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Real options and demographic decisions

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Abstract

The theory of real options is used to incorporate the influence of uncertainty on demographic decision-making. The decision to have children is formulated as an investment using portfolio theory. The timing of the decision to have a child is modelled as a real options decision, with uncertainty affecting a woman's ability to exercise the '*option to wait*' in order to delay or space births. An increase and reduction in uncertainty on this option is explored. Compared to the widely used net present value (NPV) framework, the real options approach (ROA) better explains the process of demographic decision-making in poor countries.

JEL classification: D81; J13; O12
Keywords: Option value; Demography; Uncertainty

1. Introduction

The decision to have a child is one of the most important decisions that couples take together. The decision to terminate childbearing is equally so. Women in developing countries today are both having fewer numbers of children and using an unprecedented level of permanent methods of contraception such as sterilization. In developing countries the highest rates of sterilization use, between 30% and 50%, are in Puerto Rico, Republic of Korea, China, Brazil, and India (EngenderHealth, 2003). It is predicted that the rates of

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sterilization will continue to increase until 2015, particularly in Sub-Saharan Africa, Asia and Latin America (EngenderHealth, 2003). The most paradoxical aspect about sterilization though is that in countries in which its prevalence is high, the median age at sterilization is very *low* (EngenderHealth, 2003). So why are women having fewer children and using permanent contraception at a much earlier age than their mothers or grandmothers before them?[1] What determines their decision-making?

This paper uses portfolio theory and real options in order to understand the nature and timing of demographic decision-making. We argue that demographic decisions such as whether or not to have a child are investment decisions. The decision to have a child displays the characteristics of other real options investments such as irreversibility and the necessity for flexibility (Dixit and Pindyck, 1994; Bernanke, 1983). We argue that real options reasoning is a valuable tool for demography because it allows us to extend the existing framework of the net present value analysis of childbearing, by incorporating the value of deferring a decision to have a child, as uncertainty unfolds. To this end, our paper differs substantially from existing contemporary frameworks for analysis in economic demography.

Contemporary demography emphasises the role of economic and non-economic factors in population growth, first depicted in classic studies by Bulatao and Lee (1983), the Princeton European Fertility Project (Coale and Watkins, 1986), and the 'new economic demography' which began with the ideas of Becker (1981). Collectively, this work has spawned a vast body of empirical research in economic demography which relies mainly on modelling and testing demographic choices by individuals or households as they balance the costs and benefits of childbearing and child-rearing (Schultz, 1998). Most recently, economists have been evaluating the role of reproductive externalities, networks effects and social interactions on fertility behaviour (Durlauf and Walker, 1999; Dasgupta, 2000; Kohler, 2001; Montgomery et al., 2001; Manski and Mayshar, 2002). But in essence, these additions do not alter the net present value (hereafter, NPV) framework in which demographic studies are presently conducted. Using this framework has meant that empirical studies have involved evaluating demographic decisions, but they have done so retrospectively, for example, in a number of fertility studies by taking into account family size as completed, using measures such as children ever born (CEB) and the total fertility rate (TFR). Contemporary analyses do not evaluate women's decisions to have children *as they are being made*. More importantly, there is one particular question that collectively these approaches have not addressed: *Is there a value in waiting to have a child, which may arise as a response to the influence of uncertainty*?[2] On this question, the economic demography literature has been entirely silent.

In this paper, children are viewed as an investment decision in which the presence of uncertainty affects the decision about when to have a child. To this end, within an expected utility framework, we first make use of portfolio theory to elaborate the decision to have a child and target optimal family size, and then use the theory of real options to discuss how

[1] It is recognised that a woman can also stop births by using temporary methods, but it is the decision to do so, rather than the method she uses to enforce it, which will be modelled here.

[2] As Dixit and Pindyck argue, for real investment decisions, when ignoring the influence of uncertainty, '. . . the simple NPV rule is not just wrong; it is often *very* wrong' (Dixit and Pindyck, 1994: 136).

and when the timing of births will occur to achieve this target. Viewing children as investments and consequently using the real options approach (hereafter, ROA) has merit because it explains more clearly the *process* of demographic decision-making. In this discussion, we suggest that the influence of uncertainty on demographic decision-making is exerted by its impact on the '*option to wait*' to delay or space births. The effect both of an increase and of a reduction in uncertainty on the option to wait is discussed. We argue that this approach provides a better *economic* framework in order to answer questions that lie at the heart of demography: 'How is the decision to have a child made?', 'How are the decisions about additional children, or completed family size, made?', and 'How does uncertainty affect this decision?'.

This paper consists of 5 sections. Section 2 elaborates why the use of portfolio theory and real options reasoning is necessary to understand demographic decision-making. Section 3 develops first, a formal portfolio theory of investment in children and then incorporates the option to wait in a formal real options model for the decision to have a child. In the context of this model, Section 4 discusses the impact of an increase, and then of a reduction in uncertainty on the decision to have a child. This section makes reference to observations about unusual findings in empirical demography which existing frameworks do not provide an adequate explanation for, and which, we argue, the real options framework sheds better light upon. Section 5 concludes.

