Ambivalent Investment and the Hold-Up Problem

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Abstract:

In this paper we add to the foundations of incomplete contracting literature. We study the hold-up problem with ambivalent investment, where investment benefits the investing party if ex post the right decision is undertaken but harms the investing party if the wrong decision is made. In this context, we show that the power of contracts to provide investment incentives depends on three factors: the commitment value of contracts, the amount of quasirents that the investing party can expect in the case of out-of-contract renegotiation and the degree of ambivalence of investment. First, contracts provide first-best investment incentives when parties can commit to a contract regardless of the type of investment. Second, with sufficiently ambivalent investment, if parties cannot commit not to renegotiate a contract and if the investing party’s bargaining power is intermediate, contracts cannot improve investment incentives above those provided by no contract. In contrast, a simple buyer or seller option contract is optimal when the investing party’s bargaining power is extreme.

(JEL: D23, K12, L22)
1 Introduction

Real-life contracts are often very incomplete. They contain gaps that may cause costly disputes and leave contracting parties vulnerable to expropriation by their contracting partner. This observation forms the basis of the incomplete contract approach that has resulted in many important new insights into the design of economic institutions.

The incomplete contract approach leaves the question of why contracts are incomplete largely unexplored. To address this question, references are made to “observable but unverifiable information” and “ex-ante indescribable actions”. The former refers to information that is shared by the contracting parties but unobserved by outsiders, the underlying assumption being that contractual contingencies based on this information cannot be enforced. The latter assumes that contracting parties are unable to foresee or describe future actions in sufficient detail, so that they cannot be included in a contract. However, both explanations are problematic.

The first line of reasoning neglects the possibility that parties can exchange verifiable messages regarding their information as part of their contract (see Moore (1992) for a survey on Nash Implementation). The second argument has come under attack in a recent paper by Maskin and Tirole (1999), who showed that the assumption that parties can foresee their future payoffs (a necessary assumption for rational investment decisions) is sufficient to generate very com-
plete looking contracts: Parties can write contracts based on the payoffs they would like to achieve and fill in the necessary details later.\footnote{Another strand of literature (see for instance Aghion and Bolton (1992) and Dewatripont and Tirole (1994)) considers ex-ante and ex-post noncontractible actions. With noncontractible actions throughout, who has control over these actions is important and will be part of an optimal contract.}

In light of these findings, some papers have tried to understand what constraints are imposed on contracts by the above assumptions and some further restrictions, such as the contracting parties’ inability to commit not to renegotiate a contract\footnote{See for instance Hart and Moore (1988) and Noeckele and Schmidt (1995).} or a particular assumption on their respective bargaining powers.\footnote{Aghion et al. (1994) showed that a contract can achieve the first best even with renegotiation if it can be designed to affect parties’ ex-post bargaining powers.}

In particular, Segal (1999), Hart and Moore (1999) and Che and Hausch (1999) studied versions of the classic hold-up problem and showed that ex-post unverifiable information together with ex-post contract renegotiation can make contracting useless, even with fully describable actions: parties are equally well off with an ex-ante contract as without one.\footnote{Some recent papers question the validity of the ‘contracting plus renegotiation’ approach that is adopted in these papers and most of the literature. Lyon and Rasmussen (2004) and Watson (2005) proposed various extensive forms for the renegotiation procedure that lead to quite distinct predictions about the optimality of contracts.} The main intuition gained from these papers is that, since investment affects parties’ outside options during contract renegotiation, which in turn determine how much of the investment benefit each party can claim, it is very difficult for a contract to provide the right investment...
incentives if investment affects these options adversely. This is readily seen in the paper by Che and Hausch (1999), which deals with cooperative investment, that is, investment that benefits the contracting partner and not the investor. Intuitively, cooperative investment improves the opponent’s and not the investor’s bargaining position. The papers by Segal (1999) and Hart and Moore (1999) deliver a similar result, except that they consider selfish investment in a complex environment. They show that the value of a contract vanishes as the environment’s complexity, measured by the number of possible trading opportunities ex ante, grows without bounds. This result is very appealing because it seems to imply that contracts are more likely to be incomplete in a complex world.

In the current paper we provide a simplification of the model in Hart and Moore (1999), which suggests that it is rather the particular modelling choice on how trading opportunities affect outside options that generates their result. First, we dispense with the complex trading environment in that we consider only two possible trades, one that is ex post efficient and another that is ex post inefficient. Second, we study ambivalent investment that has a positive effect on the investing party’s payoff if the right trade decision is made ex post, but makes things worse otherwise. We show that for sufficiently ambivalent investment (to be defined) and for a certain range of parties’ bargaining powers, there is no loss of surplus if parties refrain from writing an ex-ante contract.5

5Readers might wonder whether this paper is an application of Segal and Whinston (2002). Segal and Whinston provided general conditions under which a noncontingent contract specifying a fixed level of trade (which is then renegotiated) is optimal in the hold-up problem. They
Thus, the main contribution of the paper is to highlight, in a particularly transparent setting, the features that create the “no contract” result in the above paper(s). In addition, where the result in Segal (1999) and Hart and Moore (1999) only holds in the limit, that is, when complexity is unbounded, we obtain the result without reverting to a limit argument.\(^6\) Furthermore, we fully describe the optimal contract for any distribution of bargaining power. Finally, we propose an investment scenario that is different from those studied in the literature.

