

## Supervision 1

### Intertemporal Macroeconomics: Consumption and Labor Supply

#### Problems

1. Consider the two-period Fisher model of consumption and saving. Assume that initially, output and consumption are equal in period one. (For simplicity, you may assume that preferences are homothetic.)
  - (a) Explain graphically and intuitively the effect on consumption and saving of
    - i. an increase in output in period 1 (temporary change).
    - ii. an increase in output in both period 1 and 2 by the same proportion (anticipated permanent change).
    - iii. an increase in output in period 2 (anticipated change).
  - (b) Explain whether the results in (a) change if the consumer is liquidity constrained and cannot borrow against future income.
  - (c) Explain graphically the effect on consumption of an increase in the real interest rate, using the decomposition into intertemporal income and substitution effect, assuming
    - i. the consumer is a borrower.
    - ii. the consumer is a lender.

Under what conditions does saving depend positively on the real interest rate?

2. Consider the following consumption/saving decision under uncertainty. The representative consumer lives for two periods and maximizes the expected value of life-time utility

$$U = u(C_1) + \beta u(C_2)$$

where  $C_t$  denotes consumption in period  $t = 1, 2$ . Assume that  $u'(\cdot) > 0$  and  $u''(\cdot) < 0$ . The subjective intertemporal discount factor equals  $\beta = 1/(1 + \rho)$ , where  $\rho > 0$  is the rate of time preference. The intertemporal budget constraint of the consumer is

$$C_1 + \frac{C_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r}$$

where  $Y_t$  denotes labor income in period  $t = 1, 2$  and  $r$  the real interest rate. In period 1, the consumer is uncertain about labor income in period 2. Let  $E[Y_2]$  denote the expected value of  $Y_2$ . [cf Tripos 2004]

- (a) Derive the intertemporal Euler equation for consumption using the substitution method. Give a brief intuitive interpretation of the result.

- (b) Suppose now that  $r = \rho$  and that  $u(C) = C - \frac{1}{2}\alpha C^2$ , where  $C < 1/\alpha$  and  $\alpha > 0$ . Solve for the optimal levels of consumption  $C_1$  and  $C_2$ . Give a brief intuitive interpretation.
- (c) Using the result in part (b) derive the effect on consumption  $C_1$  and  $C_2$  of
- a temporary increase in labor income in period 1.
  - a permanent increase in labor income in periods 1 and 2 by  $\Delta Y$ .
  - an anticipated increase in labor income in period 2.
3. Consider the following static labour/leisure choice problem of a representative agent [cf Tripos 2013]:

$$\begin{aligned} \max_{c,l,L} \{ \ln c + \ln l \} \\ \text{s.t. } c = L(1 - \tau)w + R \\ 1 = l + L \end{aligned}$$

where  $c$  denotes consumption,  $l$  leisure,  $L$  labour,  $w$  the real wage,  $\tau$  an income tax, and  $R$  a lump-sum transfer.

- Assume initially that  $\tau = R = 0$ . Derive the optimal choice of  $c$ ,  $l$  and  $L$ . Explain intuitively how they depend on the wage  $w$ .
- Suppose now that the government levies an income tax ( $0 < \tau < 1$ ) and assume that it rebates all revenue from the income tax as a lump-sum transfer. Derive the optimal choice of  $c$ ,  $l$  and  $L$ . Explain how they are affected by the income tax  $\tau$ .
- Compute the value of  $\tau$  that maximises welfare in this setting. Explain the intuition underlying this result.

### Main readings

- Barro (1997), *Macroeconomics*, chapters 2, 3, 13.
- Doppelhofer (2009), “Intertemporal Macroeconomics”, forthcoming in McCombie and Allington (eds), *Cambridge Essays in Applied Economics*, [https://www.vle.cam.ac.uk/pluginfile.php/13437652/mod\\_resource/content/1/Intertemporal\\_Macroeconomics.pdf](https://www.vle.cam.ac.uk/pluginfile.php/13437652/mod_resource/content/1/Intertemporal_Macroeconomics.pdf)
- Williamson (2013), *Macroeconomics*, chapters 4, 5 and 9 (pp. 310-339).

### Supplementary references

- Abel, Bernanke and McNabb (1998), *Macroeconomics*, European Edition, chapter 4.
- Blanchard and Johnson (2012), *Macroeconomics*, chapter 16.
- Friedman (1957), *A Theory of the Consumption Function*. [classic on Permanent Income Hypothesis]
- Hall (1978), “Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence”, *Journal of Political Economy* 86(6), December, pp. 971-987. [classic on random walk consumption]
- Mankiw and Taylor (2014), *Macroeconomics - Second European Edition*, chapter 18.
- Modigliani (1986), “Life Cycle, Individual Thrift, and the Wealth of Nations”, *American Economic Review* 76(3), June, pp. 297-313 [seminal review of Life Cycle Hypothesis]