Business Cycle Effects of Credit Shocks in a DSGE Model with Firm Defaults

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Abstract

This paper proposes a new theoretical framework for the analysis of the relationship between credit shocks, firm defaults and volatility. The key feature of the modelling approach is to allow for the possibility of default in equilibrium. The model is then used to study the impact of credit shocks on business cycle dynamics. It is assumed that firms are identical *ex ante* but differ *ex post* due to different realizations of firm-specific technology shocks, possibly leading to default by some firms. The implications of firm defaults for the balance sheets of households and banks and their subsequent impacts on business fluctuations are investigated within a dynamic stochastic general equilibrium framework. Results from a calibrated version of the model suggest that, in the steady state, a firm's default probability rises with its leverage ratio and the level of uncertainty in the economy. A positive credit shock, defined as a rise in the loan-to-deposit ratio, increases output, consumption, hours and productivity, and reduces the spread between loan and deposit rates. Interestingly, the effects of the credit shock tend to be highly persistent, even without price rigidities and habit persistence in consumption behavior.

Keywords: Firm Defaults; Credit Shocks; Financial Intermediation; Interest Rate Spread; Uncertainty. *JEL Classification*: E32, E44, E50, G21.

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1 Introduction

The recent financial crisis and the ensuing economic recession have highlighted the importance of linkages that exist between debt, leverage, uncertainity and firm defaults. Empirical evidence suggests that bank credit has played an important role in explaining the business-cycle dynamics of output growth, inflation and interest rates in advanced economies since the late 1970s. As shown in Helbling, Huidrom, Kose, and Otrok (2011), Bagliano and Morana (2012), Xu (2012) and Eickmeier and Ng (2015), a negative shock to U.S. real credit has significant adverse effects on output and interest rates in the United States, as well as in other advanced economies such as the euro area and the United Kingdom. The contraction in credit and the subsequent decline in output during the 2007-2009 crisis was coupled with a rise in the frequency of a firm's default. In fact, according to Moody's (2012), the rolling 12-month default rate for private firms in the United States reached a peak of 5.3 per cent in September 2009, more than double the pre-crisis level. In addition, market volatility, as measured by the VIX index, reached elevated levels during the crisis, peaking at 80 toward the end of 2008, compared with an average of 20 in the five years preceding the crisis. The Federal Open Market Committee repeatedly emphasized the increased uncertainty as one of the key factors driving the 2007-2009 recession.

This paper proposes a novel theoretical framework to analyze the relationship among credit, firm defaults and uncertainty, by developing a DSGE model with financial intermediation, allowing for the possibility of firm defaults, and accounting for second-order moments (volatility) of shocks to the macroeconomic environment. The main contribution of the paper in relation to the existing literature is as follows. An alternative modelling framework is advanced for the analysis of firm defaults and financial intermediation with explicit links between credit risk and productivity. The model comprises a large number of firms, a representative household and a banking sector that operates competitively. Firms are identical *ex ante* and live from period to period. At the start of each period, firms enter the market and decide on the optimal levels of labour input and capital stock for their operations. Firms receive initial funds from the household sector, which can be interpreted as private-equity investment by the household, and augment these funds by borrowing from the banking sector. These financing arrangements are made *prior* to the realization of idiosyncratic and common technology shocks. Some firms may default if the realized technology shocks are unfavorable, such that the firm's revenue is not sufficient to repay the bank loan. We assume that the product market is competitive and, while some firms may fail at the end of each period, entry is free. The banking sector receives deposits from households before the arrival of a credit shock, which then determines the total level of loanable funds to the firms. The banking sector receives loan repayments from solvent firms and seizes the revenues of defaulted firms (if any) to partially cover losses. The equilibrium loan rate is in turn affected by the economy-wide default probability.

Unlike the literature, that uses collateral constraints to discourage defaults, we do allow for defaults and fully take into consideration the financial implications of such defaults for the representative household (the equity-holder) and the bank (the debt-holder). Prominent examples of collateral-constrained models of financial intermediations include Kiyotaki and Moore (1997), Gertler and Kiyotaki (2010), and Carlstrom, Fuerst, and Paustian (2010). In these studies, collateral constraints are introduced as a way of ensuring that borrowers can re-pay their debts, which in most cases rule out, almost by design, the possibility that firms actually default. Our paper is more closely related to the work by Bernanke, Gertler, and Gilchrist (1999, BGG), Christiano, Ilut, Motto, and Rostagno (2010) and Fiore and Tristani (2013) on modeling firm defaults and financial intermediation. Our approach differs from BGG and Christiano, Ilut, Motto, and Rostagno (2010) in that idiosyncratic shocks affect productivity rather than the return on capital, which keeps the model tractable and establishes a direct link between credit risk and productivity. We also allow households to bear part of the default risk in the face of an adverse technology shock, through their equity investment in firms; otherwise, the whole burden of firm default falls on the banking sector, resulting in excessively large spreads between loan rates and deposit rates. The timing of labour and capital decisions and the fact that firms are subject to idiosyncratic technology shocks are essential for modelling firm defaults.[1] The default probability in our model is not only related to the standard deviations (uncertainty) of the idiosyncratic and aggregate technology shocks, but also the leverage ratio of firms, which sheds further light on the relation between external finance and default probability in the steady state.

Given the importance of uncertainty and leverage for defaults, the paper also derives steady state values that reflect such uncertainties, by taking account of non-linearities and possible unit roots in the technology processes. One feature of the standard first-order perturbation approach is that the solution displays the certainty equivalence property, where the first-order approximation to the unconditional means of the endogenous variables coincides with their non-stochastic steady-state values (Schmitt-Grohe and Uribe, 2004). We propose an alternative method that relates the steady state values to their means as well as their standard deviations, without using second- or higher-order perturbation methods (see, for example, Schmitt-Grohe and Uribe, 2004, Devereux and Sutherland, 2011, Gertler, Kiyotaki, and Queralto, 2012 and de Groot, 2015). Thus we are able to study the link between credit shocks and volatility in the steady states as well as over the business cycle.

Our main findings are as follows. First, in the steady state, the firm default probability rises with firms' leverage ratio and the level of uncertainty in the economy (measured by the standard deviation of idiosyncratic and common technology shocks). As firms become more leveraged, consumption, output and capital decline, despite a rise in the level of loans, since a larger proportion of the loans become non-performing. The level of output, loans and consumption also decline with increased uncertainty.

Second, we are able to generate theoretical impulse responses to a credit shock that are in line with empirical results on the responses of output and short-term interest rates to a U.S. credit shock. A positive credit shock can be viewed as a sudden increase in the level of bank loans relative to bank deposits, possibly due to an increase in liquidity provision by the banking sector. The rise in the level of loan-to-deposit ratio leads to an increase in the available capital in the economy and, consequently, an expansion in investment and output, which is largely consistent with the empirical results in Xu (2012). The increase in the level of loanable funds also drives down the loan rate and narrows the spread between the loan rate and deposit rate. Labour hours rise on impact, which leads to higher household incomes and consequently more consumption. Our

[1] Equilibrium default models have existed in the sovereign debt literature, including Arellano (2008), Sandleris (2008) and Yue (2010), while Chatterjee, Corbae, Nakajima, and Ros-Rull (2007) studied default in the context of unsecured household debt. The focus of our paper differs, since we are interested in examining the impact of equilibrium default in the firm sector, rather than the sovereign or household sectors.

calibrated results show that the impact of a credit shock on output and consumption can be quite large, with the speed of convergence to equilibrium typically slow (more than 20 quarters). This finding is consistent with empirical studies of the output effect of financial crises, which suggest that recessions associated with financial crises have been more severe and longer lasting than recessions associated with other shocks (see, for example, IMF, World Economic Outlook, April 2009, Chapter 3 and Jorda, Schularick, and Taylor, 2013). The prolonged impact of the credit shock also reflects the high persistence in the loan-to-deposit ratio that we observe empirically.

The rest of the paper is organized as follows. Section 2 provides a brief review of the relevant literature. Section 3 presents the DSGE model of credit, leverage and default. Section 4 sets out the first-order equilibrium conditions, derives the steady states, and describes the solution of the model. Section 5 discusses the parameterizations of the model for the calibration exercises. Section 6 provides the key results on the steady state values and impulse responses of positive credit supply shocks. Section 7 offers some concluding remarks.

2 A Selected Review of the Literature

In this section, we provide a brief account of the macroeconomic literature that are closely related to the contributions of our paper. We start by reviewing the key papers that analyze firm defaults and financial intermediation. We then examine some related papers that model credit supply shocks and the banking sector.

On modelling firm defaults, it is normal to distinguish betweem microeconomic or idiosyncratic risks which can be diversified, and macroeconomic or systematic risks which cannot. Banks generally have to deal with both types of risks. Freixas and Rochet (2008) argue that defining and measuring credit risk is equivalent to determining how the market evaluates the probability of default by a particular borrower, taking into account all possibilities of diversification and hedging provided through financial intermediation.

Our modeling framework for firm defaults and financial intermediation are closest to BGG, Christiano, Ilut, Motto, and Rostagno (2010) and Fiore and Tristani (2013). In BGG, entrepreneurial loans are risky and returns on the underlying investments are subject to idiosyncratic and common shocks. A sufficiently unfavorable shock can lead to bankruptcy. The idiosyncratic shock is observed by the entrepreneur, but not by the bank, which, as in Townsend (1979), must pay a fixed monitoring cost to observe the entrepreneurs' realized return. To mitigate problems stemming from this source of asymmetric information, entrepreneurs and the bank sign a standard debt contract. Under this contract, the entrepreneur commits to paying back the loan principal plus an interest charge, unless it declares default. In case of default, the bank conducts a costly state verification of the residual value of the entrepreneur's assets and seizes the assets as partial compensation. A similar approach is also followed by Christiano, Ilut, Motto, and Rostagno (2010) who also allow for time-variations in the process generating the shocks. Our paper differs from BGG and Christiano, Ilut, Motto, and Rostagno (2010) in two main aspects. First, instead of the return on capital, idiosyncratic shocks affect productivity which keeps the model tractable and allows us to establish a direct link between technology shocks and probability of default. Second, in addition to the banking sector, households also bear part of the default risk through their equity investments in the firms after the arrival of adverse shocks, which generates a reasonable spread between the loan and deposit rates. Fiore and Tristani (2013) model firm defaults by introducing idiosyncratic shocks to the production function, which includes labor input and abstracts from the modeling of capital accumulation. The default threshold depends on the realization of the idiosyncratic productivity shock. Our modeling approach differs from Fiore and Tristani (2013) in that the default threshold is not only dependent on the idiosyncratic productivity shock, but also a function of the economy-wide aggregate technology shock, which links firm default to the level of uncertainty in the macroeconomy. In addition, the default probability in our model is related to the leverage ratio of firms, which affects the relation between external finance and default probability in the steady state.

Our modeling framework also differs from models of financial frictions that build on lending subject to collateral constraints. Examples of this line of research are given by Kiyotaki and Moore (1997), Carlstrom, Fuerst, and Paustian (2010), and Gertler and Kiyotaki (2010). Credit constraints arise because lenders cannot force borrowers to repay their debts unless the debts are secured by some form of collateral. Borrowers' credit limits are affected by the prices of the collateralized assets. These asset prices are in turn influenced by the size of the credit limits, which affects investment and demand for assets in the economy. The dynamic interaction between borrowing limits and the prices of assets amplifies the impact of a small initial shock and generates large and persistent fluctuations in output and asset prices in the economy. In these studies, collateral constraints are introduced as a way of ensuring that borrowers can re-pay their debts, which in most cases rule out firm defaults as realized outcomes.

On modeling credit shocks and the banking sector, our paper is related to the literature on financial frictions on the supply side of credit markets. In Meh and Moran (2010), the "bank capital channel" operates as depositors demand banks to invest their own net worth (bank capital) in the financing of entrepreneurial projects due to moral hazard. The capital position of banks therefore affects their ability to attract loanable funds, which could constrain the supply of credit and lead to a fall in investment and economic activity.[2] Several papers argue that imperfect competition in the banking sector, or the rate-setting strategies of banks, contribute to frictions on the supply side of credit markets, which are also important in explaining macroeconomic fluctuations, see for example, Gerali, Neri, Sessa, and Signoretti (2010) and Hulsewig, Mayer, and Wollmershaeuser (2006) (cost channel of monetary policy).[3] The models on the supply side of credit markets have largely abstracted from modeling default risks. Given that the focus of the paper is to model firms defaults and financial intermediation in a tractable manner, we adopt a more stylized approach to modeling the banking sector and credit risks, in part, following the balance sheet structure outlined in Freixas and Rochet (2008).[4]

[2] Papers that use a similar approach include Chen (2001) and Aikman and Paustian (2006).

[3] The "cost channels" of monetary policy capture the impact of interest rates and credit conditions on the short-run ability of firms to produce (by investing in working capital). See, for example, Barth and Ramey (2002), Christiano, Eichenbaum, and Evans (2005), Chowdhury, Hoffmann, and Schabert (2006), Ravenna and Walsh (2006) and Llosa and Tuesta (2009).

[4] More recently, a number of recent work also develop quantitative models to explore the effects of unconventional monetary policy instruments, such as direct lending by central banks, to capture the policy responses following the financial crisis of 2007-2009. See, for example, Christiano, Ilut, Motto, and Rostagno (2010), Gertler and Kiyotaki (2010), Curdia and Woodford (2011), Gertler and Karadi (2011, 2013) and Ellison and Tischbirek (2014). For a detailed literature review on monetary policy and financial intermediation, see for example, Beck, Colciago, and Pfajfar (2014).

3 A Model of Credit and Default

We consider an economy comprising a large number of firms, one representative household and a competitive banking sector characterized by one representative bank. Firms are identical *ex ante* and operate over a single period. At the beginning of each period, firms enter the market and decide the optimal levels of labour and capital inputs, *before* the technology and credit shocks are realized. The capital investment is financed by borrowing from the banking sector, as well as a capital injection from the household at the start of the period. The funds invested by the household can be viewed as "private equity". Technology shocks then arrive and firms combine technology with capital and labour to produce a single output. Firms may default if the technology shock is unfavorable, such that the firm's revenue is insufficient to repay its debt (principal and interest charges) to the banking sector. We assume that the product market is fully competitive and, while some firms may fail each period, entry is free. The representative household consumes, receives interest on its deposits held with the banks, wage payment for its labour services, and an *ex post* lump-sum transfer (that could be negative) from firms at the *end* of the period.[5] The banking sector takes deposits from the household at the beginning of the period, before the realization of a credit shock to the bank's balance sheet that affects the supply of loanable funds available to the firms. The banking sector receives interest and loan repayments from non-defaulted firms at the end of the period and seizes the revenue (if any) of defaulted firms to partially cover its losses.

3.1 The household sector

For the household decision, we consider the following standard optimization problem:

$$\max_{\{C_{t+j},N_{t+j},j=0,1,2...\}} E\left[\sum_{j=0}^{\infty}\beta^j U(C_{t+j},N_{t+j})|\Omega_{ct}\right], \tag{3.1}$$

subject to the budget constraint

$$D_{t+1} = (1+r_{dt})D_t + W_t N_t - C_t - S_t + \Pi_{th}, \tag{3.2}$$

where $U(C_t, N_t)$ is the one-period (instantaneous) utility function, C_t is the real consumption expenditure, N_t is labour hours and W_t is the real wage rate paid for household labour. D_t is the household's holding of real deposits with the banking sector at the beginning of time t, r_{dt} is the real return on deposits in period t, which is known at time t. S_t is household's real equity investment (private equity) in the firms at the beginning of time t, and Π_{th} is the household's lump-sum transfer from firms, realized at the end of period t.[6] Finally, β is the discount factor, where $0 < \beta < 1$, and $E(\cdot|\Omega_{ct})$ denotes the mathematical conditional expectations operator with respect to the non-decreasing information set Ω_{ct}, to be defined later. Note that we abstract from the endogenous determination of equity holdings for the household sector to keep the model tractable and assume that the household supplies an amount of equity that is determined by an exogenous

[5] Non-defaulted firms transfer any excess profits to the household sector. The transfer from defaulted firms can be negative, depending on the realization of technology shocks. Resource transfers and default settlements will be discussed in detail later.

[6] The implicit rate of return on the household's private-equity investment is given by $\Pi_{tc}/S_t - 1$.

leverage factor. As we shall see later, it is important to consider equity finance in addition to debt finance in this model; otherwise, we shall end up with excessively wide interest rate spreads and unexpectedly high default probability.

We adopt the following specification of the utility function, popularized by Greenwood, Hercowitz, and Huffman (1988, p.10),

$$U(C_t, N_t) = \frac{1}{1-\gamma}\left[\left(C_t - \frac{\chi_0}{1+\chi}N_t^{1+\chi}\right)^{1-\gamma} - 1\right], \tag{3.3}$$

where $\gamma > 0$ is the coefficient of relative risk aversion and $1/\chi$ corresponds to the intertemporal elasticity of substitution in labour supply, $\chi > 0$. Following Christiano, Eichenbaum, and Evans (1997, p.1221), we have introduced a scaling parameter $\chi_0 > 0$ in (3.3), which is calibrated with other model parameters.[7] One important property of this form of utility function is that the marginal rate of substitution between consumption and labour effort depends only on labour input. Technically, this makes it easy to solve for N_t, given the real wage:

$$-\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} = \chi_0 N_t^{\chi},$$

so that labour effort is determined independently of the inter-temporal consumption–savings choice. As we shall see later, this function implies a labour-supply schedule that depends on the real wage only and not on consumption.