The idea of ambivalent investment is best understood via an example. Consider the referral process that takes place between members of professional service partnerships. For instance, lawyers often refer cases that lie outside their range of expertise to other lawyers. Similarly, a consulting firm’s auditing branch might refer a consulting opportunity to the firm’s consulting branch. Referring a business opportunity can be viewed as a trade, in which the referring partner sells the business opportunity to his referral partner. The former bears an opportunity cost in form of lost benefits that he could have generated by dealing with the opportunity himself, whereas the latter obtains the benefits created by dealing with the referred opportunity.\(^7\)

\(^6\)Segal (1999) achieved the same result by introducing complexity cost. 
\(^7\)For an analysis of referrals that take place under asymmetric information, see Garicano and Santos (2004).
Specialization of one partner, for example by gaining expertise in a particular field of the business (a lawyer specializing in criminal law for instance), can then be viewed as an ambivalent investment. It is beneficial in the sense that it makes any potentially valuable transaction more valuable. However, it can be harmful because it makes an inefficient transaction even more inefficient.

Take the above example. A law partner who is specialized in criminal law will suffer low opportunity costs from referring a civil law case to his partner. He will be less informed about recent developments in civil law and will be on the whole less knowledgeable and therefore less successful in this area than if he had not specialized. In contrast, specialization will increase the opportunity cost of referring a case that lies within his range of expertise. If he refers a criminal law case, for instance, he will lose the high fees that he could have charged as an expert. The same applies to specialization by the partner who obtains the case. If the receiving partner specializes in civil law, he will derive a greater benefit from being referred civil law cases but will derive a smaller benefit from being referred criminal law cases.

This type of investment captures the intuition explained above: investment, in addition to being valuable because it increases the surplus from the ex-post efficient action, enhances the bargaining position of the investing party’s opponent. This party can threaten to enforce the inefficient action ex post, which is more harmful to the investing party when investment is higher. Thus, investment raises the stakes that the investing party has in the relationship and
contracts have only a very limited scope in providing incentives to undertake this investment.

This paper is structured as follows. Section 2 introduces a model of ambivalent investment. Section 3 contains the analysis and derives the no-contract result. Section 4 provides conclusions.

2 The Model

Two parties consider a future trading opportunity of one unit of a good. Ex ante, there are two possible candidates, good 1 and good 2, with uncertain costs and values. Ex post, one good, called the G good, will turn out to have a high value \( v_G \), and the other good, called the B good, will have a low value \( v_B \), with \( v_G \geq v_B \). The goods’ costs are denoted by \( c_G \) and \( c_B \) respectively. The G good will present an efficient trading opportunity, that is, it will yield a large positive surplus if traded, \( v_G - c_G > 0 \). The B good on the other hand will present an inefficient trading opportunity with \( v_B - c_B < 0 \).

Ex ante, there is complete symmetry between the goods, that is, each will turn out to be the efficient good with equal probability. For simplicity, it is assumed that \( v_G \) and \( v_B \) are fixed. In contrast, the goods’ cost structures are uncertain ex ante. We assume that there are two possible cost realizations. In the first cost state, the G good is particularly cheap and the B good is particularly

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\(^8\)For our results it is not important that \( v_B - c_B < 0 \). It is important, however, that the buyer and seller can only trade one good, for technological reasons, say.
expensive, that is, \( c_G = \bar{c} \) and \( c_B = \bar{c} \), with \( \bar{c} < \bar{c} \). In the second cost state, both goods have the same cost: \( c_G = c_B = c \). Furthermore, we assume that \( \bar{c} < c < \bar{c} \).

The former is the preferred cost state because with symmetric information the parties will always agree ex post on the efficient trade of \( G \). We refer to the latter as the normal cost state. Intuitively, this state is the default cost state, in which the seller is indifferent between producing either of the goods. In contrast, in the first cost state he has a clear preference for the valuable trading opportunity.

To summarize, an ex-post state of nature is described by a tuple \( s = (q, r) \), where the integer \( q \in \{1, 2\} \) denotes the identity of the efficient good and \( r \in \{1, 2\} \) indicates whether the preferred or the normal cost state occurs.

