

## Supervision 1 Economic Growth

### Problems

1. Shocks to an economy, such as wars, often generate large, one-time flows of workers across borders. This problem analyzes the effects of a one-time increase in the stock of labor on the economy of Paxania. [cf Tripos 2000]

A Suppose the economy of Paxania can be described by the following Solow growth model:

$$\begin{aligned} Y &= K^\alpha (AL)^{1-\alpha} \\ L &= \bar{L} \\ \dot{K} &= sY \\ \dot{A} &= gA \end{aligned}$$

where  $Y$  denotes aggregate output,  $K$  the capital stock,  $L$  labor input,  $A$  technology, and  $\dot{X} \equiv dX/dt$ . In addition,  $0 < \alpha < 1$ ,  $0 < s < 1$  and  $g > 0$ .

- (a) Derive the fundamental equation of motion for capital per effective worker  $\tilde{k} \equiv K/AL$ . Compute its balanced growth path level and the corresponding growth rate of output per worker  $y \equiv Y/L$ .
  - (b) Use the Solow diagram to show the effect of a one-time increase in the labor force  $\bar{L}$ , assuming Paxania is initially on a balanced growth path. Explain intuitively what happens to the level of per capita output  $y$  and its growth rate  $g_y \equiv \dot{y}/y$ .
  - (c) Sketch the path of  $\ln y$  over time. [Hint: The slope of  $\ln y$  is the growth rate of per capita output:  $d \ln y / dt = g_y$ .] Does the inflow of labor improve the standard of living for the people of Paxania?
- B Suppose the economy of Paxania can be described by the following Romer endogenous growth model:

$$\begin{aligned} Y &= K^\alpha (AL_Y)^{1-\alpha} \\ L_Y &= (1-a)\bar{L} \\ L_A &= a\bar{L} \\ \dot{K} &= sY \\ \dot{A} &= \beta AL_A^\theta \end{aligned}$$

where  $Y$  denotes aggregate output,  $K$  the capital stock,  $L_Y$  labor input in the production sector,  $L_A$  labor input in the technology (R&D) sector,  $A$  technology, and  $\dot{X} \equiv dX/dt$ . In addition,  $0 < \alpha < 1$ ,  $0 < a < 1$ ,  $0 < s < 1$ ,  $\beta > 0$  and  $0 < \theta < 1$ .

- (a) Compute the growth rates of technology  $g_A = \dot{A}/A$ , output  $g_Y = \dot{Y}/Y$  and capital  $g_K = \dot{K}/K$  along the balanced growth path. [Hint: On the balanced growth path,  $d \ln g_K / dt = 0$ .]
  - (b) Explain intuitively what happens to the level of per capita output  $y$  and its growth rate  $g_y$  after an increase in the labor force  $\bar{L}$ , assuming Paxania is initially on a balanced growth path.
  - (c) Sketch the path of  $\ln y$  over time. Does the inflow of labor improve the standard of living for the people of Paxania?
2. Consider the following continuous-time Solow growth model. There is a large set of identical firms indexed by  $i$ . The production technology of firm  $i$  is described by

$$Y_i(t) = A_i(t) [K_i(t)]^\alpha [L_i(t)]^{1-\alpha}$$

where  $Y_i(t)$  denotes output of firm  $i$ ,  $K_i(t)$  the capital stock used by firm  $i$ ,  $L_i(t)$  labour employed by firm  $i$ , and  $\alpha \in (0, 1)$ . The productivity factor is described by  $A_i(t) = [Y(t)]^\phi$ , where  $Y(t)$  is aggregate output and  $\phi \in (0, 1)$ . Moreover,  $\phi + \alpha < 1$ . The labour force grows at a constant rate  $n > 0$  and households save a fraction  $s \in (0, 1)$  of income. The economy is closed which implies that investment equals saving. The aggregate capital stock evolves according to the following equation of motion:

$$\dot{K}(t) = I(t) - \delta K(t)$$

where  $I(t)$  denotes aggregate investment and  $\delta$  is the depreciation rate of the capital stock, with  $\delta > 0$ . [cf Tripos 2015]

- (a) What is the intuition behind  $A_i(t) = [Y(t)]^\phi$ ?
- (b) Show that the economy exhibits a balanced growth path with a positive long-run growth rate of output per worker.
- (c) Explain whether the economy converges to this balanced growth path equilibrium.
- (d) Suppose that the economy is initially in a balanced growth path equilibrium. Consider a change in immigration laws such that it is harder for immigrants to move from another country, so the economy's underlying labour force growth rate decreases ( $n' < n$ ). Describe the effects of such a policy on the dynamics of output per worker of this economy. Be sure to distinguish between short-run and long-run effects.

### Main reading

- Jones (2002), *Introduction to Economic Growth*, ch 1-5, 10.

### Supplementary references

- Sørensen & Whitta-Jacobsen (2005), *Introducing Advanced Macroeconomics: Growth and Business Cycles*, ch 2-3, 5, 9.
- Weil (2012), *Economic Growth*, ch 1-3