Problems

1. Consider a competitive economy with an infinitely lived representative agent that maximizes the expected value of lifetime utility

\[ U_t = \sum_{s=t}^{\infty} \beta^{s-t} C_s \]

where \( C_t \) is consumption in period \( t \) and \( \beta \) the intertemporal discount factor (0 < \( \beta < 1 \)). The agent supplies a constant amount of labor \( L_t = 1 \), and produces according to the production function

\[ Y_t = Z_t K_t^\alpha L_t^{1-\alpha} \]

where \( K_t \) denotes capital in period \( t \) and \( Z_t \) the level of technology, with 0 < \( \alpha < 1 \). Capital accumulation is described by

\[ K_{t+1} = Y_t - C_t + (1 - \delta) K_t \]

where \( \delta \) denotes the rate of depreciation, with 0 < \( \delta < 1 \). Technology is stochastic and follows the process

\[ Z_t = \Psi_t Z_{t-1}^\mu \]

where \( \Psi_t \) is a technology shock such that \( \mathbb{E}[\Psi_t] = 1 \) and \( \text{Var}[\Psi_t] = \sigma^2 \), and 0 < \( \mu < 1 \). Assume that \( C_t \geq 0 \) for all technology shocks.

(a) Derive the Euler equation associated with the representative agent’s optimization problem. Interpret the result.

(b) Compute the steady state values of capital \( \bar{K} \), output \( \bar{Y} \) and consumption \( \bar{C} \). [Hint: At the steady state, random variables are at their unconditional mean.]

(c) Use the Euler equation to express \( k_{t+1} = \ln K_{t+1} \) in terms of \( z_t = \ln Z_t \), and \( y_t = \ln Y_t \) in terms of \( z_{t-1} = \ln Z_{t-1} \) and \( \psi_t = \ln \Psi_t \). Explain intuitively how an unanticipated technology shock \( \psi_t \) affects future output \( y_{t+1} \).

(d) Let \( z_0 = 0 \). Explain how the dynamic response of output \( y_t \) to a one-period technology shock \( \psi_1 = 1 \) depends on the persistence of technology shocks \( \mu \).
2. Consider the basic real business cycle model with inelastic labor supply, log utility of consumption and full capital depreciation. The equilibrium in this economy is described by the expressions: [cf Tripos 2007]

\[
\frac{1}{C_t} = \alpha \beta E_t \left[ \frac{1}{C_{t+1}} Z_{t+1} K_{t+1}^{\alpha - 1} \right] \\
K_{t+1} = Y_t - C_t \\
Y_t = Z_t K_t^\alpha \\
Z_t = \rho Z_{t-1} + \epsilon_t, \quad \epsilon_t \sim iid (0, \sigma^2)
\]

where \( C, K, Y \) and \( Z \) are consumption, capital, output and an exogenous technological shock, respectively. The parameters \( \alpha \) and \( \beta \) are the capital share and the discount factor, respectively, with \( 0 < \alpha < 1, 0 < \beta < 1 \) and \( 0 < \rho < 1 \).

(a) Give a brief interpretation of these four expressions.

(b) Work out the dynamic laws of motion for \( K_{t+1}, C_t \) and \( Y_t \) in terms of \( K_t, Z_t \)
and the model parameters.

(c) What are the sources of persistence in this model?

Essay question (1000 words max)

3. Assess the empirical plausibility of the Real Business Cycle model. [Tripos 2009]

Main readings

- Sørensen and Whitta-Jacobsen (2005), Introducing Advanced Macroeconomics: Growth and Business Cycles, chapter 14, 17-19

Supplementary references
- Romer (2011), Advanced Macroeconomics, chapter 5