Robust Incentives and Non-Exclusive Contracts

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January 11, 2008

Abstract

This paper studies a moral hazard problem in which the agent can borrow on future wages before undertaking a costly effort. The game between the agent and potential lenders is modelled as an infinite stochastic game with an exogenous stopping probability. It is shown that regardless of the initial wage contract the agent sequentially enters into multiple borrowing agreements which dilute effort incentives, so that high effort is unsustainable. This is compared to the ‘recontracting-proof’ equilibrium that most of the literature focuses on when the stopping probability in the lending game is zero. It is shown that equilibrium profits, welfare and effort in the game with a positive stopping probability are lower but that wage payments can be higher. Finally, it is shown that the two equilibria do not converge when the stopping probability is taken to zero.

*I am very grateful to Alberto Bisin, Robert Evans, Antoine Faure-Grimaud, Leonardo Felli, Piero Gottardi, George Mailath, Alexander Muermann, Volker Nocke and Michele Piccione for extensive comments. I benefited from comments of seminar participants at Birkbeck, London School of Economics, Royal Holloway, University of Cambridge, University of Edinburgh, University of Essex and the University of Pennsylvania.
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1 Introduction

Robust incentives are incentives that are immune to the possibility that the incentivized party engages in further contractual relationships to offset or partly undo those incentives. This paper asks the question whether such robust incentives exist and studies the structure of optimal incentive contracts in the context of a moral hazard problem.

To illustrate the issue, consider the problem of managerial compensation. Traditional principal-agent theory predicts that for incentive reasons pay should be tied to firm performance. To this aim a manager receives shares in his firm, is paid via stock options or is rewarded with a bonus at the end of the year. However, if profits are risky and the manager is risk-averse he will prefer to hedge some of the risk inherent in such a remuneration scheme. Similarly, if the manager is wealth constraint he might want to borrow on part of his future pay. Selling some of his stock holdings, for instance, reduces his exposure to stock price movements and increases current income. Since such hedging/borrowing activities break the link between firm performance and the manager’s wealth, the use of stock awards or other incentive schemes becomes questionable. This problem has become increasingly relevant because of the sharp increase in hedging instruments available to corporate insiders. Among such instruments are zero-cost collars, equity swaps and loans against stock holdings. Certainly, hedging or retrading incentive schemes is limited by, for instance, the use of vesting periods for stock options and there are legal limits to the possibility for executives to hedge their risk. Still, Bettis, Bizjak, and Lemmon (2001) report that high ranking insiders, such as CEO’s and members of the board of directors cover an average of 36% of their share holdings with cost-less collars. Similarly, Ofek and Yermack (2000) show that managers with high ownership shares in their company tend to sell their existing stock holdings after option or stock awards. Thus, when designing a managerial compensation scheme taking into account future retrading opportunities is of vital importance.

Recontracting is prevalent in other aspects of economic life. Borrowers often enter into multilateral contracts which impact on their “global” incentives. For instance, no legal mechanism can completely eliminate the possibility that a debtor country contracts further loans. In 2005, cardholding households in the U.S. reportedly had

\footnote{This is not so for non-executives, see the arbitration claim filed on October 28, 2003 against CISCO, which seeks compensation damages directly related to the failure to recommend hedging strategies to employee stock option plan participants.}
an average of 19.3 credit cards in total. Likewise, insurance contracts are subject not only to moral hazard but also to a recontracting hazard. In fact, the possibility that an agent retrades after entering into an insurance contract is the rule rather than the exception.

In this paper, I investigate the impact of such recontracting opportunities on the optimal compensation package offered by a firm to its manager and on the efficiency of the contracting solution. I study a simple moral hazard problem, similar to Holmstrom (1979) and Grossman and Hart (1983), in which a principal hires an agent to perform a task. After receiving his wage contract but before undertaking a costly effort, the agent can borrow on his future wage from a competitive market. He can contract with a potentially infinite number of lenders. The lending game is modelled as a stochastic game with an exogenous stopping probability, in which the agent meets lenders sequentially. With each lender he can sign a bilateral agreement after which the game either ends or moves to the next round of contracting. I characterize the unique subgame perfect equilibrium of this game for all parameter values of the exogenous stopping probability and show that the two situations when the stopping probability is zero (perfect recontracting) and when it is strictly positive (imperfect recontracting) deliver quite distinct results.

With perfect recontracting it is shown that the agent either borrows on his entire wage and exerts no effort or that he borrows on only part of his wage, keeping some residual future payments, and exerts high effort. At that recontracting-proof wage the agent is indifferent between borrowing an additional amount and quitting the lending game. Also, if the agent borrows at a wage allocation, he does so from one lender only. So there is no reason to expect multiple contracts in this context. The principal’s problem of choosing an optimal wage scheme simplifies to finding the recontracting-proof wage allocation and associated effort level. This confirms the findings of several related papers discussed below that in this model a wage that induces high effort is associated with higher risk and a higher expected payment to the agent than the conventional second-best. Also, a flat wage that induces low effort is chosen more often by the principal than in the second-best. Thus, this model predicts that the principal

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3 For simplicity, the main body of the paper focuses on intertemporal income smoothing with a risk-neutral agent. Section 6 shows how the derived results extend to an agent with risk-aversion.
can indeed design robust incentives, albeit at higher costs.

The key finding of this paper is that this is no longer true in the case of imperfect recontracting. In other words, offering a risky wage at which the agent prefers not to borrow is not a feasible strategy for the principal. Instead, the agent borrows sequentially from a finite number of lenders and stops only if he is forced to (because the re-contracting game ends for exogenous reasons) or if he has borrowed on his entire future wage. Thus, the agent always obtains multiple loans in equilibrium and no “recontracting-proof-principle” holds in a game of imperfect re-contracting. This is important empirically. Since the analysis of re-contracting games is motivated by the observed prevalence of re-contracting, models where no re-contracting takes place fail to explain the data.

The reason why the predictions of the model of perfect re-contracting differ from the one of imperfect re-contracting is the following. In the latter, in each re-contracting round the agent and the current lender perceive that this could be the last opportunity for the agent to obtain a loan. Because future borrowing imposes a negative externality on existing borrowing agreements, being the last lender is valuable. Consequently, the agent will be able to secure more favorable loans. This makes borrowing more attractive and makes a re-contracting-proof wage more difficult and, as shown, impossible to implement.

The outcome of the perfect re-contracting model is then compared to the one of an imperfect re-contracting model that is arbitrarily close. Surprisingly, it turns out that for a certain range of parameters, the two equilibria are distinct even if the exogenous stopping probability tends to zero. First, neither the agent’s effort nor his wage payment in the equilibrium of the imperfect re-contracting model approach the effort and the wage scheme in the re-contracting-proof equilibrium under perfect re-contracting. Second, the principal receives a lower payoff when re-contracting is imperfect than when it is perfect. Because the principal’s payoff from low effort is independent of the agent’s re-contracting opportunities the above result also implies that low effort will be more often implemented in a world of imperfect re-contracting. Third, the number of loans obtained by the agent goes to infinity as the stopping probability in the imperfect re-contracting game approaches zero, and the size of those loans becomes vanishingly small. Thus, there is a discontinuity in the amount of borrowing when the stopping probability is zero. Also, the wage corresponding to high output under imperfect re-
contracting is far above the second-best wage or the recontracting-proof wage under perfect recontracting, and it is infinitely costly for the principal to induce high effort with probability one. Thus, very high wage payments are more likely to be observed in a world of imperfect recontracting, which might explain some of the abnormally high salaries paid in recent years to CEO’s and other top executives.

These last findings are puzzling because one would expect the equilibria of the two models to converge. The reason for this seeming inconsistency is the following. To obtain the recontracting-proof equilibrium under perfect recontracting one needs an additional assumption on equilibrium selection. This assumption is also made implicitly in all of the literature. Namely, it is assumed that, although there is a potentially infinite number of lenders, the agent only contracts with \( N \) of them. This assumption can be justified by the unmodelled possibility that there is a small cost of contracting, so that an infinite amount of contracting agreements is infinitely costly. That is, an equilibrium with an infinite number of active lenders is precluded by assumption. Thus, by assumption, the limiting equilibrium of the imperfect recontracting model with an infinite round of borrowing cannot coincide with the equilibrium of the perfect recontracting model.

The paper is related to several papers in the literature on non-exclusive contracts, such as Bizer and DeMarzo (1992), (1999) and Kahn and Mookherjee (1998). The first two papers study a moral hazard problem, in which a wealth constraint entrepreneur can borrow sequentially from an infinite number of banks. In contrast to our paper, these two papers only consider standard debt contracts as they do not allow the borrowing contracts to be contingent on the output realization. Kahn and Mookherjee (1998) study an insurance problem, in which a risk-averse agent can buy insurance sequentially from an infinite number of insurers but they do not address the principal’s problem. In contrast to our paper all three papers investigate the perfect recontracting case, in which they show a recontracting-proofness-principle similar to the one explained above. They also show that the agent can limit borrowing or buying insurance from one bank or insurance company only.

Parlour and Rajan (2001) study an unsecured loan market, in which borrower’s can default strategically. In contrast to our paper and the ones above, the lenders make all the contracting offers. They show that equilibria exist, in which lenders make positive profits. Nevertheless, default never occurs in equilibrium and without
loss of generality the agent borrows from only one lender. Bisin and Guaitoli (2004) model a problem similar to Kahn and Mookherjee (1998) but assume that insurers simultaneously offer insurance contracts to the agent. They show that positive profit equilibria can be sustained and that multiple contracts are offered in equilibrium. Some of these contracts are actively traded and some are latent. Nevertheless, they obtain allocations that are robust to further retrading, that is, equilibria in which high effort is sustained with probability one exist. In Bisin et al. (2008) the authors use this approach to study optimal compensation packages offered to managers when managers can hedge their financial positions but firms have an imperfect monitoring technology at their disposal to monitor and punish those hedging activities. They show that a manager’s compensation is more incentivized when the monitoring technology is costly or when financial markets are more developed. In our paper, we do not consider monitoring of a manager’s hedging activities directly, but the exogenous stopping probability in the lending game can be seen as a proxy for how difficult it is to prevent the agent from hedging his position or how well developed the financial market is. In our paper, the relationship between the quality of recontracting and incentive pay are not as clear-cut. A decrease in the stopping probability goes hand in hand with a decrease in the optimal wage in case of high output that induces a fixed number of borrowing rounds. However, because a decrease in the stopping probability increases the likelihood that low effort is induced in equilibrium the principal might now offer a more incentivized wage in order to induce a higher number of borrowing rounds (and consequently a higher probability of high effort). Also, in Bisin et al., as in the other papers, hedging is prevented by the optimal compensation package, so that their paper cannot explain the findings on insider hedging activities cited above.

The paper is structured as follows. Section 2 contains the set-up, solves for the optimal effort choice and describes the structure of the equilibrium in the lending game. Section 3 solves for the optimal wage contract if borrowing can be prevented. Section 4 solves for the agent’s and principal’s equilibrium strategies when the exogenous stopping probability in the lending game is bounded away from zero. Section 5 solves for their strategies when this probability is zero. Section 6 extends the results to the case of risk-aversion. Section 7 compares the two equilibria and shows that they are distinct even if the stopping probability in the first model tends to zero. Section 8 discusses how to reconcile the two equilibria results in a larger model that entails both
contracting cost and a stopping probability. Finally, concluding remarks are provided in section 9.