The information set available to the household sector at the beginning of period t, Ω_{ct}, can be decomposed into a common component Ψ_{t-1}, and a private component Θ_{ct}, which is composed of information that is known only to the consumer at time t (but not necessarily to all the other agents), $\Omega_{ct} = \Psi_{t-1} \cup \Theta_{ct}$, where $\Theta_{ct} = \{\Pi_{th}, \Pi_{t-1,h}, ...; S_t, S_{t-1}, ...; C_t; D_{t+1}, D_t; W_t; N_t; r_{dt}\}$. The common information set Ψ_{t-1} is publicly available and will be specified later.

The solution to the consumer's optimization problem is obtained using the first-order conditions with respect to C_t, D_{t+1} and N_t. Specifically, we end up with (3.2) and the following equations:

$$E\left[\beta\left(\frac{C_{t+1} - \frac{\chi_0}{1+\chi}N_{t+1}^{1+\chi}}{C_t - \frac{\chi_0}{1+\chi}N_t^{1+\chi}}\right)^{-\gamma}(1 + r_{d,t+1})|\Omega_{ct}\right] = 1, \tag{3.4}$$

$$W_t = \chi_0 N_t^{\chi}. \tag{3.5}$$

3.2 Firms

3.2.1 Firms' optimization problem

Each firm i is endowed with the following production technology,

$$Y_{it} = Z_{it}^{\varphi} N_{it}^{1-a} K_{it}^{\alpha}, \text{ for } i = 1, 2, ..., m, \tag{3.6}$$

where K_{it} and N_{it} are capital and labour inputs, respectively, for firm i in period t; Y_{it} is output for firm i in period t; α is the share of capital; and φ is a constant to be determined subsequently.

[7] For other examples of this form of utility function, see Meng and Velasco (2003) and Chapter 3 of Heer and Maussner (2005).

The technology variable, Z_{it}, is decomposed into an idiosyncratic component, Λ_{it}, and a common business-cycle component, A_t; that is,

$$Z_{it} = \Lambda_{it} A_t. \tag{3.7}$$

It is further assumed that

$$A_t = A_{t-1} \exp(\mu + u_t), \tag{3.8}$$

where u_t is a serially correlated common technology shock that follows the first-order autoregressive process,

$$u_t = \rho_u u_{t-1} + \varepsilon_t, \qquad \text{where } \varepsilon_t \sim \mathbb{N}(0, \sigma_\varepsilon^2), \text{ and } |\rho_u| < 1. \tag{3.9}$$

The degree of serial correlation in the common technology shock, u_t, is determined by the autoregressive parameter, ρ_u.

Let $a_t = \ln A_t$, then the business-cycle component of the technology shock can be written as

$$a_t = a_{t-1} + \mu + \rho_u u_{t-1} + \varepsilon_t. \tag{3.10}$$

Also let $\lambda_{it} = \ln \Lambda_{it}$, and assume that λ_{it} is serially uncorrelated and independently and identically distributed across firms, $\lambda_{it} \sim iid(0, \sigma_\lambda^2)$. Without loss of generality we also assume that ε_t and λ_{it} are independently distributed. Thus, $z_{it} = \ln Z_{it} = \lambda_{it} + a_t$, can be viewed as a single factor model where the common factor, a_t, is assumed to follow a unit root process. In this sense the specification of technology is quite general and encompasses many other specifications entertained in the theoretical macroeconomic literature.

Firms decide on capital and labour inputs *before* the arrival of the technology shock, Z_{it}. Further, part of the capital is financed through the equity investment from the household sector at the beginning of each period, denoted by S_{it}, and the rest is borrowed from the banking sector (external finance), $L_{it} \geq 0$; therefore,

$$K_{it} = L_{it} + S_{it}. \tag{3.11}$$

The consumer's contribution to capital acquisition can be viewed as private-equity investment with possible gains/losses to be settled at the end of the period, once the shocks are realized. Note that we assume that firms are owned by the household. From the household's viewpoint, the leverage ratio of firm i is given by $\upsilon_i = K_{it}/S_{it}$, and equation (3.11) implies that $\upsilon_i \geq 1$ for non-negative L_{it}. The share of capital financed by the banking sector is then given by

$$L_{it} = \left(\frac{\upsilon_i - 1}{\upsilon_i} \right) K_{it}. \tag{3.12}$$

We assume that the leverage ratio of the firm is exogenously given and is time-invariant in this version of the paper. It is easy to allow for time variation in υ_i, as long as it is assumed exogenous. An endogenous formulation of the leverage ratio is also of interest but will not be attempted here, since it falls outside the scope of the present paper.

Note that banks cannot observe the idiosyncratic technology shocks Λ_{it}, and as a result firms are treated the same *ex ante* and receive an equal amount, L_{it}, from the banking sector. We also assume that the technology shock, Z_{it}, is not known to firm i when choosing the optimal level of labour and capital. The sequence of events is as follows: firms enter at the beginning of each period t, with commitment from the household regarding private-equity finance, borrow from the banking sector and acquire capital; then technology shocks arrive, and firms produce, sell output and pay wages to the households. Firms either default or do not default, depending on the size of the technology shocks, which we will discuss in detail later.

Having the firms acquire their entire capital stock, K_{it}, at the beginning of each period t (together with the assumption of full depreciation of capital) is a modelling device to ensure that firms are identical *ex ante* in each period t. It is also assumed that firms transfer any excess profits to the household sector, so that a favorable technology shock to firm i at time $t-1$ does not make firm i better off at the beginning of time t, relative to the other firms. The one-period nature of the firms' problem enables us to model firm defaults in a tractable manner.

For each firm i, K_{it} and N_{it} are derived by solving the following optimization problem:

$$\max_{\{K_{i,t+s},N_{i,t+s},,s=0,1,2...\}} E\left(\sum_{s=0}^{\infty} m_{t+s}\Pi_{f,i,t+s}|\Omega_{f,it}\right), \tag{3.13}$$

where $\Omega_{f,it}$ is the information set available to firm i at the beginning of time t, m_{t+s} is the stochastic discount factor. It is assumed that the representative household owns the firms.[8] The i^{th} firm's profit function, $\Pi_{f,it}$, is given by

$$\Pi_{f,it} = Y_{it} - W_t N_{it} - (1 + r_{kt})K_{it},$$

where r_{kt} is the real interest rate on capital in period t, which is known to the firm at the beginning of time t and output Y_{it} is given by equation (3.6).

We decompose the information set of firm i at the beginning of period t, $\Omega_{f,it}$ into the common component, Ψ_{t-1}, and a private (or firm-specific) component $\Theta_{f,it}$. For each firm i, namely $\Omega_{f,it} = \Psi_{t-1} \cup \Theta_{f,it}$, where $\Theta_{f,it}$ is given by

$$\begin{aligned}\Theta_{f,it} \;=\; & \{\Lambda_{it-1}, \Lambda_{it-2}, ...; Y_{i,t-1}, Y_{i,t-2}, ...; K_{it}, K_{i,t-1}, ...;\\ & N_{it}, N_{i,t-1}, ...; L_{it}, L_{i,t-1}, ...; W_t; r_{lt}; r_{kt}; \upsilon_i\}.\end{aligned}$$

The first-order conditions for firm i's optimization problem yield the optimal levels of capital and labour inputs and are given by

$$E\left[\alpha\left(\Lambda_{it}A_t\right)^{\varphi}\left(\frac{N_{it}}{K_{it}}\right)^{1-\alpha}|\Omega_{f,it}\right] = 1 + r_{kt}, \tag{3.14}$$

$$E\left[(1-\alpha)\left(\Lambda_{it}A_t\right)^{\varphi}\left(\frac{K_{it}}{N_{it}}\right)^{\alpha}|\Omega_{f,it}\right] = W_t. \tag{3.15}$$

These equations state that the expected marginal products of capital and labour are equal to the return on capital and the wage rate, respectively.

[8] The stochastic discount factor associated with the household utility function is given by $m_{t+s} = \beta^s \frac{U_C(C_{t+s},N_{t+s})}{U_C(C_t,N_t)}$.

Under our assumption that λ_{it} and a_t are distributed independently we have

$$\begin{aligned} E(\Lambda_{it}^{\varphi} A_t^{\varphi} | \Omega_{f,it}) &= E(e^{\varphi \lambda_{it}} | \Omega_{f,it}) E(e^{\varphi a_t} | \Omega_{f,it}) \\ &= M_\lambda(\varphi) M_\varepsilon(\varphi), \end{aligned}$$

where $M_\lambda(\varphi) = E(e^{\varphi \lambda_{it}} | \Omega_{f,it})$ and $M_\varepsilon(\varphi) = E(e^{\varphi \varepsilon_t} | \Omega_{f,it})$ are the moment-generating functions of λ and ε, respectively. Using equations (3.10) and (3.14), and recalling that firms are assumed to be identical *ex ante*, and N_{it} and K_{it} are chosen before the realization of technology shocks, it is easily seen that

$$\frac{K_{it}}{N_{it}} = \left(\frac{\alpha M_\lambda M_\varepsilon}{1 + r_{kt}} \right)^{\frac{1}{1-\alpha}} \exp \left[\frac{\varphi(a_{t-1} + \mu + \rho_u u_{t-1})}{1-\alpha} \right], \qquad \forall\ i, \tag{3.16}$$

where $M_\lambda = M_\lambda(\varphi)$ and $M_\varepsilon = M_\varepsilon(\varphi)$, for short. In equilibrium we must have

$$K_{it} = K_t,\ N_{it} = N_t,\ L_{it} = L_t,\ \upsilon_i = \upsilon \qquad \forall\ i, \tag{3.17}$$

where $K_t = m^{-1} \Sigma_{i=1}^m K_{it}$, $L_t = m^{-1} \Sigma_{i=1}^m L_{it}$ and $N_t = m^{-1} \Sigma_{i=1}^m N_{it}$.

To determine the optimal level of capital and labour, respectively, note that equation (3.5) in the household optimization problem and the ratio between the first-order conditions (3.14) and (3.15) imply that

$$K_t = \frac{\alpha \chi_0}{1-\alpha} \cdot \frac{N_t^{1+\chi}}{1 + r_{kt}}. \tag{3.18}$$

Using equations (3.16), (3.17) and (3.18), we also derive the following expression for optimal labour hours[9]

$$N_t = \left[\frac{1-\alpha}{\alpha \chi_0} \left(\alpha M_\lambda M_\varepsilon \right)^{\frac{1}{1-\alpha}} (1 + r_{kt})^{-\frac{\alpha}{1-\alpha}} \right]^{\frac{1}{\chi}} \exp \left[\frac{\varphi(a_{t-1} + \mu + \rho_u u_{t-1})}{\chi(1-\alpha)} \right]. \tag{3.19}$$

3.2.2 Default conditions and default probabilities

We allow for the possibility of firm defaults and examine the consequence of default on the macro economy. Firm i is expected to default if the technology shock to the i^{th} firm is sufficiently unfavorable, such that the value of the firm after wage payments, $Y_{it} - W_t N_{it}$, falls below a threshold value determined by its callable liabilities, which we take as the repayment of loan $R_{lt} L_{it}$, where $R_{lt} = 1 + r_{lt}$. See, for example, Merton (1974), and Pesaran, Schuermann, Treutler, and Weiner (2006). Our set up avoids the need for collateral or monitoring by banks since all firms are *ex ante* identical and the bank relies on the diversification of idiosyncratic shocks across firms as a form of insurance. The default condition is such that firm i defaults if and only if

$$Y_{it} - W_t N_{it} < R_{lt} L_{it}. \tag{3.20}$$

To determine the probability of default, we let $\zeta_{it} = \lambda_{it} + \varepsilon_t$, and note that by assumption,

[9] It is assumed that the rate of return on capital is identical to the rate of return on loans, $r_{kt} = r_{lt}$. A wedge can be introduced between the two rates of returns by introducing information asymmetries and monitoring costs. However, to keep the analysis simple and tractable, we abstract from these complications.

$\zeta_{it} \sim iid(0, \sigma_\zeta^2)$, where $\sigma_\zeta^2 = \sigma_\varepsilon^2 + \sigma_\lambda^2$, and ζ_{it} has the following moment-generating function

$$M_\zeta = M_\zeta(\varphi) = E(e^{\varphi \zeta_{it}} | \Omega_{f,it}) = M_\lambda M_\varepsilon.$$

Equations (3.5), (3.6), (3.12), (3.14), (3.17) and (3.18) imply that firm i defaults if and only if

$$M_\zeta^{-1} e^{\varphi \zeta_{it}} < (1 - \frac{\alpha}{\upsilon}). \tag{3.21}$$

Since $M_\zeta^{-1} e^{\varphi \zeta_{it}} > 0$, then default can occur only if $(1 - \frac{\alpha}{\upsilon}) > 0$, namely if leverage ratio, υ, is larger than α. Assuming $\upsilon > \alpha$, the default condition can be written as

$$\zeta_{it} < \frac{\ln\left(1 - \frac{\alpha}{\upsilon}\right) + \ln M_\zeta}{\varphi} \equiv \varpi_1, \tag{3.22}$$

with the default indicator, d_{it}, defined as

$$d_{it} = I(\zeta_{it} < \varpi_1), \tag{3.23}$$

where $I(A)$ takes the value of unity if A holds or zero otherwise.

Default occurs if $\upsilon > \alpha$, and the combined technology shock (idiosyncratic and common) falls below a certain threshold ϖ_1, defined in (3.22), which is common to all firms.

The probability of default depends on the probability distribution of ζ_{it}. Under the assumption that the shocks are normally distributed, we have

$$M_\varepsilon = \exp\left(\frac{\varphi^2 \sigma_\varepsilon^2}{2}\right), \quad M_\lambda = \exp\left(\frac{\varphi^2 \sigma_\lambda^2}{2}\right), \quad M_\zeta = \exp\left(\frac{\varphi^2 \sigma_\zeta^2}{2}\right), \tag{3.24}$$

and the default probability is given by

$$\kappa = P(\zeta_{it} < \varpi_1 | \Omega_{f,it}) = \Phi\left[\frac{\ln\left(1 - \frac{\alpha}{\upsilon}\right)}{\varphi \sigma_\zeta} + \frac{\varphi \sigma_\zeta}{2}\right], \tag{3.25}$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal.[10] Under our assumptions, the probability of default κ is time-invariant, but it is clear that time variation in κ can be allowed for by introducing time variation in the volatility of technology shocks. This is in line with the recent literature by Bloom (2009). The economy-wide default probability depends on the following deep structural parameters in the model: α, the share of capital; υ, the leverage ratio of firms; φ, the exponent of technological process; and σ_ζ, which depends on σ_ε and σ_λ, the standard deviation of common and idiosyncratic technology shocks, respectively.

The partial derivatives of κ, with respect to the firm's leverage factor υ and the standard

[10]The moment-generating function of a random variable X is defined as $M_X(t) = \mathbf{E}(e^{tX}), \quad t \in \mathbb{R}$, wherever this expectation exists. For a log-normal distribution where $\ln x \sim N(\mu, \sigma^2)$, all moments exist and $E(x) = e^{\mu + \frac{\sigma^2}{2}}$.

deviation of the combined technology shocks σ_ζ are given by

$$\frac{\partial\kappa}{\partial\upsilon} = \frac{\alpha}{\varphi\sigma_\zeta\upsilon(\upsilon-\alpha)}\cdot\boldsymbol{\phi}\left[\frac{\ln\left(1-\frac{\alpha}{\upsilon}\right)}{\varphi\sigma_\zeta}+\frac{\varphi\sigma_\zeta}{2}\right], \tag{3.26}$$

$$\frac{\partial\kappa}{\partial\sigma_\zeta} = \left[-\frac{\ln\left(1-\frac{\alpha}{\upsilon}\right)}{\varphi\sigma_\zeta^2}+\frac{\varphi}{2}\right]\cdot\boldsymbol{\phi}\left[\frac{\ln\left(1-\frac{\alpha}{\upsilon}\right)}{\varphi\sigma_\zeta}+\frac{\varphi\sigma_\zeta}{2}\right]. \tag{3.27}$$

It is easily seen that $\frac{\partial\kappa}{\partial\upsilon} > 0$ and $\frac{\partial\kappa}{\partial\sigma_\zeta} > 0$, $\upsilon > \alpha > 0$, $\varphi > 0$ and $\sigma_\zeta > 0$. Note also that $\ln\left(1-\frac{\alpha}{\upsilon}\right) < 0$. Therefore, the default probability rises with υ, as firms become more leveraged and more dependent on bank finance; and it rises with the volatility of combined technology shocks, σ_ζ, which is to be expected.