Whereas the nature of the goods (which one will be the \( G \) or the \( B \) good) is determined by a chance move with exogenous probabilities, the probabilities with which the cost states are realized are endogenous. More precisely, by investing \( \sigma \in \mathbb{R}^+ \) at an intermediate date, the seller can raise the probability of the desirable cost state \( 1 \). Denote by \( \pi(\sigma) \leq 1 \) the probability that cost state \( 1 \) occurs; similarly, denote by \( 1 - \pi(\sigma) \) the probability that the normal cost state, state \( 2 \), occurs. Then, \( \pi'(\sigma) > 0 \) for \( \sigma > 0 \). It is further assumed that \( \pi'(0) = \infty \) and \( \pi''(\sigma) < 0 \) to guarantee an interior solution for the first-best level of investment. Without loss of generality, investment costs are set equal to \( \sigma \).

Going back to the example on referrals in the Introduction, consider two partners in a law firm who allocate cases between them. One partner is specialized in civil law, he generates profits of \( v_G \) if he works on civil law cases and \( v_B \)
if he works on criminal law cases. His partner can then specialize in criminal law to provide a better match for his partner. He makes the investment, $\sigma = 1$, or not, $\sigma = 0$, and $\pi(1) = 1$ and $\pi(0) = 0$. Cost state 1 is the state in which he has undertaken the investment and suffers a small opportunity cost, $-c$, from referring civil law cases to his partner (selling G), but pays a high opportunity cost, $-\overline{c}$, if he refers a criminal law case (selling B). In state 2 he has not made the investment and is an all-purpose lawyer. He is indifferent between passing on a criminal or a civil law case (his opportunity cost is $-c$ for both trades).

The key idea of ambivalence is captured here by our assumption that investment has both a negative and a positive impact on the seller’s cost. Conditionally on taking the efficient decision, investment reduces his expected cost, $dE[c_G]/d\sigma = -\pi'(\sigma)(c - \bar{c}) < 0$. However, if parties choose the wrong action ex post, investment raises the expected cost, $dE[c_B]/d\sigma = \pi'(\sigma)(\bar{c} - c) > 0$.

**Definition 1** Ambivalent Investment: The seller’s investment is ambivalent if it decreases the production cost of the efficient good but increases the production cost of the inefficient good:

$$\frac{dE[c_G]}{d\sigma} < 0,$$
$$\frac{dE[c_B]}{d\sigma} > 0.$$ 

This assumption implies in particular that investment increases the opportunity for a hold-up by the buyer, that is, it raises the seller’s stake in the relationship because it raises the likelihood that the B good is very expensive. The buyer
can hold-up the seller by trying to enforce trade of the B good and this possibility will be more costly to the seller if his investment is higher. Importantly, for this to deter investment (and for contracts to be of no use) two additional conditions must be met. First, the hold-up must be severe enough, that is, B’s cost in the preferred state must be higher than G’s cost in the normal state: \( \max c_B = \bar{c} > c = \max c_G \). This is ensured in our model by the particular cost structure. If, in contrast, \( c < \bar{c} \), being held-up with the B good after successful investment is cheaper for the seller than producing the G good at high cost, and so investment is beneficial to the seller in any case. A contract can exploit this to provide some investment incentives. Second, investment benefits must be outweighed by the seller’s costs arising from the hold-up. A measure of the positive impact of investment is \( \Gamma_G = \max c_G - \min c_G = c - \bar{c} > 0 \). Similarly, a measure of the negative impact of investment is \( \Gamma_B = \max c_B - \min c_B = \bar{c} - c > 0 \). The relative impact of investment can then be measured by

\[
\Delta \Gamma := \Gamma_B - \Gamma_G.
\]

Intuitively, the seller will prefer not to invest if the potential increase in cost brought about by the investment (\( \Gamma_B \)) is larger than the potential decrease in cost (\( \Gamma_G \)), and in this case it will be difficult for a contract to enhance investment incentives. We will say the following.

**Definition 2** Investment is strongly ambivalent if \( \Delta \Gamma > 0 \).
2.1 Information Structure and Space of contracts

Information:

The investment by the seller is not contractible. The identity of the efficient good or its cost are not contractible either. Goods are distinguishable by their names (good 1 or 2) only. Whether trade of one good or another takes place is a verifiable event and so are any announcements that contracting parties make.

Contracts

We consider general message contingent contracts, that is, contracts can be made contingent on parties’ announcements concerning unverifiable information, such as the identity of the efficient good or the cost state.\(^9\) As is well known in this type of problem, there is no loss of generality in considering direct revelation mechanisms where parties only have to report their information. Here a con-

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\(^9\)For contracting with renegotiation it suffices to study contracts that are only contingent on the ex-post state of nature \(s\) and not on the investment \(\sigma\). To see this, take a mechanism, in which parties are asked to announce \((s, \sigma)\), that results in ex-post (after renegotiation) equilibrium payoffs for the seller of \(\Pi_s(\sigma)\). To be incentive compatible, parties must find it optimal to announce \(s\) and \(\sigma\) truthfully. However, because investment is payoff irrelevant ex post, announcing \(\sigma'\) (respectively \(\sigma\)) must also be optimal in state \(s\) when the true investment level is \(\sigma\) (respectively \(\sigma'\)). This follows because monotonicity of the rule to be implemented is a necessary condition for implementability (see for instance Maskin 1999). Also, as remarked by Maskin and Moore (1999), contracting in symmetric information environments with renegotiation is a constant sum game and it follows from the Minimax theorem that the equilibrium payoffs of buyer and seller are the same in all equilibria. This implies in particular that \(\Pi_s(\sigma) \equiv \Pi_s\).
tract can specify, contingent on these reports (\(\hat{s}_B\) announced by the buyer and \(\hat{s}_S\) by the seller, both in \((q, r)\)), what to trade and what payment to exchange.