2 The Model

A principal hires an agent to manage a production process that delivers an output $x \in \{\bar{x}, \underline{x}\}$, with $\bar{x} > \underline{x}$. The agent undertakes a costly effort $e$ that affects the probability distribution over output levels. He can choose between high effort $e_h$ and low effort $e_l$ that cost $c_h$ and $c_l$ respectively. High (low) effort yields the high output with probability $p_h$ ($p_l$), and it is assumed that $p_h > p_l$. High effort is also more costly, $c_h > c_l$.\(^4\) Whereas output is verifiable and accrues directly to the principal, the effort is unobservable and its cost is born by the agent. To compensate for the effort cost, the principal offers the agent an initial sign-up fee $f_0 \in \mathbb{R}$ and a wage $w_0 \in \mathbb{R}^2$. The sign-up fee is paid to the agent directly after he has been hired at date 1. The wage is paid at date 2 after production has taken place, and it can therefore be indexed on output, $w_0 = (\bar{w}_0, \underline{w}_0)$. The agent has a utility function that is separable in date 1 and date 2 consumption (money) and separable in effort. He discounts future payoffs by a discount factor $0 < \delta < 1$. For simplicity, we assume that he is risk-neutral and so his utility is

$$U(f_0, w_0, e_i) = f_0 + \delta E_{p_i}[w_0] - c_i, \quad (1)$$

where $E_{p_i}[w_0] = p_i \bar{w}_0 + (1 - p_i) \underline{w}_0$.\(^5\) The principal is also risk-neutral, and for simplicity he is assumed not to discount future profits.\(^6\) The principal’s expected returns are

$$\Pi(f_0, w_0, e_i) = -f_0 + E_{p_i}[x - w_0]. \quad (2)$$

To motivate the assumption on the parties’ differing time preferences consider the following examples. Assume, that at date 1 the agent has the possibility to invest $I$ into a project that returns $R > I$ at date 2. Assume further that only he can make this investment. For instance, to be successful the project needs the agent’s personal input as well as his capital. Then, the agent has preferences as in (1) with $\delta = \frac{I}{R} < 1$ and

\(^4\)Probabilities and costs are also denoted by $p(e_i)$ and $c(e_i)$.

\(^5\)Section 6 discusses risk-aversion.

\(^6\)All results can be established under the assumption that the principal discounts future payoffs provided he has a higher discount factor than the agent.
the principal has preferences as in (2). Alternatively, assume that the agent has some
financial needs at date 1 that are not covered by the up-front fee received as part of his
wage package. Assume in contrast that the principal has almost unlimited funds. If
there are credit market imperfections, the rate $r_1$ at which the agent can borrow money
is likely to be higher than the rate $r_2$ at which the principal can place his. Then, by
setting $δ = \frac{1 + r_2}{1 + r_1} < 1$, we obtain the above preferences.

The principal makes a take-it-or-leave-it wage offer to the agent. If the agent rejects
the offer both parties receive their reservation utilities equal to 0. The agent has no
initial wealth and is protected by limited liability. He can therefore not receive a
negative wage. Furthermore, the wage cannot be made directly contingent on his effort
because effort is unobservable. In addition, it cannot include other prescriptions, for
example it cannot forbid the agent to enter into another contract with a third party.
Implicit in this assumption is the idea that the agent can secretly solicit financing from
outsiders at date 1, that is, after he has signed the employment contract but before
the underlying uncertainty is resolved. Importantly, he can do so before he undertakes
the effort. This game is described in the next section.

2.1 The Lending Game

The fact that the agent discounts future wage payments at a rate $δ < 1$ suggests that
there are gains from trade when he exchanges part of his future earnings against an
up-front payment from a lender. The lending game is modelled as follows.

There is an infinite number of risk neutral lenders whom the agent meets sequen-
tially. A contract with lender $t = 1, 2, ...$ consists of an up-front payment $b_t ∈ \mathbb{R}$ and a
contingent payment $r_t = (\tau_t, \xi_t)$ from the lender to the agent. The up-front payment
is made at date 1 and either $\tau_t$, in case of high output, or $\xi_t$, in case of low output, is
paid at date 2. Lender $t$’s profit function is similar to the principal’s:

$$V(b_t, r_t, e_t) = -b_t - E_{p_t}[r_t],$$

that is, he is risk-neutral and does not discount future payoffs. Although this is not
assumed and will actually be proved in the subsequent analysis it is expected that
$b_t > 0$, $\tau_t < 0$ and $\xi_t ≤ 0$, that is, the agent borrows in equilibrium.

In contrast to the initial stage game in which the principal offers the wage contract,
the agent makes all contracting offers in the lending game. This can be justified by
assuming that both the financial market and the labor market are perfectly competitive. Thus, lenders earn zero profits and the agent, when being offered employment, is kept at his reservation utility.

The lending game is terminated at stage $t$ if the agent voluntarily decides to solicit no further contracts from all future lenders. This is equivalent to the agent signing the null contract with all future lenders, $(b_k, r_k) = (0, 0, 0)$ for $k \geq t$, and so the agent’s exit decision does not need to be modeled explicitly. In addition, the lending game is also terminated at stage $t$ with an exogenously given probability $1 - q$, that is, the agent meets lender $t$ (and all subsequent lenders) only with probability $q$. This probability lies strictly between 0 and 1, the case $q = 1$ is discussed in a later section, and the case $q = 0$ is the usual problem without recontracting. This modeling assumption has two advantages. First, the lending game ends with probability one. In Kahn and Mookherjee (1998) the recontracting game can go on indefinitely, and the authors address this problem by assuming that the agent’s utility is equal to negative infinity if he enters into an infinite number of contracts. They motivate this assumption with reference to contracting cost: If each contract involves a small cost an infinite round of contracting is infinitely costly. Second, the assumption of an exogenous end to the lending game allows us to study the dynamics of the recontracting game. In addition, it is reasonable to assume that the lending game will end due to some exogenous factors, which are outside the parties’ control. Consider again the motivating example in the Introduction of a manager who hedges the risk inherent in his remuneration package. It is plausible that the CEO of a large company cannot consecrate an unlimited amount of time to design an optimal portfolio, that is, he might be forced to stop searching for a further lender even if there are still gains from trade.

Neither the initial employment contract nor any of the loan contracts depend on any consecutive contract that the agent may sign. In other words, we do not consider universal mechanisms as defined in Epstein and Peters (1999) and Peters (2001), where a principal’s contract may depend on the contract that the agent writes with another principal. One way to justify this restriction is to assume that it is impossible for the principal to foresee all potential ways in which the agent can obtain financing after he has been employed. Furthermore, it is implausible that an employment contract

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7 This probability also applies to the first stage, that is, the probability that the agent enters into the lending game is $q$. 

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can prohibit, or be made contingent on the possibility that a third party related to the agent obtains a financial position in the firm’s stock. For example, assume that a CEO’s remuneration package consists partly of stock options. He could instruct some member of his family to purchase put options on the firm’s stock, which would undo the incentive effects of the remuneration scheme if the two parties’ wealths are interdependent.

Finally, it is assumed that a lender observes the agent’s wage scheme and all previously signed contracts and therefore knows the agent’s total wage allocation when they meet. Thus, we abstract from all informational problems.

2.2 Timing

To summarize the timing:

– Date 0: The principal offers a wage contract \((f_0, w_0)\) to the agent. The agent accepts or rejects. If he rejects, the game ends and both parties receive a payoff of 0. If he accepts the game moves to date 1.

– Date 1: The principal pays the sign-up fee \(f_0\) to the agent.

Lending Game: The agent meets a lender \(t = 1, 2, \ldots\) with probability \(q\). With probability \(1 - q\) he does not meet lender \(t\) and the game moves to the effort subgame. If the agent meets lender \(t\) he can either decide not to request a loan and choose his effort directly or he can ask for a loan contract \((b_t, r_t)\). The lender can either accept or reject the demand. If he accepts he pays the up-front fee \(b_t\). Then, the above described stage game is repeated.

Effort Subgame: The agent undertakes effort \(e \in \{e_h, e_l\}\).

– Date 2: Output is realized. The agent is paid \(\overline{w}_0\) or \(\underline{w}_0\) by the principal depending on the realization of output and receives \(r_t\) or \(\underline{r}_t\) from each lender \(t\) with whom he has signed a contract.

2.3 Optimal Effort

I first solve for the optimal level of effort in the effort subgame. Assume that the lending subgame is terminated after \(t\) rounds. Then, denote the agent’s total up-front payment by \(f = f_0 + \sum_{k=1}^{t} b_k\) and his final date 2 wage by \(w = w_0 + \sum_{k=1}^{t} r_k\). The agent
will choose $e = e_h$ if and only if $U(f, w, e_h) \geq U(f, w, e_l)$, or equivalently
\[ w \geq \frac{\Delta c}{\delta \Delta p} + w, \tag{4} \]
where $\Delta c = c_h - c_l$ and $\Delta p = p_h - p_l$.\(^8\) Thus, the agent’s optimal effort choice only depends on his date 2 wage $w$. Set $IC := \{ w \in \mathbb{R}^2 \mid w \geq \frac{\Delta c}{\delta \Delta p} + w \}$, and $-IC := \mathbb{R}^2 \setminus IC$. Then the agent’s equilibrium strategy in the effort subgame is
\[ e(w) = \begin{cases} e_h & \iff w \in IC \\ e_l & \iff w \in -IC. \end{cases} \]

### 2.4 Borrowing Strategies

I now turn to the lending subgame. At the beginning of round $t + 1$, the subgame has history $h_t = (h_{t-1}, (b_t, r_t, a_t))$, where $b_t$ and $r_t$ are as described in section 2.1, $a_t \in \{0, 1\}$ is the decision of the lender at stage $t$ to reject or accept the contract offer $(b_t, r_t)$, and $h_0 = (f_0, w_0)$.

Since a lender is a short-lived player who cares only about his final payoff he accepts any contract with a zero or strictly positive expected payoff and he rejects any contract with a negative expected payoff.\(^9\) When taking expectations he takes the agent’s strategy and the induced equilibrium effort and probability distribution over outputs into account. Since the agent can always offer the null contract, which regardless of his continuation strategy yields payoffs of zero to all involved parties, we can without loss of generality concentrate on contracts that are accepted in equilibrium. Also, the agent will not offer a contract that allows a lender to make a strictly positive profit. This implies that for any $(\overline{r}_t, \underline{r}_t)$ the up-front payment is $b_t = -E[r_t]$, where the expectation depends on the agent’s equilibrium strategy. Consequently, a history $h_t$ can be written more simply as a date 2 wage path $h_t = (w_0, w_1, w_2, ..., w_t)$ with $w_k = w_{k-1} + r_k$. A strategy for the agent in the lending subgame is thus a mapping from a history $h_t$ into a new wage profile $w_{t+1}$. This induces a unique wage path $h = (w_0, w_1, w_2, ...)$. Following this notation, an offer of the null contract at stage $t$ implies that $w_t = w_{t-1}$, and the decision to exit the subgame implies that $w_k = w_{t-1}$ for all $k \geq t$.

\(^8\) It is assumed that the agent chooses $e_h$ if he is indifferent between effort levels.

