paper shows, however, that the inflationary bias of monetary policy typically persists even if the market updates its expectations after it has observed the central bank's policy action. The inflation bias only disappears when there is transparency about the economic shocks that affect inflation and are reflected in the policy instrument. This suggests that the essence of the solution is not tying one's hands but being transparent about one's actions. This has important implications for the design of monetary policy and the debate about rules versus discretion.

Intuitively, when the market observes the central bank's policy action to form its inflation expectations, it is still unsure of the policy outcome because of economic disturbances. Although the policy instrument provides a signal of the central bank's intentions, uncertainty about the economic shocks to which the central bank responds makes the signal noisy, which provides scope for the central bank to create surprise inflation and boost output. The public anticipates this and increases its inflation expectations. The central bank takes this into account and pursues a higher level of inflation than socially optimal. However, transparency about the economic shocks removes the opportunity for surprise inflation and therefore eliminates the inflation bias.

The model is in the tradition of strategic monetary policy games. However, it constitutes a significant departure from the framework introduced by Kydland and Prescott (1977) and formalized by Barro and Gordon (1983*a*). In these models, the central bank takes the inflation expectations of the private sector as given, either strategically (in a Nash equilibrium), or due to the timing of events (with expectations incorporated in contracts). In addition, private sector expectations are effectively fixed while the central bank's actions take effect. Given the long transmission lags conventionally associated with monetary policy, this implies that people's expectations are not adjusted for up to one or two years. This assumption seems unrealistic. Instead, this paper assumes that people form their inflation expectations after the central bank sets the policy instrument. Thus, it considers the extensive-form game in which the central bank is Stackelberg leader.

There is an extensive literature on the inflation bias. Kydland and Prescott (1977) suggest central banks should abandon discretionary policy and commit to rules. The contribution of this paper is to show that commitment is *not* sufficient to eliminate the inflation bias when there is asymmetric information about the economic disturbances reflected in the policy instrument. The role of private information under commitment is also addressed by Canzoneri (1985). He analyzes flexible targeting rules that allow the central bank to respond to private information but reduce the inflation bias caused by dynamic inconsistency. Like Barro and Gordon (1983*b*), he relies on reputation effects generated by retaliating trigger strategies. This paper, however, analyzes how private information gives rise to an inflation bias when the central bank commits to a policy action every period, so there is no time-inconsistency problem. In addition, it follows the reputation literature of signaling and rational updating of people's expectations based on the central bank's actions (Backus and Driffill 1985, Barro 1986).

The present paper also underscores the salient role of transparency. The effects of transparency on the inflation bias are similar to the results obtained in models with dynamic inconsistency. Faust and Svensson (2001) show that greater transparency about control errors, which improves the interpretation of policy outcomes, tends to reduce the inflation bias. Geraats (2005) analyzes transparency about the economic information reflected in policy actions, like the present paper. For a more extensive survey of the literature on central bank transparency, see Geraats (2002).

The inflation bias in this paper can be eliminated by addressing its sources: the central bank's preferences or asymmetric information on the economy. To the extent that complete economic transparency is not feasible, society could appoint 'conservative' central bankers that put less weight on output stimulation (Rogoff 1985) or pursue a lower inflation target (Svensson 1997), or 'responsible' central bankers that do not aim to stimulate output beyond its natural rate (Blinder 1997) . Another possibility is to have incentive schemes or contracts for central bankers (Walsh 1995, Persson and Tabellini 1993).

Most of the literature has considered strategic monetary policy games in which the private sector and the central bank move either simultaneously, or sequentially with the private sector acting first. An exception is Goodhart and Huang (1998) who analyze an infinite-horizon model with policy lags, output persistence and/or overlapping wage contracts. They implicitly assume preference uncertainty but economic transparency. They show that a model with merely monetary policy lags eliminates the inflation bias. The present paper explains that this no longer holds when there is some economic uncertainty.

An exception to the usual Nash strategy in monetary policy games is presented by Başar and Salmon (1990). They adopt the model by Cukierman and Meltzer (1986), a repeated simultaneous-move game with private information on the central bank's preferences, but analyze the Stackelberg solution in which the central bank acts as the (strategic) leader. They presume that the central bank precommits every period to a policy rule that depends on its unobservable type and they find that the inflation bias is zero on average. In contrast, the present paper features an extensive-form game in which the central bank commits to a policy action as the Stackelberg leader but still has an incentive to create excessive inflation because asymmetric information on economic disturbances makes the policy action a noisy signal of its intentions.

The basic model in which commitment gives rise to an inflation bias is presented in section 2. It is a simple static model that features a neo-monetarist, Lucas-type transmission mechanism. Section 3 shows that the conclusion is robust to several extensions, including a traditional Keynesian interest rate transmission mechanism and an infinite-horizon model with a New Keynesian Phillips curve. The results are discussed in section 4. Section 5 concludes that in the presence of transmission lags, central banks do not need policy rules to eliminate the inflation bias. Instead, economic transparency about the shocks to which they are responding suffices while maintaining flexibility.

2 Model

The central bank has the objective function

$$W = -\frac{1}{2}(\pi - \tau)^2 + \beta y \tag{1}$$

where π is inflation, τ the central bank's inflation target, y the output gap, and β the relative weight on output stimulation ($\beta > 0$). The central bank's inflation target τ is stochastic: $\tau \sim N(\bar{\tau}, \sigma_\tau^2)$ with $\sigma_\tau^2 > 0$. The economic structure is determined by the quantity equation

$$\pi = m + v \tag{2}$$

and the Lucas supply equation

$$y = b(\pi - \pi^e) + s \tag{3}$$

where m denotes money supply growth and π^e the market's inflation expectations; s is an aggregate supply shock and v can be interpreted as a velocity shock. The economic disturbances are stochastic: $s \sim N(0, \sigma_s^2)$ and $v \sim N(0, \sigma_v^2)$, with $\sigma_s^2 > 0$ and $\sigma_v^2 > 0$; and τ, s and v are assumed to be independent. The parameter b is the extent to which surprise inflation stimulates output.

The timing is as follows. Nature draws the central bank's inflation target τ and the economic shocks s and v, which are only known to the central bank. The central bank sets the money supply growth m. Subsequently, the public observes money supply growth, and it forms its inflation expectations π^e. Finally, inflation π and the output gap y are realized.

There is asymmetric information about the central bank's preferences. So, the public uses the money supply m to infer the central bank's type τ. This is complicated by the presence of asymmetric information about the economic disturbances s and v. It is assumed that people have rational expectations. Formally, the information set available to the public when it forms its inflation expectations π^e equals $\{m, \Omega\}$, where $\Omega \equiv \{\beta, b, \bar{\tau}, \sigma_\tau^2, \sigma_s^2, \sigma_v^2\}$ summarizes the structure and parameters of the model. Moment operators are implicitly conditional on Ω.

2.1 Solution

To find the solution to this game, it is crucial to know how the public's inflation expectations π^e are affected by the central bank's actions m. It is postulated that

$$\pi^e = u_0 + u_m m \tag{4}$$

It is shown below that this is consistent with rational expectations. The central bank maximizes the objective function (1) with respect to m subject to (3) and (2), and incorporating the updating of inflation expectations (4). The first order condition with respect to m implies

$$m = \tau + (1 - u_m)\,\beta b - v \tag{5}$$

Money supply is increasing in the central bank's inflation target τ and decreasing in the velocity shock v. It does not depend on the supply shock s because the central bank does not aim to stabilize output with its objective (1). Using (2) gives

$$\pi = \tau + (1 - u_m)\,\beta b \tag{6}$$

The economic interpretation of this equation is that it equalizes the marginal costs and benefits of an increase in the money supply m. The marginal cost in terms of higher inflation is $\pi - \tau$; the marginal benefit from the stimulation of output equals $\beta b\,(\mathrm{d}\,\pi/\,\mathrm{d}\,m - \mathrm{d}\,\pi^e/\,\mathrm{d}\,m) = \beta b\,(1 - u_m)$.

Notice that the usual inflationary bias of discretionary monetary policy in the Kydland and Prescott (1977) and Barro and Gordon (1983*a*) model, $\pi = \tau + \beta b$, obtains if $u_m = 0$. This could be because the central bank is myopic in the sense that it fails to incorporate the effect of its actions m on the public's inflation expectations π^e. Alternatively, the public may not be able to use the policy instrument to update its inflation expectations, like in a simultaneous-move game, so that $\mathrm{d}\,\pi^e / \mathrm{d}\,m = 0$. In the case of commitment to a money supply rule, the public adjusts its inflation expectations fully, $\mathrm{d}\,\pi / \mathrm{d}\,m = \mathrm{d}\,\pi^e / \mathrm{d}\,m = 1$, so there is no inflation bias.

