Supervision 1
Economic Growth

Problems

1. Shocks to an economy, such as wars, often generate large, one-time flows of workers across borders. This problem analyzes the effects of a one-time increase in the stock of labor on the economy of Paxania. [cf Tripos 2000]

A Suppose the economy of Paxania can be described by the following Solow growth model:

\[ Y = K^\alpha (AL)^{1-\alpha} \]
\[ L = \bar{L} \]
\[ \dot{K} = sY \]
\[ \dot{A} = gA \]

where \( Y \) denotes aggregate output, \( K \) the capital stock, \( L \) labor input, \( A \) technology, and \( \dot{X} \equiv dX/dt \). In addition, \( 0 < \alpha < 1, 0 < s < 1 \) and \( g > 0 \).

(a) Derive the fundamental equation of motion for capital per effective worker \( \bar{k} \equiv K/AL \). Compute its balanced growth path level and the corresponding growth rate of output per worker \( y \equiv Y/L \).

(b) Use the Solow diagram to show the effect of a one-time increase in the labor force \( \bar{L} \), assuming Paxania is initially on a balanced growth path. Explain intuitively what happens to the level of per capita output \( y \) and its growth rate \( g_y \equiv \dot{y}/y \).

(c) Sketch the path of \( \ln y \) over time. [Hint: The slope of \( \ln y \) is the growth rate of per capita output: \( d\ln y/dt = g_y \).] Does the inflow of labor improve the standard of living for the people of Paxania?

B Suppose the economy of Paxania can be described by the following Romer endogenous growth model:

\[ Y = K^\alpha (AL_Y)^{1-\alpha} \]
\[ L_Y = (1-a) \bar{L} \]
\[ L_A = a \bar{L} \]
\[ \dot{K} = sY \]
\[ \dot{A} = \beta AL_A^\theta \]

where \( Y \) denotes aggregate output, \( K \) the capital stock, \( L_Y \) labor input in the production sector, \( L_A \) labor input in the technology (R&D) sector, \( A \) technology, and \( \dot{X} \equiv dX/dt \). In addition, \( 0 < \alpha < 1, 0 < a < 1, 0 < s < 1, \beta > 0 \) and \( 0 < \theta < 1 \).
(a) Compute the growth rates of technology \( g_A = \dot{A}/A \), output \( g_Y = \dot{Y}/Y \) and capital \( g_K = \dot{K}/K \) along the balanced growth path. [Hint: On the balanced growth path, \( d \ln g_K/ dt = 0 \).]

(b) Explain intuitively what happens to the level of per capita output \( y \) and its growth rate \( g_y \) after an increase in the labor force \( L \), assuming Paxania is initially on a balanced growth path.

(c) Sketch the path of \( \ln y \) over time. Does the inflow of labor improve the standard of living for the people of Paxania?

2. Consider the following continuous-time Solow growth model. There is a large set of identical firms indexed by \( i \). The production technology of firm \( i \) is described by

\[
Y_i(t) = A_i(t) [K_i(t)]^\alpha [L_i(t)]^{1-\alpha}
\]

where \( Y_i(t) \) denotes output of firm \( i \), \( K_i(t) \) the capital stock used by firm \( i \), \( L_i(t) \) labour employed by firm \( i \), and \( \alpha \in (0, 1) \). The productivity factor is described by \( A_i(t) = [Y(t)]^\phi \), where \( Y(t) \) is aggregate output and \( \phi \in (0, 1) \). Moreover, \( \phi + \alpha < 1 \). The labour force grows at a constant rate \( n > 0 \) and households save a fraction \( s \in (0, 1) \) of income. The economy is closed which implies that investment equals saving. The aggregate capital stock evolves according to the following equation of motion:

\[
\dot{K}(t) = I(t) - \delta K(t)
\]

where \( I(t) \) denotes aggregate investment and \( \delta \) is the depreciation rate of the capital stock, with \( \delta > 0 \). [cf Tripos 2015]

(a) What is the intuition behind \( A_i(t) = [Y(t)]^\phi \)?

(b) Show that the economy exhibits a balanced growth path with a positive long-run growth rate of output per worker.

(c) Explain whether the economy converges to this balanced growth path equilibrium.

(d) Suppose that the economy is initially in a balanced growth path equilibrium. Consider a change in immigration laws such that it is harder for immigrants to move from another country, so the economy’s underlying labour force growth rate decreases \( (n' < n) \). Describe the effects of such a policy on the dynamics of output per worker of this economy. Be sure to distinguish between short-run and long-run effects.

Main reading

Supplementary references
- Weil (2012), *Economic Growth*, ch 1-3