\(^9\) Without loss of generality, a lender accepts a contract with a zero payoff.
Given wage path \( h \), call \( h_{-t} = (w_t, w_{t+1}, w_{t+2}, \ldots) \) the continuation wage path after stage \( t \). A continuation wage path \( h_{-t} \) induces probability \( \rho_t \) of high output that the agent and lender \( t \) use to calculate the up-front payment \( b_t \) with

\[
\rho_t = (1 - q) \sum_{k=0}^{\infty} q^k p(e(w_{t+k}))
\]

and

\[
b_t = -E_{\rho_t}[r_t].
\]

To understand expressions (5) and (6), note that the probability that the lending subgame ends at a stage \( t + k, k \geq 0 \), conditional on it having reached stage \( t \) is \((1 - q)q^k\). Given the continuation wage path \( h_{-t} \), if the subgame ends at stage \( t + k \), the agent expects to receive a wage \( w_{t+k} \), which induces effort \( e(w_{t+k}) \) and thus high output with probability \( p(e(w_{t+k})) \).

Using (5) and (6), the cumulated up-front payment at stage \( t \) is

\[
f_t = f_0 + \sum_{k=1}^{t} E_{\rho_k}[w_{k-1} - w_k]
\]

The following definition will be useful:

**Definition 1** A wage path \( h \) is feasible if and only if it respects the agent’s limited liability constraint, that is, \( w_t \geq 0 \) and \( f_t \geq 0 \) for all \( t \), with \( f_t \) as defined in (7).

The agent’s overall expected utility of wage path \( h \) for a given initial up-front payment \( f_0 \) and can be written as

\[
EU(f_0, h) = (1 - q) \sum_{t=0}^{\infty} q^t U(f_t, w_t, e(w_t)).
\]

We can now define the equilibrium concept of Subgame Perfection for the agent’s strategy in the lending game.

**Definition 2** For a given employment contract \((f_0, w_0)\), a Subgame Perfect Equilibrium strategy for the agent in the lending game is a wage path \( h \), such that for all \( t \)

\[
h_{-t} \in \text{arg max}_{h_{-t}} EU(f_{t-1}, \hat{h}_{-t}) \text{ with } \hat{h}_{-t} \text{ feasible.}
\]
Condition (9) says that the agent chooses at each state $t$ the continuation wage path $h_{t-1}$ that maximizes his continuation payoff among all continuation wage paths that are feasible. One more tie-breaking assumption is needed:

**Assumption 1**: Assume that both $h_{t-1}$ and $h'_{t-1}$ are subgame perfect continuation wage paths for a given history $h_{t-1}$. Call $p_t$ and $p'_t$ the corresponding probabilities of high output. Then, the agent will choose $h_{t-1}$ rather than $h'_{t-1}$ if $p_t \geq p'_t$.

In the following section I solve the contracting problem between the principal and the agent when there are no borrowing possibilities. I show that if the employment contract contains a date 2 wage that induces the agent to exert high effort, then he has an incentive to acquire liquidity from a lender. Thus, an active lending market that allows the agent to borrow on his future wage constitutes a real constraint for the principal.

### 3 Benchmark: No Borrowing

Assume that the agent cannot borrow on his date 2 wage. To induce low effort, the principal pays an up-front payment just large enough to cover the effort cost and pays nothing at date 2. Thus, he sets $f_0 = c_l$ and $w_0 = (0,0)$. This is optimal since it is costly to defer paying the agent ($\delta < 1$) and the date 2 wage is beneficial only for providing incentives. In contrast, if the principal wants to induce high effort he should choose $w_0 \in IC$. To minimize wage payments the inequality in (4) must hold with equality and therefore:

$$\overline{w}_0 = \frac{\Delta c}{\delta \Delta p} + w_0. \quad (10)$$

Because deferring payment to the agent is costly, it is optimal to set

$$w_0 = 0. \quad (11)$$

Finally, the agent’s participation constraint allows to solve for the up-front fee:

$$f_0 = \max \left\{ \frac{p_h c_l - p_l c_h}{\Delta p}, 0 \right\}. \quad (10)$$

Assume that it is optimal for the principal to induce high effort in the second best and that the agent’s participation constraint is not binding.\(^{10}\)

\(^{10}\)If it is optimal to induce low effort in the second-best, recontracting is not an issue. Assuming that the agent’s participation constraint is not binding is without loss of generality.
Assumption 2:

\[ f^{sb} = 0 \text{ and } w^{sb} = \left( \frac{\Delta c}{\delta \Delta p}, 0 \right) \] (12)

I now show that with the second-best compensation scheme the agent strictly prefers to borrow against his date 2 wage. First, note that at the second-best wage the agent is indifferent between choosing high or low effort, that is, \( U(f^{sb}, w^{sb}, e_h) = U(f^{sb}, w^{sb}, e_l) \). Second, if he borrows against his entire future wage earnings, the lender, foreseeing that the agent will put in low effort, is willing to pay up to \( E_{p1}[w^{sb}] \). Thus, the agent receives \( f^{sb} + E_{p1}[w^{sb}] - c_t > U(f^{sb}, w^{sb}, e_l) \), because \( \delta < 1 \). This discussion is summarized in the following Proposition.

**Proposition 1** The second-best wage contract is vulnerable to borrowing, that is, the agent strictly prefers to borrow on his entire date 2 wage in exchange for a date 1 payment. He then chooses low effort.

4 Borrowing: The case \( q < 1 \)

I now solve for the subgame perfect equilibrium when the agent can borrow from an infinite sequence of lenders. In all of the following I will use the agent’s optimal effort choice \( e(\cdot) \) as derived in section 2.3. The analysis is split into two parts.

4.1 The \( \neg IC \) region

The equilibrium strategy within the \( \neg IC \) region is straightforward. The agent optimally pledges his entire date 2 wage against an up-front payment from a single lender, which induces the wage path

\[ h^1(w_0) := (w_0, w^0, w^0, \ldots), \text{ with } w^0 = (0, 0). \] (13)

That this is optimal for the agent is proved formally in Lemma 2 in the Appendix. The following gives an outline of the general argument and the intuition. First, it is easy to see that the agent should never voluntarily quit the lending game with a non zero date 2 wage \( w \in \neg IC \). This is worth \( \delta E_{p1}[w] \), but he can obtain \( E_{p1}[w] \) as an up-front payment from any lender. Second, borrowing from more than one lender within the \( \neg IC \) region
is strictly dominated by borrowing from only one, because the odds from any lender are the same \((p_l)\) and the agent risks not meeting all of them. Finally, one can show that exchanging a wage inside the \(\neg IC\) region against one inside the \(IC\) region is also not optimal: First, Lemma 1 in the Appendix shows that a wage path that contains a move from the \(\neg IC\) into the \(IC\) region (or vice versa) must then have the agent quit the lending game. To see this, assume that \(w_{t-1} \in \neg IC\), \(w_t \in IC\) and \(w_t \neq w_{t+1}\). Then, it is easy to show that skipping wage \(w_t\), that is, using the continuation path \((w_{t-1}, w_{t+1}, ...)\) is feasible and better for the agent. Using this result Lemma 2 in the Appendix shows that exchanging a wage allocation inside the \(\neg IC\) region against one inside the \(IC\) region and then quitting the lending game to undertake high effort is dominated by borrowing against the entire wage allocation and undertaking low effort.

### 4.2 The \(IC\) region

Obviously, the agent can follow the same strategy as in the \(\neg IC\) region and exchange his entire date 2 wage against an up-front payment. But this is not always optimal. One other possibility is not to borrow at all and undertake high effort. For wage \(w \in IC\), the associated wage path \(h^0(w)\) yields payoff:

\[
\delta E_{p_h}[w] - c_h,
\]  

whereas \(h^1(w)\) yields

\[
(1 - q)\delta E_{p_h}[w] + qE_{p_l}[w] - (1 - q)c_h - qc_l.
\]  

It is possible that (14) is larger than (15). A necessary condition for this is \(\delta p_h > p_l\).

A variant of \(h^0(w)\), call it \(h^*(w)\), can perform even better. For a given wage \(w^* \in IC\) it is defined by \(h^*(w) := (w, w^*, w^*, ...)\) with payoff

\[
(1 - q)\delta E_{p_h}[w] + qE_{p_h}[w - w^*] + q\delta E_{p_h}[w^*) - c_h.
\]

Note that the lender uses \(p_h\) to calculate the up-front payment because the agent exerts high effort for sure.

What other strategies could be optimal? The agent could borrow from two lenders. This strategy has wage path \(h^2(w) = (w, w^1, w^0, w^0, ...)\) with \(w^1 \in IC\) and \(w^0\) defined in (13). Compared to \(h^1\), this wage path has a ‘cost’ and a ‘benefit’. It is costly for the
agent because with positive probability he will not meet the second lender and will be
left with the residual wage $w^1$, which is discounted at $\delta < 1$. The benefit is that since
$w^1 \in IC$, the agent undertakes high effort with some probability (precisely in the case
where he does not meet the second lender) and this in turn implies that he receives
a better deal from the first lender. In fact, the first lender uses $\rho_1 = (1 - q)p_h + qp_l$
to calculate the up-front payment which is greater than $p_l$. Thus, the agent obtains utility

$$(1 - q)\delta E_{p_h}[w] + qE_{p_l}[w - w^1] + q(1 - q)\delta E_{p_h}[w^1] + q^2 E_{p_l}[w^1] - (1 - q^2)c_h - q^2 c_l.$$  \hspace{1cm} (17)

Comparing this expression with (15), depending on $w$ and $w^1$, path $h^2$ might perform
better than $h^1$.

Finally, consider extensions to more than 2 lenders and call the induced paths
$h^n(w) = (w, w^{n-1}, w^{n-2}, ..., w^1, w^0, ...)$, $n = 3, 4, ..., \text{with } w^i \in IC, i = 1, ..., n - 1$.
Borrowing from more lenders increases the risk of not meeting all of them and thus
increases the cost of not being able to borrow against the entire date 2 wage. At
the same time it increases the benefit of obtaining better borrowing deals from early
lenders. For example, $\rho_1 = (1 - q^{n-1})p_h + q^{n-1}p_l$ for path $h^n$, and the first lender is
consequently willing to pay a relatively high up-front transfer.

Thus, the candidate equilibrium wage paths inside the $IC$ region are as described
above with correctly specified allocations $w^n \in IC, n \geq 1$, and $w^s \in IC$.\textsuperscript{11} Two
properties of those allocations are relatively easy to establish.

**Property 1**  \hspace{1cm} $w^n = 0 \forall n$ and $w^s = 0$.

Property 1 says that after each round of borrowing the residual date 2 wage pays
nothing in case of low output. Intuitively, retaining some positive date 2 wage is only
valuable for the agent if it allows him to credibly promise high effort. This will in turn
allow him to obtain better borrowing conditions from lenders, who will receive most
of their repayments in the high output state. Put differently, without an incentive
problem the agent would strictly prefer to receive all of his remuneration at date 1.
Therefore, payments in case of low output that dilute effort incentives and are costly
to the agent, should optimally be set equal to zero.

\textsuperscript{11}This is proved formally in Lemma 3 in the appendix.
Property 2  

\[ \bar{w}^n > \bar{w}^{n-1} \quad \forall n. \]

Property 2 says that after each round of contracting the agent is left with a smaller residual date 2 wage, that is, he borrows in equilibrium. The intuition is as above. Because \( w \) is in IC the odds with which future borrowing risk is evaluated are unaffected by a contract that increases the agent’s residual wage in case of high output. Therefore, an increase in this wage only imposes a cost on the agent. The two properties are proved formally in Lemma 4 in the Appendix.