Rational expectations imply that $\pi^e = \mathrm{E}\left[\pi | m\right]$. Substituting (2) gives

$$\pi^e = m + \mathrm{E}\left[v | m\right] \tag{7}$$

Although the public forms its inflation expectations after the central bank moves, the policy action m is not fully informative about the policy outcome π because the public does not observe the velocity shock v. However, the public realizes that the money supply reflects the central bank's knowledge of the shock, and it tries to infer the velocity shock v from the money supply m. Since m has a normal distribution by (5), (7) produces

$$\pi^e = \mathrm{E}\left[v\right] + \left(1 + \frac{\mathrm{Cov}\left\{v, m\right\}}{\mathrm{Var}\left[m\right]}\right) m - \frac{\mathrm{Cov}\left\{v, m\right\}}{\mathrm{Var}\left[m\right]} \mathrm{E}\left[m\right] \tag{8}$$

Note that (8) corresponds to the postulated updating equation (4), so this is a rational expectations equilibrium. It follows from (5) that $\mathrm{Cov}\left\{v, m\right\} = -\sigma_v^2$ and $\mathrm{Var}\left[m\right] = \sigma_\tau^2 + \sigma_v^2$. Matching coefficients with (4) yields

$$u_m = \frac{\sigma_\tau^2}{\sigma_\tau^2 + \sigma_v^2}$$

so $0 < u_m < 1$.[1] This updating coefficient u_m suggests that the signal extraction problem can be recast in another way. Inflation π depends on the unknown inflation target τ by (6). To form its inflation expectations, the public uses the noisy signal m to infer τ. The updating coefficient u_m is positive as people ascribe a higher money supply to a higher inflation target and therefore expect a higher level of inflation. The magnitude of the updating coefficient reflects the accuracy of the signal m and is increasing in the signal-to-noise ratio $\sigma_\tau^2 / \sigma_v^2$.

Using (5) and (8) gives the public's inflation expectations

$$\pi^e = \bar{\tau} + \frac{\sigma_\tau^2}{\sigma_\tau^2 + \sigma_v^2} \left(\tau - \bar{\tau}\right) - \frac{\sigma_\tau^2}{\sigma_\tau^2 + \sigma_v^2} v + \frac{\sigma_v^2}{\sigma_\tau^2 + \sigma_v^2} \beta b \tag{9}$$

[1] For completeness, $u_0 = \frac{\sigma_v^2}{\sigma_\tau^2 + \sigma_v^2} \bar{\tau} + \left(\frac{\sigma_v^2}{\sigma_\tau^2 + \sigma_v^2}\right)^2 \beta b$.

A central bank with a higher inflation target τ causes higher inflation expectations, but not to the full extent because the money supply only provides a noisy signal. An increase in the velocity shock v reduces inflation expectations because the decrease in the money supply is partly attributed to a lower inflation target.

Substituting u_m into (6) gives the level of inflation

$$\pi = \tau + \frac{\sigma_v^2}{\sigma_\tau^2 + \sigma_v^2}\beta b \tag{10}$$

Clearly, there is an inflation bias even though there is no time-inconsistency problem. Although the central bank moves first, it is still able to cause surprise inflation because of the presence of asymmetric information about the velocity shock. People anticipate this and increase their inflation expectations for any level of the money supply. To prevent a drop in output, the central bank has to increase the money supply, which gives rise to the inflation bias.

Finally, using (10), (9) and (3), the output gap equals

$$y = \frac{\sigma_v^2}{\sigma_\tau^2 + \sigma_v^2} b\left(\tau - \bar{\tau}\right) + \frac{\sigma_\tau^2}{\sigma_\tau^2 + \sigma_v^2} bv + s \tag{11}$$

A central bank that has a higher than expected inflation target ($\tau > \bar{\tau}$) succeeds in boosting the output gap. However, rational expectations ensure that $\mathrm{E}\left[y\right] = 0$ so that the expected level of output equals the natural rate.

It follows from (10) that the inflation bias in this Stackelberg game has its source in (a) the objective to stimulate output beyond the natural rate ($\beta > 0$), just like in the Nash game; and (b) asymmetric information on the economic disturbances that affect inflation and are reflected in the policy instrument ($\sigma_v^2 > 0$), or simply lack of 'economic transparency'.[2] The size of the inflation bias depends on the degree of economic uncertainty σ_v^2 and preference uncertainty σ_τ^2.

A lower variance of velocity shocks σ_v^2 reduces the inflation bias since there is less opportunity for surprises. An alternative explanation is that a reduction in the uncertainty about the economic disturbances that affect the central bank's actions makes the policy instrument a more accurate signal of the inflation target. So, people adjust their inflation expectations more in response to the policy instrument. This makes it more beneficial for the central bank to mimic the behavior of a low-inflation type. As a result, the inflation bias is lower. This argument is analogous to the effect of reputation in

[2]In case of a monetary transmission mechanism, (2) implies that only the velocity shock v matters so that uncertainty about the supply shock s is immaterial. However, when the central bank faces a real-interest rate transmission mechanism, economic transparency requires symmetric information about both aggregate demand and supply shocks (see appendix A.2).

dynamic monetary policy games: The central bank changes its current actions to affect (future) inflation expectations and obtain a more favorable output-inflation trade-off.

Similarly, greater uncertainty about the central bank's preferences σ_τ^2 increases the responsiveness of inflation expectations to the money supply. This reduces the payoff to increasing the money supply and leads to a lower inflation bias. In the limit, as $\sigma_\tau^2 \to \infty$, the inflation bias vanishes. On the other hand, for $\sigma_v^2 \to \infty$ the money supply becomes so unreliable that people no longer pay attention to it. As a consequence, the simultaneous-move outcome obtains: $\pi = \tau + \beta b$.

In short, there is an inflation bias despite the fact that the central bank commits. The size of the inflation bias increases with economic uncertainty and decreases with preference uncertainty faced by the private sector.

2.2 Transparency

It is also interesting to consider the special cases of economic transparency and/or preference transparency. Transparency of monetary policy refers to a situation of symmetric information about (aspects of) the monetary policymaking process (Geraats 2002). In particular, the private sector observes the economic disturbances to which the central bank responds (v) under economic transparency, and the inflation target (τ) under preference transparency. So, it faces less uncertainty when forming its inflation expectations π^e. Note that the special case of $\sigma_v^2 = 0$ implies economic transparency and $\sigma_\tau^2 = 0$ corresponds to preference transparency. The formal derivations of the transparency cases are in appendix A.1.

In the case of economic transparency, the policy instrument m is a perfect signal of the central bank's type, so $u_m = 1$. This provides the maximum incentive for the central bank to reduce the money supply to lower inflation expectations. It appears that the disadvantage of higher inflation expectations exactly offsets the temptation to boost output by creating surprise inflation. As a result, there is no inflation bias. An alternative interpretation is that the absence of economic uncertainty eliminates the possibility of surprise inflation and thereby the inflation bias.

In the case of preference transparency, the private sector directly observes the central bank's inflation target τ. When there is some economic uncertainty, the public no longer relies on the noisy policy instrument m to update its expectations on τ, so $u_m = 0$. This means that the outcome is the same as in the simultaneous-move game: $\pi = \tau + \beta b$. Intuitively, the central bank realizes that people do not pay attention to its policy actions, so it feels tempted to generate inflation surprises. The public anticipates this and increases its inflation expectations accordingly. The result is the full inflation bias. From an economic perspective, the central bank can no longer benefit from the reputation effect of a reduction in the money supply, so there is nothing to counteract the incentive to create surprise inflation.

Finally, there is the case of perfect economic and preference transparency. Given that merely economic transparency gives no inflation bias, but merely preference transparency gives the full bias, this really is a knife-edged case. Appendix A.1.3 shows that the combination of economic and preference transparency eliminates the inflation bias. Intuitively, although preference transparency removes the possibility of mimicking, the presence of economic transparency makes inflation surprises impossible.

However, as shown in the appendix, this outcome of no inflation bias is extremely sensitive to the assumptions made. Assuming that the public incurs tiny costs associated with the verification of a state variable gives the worst outcome of a full inflation bias. Introducing the slightest economic uncertainty, the Stackelberg outcome also produces the full inflation bias.[3] The only way in which the result is robust is that introducing some preference uncertainty does not affect the outcome.