3.2.3 Financial implications of defaults

Two outcomes can arise after the realization of technology shocks.

Outcome 1: Firm i does not default ($Y_{it} - W_tN_{it} - R_{lt}L_{it} \geqslant 0$). As shown above, this outcome occurs with probability of $1-\kappa$, and the bank is repaid the principal and interest on the loan, $R_{lt}L_{it}$, and the household receives a non-negative transfer from the firm after wage payment. More specifically, the profit share of the household, $\Pi_{i,th}$, and the share of the bank are given by

$$\begin{aligned}\Pi_{i,th} &= Y_{it} - W_tN_{it} - R_{lt}L_{it}, \text{ with probability } 1-\kappa,\\ \Pi_{i,tb} &= R_{lt}L_{it}, \qquad\qquad \text{with probability } 1-\kappa.\end{aligned}$$

Outcome 2: Firm i defaults ($Y_{it} - W_tN_{it} - R_{lt}L_{it} < 0$). When firm i defaults, it is unable to repay the loan, $R_{lt}L_{it}$, to the banking sector. The bank instead seizes the revenue of the defaulted firm after wage payments, if this value $(Y_{it} - W_tN_{it})$ is positive; otherwise, the bank gets no payment. The household bears the rest of default losses and receives a zero or negative transfer from the firm after wage payments. More specifically,

$$\begin{aligned}\Pi_{i,th} &= Min(0, Y_{it} - W_tN_{it}), \text{ with probability } \kappa,\\ \Pi_{i,tb} &= Max(0, Y_{it} - W_tN_{it}), \text{ with probability } \kappa.\end{aligned}$$

Depending on the realization of technology shocks, there are two subcases to distinguish under the default outcome, depending on whetehr $Y_{it} - W_tN_{it} < 0$, or not. Using equations (3.5), (3.6), (3.14), (3.17) and (3.18), $Y_{it} - W_tN_{it} < 0$ if and only if

$$\frac{e^{\varphi\zeta_{it}}}{M_\zeta} < (1-\alpha). \tag{3.28}$$

But using (3.24) and noting that $\varphi > 0$, we also have

$$\zeta_{it} < \frac{\ln(1-\alpha)}{\varphi} + \frac{\varphi\sigma_\zeta^2}{2} \equiv \varpi_2.$$

Therefore, the probability that the household receives a negative transfer is given by

$$\tau = P(\zeta_{it} < \varpi_2 | \Omega_{f,it}) = \Phi \left[\frac{\ln(1-\alpha)}{\varphi \sigma_\zeta} + \frac{\varphi \sigma_\zeta}{2} \right]. \quad (3.29)$$

The household and the bank, respectively, receive a "negative" and zero transfer from the firms, namely in this case $\Pi_{i,th} = Y_{it} - W_t N_{it} < 0$ and $\Pi_{i,tb} = 0$.

In the second sub-case, firm i defaults and the revenue generated is sufficient to cover wage payments, $(0 < Y_{it} - W_t N_{it} < R_{lt} L_{it})$. This scenario arises if and only if the technology shock, ζ_{it}, lies in the range $\varpi_2 < \zeta_{it} < \varpi_1$, which occurs with probability of $\kappa - \tau$. The household receives zero transfer after the wage payment and the bank seizes the revenue of the defaulted firm after wage payments. In this sub-case $\Pi_{i,th} = 0$ and $\Pi_{i,tb} = Y_{it} - W_t N_{it} > 0$.

3.2.4 Aggregate outcomes

For a given number of sufficiently large firms, the aggregate output defined by

$$Y_t = m^{-1} \sum_{i=1}^{m} Y_{it} = \left(m^{-1} \sum_{i=1}^{m} e^{\varphi \lambda_{it}} \right) A_t^{\varphi} N_t^{1-\alpha} K_t^{\alpha}.$$

But since λ_{it} is assumed to be identically and independently distributed and $E(e^{\varphi \lambda_{it}})$ exists, then according to the law of large numbers we have

$$\frac{\sum_{i=1}^{m} e^{\varphi \lambda_{it}}}{m} \xrightarrow{p} E_c(e^{\varphi \lambda_{it}}) = M_\lambda(\varphi) = M_\lambda,$$

where $E_c(e^{\varphi \lambda_{it}})$ is the cross-sectional expectation of $e^{\varphi \lambda_{it}}$. Therefore, aggregate output is given by

$$Y_t = M_\lambda A_t^{\varphi} N_t^{1-\alpha} K_t^{\alpha}. \quad (3.30)$$

Recall that the household and the bank receive a transfer from the firms after production, the amount of which depends on the realization of technology shocks. Denote the average transfer to the household by Π_{th} and to the bank by Π_{tb}, then

$$Y_t - W_t N_t = \Pi_{th} + \Pi_{tb}. \quad (3.31)$$

Note alos that Π_{th} and Π_{tb} are obtained by avergaing the transfers (if any) across the firms comprised of defaulted and non-defaulted firms:

$$\Pi_{th} = \frac{\sum_{i=1}^{m}(1-d_{it})(Y_{it} - W_t N_{it} - R_{lt} L_{it}) + \sum_{i=1}^{m} d_{it} Min(0, Y_{it} - W_t N_{it})}{m}, \quad (3.32)$$

$$\Pi_{tb} = \frac{\sum_{i=1}^{m} d_{it} Max(0, Y_{it} - W_t N_{it}) + \sum_{i=1}^{m}(1-d_{it}) L_{it} R_{lt}}{m}. \quad (3.33)$$

We evaluate Π_{th} in equation (3.32) by first noting that $Min(0, Y_{it} - W_t N_{it})$ can be written in terms of the following indicator function:

$$\begin{aligned} & Min(0, Y_{it} - W_t N_{it}) \\ = \; & I(\zeta_{it} < \varpi_2) \cdot (Y_{it} - W_t N_{it}) + I(\varpi_2 < \zeta_{it} < \varpi_1) \cdot 0. \end{aligned}$$

Recall that $d_{it} = I(\zeta_{it} < \varpi_1)$, therefore

$$d_{it} Min(0, Y_{it} - W_t N_{it}) = I(\zeta_{it} < \varpi_2) \cdot (Y_{it} - W_t N_{it}).$$

Hence,

$$\frac{\sum_{i=1}^m d_{it}}{m} \xrightarrow{p} E_c(d_{it}) = \kappa \quad \forall\ i \text{ and } t,$$

where κ is the probability of default.

The average output of the defaulted firms can be expressed as

$$\frac{\sum_{i=1}^m d_{it} Y_{it}}{m} = \frac{\sum_{i=1}^m d_{it} e^{\varphi \lambda_{it}}}{m} A_t^{\varphi} N_t^{1-\alpha} K_t^{\alpha}.$$

Note also that $d_{it} = I(\lambda_{it} + \varepsilon_t < \varpi_1) = I(\lambda_{it} < \varpi_1 - \varepsilon_t)$, and

$$\frac{\sum_{i=1}^m e^{\varphi \lambda_{it}} I(\lambda_{it} < \varpi_1 - \varepsilon_t)}{m} \xrightarrow{p} \int_{-\infty}^{\varpi_1 - \varepsilon_t} e^{\varphi x} f_{\lambda}(x) dx, \text{ as } m \to \infty,$$

where $f_{\lambda}(x)$ is the probability density function of λ_{it}.

Lemma 1 *In the case where $\lambda_{it}/\sigma_{\lambda} \sim N(0,1)$, and hence $f_{\lambda}(x) = \phi(x/\sigma_{\lambda})$ is the normal density, we have*

$$\int_{-\infty}^{\varpi_1 - \varepsilon_t} e^{\varphi x} f_{\lambda}(x) dx = M_{\lambda} \varsigma_1(\varepsilon_t),$$

where M_{λ} is given by (3.24) and

$$\varsigma_1(\varepsilon_t) = \Phi\left(\frac{\varpi_1 - \varepsilon_t - \sigma_{\lambda}^2 \varphi}{\sigma_{\lambda}}\right). \tag{3.34}$$

Proof. *See Appendix A1.* ■

Following from Lemma 1, for large m,

$$\frac{\sum_{i=1}^m d_{it} Y_{it}}{m} \xrightarrow{p} M_{\lambda} \varsigma_1(\varepsilon_t) A_t^{\varphi} N_t^{1-\alpha} K_t^{\alpha} = \varsigma_1(\varepsilon_t) Y_t.$$

Finally, using the law of large numbers and Lemma 1, it can be shown that

$$\frac{\sum_{i=1}^m I(\zeta_{it} < \varpi_2)}{m} \xrightarrow{p} \tau,$$
$$\frac{\sum_{i=1}^m I(\lambda_{it} < \varpi_2 - \varepsilon_t) Y_{it}}{m} \xrightarrow{p} \varsigma_2(\varepsilon_t) Y_t,$$

where τ is given by (3.29) and

$$\varsigma_2(\varepsilon_t) = \Phi\left(\frac{\varpi_2 - \varepsilon_t - \sigma_{\lambda}^2 \varphi}{\sigma_{\lambda}}\right). \tag{3.35}$$

The transfer of resources from firms to the household sector, Π_{th}, and the banking sector, Π_{tb}, are therefore given by

$$\Pi_{th} = [1-\varsigma_1(\varepsilon_t)]\,Y_t - (W_tN_t + R_{lt}L_t)(1-\kappa) + \varsigma_2(\varepsilon_t)Y_t - \tau W_tN_t, \quad (3.36)$$

$$\Pi_{tb} = R_{lt}L_t(1-\kappa) - (\kappa-\tau)W_tN_t + [\varsigma_1(\varepsilon_t) - \varsigma_2(\varepsilon_t)]\,Y_t, \quad (3.37)$$

where Y_t, κ, $\varsigma_1(\varepsilon_t)$ and $\varsigma_2(\varepsilon_t)$ are given in (3.30), (3.25), (3.34) and (3.35), respectively. As a check on our derivations it is easily verified that $\Pi_{th} + \Pi_{tb} = Y_t - W_tN_t$.

3.3 The banking sector

The banking sector acts as the financial intermediary between the household and the firms. It receives deposits, D_t, from the household at the beginning of time t and channels these deposits to loans, L_t, extended to the firms. We postulate the relationship between L_t and D_t as

$$L_t = \theta_t D_t, \quad (3.38)$$

where θ_t is assumed to be exogenously given. Equation (3.38) allows us to introduce shocks that originate on the supply side of credit and to study their propagation in the real economy, in a tractable manner.[11]

The credit supply shock can be attibuted to a number of factors. One interpretation of (3.38) is that the banking sector is required to deposit some reserves, B_t, with the central bank, through which the central bank is able to influence the amount of bank credit available in the economy (see, for example, the bank balance sheet in Freixas and Rochet, 2008). In this case, $L_t + B_t = D_t$, where $B_t = (1-\theta_t)D_t$. The purpose of a compulsory reserve requirement as a policy instrument can be twofold. When the economy is overheating and the level of fixed investment is high, the central bank can raise the reserve-requirement ratio $(1-\theta_t)$, to curb credit expansion and reduce the inflationary pressure in the economy, in which case the reserve requirement acts as a countercyclical policy tool. Alternatively, when lending risk is high (for example, owing to an increase in firm default probability), the central bank may raise the reserve-requirement ratio to ensure that the banking sector puts aside sufficient reserves to cushion the impact of higher bank losses due to firm defaults. Further, θ_t can be interpreted as a macroprudential policy tool, where the financial regulatory authority targets the volume of loans extended to the real economy, to dampen procyclicality in the credit cycle. In both cases, θ_t will be less than 1.

However, one could also consider the case where θ_t is greater than 1. This is possible when banks are allowed to issue securities (IOUs) that are not backed by deposits. These securities could potentially be guaranteed by the central bank in event of a bank run (not modeled in our framework). The central bank can also be a source of additional liquidity to the banking sector, as seen in the recent financial crisis. To model this possibility explicitly, one would need to introduce the price level and inflation into our framework, since the central bank's credit provision

[11] For alternative approaches of modeling credit supply shocks, see for example, Hulsewig, Mayer, and Wollmershaeuser (2006) (cost channel of monetary policy), Meh and Moran (2010) (bank capital channel), Gerali, Neri, Sessa, and Signoretti (2010) (imperfectly competitive banking sector) and Gertler and Karadi (2011) (unconventional monetary policy). Given that the focus of the paper is to examine the linkages between firm defaults, credit and volatility, we adopt a more stylized approach to modeling the banking sector and credit risks, in part, following the balance sheet structure in Freixas and Rochet (2008).

could lead to inflationary pressure in the economy.[12] Given the relatively simple and canonical characterization of the banking sector in our model, we abstract from pinpointing the exact source of the credit shock; instead, we investigate all three different scenarios where the *mean* of θ_t is less than, equal to, and greater than unity in our calibration and simulation exercises. These three scenarios are motivated by time series evidence in the US, where the historical loan-to-deposit ratio has fluctuated around one (see Section 5).

For the banking sector to be solvent, the end-of-period asset position of the banking sector must be greater than or equal to the liabilities of the banking sector. Assuming that the banking sector makes zero profit, then

$$(1 + r_{dt})D_t = \Pi_{tb}, \tag{3.39}$$

where Π_{tb} is given by equation (3.37).

The exogenous process for the loan-to-deposit ratio, θ_t, is assumed to follow

$$\ln \theta_t = \rho_\theta \ln \theta_{t-1} + \eta_t, \tag{3.40}$$

where $|\rho_\theta| < 1$, and $\eta_t \sim N(\mu_\eta, \sigma_\eta^2)$. The distribution of $\ln \theta_t$ is therefore given by $\ln \theta_t \sim N(\frac{\mu_\eta}{1-\rho_\theta}, \frac{\sigma_\eta^2}{1-\rho_\theta^2})$. Using the properties of the log-normal distribution, we have

$$E(\theta_t) = E(e^{\ln \theta_t}) = \exp\left[\frac{\mu_\eta}{1-\rho_\theta} + \frac{1}{2}\frac{\sigma_\eta^2}{(1-\rho_\theta^2)}\right] = \mu_\theta,$$

where μ_θ is the mean of the loan-to-deposit ratio. μ_η and $E(ln\theta_t)$ can be expressed in terms of μ_θ, σ_η and ρ_θ, as follows

$$\mu_\eta = (1-\rho_\theta)ln(\mu_\theta) - \frac{1}{2}\frac{\sigma_\eta^2}{1+\rho_\theta}, \tag{3.41}$$

$$E(ln\theta_t) = ln(\mu_\theta) - \frac{1}{2}\frac{\sigma_\eta^2}{1-\rho_\theta^2}. \tag{3.42}$$

Finally, for completeness, the information set for the bank Ω_{bt} can be written as $\Omega_{bt} = \Psi_{t-1} \cup \Theta_{bt}$, where Θ_{bt} contains information that is known to the bank at the beginning of time t and $\Theta_{bt} = \{\Pi_{tb}, \Pi_{t-1,b}, ...; r_{dt}; r_{lt}; L_t; D_t; \theta_t\}$. As in Binder and Pesaran (1998, 2001), Ψ_{t-1} is a common information set, containing all the publicly available information at the beginning of period t that is common to the household, firms and the bank:

$$\begin{aligned}\Psi_{t-1} = \{&C_{t-1}, C_{t-2}, ...; K_{t-1}, K_{t-2}, ...; Y_{t-1}, Y_{t-2}, ...; L_{t-1}, L_{t-2}, ...; D_{t-1}, D_{t-2}, ...;\\ &N_{t-1}, N_{t-2}, ...; W_{t-1}, W_{t-2}, ...; r_{d,t-1}, r_{d,t-2}, ...; r_{l,t-1}, r_{l,t-2}, ...; \varepsilon_{t-1}, \varepsilon_{t-2}, ...;\\ &\theta_{t-1}, \theta_{t-2}, ...\}.\end{aligned}$$

[12] In addition, θ_t could also be related to the recently introduced target on the loan-to-deposit ratio, which addresses liquidity mismatches in the banking sector, in part, due to the funding mix between retail and wholesale funding, see for example, van den End (2013).