Renegotiation allows us to simplify the structure of the relevant contracts to consider. First, we only need to consider balanced mechanisms, that is, mechanisms that do not involve payments to third parties because they would be voided by renegotiation. In addition, because renegotiation achieves ex-post efficiency, total surplus is either \(v_G - c\) or \(v_G - c\), and we can, without loss of generality, concentrate on mechanisms in which the efficient good is traded with probability one in equilibrium. Therefore, a contract has the form \([q_s, p_s, \alpha_j(\hat{s}_B, \hat{s}_S), t(\hat{s}_B, \hat{s}_S)]\), where \(q_s \in \{1, 2\}\) designates the identity of the efficient good to be traded and \(p_s\) is the price paid by the buyer when parties agree on the state of nature \(s\). If they disagree, that is, \(\hat{s}_S \neq \hat{s}_B\), \(\alpha_j(\hat{s}_B, \hat{s}_S) \in [0, 1]\) is the probability with which good \(j\) is traded. No trade may occur, that is, it is possible that \(\alpha_1(\hat{s}_B, \hat{s}_S) + \alpha_2(\hat{s}_B, \hat{s}_S) < 1\). The final contractual component is a disagreement transfer \(t(\hat{s}_B, \hat{s}_S)\) from the buyer to the seller.

### 2.2 Timing

The time structure in the game is as follows.

- At date 0, parties can write a contract that governs their relationship.
- At date 1/2, the seller makes his investment \(\sigma\).
- At date 1, all relevant uncertainty is resolved. Both parties observe the state of nature \(s\), which determines the identity of the efficient and inefficient good
and the cost state.

- At date 2, parties can send messages about the state of nature. The contract determines an outcome contingent on these messages.

- Finally, parties can renegotiate any remaining inefficiencies before trade at date 3. The additional surplus resulting from renegotiation is split according to some exogenous factor $\lambda$, where $\lambda$ is the fraction received by the seller and $1 - \lambda$ is the fraction received by the buyer.\(^{10}\)

### 2.3 Benchmarks

We first characterize the optimal contract and the outcome of the transaction under the assumption of full commitment. We show here that if parties can commit not to renegotiate, there exists a contractual arrangement that achieves full efficiency. The first-best level of investment $\sigma^*$ is found by maximizing total expected surplus, that is,

$$
\sigma^* \in \arg \max_\sigma \pi(\sigma)(v_G - c) + (1 - \pi(\sigma))(v_G - c) - \sigma.
$$

Rearranging the first-order condition of this maximization problem yields

$$
\pi'(\sigma^*) = \frac{1}{c - c'}.
$$

Second-order conditions are satisfied because of the strict concavity of the function $\pi(\sigma)$. Moreover, the solution $\sigma^*$ is unique.

\(^{10}\)This outcome can be achieved by a noncooperative bargaining game in which the seller (respectively the buyer) makes a take-it-or-leave-it offer with probability $\lambda$ (respectively $1 - \lambda$).
If parties can commit not to renegotiate, a very simple contract achieves this result. With a contract that allows the seller to make a take-it-or-leave-it offer to the buyer at date 2, he will offer to sell the good G for a price of $v_G$. This offer is accepted by the buyer and the seller will indeed invest $\sigma^*$ because he is the residual claimant of his investment.

**Proposition 1** *If parties can commit not to renegotiate a contract, the optimal contract achieves first-best investment incentives.*

The efficiency of this contract relies heavily on the fact that the leave-it part of the offer is credible. Suppose instead that after a rejection of the offer by the buyer at date 2, parties can reopen bargaining at date 3, and assume further that in this bargaining game the buyer has all the bargaining power: $\lambda = 0$. He will then offer to buy the G good at a price equal to the production cost, that is, either $c$ or $\underline{c}$, depending on the realization of the cost state. Expecting this outcome, the seller has no incentive to invest in reducing the efficient good’s cost and sets $\sigma = 0$.