In all (economic and/or preference) transparency cases considered, the public has perfect foresight: $\pi^e = \pi$. It may be tempting to simply substitute (3) and $\pi^e = \pi$ into (1) to get $W = -\frac{1}{2}(\pi - \tau)^2 + \beta s$ and conclude that $\pi = \tau$. However, this reasoning is too simplistic. First, the central bank does not directly control inflation π but only its policy instrument m. Furthermore, the optimal setting of its policy instrument given by (5) depends on *how* the private sector forms its inflation expectations. To be precise, the relevant objective is $W = -\frac{1}{2}(m + v - \tau)^2 + \beta b(m + v - \pi^e) + \beta s$ and the optimal policy m depends on $\mathrm{d}\,\pi^e/\,\mathrm{d}\,m$. The inflation bias only disappears ($\pi = \tau$) for $\mathrm{d}\,\pi^e/\,\mathrm{d}\,m = 1$. The full inflation bias ($\pi = \tau + \beta b$) obtains for $\mathrm{d}\,\pi^e/\,\mathrm{d}\,m = 0$. This means that the public does not use the policy instrument m to update its inflation expectations π^e. This occurs when there is preference transparency with economic uncertainty. It is also the outcome of the model if the central bank and public were to move simultaneously, or if the public were to form its inflation expectations before the central bank sets monetary policy.

To summarize, when the central bank commits to a policy action, there is still an inflation bias whenever there is lack of economic transparency in the sense that the private sector is uncertain about the economic disturbances to which the central bank

[3] This reflects a more general property of Stackelberg outcomes. Bagwell (1995) shows that the first-mover advantage that prevails in games of perfect information vanishes when the follower observes the leader's action with even a slight amount of imprecision.

responds. In that case, less uncertainty about the central bank's preferences actually increases the inflation bias. On the other hand, less uncertainty about the economic shocks reduces the inflation bias, and it completely vanishes in the case of perfect economic transparency.

3 Extensions

It is important to assess whether the conclusions of the basic model in section 2 are robust. So, four variations on the basic model are considered.

First, the neo-monetarist transmission is replaced by a traditional Keynesian interest rate mechanism for which the monetary policy instrument is the nominal interest rate i. As before, the monetary policy action i reflects the central bank's inflation target τ and the economic shocks to which the central bank responds, which are now aggregate demand shocks d and aggregate supply shocks s. Appendix A.2 shows that the level of inflation is given by

$$\pi = \tau + \frac{\sigma_d^2 + \sigma_s^2}{\sigma_d^2 + \sigma_s^2 + b^2\sigma_\tau^2}\beta b$$

Clearly, there is an inflation bias ($\pi > \tau$) despite the fact that the central bank commits to a policy action that is incorporated in private sector inflation expectations. Again, the reason is opacity about the economic shocks to which the central bank responds, which makes the policy instrument i a noisy signal of the central bank's intentions. Greater political uncertainty σ_τ^2 reduces the inflation bias as the public becomes more sensitive to the central bank's actions. The inflation bias disappears when the private sector faces no uncertainty about the economic shocks ($\sigma_d^2 = \sigma_s^2 = 0$). Note that for the interest rate transmission mechanism, economic transparency requires symmetric information about aggregate demand and supply shocks (d and s), since both are reflected in the policy action i.

Second, suppose that the private sector uncertainty about the central bank's preferences pertains not to the inflation target τ, but to the output gap stimulation parameter β, similar to Cukierman and Meltzer (1986). Then, people use the policy action m to infer β and update their inflation expectations. In appendix A.3 it is shown that (6) still applies and that the updating coefficient in this case also satisfies $0 < u_m < 1$. Again, there is an inflation bias ($\pi > \tau$), which disappears when the private sector faces no economic uncertainty.

Third, assume that the central bank cares about output gap stabilization and has the objective function

$$W = -\frac{1}{2}(\pi - \tau)^2 - \frac{1}{2}\beta(y - \kappa)^2 \tag{12}$$

where κ is the central bank's output gap target ($\kappa \geq 0$), which is known to the private sector. Since the central bank is now concerned about stabilizing the output gap, it adjusts the money supply to partly offset the effect of aggregate supply shocks s. So, the monetary policy action m now reflects both velocity and aggregate supply shocks (v and s). In appendix A.4 it is derived that expected inflation equals

$$\mathrm{E}[\pi] = \bar{\tau} + \beta b(1 - u_m)\kappa$$

where the updating coefficient again satisfies $0 < u_m < 1$. As before, there is an inflationary bias ($\mathrm{E}[\pi] > \bar{\tau}$). But when the private sector faces no uncertainty about the economic shocks to which the central bank responds ($\sigma_v^2 = \sigma_s^2 = 0$), $u_m = 1$ and the average inflation bias disappears.

Last but not least, the effect of economic transparency is analyzed for an infinite-horizon model with a New Keynesian Phillips curve.

3.1 New Keynesian Phillips Curve

The central bank maximizes the expected value of

$$\bar{U} = (1 - \delta)\sum_{t=1}^{\infty}\delta^{i-1}U_t$$

where δ is the subjective intertemporal discount factor ($0 < \delta < 1$) and the objective function U_t is given by

$$U_t = -\frac{1}{2}\alpha(\pi_t - \tau_t)^2 - \frac{1}{2}(1 - \alpha)(y_t - \kappa)^2 \tag{13}$$

The central bank's inflation target τ_t follows an autoregressive process:

$$\tau_t = \rho\tau_{t-1} + \eta_t \tag{14}$$

where $0 \leq \rho < 1$ and η_t is i.i.d. white noise with $\eta_t \sim N(0, \sigma_\eta^2)$. So, $\mathrm{E}[\tau_t] = 0$ and $\mathrm{Var}[\tau_t] = \sigma_\tau^2 = \frac{1}{1-\rho^2}\sigma_\eta^2$. The central bank's output gap target κ satisfies $\kappa \geq 0$ and it is assumed to be deterministic and known to the private sector.[4] The economy is described by the New Keynesian Phillips curve:

$$\pi_t = \mathrm{E}_t[\pi_{t+1}|y_t] + y_t + s_t \tag{15}$$

[4]Alternatively, it could be assumed that the inflation target τ is deterministic and the output gap target κ stochastic. This would give similar results but it makes the effect of economic transparency on the inflation bias less transparent.

where $\mathrm{E}_t[.]$ denotes the expectation of the private sector at the beginning of period t, which is (implicitly) conditional on all variables observed in periods $t-k$ for $k \in \{1, 2, ...\}$. The supply shock s_t is i.i.d. white noise with $s_t \sim N(0, \sigma_s^2)$. For simplicity it is assumed that the central bank directly controls the output gap y_t.

The timing of events is as follows. At the beginning of period t, nature draws the central bank's inflation target shock η_t and the aggregate supply shock s_t, which are only observed by the central bank. Then the central bank sets the output gap y_t, which is observed by the public. Subsequently, the public forms its inflation expectations $\mathrm{E}_t[\pi_{t+1}|y_t]$. Finally, the level of inflation π_t is realized. Monetary policy is conducted under pure discretion, but the central bank commits to either transparency or opacity about supply shocks s_t.

The private sector faces imperfect information about the supply shock s_t when it forms its inflation expectations (except when there is economic transparency). The information set available to the private sector when it forms $\mathrm{E}_t[\pi_{t+1}|y_t]$ includes the monetary policy action y_t, and the history of the output gap y_{t-k}, inflation π_{t-k}, the inflation target τ_{t-k} and supply shocks s_{t-k}, for $k \in \{1, 2, ...\}$. Note that the supply shock s_t can always be deduced by the private sector at the end of period t from y_t, π_t and $\mathrm{E}_t[\pi_{t+1}|y_t]$ using (15). This means that the private sector can also infer the inflation target τ_t at the end of period t.

The private sector rationally uses the monetary policy action y_t to form its inflation expectations for the next period. It is postulated that

$$\mathrm{E}_t[\pi_{t+1}|y_t] = u_0 + u_y y_t + u_\tau \tau_{t-1} + u_s s_t \tag{16}$$

where $u_s = 0$, unless there is economic transparency and the supply shock s_t is observed by the public at the beginning of period t.

The results for the New Keynesian case are derived in appendix A.5. On average, output is equal to its natural rate so that $\mathrm{E}[y_t] = 0$. The expected value of inflation is equal to

$$\mathrm{E}[\pi_t] = \frac{1-\alpha}{\alpha(u_y+1)}\kappa$$

So, an average there is an inflation bias ($\mathrm{E}[\pi_t] > 0$). Under economic opacity, $0 < u_y < \frac{\rho}{1-\rho}$, but in the case of economic transparency $u_y = \frac{\rho}{1-\rho}$. Intuitively, economic opacity makes the policy action y_t a noisy signal of the inflation target so the private sector is less responsive to it ($u_y < \frac{\rho}{1-\rho}$). As a result, average inflation $\mathrm{E}[\pi_t]$ is higher under economic opacity.