2. Conceptual framework

Conventional economic theory argues that men and women evaluate the costs and benefits of childbearing in a net present value framework taking into account economic factors such as the contributions to current and future income that children may make, but also the psychological and societal benefits of having children (Becker, 1981; Schultz, 1998; Dasgupta, 2000). In developing countries, parents devote both their time and money in order to nurture a child, with a view of obtaining a return on their investment. The return from a child can be expressed in the form of emotional happiness, current labour income, as well as monetary support in old age. To this extent, economists have traditionally viewed children as consumer goods, producer goods and investment goods (see, for example, Dasgupta, 1993). This implies that the net benefit of having children incorporates an insurance element.[3]

The focus on net benefits or expected returns from children is central to economic theories of fertility (Becker, 1981). If children are considered as normal goods, then the simple income effect on childbearing would imply higher fertility associated with rising incomes. However, the simple income effect on fertility is usually outweighed by two other effects: first, price effects specifically in terms of the rising opportunity cost of parental time; and secondly, substitution effects that make couples substitute away from

[3] The net benefit (or expected return) from having children would include the balance between factors such as the psychological and economic benefits of having children, the opportunity costs of parental time, the economic cost of obtaining a contraceptive method, the social or psychological costs (for example, the distance from a health centre or resistance from other family members), and the economic costs of having children.

greater 'child-quantity' and towards increased 'child-quality' (Becker and Lewis, 1973; Schultz, 1997). The collective impact of these effects is that with higher incomes, fertility falls. For different women, depending upon the balance of individual factors, such as their level of education or employment which determines the opportunity costs of their time, the location of the net benefit curve will be different. However it is very important to note that for all women the utility function is most likely to be concave exhibiting diminishing returns. This is because while the net benefits initially increase when having the first couple of children; having many children diminishes the net benefits substantially. Any increase in the net benefits of childbearing caused initially by factors such as greater companionship engendered by the first few children may soon be followed by reduced net benefits overall if there are, for example, many mouths to feed, clothe and house.

In this paper, we continue to extend the expected utility framework of Beckerian models of fertility, by emphasising the investment and the returns from having children. In many developing countries, the return on the investment in children can be highly uncertain due to the risk of child mortality and other factors. Consequently, a couple's decision making framework then concerns their need to allocate limited resources to having children who may yield an uncertain return, compared to investing in other relatively more certain investment opportunities. In this respect, the decision to have a child can be viewed in the same light as any other investment decision. We argue therefore that the decision problem that a couple faces is very similar to the lifetime portfolio selection choice under uncertainty developed by Merton (1969). Consequently in this paper, the Merton model is the basis upon which we develop a model that reflects the process of decision-making about completed family size. We argue however that the *timing* of the decision to have a child will be significantly influenced by uncertainty. Once the target family size has been established, we develop a real options framework to determine when couples choose to have children.

An option is the right but not the obligation to take an action in the future. Options reasoning has been applied most commonly to financial modelling and real investment decisions in management science and natural resource management (Dixit and Pindyck, 1994; Hull, 1993; Merton, 1973; Black and Scholes, 1973; Amram and Kulatilaka, 1999). Real options reasoning is a theory of investment which identifies the factors that influence 'the point at which investors choose whether to invest or not' (Miller and Folta, 2002: 656). The real options approach traditionally has been used to understand the risk of new technology projects (McGrath and MacMillan, 2001) but has also been applied more widely to real estate markets (Grenadier, 1996), patents (Bloom and Van Reenen, 2002), nuclear waste management (Louberge et al., 2002), migration (Burda, 1995), and family farm transfers in less developed countries (Miljkovic, 2000). It has not, to the best of our knowledge, been applied to demographic decisions.

For demography, we argue that the ROA has many advantages over the NPV framework. First, the NPV framework does not take into account the effect of *uncertainty* (Kalemli-Ozcan, 2003). Uncertainty implies that couples need to be flexible as to whether they have the next child, and to react to factors such as child mortality and the uncertainties that affect economic circumstances. At the heart of these decisions are implicit trade-offs between the commitment not to have another child and flexibility in choosing when to have the next child (Ghemawat, 1991).

Second, real options theory is a useful tool when decision-making displays *irreversibility* (Bernanke, 1983; Dixit and Pindyck, 1994). Surviving children are 'irreversible' investments in that once born, they cannot be 'reversed' unlike the case of more traditional investments. The difference between children and other firm- or industry-specific investments is also that children cannot be delayed until the majority of the uncertainty is resolved (due to biological constraints on women's fecundity), or by breaking-up the investment into stages.

Third, real options reasoning is useful when decision-making demands *flexibility*. For demographic decisions in poor societies, whether you have an additional child is highly uncertain—it depends upon existing children, a woman's fecundity, marriage status, social milieu, and so forth. Given that flexibility is an integral part of demographic decision-making, we argue that the ROA should be the logical approach to pursue.

Fourth, and most importantly, the decision to have a child can be evaluated at different decision points, for example, at the birth of each additional child. We can define therefore an '*option to wait*' for the woman at each decision node in her reproductive life-span: this is the marginal benefit of keeping open the decision to have a child, versus the alternative either to delay or to stop childbearing. So the options framework for demography incorporates *path dependency*—the sequence of when and in what circumstances women exercise an option to wait. This allows us to evaluate the decision to have a child, or to exercise the option to wait, as these decisions are being made.

3. Modelling completed family size

This section develops more formal models of portfolio theory and real options for economic demography. First, the portfolio theory model examines how couples take the decision to have a child and their desired family size, in an expected utility framework. We then incorporate the real options model to elaborate upon the timing of these decisions by incorporating the effect of uncertainty on the option to wait. Together, these models provide an economic framework within which to consider both the nature and timing of fertility decisions.