In what follows we are interested in comparing the seller’s investment incentives when a contract is put into place at date 0 with his incentives when there is no contract. We also refer to this situation as the null contract. In this case, the seller’s investment level $\sigma^0$ maximizes

$$\pi(\sigma)\lambda(v_G - \underline{c}) + (1 - \pi(\sigma))\lambda(v_G - c) - \sigma$$
and is therefore given by

\[ \pi'(\sigma^0) = \frac{1}{\lambda(c - c)}. \]

Note that \( \sigma^0 \to 0 \) when \( \lambda \to 0 \) and \( \sigma^0 \to \sigma^* \) when \( \lambda \to 1 \).

The following section characterizes the situations in which these two benchmarks are obtained, when parties can draw contracts but cannot commit not to renegotiate.

3 Contracts, Hold-up and Ambivalence

3.1 The Role of Ambivalence

To highlight the importance of ambivalence in determining the outcome of the contractual relationship, suppose that instead both goods always have either low cost \( c \) or high cost \( c \) and that investment decreases expected costs of both goods. Then, there is a simple mechanism that implements the first best even if parties cannot commit not to renegotiate: the contract says that good 1 has to be traded at a price \( p \), no matter what. Obviously, when good 1 is not the efficient good, renegotiation will take place. The surplus increase generated by renegotiation is equal to \( (v_G - c_G) - (v_B - c_B) \). Crucially, this expression does not depend on the cost state because \( c_G = c_B \) in both states. The seller’s overall expected payoff is his contractual payoff plus the expected share of the surplus generated by renegotiation, that is,

\[ p - (\pi(\sigma)c + (1 - \pi(\sigma))c) - \sigma + \frac{1}{2}\lambda[v_G - v_B]. \]
It follows immediately that the seller has first-best incentives to invest.

Let us go back to the case of ambivalent investment and see under what conditions the previous simple contract fails to implement the first best. The seller’s expected payoff is

\[
p - \frac{1}{2}(\pi(\sigma)c + (1 - \pi(\sigma))c) - \frac{1}{2}(\pi(\sigma)c + (1 - \pi(\sigma))c) - \sigma
\]

\[
+ \frac{\lambda}{2} [(v_G - \pi(\sigma)c - (1 - \pi(\sigma))c) - (v_B - \pi(\sigma)c - (1 - \pi(\sigma))c)].
\]

Rearranging terms, the expected payoff for the seller is

\[
p - \lambda(\pi(\sigma)c + (1 - \pi(\sigma))c) - \frac{1}{2}(1 - \lambda)\pi(\sigma)\Delta\Gamma - \sigma + \frac{1}{2}\lambda(v_G - v_B) - (1 - \lambda)c.
\]

We can see that the simple performance contract only performs better than the null contract if \(\Delta\Gamma < 0\). Also, its benefit vanishes as \(\Delta\Gamma\) approaches zero, that is, as investment becomes more ambivalent, and it provides worse investment incentives than the null contract (as long as \(\lambda \neq 1\)) if investment is strongly ambivalent: \(\Delta\Gamma > 0\). Intuitively, if \(\Delta\Gamma > 0\), the positive impact of investment on the seller’s payoff if good 1 turns out to be the G good is outweighed by its negative impact if it turns out to be the B good. Although in the latter case renegotiation will ensure that G is traded in the end, the buyer will capture a large share of the renegotiation surplus because of the ambivalent nature of investment, and this is bad for the seller’s incentives to invest. Finally, note that first-best incentives are provided if \(\lambda = 1\). The following section shows that these insights are quite general.
3.2 Optimal Contracts

How effective a contract will be for investment purposes depends on the ease with which one party or the other can be induced to report truthfully the state of the world that prevails ex post. This in turn depends both on $\lambda$, the allocation of bargaining power, and $\Delta \Gamma$, the degree of ambivalence. The most striking implication of ambivalent investment for contracting appears when the seller’s bargaining power is intermediate, as detailed in the following proposition.

**Proposition 2** Assume that investment is strongly ambivalent. Then, there exist $0 < \lambda_0 < \lambda_0 < 1$, so that whenever $\lambda \in [\lambda_0, \lambda_0]$, the best contract is the null contract. That is, for these values of $\lambda$, the possibility to write ex-ante contracts offers no protection against the hold-up problem.

To provide investment incentives, a contract can either reward the seller when the cost of the efficient good turns out to be low or punish him when it turns out to be high. Let us consider the first alternative. A high equilibrium payoff for the seller in that case is a good way to increase his incentive to invest. However, as ex post the seller and the buyer play a constant sum game, increasing the seller’s equilibrium payoff necessarily goes with a decrease in the buyer’s equilibrium payoff in this state. To avoid this low payoff, the buyer can ex post pretend that the wrong good is efficient and that the normal cost state, in which both goods cost $c$, has occurred. Indeed, if he is believed and the wrong good is traded this will impose a high cost $\overline{c}$ on the seller, by the ambivalence of investment.
Although the buyer does not value the inefficient good per se, it is valuable to him in the renegotiation game. In fact, to avoid having to produce the expensive good, the seller is willing to accept a very small part of the renegotiation surplus and this is more valuable to the buyer if the seller’s share in the renegotiation surplus is lower, that is, for $\lambda \leq \lambda_0$. To deter such a deviation, one has to lower the buyer’s *out-of-equilibrium* payoff.