In the case of economic transparency,

$$\mathrm{E}\left[\pi_t\right] = \frac{1-\alpha}{\alpha}\left(1-\rho\right)\kappa$$

There is generally still a positive inflation bias, which is decreasing in the degree of persistence ρ of the inflation target. When the inflation target is white noise ($\rho = 0$), the full inflation bias $\frac{1-\alpha}{\alpha}\kappa$ emerges. Intuitively, the current policy action y_t is completely uninformative about future inflation, so private sector inflation expectations are not sensitive to it. But when the inflation target is persistent, private sector inflation expectations depend positively on the policy action y_t, which gives the central bank an incentive to pursue less expansionary monetary policy. Greater persistence ρ of the inflation target makes the policy action y_t more informative about future inflation, which increases the responsiveness of private sector inflation expectations and reduces the inflation bias. In the limiting case of a completely persistent inflation target ($\rho \to 1$) the inflation bias completely disappears under economic transparency.

4 Discussion

The monetary policy game introduced in this paper assumes that the central bank commits itself by moving first and setting the monetary policy action. This reflects the implicit assumption of transmission lags, which are considered to be significant in monetary policy. This gives the private sector the opportunity to respond to the central bank's actions, which in turn affects the policy outcome. Thus, the model captures the important feature that policymakers need to incorporate the effect of their policy actions on the public's expectations. In contrast to previous literature on reputation in a multi-period context, it is assumed that the adjustment of expectations influences the effect of *current* policy actions on the policy outcome. In other words, the response of private sector expectations is considered an integral part of the policy transmission mechanism.

The model starts from the usual premise in the time-inconsistency literature that the central bank has an objective function that is (at least locally) increasing in output. Furthermore, it is assumed that there is asymmetric information between the central bank and the private sector.

First, the private sector is uncertain about the central bank's preferences. This could be interpreted as a fundamental credibility problem inherent to the impossibility to observe intentions directly. However, the inflation bias under commitment is not caused by preference uncertainty.

Second, there is asymmetric information about economic disturbances. This is the driving force of the inflation bias. Romer and Romer (2000) provide evidence for such asymmetric information. They show that (confidential) Federal Reserve forecasts of inflation are superior to those by commercial forecasters, even at a short horizon of one or two quarters ahead. This suggests that central banks may indeed have private information about the economy. However, it should be emphasized that the results in this paper do not rely on the central bank having superior information. The only thing that matters is that the private sector perceives that there is uncertainty about the economic information that the central bank uses for its policy decisions.[5]

The model in section 2 adopts a neo-monetarist transmission mechanism in which the central bank directly controls inflation by setting money supply growth. Output can only be affected through inflation surprises. However, the signaling intuition suggests that the results do not depend on the need for inflation surprises to stimulate output; instead, they are driven by the rewards of investing in 'reputation' in the form of lower inflation expectations. This is confirmed by the extensions of the model discussed in section 3 that feature a traditional Keynesian real interest rate mechanism and a New Keynesian Phillips curve.

The paper suggests which kind of transparency is needed to eliminate the inflation bias. A central bank should disclose the economic shocks that affect its policy decision. So, the relevant information depends on the policy instrument that the central bank adopts; velocity and aggregate supply shocks for the money supply, and aggregate demand and supply shocks for the nominal interest rate. The latter could be conveniently conveyed through the publication of (conditional) central bank forecasts of output and inflation that are based on an explicit nominal interest rate (path) and private sector inflation expectations.[6]

Notice that the model presumes that there is perfect information about the structure of the economy. If there is asymmetric information about the economic model, the policy instrument typically becomes a noisy signal of the central bank's intentions and the inflation bias reappears. More generally, the inflation bias vanishes only if there is complete economic transparency, that is, symmetric information about the economic information (data, models, forecasts) on which policy actions are based.

In addition, the model assumes that the central bank is able to observe (or forecast)

[5]Although the critical updating coefficients u_m, u_i and u_y appear to depend on the actual variances of the inflation target and economic shocks (σ_τ^2, σ_v^2, σ_s^2, σ_d^2), they are actually determined by the private sector's perceived uncertainty about the shocks (Geraats 2007).

[6]This follows directly from (19) and (20).

economic shocks perfectly. Introducing unanticipated control errors and transmission disturbances would not affect the results when there is transparency about such operational shocks. But under operational opacity the New Keynesian model would no longer be analytically tractable, except when there is transparency about the economic shocks to which the central bank responds, in which case operational transparency is immaterial. In general, transparency about unanticipated shocks allows the public to infer the central bank's (past) intentions from macroeconomic outcomes. In contrast, this paper analyzes the effect of transparency about anticipated economic shocks that are reflected in the monetary policy action, which allows the public to infer the central bank's current intentions that determine (future) macroeconomic outcomes. Since monetary transmission lags tend to be considerable, the latter type of transparency is likely to be more important in practice for the formation of private sector expectations of (future) inflation. Nevertheless, in a dynamic context with repeated games under economic opacity, greater operational transparency could be useful and reduce the inflation bias like in the model by Faust and Svensson (2001).

Finally, it is important to point out how the commitment to a policy action in this paper differs from commitment to a policy rule. In both cases, the central bank moves first and decides about the setting of the policy instrument, which is observed by the public and incorporated into its inflation expectations. But commitment to a policy action preserves the flexibility associated with discretionary monetary policy because the central bank decides about the policy action in every period and has the opportunity to respond to shocks. These economic disturbances make the policy action a noisy signal of the policy outcome. The central bank has the opportunity to exploit this to cover up expansionary policy, which leads to the inflation bias.

Commitment eliminates the inflation bias only if the policy decision is made under symmetric information about the economic disturbances that are reflected in the policy instrument. This could be obtained by policy rules that are fixed for long periods (so that the shocks average out), or set well in advance (so that economic shocks cannot be anticipated), or that are only based on public information (so that surprises are not possible). Examples of the former include Friedman's fixed money growth rule and the Taylor rule for setting the interest rate. Alternatively, the inflation bias could be eliminated by discretionary monetary policy that incorporates the effect of inflation expectations as part of the monetary transmission process, combined with economic transparency such that the central bank communicates the economic disturbances to which it responds.[7]

[7]This provides formal support for critics of the time-inconsistency literature, most notably, Blinder

5 Conclusion

The time-inconsistency literature suggests that commitment eliminates the inflationary bias of discretionary monetary policy. This paper shows, however, how an inflation bias still arises when the central bank moves first and commits to a policy action, after which the private sector forms its inflation expectations before the policy action takes effect. Thus, the model captures an implicit policy lag. Moreover, it incorporates the private sector's response to policy actions into the policy transmission process.

The public uses the policy action to infer the central bank's intentions. However, economic disturbances make the policy action a noisy signal. This provides an opportunity for expansionary policy without detection and is the source of the inflation bias.

Greater transparency about the economic shocks to which the central bank responds makes the public pay closer attention to the central bank's actions to update its expectations. The central bank takes into account the effect of the private sector's inflation expectations on the policy outcome. This exerts discipline on the central bank's actions and reduces its incentive to pursue expansionary monetary policy. More economic transparency gives the central bank less scope to stray. In the case of perfect economic information, the feedback from private sector inflation expectations is so strong that it could completely offset the tendency to create an inflation bias.

As a result, commitment tends to give rise to an inflation bias unless it is based on symmetric information about the shocks that are reflected in the policy instrument. Less uncertainty about the central bank's preferences worsens the inflation bias because it reduces the need for the private sector to focus on the central bank's actions, and thereby reduces the disciplining effect of private sector expectations.

The implication for monetary policy is that central bank communication should not aim to completely eliminate uncertainty about the central bank's preferences, but instead focus on explaining its policy actions. Furthermore, this paper suggests that central banks don't necessarily need rules; instead, economic transparency could eliminate the inflation bias, while maintaining discretionary flexibility. Perhaps, this explains why in practice, central banks that redesign their monetary policy framework do not commit to policy rules, but to inflation reports.

(1998) who states that the academic debate on rules versus discretion "has been barking up the wrong – or, rather, nonexistent – trees" and has made "insufficient contact with reality", and McCallum (1995, 1997) who argues that somehow central banks can 'just do it'.

A Appendix

The appendix analyzes special cases and variations on the basic model in section 2. Section A.1 analyzes the model for the case in which there is economic, preference or perfect transparency. Section A.2 derives the results for a real-interest rate transmission mechanism. Section A.3 considers uncertainty about the central bank's preference for output stimulation. Section A.4 analyzes the result of the basic model when the central bank also cares about output stabilization. Last but not least, section A.5 derives the effect of economic transparency in an infinite-horizon model with a New-Keynesian Phillips curve.

A.1 Transparency

This section analyzes the model in section 2 in the special case of economic and/or preference transparency. It is useful to adopt a solution approach that is similar to the one in section 2 and postulate the general updating equation

$$\pi^e = u_0 + u_m m + u_\tau \tau + u_v v + u_s s \tag{17}$$

When the public updates its inflation expectations, it can only use variables it directly observes. So, in the benchmark case, $u_\tau = u_v = u_s = 0$, whereas under economic and preference transparency, $u_\tau = 0$ and $u_v = u_s = 0$, respectively.