4 Equilibrium Conditions and Long-Run Steady States

4.1 Equilibrium conditions

The complete set of equations that characterize the equilibrium conditions of the model is given by equations (3.2), (3.4), (3.5), (3.18), (3.19), (3.30), (3.31), (3.37), (3.38), (3.39) and the aggregate version of equations (3.11) and (3.12). We set out below the key equations of the complete macroeconomic framework again for convenience:

$$
\begin{aligned}
1 &= E\left[\beta\left(\frac{C_{t+1}-\frac{\chi_0}{1+\chi}N_{t+1}^{1+\chi}}{C_t-\frac{\chi_0}{1+\chi}N_t^{1+\chi}}\right)^{-\gamma} R_{d,t+1}|\Omega_{ct}\right], && (4.1)\\
W_t &= \chi_0 N_t^{\chi}, && (4.2)\\
D_{t+1} &= R_{dt}D_t + W_tN_t - C_t - S_t + \Pi_{th}, && (4.3)\\
K_t &= \frac{\alpha\chi_0}{1-\alpha}\cdot\frac{N_t^{1+\chi}}{R_{lt}}, && (4.4)\\
N_t &= \left[\frac{1-\alpha}{\alpha\chi_0}(\alpha M_\lambda M_\varepsilon)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\chi}}(R_{lt})^{-\frac{\alpha}{(1-\alpha)\chi}}\exp\left[\frac{\varphi(a_{t-1}+\mu+\rho_u u_{t-1})}{\chi(1-\alpha)}\right], && (4.5)\\
Y_t &= M_\lambda A_t^{\varphi}{N_t}^{1-\alpha}K_t^{\alpha}, && (4.6)\\
L_t &= (1-\frac{1}{\upsilon})K_t, && (4.7)\\
\Pi_{tb} &= R_{dt}D_t, && (4.8)\\
K_t &= L_t + S_t, && (4.9)\\
L_t &= \theta_t D_t, && (4.10)\\
\Pi_{tb} &= (1-\kappa)R_{lt}L_t - (\kappa-\tau)W_tN_t + \varsigma(\varepsilon_t)Y_t, && (4.11)\\
\Pi_{th} &= Y_t - W_tN_t - \Pi_{tb}, && (4.12)
\end{aligned}
$$

where

$$
\begin{aligned}
\kappa &= \Phi\left(\frac{\varpi_1}{\sigma_\zeta}\right),\quad \tau=\Phi\left(\frac{\varpi_2}{\sigma_\zeta}\right),\quad \varsigma(\varepsilon_t)=\varsigma_1(\varepsilon_t)-\varsigma_2(\varepsilon_t),\\
\varsigma_1(\varepsilon_t) &= \Phi\left(\frac{\varpi_1-\varepsilon_t-\sigma_\lambda^2\varphi}{\sigma_\lambda}\right),\quad \varsigma_2(\varepsilon_t)=\Phi\left(\frac{\varpi_2-\varepsilon_t-\sigma_\lambda^2\varphi}{\sigma_\lambda}\right),\\
\varpi_1 &= \frac{\ln\left(1-\frac{\alpha}{\upsilon}\right)}{\varphi}+\frac{\varphi\sigma_\zeta^2}{2},\quad \varpi_2=\frac{\ln(1-\alpha)}{\varphi}+\frac{\varphi\sigma_\zeta^2}{2},\\
M_\lambda &= \exp\left(\frac{\varphi^2\sigma_\lambda^2}{2}\right),\quad M_\epsilon=\exp\left(\frac{\varphi^2\sigma_\epsilon^2}{2}\right),\\
R_{lt} &= 1+r_{lt}\ \text{ and }\ R_{dt}=1+r_{dt}.
\end{aligned}
$$

There are 12 equations governing the macroeconomy, (4.1) to (4.12), in 12 endogenous variables C_t, W_t, N_t, D_t, S_t, L_t, K_t, Y_t, Π_{tb}, Π_{th}, R_{dt} and R_{lt}. The model is subject to two exogenously determined processes, the technological process, a_t, and the credit shock to θ_t, governed by

$$
\begin{aligned}
a_t &= a_{t-1}+\mu+u_t,\quad \text{where}\ \ u_t=\rho_u u_{t-1}+\varepsilon_t, && (4.13)\\
\ln\theta_t &= \rho_\theta\ln\theta_{t-1}+\eta_t. && (4.14)
\end{aligned}
$$

We combine equations (4.3), (4.8) and (4.12) to obtain the economy-wide budget constraint in our model

$$Y_t - C_t = S_t + D_{t+1}, \tag{4.15}$$

which shows that the composition of output net of consumption (savings) is in the form of 'private-equity' investment, S_t, and deposits, D_{t+1}.

From simulation exercises, we find that $\varsigma(\varepsilon_t)$ is very small for reasonable parameter values of $\alpha, \upsilon, \chi, \sigma_\varepsilon^2$ and σ_λ^2. As a result, we approximate equation (4.11) with the following expression[13]:

$$\Pi_{tb} = R_{lt}L_t(1-\kappa) - (\kappa - \tau)W_tN_t.$$

We assume that a central planner with a common information set, Ω_t, solves the system of equilibrium conditions. The common information set, Ω_t, which is known to the planner at time t is defined by

$$\Omega_t = \Psi_{t-1} \cup \Theta_{bt} \cup \Theta_{ct} \cup \left(\cup_{i=1}^{m} \Theta_{f,it}\right).$$

Note that the variables C_t, D_{t+1}, L_t, K_t, S_t, N_t, W_t, R_{dt}, R_{lt}, Y_t, Π_{th}, Π_{tb}, υ, θ_t and ε_t are included in the planner's information set Ω_t at time t.

4.2 Derivation of steady states

Since this model depicts a growing economy where the technological process, a_t, contains a unit root as well as a deterministic growth component, μ, we must scale the endogenous variables C_t, L_t, D_t, S_t, K_t, W_t, N_t and Y_t in the system of equilibrium conditions by an appropriate factor of technology, A_{t-1}, so that the transformed variables are stationary on a balanced growth path, to guarantee that the model possesses steady states. We shall also assume that the real interest rates are stochastically bounded (bounded in probability), that is, $R_{lt} = O_p(1)$ and $R_{dt} = O_p(1)$, which is in line with the empirical evidence on U.S. real interest rates series.[14]

Proposition 1 *To guarantee the existence of steady states in the economy defined by equations (4.1) to (4.12), and the processes of the exogenous variables given in equations (4.13) and (4.14), the exponent of the technology process, Z_{it}, in the firm's production function (3.6) must be restricted as*

$$\varphi = \frac{(1-\alpha)\chi}{1+\chi}. \tag{4.16}$$

Proof. *First note that (4.5) can be rewritten as*

$$N_t = \left[\frac{1-\alpha}{\alpha\chi_0}\left(\alpha M_\lambda M_\varepsilon\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\chi}} (R_{lt})^{-\frac{\alpha}{(1-\alpha)\chi}} \left[A_{t-1}^{\frac{\varphi}{(1-\alpha)\chi}}\right] \exp\left[\frac{\varphi(\mu + \rho_u u_{t-1})}{\chi(1-\alpha)}\right]. \tag{4.17}$$

Assuming that real interest rates are bounded in probability and u_t is stationary, we have

$$N_t = O_p\left[A_{t-1}^{\frac{\varphi}{(1-\alpha)\chi}}\right]. \tag{4.18}$$

[13] Under the baseline parametrization of the model, the average value of $\varsigma(\varepsilon_t)$ is very small, at 0.0062 for 100,000 simulations.

[14] "Boundedness in probability" is defined as follows: the sequence X_n is bounded in probability, written $X_n = O_p(1)$, if for every $\varepsilon > 0$ there exists $\delta(\varepsilon) \in (0, \infty)$ such that $Pr\left[|X_n| > \delta(\varepsilon)\right] < \varepsilon$ for all n.

If we scale K_t by A_{t-1}, equation (4.4) becomes

$$\frac{K_t}{A_{t-1}} = \left(\frac{\alpha\chi_0}{1-\alpha}\right)\frac{N_t^{1+\chi}A_{t-1}^{-1}}{R_{lt}} \tag{4.19}$$

Therefore, to ensure that $\frac{K_t}{A_{t-1}}$ is bounded in probability, (4.19) implies that

$$N_t = O_p\left(A_{t-1}^{\frac{1}{1+\chi}}\right). \tag{4.20}$$

But, for the results of (4.18) and (4.20) to be compatible, we must have the condition $\frac{\varphi}{(1-\alpha)\chi} = \frac{1}{1+\chi}$ or $\varphi = \frac{(1-\alpha)\chi}{1+\chi}$, as required. ■

Remark 1 *The expressions for the endogenous variables (except for the interest rates) in efficiency units are given by $\mathring{C}_t = \frac{C_t}{A_{t-1}}$, $\mathring{L}_t = \frac{L_t}{A_{t-1}}$, $\mathring{S}_t = \frac{S_t}{A_{t-1}}$, $\mathring{D}_t = \frac{D_t}{A_{t-1}}$, $\mathring{K}_t = \frac{K_t}{A_{t-1}}$, $\mathring{W}_t = \frac{W_t}{A_{t-1}^{\chi/(1+\chi)}}$, $\mathring{N}_t = \frac{N_t}{A_{t-1}^{1/(1+\chi)}}$ and $\mathring{Y}_t = \frac{Y_t}{A_{t-1}}$. In particular, equation (4.2) implies $W_t = \chi_0 N_t^{\chi} = O_p(A_{t-1}^{\frac{\chi}{1+\chi}})$, which implies that W_t must be scaled by $A_{t-1}^{\frac{\chi}{1+\chi}}$, to ensure that $\mathring{W}_t$ is $O_p(1)$.*

The growth rate of technology is given by

$$g_t = \frac{e^{a_t}}{e^{a_{t-1}}} = e^{\mu+u_t} = (1+g)e^{u_t}, \tag{4.21}$$

where $u_t = \rho u_{t-1} + \varepsilon_t$, and $e^{\mu} \equiv 1+g$. In what follows, we denote the natural logarithm of the variables with lowercase letters, for example $\mathring{c}_t = ln\mathring{C}_t$, and denote the associated steady state values by lowercase letters with an asterisk, namely $\mathring{c}^*$.

When deriving the steady states, standard macro models typically set the shocks equal to zero and, in effect, abstract from any possible non-linearities in the steady state relations of the model. One property of the standard first-order perturbation approach is that the solution displays the certainty equivalence property, where the first-order approximation to the unconditional means of endogenous variables coincides with their non-stochastic, steady-state values, and therefore, by construction, rules out a risk premium in the steady state (Schmitt-Grohe and Uribe, 2004). Here we propose an alternative method where the steady-state relations of the model are derived from unconditional expectations of the model's relations in terms of the log of the variables measured in efficiency units. In our solution approach, the steady state of log consumption in efficiency units is defined as $\mathring{c}^* = E(ln\mathring{C}_t)$, and the steady-state loan rate is given by $r_l^* = E(\ln R_{lt}) \approx E(r_{lt})$, which differ from the standard first-order perturbation approach in two aspects. First, we define steady states as the unconditional expectation of the *log* of the variables in efficiency units, rather than the variables themselves. Owing to the non-linear nature of the log operator, certainty equivalence no longer applies in the model solution, which allows us to study the role of the risk premium explicitly. Second, we consider in efficiency units all variables (except for interest rates), scaled by technology shocks, so that the transformed variables are ergodic, despite the presence of a unit root in the technology process. Using this approach and the properties of the log normal distribution, we are able to express the steady state of shocks as a function of their mean and standard deviation, therefore allowing for explicit consideration of risks in the steady state, without using second- or higher-order perturbation methods (see, for example, Schmitt-Grohe and Uribe, 2004 and Devereux and Sutherland, 2011).

The steady state of the system of equilibrium conditions is therefore given by[15]

$$E(ln\theta_t) + \mathring{d}^* = ln\chi_0 + ln\left[\frac{\alpha(1-\frac{1}{\upsilon})}{1-\alpha}\right] + (1+\chi)\mathring{n}^* - r_l^*, \tag{4.22}$$

$$\mathring{n}^* = -\frac{ln\chi_0}{\chi} + \frac{1}{\chi}ln\left[\frac{1-\alpha}{\alpha}(\alpha M_\lambda M_\varepsilon)^{\frac{1}{1-\alpha}}\right] + \frac{\mu}{1+\chi} - \frac{\alpha}{(1-\alpha)\chi}r_l^*, \tag{4.23}$$

$$\mathring{y}^* = ln\left[\frac{M_\lambda}{(1-\frac{1}{\upsilon})^\alpha}\right] + \varphi\mu + (1-\alpha)\mathring{n}^* + \alpha E(ln\theta_t) + \alpha\mathring{d}^*, \tag{4.24}$$

$$e^{\mathring{y}^*} - e^{\mathring{c}^*} = e^{\mathring{d}^*+\mu} + \frac{1}{\upsilon-1}e^{E(ln\theta_t)+\mathring{d}^*}, \tag{4.25}$$

$$r_d^* = \gamma\mu - \ln\beta, \tag{4.26}$$

$$e^{r_d^*+\mathring{d}^*} = e^{r_l^*+E(ln\theta_t)+\mathring{d}^*}(1-\kappa) - \chi_0(\kappa-\tau)e^{(1+\chi)\mathring{n}^*}, \tag{4.27}$$

where α, υ, χ, μ, γ, β and χ_0 are parameters of the model, which will be calibrated at a later stage. M_λ and M_ε are defined in (3.24), κ and τ are given by equations (3.25) and (3.29), respectively, $E(ln\theta_t)$ is given by equation (3.42), and φ must satisfy equation (4.16) in Proposition 1.

4.3 Log-liberalized system of equations

Consistent with the above derivation of steady states, we log-linearize the system of equilibrium equations around the log steady state values obtained by solving (4.22) to (4.27). We denote the log deviations from the steady state as $\tilde{\mathring{c}}_t = \mathring{c}_t - \mathring{c}^*$, where $\mathring{c}_t = \ln \mathring{C}_t$, then

$$\mathring{C}_t = e^{\mathring{c}_t} \approx e^{\mathring{c}^*} + e^{\mathring{c}^*}(\mathring{c}_t - \mathring{c}^*) = e^{\mathring{c}^*}(1+\tilde{\mathring{c}}_t).$$

Also $\widetilde{\ln g_t} = \ln g_t - E(\ln g_t)$, where using (4.21) we have $\ln g_t = \mu + u_t$. Hence, $\widetilde{\ln g_t} = u_t = \rho_u u_{t-1} + \varepsilon_t$. Similarly, for the logarithm of the loan-to-deposit ratio, $\widetilde{\ln\theta_t} = \ln\theta_t - E(ln\theta_t)$, and since $\ln\theta_t = \rho_\theta \ln\theta_{t-1} + \eta_t$, then $\widetilde{\ln\theta_t} = \rho_\theta\widetilde{\ln\theta}_{t-1} + \tilde{\eta}_t$, where $\tilde{\eta}_t = \eta_t - (1-\rho_\theta)E(ln\theta_t)$. However, using (3.42), and recalling that by assumption $\tilde{\eta}_t \sim \mathbb{N}(\mu_\eta, \sigma_\eta^2)$, it then follows readily that, $\tilde{\eta}_t \sim \mathbb{N}(0, \sigma_\eta^2)$.

The log-linearized approximation of the equilibrium conditions of the model are therefore given by[16]

$$\tilde{\mathring{c}}_t - a_1\tilde{\mathring{n}}_t = E\left(\tilde{\mathring{c}}_{t+1} - a_1\tilde{\mathring{n}}_{t+1} - a_3\tilde{r}_{d,t+1}|\Omega_{ct}\right) + a_2 u_t, \tag{4.28}$$

$$-\tilde{\mathring{c}}_t + (1+a_4+a_6)\tilde{\mathring{y}}_t - a_6\tilde{\mathring{d}}_{t+1} = a_4\tilde{\mathring{d}}_t + a_6 u_t + a_4\widetilde{\ln\theta}_t, \tag{4.29}$$

$$\tilde{\mathring{n}}_t + \frac{\alpha}{(1-\alpha)\chi}\tilde{r}_{lt} = \frac{\rho_u}{1+\chi}u_{t-1}, \tag{4.30}$$

$$\tilde{r}_{dt} - \tilde{r}_{lt} = \widetilde{ln\theta}_t, \tag{4.31}$$

$$\tilde{\mathring{y}}_t - (1+\alpha\chi)\tilde{\mathring{n}}_t + \alpha\tilde{r}_{lt} = \varphi u_t, \tag{4.32}$$

$$(1+\chi)\tilde{\mathring{n}}_t - \tilde{r}_{lt} = \tilde{\mathring{d}}_t + \widetilde{\ln\theta}_t, \tag{4.33}$$

[15]The detailed derivation of the steady-state conditions can be found in Appendix A2.

[16]Note that $E(u_t|\Omega_{ct}) = u_t$. The derivation of the log-linearized approximation of the equilibrium conditions can be found in Appendix A2.

where

$$a_1 = b_1(1+\chi),\ a_2 = 1-b_1,\ a_3 = \frac{1-b_1}{\gamma},$$
$$a_4 = \frac{1}{\upsilon-1}e^{E(ln\theta_t)+\mathring{d}^*-\mathring{c}^*},\ a_6 = e^{\mathring{d}^*+\mu-\mathring{c}^*},\ b_1 = \frac{\chi_0}{1+\chi}e^{(1+\chi)\mathring{n}^*-\mathring{c}^*}.$$

There are six equations as set out in (4.28) to (4.33) for six endogenous variables, $\widetilde{\mathring{c}}_t$, $\widetilde{\mathring{d}}_t$, $\widetilde{\mathring{y}}_t$, $\widetilde{\mathring{n}}_t$, $\widetilde{r}_{dt}$ and $\widetilde{r}_{lt}$, which are all known at time t.

4.4 Impulse responses

In our simulation exercises, we are interested in the impact of a positive credit shock on business-cycle dynamics, in particular, the movements of the interest rate spread, output, bank loans and deposits. We compute the impulse responses to a credit shock following Koop, Pesaran, and Potter (1996) and Pesaran and Shin (1998).[17]

Proposition 2 *Under the system of equilibrium conditions set out in equations (4.28) to (4.33), it follows that: a credit shock has a negative impact on the interest rate premium,* $\widetilde{r}_{lt} - \widetilde{r}_{dt}$.