3.1. A portfolio theory model of investment in children

Let us assume first that a couple's lifetime wealth can be stated in present value terms as W. Total lifetime wealth is the present value in monetary terms of the money costs and the opportunity costs of time over the lifetime of the couple. Secondly, let us assume that there exists an investment opportunity in a sure or risk-less asset.[4] The couple can then choose

[4] The 'risk-less' asset can be thought of as land or other investment opportunity where the relative return, compared to that from having children, is assumed to be fairly certain.

to allocate their personal lifetime wealth among current consumption, investment in a risk-less asset, and investment in a child, the latter we suggest can be viewed as a risky asset with an uncertain return. Then, let

n = fraction of wealth in the risky asset (this is the desired number of children)
s = return on the sure asset
μ = expected return on the risky asset, where $\mu > s$
r = the opportunity cost of investment (or the value of parental time)
σ^2 = variance per unit time on the return from the risky asset
c = consumption
$U(C) = c^b/b$ is the utility function, where $b < 1$.

In the model that is developed, we will neglect integer restrictions on the number of children. The utility function specification[5] allows for a constant relative risk aversion with a Pratt (1964) measure of relative risk aversion: $-U''(C)C/U'(C) = 1 - b = \delta$.

The change in wealth is given by

$$\mathrm{d}W = [s(1-n)W + \mu nW - c]\mathrm{d}t + nW\sigma\mathrm{d}z \tag{1}$$

The deterministic portion of the change in wealth is composed of the return on the risk-less asset plus the expected return from the investment in children less consumption. The stochastic component of the change in wealth will be a function of the proportion of investment in children, the standard deviation of the return on the investment in children and dz, an increment to a Weiner process (a continuous time Markov process with independent and normally distributed increments): $\mathrm{d}z = \varepsilon_t\sqrt{\mathrm{d}t}$ with $\varepsilon_t = N(0,1)$. The objective is maximization of the expected discounted utility stream.

For convenience, we will assume an infinite horizon:[6]

$$\max \int_0^\infty \left(e^{-rt}c^b/b\right)\mathrm{d}t \tag{2}$$

Subject to (1) and $W(0) = W_0$.

This is an infinite horizon autonomous problem with one state variable W and two controls c and n. Using the specifications of (1) and (2) and stochastic optimal control theory, we can restate this as follows:

$$rV(W) = \max_{c,n}\left(c^b/b + V'(W)[s(1-n)W + \mu nW - c] + (1/2)n^2W^2\sigma^2V''(W)\right) \tag{3}$$

[5] This specification to the utility function implies a constant relative risk aversion (i.e. isoelastic marginal utility) which allows for the model to be solved explicitly.

[6] Because we are interested in studying the qualitative changes in the investment in children with respect to the change in the parameters, the simpler form of the infinite time horizon case is examined. The results will not be substantially different with a finite time horizon.

where $V(W)$ is an unknown function to be determined. Calculus gives us the maximizing values of c and n in terms of the parameters, the state W and the function V:

$$c = [V'(W)]^{1/(b-1)}, \qquad n = V'(W)(s-\mu)/\sigma^2 W V''(W) \tag{4}$$

The optimal solution involves investment in both assets at all times. Substituting from (4) and (3) and simplifying yields

$$rV(W) = (V')^{(b/b-1)}(1-b)/b + sWV' - (s-\mu)^2 (V')^2 / 2\sigma^2 V'' \tag{5}$$

Let us assume a solution to this nonlinear second order differential equation of the form

$$V(W) = AW^b \tag{6}$$

where A is a positive parameter to be determined. By computing the first and second derivatives of (6) and substituting the results into (5) and simplifying, we get

$$Ab = \left\{ \left[r - sb - (s-a)^2 b / 2\sigma^2 (1-b) \right] / (1-b) \right\}^{b-1} \tag{7}$$

Hence the optimal current value function is (6), where A is as specified in (7). By substituting (6) and (7) in (4) we get the optimal control functions:

$$c = W(Ab)^{1/(b-1)} \tag{8}$$

$$n = (\mu - s)/(1-b)\sigma^2 = (\mu - s)/\delta\sigma^2 \tag{9}$$

The optimal division of lifetime wealth between children and the risk-less asset is a constant and independent of total lifetime wealth.[7] The proportion devoted to the risky asset represents the desired number of children that the couple intends to have. The desired number of children varies directly with the return on the children as $\partial n/\partial \mu > 0$. On the other hand, the desired number of children varies inversely with the variance of the return on the investment in children as $\partial n/\partial \sigma^2 < 0$.

To examine the relative effect on the desired number of children, of an upward shift in the mean compared to a downward shift in the variance, we need to examine the relevant elasticity. Let us define the elasticity of the desired number of children with respect to the mean as

$$E_a = \frac{\partial n}{\partial \mu}\frac{\mu}{n} = \frac{1}{\delta\sigma^2} \times \frac{\mu}{n}$$

[7] The specification of a utility function with constant relative risk aversion results in the desired number of children being a constant and independent of the wealth level. In contrast, a utility specification of the form $U(c) = -\exp(-\eta c)/\eta$, $\eta > 1$ gives a constant Pratt (1964) measure of absolute risk aversion, $-U''(c)/U(c) = \eta$. This provides the result where the desired number of children falls as the wealth level increases, where $n = \frac{\mu - s}{\eta s \sigma^2 W(t)}$. Although this specification conforms to the income effect reducing the number of children as postulated by Becker and Lewis (1973), for analytical tractability we will use the case of constant relative risk aversion as formulated above.

By substituting (9) into the above equation and rearranging the terms, we get

$$E_{\mu} = \frac{\mu}{(\mu - s)} \tag{10}$$

Similarly, the elasticity of the desired number of children with respect to the variance is

$$E_{\sigma^2} = \frac{\partial n}{\partial \sigma^2}\frac{\sigma^2}{n} = \frac{(\mu - s)(-1)}{\delta(\sigma^2)^2} \times \frac{\sigma^2}{n}$$

By substituting (9) into the above equation and rearranging the terms, we get

$$E_{\sigma^2} = -1 \tag{11}$$

The elasticity of the desired number of children with respect to the variance is a constant. The elasticity of desired children with respect to the mean, E_a will always be larger than the elasticity of desired children with respect to the variance, E_{σ^2} as $\mu > s$. However, as the difference between the expected return on children gets larger relative to the return on the safe asset, the elasticity of desired children with respect to the mean approaches the elasticity of desired children with respect to the variance. This is shown more clearly in Fig. 1.