The central bank maximizes the objective function (1) subject to (3) and (2), and incorporating (17). The first order condition with respect to m yields (5) and (6) as before:

$$\begin{aligned} m &= \tau + (1 - u_m)\,\beta b - v \\ \pi &= \tau + (1 - u_m)\,\beta b \end{aligned}$$

To compute the critical updating coefficient u_m it is necessary to specify the variables that can be used for updating in each transparency case.

A.1.1 Economic Transparency

In the case of economic transparency, the public has the same information about the economic shocks v and s as the central bank, but it cannot directly observe the inflation target τ. So, $u_\tau = 0$ and the public can use m, v and s to update its inflation expectations. Rational expectations imply $\pi^e = \mathrm{E}\left[\pi | m, v, s\right]$. The public uses the policy instrument m to deduce the inflation target τ from (5). Substituting this into (6) gives

$$\pi^e = m + v$$

Matching coefficients with the postulated updating equation (17) yields $u_0 = 0$, $u_m = 1$, $u_\tau = 0$, $u_v = 1$ and $u_s = 0$. So, this corresponds to a rational expectations equilibrium. Substituting $u_m = 1$, it follows that $m = \tau - v$, $\pi^e = \tau$ and[8]

$$\pi = \tau$$

As a result, there is no inflation bias with economic transparency. People correctly anticipate the level of inflation ($\pi^e = \pi$), so output equals $y = \bar{y} + s$.

These results could also be obtained from the expressions in section 2 by noting that under economic transparency, $\mathrm{E}[v] = v$ and $\sigma_v^2 = 0$ for the public when it updates its inflation expectations. For instance, as $\sigma_v^2 \to 0$, $u_m = \frac{\sigma_\tau^2}{\sigma_\tau^2 + \sigma_v^2}$ reduces to $u_m = 1$.

Furthermore, the conclusion that economic transparency eliminates the inflation bias even holds if the public is not sure whether the central bank actually engages in optimizing behavior. Suppose that the public is uncertain about the behavior of the central bank and therefore cannot rely on the optimizing relations (5) and (6). Then using (2), the private sector again sets $\pi^e = m + v$, so $\pi = \tau$ and there is still no inflation bias.

A.1.2 Preference Transparency

In the case of preference transparency, the public has the same information about the inflation target τ as the central bank, but it cannot directly observe the economic shocks v and s. So, $u_v = u_s = 0$ and the public can use m and τ to update its inflation expectations. Rational expectations imply $\pi^e = \mathrm{E}[\pi | m, \tau]$. The public is able to directly observe the inflation target τ, so using (6) gives

$$\pi^e = \tau + (1 - u_m)\,\beta b$$

Matching coefficients with the postulated updating equation (17) yields $u_0 = \beta b$, $u_m = 0$, $u_\tau = 1$, $u_v = 0$ and $u_s = 0$. So, this corresponds to a rational expectations equilibrium. Substituting $u_m = 0$, it follows that $m = \tau + \beta b - v$, $\pi^e = \tau + \beta b$ and

$$\pi = \tau + \beta b$$

[8]Note that $u_m = 0$ and $u_\tau = 1$ would not be consistent with a rational expectations equilibrium because the inflation target τ cannot be directly observed but only inferred from m and v. So, if the public (to its unbeknownst) were to make a measurement error while observing the money supply m, then this would clearly affect π^e, which implies $u_m \neq 0$.

As a result, preference transparency gives rise to the full inflation bias. People correctly anticipate the level of inflation ($\pi^e = \pi$), so output equals $y = \bar{y} + s$.

These results could also be obtained from the expressions in section 2 by noting that under preference transparency, $\mathrm{E}[\tau] = \tau$ and $\sigma_\tau^2 = 0$ for the public when it updates its inflation expectations.

A.1.3 Perfect Transparency

In the case of perfect economic and preference transparency, the public has the same information as the central bank about the inflation target τ and the economic shocks v and s. So, the public can use m, τ, v and s to update its inflation expectations. Rational expectations imply $\pi^e = \mathrm{E}[\pi | m, \tau, v, s]$. Clearly, the public is able to perfectly forecast inflation. But this time, it has infinitely many ways of doing so, which make the derivation of the perfect transparency case more involved.

Since the private sector has rational expectations, its implicit objective is to maximize

$$W_P = -\frac{1}{2} \mathrm{E}\left[(\pi^e - \pi)^2 | \Omega_P \right] \tag{18}$$

where $\Omega_P = \{m, \tau, v, s, \Omega\}$ denotes the information set available to the private sector when it forms its inflation expectations. The private sector adopts the updating equation (17) and incorporates the fact that the central bank's optimizing behavior implies (5) and (6), so

$$W_P = -\frac{1}{2} \left(u_0 + (u_m + u_\tau - 1)\, \tau + (u_v - u_m)\, v + u_s s - (1 - u_m)^2\, \beta b \right)^2$$

Since the updating equation should hold for any value of τ, v and s, the private sector's objective W_P is maximized for $u_0 = (1 - u_m)^2 \beta b$, $u_\tau + u_m = 1$, $u_v = u_m$ and $u_s = 0$. This means that the private sector is indifferent between using τ directly ($u_\tau = 1$), or using m and v to infer τ ($u_m = u_v = 1$), or any combination thereof. They all lead to perfect foresight: $\pi^e = \pi$. The central bank realizes this and it has the advantage of moving first. Substituting its optimal inflation outcome (6), (3) and the public's optimal expectations formation,

$$W = -\frac{1}{2} (1 - u_m)^2 \beta^2 b^2 + \beta s$$

As a result, the central bank prefers $u_m = 1$. Since it has the advantage of moving first, it acts accordingly and there is no inflation bias:

$$\pi = \tau$$

However, this outcome is not very robust. The source of this sensitivity lies in the private sector's indifference among all methods μ. Introducing slight changes to the private sector payoff or information structure could easily change the result.

Suppose the public experiences a tiny cost $\gamma > 0$ for each state variable it uses from the set $\mathcal{S} = \{m, \tau, v, s\}$. This could arise from costs of data collection or information processing. Let n denote the number of state variables from $\mathcal{S}$ that the public decides to include in its information set $\Omega_P \subset \mathcal{S} \cup \Omega$. Assume that the public's objective is to minimize the sum of the mean square forecast error and the information costs: $W_P^c = -\frac{1}{2} \mathrm{E}\left[(\pi^e - \pi)^2 |\Omega_P\right] - \gamma n$. Since the public is able to forecast inflation perfectly ($\pi^e = \pi$) in infinitely many ways, it chooses the method that relies on the smallest number of state variables.[9] So, the public only uses the inflation target τ, which amounts to $u_0 = \beta b$, $u_m = 0$, $u_\tau = 1$ and $u_v = u_s = 0$. As a result, the outcome is $\pi = \tau + \beta b$ and the inflation bias reemerges with this tiny change in the public's payoff.

Now consider another slight variation on the perfect transparency case and suppose that the private sector is a little bit uncertain about the accuracy of its observation of the velocity shock v. In particular, it believes it observes $\tilde{v} = v + \varepsilon$, where $\mathrm{E}[\varepsilon|\Omega_P] = 0$, $\mathrm{Var}[\varepsilon|\Omega_P] = \sigma_\varepsilon^2 > 0$ and $\Omega_P = \{m, \tau, \tilde{v}, s, \Omega\}$. This means that v should be replaced by $\tilde{v}$ in (17). Substituting this, (5) and (6) into (18) gives

$$W_P = -\frac{1}{2}\left(u_0 + (u_m + u_\tau - 1)\tau + (u_{\tilde{v}} - u_m)\tilde{v} + u_s s - (1 - u_m)^2 \beta b\right)^2 - \frac{1}{2} u_m^2 \sigma_\varepsilon^2$$

Then, $u_m = 0$, $u_\tau = 1$, $u_{\tilde{v}} = 0$, $u_s = 0$ and $u_0 = \beta b$. So, $\pi = \tau + \beta b$ and the inflation bias rears its ugly head again with this small change in the information structure.

Note that the solution approach focusing on W_P applies more generally and could be used to verify all the other results of the model. In particular:

· With economic transparency, $\Omega_P = \{m, v, s, \Omega\}$ and

$$W_P = -\frac{1}{2}\left(u_0 + (u_m - 1)\bar{\tau} + (u_v - u_m) v + u_s s - (1 - u_m)^2 \beta b\right)^2 - \frac{1}{2}(u_m - 1)^2 \sigma_\tau^2$$

so $u_m = 1$, $u_v = 1$, $u_s = 0$ and $u_0 = 0$.

· With preference transparency, $\Omega_P = \{m, \tau, \Omega\}$ and

$$W_P = -\frac{1}{2}\left(u_0 + (u_m + u_\tau - 1)\tau - (1 - u_m)^2 \beta b\right)^2 - \frac{1}{2} u_m^2 \sigma_v^2$$

so $u_m = 0$, $u_\tau = 1$ and $u_0 = \beta b$.

