Supervision 2
Growth and Capital

Problems

1. Consider an aggregate production function given by [cf Tripos 2002]

\[ Y_t = K_t^\alpha H_t^\beta (A_t L_t)^{1-\alpha-\beta} \]

where \( Y_t \) is aggregate output, \( K_t \) the aggregate stock of physical capital, \( H_t \) the aggregate stock of human capital and \( L_t \) labor at time \( t \), and \( \alpha > 0, \beta > 0 \) and \( \alpha + \beta < 1 \). Suppose that labor \( L_t \) and technology \( A_t \) grow at an exogenous rate of \( n \) and \( g \), respectively, so that \( L_t = L_0 e^{nt} \) and \( A_t = A_0 e^{gt} \). Suppose that individuals invest constant shares of their income in physical capital \( (s_K) \) and in human capital \( (s_H) \) and that both types of capital depreciate at a constant rate \( \delta \). Physical and human capital therefore evolve as follows:

\[
\dot{K}_t = dK_t/dt = s_K Y_t - \delta K_t \\
\dot{H}_t = dH_t/dt = s_H Y_t - \delta H_t
\]

(a) Show that, in the long run, the growth rate of output per worker \( (Y_t/L_t) \) is unaffected by changes in the proportions of output invested in physical capital \( (s_K) \) and human capital \( (s_H) \). What is the intuition behind this result?

(b) Suppose government policy is able to permanently increase the proportion of output allocated to human capital \( (s_H) \). Explain the long-run impact on

i. the level of output per worker.
ii. the returns to human capital.
iii. the returns to physical capital.
iv. the share of income going to the total workforce, i.e. human capital and labor combined.

2. Consider a continuous-time Solow growth model. The production technology is represented by:

\[ Y(t) = K(t)^\alpha L(t)^{1-\alpha}, \alpha \in (0,1) \]

where \( Y(t) \) corresponds to output, \( K(t) \) is the capital stock, and \( L(t) \) is labor. Population grows at constant rate \( n \) and agents save a fraction \( s \in (0,1) \) of income. The economy is closed which implies that investment equals savings. Capital evolves according to the following equation of motion:

\[ \dot{K}(t) = q(t) I(t) - \delta K(t), \delta > 0 \]

where \( I(t) \) denotes investment and \( \delta \) is the depreciation of the capital stock. Variable \( q(t) \) corresponds to the inverse of the relative price of machinery to output. Assume that this relative price is declining over time, such that \( \frac{\dot{q}(t)}{q(t)} = \gamma > 0 \). [cf Tripos 2014]
(a) Derive the equilibrium equation that describes the evolution of capital per unit of labour. What is the long-run growth rate of output per unit of labour along the Balanced Growth Path? Explain.

(b) Does the economy converge to this Balanced Growth Path equilibrium? Explain. (Hint: Show how the system can be made stationary.)

(c) What is the equilibrium level of output per unit of labour along the Balanced Growth Path equilibrium?

(d) Show the dynamics of output per unit of labour after a permanent increase in $\gamma$. Explain.

Essay question (1000 words max)

3. Thomas Piketty in his best-seller book, “Capital in the Twenty-First Century”, has documented that the wealth-income ratio in rich countries has increased from roughly 200-300% in 1970 to 400-600% in 2010. He uses standard analytical results of the Solow growth model to make predictions about inequality (i.e. the division of income between labour and capital) in the future. Evaluate these findings and predictions. [Tripos 2015]

Main reading

- Jones (2002), Introduction to Economic Growth, ch 1, 3, 7-10.

Supplementary references