Supervision 2
Intertemporal Macroeconomics:
Government, Investment and Business Cycles

Short question (250 words max)

1. What is Ricardian Equivalence and under what conditions does it hold?

Problems

2. Consider the following two-period neoclassical model of investment. The representative firm faces perfect competition and produces output $Y_t$ according to

$$Y_t = F(K_t, L_t)$$

where $F(.)$ satisfies positive but diminishing marginal product with respect to capital $K_t$ and labor $L_t$, with $L_t = \bar{L}$. The subscript $t = 1, 2$ indicates the time period. Capital accumulation is described by

$$K_{t+1} = (1 - \delta) K_t + I_t$$

where $I_t$ is the level of investment, $\delta$ the depreciation rate ($0 < \delta < 1$), $K_1$ is predetermined, and $K_3 = 0$. The firm maximizes the present value of its profits

$$\Pi = Y_1 - p_{K,1} I_1 - w_1 L_1 + \frac{1}{1 + r} (Y_2 - p_{K,2} I_2 - w_2 L_2)$$

where $r$ is the real interest rate, $p_{K,t}$ the relative price of capital goods and $w_t$ the real wage. Explain the effect on investment $I_1$ of:

(a) An increase in the real interest rate due to disinflationary monetary policy.
(b) A partial destruction of the capital stock in period 1 due to an earthquake.
(c) An increase in the size of the labor force $\bar{L}$ stemming from the immigration of foreign workers.
(d) The (credible) announcement in period 1 of the introduction of an investment tax credit starting in period 2.

3. Consider the market-clearing, neoclassical model in which the supply of labor depends positively on the real interest rate, and consumption and investment demand depend negatively on the real interest rate. Explain the effects on labor supply, consumption demand, investment demand, output and the real interest rate of the following sudden changes:

(a) A temporary productivity increase this period.
(b) A permanent productivity increase this period.
(c) A permanent productivity increase next period.
4. Consider a price-taking representative agent who faces the following optimisation problem

\[
\begin{align*}
\max & \{ \ln c_1 + \ln \ell_1 + \beta (\ln c_2 + \ln \ell_2) \}, \\
\text{subject to} & \quad c_1 + b_2 = w_1 (1 - \ell_1) - T_1, \\
& \quad c_2 = w_2 (1 - \ell_2) + b_2 (1 + r_2) - T_2,
\end{align*}
\]

where \( c_t \) denotes consumption, \( \ell_t \) leisure, \( b_t \) savings, \( T_t \) lump-sum taxes, \( w_t \) the real wage, \( r_t \) the real interest rate, and \( \beta \) the intertemporal discount factor (0 < \( \beta \) < 1). The subscript \( t = 1, 2 \) denotes the time period. [Tripos 2016]

(a) Derive the agent’s intertemporal budget constraint. Give an economic interpretation of the result.

(b) Find the optimality conditions for \( c_1, c_2, b_2, \ell_1 \) and \( \ell_2 \). Derive the intertemporal Euler equation for consumption, and the intratemporal (i.e. within period) Euler equation characterizing the optimal trade-off between consumption and leisure. Give an intuitive explanation of these Euler equations.

Assume that the economy is closed and that there is no investment. So the GDP identity is given by \( y_t = c_t + g_t \), where \( g_t \) denotes government purchases, which are exogenous, and \( y_t \) denotes aggregate output, which is equal to \( y_t = w_t (1 - \ell_t) \). Suppose that \( w_t = A_t \), where \( A_t \) represents productivity in period \( t = 1, 2 \). In addition, assume that the government runs a balanced budget such that \( T_t = g_t \).

(c) Solve for the equilibrium level of output \( y_t \) in terms of \( A_t \) and \( g_t \) for \( t = 1, 2 \). Derive the fiscal multiplier \( \partial y_t / \partial g_1 \) for a permanent increase in \( g_1 \). Provide an intuitive explanation of the results.

(d) Now suppose that \( A_1 = A_2 = 1 \) and \( g_2 = \rho g_1 \), where \( 0 \leq \rho \leq 1 \). What are the fiscal multipliers \( \partial y_t / \partial g_t \) for \( t = 1, 2 \)? Compare the result to part (c). Derive the equilibrium real interest rate \( r_2 \) and explain how it is affected by a rise in \( g_1 \) when \( \rho = 0 \) and when \( \rho = 1 \). Explain the intuition underlying these results.

Main readings


Supplementary references

- Blanchard and Johnson (2012), *Macroeconomics*, chapter 16 and 23
- Burda and Wyplosz (2009), *Macroeconomics: A European Text*, chapter 7 and 8