Proof. *The impulse response implied by equation (4.31) can be written as*

$$GI_{r_d}(h) - GI_{r_l}(h) = GI_{ln\theta}(h), \tag{4.34}$$

where $GI_x(h)$ *denotes the impulse responses for variable* x *at time horizon* h. *It follows from equation (4.34) that the impulse response of the interest rate spread depends on the credit shock, and that a credit shock has a negative impact on the interest rate premium.* ■

Also, recalling that $L_t = \theta_t D_t$, then $\widetilde{\mathring{l}}_t = \widetilde{\ln\theta}_t + \widetilde{\mathring{d}}_t$, and the impulse response of $\widetilde{\mathring{l}}_t$ is given by

$$GI_l(h) = GI_{\ln\theta}(h) + GI_d(h-1),$$

which implies that the level of bank loans is dependent on the the credit supply shock and the level of deposits in the economy.

5 Parameter calibration

Following much of the literature, the capital share, α, is set to 0.35, the discount rate r to 1.6 per cent per annum (0.4 per cent per quarter), which gives $\beta = 0.996$. Following Greenwood, Hercowitz, and Huffman (1988), Christiano, Eichenbaum, and Evans (1997), and as is standard in business-cycle analysis, the coefficient of risk aversion, γ, is set to 1. For $1/\chi$, the intertemporal elasticity of substitution in labour supply, Greenwood, Hercowitz, and Huffman (1988) argue that, for a representative household, $1/\chi$ should summarize the variation in labour of all members of such a unit, both at the intensive and extensive margins. These authors suggest that a reasonable value of $1/\chi$ should lie in the range of 0.3 to 2.2. In our calibrated exercise, we select a mid-point

[17] See Mathematical Appedix A2.6 for detailed definition and derivation.

value in this range and set $1/\chi = 1.4$, or $\chi = 0.7$. The scaling parameter, χ_0, in (3.3) is chosen so that the steady-state value of labour hours in efficiency units is set to unity, namely $\mathring{N}^* = e^{\mathring{n}^*} = 1$.

Table 1: Model Parameters

Preference		Value	Source
γ	coefficient of risk aversion	1	Christiano *et al.* (1997)
χ	inverse of the intertemporal elasticity of substitution in labour supply	0.7	Greenwood *et al.* (1988)
β	discount factor	0.996	
Technology			
ρ_u	coefficient of autoregression in a common technology shock	0.439	U.S. data
σ_ε	standard deviation of a common technology shock	0.011	U.S. data
μ	deterministic trend in technology growth	0.003	U.S. data
Production and firm financing			
α	share of capital	0.35	BGG
υ	firm's leverage factor	1.43	U.S. data
κ	probability of default	0.086	Moody's (2012) and BGG
Credit			
ρ_θ	coefficient of autoregression in a credit shock	0.848	U.S. data
σ_η	standard deviation of a credit shock	0.011	U.S. data
μ_θ	mean of loan-to-deposit ratio	0.95, 1, 1.05	

The remaining parameters, ρ_u, σ_ε, μ, υ, ρ_θ, σ_η and σ_λ are calibrated using U.S. quarterly time series data covering the period 1985Q1 to 2009Q4.[18] Following the literature, we use the Hodrick-Prescott (1997) filter to extract the cyclical components from the data series, with a smoothing parameter value of 1600, which is recommended for quarterly data.[19] To derive the standard deviation of common technology shock, σ_ε, from U.S. data, we first detrend the log of the per capita real output series. The standard deviation of the cyclical component (as a proxy for the innovation to common technology shock, ε_t) is found to be 0.011, similar to the value used in Romer (2006). The deterministic trend in technology growth, μ, is obtained by calculating the mean of the growth rate of per capita real GDP in logarithm in the United States between 1985Q1 and 2009Q4. μ is found to be around 0.003, which implies a per capita output growth rate of around 1.2%, per annum. The autoregressive coefficient of the growth rate of per capita real GDP (in logarithm) is 0.439, which is taken as the coefficient of autoregression in the common technology shock, ρ_u. To match an annualized default probability of around 3.4 per cent (a quarterly rate of

[18] A detailed description of the U.S. data series used for calibration can be found in Appendix B.

[19] For details on the HP filter, see, for example, King and Rebelo (1993) and Hodrick and Prescott (1997). The cyclical component y_t^c of the series extracted by an HP filter is defined by (in the infinite sample version of the HP filter) $y_t^c = \frac{\lambda(1-L)^2(1-L^{-1})^2}{1+\lambda(1-L)^2(1-L^{-1})^2} y_t$, where y_t is the original time series, L is the lag operator and λ is the smoothing parameter. Alternative approaches for permanent–transitory decomposition include the Beveridge-Nelson procedure (see, for example, Garratt, Lee, Pesaran, and Shin, 2006).

0.86 per cent), consistent with BGG and the rolling 12-month quarterly default rate of U.S. private firms provided by Moody's from 2000 to 2009, we set the standard deviation of the idiosyncratic technology shock, σ_λ, to 0.43.[20]

We calibrate the values for the standard deviation and the autoregressive coefficient of the credit shock using U.S. data on loans and deposits between 1985Q1 to 2009Q4. Loans data are taken from the "U.S. Commercial Bank Assets–Bank Credit" series from the Federal Reserve H.8 Table, comprising securities, loans and leases for all commercial banks in the U.S. Among which, loans and leases include commercial and industrial loans, real estate and commercial loans. The Federal Reserve series "U.S. Commercial Bank Liabilities–Deposits and Borrowing" (H.8 Table) is used as a measure for bank deposits, which captures large time deposits and other deposits for all commercial banks. Historically, the loan-to-deposit ratio has fluctuated around one, with the ratio above one between 1993Q3 and 2001Q3, 2005Q1 and 2008Q1, and below one for the remaining sample period.[21] This time series evidence supports the calibration of mean loan-to-deposit ratio (μ_θ) at 0.95, 1 and 1.05 in our analysis. To calibrate the the key parameters for the credit shock, first, we define a series that is the difference between the logarithm of per capita loans and the logarithm of per capita deposits. We then detrend the series using an HP filter (with the smoothing parameter of 1600) and take the cyclical components of the series as a proxy for η_t. The standard deviation of the series, σ_η, is found to be 0.011, and the autoregressive coefficient of the credit shock process, ρ_θ, is estimated to be 0.848.

The leverage factor of firms is derived using the Federal Reserve Flow of Funds data, Table L.102 (levels data) on U.S. non-farm non-financial corporate business, following Fiore and Uhlig (2011). Debt is defined as bank loans and corporate bonds (lines 39+26), and equity is defined as the market value of equities outstanding (line 37). The proportion of debt finance in total finance between 1985Q1 to 2009Q4 is around 30 per cent, which implies a firm leverage ratio of around 1.43.[22]

6 Findings

This section presents our main findings. First, we show that, in the steady state, firms' default probability rises with firms' leverage ratio and the volatility of shocks to the economy. A rise in default probability would then drive up the interest rate premium and lead to a fall in economy-wide output and consumption. Second, higher loan-to-deposit ratio and higher banking sector leverage could lead to a decline in the interest rate premium in the steady state. Third, a positive credit supply shock increases output, consumption, hours and productivity, and reduces the spread between loan and deposit rates. The effects of the credit shock tend to be highly persistent and

[20] See, for example, Moody's (2012). The data source is Moody's Analytics Credit Research Database (CRD), which collects quarterly data from 15 U.S. lending organizations, representing both large institutions and smaller regional banks. The CRD definition of default is consistent with the Basel II directive.

[21] Fluctuations in the loan-to-deposit ratios can be attributed to a number of factors. From a liquidity perspective, this ratio reflects the funding mix of banks between retail and wholesale funding markets. The loan-to-deposit ratio tends to rise in good times, when market funding is abundantly available to finance credit growth, and usually levels off in stressed market conditions, when wholesale funding is substituted for retail savings and credit growth diminishes, see for example, van den End (2013).

[22] This result is very similar to Fiore and Uhlig (2011), who find that the debt to total finance ratio is around 0.3 and a leverage ratio of around 1.43 for US non-farm, non-financial corporate business sector, using the shorter sample period of 1997 to 2003.

the impact on the real economy would increase with elevated market volatility.

6.1 Steady state values, default probability and the loan-to-deposit ratio

6.1.1 Default probability, firm leverage ratio and volatility

Since an important focus of this paper is to examine the impact of firm defaults on macroeconomic conditions, we compute the steady-state values of the model economy by varying some of the key parameters that determine the equilibrium default probability, namely, the leverage ratio of the firms, υ, and the standard deviations (volatility) of common and idiosyncratic technology shocks, σ_ϵ and σ_λ, respectively. Recall from (3.26) and (3.27) that the model predicts that default probability rises with a firms' leverage ratio and the volatility of technology shocks. The results in Table 2 confirm this prediction in the steady states. For the firms' leverage ratio, υ, we consider three scenarios with $\upsilon = \{1.25, 1.43, 1.67\}$, where the proportion of debt finance to total finance is 20 per cent, 30 per cent (U.S. data), and 40 per cent, respectively. As the results in Table 2 show, the probability of default rises from 0.086 per cent to 2.33 per cent per quarter when the leverage ratio increases from 1.43 to 1.67, and the interest rate on loans doubles from 7 per cent to 14.8 per cent, with a sharp increase in the interest rate premium between the loan rate and the deposit rate. We also observe a fall in the steady-state levels of per capita consumption, output and capital, when the firm's leverage ratio rises, despite an increase in the level of loans per capita, since a larger proportion of the loans become non-performing and the steady-state value of private equity per capita is falling (the substitution effect). As expected, the probability of default rises with the volatility of common and idiosyncratic technology shocks (see Panels B and C of Table 2). The steady-state levels of per capita output, consumption and loans decline with increased volatility or uncertainty in the economy. Similarly, increased volatility is accompanied by a rise in the interest rate premium between the loan and the deposit rates.[23]

6.1.2 Loan-to-deposit ratio, bank leverage and interest rate premium

It is interesting to note that, in the steady state, output per capita, consumption per capita and capital per capita in efficiency units rise with the loan-to-deposit ratio, μ_θ, which measures the availability of loans and the extent of leverage in the banking sector (Table 3).[24] This finding suggests that, as the banking sector becomes more leveraged and the extent of financial intermediation increases, the steady-state level of output per capita in the economy tends to be higher. This result is consistent with empirical studies of the relationship between finance and development, where a more developed banking sector is often associated with faster economic development (see, for example, Levine, 2005). Note also that, when the banking sector becomes more leveraged, the interest rate charged on loans tends to fall. For example, when μ_θ increases from 1 (no leverage) to 1.05, the real loan rate falls from an implausibly high rate of 7 per cent per quarter (28 per cent per annum) to around 8 per cent per annum, yielding an interest rate spread of around 5 per cent per annum, which is more reasonable. The results suggest that, in order to meet the break-even condition, in equilibrium the banking sector could either charge a high interest rate on loans to

[23]Note also that in the steady state, $\mathring{Y}^* = \mathring{C}^* + \mathring{S}^* + \mathring{D}^*$; that is, output is divided into consumption and savings, comprising private-equity investment, $\mathring{S}^*$, and bank deposits, $\mathring{D}^*$, as implied by equation (4.15).

[24]The steady-state values of the model variables in efficiency units provided in Table 3 are obtained using equations (4.22) to (4.27).

Table 2: Steady-State Values and Default Probability

		Steady-state values										Default Prob
		r_d^*	r_l^*	$\mathring{N}^*$	$\mathring{W}^*$	$\mathring{C}^*$	$\mathring{Y}^*$	$\mathring{K}^*$	$\mathring{L}^*$	$\mathring{S}^*$	$\mathring{D}^*$	κ
Panel A												
	1.25	0.007	0.033	1	0.367	0.373	0.564	0.191	0.038	0.153	0.038	0.0026
υ	1.43	0.007	0.070	1	0.360	0.373	0.553	0.181	0.054	0.126	0.054	0.0086
	1.67	0.007	0.148	1	0.345	0.370	0.531	0.160	0.064	0.096	0.064	0.0233
Panel B												
	0.001	0.007	0.070	1	0.360	0.373	0.554	0.181	0.054	0.126	0.054	0.0086
σ_ϵ	0.011	0.007	0.070	1	0.360	0.373	0.553	0.181	0.054	0.126	0.054	0.0086
	0.110	0.007	0.085	1	0.357	0.372	0.550	0.177	0.053	0.124	0.053	0.0105
Panel C												
	0.33	0.007	0.013	1	0.369	0.372	0.568	0.196	0.059	0.137	0.059	0.0009
σ_λ	0.43	0.007	0.070	1	0.360	0.373	0.553	0.181	0.054	0.126	0.054	0.0086
	0.53	0.007	0.221	1	0.333	0.369	0.513	0.144	0.043	0.101	0.043	0.0281

Note: The steady-state values in Panels A, B and C are computed based on the parameter values given in Table 1, with μ_θ=1 and the values of υ, σ_ϵ and σ_λ given in this table, respectively. The steady state of labour hours in efficiency unit $\mathring{N}^*$ is normalized to 1.

cover the losses resulting from firm defaults, or take on more risks by increasing leverage, in the form of security issuance, for example.

Table 3: Steady-State Values and the Loan-to-Deposit Ratio

		Steady-state values									
		r_d^*	r_l^*	$\mathring{N}^*$	$\mathring{W}^*$	$\mathring{C}^*$	$\mathring{Y}^*$	$\mathring{K}^*$	$\mathring{L}^*$	$\mathring{S}^*$	$\mathring{D}^*$
	0.95	0.007	0.121	1	0.350	0.369	0.538	0.167	0.050	0.117	0.053
μ_θ	1	0.007	0.070	1	0.360	0.373	0.553	0.181	0.054	0.126	0.054
	1.05	0.007	0.021	1	0.369	0.376	0.568	0.195	0.058	0.136	0.056

Note: The steady-state values are computed based on the parameter values given in Table 1. The steady state of labour hours in efficiency unit $\mathring{N}^*$ is normalized to 1.

The steady state findings highlight two channels through which interest rate premium (the spread between loan and deposit rates) is affected. The first channel operates through firm defaults, as banks charge a loan rate that reflects a compensation for their expected losses due to firm defaults. The rise in the loan rate, in turn, influences the amount of default in equilibrium. The second channel operates through the loan-to-deposit ratio of the banking sector. As loans are necessary for production, lower mean loan-to-deposit ratio leads to higher scarcity of capital in the economy, which would then imply higher returns on capital and a higher loan rate.

6.2 Dynamic effects of credit supply shocks

6.2.1 Credit supply shock, output and interest rate spread

Here we consider the dynamic effects of a positive credit supply shock on the macro economy. [25] As can be seen from Figure 1, a positive credit shock results in an increase in loans of around 1 per cent on impact.[26] The rise in the level of loans leads to an increase in available capital in the economy and a rise in output level of around 0.6 per cent. The increase in the supply of funds also drives down the interest rate on loans by almost half a per cent and leads to a fall in the interest rate spread of around 1 per cent, as predicted by Proposition 2. The deposit rate rises by about 0.6 per cent on impact, which in turn implies an increase in the level of deposits of around 1 per cent at its peak, consistent with the zero-profit condition imposed on the banking sector. An increase in labour income raises household consumption by around 0.4 per cent on impact. Further, productivity rises, since output increases more than labour hours on impact.

