Supervision 6
International Financial System

Problems

1. Consider the following two-country, two-period endowment economy. The representative household in the Home country maximizes lifetime utility

\[ U = \ln C_1 + \beta \ln C_2 \]

where \( C_t \) denotes Home consumption in period \( t \), with \( \beta \in (0, 1) \). The representative household in the Foreign country, indicated by an asterisk, has the same preferences over \( \{C_1^*, C_2^*\} \). There is a single tradable good, where \( \{Y_1, Y_2\} \) and \( \{Y_1^*, Y_2^*\} \) denote the Home and Foreign endowments in the two periods. Home and Foreign households can freely trade in an international bond \( B \) with world gross real interest rate \( R \). Home and Foreign households do not have any initial assets.

(a) Derive the level of Home consumption \( C_1 \) and Home savings \( S_1 \) in the first period for a given level of \( R \).

(b) Solve for the equilibrium world real interest rate \( R \).

(c) Show that the world real interest rate \( R \) lies in between the autarky real interest rates in the two countries. Explain whether the country with the autarky real interest rate below the world real interest rate will run a current account deficit or surplus in period 1.

(d) Explain the effect of an increase in the growth rate of Foreign output on Home’s welfare.

2. Consider a small open economy that exists for two periods and is populated by many identical agents that maximize their lifetime utility

\[ U = \ln C_1 + \beta \ln C_2 \]

where \( C_t \) denotes domestic consumption in period \( t \), with \( \beta \in (0, 1) \). At time 1, the agents are endowed with an exogenous amount \( Y_1 \) of output. They can invest their savings either in foreign bonds \( B_1 \), which yield the world gross real interest rate \( 1 + r \), or in domestic projects \( i_1 \), which yield output \( Y_2 \) in period 2 with decreasing returns:

\[ Y_2 = Ai_1^\alpha \]

where \( A \) and \( \alpha \) are positive parameters, with \( 0 < \alpha < 1 \). Investment in projects \( i_1 \) requires a specialised imported input with a price of \( p \) units of output, so it costs \( pi_1 \) units of output (i.e. consumption). Assume that initial net foreign wealth is zero, so \( B_0 = 0 \).
(a) Derive the intertemporal production possibilities frontier (IPPF) of available consumption bundles under financial autarky. Graphically show the IPPF, comment on its properties, and explain how it is affected by a decline in the price \( p \) of the imported input.

(b) Write down the budget constraints of the country in the two periods and the optimization problem of the representative agent. Derive the first order conditions characterizing the optimal consumption and investment decisions.

(c) Define and derive the real interest rate \( r_{FA} \) under financial autarky. Explain how it depends on \( A \) and on \( p \).

(d) Suppose that preferences and technology are such that \( 1 + r_{FA} > 1 + r \). Explain whether the current account is going to be positive or negative.

(e) Derive the consumption and investment plans when the agents can trade in the international bond. Explain how consumption, investment and the current account are affected by a decline in the price \( p \) of the imported input.

3. Consider two economies, Home and Foreign, each populated with a continuum of identical agents who live for two periods and maximize expected lifetime utility

\[
U^{(*)} = \ln C_1^{(*)} + \mathbb{E} \left[ \ln C_2^{(*)} \right]
\]

where \( C_t \) denotes Home consumption in period \( t \), and Foreign variables are indicated by an asterisk (*)`. The budget constraints with free trade are

\[
C_1^{(*)} = Y_1^{(*)} - B^{(*)}
\]

\[
C_2^{(*)} = Y_2^{(*)} + RB^{(*)}
\]

where \( Y_t \) denotes (tradeable) output in period \( t \), \( B \) denotes holdings of the internationally traded bond at the end of the first period and \( R \) its gross real interest rate. Assume that output in period 1 is the same in both countries so that \( Y_1 = Y_1^* = 8 \). In period two, the countries have different growth prospects such that

\[
Y_2 = \begin{cases} 
Y_{2H} = 12 & \text{with probability } 0.6 \\
Y_{2L} = 8 & \text{with probability } 0.4 
\end{cases}
\]

\[
Y_2^* = \begin{cases} 
Y_{2H}^* = 18 & \text{with probability } 0.6 \\
Y_{2L}^* = 6 & \text{with probability } 0.4
\end{cases}
\]

So, expected output is higher in Foreign, but the macroeconomic uncertainty it faces is also higher.

(a) Derive the real interest rates under financial autarky \( R_{FA} \) and \( R_{FA}^* \) in Home and in Foreign. Explain what factors determine the level of \( R_{FA}^{(*)} \).

(b) For a given international real interest rate \( R \), explain under what condition Home/Foreign would run a current account deficit.
(c) Assume the world economy consists of the two countries, Home and Foreign. Determine the level of the world equilibrium real interest rate $R_W^*$ and explain whether Home/Foreign runs a current account surplus/deficit.

(d) Now suppose that a revision of Foreign growth prospects changes $Y_2^*$ to

$$Y_2^* = \begin{cases} 
Y_{2H}^* = 20 & \text{with probability 0.6} \\
Y_{2L}^* = 2 & \text{with probability 0.4}
\end{cases}$$

Compute Foreign’s autarky interest rate $R_{FA}^*$ for this case. Explain qualitatively how this affects the world equilibrium real interest rate $R_W^*$, and the current account balances of Home and Foreign.

Main readings


Supplementary references