The relevance of the discussion of the elasticity with respect to the mean and with respect to the variance in Eqs. (10) and (11) for empirical demography is discussed more fully in Section 4 below. Suffice to comment at this stage that the balance between these two elasticities becomes important in situations of a change in the levels of uncertainty.

In summary, portfolio theory is useful because it enables couples to identify their desired family size. Once the desired number of children has been established, couples then need to take the decision about when to have their children, i.e. the timing of the fertility decision. In this paper we argue that because the timing of fertility decisions is influenced by uncertainty, it is necessary to use a real options model. The section which follows discusses this argument in greater detail.

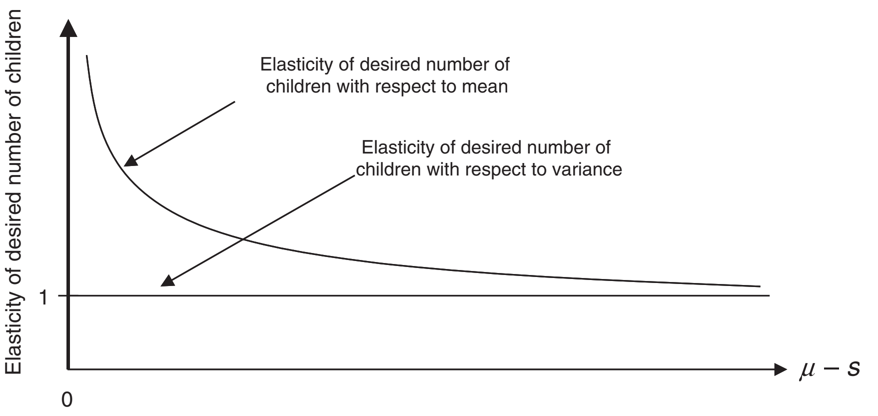

Fig. 1. Elasticity of the desired number of children.

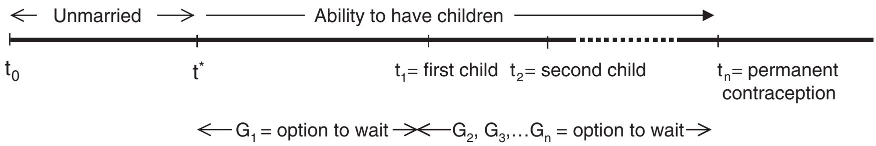

Fig. 2. The fertility decision.

3.2. A real options model for the decision to have a child

In a demographic context, let us define C[8] as the costs of children and R as the benefits.[9] Let the future benefits and costs be discounted at a positive rate $r>0$, the opportunity cost of parental time specified exogenously. Then the net benefits of children (let us call this B) are:

$$B = R - C$$

This is the conventional net present value (NPV) decision framework. Now consider Fig. 2, which illustrates a representative woman's fertility decisions over her reproductive life span.

For the woman, we assume that biological constraints dictate that she does not bear any children from the time of her birth, t_0, until the age at which she attains menarche. In many societies, she would be married considerably later than the age at menarche, and this age is denoted by t^*. Note that we assume that childbearing is taking place within marriage. This is not an unrealistic assumption to make, as in many developing countries childbearing continues to take place largely within marriage. But the assumption is not strictly necessary. Married or not, the woman embarks upon her reproductive life span at t^*. In a net present value framework, the woman may choose to have her first child at time t_1, her second child at time t_2, and so forth, where it is argued, these decisions are dictated by the balance between the costs and benefits of children ($R-C$).

Now, the conventional NPV framework can be modified to incorporate the option to wait (F) using the real options approach. Assume that the benefits of having children are subject to uncertainty. Then there is value in waiting to have the next child, which exists over and above the net benefit of having the child. This value is derived from two sources. First, the value from delaying having the next child in order to see whether, and how, the uncertainty surrounding the benefits of having the next child resolves. Second, the value from the benefit of delaying the current cost of the investment to have the child, X[10]. The second condition will hold as long as the reduction in the net benefit from having the child is smaller than the savings generated from delaying the investment in having the child. This results in the decision to have the next child being undertaken at a date *later* than that which conventional NPV analysis would predict. Let us show this result more formally.

Let us denote the value of waiting to have the next child by $F(B)$. The benefits of having children are subject to uncertainty. The level of uncertainty will be influenced by

[8] This is the cost of child rearing.
[9] Both the costs and benefits are stated in present value terms.
[10] This current cost of investment is the cost of childbearing that includes hospital and other medical expenses.

two types of risks, namely, private risks to women in childbearing and the collective risks or market risks to women as a group. Private risks to the woman may include such factors as women's own education and employment, existing children and their characteristics; and the existing household structure (whether or not the household is nuclear or joint). On the other hand, the collective risks to women or market risks include general infrastructure provision, access to health services, communal relations, the status of women in general locally, the volatility of agricultural prices, how prosperous the region is generally, weather, climate and so forth, that are common to women living in a particular region more generally.[11]