The second alternative is to punish the seller when the efficient good’s cost is high, that is, to lower his equilibrium payoff in the normal cost state. To avoid this punishment, the seller can ex post pretend that the wrong good is the efficient one but that it can be produced at low cost. If, after this deviation, the wrong good ends up being traded, the seller does not suffer a loss because both goods cost the same to produce. However, he gains in renegotiation: renegotiation will generate a positive surplus of $v_G - v_B$, of which a fraction $\lambda$ goes to the seller. This deviation is particularly attractive if this fraction is not too small, that is, if $\lambda \geq \lambda_0$. To deter this deviation, the seller’s out-of-equilibrium payoff must be reduced, which, by the same token as before, requires an increase in the buyer’s out-of-equilibrium payoff.

This discussion shows that the key feature for designing better incentives than those provided by the null contract is to reduce the buyer’s payoff in the first kind of disagreement while increasing it in the second (and vice versa for the seller). There is not much room for doing so because the contract can only depend on parties’ announcements and those are the same in these two types
of disagreement. True, the preferences in the two true states are different and the optimal contract can exploit that fact to improve incentives. Note, however, that providing incentives would be much easier if investment was not ambivalent. For instance, if it was the case that the costs of the two goods were either both low or both high, the first type of disagreement, the buyer wrongly pretending that costs are high and the seller rightly claiming that costs are low, would be trivial to resolve: one could let the buyer choose a good that is traded at a fixed price. Indeed, with this cost structure the buyer has no incentive to designate the wrong good, because the inefficient good has a low value but costs the same as the efficient good. The ambivalence of investment reduces greatly the usefulness of such a buyer-friendly contract.

For an ex-ante contract to be of no value, $\lambda \leq \lambda_0$ and $\lambda \geq \lambda_0$ must hold simultaneously, that is, we need $\lambda_0 < \lambda_0$. This is ensured by the assumption that investment is strongly ambivalent. Otherwise, the positive effect of investment on the seller’s inside option (trade of G) is strong enough to outweigh the negative effect on his outside option (trade of B) and a contract can exploit this to improve investment incentives for all levels of the seller’s bargaining power.

We now show that a simple contract is optimal when the seller’s bargaining power is extreme.

**Proposition 3** Assume that investment is strongly ambivalent. Whenever $\lambda \in [0, \lambda_0)$ or $\lambda \in (\lambda_0, 1]$ parties are better off signing an ex-ante contract. One possible optimal contract is a seller option contract (in the case $\lambda \in [0, \lambda_0)$) or
a buyer option contract (in the case $\lambda \in (\overline{\lambda}, 1]$). Such a contract stipulates a fixed price and gives to the seller (buyer) the right to choose which good to trade.

Investment is first best only if $\lambda = 0$ or if $\lambda \geq \lambda^*$, where $\overline{\lambda} < \lambda^* < 1$.

Proposition 3 says that in the case of a valuable ex-ante contract, it should give power to the party who is weak in the ex-post bargain, that is, to the seller if $\lambda$ is small and to the buyer if it is large. To see the intuition for Proposition 3, let us first consider what happens in the region where the seller’s bargaining power is relatively high, that is, $\lambda \in [\lambda^*, 1]$. In this case a buyer option contract as described in Proposition 3 achieves first-best efficiency because the buyer has very little incentive to lie. If he were to misreport the identity of the efficient good, he would trigger further renegotiation of the contractual terms with the seller. However, for high $\lambda$ the buyer cannot expect much from renegotiation. Therefore, he prefers to behave in the first place and this implies that the seller invests efficiently. Now consider the case where $\lambda$ is intermediate, that is, $\lambda \in (\overline{\lambda}, \lambda^*)$. Then, this contract is no longer first best. In fact, in the low-cost state the buyer will now want to trade the inefficient good because his share of the renegotiation surplus $(1 - \lambda)(v_G - e - (v_B - e))$ is sufficiently large to make this lie profitable. In the normal-cost state on the other hand, he will choose the efficient good. In the low-cost state this contract is renegotiated leading to a lower payoff to the seller than in the first best. Consequently, investment incentives are only second best. Finally, if $\lambda$ drops below $\overline{\lambda}$, the seller’s payoff in the low-cost state is so small that this contract provides worse investment incentives than the null contract.
Now let us consider what happens in the other region where the seller’s bargaining power is low. In the case $\lambda = 0$, a seller option contract is first best. He will always want to trade the cheapest good, which is the efficient good in both cost states (in the normal-cost state the seller is indifferent). The reason why the first best cannot be implemented for $\lambda$ small but greater than zero is that whenever $\lambda \neq 0$, the seller has a strict preference to choose the inefficient good in the normal cost state. Although both goods cost the same in this state, the inefficient good yields an additional value in renegotiation, $\lambda(v_G - v_B)$, which is strictly positive unless $\lambda = 0$. Here, the seller option contract is renegotiated in the normal cost state and is optimal as long as $\lambda < \lambda_0$. Otherwise, it is worse than the null contract.\textsuperscript{11}

