Supervision 5
Monetary Policy

Short questions (250 words max)

1. In the AS-AD model with rational expectations, an increase in aggregate demand due to a monetary expansion could decrease output, ceteris paribus. True or false? Explain.

2. According to the policy ineffectiveness proposition, anticipated monetary policy has no effect on the economy. True or false? Explain.

Problems

3. Consider the following macroeconomic model with rational expectations. Goods market equilibrium is described by the IS relation

\[ y_t = -\alpha (r_t - \bar{r}) + \eta_t \]

where \( y_t \) denotes the output gap, \( r_t \) the real interest rate, \( \bar{r} \) the natural real interest rate, \( \eta_t \) a white noise shock, \( t \) a time subscript, and \( \alpha \) a positive parameter. Monetary policy is set according to the following reaction function:

\[ r_t = \bar{r} + \mu_\pi (\pi_t - \pi^*) + \mu_y y_t + \nu_t \]

where \( \pi_t \) denotes inflation, \( \pi^* \) the inflation target, \( \nu_t \) a white noise monetary policy shock, and \( \mu_\pi \) and \( \mu_y \) positive parameters. The Phillips curve is described by

\[ \pi_t = \pi^c_t + \theta y_t + \varepsilon_t \]

where \( \pi^c_t \) denotes private sector inflation expectations, \( \varepsilon_t \) a white noise cost-push shock, and \( \theta \) a positive parameter.

(a) Derive the aggregate demand equation. Explain how it is affected by \( \pi^* \), \( \nu_t \) and \( \eta_t \).

(b) Assume perfect foresight and flexible prices. Derive the equilibrium level of the output gap \( y_t \) and inflation \( \pi_t \). Explain how they are affected by \( \pi^* \), \( \nu_t \) and \( \eta_t \).

Now assume private sector inflation expectations are pre-set so that \( \pi^c_t = E_{t-1} [\pi_t] \), and that the shocks \( \eta_t \), \( \nu_t \) and \( \varepsilon_t \) are independent white noise, so that \( E_{t-1} [\eta_t] = E_{t-1} [\nu_t] = E_{t-1} [\varepsilon_t] = 0 \).

(c) Use the Phillips curve and aggregate demand relation to compute \( E_{t-1} [y_t] \) and \( E_{t-1} [\pi_t] \). Give an economic interpretation of the results.

(d) Solve for the equilibrium level of the output gap \( y_t \) and inflation \( \pi_t \). Explain how they are affected by \( \pi^* \), \( \nu_t \) and \( \eta_t \). Explain whether the parameters \( \mu_\pi \) and \( \mu_y \) of the monetary policy reaction function contribute to the stabilization of the \( \nu_t \) and \( \eta_t \) shocks.
4. Suppose a central bank using monetary targeting minimizes the social welfare loss function
\[ L = \frac{1}{2} (\pi - \pi^*)^2 + \frac{1}{2} \lambda (y - y^*)^2 \]
where \( \pi \) is inflation, \( y \) the output gap, \( \pi^* \) the inflation target, \( y^* \) the output gap target and \( \lambda \) a positive parameter. Assume that \( \pi^* > 0 \) and \( y^* > 0 \). The central bank sets the rate of money growth \( m \), which affects inflation:
\[ \pi = m + v \]
where \( v \) is a velocity shock. The aggregate supply relation is described by
\[ y = \theta (\pi - \pi^e) + s \]
where \( \pi^e \) denotes private sector inflation expectations, \( s \) is an aggregate supply shock, and \( \theta \) is a positive parameter. Assume that the shocks \( v \) and \( s \) are i.i.d. white noise with variance \( \sigma_v^2 \) and \( \sigma_s^2 \), respectively.

At the beginning of the period, the private sector forms its inflation expectations \( \pi^e \) using rational expectations. Subsequently, the supply shock \( s \) and velocity shock \( v \) are observed. Then, the central bank adjusts its monetary policy instrument \( m \), after which inflation \( \pi \) and the output gap \( y \) are realized. [cf Tripos 2016]

(a) Derive the rate of money growth \( m \) that the central bank sets for a given level of private sector inflation expectations \( \pi^e \). Explain intuitively how \( m \) depends on \( \pi^* \), \( y^* \), \( \pi^e \), \( s \) and \( v \).

(b) Derive the level of private sector inflation expectations \( \pi^e \), and the outcome for inflation \( \pi \) and the output gap \( y \). Compute the expected value and variance of inflation and the output gap: \( E[\pi] \), \( E[y] \), \( \text{Var}[\pi] \) and \( \text{Var}[y] \). Give a brief economic interpretation of the results.

(c) Suppose the government is dissatisfied with macroeconomic performance and considering whether to require the central bank to announce and commit to a rate of money growth \( m_C \) before the start of the period. Derive the level of \( m_C \) that minimizes expected social welfare losses \( L \), and the resulting macroeconomic outcomes \( \pi_C \), \( y_C \), \( E[\pi_C] \), \( E[y_C] \), \( \text{Var}[\pi_C] \) and \( \text{Var}[y_C] \). Would commitment improve macroeconomic outcomes?

(d) Alternatively, the government could delegate monetary policy to a new central banker (indicated by \( CB \)) who minimizes \( L \) but with a different \( \lambda \) or \( y^* \), such that either (i) \( \lambda_{CB} = 0 \), or (ii) \( y^*_{CB} = 0 \). Explain whether these delegation options would improve macroeconomic outcomes compared to part (b). In addition, analyse which of the three alternatives would be best: commitment as in part (c), delegation (i) or delegation (ii).

Main readings


Supplementary references
- Sheffrin (1996), *Rational Expectations*, chapters 1, 2 and 3 (pp. 79-88).