It remains to determine the payments for high output \( \bar{w}^n, n \geq 1 \), and \( \bar{w}^s \). Two conditions must be met. First, at wage \( w^n \) the path \( h^{n-1}(w^n) = (w^n, w^{n-1}, w^{n-2}, ...) \) must be optimal. Second, \( \bar{w}^n \) must be as small as possible. The intuition for the latter is similar as for Properties 1 and 2. Retaining positive date 2 wage is costly for the agent and so \( \bar{w}^n \) should be minimal. Similarly, at \( w^s \) the wage path \( h^0(w^s) = (w^s, w^s, ...) \) must be optimal and \( \bar{w}^s \) should be as small as possible.

This suggests the following recursive method. First, set \( \bar{w}^1 = \bar{w}^{sb} \). (18)

As shown in section 3, the continuation path \( h^1(w^{sb}) = (w^{sb}, w^0, w^0, ...) \) is optimal and \( \bar{w}^{sb} \) is the lowest payment such that \( w^{sb} \in IC \).

With wage \( \bar{w}^1 \) defined, set \( \bar{w}^2(w) := (w, w^1, w^0, w^0, ...) \). Then, from \( \bar{w}^1 \) move upwards on the vertical axis, that is, increase \( \bar{w} \). Using (17) we see that \( h^2 \)'s payoff increases by \( (1 - q)\delta p_h + q p_1 \), whereas \( h^1 \)'s payoff increases by \( (1 - q)\delta p_h + q p_1 \) (see (15)). Since \( p_1 > p_l \), the marginal increase in \( h^2 \)'s payoff from an increase in \( \bar{w} \) is larger than the marginal increase in \( h^1 \)'s payoff. Thus, there exists an allocation \( \bar{w}^2 \) on the vertical axis with \( \bar{w}^2 > \bar{w}^1 \), at which paths \( h^2 \) and \( h^1 \) yield the same payoff. Everywhere above \( \bar{w}^2 \) path \( h^2 \) dominates \( h^1 \), and everywhere below \( h^1 \) dominates \( h^2 \). The wage \( \bar{w}^2 \) is found by by equating (15) and (17) and setting \( \bar{w}^2 = 0 \), so

\[ \bar{w}^2 = \frac{(1 - \delta)p_l}{\delta \Delta p^2} \Delta c + \bar{w}^1. \]

(19)

From Property 2 and its discussion it should be clear that between \( \bar{w}^1 \) and \( \bar{w}^2 \), \( h^1 \) also dominates all paths \( h^n \) with \( n > 2 \).  

It remains to evaluate path \( h^s \) on this section, that is, to find a wage \( w^s \), such that \( h^0 \) is the optimal continuation wage path at \( w^s \). Proceed as above. Moving upwards

---

12See Lemma 4 in the Appendix.
from \( w^1 \) on the vertical axis, \( h^0 \)'s payoff increases by \( \delta p_h \) (see (14)), whereas \( h^1 \)'s payoff increases by \( (1-q)\delta p_h + q p_l \). If \( \delta p_h \leq p_l \), there is never an allocation at which \( h^0 \) dominates \( h^1 \). Assume the contrary. Then, \( h^0 \)'s payoff increases faster than \( h^1 \)'s payoff and there is a wage, such that everywhere above it \( h^0 \) dominates and everywhere below it \( h^1 \) dominates. This allocation \( w^{1s} \) is found by equating (15) and (14) and setting \( w^{1s} = 0 \):

\[
\frac{1}{\delta p_h - p_l} \Delta c.
\]

Comparing (19) and (20), it is easy to see that \( \overline{w}^{1s} > \overline{w}^2 \), and so there is no wage \( w^s \) between \( w^1 \) and \( w^2 \) at which \( h^0 \) is the optimal continuation path. This finalizes the proof that \( h^1 \) is the agent’s subgame perfect equilibrium wage path on the vertical axis below \( w^2 \). The argument can be extended to find the entire subset of the \( IC \) region on which \( h^1 \) is optimal: \( \Omega_1 = \{ w \in IC \mid \overline{w}^1 \leq \Delta w < \overline{w}^2 \} \).

An inductive argument establishes the remaining wages \( w^n \) and the regions \( \Omega_n \), on which paths \( h^n \) are the subgame perfect equilibrium wage paths for \( n > 2 \). It turns out that path \( h^s \) is never optimal because there is no wage \( w^s \) at which \( h^0 \) is the optimal continuation wage path. The explicit solutions for the high payments in \( w^n \), for \( n > 2 \), are given below

\[
\overline{w}^n = \frac{(1-\delta)p_l \Delta c}{\delta \Delta^2 p} \sum_{i=1}^{n-2} \left( \frac{(1-\delta)p_h}{q^{(i+1)/2} \Delta p} \right)^i + \overline{w}^2.
\]

### 4.3 The Equilibrium

The following proposition contains the formal description of the agent’s subgame perfect equilibrium strategy in the lending game:

**Proposition 2** The agent’s subgame perfect equilibrium strategy in the lending game is the wage path:

\[
h(w) := \begin{cases} 
  h^1(w) & \text{for all } w \in \neg IC \\
  h^n(w) & \text{for all } w \in \Omega_n
\end{cases}
\]

with \( \Omega_n = \{ w \in IC \mid \overline{w}^n \leq \Delta w < \overline{w}^{n+1} \} \), \( 1 \leq n \).

**Proof.** See Appendix. □
Figure 1 illustrates the equilibrium. The wage $w$ in $\Omega_3$ induces the agent to contract with the first lender to receive wage $w^2$, with the second to receive $w^1$, and finally he borrows against all of $w^1$ to receive $w^0$ from the third recontractor.

Several conclusions can be drawn from Proposition 2. First, recontracting is an equilibrium phenomenon. Contrary to most results in the literature (for example, Bizer and DeMarzo (1992), (1999) and Kahn and Mookherjee (1998)), the principal cannot prevent the agent from requesting and receiving subsequent financing by choosing an appropriate wage allocation. Instead, the agent will always choose to borrow from a finite number of lenders. The intuition for this can be seen as follows. In order for $h^0$ to be the optimal path at some wage $w$, it must be better than the ‘best’ path $h^n$, which in turn is better than path $h^{n+1}$, so

$$EU(f, h^0(w)) \geq EU(f, h^n(w)) \geq EU(f, h^{n+1}(w)).$$

However, this implies that also path $\hat{h}^{n+1}(w) := (w, w, w^{n-1}, w^{n-2}, ...)$ is better than $h^{n+1}(w)$, since

$$EU(f, \hat{h}^{n+1}(w)) = qEU(f, h^0(w)) + (1 - q)EU(f, h^n(w)).$$

It is easy to see that this cannot be true, since $h^{n+1}$ and $\hat{h}^{n+1}$ are identical starting with the second lender but $h^{n+1}$ has the advantage that part of the payment in $w$ is
brought forward because the agent borrows on part of his wage from the first lender who gives better odds.

Second, the more incentivized the initial wage allocation the higher the incidence of borrowing in equilibrium and also, the higher the likelihood of high equilibrium effort. Thus, the principal when choosing the optimal incentive scheme trades off large rewards in case of high output against high equilibrium effort. The principal’s problem is solved formally in the next section. In section 7 I will compare the result on recontracting found here with the equilibria usually exhibited in other papers in the literature.

4.4 The Principal’s Problem

Following the analysis in the preceding section, a wage \( w \in \Omega_n \) induces the agent to follow wage path \( h^n(w) \), which leads to an expected effort \( e_n = (1 - q^n)e_h + q^n e_l \), where \( e_0 = e_l \) and \( e_\infty = e_h \). Set
\[
\rho^n := (1 - q^n)p_h + q^n p_l. \tag{23}
\]

Similar to the analysis in Grossman and Hart (1986) the principal’s problem can be divided into two parts. First, for each \( n \) the principal chooses a wage in \( \Omega_n \) to minimize expected wage payments. Second, the principal maximizes his expected payoff over \( n \).

Trivially, the wage that induces \( e_0 \) is \( w^0 \), see Section 3. For \( n \geq 1 \), the principal solves
\[
\min_{w \in \Omega_n} E_{\rho^{n-1}}[w] \text{ s.t. } EU(0, h^n(w)) \geq 0. \tag{24}
\]

It is easy to show that the date 2 wage schemes solving (24) are the allocations \( w^n \), \( n \geq 1 \). Then, the principal solves
\[
\max_{\tilde{n}} E_{\rho^{n-1}}[x - w^{\tilde{n}}]. \tag{25}
\]

and we have

Proposition 3 The principal sets a wage \( w^n \), where \( n \) solves (25).

5 Borrowing: The Case \( q = 1 \).

All earlier models of moral hazard with recontracting (see for instance, Kahn and Mookherjee (1998), Bizer and DeMarzo (1992)) only consider the special case of perfect recontracting, that is, the situation where the agent meets subsequent lenders with
probability 1. To compare the results in this paper with the ones in the literature, I now solve for the equilibrium of the perfect recontracting version of the model. Under this assumption it is possible for the agent to enter into an infinite number of lending agreements. As this is impracticable in reality, the literature usually focuses on equilibria that involve only a finite number of lenders. One way to justify this restriction is to assume that the agent’s utility from an infinite series of transactions is equal to $-\infty$. For instance, if each transaction imposes a small cost on the agent, signing an infinite number of contracts becomes prohibitively costly. Alternatively, one can assume that the agent has only a limited amount of time to allocate between different tasks. Therefore, spending all of it soliciting funds leaves him with no time for the job for which he has been hired. Consequently, no output is realized and the principal pays a zero wage. Anticipating this outcome, lenders are only willing to sign the null contract, which is a trivial equilibrium. Following the literature, in the subsequent discussion infinite subgame perfect equilibria are excluded and it is shown that this has significant impact on the results.

It is easy to see that if $q = 1$ the agent does not need to borrow from more than one lender in equilibrium. Assume to the contrary that he borrows sequentially from two sources. Because he meets the second lender with probability 1, the first lender offers exactly the same loan as the second lender. Thus, the agent can achieve the same payoff by borrowing from only one lender.

Then, there are only two situations to consider. The agent either immediately borrows against his entire wage and exerts low effort (he follows wage path $h^1$), or he borrows against only part of his wage, retains a wage $w^s := w^{1s}$ as defined in (20) and then undertakes high effort (he follows wage path $h^s$). The first strategy is optimal in $\neg IC$ and in $\Omega_1 := \{w \in IC \mid \bar{w}^1 < \Delta w < \bar{w}^s\}$ and the latter is optimal in $\Omega_s := \{w \in IC \mid \bar{w}^s \leq \Delta w\}$. For $w^s$ to be well defined we need\footnote{Kahn and Mookherjee (1998) acknowledge this restriction, but most papers neglect to do so.}

**Assumption 3:** $\delta p_h > p_l$.

Assumption 3 is made for the remainder of the paper. The first strategy leads to low effort in equilibrium and the principal implements this effort optimally by offering the wage $w^0$. The agent does not borrow in equilibrium. If the principal wants to
induce high effort he optimally offers the wage $w^s$. Remark, that again at this wage no borrowing occurs in equilibrium. Thus, the principal’s overall problem reduces to a third-best problem, in which to the usual incentive and participation constraints is added a recontracting proofness constraint. To ease comparison with the limiting equilibrium that will be derived in the following section, call the equilibrium wage and effort if $q = 1$, $w(1)$ and $e(1)$ respectively. Summarizing, we have:

**Proposition 4** The equilibrium is characterized by either an incentivized wage and high effort or by a flat wage and low effort, that is, either $w(1) = w^s$ and $e(1) = e_h$, or $w(1) = w^0$ and $e(1) = e_l$. There is no borrowing in equilibrium.