4 Concluding Remarks

This paper adds to the foundations of incomplete contracts literature. It proposes a simple investment problem in which investment is beneficial, but at the same time worsens the hold-up problem that the investing party faces: it decreases the cost of an efficient trading opportunity and increases the cost of an inefficient trading opportunity. If this hold-up is severe enough and if investment

\textsuperscript{11}This argument also shows that $v_G \geq v_B$ is necessary for Proposition 2. Indeed, if $v_G < v_B$, the seller has no incentive to lie in a seller option contract: the G good is cheaper and choosing the B good puts the seller in a weak position for renegotiation because the buyer has to be compensated for his large loss of $v_B$ when the parties renegotiate to the efficient trade.
is sufficiently ambivalent, the hold-up problem may appear in its starkest form: contracts may turn out to be of no use for providing investment incentives. The fact that contracts can always be renegotiated and that investment is ambivalent makes hold-up problems particularly severe because investment raises the stakes that the investing party has in the relationship. In addition, we have shown that the bargaining power of the investing party, $\lambda$, has an interesting effect on the nature of contracting. With increasing $\lambda$, first a simple seller option contract, then no contract and finally a buyer option contract is optimal. Thus, for a contract to be valuable it needs to give power to the party that is weak in the ex-post bargain.

**Appendix**

We first introduce some notation. The seller’s equilibrium payoff in state $s = (q, 1)$ can be written as $\Pi_{q1} = p_{q1} - c$, where $p_{q1}$ is the final payment that the buyer makes to the seller. It includes the payment contained in the contract plus the additional payment that he makes at renegotiation. Similarly, the buyer’s equilibrium payoff in state $(q, 1)$ is $v_{G} - p_{q1}$. In state $(q, 2)$, the seller and buyer obtain $\Pi_{q2} = p_{q2} - c$ and $v_{G} - p_{q2}$, respectively.

There are several ex-post incentive constraints to ensure truthful revelation of the state of the world. Among those, consider the disagreement where the seller claims that the state is $(q, 1)$ and the buyer claims that it is $(q', 2)$ with $q \neq q'$. Without loss of generality, assume that the seller announces $(1, 1)$ and
the buyer announces $(2, 2)$. Then, denote the probabilities with which goods 1 and 2 are traded by $\alpha^1$ and $\alpha^2$, respectively. Similarly, denote the transfer payment from buyer to seller prescribed by the contract by $t$.

For the contract to be incentive compatible, the seller must prefer his equilibrium payoff in state $(2, 2)$ to his out-of-equilibrium payoff if he announces $(1, 1)$, that is,

$$p_{22} - c \geq - (\alpha^1 + \alpha^2)c + t + \lambda[\alpha^1((v_G - c) - (v_B - c)) + (1 - \alpha^1 - \alpha^2)(v_G - c)]. \tag{3}$$

The seller’s out-of-equilibrium payoff on the right-hand-side can be understood as follows. The first two terms represent his payoff from the contract. Trade (of either good) occurs with probability $\alpha^1 + \alpha^2$, which imposes a cost $c$ on the seller, and he receives a payment $t$. The last term in brackets represents the fraction $\lambda$ of the renegotiation surplus that goes to the seller. If trade of good 1 is prescribed by the contract, the additional surplus is equal to the surplus from trade of the G good minus the surplus from trade of the B good. If trade of good 2 is prescribed by the contract, there is nothing to renegotiate. If no trade is prescribed (with probability $1 - \alpha^1 - \alpha^2$) the increase in surplus is equal to the total surplus resulting from trade of good G.

Similarly, the buyer must prefer his equilibrium payoff in state $(1, 1)$ to his out-of-equilibrium payoff if he announces $(2, 2)$ and

$$v_G - p_{11} \geq \alpha^1 v_G + \alpha^2 v_B - t + (1 - \lambda)[\alpha^2((v_G - \underline{v}) - (v_B - \underline{v})) + (1 - \alpha^1 - \alpha^2)(v_G - \underline{v})], \tag{4}$$
which follows from a similar argument as that presented above for (3).