It is worth highlighting that our findings are in line with the empirical evidence of responses to a U.S. credit shock, where output and short-term interest rate (deposit rate in our model) move in the same direction as the credit shock (see, for example, Helbling, Huidrom, Kose, and Otrok, 2011 and Xu, 2012). Our calibrated results show that the speed of convergence to equilibrium is slow for the credit shock (around 20 quarters), while the peak impacts of a credit shock on output and consumption are much larger than the effects of a typical technology shock. This observation is consistent with empirical studies on the output effect of financial crises, which suggest that recessions associated with financial crises have been much longer lasting than recessions associated with other shocks (see, for example, IMF, World Economic Outlook, April 2009, Chapter 3). The prolonged effects of the credit shock also reflect the high persistence in the loan-to-deposit ratio that we observe empirically, where the autoregressive coefficient of the credit shock is found to be 0.848, while the autoregressive coefficient of the technology shock is found to be around 0.44 based on time series evidence.[27]

6.2.2 Credit shocks and volatility

As we have seen earlier, the steady state output and interest rate premium are sensitive to the volatility of idiosyncratic and aggregate technology shocks. Here, we examine the impact of higher volatity of the credit shock on the dynamic responses of the model to a positive credit supply shock. In particular, we increase the volatility of the credit shock, as measured by σ_η, by 50 per cent above the value implied by U.S. data, thus increasing σ_η from 1.1 per cent to 1.65 per cent. As shown in Figure 2, the peak response in output, consumption and the level of loans is around 50 per cent higher when $\sigma_\eta = 1.65\%$, accompanied by a slower rate of convergence to equilibrium. Consistent with the steady state results, the loan rate is sensitive to changes in volatility, which results in a widening of the interest rate spread by about 50 per cent on impact. [28] This result confirms that a credit shock can have a profound impact on the real economy, in terms of both the magnitude

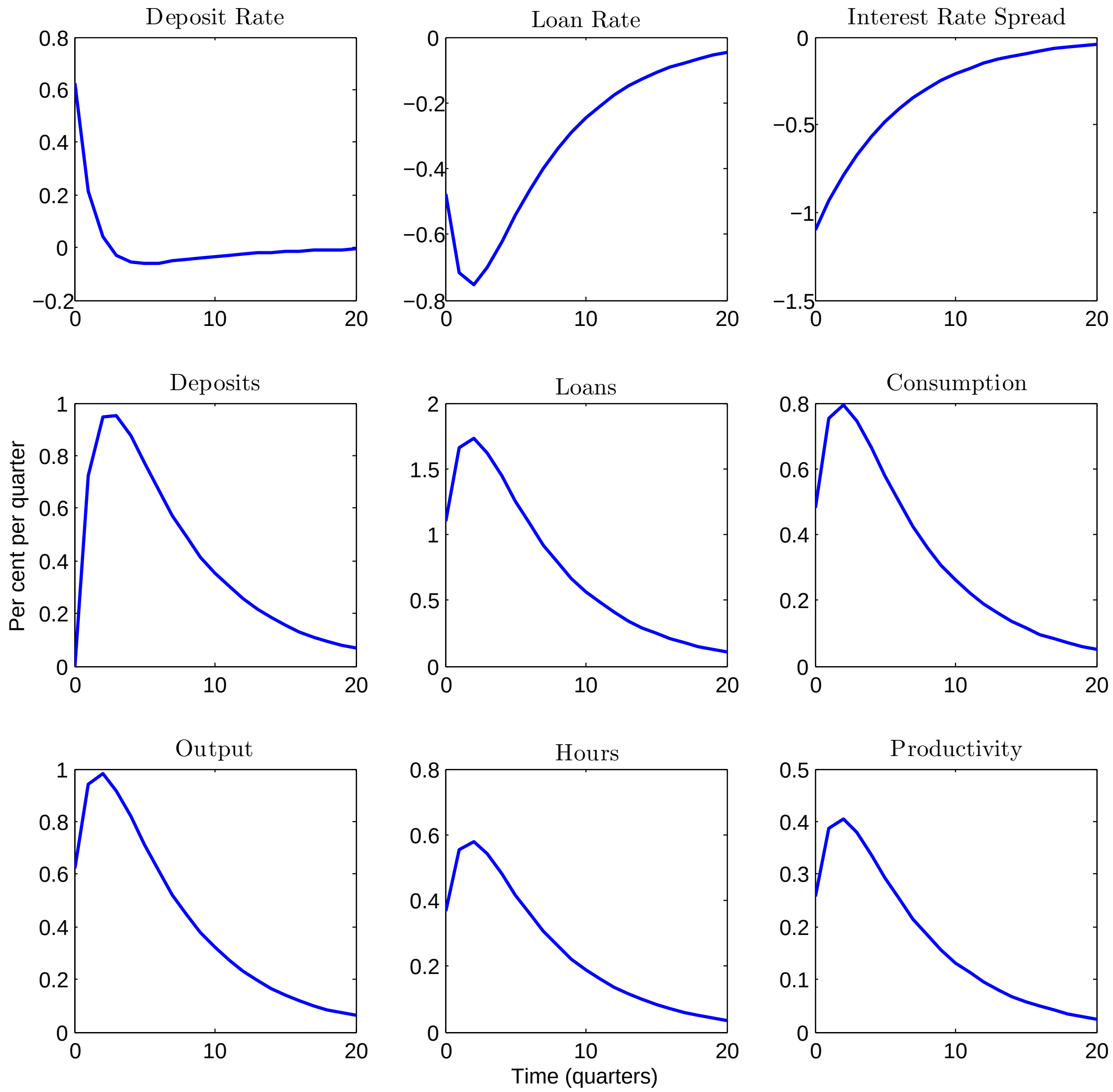
[25] The parameter values are given in Table 1, with the mean of the loan-to-deposit ratio, μ_θ, set at one.

[26] A negative credit shock can be interpreted as a credit crunch.

[27] Owing to space considerations, the impulse responses to a positive technology shock are not presented here, but are available upon request.

[28] The volatility of the credit shock, σ_η, has an impact on the dynamics of the model, since it affects the steady-state conditions of the model through $E(ln\theta_t)$, given by equation (3.42), and therefore enters the coefficients of the log-linearized approximation of the equilibrium conditions.

Figure 1: Impulse responses of one-standard-deviation positive credit shock (Benchmark Calibration, $\mu_\theta = 1$, per cent per quarter)



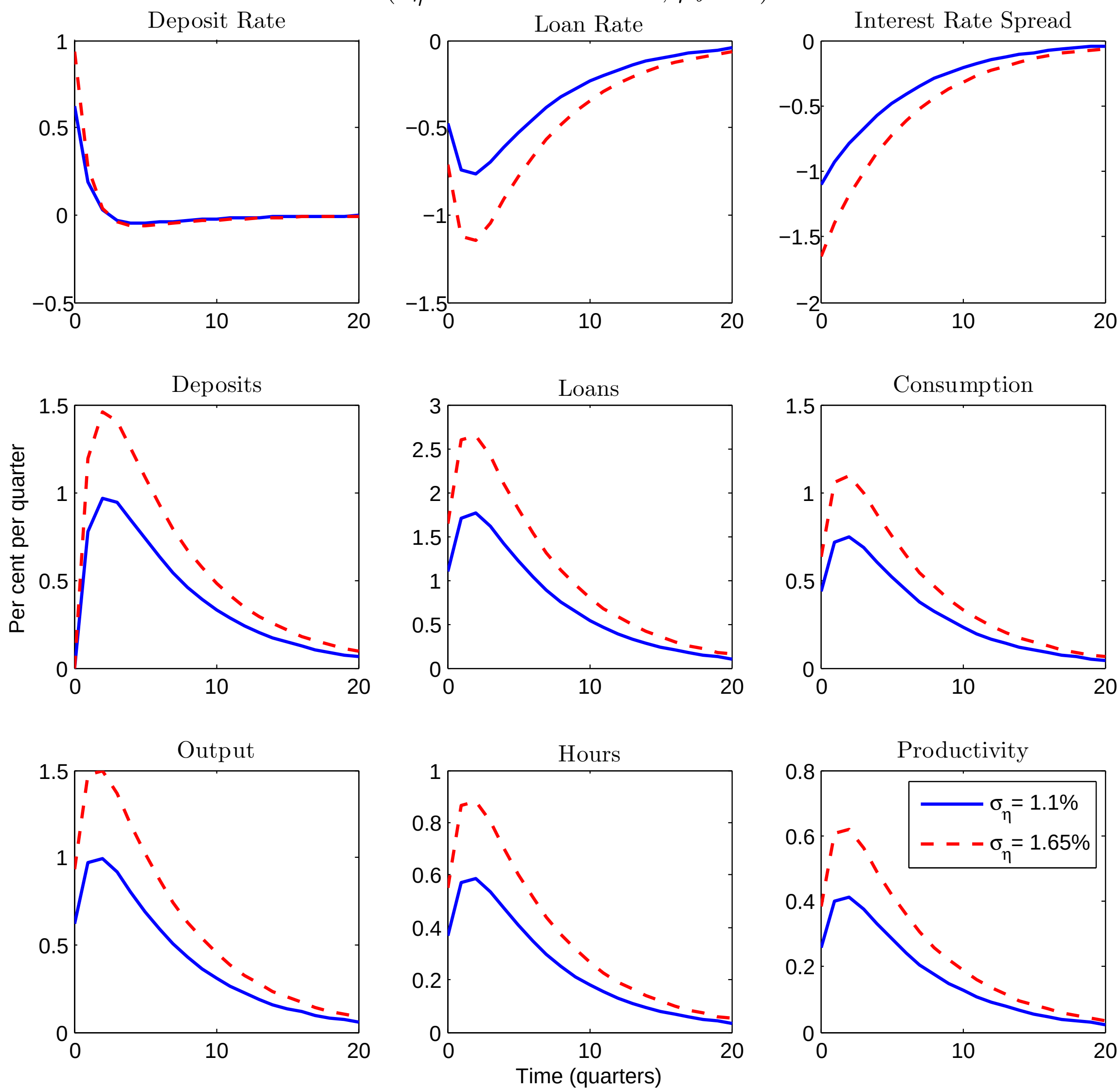
and duration of the responses, and the impact could be even more severe during a banking crisis that is combined with elevated market volatility. As the volatility of credit shocks rises, the peak impact on the interest rate spread, output, and consumption would increase notably, accompanied by an increase in the time required for the macro economy to return to equilibrium.

6.2.3 Credit shocks and persistence

Finally, we considered the robustness of our results to the value of ρ_θ, the autoregressive coefficient of the credit shock. We reduced the benchmark estimate of ρ_θ from 0.848 to 0.678 (by 20 per cent). The results are displayed in Figure 3. As expected, we observe faster convergence in the impact of the credit shock when the autoregressive coefficient is reduced; in particular, the impact on output takes approximately 8 to 10 quarters to vanish, compared with around 20 quarters in the benchmark case. While the magnitude of the response in the levels of loans, output and consumption is robust on impact, the peak impact of credit supply shock is reduced when credit

Figure 2: Impulse responses of one-standard-deviation positive credit shock with alternative volatility

($\sigma_\eta = 1.1\%$ and 1.65%, $\mu_\theta = 1$)

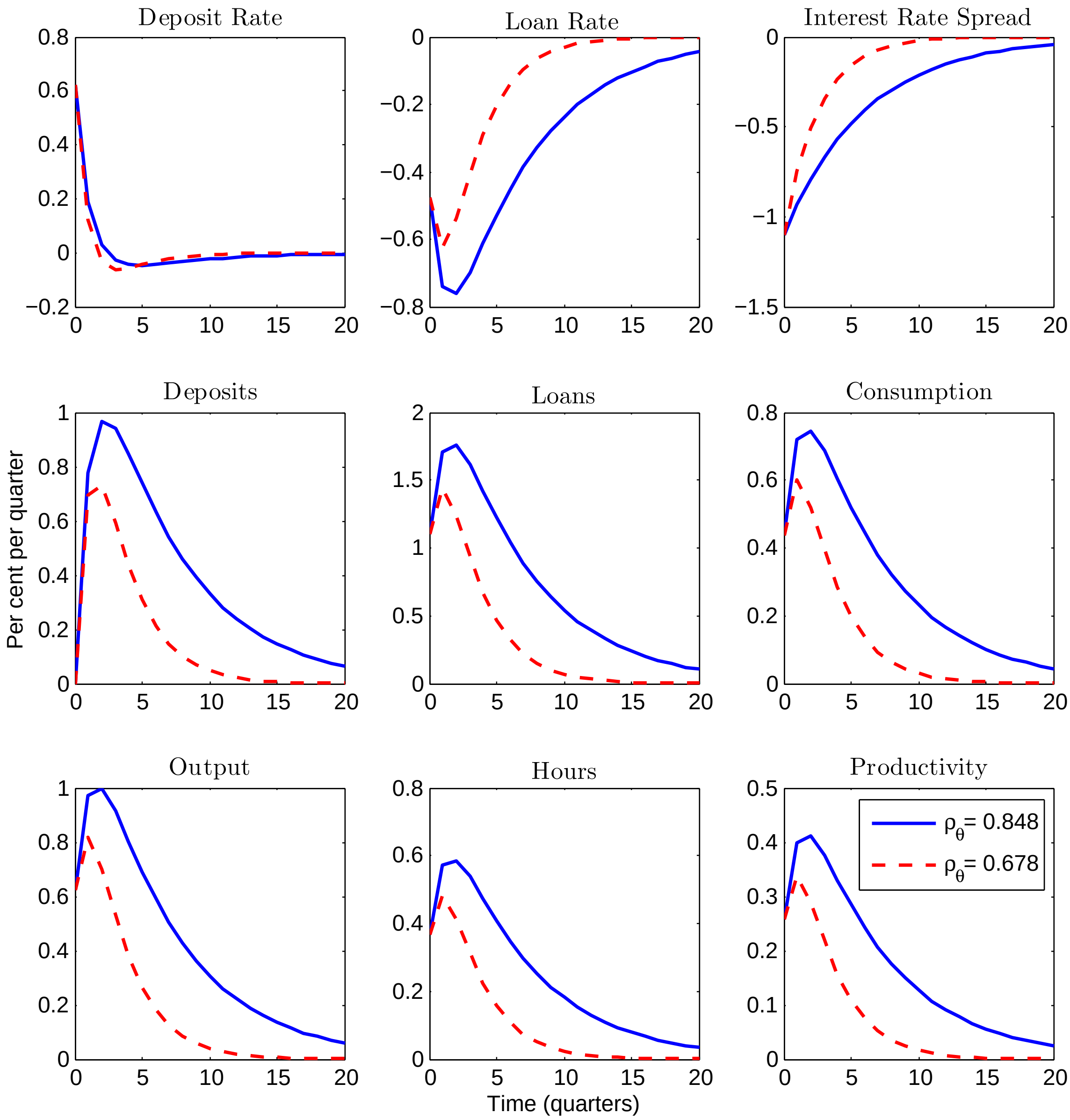


shocks are less persistent. The dynamics of the impact on the endogenous variables remain similar, as a positive credit supply shock leads to a decline in loan interest rate, and consequently the interest rate spread, and an increase in output and consumption.

7 Conclusion

This paper develops an analytically tractable theoretical model to analyze the relationship between credit, firm default and volatility, with the aim of studying the effects of credit shocks on the real economy. It advances a new approach to modelling firm defaults and financial intermediation, and examines the financial implications of such defaults on the behavior of the household and the banking sector. It also incorporates growth into a DSGE framework and proposes a new method of computing steady states, allowing the steady-state values of the model to depend on the volatility of technology and credit shocks.

Figure 3: Impulse responses of one-standard-deviation positive credit shock with alternative persistence level ($\rho_\theta = 0.848$ and 0.678, $\mu_\theta = 1$)



We show that, in the steady state, the probability of firm default rises with firm's leverage ratio and the volatility of the shocks. A positive credit shock, defined as a rise in the loan-to-deposit ratio, leads to an increase in available capital and a rise in output, which is largely consistent with the empirical findings in Xu (2012) and Eickmeier and Ng (2015). The positive credit shock also drives down the spread between the loan rate and the deposit rate. The effects of the credit shock are found to be highly persistent, consistent with empirical studies on the output effects of financial crises, which suggest that recessions associated with financial crisis have lasted much longer than recessions associated with other shocks.

The current modelling framework can be extended and enhanced along several dimensions. First, a more elaborate banking sector including bank capital can be considered, to allow for endogenous credit and leverage shocks. Second, it would be important to consider the risks of high leverage as well as the benefits, which were highlighted in the current framework. One way to introduce the potential costs of leverage is to augment the model with price rigidities and a central bank operating under a monetary policy rule such as the Taylor rule. This would allow one to capture possible inflationary pressures from high leverage and to study the policy implications of credit shocks. Alternatively, one could incorporate leverage costs in the production function, to establish a direct link between excess leverage and low average productivity in the economy. Third, it would be interesting to extend the model to incorporate time-varying volatility, and to study the evolution of the probability of firm default over the business cycle.