In option theory the uncertainty can be modelled as a simple Brownian motion with drift:[12]

$$\mathrm{d}B = \mu \mathrm{d}t + \sigma \mathrm{d}z \tag{12}$$

where μ is the instantaneous conditional expected percentage change in B per unit time, σ is the instantaneous conditional standard deviation per unit of time, and dz is an increment to a Weiner process (a continuous time Markov process with independent and normally distributed increments): $\mathrm{d}z = \varepsilon_t \sqrt{\mathrm{d}t}$ with $\varepsilon_t = N(0,1)$. If the current cost of childbearing X is normalized to 1, then μ is the same as the expected return on children as defined in Section 3.1 where $\mu = E(\mathrm{d}B/X)$.[13]

It follows then that,

$$E(\mathrm{d}B) = \mu \mathrm{d}t \tag{13}$$

$$Var(\mathrm{d}B) = \sigma^2 \mathrm{d}t \tag{14}$$

and

$$E\left[(\mathrm{d}B)^2\right] = Var(\mathrm{d}B) + [E(\mathrm{d}B)]^2 = \sigma^2 + [\mu \mathrm{d}t]^2 \tag{15}$$

In equilibrium, the 'assets' are priced to return the opportunity cost of parental time, r. By Ito's Lemma, the expected capital gain from holding the 'asset' $F(B)$ can be expressed as:

$$\mathrm{d}B = F'\mathrm{d}B + 0.5F''(\mathrm{d}B)^2 \tag{16}$$

As the option to wait does not have any interim payoff, the total return to holding this option comes in the form of capital gains, dF/F. Therefore, the expected return on F per unit of time, α_F, is

$$\alpha_F = \left((\sigma^2/2)F'' + \mu F'\right)/F \tag{17}$$

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[11] Some of these private and market risks, such as inadequate access to water and fuel infrastructure, raise the level of uncertainty (Aggarwala et al., 2001); other factors, such as rising rural wages, would significantly decrease it (Khandker et al., 1998).

[12] The set-up and solution of the model derived here is based upon standard real options methods and solutions, as presented in Dixit and Pindyck (1994).

[13] This relates to a one period return where dt = 1.

In equilibrium, this expected return must equal the normal return rF, hence:

$$\left(\sigma^2/2\right)F'' + \mu F' - rF = 0 \tag{18}$$

Eq. (18) will consist of linear combinations of the form with the solution

$$F(B) = A_1 e^{\beta_1 B} + A_2 e^{\beta_2 B} \tag{19}$$

A_1 and A_2 are constants to be determined and β_1 and β_2 are the solutions to the quadratic equation:

$$\left(\sigma^2/2\right)\beta^2 + \mu\beta - r = 0 \tag{20}$$

The solutions for β_1 and β_2 are:

$$\beta_1 = \left[-\mu + \left(\mu^2 + 2\sigma^2 r\right)^{0.5}\right] \Big/ \sigma^2 > 0 \tag{21}$$

$$\beta_2 = \left[-\mu - \left(\mu^2 + 2\sigma^2 r\right)^{0.5}\right] \Big/ \sigma^2 < 0 \tag{22}$$

In addition, we impose the condition that the value of the option to wait goes to zero when the benefit of having the next child goes to zero. In other words the first boundary condition imposes the constraint,

$$F(0) = 0 \tag{23}$$

Boundary condition (23) implies that $A_2 = 0$. We can then simplify the option value of waiting as

$$F(B) = A_1 e^{\beta_1 B} \tag{24}$$

(24) is unambiguously positive as long as $A_1 > 0$. To determine the two unknowns, A_1 and the trigger point B_t, where the decision is taken to have the next child, two additional boundary conditions are necessary.

The first of these boundary conditions is the 'value matching' condition. Intuitively, this states that for the couple to have the child, the net benefit of having the child must be equal to giving up the option to wait. This condition therefore equates the 'value of waiting' with the net present value of having the child at $B = B_t$:

$$F(B) = (r\beta_1)^{-1} e^{-\beta_1 B_t} e^{\beta_1 B_t} = (r\beta_1)^{-1} = (B_t + \mu/r)r - X \Rightarrow B_t = \beta_1^{-1} + rX - \mu/r \tag{25}$$

The second boundary condition is the 'smooth pasting condition' which ensures the optimality of the trigger point by setting the derivative of F with respect to B equal to the NPV valuation, when B equals B_t:[14]

$$F'(B) = A_1 \beta_1 e^{\beta_1 B_t} = 1/r \Rightarrow A_1 = (r\beta_1)^{-1} e^{-\beta_1 B_t} \tag{26}$$

[14] For further detail about the smooth pasting condition, see Dixit and Pindyck (1994).

Hence, the solution can be written as,

$$F(B) = (r\beta_1)^{-1} e^{\beta_1(B - rF + \mu/r) - 1} \quad \text{for } B < B_t \tag{27}$$

$$F(B) = (B + \mu/r)/r - X \quad \text{for } B \geq B_t \tag{28}$$

In a Marshallian world which evaluates decisions on the basis of positive NPV, the decision to have the next child will occur when,[15]

$$B_{\mathrm{NPV}} \geq rX - \mu/r \tag{29}$$

From Eqs. (25) and (29), the trigger point for the exercise of the option to have the next child occurs at a later date than the pure Marshallian NPV case because $B_t > B_{\mathrm{NPV}}$, as β_1^{-1} is positive.