![Figure 2](image)

Figure 2 shows two possible wages, $w$ in $\Omega_s$ at which $h^s$ is optimal, and $w'$ in $\Omega_1$ at which $h^1$ is optimal.

## 6 Risk-Aversion

Since the main body of the literature on moral hazard with non-exclusive contracts deals with risk-aversion it is important to extend the conclusions of the above model with imperfect recontracting to that case. For this, consider the same set-up as above
except that the agent has the following utility function, defined over the date 2 wage and his effort choice only

\[ U(w_0, e_i) = E_p[u(w_0)] - c_i, \]

where \( u(\cdot) \) is a strictly concave function. Similarly, the principal's profit function is

\[ \Pi(w_0, e_i) = E_p[x - w]. \]

The intertemporal aspect of the relationship (the agent prefers early to late payments) is replaced by the usual incentive insurance trade-off: The agent prefers flat wage payments but puts in no effort unless he is subject to some risk. Correspondingly, recontractors are insurers rather than lenders in this context. Facing a risky wage, the agent buys insurance before undertaking effort, and each insurance contract dilutes effort incentives. An insurance contract at stage \( t \) is a payment \( r_t = (\tau_t, r_d) \), and an insurer's profits are

\[ V(r_t, e_i) = -E_p[r_t]. \]

Otherwise, everything is as in the preceding sections.

First, it is easy to establish that at the second-best wage \( w_{sb} \) as defined in (12) the agent buys insurance. At that wage the agent is indifferent between the two efforts

\[ E_p[u(w_{sb})] - c_h = E_p[u(w_{sb})] - c_l \]

\[ > u(E_p[w_{sb}]) - c_l, \]

where the second line is his utility from insuring fully and exerting low effort.

Second, the proof of the preceding sections can be adapted to show that the wage region \((\bar{w}, w) \in \mathbb{R}_+^2\) can be divided into regions \( \Omega_n, n \in \mathbb{N} \), such that the agent will insure completely if his wage lies in either \( \neg IC \) or \( \Omega_1 \), and that he will follow a path \( h^n \) if it lies in \( \Omega_n, n > 1 \), where \( h^n \) involves \( n \) insurers who insure the agent successively. The following figure illustrates the regions and a typical wage path for \( u(\cdot) = \ln(\cdot) \).
At wage $w$ in $\Omega_3$, the agent obtains insurance with the first insurer up to wage $w^2$ that lies at the frontier between $\Omega_2$ and $\Omega_1$. He buys more insurance from the second insurer up to wage $w^1$ that lies at the frontier between the $IC$ and $\neg IC$ region, and at that wage he obtains full insurance with the third insurer (he attains wage $w^0$). Note that, in contrast to the model with a risk-neutral agent, the wage allocations $w^i$ that the risk-averse agent attains within a wage path $h^n(w)$ are a function of the initial wage $w$. The frontiers between regions $\Omega_n$ and $\Omega_{n-1}$ are defined similarly as in the model with risk-neutrality, that is, they are composed of those wages at which the agent is indifferent between following path $h^n$ or path $h^{n-1}$. The slope of the lines that successively connect wages $w$, $w^2$, $w^1$ and $w^0$ are $-\frac{1-\rho^2}{\rho^2}$, $-\frac{1-\rho^3}{\rho^3}$ and $-\frac{1-\rho^4}{\rho^4}$ respectively, with $\rho^n$ defined in (23). They are the ‘slopes of the insurance contracts’ $r_1$, $r_2$ and $r_3$, that is, equal to $\frac{p_t}{\zeta_t}$. To see this note, that since all insurers make zero profits in equilibrium, $\rho_t \tau_t + (1 - \rho_t) \zeta_t = 0$, or $\frac{p_t}{\zeta_t} = -\frac{1-\rho_2}{\rho_3}$. With wage path $h^n$, the probability $\rho_t$ of high output that is used by insurer $t$ is equal to $\rho^{n-t}$.

Finally, it is easy to establish that just as in the model with risk-neutrality, a risk-averse agent, if given the opportunity, always buys some insurance. Consequently, the principal cannot prevent recontracting (and ensure high effort) by choosing an appropriate wage contract. To see this, take a wage $w$ at which the agent is indifferent
between following wage path $h^n$ and not obtaining any insurance:

$$U(w, e_h) = EU(h^n(w)),$$

where, as before, $EU(h^n(w))$ is the expected utility of the wage path $h^n(w)$. But then, it is easy to see that wage path $h^{n+1}(w)$ is strictly better since it gives utility

$$(1 - q)U(w, e_h) + q(1 - q)U(w^n, e_h) + q^2EU(h^n(w^n)),$$

where $w^n$ is the intermediate wage attained with the first insurer on the path $h^{n+1}$. This expression is larger than the expression in (26) since $U(w^n, e_h) > U(w, e_h)$ ($w^n$ is less risky than $w$) and $EU(h^n(w^n)) > EU(h^n(w))$, since all intermediate wages attained along the two paths, including the initial wages, are less risky on path $h^n(w^n)$ than on path $h^n(w)$. So, at no wage is it a credible strategy for the agent to forgo insurance.

Summarizing, most of the qualitative conclusions of the preceding sections carry over to the model with risk-aversion.\(^\text{15}\)

### 7 Non Convergence

The aim of this section is to investigate whether the equilibrium described in Propositions 2 and 3 for $q < 1$ converges to the equilibrium described in Proposition 4.

#### 7.1 The limiting equilibrium

To denote the dependency of the equilibrium described in Propositions 2 and 3 on $q$, let $n(q)$ be the optimal amount of lenders solved for in (25) and $w(q) = w^{n(q)}$ the corresponding date 2 wage. Call the effort induced by this wage $e(q) = (1 - q^{n(q)})e_h + q^{n(q)}e_l$.

The first step is to see whether the number of active lenders $n(q)$ remains finite when $q$ approaches 1. From the definition of $e(q)$ it is immediate that if $n(q)$ remains finite, equilibrium effort will approach $e_l$. But then the principal is better off inducing no borrowing at all by setting the date 2 wage equal to $w^0$.

The next step is to see what happens if the number of active lenders goes to infinity when $q$ approaches 1. If $n(q)$ converges more quickly to infinity than $q$ goes to 1,

\(^\text{15}\)Formal statements and proofs of the issues discussed in this section can be obtained from the author.
\[
\lim_{q \to 1} q^{n(q)} = 0, \text{ and consequently } \lim_{q \to 1} e(q) = e_h. \text{ Consider the expression derived for the high wage payment } \bar{w}^{n(q)} \text{ in (21)}
\]
\[
\bar{w}^{n(q)} = \frac{(1 - \delta)p_h \Delta c}{\Delta^2 p} \sum_{i=1}^{n-2} \frac{(1 - \delta)p_h}{q^{(i+1)/2} \Delta p}^i + \bar{w}^2. \tag{27}
\]

From (27), if \(\lim_{q \to 1} q^{n(q)} = 0\), \(\bar{w}^{n(q)}\) will tend to infinity. Surely this cannot be optimal for the principal. In fact, for large \(q\), a necessary condition for \(\bar{w}^{n(q)}\) to remain bounded is
\[
\frac{(1 - \delta)p_h}{q^{(n(q)-1)/2} \Delta p} \leq 1,
\]
which puts a strictly positive lower bound on \(\lim_{q \to 1} q^{n(q)}\):
\[
\lim_{q \to 1} q^{n(q)} > \frac{(1 - \delta)^2 p_h^2}{\Delta^2 p}.
\]

It can be shown that this inequality is satisfied with equality (see the proof of the following proposition), which implies that the wage for high output approaches
\[
\bar{w}^{\infty} = \frac{(1 - \delta)p_h \Delta c}{\Delta^2 p} \lim_{q \to 1} \sum_{i=1}^{\log_q \left( \frac{(1 - \delta)^2 p_h^2}{\Delta^2 p} \right) - 2} \left( \frac{(1 - \delta)p_h}{q^{(i+1)/2} \Delta p} \right)^i + \bar{w}^2. \tag{28}
\]

Set \(w^{\infty} = (\bar{w}^{\infty}, 0)\). This discussion is summarized in the following Proposition:

**Proposition 5** The limiting equilibrium for \(q\) tending to 1 is characterized by either an incentivized wage and an intermediate effort level or by a flat wage and a low effort level, that is, either \(\lim_{q \to 1} w(q) = w^{\infty}\) and \(\lim_{q \to 1} e(q) = (1 - \kappa)e_h + \kappa e_1\) with \(\kappa = \frac{(1 - \delta)^2 p_h^2}{\Delta^2 p}\), or \(\lim_{q \to 1} w(q) = w^0\) and \(\lim_{q \to 1} e(q) = e_1\). In the first type of equilibrium the number of potential recontracting rounds approaches infinity at a rate \(\log_q \left( \frac{(1 - \delta)^2 p_h^2}{\Delta^2 p} \right) - 2\). In the second type of equilibrium there is no recontracting.

**Proof.** See Appendix. ■

I now turn to a comparison between the equilibrium for \(q = 1\) and the limiting equilibrium for \(q\) tending to 1.

### 7.2 Comparing the two equilibria

Assume first that \(w(1) = w^s\) and \(e(1) = e_h\). Remark that (20) can be written as
\[
\bar{w}^s = \frac{(1 - \delta)p_h \Delta c}{\Delta^2 p} \sum_{i=1}^{\infty} \left( \frac{(1 - \delta)p_h}{\Delta p} \right)^i + \bar{w}^2. \tag{29}
\]
Therefore, since the last element in the series in (28) converges to 1, whereas the last element in (29) is \( \frac{(1-\delta)p_n}{\Delta p} < 1 \), the following inequality holds

\[ \bar{w}^\infty > \bar{w}^s. \]

Therefore, if the limiting equilibrium of the model with imperfect recontracting is of the first type described in Proposition 5, the limit wage is higher and more incentivized, effort is lower and total surplus and the principal’s payoff are lower than in the equilibrium with perfect recontracting. Moreover, the agent is better off with imperfect recontracting. This is so, because he is paid a higher wage and can always mimic his equilibrium behavior under perfect recontracting, namely recontract with no-one and undertake high effort. He must therefore obtain a higher payoff by following his actual equilibrium strategy.

Second, since the principal’s payoff is lower when he pays an incentivized wage in a world of imperfect recontracting, it is possible that he foregoes incentives altogether and pays a flat wage. Thus, even if in the model with perfect recontracting the equilibrium is characterized by an incentivized wage, the limiting equilibrium of the model with imperfect recontracting can contain a flat wage as in the second type of equilibrium described in Proposition 5. Consequently, the limit wage is lower and less incentivized, effort, total surplus and the principal’s and the agent’s payoffs are lower. Summarizing, in neither case does the equilibrium for \( q < 1 \) converge to the one for \( q = 1 \).