· For the basic (Stackelberg) model with economic and preference opacity, $\Omega_P =$

[9] This amounts to the minimum state variable condition that McCallum (1983) proposes in the case of multiple rational expectations equilibria.

$\{m, \Omega\}$ and

$$W_P = -\frac{1}{2}\left(u_0 + (u_m - 1)\,\bar{\tau} - (1 - u_m)^2\,\beta b\right)^2 - \frac{1}{2}\,(u_m - 1)^2\,\sigma_\tau^2 - \frac{1}{2}u_m^2\sigma_v^2$$

so $u_m = \frac{\sigma_\tau^2}{\sigma_\tau^2+\sigma_v^2}$ and $u_0 = \frac{\sigma_v^2}{\sigma_\tau^2+\sigma_v^2}\bar{\tau} + \left(\frac{\sigma_v^2}{\sigma_\tau^2+\sigma_v^2}\right)^2 \beta b$.

· For the perfect information simultaneous-move (Nash) case, $\Omega_P = \{\tau, s, v, \Omega\}$ and

$$W_P = -\frac{1}{2}\,(u_0 + (u_\tau - 1)\,\tau + u_v v + u_s s - \beta b)^2$$

so $u_\tau = 1$, $u_v = 0$, $u_s = 0$ and $u_0 = \beta b$.

A.2 Real Interest Rate Transmission

This section analyzes the basic model in section 2 under a different transmission mechanism. Instead of the monetary, Lucas-type mechanism, it employs a real interest rate transmission. The structure of the economy is summarized by the IS relation

$$y = -a\,(i - \pi^e - \bar{r}) + d \tag{19}$$

and the price-adjustment equation

$$\pi = \pi^e + \frac{1}{b}\,(y - \bar{y}) - \frac{1}{b}s \tag{20}$$

where i is the nominal interest rate, $\bar{r}$ the long run real interest rate, d a demand shock, and a the sensitivity of the output gap to the ex ante real interest rate ($a > 0$). Assume that $d \sim N\,(0, \sigma_d^2)$, $s \sim N\,(0, \sigma_s^2)$ and $\tau \sim N\,(\bar{\tau}, \sigma_\tau^2)$, and that d, s and τ are independent.

The timing is as follows. Nature draws the central bank's inflation target τ and the economic shocks d and s, which are only known to the central bank. Then, the central bank sets the interest rate i. Subsequently, the public observes the interest rate, and it forms its inflation expectations π^e. Finally, the output gap y and inflation π are realized. Formally, the information set available to the public when it forms its inflation expectations π^e equals $\{i, \Omega_r\}$, where $\Omega_r \equiv \{\beta, a, b, \bar{r}, \bar{\tau}, \sigma_\tau^2, \sigma_d^2, \sigma_s^2\}$.

Again, the updating of inflation expectations based on the policy instrument i plays a crucial role. It is postulated that

$$\pi^e = u_0 + u_i i \tag{21}$$

which appears to be consistent with a rational expectations equilibrium. The central bank maximizes its objective (1) subject to (19) and (20), and incorporates the effect of its policy actions on the public's inflation expectations through (21). The first order condition implies

$$i=\frac{1}{(1-u_i)\,a-u_ib}\left[(a+b)\,u_0-\frac{a\beta b^2\,(1-u_i)}{(1-u_i)\,a-u_ib}+a\bar{r}-b\tau+d-s\right] \tag{22}$$

Substituting this into (21), (19) and (20) gives the level of inflation

$$\pi=\tau+\beta b\frac{(1-u_i)\,a}{(1-u_i)\,a-u_ib} \tag{23}$$

The usual inflation bias, $\pi=\tau+\beta b$, arises if the policy instrument i has no effect on inflation expectations ($u_i=0$).

Rational expectations imply that $\pi^e=\mathrm{E}\,[\pi|i]$. Substituting (23) and using the fact that i is normally distributed by (22),

$$\pi^e=\mathrm{E}\,[\tau]+\frac{\mathrm{Cov}\,\{\tau,i\}}{\mathrm{Var}\,[i]}\,(i-\mathrm{E}\,[i])+\beta b\frac{(1-u_i)\,a}{(1-u_i)\,a-u_ib}$$

Using (22), $\mathrm{Cov}\,\{\tau,i\}=\frac{-b}{(1-v)a-u_ib}\sigma_\tau^2$ and $\mathrm{Var}\,[i]=\frac{1}{((1-u_i)a-u_ib)^2}\,(b^2\sigma_\tau^2+\sigma_d^2+\sigma_s^2)$. Matching coefficients with (21) and rearranging gives the updating coefficient in the rational expectations equilibrium[10,11]

$$u_i=-\frac{ab\sigma_\tau^2}{\sigma_d^2+\sigma_s^2-ab\sigma_\tau^2}$$

The sign of the updating coefficient depends on the relative uncertainty about economic disturbances. When there is a lot of economic uncertainty ($\sigma_d^2+\sigma_s^2>ab\sigma_\tau^2$), the nominal interest rate is a poor signal of the central bank's inflation target, so inflation expectations are not very responsive. This means that an increase in the nominal interest rate leads to a higher ex ante real interest rate, which reduces inflation. As a result, there is a negative relation between the nominal interest rate and inflation expectations ($u_i<0$). When there is relatively little economic uncertainty ($\sigma_d^2+\sigma_s^2<ab\sigma_\tau^2$), a higher nominal interest rate is associated with a higher inflation target which induces an increase in inflation expectations ($u_i>0$). In fact, inflation expectations are so responsive that they rise by more than the nominal interest rate ($u_i>1$) and depress the ex ante real interest rate $r\equiv i-\pi^e=(1-u_i)\,i-u_0$.

Substituting u_i into (23) produces

$$\pi=\tau+\frac{\sigma_d^2+\sigma_s^2}{\sigma_d^2+\sigma_s^2+b^2\sigma_\tau^2}\beta b$$

This shows that there is also generally an inflation bias with a Keynesian interest rate transmission mechanism.

[10] In the special case of $\sigma_d^2+\sigma_s^2=ab\sigma_\tau^2$, no (pure-strategy) rational expectations equilibrium exists.

[11] For completeness, $u_0=(1-u_i)\,\bar{\tau}-u_i\bar{r}+a\beta b\frac{(1-u_i)^2}{(1-u_i)a-u_ib}$.

A.3 Uncertainty About Output Preferences

Consider the monetary transmission mechanism described by (3) and (2). Assume that the objective function is linear in output (1) but that there is asymmetric information about the output stimulation parameter β, where $\beta \sim N\left(\bar{\beta}, \sigma_\beta^2\right)$ and $\sigma_\beta^2 > 0$. Besides that, the timing is the same as in the basic model. Postulate the updating equation (4). The first order condition yields (5) as before, so that (6) still applies. Rational expectations imply

$$\pi^e = \mathrm{E}\left[\pi|m\right] = \tau + \left(1 - u_m\right) b\, \mathrm{E}\left[\beta|m\right]$$

where $\mathrm{E}\left[\beta|m\right] = \bar{\beta} + \frac{\mathrm{Cov}\{\beta,m\}}{\mathrm{Var}[m]}\left(m - \mathrm{E}\left[m\right]\right)$ because of joint normality of β and m. Using (5) and matching coefficients with (4) gives

$$u_m = \left(1 - u_m\right) b \frac{\left(1 - u_m\right) b\sigma_\beta^2}{\left(1 - u_m\right)^2 b^2\sigma_\beta^2 + \sigma_v^2}$$

So, u_m is given by $f\left(u_m\right) = 0$, where

$$f\left(u\right) = \left(\left(1 - u\right)^2 b^2\sigma_\beta^2 + \sigma_v^2\right) u - \left(1 - u\right)^2 b^2\sigma_\beta^2$$

This yields one real solution:

$$u_m = R - \frac{\sigma_v^2}{3b^2\sigma_\beta^2 R} + 1$$

where $R \equiv -\frac{1}{6b\sigma_\beta}\sqrt[3]{12\sigma_v^2\left(9b\sigma_\beta - \sqrt{81b^2\sigma_\beta^2 + 12\sigma_v^2}\right)} > 0$. Note that $f\left(0\right) = -b^2\sigma_\beta^2 < 0$ and $f\left(1\right) = \sigma_v^2 > 0$. By continuity, $0 < u_m < 1$. As a result, (6) shows that $\pi > \tau$, so that again there is an inflation bias. When the private sector faces no economic uncertainty ($\sigma_v^2 = 0$), $u_m = 1$ and the inflation bias vanishes.

