Macroeconomics, Part IIB
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Supervision 2
Growth and Capital

Problems

1. Consider an aggregate production function given by [cf Tripos 2002]

$$Y_t = K_t^{\alpha} H_t^{\beta} \left(A_t L_t\right)^{1-\alpha-\beta}$$

where Y_t is aggregate output, K_t the aggregate stock of physical capital, H_t the aggregate stock of human capital and L_t labor at time t, and $\alpha > 0$, $\beta > 0$ and $\alpha+\beta < 1$. Suppose that labor L_t and technology A_t grow at an exogenous rate of n and g, respectively, so that $L_t = L_0 e^{nt}$ and $A_t = A_0 e^{gt}$. Suppose that individuals invest constant shares of their income in physical capital (s_K) and in human capital (s_H) and that both types of capital depreciate at a constant rate δ. Physical and human capital therefore evolve as follows:

$$\begin{aligned} \dot{K}_t &= \mathrm{d}K_t/\mathrm{d}t = s_K Y_t - \delta K_t \\ \dot{H}_t &= \mathrm{d}H_t/\mathrm{d}t = s_H Y_t - \delta H_t \end{aligned}$$

(a) Show that, in the long run, the growth rate of output per worker (Y_t/L_t) is unaffected by changes in the proportions of output invested in physical capital (s_K) and human capital (s_H). What is the intuition behind this result?

(b) Suppose government policy is able to permanently increase the proportion of output allocated to human capital (s_H). Explain the long-run impact on

i. the level of output per worker.

ii. the returns to human capital.

iii. the returns to physical capital.

iv. the share of income going to the total workforce, i.e. human capital and labor combined.

2. Consider a continuous-time Solow growth model. The production technology is represented by:

$$Y(t) = K(t)^{\alpha} L(t)^{1-\alpha}, \ \alpha \in (0,1)$$

where $Y(t)$ corresponds to output, $K(t)$ is the capital stock, and $L(t)$ is labour. Population grows at constant rate n and agents save a fraction $s \in (0,1)$ of income. The economy is closed which implies that investment equals savings. Capital evolves according to the following equation of motion:

$$\dot{K}(t) = q(t)\, I(t) - \delta K(t), \ \delta > 0$$

where $I(t)$ denotes investment and δ is the depreciation of the capital stock. Variable $q(t)$ corresponds to the inverse of the relative price of machinery to output. Assume that this relative price is declining over time, such that $\frac{\dot{q}(t)}{q(t)} = \gamma > 0$. [cf Tripos 2014]

(a) Derive the equilibrium equation that describes the evolution of capital per unit of labour. What is the long-run growth rate of output per unit of labour along the Balanced Growth Path? Explain.

(b) Does the economy converge to this Balanced Growth Path equilibrium? Explain. (Hint: Show how the system can be made stationary.)

(c) What is the equilibrium level of output per unit of labour along the Balanced Growth Path equilibrium?

(d) Show the dynamics of output per unit of labour after a permanent increase in γ. Explain.

Essay question (1000 words max)

3. Thomas Piketty in his best-seller book, "*Capital in the Twenty-First Century*", has documented that the wealth-income ratio in rich countries has increased from roughly 200-300% in 1970 to 400-600% in 2010. He uses standard analytical results of the Solow growth model to make predictions about inequality (i.e. the division of income between labour and capital) in the future. Evaluate these findings and predictions. [Tripos 2015]

Main reading

- Jones (2002), *Introduction to Economic Growth*, ch 1, 3, 7-10.
- Lucas (1990), "Why Doesn't Capital Flow from Rich to Poor Countries?", *American Economic Review Papers and Proceedings* 80(2), May, pp. 92-96.
- Piketty & Zucman (2014), "Capital is Back: Wealth-Income Ratios in Rich Countries, 1700-2010", *Quarterly Journal of Economics* 129(3), pp. 1255-1310.

Supplementary references

- Acemoglu, Johnson and Robinson (2004), "Institutions as the Fundamental Cause of Long-Run Growth", in: Durlauf and Aghion (eds.), *Handbook of Economic Growth.*
- Hall, and Jones (1999), "Why Do Some Countries Produce so much more Output per Worker than Others?", *Quarterly Journal of Economics* 114, February, pp. 83-116.
- Jones (2004), "Growth and Ideas", in: Durlauf and Aghion (eds.), *Handbook of Economic Growth.*
- Krusell & Smith (2015), "Is Piketty's 'Second Law of Capitalism' Fundamental?", *Journal of Political Economy* 123(4), August, pp. 725-748.
- Mankiw, Romer and Weil (1992), "A Contribution to the Empirics of Economic Growth", *Quarterly Journal of Economics* 107, May, pp. 407-38.
- Parente and Prescott (2007), "What a country must do to catch up with the industrial leaders", in: Balcerowicz and Fischer (eds), *Living Standards and The Wealth of Nations: Successes and Failures in Real Convergence.*
- Piketty (2014), *Capital in the Twenty-First Century*, ch 4.

- Sørensen & Whitta-Jacobsen (2005), *Introducing Advanced Macroeconomics: Growth and Business Cycles*, ch 2, 6-10.
- Symposium on Distribution of World Income, *Journal of Economic Perspectives*, Summer 1997.
- Symposium on New Growth Theory, *Journal of Economic Perspectives*, Winter 1994.
- Temple (1999), "The New Growth Evidence", *Journal of Economic Literature* 37(1), March, pp. 112-156.