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A Mathematical Appendix

A1 Proof of Lemma 1

Proof. Recall that λ_{it} is independently and identically distributed across i and t and $\lambda_{it} \sim N(0, \sigma_\lambda^2)$. Then

$$\int_{-\infty}^{\varpi_1-\varepsilon_t} e^{\varphi x} f_\lambda(x)dx = \int_{-\infty}^{\varpi_1-\varepsilon_t} \exp(\varphi x)\frac{1}{\sqrt{2\pi\sigma_\lambda^2}}\exp\left(-\frac{x^2}{2\sigma_\lambda^2}\right)dx$$

$$\begin{aligned}
&= \frac{1}{\sigma_\lambda}\int_{-\infty}^{\varpi_1-\varepsilon_t}\frac{1}{\sqrt{2\pi}}\exp\left(\varphi x - \frac{x^2}{2\sigma_\lambda^2}\right)dx \\
&= \frac{1}{\sigma_\lambda}\int_{-\infty}^{\varpi_1-\varepsilon_t}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2\sigma_\lambda^2}\left[x^2 - 2\sigma_\lambda^2\varphi x + (\sigma_\lambda^2\varphi)^2\right] + \frac{\sigma_\lambda^2\varphi^2}{2}\right]dx \\
&= \frac{1}{\sigma_\lambda}\exp\left(\frac{\sigma_\lambda^2\varphi^2}{2}\right)\int_{-\infty}^{\varpi_1-\varepsilon_t}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{(x-\sigma_\lambda^2\varphi)^2}{2\sigma_\lambda^2}\right]dx. \qquad \text{(A1)}
\end{aligned}$$

Now let $\varrho = \left(x - \sigma_\lambda^2\varphi\right)/\sigma_\lambda$, then

$$\int_{-\infty}^{\varpi_1-\varepsilon_t}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{(x-\sigma_\lambda^2\varphi)^2}{2\sigma_\lambda^2}\right]dx = \int_{-\infty}^{\frac{\varpi_1-\varepsilon_t-\sigma_\lambda^2\varphi}{\sigma_\lambda}}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\varrho^2}{2}\right)\sigma_\lambda d\varrho$$

$$= \sigma_\lambda\Phi\left(\frac{\varpi_1-\varepsilon_t-\sigma_\lambda^2\varphi}{\sigma_\lambda}\right),$$

where $\Phi(\cdot)$ is the cumulative distribution of a standard normal. Then (A1) becomes

$$\int_{-\infty}^{\varpi_1-\varepsilon_t} e^{\varphi x} f_\lambda(x)dx = M_\lambda\Phi\left(\frac{\varpi_1-\varepsilon_t-\sigma_\lambda^2\varphi}{\sigma_\lambda}\right),$$

where $M_\lambda = \exp\left(\frac{\sigma_\lambda^2\varphi^2}{2}\right)$ and $\varpi_1 = \frac{\ln\left(1-\frac{\alpha}{\upsilon}\right)}{\varphi} + \frac{\varphi\sigma_\zeta^2}{2}$ from equations (3.24) and (3.22). ■

A2 Model Derivation and Solution

A2.1 Equilibrium conditions in efficiency units

The system of equations in (4.1) to (4.12) can be further simplified into a nine-equation system in nine unknowns C_t, N_t, D_t, L_t, S_t, K_t, Y_t, R_{lt}, and R_{dt} by eliminating W_t, Π_{tb} and Π_{th}. As stated earlier, since this model depicts a growing economy where technology grows with a deterministic trend μ, we must scale the endogenous variables C_t, L_t, D_t, S_t, K_t, N_t and Y_t in the system of equilibrium conditions by an appropriate factor of technology, A_{t-1}, so that they are stationary on a balanced growth path, to guarantee the existence of a steady state in solving the model. We denote the variables in efficiency units by capital letters with a dot, $\mathring{C}_t = \frac{C_t}{A_{t-1}}$, $\mathring{L}_t = \frac{L_t}{A_{t-1}}$, $\mathring{S}_t = \frac{S_t}{A_{t-1}}$, $\mathring{D}_t = \frac{D_t}{A_{t-1}}$, $\mathring{K}_t = \frac{K_t}{A_{t-1}}$, $\mathring{N}_t = \frac{N_t}{A_{t-1}^{1/(1+\chi)}}$, $\mathring{Y}_t = \frac{Y_t}{A_{t-1}}$.

The equilibrium conditions in efficiency units can be written as

$$
\begin{aligned}
1 &= E\left[\beta\left(\frac{\mathring{C}_{t+1}-\frac{\chi_0}{1+\chi}\mathring{N}_{t+1}^{1+\chi}}{\mathring{C}_t-\frac{\chi_0}{1+\chi}\mathring{N}_t^{1+\chi}}\right)^{-\gamma} g_t^{-\gamma}R_{d,t+1}|\Omega_{ct}\right],\\
\mathring{Y}_t-\mathring{C}_t &= \mathring{S}_t+\mathring{D}_{t+1}g_t,\\
\mathring{K}_t &= \frac{\alpha\chi_0}{1-\alpha}\cdot\frac{\mathring{N}_t^{1+\chi}}{R_{lt}},\\
\mathring{N}_t &= \left[\frac{1-\alpha}{\alpha\chi_0}\left(\alpha M_\lambda M_\varepsilon\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\chi}}\exp\left[\frac{\mu(1-\rho_u)}{1+\chi}\right](R_{lt})^{-\frac{\alpha}{(1-\alpha)\chi}}g_{t-1}^{\frac{\rho_u}{1+\chi}},\\
\mathring{Y}_t &= M_\lambda g_t^{\varphi}\mathring{N}_t^{1-\alpha}\mathring{K}_t^{\alpha},\\
\mathring{L}_t &= (1-\frac{1}{\upsilon})\mathring{K}_t,\ \ \mathring{K}_t=\mathring{L}_t+\mathring{S}_t, \mathring{L}_t=\theta_t\mathring{D}_t,\\
R_{dt}\mathring{D}_t &= R_{lt}\mathring{L}_t(1-\kappa)-\chi_0(\kappa-\tau)\mathring{N}_t^{1+\chi}.
\end{aligned}
$$

The above system of equations can be further simplified to a system of six equations by eliminating $\mathring{L}_t$, $\mathring{S}_t$ and $\mathring{K}_t$. The equilibrium conditions in terms of $\mathring{C}_t$, $\mathring{D}_t$, $\mathring{N}_t$, $\mathring{Y}_t$, R_{dt} and R_{lt} are then given by

$$1 = E\left[\beta\left(\frac{\mathring{C}_{t+1}-\frac{\chi_0}{1+\chi}\mathring{N}_{t+1}^{1+\chi}}{\mathring{C}_t-\frac{\chi_0}{1+\chi}\mathring{N}_t^{1+\chi}}\right)^{-\gamma} g_t^{-\gamma}R_{d,t+1}|\Omega_{ct}\right], \tag{A2}$$

$$\mathring{Y}_t-\mathring{C}_t = \frac{1}{\upsilon-1}\theta_t\mathring{D}_t+\mathring{D}_{t+1}g_t, \tag{A3}$$

$$\theta_t\mathring{D}_t = \frac{\alpha\chi_0(1-\frac{1}{\upsilon})}{1-\alpha}\cdot\frac{\mathring{N}_t^{1+\chi}}{R_{lt}}, \tag{A4}$$

$$\mathring{N}_t = \left[\frac{1-\alpha}{\alpha\chi_0}\left(\alpha M_\lambda M_\varepsilon\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\chi}}\exp\left[\frac{\mu(1-\rho_u)}{1+\chi}\right](R_{lt})^{-\frac{\alpha}{(1-\alpha)\chi}}g_{t-1}^{\frac{\rho_u}{1+\chi}}, \tag{A5}$$

$$\mathring{Y}_t = M_\lambda g_t^{\varphi}\mathring{N}_t^{1-\alpha}\left[\frac{\theta_t\mathring{D}_t}{(1-\frac{1}{\upsilon})}\right]^{\alpha}, \tag{A6}$$

$$R_{dt}\mathring{D}_t = R_{lt}\theta_t\mathring{D}_t(1-\kappa)-\chi_0(\kappa-\tau)\mathring{N}_t^{1+\chi}. \tag{A7}$$

A2.2 Derivation of the steady states

We denote the variables in steady states by the lowercase letters with an asterisk, for example, the steady state of consumption is given by $\mathring{c}^* = E(ln\mathring{C}_t)$ and the steady state of the loan rate is $r_l^* = E(\ln R_{lt}) \approx E(r_{lt})$. To derive the steady state, we first take logarithms of the equilibrium conditions (A4) to (A6) and then the unconditional expectations of the resulting equations:

$$
\begin{aligned}
E(ln\theta_t)+\mathring{d}^* &= ln\chi_0+ln\left[\frac{\alpha(1-\frac{1}{\upsilon})}{1-\alpha}\right]+(1+\chi)\mathring{n}^*-r_l^*,\\
\mathring{n}^* &= -\frac{ln\chi_0}{\chi}+\frac{1}{\chi}ln\left[\frac{1-\alpha}{\alpha}(\alpha M_\lambda M_\varepsilon)^{\frac{1}{1-\alpha}}\right]+\frac{\mu}{1+\chi}-\frac{\alpha}{(1-\alpha)\chi}r_l^*,\\
\mathring{y}^* &= ln\left[\frac{M_\lambda}{(1-\frac{1}{\upsilon})^{\alpha}}\right]+\varphi\mu+(1-\alpha)\mathring{n}^*+\alpha E(ln\theta_t)+\alpha\mathring{d}^*,
\end{aligned}
$$

since $ln(g_t) = \mu + u_t$ and $E(lng_t) = \mu$.

To obtain the steady-state conditions for equation (A3), first note that $\mathring{C}_t$ can be approximated as follows:

$$\mathring{C}_t = e^{\mathring{c}_t} \approx e^{\mathring{c}^*}(1+\widetilde{\mathring{c}}_t),$$

where $\widetilde{\mathring{c}}_t = \mathring{c}_t - \mathring{c}^*$. Equation (A3) can then be approximated by

$$e^{\mathring{y}^*}(1+\widetilde{\mathring{y}}_t) - e^{\mathring{c}^*}(1+\widetilde{\mathring{c}}_t) = \frac{e^{E(ln\theta_t)+\mathring{d}^*}}{\upsilon-1}(1+\widetilde{ln\theta_t})(1+\widetilde{\mathring{d}}_t) + e^{\mathring{d}^*}(1+\widetilde{\mathring{d}}_{t+1})e^{\mu}(1+\widetilde{lng_t}),$$

where $\widetilde{\ln g_t} = \ln g_t - E(\ln g_t)$ and $\widetilde{\ln\theta_t} = \ln\theta_t - E(\ln\theta_t)$. If we take the unconditional expectations on both sides of the above equation, we have that, in steady state,

$$e^{\mathring{y}^*} - e^{\mathring{c}^*} = e^{\mathring{d}^*+\mu} + \frac{1}{\upsilon-1}e^{E(ln\theta_t)+\mathring{d}^*}.$$

Similarly, the steady-state condition for equation (A7) is given by

$$e^{r_d^*}e^{\mathring{d}^*} = e^{r_l^*+E(ln\theta_t)+\mathring{d}^*}(1-\kappa) - \chi_0(\kappa-\tau)e^{(1+\chi)\mathring{n}^*}.$$

Finally, to derive the steady-state condition for equation (A2), we can approximate $\mathring{C}_t$ and $\mathring{N}_t^{1+\chi}$ as follows:

$$\frac{\mathring{C}_{t+1} - \frac{\chi_0}{1+\chi}\mathring{N}_{t+1}^{1+\chi}}{\mathring{C}_t - \frac{\chi_0}{1+\chi}\mathring{N}_t^{1+\chi}} = \frac{1+\frac{1}{1-b_1}\widetilde{\mathring{c}}_{t+1} - \frac{b_1}{1-b_1}(1+\chi)\widetilde{\mathring{n}}_{t+1}}{1+\frac{1}{1-b_1}\widetilde{\mathring{c}}_t - \frac{b_1}{1-b_1}(1+\chi)\widetilde{\mathring{n}}_t},$$

where $b_1 = \frac{\chi_0}{1+\chi}e^{(1+\chi)\mathring{n}^*-\mathring{c}^*}$. Taking first-order Taylor expansion, we have that following approximation:

$$\begin{aligned}&\left(\frac{\mathring{C}_{t+1} - \frac{\chi_0}{1+\chi}\mathring{N}_{t+1}^{1+\chi}}{\mathring{C}_t - \frac{\chi_0}{1+\chi}\mathring{N}_t^{1+\chi}}\right)^{-\gamma}\\ \approx\ & 1 - \frac{\gamma}{1-b_1}\widetilde{\mathring{c}}_{t+1} + \frac{b_1\gamma}{1-b_1}(1+\chi)\widetilde{\mathring{n}}_{t+1} + \frac{\gamma}{1-b_1}\widetilde{\mathring{c}}_t - \frac{b_1\gamma}{1-b_1}(1+\chi)\widetilde{\mathring{n}}_t.\end{aligned}$$

Furthermore, we can approximate $g_t^{-\gamma}R_{d,t+1}$ in equation (A2) as follows:

$$g_t^{-\gamma}R_{d,t+1} \approx b_2(1-\gamma\widetilde{\ln g_t} + \widetilde{r}_{d,t+1}),$$

where $b_2 = e^{-\gamma\mu+r_d^*}$. Therefore, equation (A2) is approximated by

$$\frac{1}{\beta b_2} - 1 = E\left[-\frac{\gamma}{1-b_1}\left(\widetilde{\mathring{c}}_{t+1} - \widetilde{\mathring{c}}_t\right) + \frac{b_1\gamma(1+\chi)}{1-b_1}\left(\widetilde{\mathring{n}}_{t+1} - \widetilde{\mathring{n}}_t\right) - \gamma\widetilde{\ln g_t} + \widetilde{r}_{d,t+1}|\Omega_{ct}\right]. \qquad \text{(A8)}$$

Now, if we take the unconditional expectations on both sides of (A8), by the law of iterated expectations, the right hand side of equation (A8) is equal to zero. In steady state, we then have $\beta b_2 = 1$, which together with $b_2 = e^{-\gamma\mu+r_d^*}$, we obtain $r_d^* = \gamma\mu - \ln\beta$.

A2.3 Log-linearization

We log-linearize the system of equilibrium equations around the steady state of the log of the variables and denote the variables with a tilde, e.g. $\widetilde{\mathring{c}}_t = \mathring{c}_t - \mathring{c}^*$. Note again that $\mathring{C}_t$ can be approximated as $\mathring{C}_t = e^{\mathring{c}_t} \approx e^{\mathring{c}^*}(1+\widetilde{\mathring{c}}_t)$. For equations (A4) to (A6), the log approximations are given by

$$
\begin{aligned}
\widetilde{\ln\theta}_t + \tilde{\mathring{d}}_t + \tilde{r}_{lt} &= (1+\chi)\,\tilde{\mathring{n}}_t \\
\tilde{\mathring{n}}_t &= -\frac{\alpha}{(1-\alpha)\chi}\tilde{r}_{lt} + \frac{\rho_u}{1+\chi}\widetilde{\ln g}_{t-1} \\
\tilde{\mathring{y}} &= \varphi\widetilde{\ln g}_t + (1-\alpha)\,\tilde{\mathring{n}}_t + \alpha\widetilde{\ln\theta}_t + \alpha\tilde{\mathring{d}}_t.
\end{aligned}
$$

For equation (A3), it can be approximated by

$$
\begin{aligned}
& e^{\mathring{y}^*}(1+\tilde{\mathring{y}}_t) - e^{\mathring{c}^*}(1+\tilde{\mathring{c}}_t) \\
= \; & e^{\mathring{d}^*}(1+\tilde{\mathring{d}}_{t+1})e^{\mu}(1+\widetilde{\ln g}_t) + \frac{1}{\upsilon-1}e^{E(ln\theta_t)}(1+\widetilde{\ln\theta}_t)e^{\mathring{d}^*}(1+\tilde{\mathring{d}}_t),
\end{aligned}
$$

while in the steady state,

$$
e^{\mathring{y}^*} - e^{\mathring{c}^*} = e^{\mathring{d}^*+\mu} + \frac{1}{\upsilon-1}e^{E(ln\theta_t)+\mathring{d}^*}.
$$

The log approximation is then given by

$$
e^{\mathring{y}^*-\mathring{c}^*}\tilde{\mathring{y}}_t - \tilde{\mathring{c}}_t = e^{\mathring{d}^*+\mu-\mathring{c}^*}\left(\tilde{\mathring{d}}_{t+1} + \widetilde{\ln g}_t\right) + \frac{1}{\upsilon-1}e^{E(ln\theta_t)+\mathring{d}^*-\mathring{c}^*}\left(\widetilde{\ln\theta}_t + \tilde{\mathring{d}}_t\right).
$$

Similarly, the log-linearized approximation for equation (A7) is given by

$$
\begin{aligned}
& e^{r_d^*+\mathring{d}^*}\left(\tilde{r}_{dt} + \tilde{\mathring{d}}_t\right) \\
= \; & e^{r_l^*+E(ln\theta_t)+\mathring{d}^*}(1-\kappa)\left(\tilde{r}_{lt} + \tilde{\mathring{d}}_t + \widetilde{ln\theta_t}\right) - \chi_0(\kappa-\tau)(1+\chi)e^{(1+\chi)\mathring{n}^*}\tilde{\mathring{n}}_t.
\end{aligned}
$$

Finally, to log-linearize equation (A2), first recall that it can be approximated by equation (A8). Note that $\frac{1}{\beta b_2} - 1 = 0$ in steady state, and we have the following log approximation:

$$
\tilde{\mathring{c}}_t - b_1(1+\chi)\tilde{\mathring{n}}_t = E\left[\tilde{\mathring{c}}_{t+1} - b_1(1+\chi)\tilde{\mathring{n}}_{t+1} + (1-b_1)\widetilde{\ln g}_t - \frac{1-b_1}{\gamma}\tilde{r}_{d,t+1}|\Omega_{ct}\right].
$$