$$\beta_1^{-1} + rX - \mu/r > rX - \mu/r \tag{30}$$

This result is due to the delay engendered by the benefit arising from being able to keep the 'option to wait' open. The value β_1^{-1} drives a wedge between the trigger point of the Marshallian NPV and the case with uncertainty. It is important to examine the impact of higher uncertainty on this trigger value. This is done, using standard comparative statics, by differentiating the quadratic expression (20) totally:[16]

$$(\partial Q/\partial \beta)/(\partial \beta_1/\partial \sigma) + \partial Q/\partial \sigma = 0 \tag{31}$$

where the derivatives are evaluated at β_1. Since $\beta_1 > 0$, $\partial Q/\partial \beta = \sigma^2 \beta + \mu > 0$ and $\partial Q/\partial \sigma = \sigma \beta^2 > 0$. Therefore for Eq. (31) to hold, $\partial \beta_1/\partial \sigma < 0$. In other words, as σ increases, β_1 decreases and therefore β_1^{-1} increases. The greater is the uncertainty over the future net benefits of having children, the larger is the wedge between the Marshallian trigger and the trigger taking into account the uncertainty. Intuitively, the greater is the uncertainty over the future net benefits of having children, the greater will be the net benefit demanded by couples before they are willing to have the next child. Fig. 3 shows $F(B)$ as a function of B for $X = 1$. The value of $F(B)$ is larger than the net benefit, B, due to the value of uncertainty and the value from delaying the investment cost of having the child, as discussed above. This creates the value of waiting. This value is higher when the uncertainty is higher as denoted by $F^1(B) > F^2(B)$. Hence, increased uncertainty can increase the value of waiting and thereby delay having the next child. For example, in Fig. 3, the increased uncertainty results in a requirement for a larger net benefit, from 1.5 to 2.0, in order to compensate for giving up the greater opportunity cost of waiting.

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[15] The present discounted value of expected future benefit is $\int_0^\infty e^{-rt} E(B_t)\mathrm{d}t$. But $E(B_t) = \int_0^t E(\mathrm{d}B_t)\mathrm{d}t = B_0 + t\mu$, so $\int_0^\infty e^{-rt} E(B_t)\mathrm{d}t = (B_0 + \mu/r)/r$.

[16] Where $Q = (\sigma^2/2)\beta^2 + \mu\beta - r$.

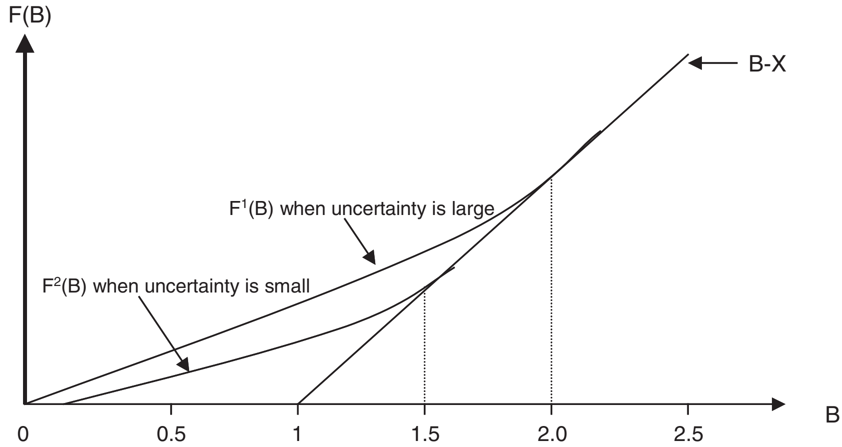

Fig. 3. Value of the opportunity of having a child.

3.3. Under what conditions will the option to wait be exercised and the decision taken to terminate fertility?

Let us assume that women have the opportunity to terminate the decision to have an additional child by using a permanent method of contraception. Within this framework, the incremental value associated with moving from holding the option to wait to taking the decision to have a child is given by

$$M = f(B, X, F_1, F_2, \ldots, F_n,),$$

where F_1, F_2 refers to the 'option to wait' to have the first, second,. . .n children, respectively. The existing literature from economic demography does not take into account the value of the option to wait, F. The real options approach is based on maximizing M at each point of decision-making. In this case, it would be at each node that a couple decides whether or not to have another child. A couple would naturally only take the decision to have a child when the benefit of having the child exceeds the total cost of having the child, that is when $B=X+F$. The difference between this approach and the conventional demographic one is that here we are explicitly taking into account the opportunity cost of foregoing the option to wait, F, in the decision framework.

At some point in the future, say t_n, when the desired number of children has been attained, the costs of an additional child will exceed the benefits and hence the woman may seek a method of contraception such as contraceptive sterilization, if she does not want to bear further the risk of unwanted pregnancies. She could also choose a temporary method to achieve the same outcome, but let us assume for the sake of argument that childbearing is terminated by the adoption of a permanent method. Then in this case, the cost of foregoing having another child indefinitely includes first, the cost of undertaking the sterilization operation (A) and second, surrendering the option to wait (F). Therefore, we argue that it is possible that a permanent contraceptive method will be used when the desired number of children has been attained and the woman is content with surrendering the option to wait.

4. How does uncertainty affect the value of the option to wait?

As outlined in Section 3.1 above, the decision to terminate childbearing altogether will be a function of portfolio investment choice whereas the value of the option to wait will be a function of uncertainty. The higher the uncertainty, the higher is the value of this option. The model set out in Sections 3.1 and 3.2 is developed in two stages. First, we investigate the effect on demographic decisions for the case in which there is an *increase* in uncertainty. Second, we investigate the case in which there is a significant *reduction* in uncertainty. In this context, we also discuss some observations on empirical fertility outcomes in poor countries.

