1. Consider the two-period Fisher model of consumption and saving. Assume the consumer has no initial assets. (For simplicity, you may assume that preferences are homothetic.)

(a) Assume that output and consumption are initially equal in period 1. Explain graphically and intuitively the effect on consumption and saving of
   i. an increase in output in period 1 (temporary change).
   ii. an increase in output in both period 1 and 2 by the same proportion (anticipated permanent change).
   iii. an increase in output in period 2 (anticipated change).

(b) Explain whether the results in (a) change if the consumer is liquidity constrained and cannot borrow against future income.

(c) Explain graphically the effect on consumption of an increase in the real interest rate, using the decomposition into intertemporal income and substitution effect, assuming
   i. the consumer is a borrower.
   ii. the consumer is a lender.

Under what conditions does saving depend positively on the real interest rate?

2. Consider the following consumption/saving decision under uncertainty. The representative consumer lives for two periods and maximizes the expected value of life-time utility

$$ U = u(C_1) + \beta u(C_2) $$

where $C_t$ denotes consumption in period $t = 1, 2$. Assume that $u'(.) > 0$ and $u''(.) < 0$. The subjective intertemporal discount factor equals $\beta = 1/(1 + \rho)$, where $\rho > 0$ is the rate of time preference. The intertemporal budget constraint of the consumer is

$$ C_1 + \frac{C_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r} $$

where $Y_t$ denotes labor income in period $t = 1, 2$ and $r$ the real interest rate. In period 1, the consumer is uncertain about labor income in period 2. Let $E[Y_2]$ denote the expected value of $Y_2$. [cf Tripos 2004]
(a) Derive the intertemporal Euler equation for consumption using the substitution method. Give a brief intuitive interpretation of the result.

(b) Suppose now that \( r = \rho \) and that \( u(C) = C - \frac{1}{2}\alpha C^2 \), where \( C < 1/\alpha \) and \( \alpha > 0 \). Solve for the optimal levels of consumption \( C_1 \) and \( C_2 \). Give a brief intuitive interpretation.

(c) Using the result in part (b) derive the effect on consumption \( C_1 \) and \( C_2 \) of

i. a temporary increase in labor income in period 1.

ii. a permanent increase in labor income in periods 1 and 2 by \( \Delta Y \).

iii. an anticipated increase in labor income in period 2.

Briefly interpret the results.

3. Consider the following static labour/leisure choice problem of a representative agent [cf Tripos 2013]:

\[
\max_{c,l,L} \{ \ln c + \ln l \} \\
\text{s.t. } c = L (1 - \tau) w + R \\
1 = l + L
\]

where \( c \) denotes consumption, \( l \) leisure, \( L \) labour, \( w \) the real wage, \( \tau \) an income tax, and \( R \) a lump-sum transfer.

(a) Assume initially that \( \tau = R = 0 \). Derive the optimal choice of \( c \), \( l \) and \( L \). Explain intuitively how they depend on the wage \( w \).

(b) Suppose now that the government levies an income tax \( (0 < \tau < 1) \) and assume that it rebates all revenue from the income tax as a lump-sum transfer. Derive the optimal choice of \( c \), \( l \) and \( L \). Explain how they are affected by the income tax \( \tau \).

(c) Compute the value of \( \tau \) that maximises welfare in this setting. Explain the intuition underlying this result.

Main readings


Supplementary references
- Friedman (1957), *A Theory of the Consumption Function*. [classic on Permanent Income Hypothesis]