A.4 Preference For Output Stabilization

Suppose the central bank cares about output stabilization and has the objective function (12), where the output gap target $\kappa \geq 0$ is known to the private sector. The structure of the economy is still given by (2) and (3). The private sector faces uncertainty about the central bank's inflation target τ and about the economic shocks v and s. The timing of events is the same as before. Postulate the updating equation (4). The central bank maximizes (12) with respect to m subject to (2), (3) and (4). The first order condition implies

$$m = \frac{1}{1+\beta b^2(1-u_m)^2}\left\{\tau - \left[1+\beta b^2(1-u_m)\right]v - \beta b(1-u_m)s - \beta b(1-u_m)[\kappa - bu_0]\right\} \tag{24}$$

Using rational expectations and joint normality of v and m, (7) and (8) continue to hold, but now $\mathrm{Cov}\{v,m\} = -\frac{1+\beta b^2(1-u_m)}{1+\beta b^2(1-u_m)^2}$ and
$\mathrm{Var}[m] = \frac{1}{\left[1+\beta b^2(1-u_m)^2\right]^2}\left\{\sigma_\tau^2 + [1+\beta b^2(1-u_m)]^2\sigma_v^2 + \beta^2 b^2(1-u_m)^2\sigma_s^2\right\}$.
Matching coefficients with (4) then yields

$$u_m = 1 - \frac{[1+\beta b^2(1-u_m)]\left[1+\beta b^2(1-u_m)^2\right]\sigma_v^2}{\sigma_\tau^2 + [1+\beta b^2(1-u_m)]^2\sigma_v^2 + \beta^2 b^2(1-u_m)^2\sigma_s^2} \tag{25}$$

Solving for u_m gives one real solution. Rearranging and simplifying shows that u_m satisfies $g(u_m) = 0$ where

$$g(u) = (1-u)\sigma_\tau^2 - u\left[1+\beta b^2(1-u)\right]\sigma_v^2 + \beta^2 b^2(1-u)^3\sigma_s^2$$

Note that $g(0) = \sigma_\tau^2 + \beta^2 b^2\sigma_s^2 > 0$ and $g(1) = -\sigma_v^2 < 0$. By continuity, $0 < u_m < 1$.

Regarding the inflation bias, substituting (24) into (2) and taking expectations gives

$$\mathrm{E}[\pi] = \frac{1}{1+\beta b^2(1-u_m)^2}\{\bar{\tau} - \beta b(1-u_m)[\kappa - bu_0]\} \tag{26}$$

To compute u_0, match coefficients between (4) and (8), using (24) to get

$$u_0 = \frac{1-u_m}{1+\beta b^2(1-u_m)^2}\{\bar{\tau} - \beta b(1-u_m)[\kappa - bu_0]\}$$

Solving for u_0 yields

$$u_0 = (1-u_m)\bar{\tau} - \beta b(1-u_m)^2\kappa$$

Substituting this into (26),

$$\mathrm{E}[\pi] = \bar{\tau} + \beta b(1-u_m)\kappa$$

Generally, there is an inflationary bias ($\mathrm{E}[\pi] > \bar{\tau}$). In the absence of economic uncertainty ($\sigma_v^2 = \sigma_s^2 = 0$), $u_m = 1$ and the inflation bias disappears.

A.5 New-Keynesian Phillips Curve

This section derives the results for the infinite-horizon model with the quadratic central bank objective function (13) and the New-Keynesian Phillips curve (15). In every period, the central bank faces the same infinite-horizon problem but with a different state variable τ_{t-1}. Since the current policy decision y_t has no effect on future outcomes, the central bank simply sets y_t to maximize U_t every period subject to (15) and taking into account (16). This yields the optimal levels of the output gap and inflation under discretion:

$$\begin{aligned} y_t &= \frac{1}{1+\alpha\left(u_y+2\right)u_y}\left[\left(1-\alpha\right)\kappa-\alpha\left(u_y+1\right)u_0-\alpha\left(u_y+1\right)\left(u_s+1\right)s_t\right. \\ &\quad \left.+\alpha\left(u_y+1\right)\tau_t-\alpha\left(u_y+1\right)u_\tau\tau_{t-1}\right] && (27) \\ \pi_t &= \frac{1}{1+\alpha\left(u_y+2\right)u_y}\left[\left(1-\alpha\right)u_0+\left(u_y+1\right)\left(1-\alpha\right)\kappa+\left(1-\alpha\right)\left(u_s+1\right)s_t\right. \\ &\quad \left.+\alpha\left(u_y+1\right)^2\tau_t+\left(1-\alpha\right)u_\tau\tau_{t-1}\right] && (28) \end{aligned}$$

Using (14), inflation in the next period equals

$$\begin{aligned} \pi_{t+1} &= \frac{1}{1+\alpha\left(u_y+2\right)u_y}\left\{\left(1-\alpha\right)u_0+\left(u_y+1\right)\left(1-\alpha\right)\kappa+\left(1-\alpha\right)\left(u_s+1\right)s_{t+1}\right. \\ &\quad \left.+\left[\alpha\left(u_y+1\right)^2\rho+\left(1-\alpha\right)u_\tau\right]\tau_t+\alpha\left(u_y+1\right)^2\eta_{t+1}\right\} && (29) \end{aligned}$$

This shows that y_t and π_{t+1} are correlated because of their common dependence on the inflation target τ_t. So, the private sector rationally uses y_t when it forms its expectations for π_{t+1}.

A.5.1 Economic Opacity

When there is economic opacity, the supply shock s_t is not observed by the private sector. Using the fact that y_t and π_{t+1} are bivariate normal

$$\mathrm{E}_t\left[\pi_{t+1}|y_t\right]=\mathrm{E}_t\left[\pi_{t+1}\right]+\frac{\mathrm{Cov}_t\left\{\pi_{t+1},y_t\right\}}{\mathrm{Var}_t\left[y_t\right]}\left(y_t-\mathrm{E}_t\left[y_t\right]\right)$$

where

$$\begin{aligned} \mathrm{Cov}_t\left\{\pi_{t+1},y_t\right\} &= \frac{\alpha\left(u_y+1\right)^2\rho+\left(1-\alpha\right)u_\tau}{1+\alpha\left(u_y+2\right)u_y}\frac{\alpha\left(u_y+1\right)}{1+\alpha\left(u_y+2\right)u_y}\sigma_\tau^2 \\ \mathrm{Var}_t\left[y_t\right] &= \left(\frac{\alpha\left(u_y+1\right)\left(u_s+1\right)}{1+\alpha\left(u_y+2\right)u_y}\right)^2\sigma_s^2+\left(\frac{\alpha\left(u_y+1\right)}{1+\alpha\left(u_y+2\right)u_y}\right)^2\sigma_\tau^2 \end{aligned}$$

$$\mathrm{E}_t\left[\pi_{t+1}\right]=\frac{1}{1+\alpha\left(u_y+2\right)u_y}\left\{\left(1-\alpha\right)u_0+\left(u_y+1\right)\left(1-\alpha\right)\kappa+\left[\alpha\left(u_y+1\right)^2\rho+\left(1-\alpha\right)u_\tau\right]\rho\tau_{t-1}\right\}$$

$$\mathrm{E}_t\left[y_t\right]=\frac{1}{1+\alpha\left(u_y+2\right)u_y}\left[\left(1-\alpha\right)\kappa-\alpha\left(u_y+1\right)u_0+\alpha\left(u_y+1\right)\left(\rho-u_\tau\right)\tau_{t-1}\right]$$

Note that τ_{t-1} can be inferred from y_{t-1} and π_{t-1} at the end of period $t-1$, so it is in the public's information set at the beginning of period t.

Matching coefficients with the postulated updating equation (16) yields

$$\begin{aligned}
u_0 &= \frac{1}{1+\alpha\left(u_y+2\right)u_y}\left\{\left(1-\alpha\right)u_0+\left(u_y+1\right)\left(1-\alpha\right)\kappa-u_y\left[\left(1-\alpha\right)\kappa-\alpha\left(u_y+1\right)u_0\right]\right\}\\
u_y &= \frac{\left[\alpha\left(u_y+1\right)^2\rho+\left(1-\alpha\right)u_\tau\right]\alpha\left(u_y+1\right)\sigma_\tau^2}{\left[\alpha\left(u_y+1\right)\left(u_s+1\right)\right]^2\sigma_s^2+\left[\alpha\left(u_y+1\right)\right]^2\sigma_\tau^2}\\
u_\tau &= \frac{1}{1+\alpha\left(u_y+2\right)u_y}\left[\alpha\left(u_y+1\right)^2\rho^2+\left(1-\alpha\right)\rho u_\tau-u_y\alpha\left(u_y+1\right)\left(\rho-u_\tau\right)\right]\\
u_s &= 0
\end{aligned}$$

Rearranging and simplifying gives

$$\begin{aligned}
u_0 &= \frac{1-\alpha}{\alpha\left(u_y+1\right)}\kappa\\
u_\tau &= \frac{\left[\left(u_y+1\right)\rho-u_y\right]\alpha\left(u_y+1\right)\rho}{1+\alpha u_y-\left(1-\alpha\right)\rho}
\end{aligned}\tag{30}$$

Substituting u_τ and u_s gives

$$\begin{aligned}
u_y &= \left[\left(u_y+1\right)+\left(1-\alpha\right)\frac{\left(u_y+1\right)\rho-u_y}{1+\alpha u_y-\left(1-\alpha\right)\rho}\right]\frac{\rho\sigma_\tau^2}{\sigma_s^2+\sigma_\tau^2}\\
&= \frac{\alpha u_y^2+2\alpha u_y+1}{1+\alpha u_y-\left(1-\alpha\right)\rho}\frac{\rho\sigma_\tau^2}{\sigma_s^2+\sigma_\tau^2}
\end{aligned}$$

In the special case in which the inflation target τ_t is white noise ($\rho=0$), the private sector uses neither τ_{t-1} nor y_t to update its future inflation expectations ($u_\tau=u_y=0$) because the past inflation target and current policy action are uninformative about the future inflation target. In addition, in the limiting case in which the central bank's inflation target is constant and known ($\sigma_\tau^2=0$), $u_y=0$ because the public does not use the noisy policy action y_t under perfect preference transparency.

