Supervision 3  
Labour and Unemployment

Short questions (250 words max)

1. Show graphically and explain intuitively how the reservation wage in the McCall model of job search is affected by:

   (a) A decrease in the unemployment benefit.
   (b) An increase in the patience of workers.

2. Suppose a representative, profit maximizing firm produces output $Y$ according to the production function

   $$Y = A_U L_U^\alpha + A_S (e_S L_S)^\alpha$$

   where $L_U$ and $L_S$ denote the employment of unskilled and skilled labour, respectively, with parameters $A_S > A_U > 0$ and $0 < \alpha < 1$. The skilled effort level satisfies

   $$e_S = \beta (w_S - \bar{w})^\gamma$$

   where $w_S$ denotes the real wage for skilled labour and $\bar{w}$ the minimum wage set by the government, with parameters $\beta > 0$ and $0 < \gamma < 1$. The supply of skilled and unskilled labour is $L_S$ and $L_U$, respectively. Assume that there is a competitive labour market. [cf Tripos 2011]

   Set up the firm’s profit maximization problem and derive the level of the real wage for (i) unskilled labour ($w_U$) and (ii) skilled labour ($w_S$) in this economy. Explain what determines the ‘skill premium’ $w_S/w_U$.

Problems

3. Consider a one-period economy in which the representative agent maximizes the following utility function

   $$U (C, \ell) = \frac{C^\gamma - 1}{\gamma} + \frac{\ell^\gamma - 1}{\gamma}$$

   where $C$ is consumption, $\ell$ is leisure, and $\gamma < 1$. The agent is subject to the time constraint:

   $$\ell + L = 1$$

   where $L$ is labor supplied by the agent at the exogenously given real wage $w$. The agent has to pay a proportional tax $\tau$ on wage income as well as a lump-sum tax $T$, with $0 < \tau < 1$. In addition, the agent has some non-labor income $\Pi$. Assume that $T \leq \Pi$. 

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(a) Assume that \( \tau = 0 \) and \( \gamma = 1/2 \). Solve for the optimal labor supply as a function of the real wage. Explain the effect of \( \Pi \) on labor supply.

(b) Now assume instead that \( T = 0 \) and \( \Pi = 0 \). Solve for the optimal labor supply and show the effect of an increase in the tax rate \( \tau \). Does the qualitative effect depend on the value of \( \gamma \)? Provide an intuitive explanation.

4. Consider the following variation on the Shapiro-Stiglitz model. The representative firm sets the real wage \( w \) to maximize profits \( \pi = AF(eL) - wL \), where \( F(.) \) is the production function, which satisfies \( F''(.) > 0 \) and \( F'''(.) < 0 \), \( L \) the number of employees, \( w \) the real wage, \( A \) technology, and \( e \) employee effort, with \( e = 1 \) if working and \( e = 0 \) if shirking. The utility of an employee equals \( w - e \) when working, and \((1 - q) w + qv \) when shirking, where \( q \) is the probability that shirking is detected, and \( v \) the expected utility of being unemployed. Let \( v = ub + (1 - u) w \), where \( b \) is the unemployment benefit and \( u \) the unemployment rate, which equals \( u = (\bar{L} - L) / \bar{L} \), with \( \bar{L} \) the labor force.

(a) Derive the no-shirking condition. Give an intuitive explanation of this relation between the unemployment rate and the real wage.

(b) Show graphically and explain intuitively the effect on the real wage and employment of

i. a decrease in the unemployment benefit.

ii. an economic boom which improves productivity.

iii. an increase in the probability that a shirker is detected.

Main readings

- Blanchard (2017), *Macroeconomics*, chapter 7 and 13

Supplementary references

- Carlin and Soskice (2006), *Macroeconomics: Imperfections, Institutions and Policies*, chapter 2.5-2.6, 4