Assume second that \( w(1) = w^0 \) and \( e(1) = e_l \). Then, it is easy to verify that the limiting equilibrium of the model with imperfect recontracting coincides with this equilibrium. This discussion is summarized in the following proposition:

**Proposition 6** Assume that the equilibrium of the model with perfect recontracting contains an incentivized wage. Then this equilibrium will differ from the limiting equilibrium of the model with imperfect recontracting. The latter will either contain a higher powered incentive scheme, lower effort and a non-negligible amount of borrowing or a flat wage, the minimum effort and no borrowing. If the equilibrium of the model of perfect recontracting is characterized by a flat wage, it will coincide with the limiting equilibrium of the model with imperfect recontracting.
8 Costly Contracting

A companion paper Reiche (2007) studies a model of recontracting with transaction costs that nests the two models of perfect and imperfect recontracting. It is shown in this paper that if both the contracting cost and the stopping probability go to zero, which of the above equilibria obtain depends on the speed with which the two parameters approach zero. That is, for fixed contracting cost, taking the stopping probability to zero results in the equilibrium of the perfect recontracting model. In contrast, for a fixed stopping probability, taking the contracting cost to zero results in the equilibrium of the imperfect recontracting model. Additional comparative static results can be obtained as well. In particular, the model’s framework can be used to study the impact of both a change in the quality of recontracting and the size of transaction costs on payoffs and overall welfare. This can then be used to answer normative questions on the optimal quality of recontracting and size of transaction cost. It is possible to derive potentially testable conclusions about the link between the quality of recontracting and the occurrence of recontracting and its effect on wages. Such questions on the optimal amount of recontracting can not be asked meaningfully in any of the earlier models of recontracting.

9 Conclusion

This paper has developed a model in which sequential borrowing impacts on an agent’s incentive to undertake a costly effort. It studies the principal’s problem of designing an optimal incentive scheme for the agent if such borrowing opportunities are present and shows that in contrast to existing results in the literature, the principal cannot design an incentive scheme that is immune to borrowing. This model is therefore able to explain the large evidence of recontracting (borrowing from multiple lenders, hedging of financial positions by corporate insiders etc.) in the real world. It also shows that the perfect \( q = 1 \) and imperfect \( q < 1 \) recontracting model deliver distinct results even if the parameter measuring the imperfection is taken to 1. Furthermore, it is shown that the incidence of borrowing and the bonus payments for high output are higher, the better developed are capital markets (the higher is \( q \)). One interesting way to extend the research presented in this paper would be to relax some of the simplifying
informational and institutional assumptions made in the model. First, it was assumed
that all lenders observe earlier contracts. Second, lending agreements are assumed to
be bilateral and priced optimally, that is, while taking into account all subsequent
borrowing by the agent. This way of modelling is quite distinct from the conditions
in real world financial markets in which a large number of investors trade financial
contracts anonymously and in which market makers price contracts knowing that some
of their customers are insiders. A fully-flexed model that mimics real world financial
market conditions is unfortunately beyond the scope of this paper.

10 Appendix

Lemma 1 For any subgame perfect equilibrium wage path \( h \) if either (i) \( w_{t-1} \in IC \) and \( w_t \in \neg IC \), or (ii) \( w_{t-1} \in \neg IC \) and \( w_t \in IC \) for some \( t \), then \( w_t \equiv w_{\tilde{t}} \) for all \( t > \tilde{t} \).

Proof. The proof is in two steps.

1. Assume that for a sequence of wages in \( h \), \( w_{t-1} \neq w_t \neq w_{t+1} \). Then, either (a) \( w_t \in \neg IC \) and \( \overline{w}_{t-1} - \overline{w}_t \leq w_{t-1} - w_t \), or (b) \( w_t \in IC \) and \( \overline{w}_{t-1} - \overline{w}_t \geq w_{t-1} - w_t \).

Proof by contradiction. Assume otherwise. I show that the path \( h' = (w_0, w_1, ..., w_{t-1}, w_{t+1}, ...) \), which is identical to \( h \) except that it leaves out wage \( w_t \), is feasible and gives a higher expected utility to the agent. Since \( h' \neq h \), it constitutes a profitable deviation, which contradicts the assumption that \( h \) is subgame perfect.

Path \( h \) feasible requires that \( f_k \geq 0 \) for all \( k \), where \( f_k \) is defined as in (7). To show that \( h' \) feasible, we need to show that \( f'_k \geq 0 \) for all \( k \). First, \( f'_k = f_k \) for all \( k \leq t - 1 \) because the two paths are identical up to wage \( w_{t-1} \). Second

\[
f'_t = f_{t-1} + E_{\rho_{t+1}}[w_{t-1} - w_{t+1}]
= f_{t+1} + E_{\rho_{t+1}}[w_{t-1} - w_t] - E_{\rho_t}[w_{t-1} - w_t].
\]

So, \( f'_t \geq 0 \) follows from \( f_{t+1} \geq 0 \) provided that

\[
E_{\rho_{t+1}}[w_{t-1} - w_t] \geq E_{\rho_t}[w_{t-1} - w_t].
\] (30)

It is easy to see that if \( w_t \in \neg IC \) (or \( w_t \in IC \)), then \( \rho_{t+1} \geq \rho_t \) (or \( \rho_{t+1} \leq \rho_t \)). Therefore, if neither (a) nor (b) hold,

\[
(\rho_{t+1} - \rho_t)(\overline{w}_{t-1} - \overline{w}_t - (w_{t-1} - w_t)) \geq 0,
\]
which implies (30). Finally,

\[ f_k' = f_{k-1} + E_{\nu_{t+1}}[w_{k+1} - w_k] \]

\[ \geq f_k + E_{\nu_{t+1}}[w_{k+1} - w_k] = f_{k+1} \geq 0 \]

for all \( k \geq t + 1 \). So, \( h' \) is feasible.

We now show that the continuation path \( h'_{-(t-1)} = (w_{t-1}, w_{t+1}, w_{t+2}, \ldots) \) yields a higher payoff than the continuation path \( h_{-(t-1)} = (w_{t-1}, w_t, w_{t+1}, \ldots) \), which implies that \( h' \) is better for the agent than \( h \). The payoff of \( h_{-(t-1)} \) can be written as

\[ EU(f_{t-1}, h_{-(t-1)}) = (1 - q)U(f_{t-1}, w_{t-1}, e(w_{t-1})) + (1 - q)qU(f_t, w_t, e(w_t)) + q^2 EU(f_{t+1}, h_{-(t+1)}) \]

and the payoff of \( h'_{-(t-1)} \) can be written as

\[ EU(f_{t-1}, h'_{-(t-1)}) = (1 - q)U(f_{t-1}, w_{t-1}, e(w_{t-1})) + qEU(f_t', h_{-(t+1)}). \]

Since the sequence \( (\ldots, w_t, w_{t+1}, \ldots) \) is part of a subgame perfect equilibrium wage path it must be that

\[ U(f_t, w_t, e(w_t)) \leq EU(f_{t+1}, h_{-(t+1)}). \]

Therefore, (31) is smaller than (32) if

\[ EU(f_{t+1}, h_{-(t+1)}) \leq EU(f_t', h_{-(t+1)}), \]

which is true because \( f_{t+1} \leq f_t' \).

2. Take \( h \) with either \( w_{t-1} \in IC \) and \( w_t \notin IC \) or \( w_{t-1} \notin IC \) and \( w_t \in IC \). That is, either \( \bar{w}_{t-1} - \bar{w}_t > w_{t-1} - w_t \) or \( \bar{w}_{t-1} - \bar{w}_t < w_{t-1} - w_t \). Then, 1a. or 1b. imply that \( w_{t+1} = w_t \). By reiterating the argument in 1. it follows that also all the subsequent wage allocations must be equal to \( w_t \).

**Lemma 2** For any subgame perfect equilibrium wage path \( h \), if \( w_\bar{t} \in \neg IC \) for some \( \bar{t} \), then \( w_t \equiv w^0 \) for all \( t > \bar{t} \), where \( w^0 = (0, 0) \).
Proof. Assume first that \( w_t \in \neg IC \) for all \( t > \bar{t} \). The expected utility from the continuation wage path \( h_{\bar{t}} \) is

\[
EU(f_{\bar{t}}, h_{\bar{t}}) = (1 - q) \sum_{t=\bar{t}}^{\infty} Q^{t-\bar{t}} U(f_t, w_t, e(w_t))
\]

\[
= (1 - q) \sum_{t=\bar{t}}^{\infty} Q^{t-\bar{t}} \left( f_t + \sum_{k=\bar{t}+1}^{t} E_{p_{t}}[w_{k-1} - w_{k}] + \delta E_{p_{t}}[w_{t}] - c_t \right)
\]

\[
= (1 - q) \sum_{t=\bar{t}}^{\infty} Q^{t-\bar{t}} \left( f_t + E_{p_{t}}[w_{t}] - (1 - \delta) E_{p_{t}}[w_{t}] - c_t \right)
\]

\[
= f_t + E_{p_{t}}[w_{\bar{t}}] - c_t - (1 - q)(1 - \delta) \sum_{t=\bar{t}}^{\infty} Q^{t} E_{p_{t}}[w_{t}].
\]

Clearly, this is maximized by setting \( w_t = w^0 \) for all \( t > \bar{t} \).

Assume second that \( w_t \in IC \) for some \( t > \bar{t} \), and let this be the smallest such \( t \), that is, \( w_{t-1} \in \neg IC \). From Lemma 1 it follows that \( w_k = w_t \) for all \( k > t \), that is, the agent after stage \( t \) obtains no further loans and exerts effort \( e_h \). In particular, he must prefer this strategy to borrowing against all of \( w_t \), or

\[
\delta E_{p_{t}}[w_{t}] - c_h \geq E_{p_{t}}[w_{t}] - c_t.
\]

But then one can show that at stage \( t - 1 \) he strictly prefers to borrow against all of \( w_{t-1} \) rather than follow the wage path \( h \), that is,

\[
E_{p_{t}}[w_{t-1}] - c_t \geq E_{p_{t}}[w_{t-1} - w_{t}] + \delta E_{p_{t}}[w_{t}] - c_h,
\]

or

\[
E_{p_{t}}[w_{t-1}] - E_{p_{t}}[w_{t-1}] - (1 - \delta) E_{p_{t}}[w_{t}] \leq \Delta c.
\]

Since \( w_{t-1} \in \neg IC \),

\[
\delta(E_{p_{t}}[w_{t-1}] - E_{p_{t}}[w_{t-1}]) \leq \Delta c
\]

so one needs to show that

\[
(1 - \delta)(E_{p_{t}}[w_{t-1}] - E_{p_{t}}[w_{t-1}]) \leq (1 - \delta) E_{p_{t}}[w_{t}].
\]

Inequalities (35) and (33) together imply that

\[
\delta(E_{p_{t}}[w_{t-1}] - E_{p_{t}}[w_{t-1}]) \leq \delta E_{p_{t}}[w_{t}] - E_{p_{t}}[w_{t}],
\]

which implies (36).
Lemma 3 For any subgame perfect equilibrium wage path $h$ with $w_0 \in IC$, either $h = (w_0, w^{n-1}, w^{n-2}, \ldots, w^1, w^0, w^0, \ldots)$ for some $n \in \mathbb{N}$ and appropriately defined allocations $w^i \in IC$, $i = 1, \ldots, n-1$, or $h = (w_0, w^s, w^s, \ldots)$ for some appropriately defined $w^s \in IC$.