4.1. Increase in uncertainty

Let us consider first a situation in which there is a significant increase in uncertainty. The increase in uncertainty will require a larger buffer against unforeseen circumstances. The value of the insurance provided by having a child, or additional children, will increase substantially. The increase in uncertainty will have an impact on the option value to wait and the decision to terminate childbearing. This influences both the timing of the decision to have children, as well as the decision to use, for example, a permanent method of contraception. The increase in uncertainty will have two opposing effects on the desired number of children. First, the increased uncertainty will reduce the value of n, the desired number of children. However, the increased uncertainty will also increase the insurance value of having children and hence increase the return or net benefit of having children. This in turn will have the effect of increasing the desired number of children. The net impact of these two forces will depend upon the relative increase in uncertainty, the increased benefit from the insurance effect and the respective elasticities as shown in Eqs. (10) and (11) above. The increased uncertainty will increase the value of the option to wait. It must be noted that there are two opposing forces at work here: the increase in uncertainty will increase the option to wait and reduce the demand for children—this will delay the decision to adopt a permanent method of contraception. On the other hand, the increased benefits of having children imply that there will be greater demand to have the next child, and faster. The *net* impact will depend upon the relative strength of these opposing forces.

For example, in a situation of famine where there is a sudden and rapid increase in child mortality, and if there are many child deaths in a particular family, then the reduction in the existing number of surviving children makes the benefit of having an additional child very high indeed. In this extreme situation, the increased net benefits may more than compensate for the delay caused by the increased value of the option to wait, in order to reduce uncertainty. Hence, the net impact on fertility in a famine situation would be a spurt in the birth rate in order to replace those children that do not survive, coupled with the desire to create a strong buffer as a result of the increased uncertainty. An options-based approach would therefore predict that in a situation of an extreme increase in uncertainty, such as a famine, we would expect an increase in the birth rate. On the other hand, when a family loses a child, say due to illness, the increased benefit of replacing the child may not be compensated for sufficiently by giving up the option to wait as a result of the increased uncertainty. In this case, we may observe that the family waits before taking the decision to replace the child, resulting in either a longer birth interval before the child is replaced, or a situation in which the child is never replaced.

Therefore, in situations of an increase in uncertainty the options-based approach would argue that the outcome will be ambiguous, depending upon the source of the increase in uncertainty. In situations of an extreme increase in uncertainty, such as a famine situation, fertility is likely to rise; if the source of the increase in uncertainty is not as extreme, fertility may remain at the same level.

4.2. Reduction in uncertainty

Having considered an increase in uncertainty, let us now consider the converse: a situation in which the uncertainty is reduced. Families often have to consider the need to have children as an insurance against various uncertainties. These uncertainties can manifest themselves in various forms, for example, risk to family income and mortality risks. Historically, the experience of most developed and developing countries (with the exception of France) has shown that a decline in mortality usually precedes a decline in fertility (Chesnais, 1992). In order to protect themselves against risks and unforeseen circumstances, families often need to have additional children as insurance. However as these risks subside as a result of improved education and health services, there is a reduction in risks from income uncertainty. In such a situation, for each additional child, the insurance benefit is reduced.

The reduction in uncertainty will reduce the expected return from children. The decrease in uncertainty will have two opposing effects on the desired number of children. First, the decreased uncertainty will increase the value of n, the desired number of children.[17] However, the decreased uncertainty will also reduce the insurance value of having children and hence decrease the return or net benefit of having children. This in turn will have the effect of decreasing the desired number of children. The net impact of these two forces will depend on the relative decrease in uncertainty, the decreased benefit from the insurance effect and the respective elasticities as shown in Eqs. (10) and (11) above. Typically, one would expect that in a situation of reduced uncertainty, the decrease in the demand for children as a result of the decreased insurance benefit more than compensates the decrease in the demand for children as a direct result of the reduction in the uncertainty. This is due to the fact that the elasticity of the demand for children with respect to the mean is higher than the elasticity of the demand for children with respect to the variance (as also shown in Fig. 1). At lower levels of uncertainty, this difference in the elasticities is larger, as the difference between the return from children and the return from the safe asset is smaller. Thus, a reduction in uncertainty will have an impact on the option value and hence on the timing of the ‘investment’ in children.

First the reduction in risk will reduce the value of the option to wait to have the next child, F. This implies that birth intervals between having successive children will be shorter.

[17] The relationship between mortality and fertility is not always straightforward: for example, Becker and Barro (1988) show that declining mortality may increase the demand for surviving children because the cost of raising surviving children is reduced.

Second, as outlined above, the net benefit of an additional child at each level will reduce. Hence, the desired or targeted number of children will also fall. With a smaller desired family size and shorter birth intervals between children, target fertility is achieved more quickly. As a result, the decision to terminate childbearing altogether will occur earlier in a woman's reproductive lifetime. So an options approach would predict that a reduction in uncertainty will have three effects on demographic decisions: *it will decrease the desired number of children that women want, it will reduce the birth intervals between each child as women will be reluctant to exercise the option to wait, and it will reduce the age at which women decide to terminate childbearing and use a permanent contraceptive method*.

For example, in developing countries, sterilization is one of the oldest and most widely used methods of fertility control. The rates of sterilization use are between 30% and 50% in many countries, and it is predicted that these rates will continue to increase until 2015, particularly in Sub-Saharan Africa, Asia and Latin America (EngenderHealth, 2003; Arends-Kuenning, 2002; Säävälä, 1999). Paradoxically though, in countries in which the prevalence of sterilization is high, the median age at sterilization is very *low* (EngenderHealth, 2003). Women in South Asia, Africa and other developing societies, which are often characterised by limited access to publicly provided social security, are increasingly adopting contraception at early ages after completing a family size of three or four children (EngenderHealth, 2003). This would not appear 'rational' in a net present value context, for them to do so. More importantly, why should a woman, on average, adopt a permanent method of contraception when she is still relatively young? For example, both Census and sample survey data from India show that the median age at sterilization is about 25.7 years (IIPS and ORC Macro, 2000). Why does a typical Indian woman therefore adopt a permanent method of contraception at the age of 25 (or earlier) rather than when she is 35 or 45?