Rearranging the expression for u_y produces

$$\alpha\left[\sigma_s^2+\left(1-\rho\right)\sigma_\tau^2\right]u_y^2+\left\{\left[1-\left(1-\alpha\right)\rho\right]\sigma_s^2+\left[1-\left(1+\alpha\right)\rho\right]\sigma_\tau^2\right\}u_y-\rho\sigma_\tau^2=0$$

This quadratic equation for u_y generally has two roots, $u_y^-<0$ and $u_y^+>0$. However, u_y^- can be excluded based on an argument by McCallum (1983) because it is not valid for all admissible parameter values. In particular, $\lim_{\rho\to 0}u_y^-=-1/\alpha\neq 0$

and $\lim_{\sigma_\tau^2 \to 0} u_y^- = -[1-(1-\alpha)\rho]/\alpha \neq 0$. In these special cases it is clear that u_y^- is a sunspot equilibrium that violates McCallum's (1983) minimum state variable condition. The remaining positive root equals

$$\begin{aligned} u_y &= \frac{1}{2\alpha\left[\sigma_s^2+(1-\rho)\sigma_\tau^2\right]}\left\{-\left[(1-(1-\alpha)\rho)\right]\sigma_s^2-\left[1-(1+\alpha)\rho\right]\sigma_\tau^2\right. \\ &\quad \left. +\sqrt{\left\{\left[1-(1-\alpha)\rho\right]\sigma_s^2+\left[1-(1+\alpha)\rho\right]\sigma_\tau^2\right\}^2+4\alpha\left[\sigma_s^2+(1-\rho)\sigma_\tau^2\right]\rho\sigma_\tau^2}\right\} \end{aligned}$$

After some clever rearranging, the square root term can be written as follows

$$\begin{aligned} &\left\{\left[1-(1-\alpha)\rho\right]\sigma_s^2+\left[1-(1+\alpha)\rho\right]\sigma_\tau^2\right\}^2+4\alpha\left[\sigma_s^2+(1-\rho)\sigma_\tau^2\right]\rho\sigma_\tau^2 = \\ &\qquad \left\{\left[(1-\rho)(1-\alpha)+\frac{\alpha(1+\rho)}{1-\rho}\right]\sigma_s^2+\left[1-(1-\alpha)\rho\right]\sigma_\tau^2\right\}^2 \\ &\qquad -4\alpha\frac{\left[(1-\rho)^2(1-\alpha)+\alpha\right]\sigma_s^2+(1-\rho)\left[(1-\rho)^2(1-\alpha)+\alpha\right]\sigma_\tau^2}{(1-\rho)^2}\rho\sigma_s^2 \end{aligned}$$

Using the fact that the second term on the right-hand side is negative,

$$u_y < \frac{-\left[(1-(1-\alpha)\rho)\right]\sigma_s^2-\left[1-(1+\alpha)\rho\right]\sigma_\tau^2+\left[(1-\rho)(1-\alpha)+\frac{\alpha(1+\rho)}{1-\rho}\right]\sigma_s^2+\left[1-(1-\alpha)\rho\right]\sigma_\tau^2}{2\alpha\left[\sigma_s^2+(1-\rho)\sigma_\tau^2\right]}$$

Simplifying yields

$$u_y < \frac{\frac{2\alpha\rho}{1-\rho}\sigma_s^2+2\alpha\rho\sigma_\tau^2}{2\alpha\left(\sigma_s^2+(1-\rho)\sigma_\tau^2\right)} = \frac{\rho}{1-\rho}$$

Hence, under economic opacity $0 < u_y < \frac{\rho}{1-\rho}$.

Substituting (30) into (27) it immediately follows that $\mathrm{E}[y_t] = 0$ so that aggregate output is on average equal to its natural rate. Similarly, substituting (30) into (28) yields

$$\mathrm{E}[\pi_t] = \left[\frac{1-\alpha}{\alpha(u_y+1)}+(u_y+1)\right]\frac{(1-\alpha)\kappa}{1+\alpha(u_y+2)u_y} = \frac{1-\alpha}{\alpha(u_y+1)}\kappa \qquad (31)$$

As a result, there is a positive inflation bias.

A.5.2 Economic Transparency

When there is economic transparency, the supply shock s_t is observed by the private sector. So the private sector uses the policy action y_t to infer the inflation target τ_t. In particular, rearranging (27) gives

$$\tau_t = \frac{1+\alpha(u_y+2)u_y}{\alpha(u_y+1)}y_t+u_0-\frac{(1-\alpha)\kappa}{\alpha(u_y+1)}+(u_s+1)s_t+u_\tau\tau_{t-1}$$

Substituting this into (29) yields

$$\begin{aligned}
\mathrm{E}_t\left[\pi_{t+1}|y_t\right] &= \frac{(1-\alpha)\,u_0+(u_y+1)\,(1-\alpha)\,\kappa}{1+\alpha\,(u_y+2)\,u_y} \\
&\quad+\frac{\alpha\,(u_y+1)^2\,\rho+(1-\alpha)\,u_\tau}{\alpha\,(u_y+1)}\left\{y_t+\frac{\alpha\,(u_y+1)}{1+\alpha\,(u_y+2)\,u_y}\left[u_0-\frac{(1-\alpha)\,\kappa}{\alpha\,(u_y+1)}\right]\right. \\
&\quad\left.+\frac{\alpha\,(u_y+1)}{1+\alpha\,(u_y+2)\,u_y}\left[(u_s+1)\,s_t+u_\tau\tau_{t-1}\right]\right\}
\end{aligned}$$

Matching coefficients with the postulated updating equation (16) produces

$$\begin{aligned}
u_0 &= \frac{1}{1+\alpha\,(u_y+2)\,u_y}\left\{(1-\alpha)\,u_0+(u_y+1)\,(1-\alpha)\,\kappa+u_y\left[\alpha\,(u_y+1)\,u_0-(1-\alpha)\,\kappa\right]\right\} \\
u_y &= \frac{\alpha\,(u_y+1)^2\,\rho+(1-\alpha)\,u_\tau}{\alpha\,(u_y+1)} \\
u_\tau &= u_y\frac{\alpha\,(u_y+1)}{1+\alpha\,(u_y+2)\,u_y}u_\tau \\
u_s &= u_y\frac{\alpha\,(u_y+1)}{1+\alpha\,(u_y+2)\,u_y}\,(u_s+1)
\end{aligned}$$

Rearranging and simplifying gives (30) and[12]

$$\begin{aligned}
u_\tau &= 0 \\
u_s &= \frac{u_y\alpha\,(u_y+1)}{1+\alpha u_y}
\end{aligned}$$

Substituting u_τ gives $u_y=(u_y+1)\,\rho$ so that

$$u_y=\frac{\rho}{1-\rho}$$

Under economic transparency it again holds that $\mathrm{E}\left[y_t\right]=0$ and $\mathrm{E}\left[\pi_t\right]$ is given by (31), which reduces to

$$\mathrm{E}\left[\pi_t\right]=\frac{1-\alpha}{\alpha}\,(1-\rho)\,\kappa$$

So, there is a positive average inflation bias, which is decreasing in the degree of persistence ρ of the inflation target. In the limiting case of a completely persistent inflation target ($\rho\to1$) the inflation bias completely disappears.

[12]Note that the alternative solution to the equation for u_τ is that $u_y=-1/\alpha$ which implies $u_\tau=\left[1-(1-\alpha)\,\rho\right]/\alpha$. But this solution can be excluded since it is not valid for all admissible parameter values (in particular, $\rho=0$) and violates the minimum state variable condition (McCallum 1983).

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