Equations (3) and (4) can be simplified to yield

\[ p_{22} - c \geq \lambda (v_G - c) - \alpha^1 (1 - \lambda)c + \lambda v_B - \alpha^2 (1 - \lambda)c + \lambda v_G + t \]

\[ v_G - p_{11} \geq -\lambda (v_G - c) + \alpha^1 (1 - \lambda)c + \lambda v_G - \alpha^2 (1 - \lambda)c + \lambda v_B - t. \]

Then, these equations together imply that

\[ \Pi_{22} - \Pi_{11} \geq \lambda (c - c) + \alpha^1 (1 - \lambda)(c - c) + \lambda (v_G - v_B) + \alpha^2 (1 - \lambda)(c - c) + \lambda (v_B - v_G). \]

(5)

A similar condition would be obtained when considering states (1, 2) and (2, 1) where \( \alpha^{1'} \) and \( \alpha^{2'} \) would play the role of \( \alpha^2 \) and \( \alpha^1 \), respectively. Of course, there are other incentive constraints, but to establish the results below, it is enough to concentrate on (3) and (4).

**Proof of Proposition 2.**

The proof consists in showing that it is impossible to relax (5) more than by choosing \( \alpha^i = 0 \). For that to be the case, we need the last two terms in brackets in (5) to be positive. The first term is positive if

\[ \lambda \geq \frac{c - c}{v_G - v_B + c - c} := \lambda_0. \]

(6)

The second term is positive if

\[ \lambda \leq \frac{c - c}{v_G - v_B + c - c} := \lambda_0. \]

(7)

For the two constraints to be satisfied simultaneously we need \( \lambda_0 \leq \lambda_0 \), or
equivalently\textsuperscript{12} $\Gamma_G \leq \Gamma_B$, that is, investment must be strongly ambivalent. Then, the constraints put a lower bound on $\Pi_2 - \Pi_1 := E_q[\Pi_{q2}] - E_q[\Pi_{q1}]$: (5) shows that if $\lambda \in [\lambda_0, \lambda_0]$, then

$$
\Pi_2 - \Pi_1 = \left( \frac{1}{2} \Pi_{12} + \frac{1}{2} \Pi_{22} \right) - \left( \frac{1}{2} \Pi_{11} + \frac{1}{2} \Pi_{21} \right)
= \frac{1}{2}(\Pi_{12} - \Pi_{21}) + \frac{1}{2}(\Pi_{22} - \Pi_{11}) \geq \lambda(\underline{c} - c).
$$

Therefore, as the null contract results in efficient trade ex post and in payoffs with $\Pi_2 - \Pi_1 = \lambda(\underline{c} - c)$, it is the most efficient. \rule{1cm}{0.1em}

**Proof of Proposition 3.**

Assume first that $\lambda < \lambda_0$. Then, the first term in (5) is negative but the second term is positive, and so setting $\alpha^1 = 1$ and $\alpha^2 = 0$ is optimal. We have

$$
\Pi_2 - \Pi_1 \geq \underline{c} - c + \lambda(v_G - v_B).
$$

(8)

We now show that a seller option contract with a fixed price $p$ achieves this bound and is therefore optimal. With this contract, the seller chooses the G good in cost state 1 if

$$
p - \underline{c} \geq p - \bar{c} + \lambda(v_G - \underline{c} - (v_B - \bar{c})),
$$

or $\lambda \leq \lambda^*$ with

$$
\lambda^* := \frac{\underline{c} - \bar{c}}{v_G - v_B + \bar{c} - \underline{c}}.
$$

\textsuperscript{12}This is independent of the particular parametrization of the model. Given the assumption $v_G \geq v_B$ and $c_G \leq c_B$ if we call $c_{jr}$, with $j = G$, $B$ and $r = 1, 2$, the cost of the $j$ good in state $r$, we need $c_{B1} - c_{G2} \geq c_{B2} - c_{G1}$, which is equivalent to $\Gamma_B = c_{B1} - c_{B2} \geq c_{G2} - c_{G1} = \Gamma_G$. 26
This is true because $\lambda_0 \leq \lambda^*$. He chooses the B good in state 2 (unless $\lambda = 0$) because

$$p - c + \lambda(v_G - v_B) \geq p - c.$$ 

Therefore, $\Pi_2 - \Pi_1 = \xi - c + \lambda(v_G - v_B)$ and (8) is fulfilled with equality.

Second, assume that $\lambda_0 < \lambda$. Then, the first term in (5) is positive, but the second is negative, and so setting $\alpha^1 = 0$ and $\alpha^2 = 1$ is optimal. We have

$$\Pi_2 - \Pi_1 \geq \xi - c + \lambda(\xi - \xi + v_B - v_G).$$

(9)

In this case a buyer option contract with a fixed price $p$ is optimal. Consider first the case of $\lambda \leq \lambda^*$. The buyer will choose the inefficient good in cost state 1 because

$$v_B - p + (1 - \lambda)(v_G - \xi - (v_B - \xi)) \geq v_G - p.$$ 

He will choose the efficient good in cost state 2 because

$$v_G - p \geq v_B - p + (1 - \lambda)(v_G - v_B).$$ 

Easy calculations therefore show that (9) is satisfied with equality.

Finally, if $\lambda^* \leq \lambda$, it is easy to see that the buyer will pick the efficient good in both cost states, which leads to first-best investment incentives for the seller.

$\blacksquare$

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References