Proof. The first part of the statement follows from combining Lemma 1 and Lemma 2. To prove the second part of the statement, take a wage path $h$ with $w_t \in IC$ for all $t$. The expected utility of this path is

$$EU(f, h) = f - c_h + E_{p_{h}}[w_0] - (1 - q)(1 - \delta) \sum_{t=0}^{\infty} q^{t} E_{p_{h}}[w_t].$$

(37)

A similar expression was derived in the proof of Lemma 2 for the case $w_t \in \neg IC$ for all $t$, and the proof for (37) is identical. For this path to be subgame perfect $E_{p_{h}}[w_t]$ must be the same for all $t$. Assume otherwise, that is, assume that there exists a $k$ with $E_{p_{h}}[w_k] > E_{p_{h}}[w_1]$ for all $t \neq k$. Then, it is easy to construct an argument as in Lemma 1 to show that a path that skips $w_k$, that is, a path $(w_0, \ldots, w_{k-1}, w_{k+1}, \ldots)$, is feasible and preferred by the agent. Finally, since after each stage $t$ the continuation wage path $h_{-t} = (w_t, w_{t+1}, w_{t+2}, \ldots)$ is optimal, the continuation wage path $(w_t, w_t, w_t, \ldots)$, which gives the same payoff, is also optimal. Therefore, without loss of generality any wage path $h$ with $w_t \in IC$ for all $t$ is of the form $(w_0, w^s, w^s, \ldots)$.

Lemma 4 $\overline{w}^n = 0$, $\overline{w}^s = 0$, and $\overline{w}^n \geq \overline{w}^{n-1}$ for all $n$.

Proof. First, I show that $\overline{w}^n = 0$ for all $n$. Assume to the contrary that $\overline{w}^n > 0$ for some $n$ and consider the path $h = (w, w^n, w^{n-1}, \ldots, w^1, w^0, \ldots)$. Then, I show that path $h' = (w, w^n, w^{n-1}, \ldots, w^1, w^0, \ldots)$, where $\overline{w}^n = \overline{w}^n - \overline{w}^n$ and $\overline{w}^n = 0$, dominates path $h$. A similar argument shows that $\overline{w}^s = 0$.

First, I show that $h'$ is feasible. Note that $w^n \in IC$ implies $w^n \in IC$ and so $\rho_1$, the probability that the first lender attaches to high output in wage path $h$, is equal to $\rho_1'$, the probability that he uses in wage path $h'$. Since $f_1 = f_0 + E_{\rho_1}[w - w^n]$ and $f_1' = f_0 + E_{\rho_1}[w - w^n]$, it is easy to see that $f_1' = f_0 + \overline{w}^n$. Finally, it follows from there that $f_1' = f_1$ for all $i > 1$. So, the feasibility of path $h'$ follows from the feasibility of path $h$. 
Next, I show that path $h'$ yields a higher payoff than path $h$. The difference in the two payoffs is

$$EU(f_0, h) - EU(f_0, h') = (1 - q)q(U(f_1, w^n, e(w^n)) - U(f'_1, w^n, e(w^n)))$$

$$+ q^2(EU(f_2, h_{-2}) - EU(f'_2, h'_{-2}))$$

Since $f'_2 = f_2$ and $h'_{-2} = h_{-2}$ the second part of this expression is equal to zero. The first part is negative if, substituting for $f'_1$ and $w^n$,

$$f_n + \delta[p_n\bar{w}^n + (1 - p_n)w^n] < f_n + w^n + \delta[p_n(\bar{w}^n - w^n) + (1 - p_n)\cdot 0],$$

which is true for all $\delta \leq 1$.

Second, I show that $\bar{w}^n \geq \bar{w}^{n-1}$. Assume to the contrary that $\bar{w}^n < \bar{w}^{n-1}$ for some $n$ and consider the path $h = (w^n, w^{n-1}, w^{n-2}..., w^1, w^0, ...).$ Then, I show that path $h' = (w^n, w^n, w^{n-2}..., w^1, w^0, ...)$ dominates path $h$. Since all $w^i \in IC$ for $i = 1, ..., n,$ $\rho_t = \rho'_t$ for all $t$, where $\rho_t (\rho'_t)$ is the probability that lender $t$ attaches to high output in path $h$ ($h'$). Also, from the first part of this lemma, $w^k = 0$ for all $k$. Therefore, the cumulative up-front payments in wage path $h'$ are $f'_1 = f_0$, $f'_2 = f_0 + E_{\rho_2}[w^n - w^{n-2}] = f_2 + (\rho_2 - \rho_1)(\bar{w}^n - \bar{w}^{n-1})$ and $f'_k = f_k + (\rho_2 - \rho_1)(\bar{w}^n - \bar{w}^{n-1})$ for all $k > 2$. Because $\rho_2 < \rho_1$ and $\bar{w}^n < \bar{w}^{n-1}$, it follows that $f'_k > f_k$, and therefore the feasibility of path $h'$ follows from the feasibility of path $h$.

Next, I show that path $h'$ yields a higher payoff than path $h$. The difference in the two payoffs is

$$EU(f_0, h) - EU(f_0, h') = (1 - q)q(U(f_1, w^{n-1}, e(w^{n-1})) - U(f'_1, w^n, e(w^n)))$$

$$+ q^2(EU(f_2, h_{-2}) - EU(f'_2, h'_{-2}))$$

$$= [q(1 - q)(\rho_1 - \delta p_h) - q^2(\rho_2 - \rho_1)](\bar{w}^n - \bar{w}^{n-1})$$

$$= q(1 - q)(1 - \delta)p_h(\bar{w}^n - \bar{w}^{n-1}) \leq 0$$

where the second equality follows from the expressions for $f_1$, $f'_1$, $f_2$ and $f'_2$ and the fact that $h_{-2} = h'_{-2}$, and the third equality follows from replacing the expressions for $\rho_1$ and $\rho_2$. □

Before proving Proposition 2 I derive a useful simplification of $EU(f_0, h^n(w))$ for $n \geq 1$ and $w \in IC$. The wage payments in $h^n(w)$ are $w_t = w^{n-t}$ for $1 \leq t \leq n$ and
\(w_t = w^0\) for \(n < t\). The probability of high output that lender \(t\) uses to calculate his up-front payment is \(\rho^{n-t} = (1 - q^{n-t})p_h + q^{n-t}p_l\). Consequently, for \(t = 2, \ldots, n\):

\[
f_t = f_0 + E_{\rho^{n-1}}[w - w^{n-1}] + \sum_{k=2}^{t} E_{\rho^{n-k}}[w^{n-k+1} - w^{n-k}].
\]

Then,

\[
EU(f_0, h^{n}(w))
\]

\[
= (1 - q)U(f_0, w, e(w)) + (1 - q) \sum_{t=1}^{n-1} q^t U(f_t, w^{n-t}, e(w^{n-t})) + q^n U(f_n, w^0, e(w^0))
\]

\[
= f_0 + qE_{\rho^{n-1}}[w - w^{n-1}] + \sum_{t=2}^{n} q^t E_{\rho^{n-t}}[w^{n-t+1} - w^{n-t}]
\]

\[
+ (1 - q)\delta E_{p_h}[w] + (1 - q) \sum_{t=1}^{n-1} q^t \delta E_{p_h}[w^{n-t}] - (1 - q^n)c_h - q^n c_l
\]

\[
= f_0 + E_{\tilde{\rho}^n}[w] - (1 - q)(1 - \delta)p_h \sum_{t=1}^{n-1} q^t \tilde{w}^{n-t} - (1 - q^n)c_h - q^n c_l,
\]

where

\[
\tilde{\rho}^n = \rho^n - (1 - q)(1 - \delta)p_h.
\]

Line (40) follows from replacing the expressions for the up-front payments in (39) and noting that

\[
(1 - q) \sum_{k=t}^{n-1} q^k + q^n = q^t.
\]

Line (41) follows from

\[
q^{t+1} E_{\rho^{n-t-1}}[w^{n-t}] - q^t E_{\rho^{n-t}}[w^{n-t}] + (1 - q)\delta q^t E_{p_h}[w^{n-t}] = -(1 - q)(1 - \delta)p_h q^t \tilde{w}^{n-t}.
\]

**Proof.** (Proposition 2) The proof is via induction. For all \(n \geq 2\):

**I**(\(n\)) : The allocations \(w^n\), defined by \(\overline{w}^n = 0\) and \(EU(f_0, h^n(w^n)) = EU(f_0, h^{n-1}(w^n))\) are:

\[
\overline{w}^n = \frac{(1 - \delta)^{n-1}p_h^{n-2}p_l \Delta c + \overline{w}^{n-1}}{\delta q^{(n-1)(n-2)/2} \Delta p^n \Delta c}.
\]

**II**(\(n\)) : \(h^n(w)\) is optimal if \(\overline{w} = 0\) and \(\overline{w}^n \leq \overline{w} \leq \overline{w}^{n+1}\).

**III**(\(n\)) : \(h^n(w)\) is better than \(h^i(w)\), \(i < n\), if \(\overline{w} = 0\) and \(\overline{w}^n \leq \overline{w}^i\).
IV(n) : If \( \theta_n := \delta p_h - \rho^{n-1} > 0 \), the allocations \( w^{n s} \) defined by \( w^{n s} = 0 \) and 
\[ EU(f_0, h^n(w^{n s})) = EU(f_0, h^0(w^{n s})) \]
is well defined and
\[
\overline{w}^{n s} = \frac{q^{n-1} \Delta p}{\theta_n} \left( 1 - \delta \right) \left( 1 - \delta \right)^{n-2} p_h \Delta c + \overline{w}^{n-1}.
\] (44)

The discussion preceding Proposition 2 shows I(2), II(1) and III(2). A similar argument that was used to obtain \( w^{1 s} \) shows IV(2) and the expression for \( \overline{w}^{2 s} \). It is provided below for general \( n \). For now, note that \( EU(f_0, h^2(w^{2 s})) = EU(f_0, h^0(w^{2 s})) \) is equivalent to
\[
\theta_2 \overline{w}^{2 s} = -(1 - q)(1 - \delta) \overline{w}^1 + q \Delta c
\]
Therefore, only if \( \theta_2 > 0 \) is \( w^{2 s} \) well defined, and \( \overline{w}^{2 s} \) is given by
\[
\overline{w}^{2 s} = \frac{q(1 - \delta)p_1}{\theta_2 \delta \Delta p} \Delta c + \overline{w}^1.
\] (45)

The Inductive Step shows that if I(\( n \)), II(\( n - 1 \)), III(\( n \)), and IV(\( n \)) are true for some \( n \), then I(\( n + 1 \)), II(\( n \)), III(\( n + 1 \)), and IV(\( n + 1 \)) are also true.

