Supervision 3
Monetary Policy

Short question

1. Suppose that the output gap $y$ is determined by

$$y = \alpha - (\kappa + \varepsilon) r + \nu$$

where $r$ is the policy instrument controlled by the policymaker; $\alpha$ is an anticipated shock that is observed by the policymaker when it sets $r$; $\kappa$ is a positive parameter capturing the effectiveness of the policy instrument; and $\varepsilon$ and $\nu$ are independent shocks that are not observed by the policymaker, with zero means and positive variances $\sigma^2_\varepsilon$ and $\sigma^2_\nu$, respectively. The policymaker minimizes the loss function $L = E[y^2]$.

Derive the optimal policy $r^*$, and explain how the optimal policy response to $\varepsilon$ depends on $\sigma^2_\varepsilon$ and $\sigma^2_\kappa$. Give an economic interpretation of the results.

Problems

2. The following model describes the process for output and inflation:

$$y_t = \beta (i_t - \pi^e_t) + \eta_t \quad \beta > 0$$

$$\pi_t = \pi^e_t + \alpha y_t + \varepsilon_t \quad \alpha > 0$$

where $y_t$ is the output gap, $i_t$ is the nominal interest rate, $\pi_t$ is the inflation rate, and $\pi^e_t$ is the expected inflation rate for period $t$, which is formed at the end of period $t - 1$ using rational expectations. The additive aggregate demand and supply disturbances $\eta_t$ and $\varepsilon_t$ are zero mean, serially uncorrelated processes with variances $\sigma^2_\eta$ and $\sigma^2_\varepsilon$, respectively. These shocks are observed in period $t$ by the monetary policymaker, who sets the nominal interest rate $i_t$ to minimise the quadratic loss function

$$L_t = \lambda y_t^2 + \pi_t^2$$

where $\lambda > 0$. [cf Tripos 2003]

(a) Derive the optimal interest rate rule $i_t = \theta \eta_t + \gamma \varepsilon_t$, where $\theta$ and $\gamma$ are constant parameters. Give a brief intuitive explanation.

(b) Derive the efficient policy frontier and explain why different types of shock pose different problems for the policymaker.

(c) If both output and inflation became more volatile would this affect the policy rule derived in part (a)? Briefly discuss what other forms of uncertainty a policymaker might face in practice and how these affect the conduct of policy.
3. Consider the following model for output and inflation

\[ y_t = -\beta r_t + \eta_t \quad \beta > 0 \]  \hspace{1cm} (1)

\[ \pi_t = \pi_t^e + \alpha y_t + \varepsilon_t \quad \alpha > 0 \]  \hspace{1cm} (2)

where \( y_t \) is the output gap, \( r_t \) is the real interest rate, \( \pi_t \) is the inflation rate, and \( \pi_t^e \) is the expected inflation rate for period \( t \), which is assumed to equal zero. The shocks \( \eta_t \) and \( \varepsilon_t \) are zero mean, independent and serially uncorrelated random variables with variances \( \sigma_\eta^2 \) and \( \sigma_\varepsilon^2 \), respectively. Assume that the policymaker has rational expectations and minimizes the expected value of the loss function

\[ L_t = \pi_t^2 + \lambda y_t^2 \quad \lambda \geq 0 \]  \hspace{1cm} (3)

Suppose that in period \( t \) the monetary policymaker perfectly observes \( \eta_t \) and \( \varepsilon_t \). [cf Tripos 2006]

(a) Give a brief economic explanation of equations (1), (2) and (3).
(b) Derive the optimal real interest rate \( r_t \). Briefly comment on its properties.

Now suppose that the policymaker faces uncertainty and cannot perfectly observe the shock \( \eta_t \). The policymaker has two independent forecasts of \( \eta_t \), one produced by its own staff economists, \( \eta_{S,t} \), and one by the International Monetary Fund (IMF), \( \eta_{IMF,t} \):

\[ \eta_{S,t} = \eta_t + \nu_{S,t} \]

\[ \eta_{IMF,t} = \eta_t + \nu_{IMF,t} \]

where \( \nu_{S,t} \) and \( \nu_{IMF,t} \) are zero mean, independent forecast errors with variances \( \sigma_{S}^2 \) and \( \sigma_{IMF}^2 \), respectively. Both forecasts produce unbiased estimates of the true shock \( \eta_t \), but the staff economists have a better forecasting record than the IMF, so \( \sigma_{S}^2 < \sigma_{IMF}^2 \).

(c) Derive the best unbiased forecast of \( \eta_t \) that the policymaker can make. Provide an intuitive explanation of the result.
(d) Explain what the optimal real interest rate \( r_t \) is in this case. Compare your answer to part (b).

Main readings

Supplementary references