Based upon the ROA, we predict that the empirical demographic consequences of the reduction in uncertainty in poor societies will be three-fold: there will be smaller family sizes, shorter birth intervals between children, and a lower age at which women exercise the option to terminate births altogether. Pursuing the example of India, fertility here has been falling from a TFR of 3.39 in 1992–1993 to 2.85 in 1998–1999 (IIPS and ORC Macro, 2000: 83). What about birth intervals? In India, 13% of births occur within 18 months of a previous birth and 28% occur within 24 months. The median closed birth interval (the length of time between two successive live births) for women aged 15–19 is 24 months, substantially less than the median interval of 36 months for women aged 30–39 (IIPS and ORC Macro, 2000: 100). This finding is important: this particular survey is a nationally representative cross-sectional survey conducted in 1998–1999 of 90,000 ever-married women aged 15–49, drawn from 26 Indian states. It indicates that women sampled in the survey who would have been bearing children more recently (the 15–19 year olds) had, on average, shorter birth intervals between children, than older women (30–39 year olds). Between 1983–1984 and 1998–1999, the all-India women's median age at sterilization declined by 1 1/2 years. The 1998–1999 *National Family Health Survey* data from India, shows that 79% of sterilizations take place before the wife reaches age 30, and only less than 1% of sterilizations take place when the wife is in her 40s (IIPS and ORC Macro, 2000: 146). More significantly, there are very interesting inter-state variations: the median age at sterilization is lowest in the southern Indian states of

Andhra Pradesh (23.6), Karnataka (24.2), Kerala (26.4) and Tamil Nadu (25.3), compared to much higher ages in the north Indian states of Uttar Pradesh (28.3), Bihar (27.7) or Punjab (27.1). These states have also witnessed the greatest declines in fertility with TFRs very low in Kerala (1.5), Karnataka (1.89), Tamil Nadu (2.1) and Andhra Pradesh (2.1) (IIPS and ORC Macro, 2000: 87). So sterilization remains one of the most popular methods of fertility control used in India, and the age at which women undertake a sterilization operation, is very low.

We would argue that the real options reasoning presented in this paper can explain these findings due to the significant reduction in uncertainty in their demographic decisions which women in southern India have experienced over time, and which has reduced the value of the option to wait. There is much empirical evidence from South India that the uncertainty associated with having a child has reduced substantially due to employment in small-scale industry and developed local markets (Kapadia, 2000; Harriss, 1991; Desai and Jain, 1994). Access to health services for women and children through primary health centres is very widespread (Drèze and Sen, 1997); indeed the southern states have a very good record particularly in delivering antenatal care.[18] Better access to maternal and child health-care facilities considerably alters the risk posed by infant and child mortality, as does access to education and literacy schemes,[19] which have all contributed significantly to reducing the levels of uncertainty (Drèze and Sen, 1997; Iyer, 2002). So we argue that women in poor societies increasingly prefer to adopt permanent methods of contraception at relatively young ages because the real option value to women of having additional children is falling very rapidly to zero in societies where improvements in literacy and public health make the uncertainty surrounding the birth of children fall substantially. Mothers therefore decide to have their children quickly, adopting a permanent contraceptive method at an age very much younger compared to their mothers or grandmothers before them.

In summary, both the option to wait and the eventual decision to terminate will be influenced significantly by the effect of uncertainty. An increase in uncertainty would increase the value of the option to wait, possibly causing an increase in fertility if the net benefits of children are outweighed; a reduction in uncertainty would decrease the value of the option to wait, lowering desired family size, reducing birth intervals, and prompting women to terminate childbearing earlier in their reproductive life span.

5. Conclusion

This paper uses the theory of real options in order to understand the nature and timing of demographic decision-making. It is argued that demographic decisions, such as whether

[18] On average, 94% of southern Indian mothers sampled in the *National Family Health Survey* had received at least one antenatal check-up during the 3 years preceding the survey (IIPS and ORC Macro, 2000: 293). Among them, there were also 49% of southern Indian mothers who had received, during the 3 years preceding the survey, 'all recommended types of antenatal care' which includes three or more antenatal check-ups, two or more tetanus toxoid injections, and iron and folic acid tablets or syrup for 3 or more months (IIPS and ORC Macro, 2000: 305).

[19] There is also evidence that education in India is a very significant factor that has reduced fertility over time (see for example, Drèze and Murthi, 2001).

to delay having an additional child (the option to wait), can be viewed as a real options decision, in particular because investments in children display the characteristics of other real investments such as irreversibility and flexibility. A weakness of existing demographic analyses is that they are located in a net present value context, and need to be extended to take into account the uncertainty surrounding childbearing as it unfolds for women, as they proceed through their reproductive years. As this paper has shown, the timing of the decision made to have a child based upon the real options framework is different to the outcome of the conventional NPV framework.

The analysis conducted in this paper first examined a portfolio theory model of investment in children to establish how couples decide upon desired completed family size within an expected utility framework. It then proceeded to examine the timing of the decision to have children to attain desired family size, by setting out a real options model that incorporated the 'option to wait' to delay or space births. The effect of an increase and then of a decrease in uncertainty was considered for this option and consequently, for its effect on the decision to have another child. We observe that the value of thinking about demographic decisions in terms of real options is that it may allow us to explain seemingly unusual empirical findings in demography such as the substantial increase in the use of permanent methods of fertility control in poor countries.

We believe that the value of thinking about demographic decisions in terms of real options reasoning is that it provides a more rigorous economic framework within which to understand demographic decision-making, allowing economists and demographers to evaluate, better than has been done previously, the determinants of couples' decisions to have children, as these decisions are being made.

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