Given \( w^n \), path \( h^{n+1}(w) \) is well defined. Then, since \( h^n \) and \( h^{n-1} \) yield the same payoff at \( w^n \), and \( h^{n-1} \)'s payoff is maximal at \( w^n \) so is \( h^n \)'s. In particular, its payoff must be higher than \( h^{n-1} \)'s payoff. Then, from \( w^n \) increasing \( \overline{w} \) increases \( h^{n+1} \)'s payoff by \( \tilde{\rho}^{n+1} \) (see (42)), whereas \( h^n \)'s payoff increases by \( \tilde{\rho}^n \). Since \( \tilde{\rho}^{n+1} > \tilde{\rho}^n \), the marginal increase in \( h^{n+1} \)'s payoff is larger than the marginal increase in \( h^n \)'s payoff. Thus, there exists an allocation \( w^{n+1} \) on the vertical axis with \( \overline{w}^{n+1} > \overline{w}^n \), at which paths \( h^{n+1} \) and \( h^n \) yield the same payoff. Everywhere above \( h^{n+1} \) dominates \( h^n \). This proves III(\( n + 1 \)) since by the Inductive Hypothesis III(\( n \)), \( h^n \) dominates all \( h^i \) with \( i < n \).

Using (41), \( \overline{w}^{n+1} \) is defined by
\[
(\tilde{\rho}^{n+1} - \tilde{\rho}^n)\overline{w}^{n+1} = (1 - q)(1 - \delta)p_h \sum_{i=1}^n q^{n+1-i}(\overline{w}^i - \overline{w}^{i-1}) + (1 - q)q^n \Delta c.
\]
So,
\[
\overline{w}^{n+1} = \sum_{i=2}^n \frac{(1 - \delta)^{i-1} p_h^{i-1} p_l}{\delta q(i-1)/2 \Delta p^{i+1}} \Delta c + \overline{w}^{n+1} = \sum_{i=3}^{n+1} \frac{(1 - \delta)^{i-1} p_h^{i-2} p_l}{\delta q(i-1)(i-2)/2 \Delta p^{i}} \Delta c + \overline{w}^2
\]
\[
= \frac{(1 - \delta)^n p_h^{n-1} p_l}{\delta q(n-1)/2 \Delta p^{n+1}} \Delta c + \overline{w}^n
\]
where the first equality follows from the definition of \( \overline{w}^i \) for \( i = 1, \ldots, n \), from \( \tilde{\rho}^{n+1} - \tilde{\rho}^n = (1 - q)q^n \Delta p \) and \( \frac{q^{n+1-i}}{q^n q(i-1)(i-2)} = \frac{1}{q(i-1)(i-2)} \). This concludes I(\( n + 1 \)).
Consider now the section of the vertical axis between \(w^n\) and \(w^{n+1}\). From the above discussion, \(h^n\) dominates \(h^{n+1}\) on this section of the vertical axis. By the Inductive Hypothesis \(III(n)\), \(h^n\) also dominates all \(h^i, i < n\). Property 2 and the proof of Lemma 4 imply that \(h^n\) also dominates all \(h^i, i > n + 1\). To show that it is indeed the optimal path one needs to show that \(w^{ns}\), the allocation at which paths \(h^0\) and \(h^n\) yield the same payoff, lies above \(w^{n+1}\).

The expression for \(w^{ns}\) is given in (44), which is true by the Inductive Hypothesis. Then, \(w^{n+1} < w^{ns}\) is equivalent to

\[
\frac{(1 - \delta)^n p_h^{n-1} p_i}{\delta q^{(n-2)/2} \Delta p^n} \Delta c + \frac{q^{n-1} \Delta p}{\theta_n} \frac{(1 - \delta)^{n-1} p_h^{n-2} p_i}{\delta q^{(n-2)/2} \Delta p^n} \Delta c + w^{n-1} = 0,
\]

which is equivalent to

\[
\frac{(1 - \delta)^n p_h^{n-1} p_i}{\delta q^{(n-2)/2} \Delta p^n} \Delta c + \frac{q^{n-1} \Delta p}{\theta_n} \frac{(1 - \delta)^{n-1} p_h^{n-2} p_i}{\delta q^{(n-2)/2} \Delta p^n} \Delta c < 0.
\]

Since \(\theta_n = \delta p_h - \rho^{n-1}\), this is equivalent to

\[-(1 - \delta)^2 p_h^2 < 0,
\]

which is trivially true. This concludes \(II(n)\).

The last step is to show \(IV(n+1)\). At \(w^{n+1}\), path \(h^{n+1}\)'s payoff is equal to \(h^n\)'s payoff and consequently higher than \(h^0\)'s payoff. Increasing \(w\) from \(w^{n+1}\), \(h^0\)'s payoff increases by \(\delta p_h\) (see (14)), whereas \(h^{n+1}\)'s payoff increases by \(\rho^{n+1}\). If \(\delta p_h \leq \rho^{n+1}\), that is, \(\theta_{n+1} \leq 0\), there is no allocation at which \(h^0\) dominates \(h^{n+1}\). If \(\theta_{n+1} > 0\), \(h^0\)'s payoff increases faster than \(h^{n+1}\)'s payoff, and there is a wage \(w^{n+1s}\), such that everywhere above it \(h^0\) dominates and everywhere below it \(h^{n+1}\) dominates. It is first useful to note that by equating (41) and (14), one obtains for all \(j\):

\[
\theta_j \bar{w}^{j s} = -(1 - q)(1 - \delta) p_h \sum_{t=1}^{j-1} q^{t-1} w^{j-t} + q^{j-1} \Delta c.
\]
Therefore,

\[
\theta_{n+1} \overline{w}^{n+1} = -(1-q)(1-\delta)p_h \sum_{t=1}^{n} q^{t-1} \overline{w}^{n+1-t} + q^n \Delta c
\]  \hspace{1cm} \text{(47)}

\[
= -(1-q)(1-\delta)p_h \overline{w}^n + q\theta_n \overline{w}^n
\]  \hspace{1cm} \text{(48)}

\[
= -(1-q)(1-\delta)p_h \overline{w}^n + q^n \Delta p \overline{w}^n - q(1-\delta)p_h \overline{w}^{n-1} + q^n \Delta p \overline{w}^n
\]  \hspace{1cm} \text{(49)}

\[
= \theta_{n+1} \overline{w}^n + q^n \frac{(1-\delta)^n p_h^{n-1} p_l}{p_h (n-1)(n-2)/2 \Delta p} + \frac{q^n}{q^{(n-2)(n-1)/2}}\Delta p
\]  \hspace{1cm} \text{(50)}

where (47) and (48) follow from (46) for \( j = n + 1, n \), (49) follows from the Inductive Hypothesis, and (50) and (51) both follow from the definition of \( \overline{w}^n \) and \( \overline{w}^{n-1} \) and \( \theta_{n+1} = \delta p_h - \rho^n \). The expression (44) for \( n + 1 \) then follows because \( \frac{q^n}{q^{(n-2)(n-1)/2}} = \frac{q^n}{q^{(n-2)(n-1)/2}} \). This proves IV\((n + 1)\).

This describes the subgame perfect equilibrium wage paths starting at a wage \( w \in IC \) with \( w = 0 \). To extend this result to the entire \( IC \) region, the same proof can be repeated (including the discussion preceding Proposition 2 to prove the Inductive Hypothesis) by starting at an allocation \( w = (\overline{w}^1 + w, w) \) with an arbitrary \( w > 0 \). ■

**Proof.** (Proposition 5) It remains to prove \( \kappa := \lim_{q \to 1} q^n(q) = \frac{(1-\delta)^2 \rho_h^2}{\Delta^2 p} \). Assume to the contrary that the equilibrium for all \( q < 1 \) is characterized by a wage \( w(q) \) and a corresponding number of lenders \( n(q) \), such that \( \kappa = \lim_{q \to 1} q^n(q) > \frac{(1-\delta)^2 \rho_h^2}{\Delta^2 p} \).

Then, I show that for \( q \) sufficiently close to 1, the principal can profitably deviate from this equilibrium by offering a more incentivized wage \( w'(q) \) with \( \overline{w}'(q) = \overline{w}'(q)(q) > \overline{w}'(q)(q) = \overline{w}(q) \) and by inducing a higher number of lenders \( n'(q) > n(q) \). For this choose \( n'(q) \) such that \( \kappa' := \lim_{q \to 1} q^n(q) > \frac{(1-\delta)^2 \rho_h^2}{\Delta^2 p} \). Since \( n'(q) > n(q) \), \( q^n(q) < q^n(q) \) and consequently \( \kappa' < \kappa \). To show that it is a profitable deviation for the principal I show that it raises his payoff. Making the simplifying assumption \( f_0 = 0 \), his payoff from a wage \( \overline{w}'(q) \) can be written as

\[
\Pi'(q) = \rho^n(\overline{x} - \overline{w}'(q)) + (1-\rho^n)\overline{x} = \overline{x} + (p_h - q^n \Delta p)(\Delta x - \overline{w}'(q)).
\]  \hspace{1cm} \text{(52)}

The deviation increases the first bracket in (52) since it increases the probability of high effort and thus the probability of high output. It decreases the second bracket in (52) since the principal pays a higher wage in case of high output. It is shown that
the second effect can be made arbitrarily small while keeping the first effect bounded away from 0 for \( q \) close to 1.

Write the difference in the principal’s payoff when inducing \( n'(q) \) as opposed to \( n(q) \) recontracting agreements as

\[
\Pi^{n'(q)}(q) - \Pi^n(q) = (q^{n'(q)} - q^n(q)) + (q^{n'} - q^n) \Delta p (\Delta x - \overline{w}^{n'}(q))
\]

The second term on the right-hand-side of this expression is positive and stays bounded away from zero when \( q \) approaches 1, since \( \Delta x - \overline{w}^{n'}(q) > 0 \) for all \( q \) (a necessary condition for \( \overline{w}^{n'}(q) \) to be the equilibrium wage) and \( q^n(q) - q^{n'}(q) \) approaches \( \kappa - \kappa' > 0 \). The first term on the right-hand-side is negative, since \( \overline{w}^{n'}(q) > \overline{w}^{n}(q) \) and \( q^{n'} \Delta p - p_h < 0 \) (necessary for \( \Pi^{n'(q)}(q) \) to be a profitable deviation, see (52)). I show that it can be made arbitrarily small. To see this, note first that the first bracket remains bounded by \( \kappa' \Delta p - p_h \). The second bracket can be written as follows (see (27)):

\[
\overline{w}^{n'}(q) - \overline{w}^{n}(q) = \frac{1 - \delta}{\Delta p^2} \sum_{i=n(q)-1}^{n'(q)-2} \left( \frac{1 - \delta}{\kappa' \Delta p} \right)^i.
\]  

(53)

Since by definition \( \lim_{q \to 1} q^{n'(q)} = \kappa' \), there must exist a neighborhood around \( q = 1 \), such that for all \( q < 1 \) in this neighborhood the sum in (53) is bounded from above by

\[
\frac{1 - \delta}{\Delta p^2} \sum_{i=n(q)-1}^{n'(q)-2} \left( \frac{1 - \delta}{\kappa' \Delta p} \right)^i = \frac{(1 - \delta) p_h (1 - \delta) p_h}{\Delta p^2} \frac{1 - \frac{(1 - \delta) p_h}{\kappa' \Delta p}}{1 - \frac{(1 - \delta) p_h}{\kappa' \Delta p}}
\]

For \( q \) tending to 1, the denominator of this expression tends to 0, whereas the numerator is fixed and lies between 0 and 1, which proves that (53) can be made arbitrarily small.

Finally, since this argument can be applied to any \( n(q) \) with \( \lim_{q \to 1} q^n(q) > \frac{(1 - \delta) p_h^2}{\Delta p^2} \), in equilibrium \( \kappa = \lim_{q \to 1} q^n(q) = \frac{(1 - \delta)^2 p_h^2}{\Delta p^2} \).
References