The log-linearized approximation of the equilibrium conditions of the model are therefore, noting $\widetilde{\ln g}_t = u_t$,

$$
\begin{aligned}
\tilde{\mathring{c}}_t - a_1\tilde{\mathring{n}}_t &= E\left(\tilde{\mathring{c}}_{t+1} - a_1\tilde{\mathring{n}}_{t+1} - a_3\tilde{r}_{d,t+1}|\Omega_{ct}\right) + a_2u_t, && \text{(A9)} \\
-\tilde{\mathring{c}}_t + a_5\tilde{\mathring{y}}_t - a_6\tilde{\mathring{d}}_{t+1} &= a_4\tilde{\mathring{d}}_t + a_6u_t + a_4\widetilde{\ln\theta}_t, && \text{(A10)} \\
\tilde{\mathring{n}}_t + \frac{\alpha}{(1-\alpha)\chi}\tilde{r}_{lt} &= \frac{\rho_u}{1+\chi}u_{t-1}, && \text{(A11)} \\
a_9\tilde{\mathring{n}}_t + a_7\tilde{r}_{dt} - a_8\tilde{r}_{lt} &= -(a_7-a_8)\,\tilde{\mathring{d}}_t + a_8\widetilde{ln\theta_t}, && \text{(A12)} \\
\tilde{\mathring{y}}_t - (1-\alpha)\,\tilde{\mathring{n}}_t &= \alpha\tilde{\mathring{d}}_t + \varphi u_t + \alpha\widetilde{\ln\theta}_t, && \text{(A13)} \\
(1+\chi)\,\tilde{\mathring{n}}_t - \tilde{r}_{lt} &= \tilde{\mathring{d}}_t + \widetilde{\ln\theta}_t, && \text{(A14)}
\end{aligned}
$$

where

$$\begin{aligned} a_1 &= b_1(1+\chi),\ a_2 = 1-b_1,\ a_3 = \frac{1-b_1}{\gamma}, \\ a_4 &= \frac{1}{\upsilon-1}e^{E(ln\theta_t)+\mathring{d}^*-\mathring{c}^*},\ a_5 = e^{\mathring{y}^*-\mathring{c}^*},\ a_6 = e^{\mathring{d}^*+\mu-\mathring{c}^*}, \\ a_7 &= e^{r_d^*+\mathring{d}^*},\ a_8 = e^{r_l^*+E(ln\theta_t)+\mathring{d}^*}(1-\kappa), \\ a_9 &= \chi_0(\kappa-\tau)(1+\chi)e^{(1+\chi)\mathring{n}^*},\ b_1 = \frac{\chi_0}{1+\chi}e^{(1+\chi)\mathring{n}^*-\mathring{c}^*}. \end{aligned}$$

Using the expressions for a_4, a_5, a_6, a_7, a_8 and a_9 above, the steady-state conditions (4.25) and (4.27) can be re-written as

$$a_5 = 1 + a_4 + a_6,\ a_9 = (1+\chi)(a_8 - a_7), \text{respectively.}$$

Note also that the above system of equations can be simplified by substituting $\widetilde{\mathring{d}}_t$ from equation (A14) in equations (A12) and (A13). The log-linearized equations (A10), (A12) and (A13) can then be re-written as the expressions in the main body of the paper.

A2.4 Model solution

We use the quadratic determinantal equation (QDE) approach of Binder and Pesaran (1995, 1997) to solve the rational expectations equations given by (4.28) to (4.33). Let $\mathbf{x}_t = (\widetilde{\mathring{c}}_t, \widetilde{\mathring{d}}_{t+1}, \widetilde{r}_{lt}, \widetilde{r}_{dt}, \widetilde{\mathring{y}}_t, \widetilde{\mathring{n}}_t)'$, and the above system of equations are written as

$$\mathbf{H}_0\mathbf{x}_t = \mathbf{H}_1\mathbf{x}_{t-1} + \mathbf{H_2}E(\mathbf{x}_{t+1}|\Omega_t) + \mathbf{v}_t, \tag{A15}$$

where

$$\mathbf{v}_t = \tilde{\mathbf{G}}_0\boldsymbol{\xi}_t + \tilde{\mathbf{G}}_1\boldsymbol{\xi}_{t-1}, \quad \boldsymbol{\xi}_t = \begin{pmatrix} u_t \\ \widetilde{\ln\theta_t} \end{pmatrix}, \tag{A16}$$

and

$$\boldsymbol{\xi}_t = \mathbf{R}\boldsymbol{\xi}_{t-1} + \boldsymbol{\psi}_t, \quad \mathbf{R} = \begin{pmatrix} \rho_u & 0 \\ 0 & \rho_\theta \end{pmatrix}, \quad \boldsymbol{\psi}_t = \begin{pmatrix} \varepsilon_t \\ \widetilde{\eta}_t \end{pmatrix}, \tag{A17}$$

with $\varepsilon_t \sim \mathbb{N}(0, \sigma_\varepsilon^2)$, and $\widetilde{\eta}_t \sim \mathbb{N}(0, \sigma_\eta^2)$.

The matrices $\mathbf{H}_0$, $\mathbf{H}_1$, $\mathbf{H}_2$, $\tilde{\mathbf{G}}_0$, $\tilde{\mathbf{G}}_1$ and $\mathbf{x}_t$ are given by

$$\mathbf{H}_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -a_1 \\ -1 & -a_6 & 0 & 0 & 1+a_4+a_6 & 0 \\ 0 & 0 & \frac{\alpha}{(1-\alpha)\chi} & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 1 & -(1+\alpha\chi) \\ 0 & 0 & -1 & 0 & 0 & (1+\chi) \end{pmatrix}, \mathbf{H}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathbf{H}_2 = \begin{pmatrix} 1 & 0 & 0 & -a_3 & 0 & -a_1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tilde{\mathbf{G}}_0 = \begin{pmatrix} a_2 & 0 \\ a_6 & a_4 \\ 0 & 0 \\ 0 & 1 \\ \varphi & 0 \\ 0 & 1 \end{pmatrix}, \tilde{\mathbf{G}}_1 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{\rho_u}{1+\chi} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{x}_t = \begin{pmatrix} \widetilde{\mathring{c}}_t \\ \widetilde{\mathring{d}}_{t+1} \\ \widetilde{r}_{lt} \\ \widetilde{r}_{dt} \\ \widetilde{\mathring{y}}_t \\ \widetilde{\mathring{n}}_t \end{pmatrix}.$$

Note that $\boldsymbol{\psi}_t$ is a serially uncorrelated vector process with zero mean. Note also that $\mathbf{H}_0$ is non-singular, and using equation (A16), (A15) can be written as

$$\mathbf{x}_t = \mathbf{H}_0^{-1}\mathbf{H}_1\mathbf{x}_{t-1} + \mathbf{H}_0^{-1}\mathbf{H_2}E(\mathbf{x}_{t+1}|\Omega_t) + \mathbf{H}_0^{-1}\tilde{\mathbf{G}}_0\boldsymbol{\xi}_t + \mathbf{H}_0^{-1}\tilde{\mathbf{G}}_1\boldsymbol{\xi}_{t-1},$$

or

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{B}E(\mathbf{x}_{t+1}|\Omega_t) + \mathbf{G}_0\boldsymbol{\xi}_t + \mathbf{G}_1\boldsymbol{\xi}_{t-1}, \tag{A18}$$

where $\mathbf{A} = \mathbf{H}_0^{-1}\mathbf{H}_1$, $\mathbf{B} = \mathbf{H}_0^{-1}\mathbf{H}_2$, $\mathbf{G}_0 = \mathbf{H}_0^{-1}\tilde{\mathbf{G}}_0$, and $\mathbf{G}_1 = \mathbf{H}_0^{-1}\tilde{\mathbf{G}}_1$.

The rational expectations solution of (A18) is given by[29]

$$\mathbf{x}_t = \mathbf{C}\mathbf{x}_{t-1} + \mathbf{D}_0\boldsymbol{\xi}_t + \mathbf{D}_1\boldsymbol{\xi}_{t-1}, \tag{A19}$$

where

$$\begin{aligned}
\mathbf{B}\mathbf{C}^2 - \mathbf{C} + \mathbf{A} &= \mathbf{0}, && \text{(A20)}\\
\mathbf{D}_1 &= (\mathbf{I} - \mathbf{B}\mathbf{C})^{-1}\mathbf{G}_1, && \text{(A21)}\\
(\mathbf{I} - \mathbf{B}\mathbf{C})\mathbf{D_0} - \mathbf{B}\mathbf{D}_0\mathbf{R} &= \mathbf{G}_0 + \mathbf{B}(\mathbf{I} - \mathbf{B}\mathbf{C})^{-1}\mathbf{G}_1. && \text{(A22)}
\end{aligned}$$

Following Binder and Pesaran (1995, 1997), we use the quadratic determinantal equation (QDE) in (A20) to solve for $\mathbf{C}$. After obtaining $\mathbf{C}$, (A21) can be used to obtain $\mathbf{D}_1$. To solve for $\mathbf{D}_0$, first write equation (A22) as

$$\mathbf{D}_0 - \mathbf{Q}_0\mathbf{D}_0\mathbf{R} = \mathbf{Q}_1, \tag{A23}$$

where

$$\begin{aligned}
\mathbf{Q}_0 &= (\mathbf{I} - \mathbf{B}\mathbf{C})^{-1}\mathbf{B},\\
\mathbf{Q}_1 &= (\mathbf{I} - \mathbf{B}\mathbf{C})^{-1}\mathbf{G}_0 + (\mathbf{I} - \mathbf{B}\mathbf{C})^{-1}\mathbf{B}(\mathbf{I} - \mathbf{B}\mathbf{C})^{-1}\mathbf{G_1}.
\end{aligned}$$

Then, using results in Magnus and Neudecker (1988) (pp. 30-31), we find

$$vec(\mathbf{D}_0) - \left(\mathbf{R}' \otimes \mathbf{Q}_0\right) vec(\mathbf{D}_0) = vec\left(\mathbf{Q}_1\right),$$

which yields

$$vec(\mathbf{D}_0) = \left[\mathbf{I} - \left(\mathbf{R}' \otimes \mathbf{Q}_0\right)\right]^{-1} vec\left(\mathbf{Q}_1\right).$$

A2.5 Solution of the Canonical Rational Expectations model

Proof. We show that $\mathbf{x}_t = \mathbf{C}\mathbf{x}_{t-1} + \mathbf{D}_0\boldsymbol{\xi}_t + \mathbf{D}_1\boldsymbol{\xi}_{t-1}$ is indeed a solution of

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{B}E(\mathbf{x}_{t+1}|\Omega_t) + \mathbf{G}_0\boldsymbol{\xi}_t + \mathbf{G}_1\boldsymbol{\xi}_{t-1}. \tag{A24}$$

First, note that the left-hand side of (A24) can be written as

$$\begin{aligned}
\mathbf{x}_t &= \mathbf{C}\mathbf{x}_{t-1} + \mathbf{D}_0\boldsymbol{\xi}_t + \mathbf{D}_1\boldsymbol{\xi}_{t-1}\\
&= \mathbf{C}(\mathbf{C}\mathbf{x}_{t-2} + \mathbf{D}_0\boldsymbol{\xi}_{t-1} + \mathbf{D}_1\boldsymbol{\xi}_{t-2}) + \mathbf{D}_0\boldsymbol{\xi}_t + \mathbf{D}_1\boldsymbol{\xi}_{t-1}\\
&= \mathbf{C}^2\mathbf{x}_{t-2} + \mathbf{D}_0\boldsymbol{\xi}_t + (\mathbf{C}\mathbf{D}_0 + \mathbf{D}_1)\boldsymbol{\xi}_{t-1} + \mathbf{C}\mathbf{D}_1\boldsymbol{\xi}_{t-2}.
\end{aligned}$$

To evaluate the right-hand side of (A24), note that

$$\begin{aligned}
E(\mathbf{x}_{t+1}|\Omega_t) &= E(\mathbf{C}\mathbf{x}_t + \mathbf{D}_0\boldsymbol{\xi}_{t+1} + \mathbf{D}_1\boldsymbol{\xi}_t|\Omega_t)\\
&= \mathbf{C}\mathbf{x}_t + \mathbf{D}_0E(\boldsymbol{\xi}_{t+1}|\Omega_t) + \mathbf{D}_1\boldsymbol{\xi}_t\\
&= \mathbf{C}(\mathbf{C}\mathbf{x}_{\mathbf{t-1}} + \mathbf{D_0}\boldsymbol{\xi}_{\mathbf{t}} + \mathbf{D_1}\boldsymbol{\xi}_{\mathbf{t-1}}) + \mathbf{D_0}\mathbf{R}\boldsymbol{\xi}_{\mathbf{t}} + \mathbf{D}_1\boldsymbol{\xi}_t\\
&= \mathbf{C}^2\mathbf{x}_{t-1} + (\mathbf{C}\mathbf{D_0} + \mathbf{D_0}\mathbf{R} + \mathbf{D}_1)\,\boldsymbol{\xi}_t + \mathbf{C}\mathbf{D_1}\boldsymbol{\xi}_{t-1}\\
&= \mathbf{C}^2(\mathbf{C}\mathbf{x}_{t-2} + \mathbf{D}_0\boldsymbol{\xi}_{t-1} + \mathbf{D}_1\boldsymbol{\xi}_{t-2}) + (\mathbf{C}\mathbf{D_0} + \mathbf{D_0}\mathbf{R} + \mathbf{D}_1)\,\boldsymbol{\xi}_t + \mathbf{C}\mathbf{D_1}\boldsymbol{\xi}_{t-1}\\
&= \mathbf{C}^3\mathbf{x}_{t-2} + (\mathbf{C}\mathbf{D_0} + \mathbf{D_0}\mathbf{R} + \mathbf{D}_1)\,\boldsymbol{\xi}_t + (\mathbf{C}^2\mathbf{D}_0 + \mathbf{C}\mathbf{D_1})\boldsymbol{\xi}_{t-1} + \mathbf{C}^2\mathbf{D}_1\boldsymbol{\xi}_{t-2}.
\end{aligned}$$

[29] The proof that (A19) is indeed a solution to (A18) is given in the Appendix A2.5.

Therefore, the right-hand side of (A24) is given by

$$\begin{aligned}
& \mathbf{A}\mathbf{x}_{t-1}+\mathbf{B}E(\mathbf{x}_{t+1}|\Omega_t)+\mathbf{G}_0\boldsymbol{\xi}_t+\mathbf{G}_1\boldsymbol{\xi}_{t-1} \\
= \; & \mathbf{A}(\mathbf{C}\mathbf{x}_{t-2}+\mathbf{D_0}\boldsymbol{\xi}_{t-1}+\mathbf{D_1}\boldsymbol{\xi}_{t-2})+\mathbf{B}E(\mathbf{x}_{t+1}|\Omega_t)+\mathbf{G}_0\boldsymbol{\xi}_t+\mathbf{G}_1\boldsymbol{\xi}_{t-1} \\
= \; & \mathbf{A}(\mathbf{C}\mathbf{x}_{t-2}+\mathbf{D_0}\boldsymbol{\xi}_{t-1}+\mathbf{D_1}\boldsymbol{\xi}_{t-2})+\mathbf{G}_0\boldsymbol{\xi}_t+\mathbf{G}_1\boldsymbol{\xi}_{t-1} \\
& +\mathbf{B}\mathbf{C}^3\mathbf{x}_{t-2}+(\mathbf{BCD_0}+\mathbf{BD_0R}+\mathbf{BD}_1)\,\boldsymbol{\xi}_t+(\mathbf{BC}^2\mathbf{D}_0+\mathbf{BCD_1})\boldsymbol{\xi}_{t-1}+\mathbf{BC}^2\mathbf{D}_1\boldsymbol{\xi}_{t-2} \\
= \; & (\mathbf{AC}+\mathbf{BC}^3)\mathbf{x}_{t-2}+(\mathbf{G}_0+\mathbf{BCD_0}+\mathbf{BD_0R}+\mathbf{BD}_1)\boldsymbol{\xi}_t \\
& +(\mathbf{AD_0}+\mathbf{G}_1+\mathbf{BC}^2\mathbf{D}_0+\mathbf{BCD_1})\boldsymbol{\xi}_{t-1}+(\mathbf{AD_1}+\mathbf{BC}^2\mathbf{D}_1)\boldsymbol{\xi}_{t-2}.
\end{aligned}$$

Equating the coefficients of $\mathbf{x}_{t-2}$, $\boldsymbol{\xi}_t$, $\boldsymbol{\xi}_{t-1}$ and $\boldsymbol{\xi}_{t-2}$ on both sides of (A24), we have

$$\begin{aligned}
\mathbf{x}_{t-2} &: \; \mathbf{C}^2=\mathbf{AC}+\mathbf{BC}^3, \\
\boldsymbol{\xi}_t &: \; \mathbf{D}_0=\mathbf{G}_0+\mathbf{BCD_0}+\mathbf{BD_0R}+\mathbf{BD}_1, \\
\boldsymbol{\xi}_{t-1} &: \; \mathbf{CD}_0+\mathbf{D}_1=\mathbf{AD_0}+\mathbf{G}_1+\mathbf{BC}^2\mathbf{D}_0+\mathbf{BCD_1}, \\
\boldsymbol{\xi}_{t-2} &: \; \mathbf{CD}_1= \; \mathbf{AD_1}+\mathbf{BC}^2\mathbf{D}_1.
\end{aligned}$$

The above conditions can be simplified to

$$\begin{aligned}
\mathbf{BC}^2+\mathbf{A}-\mathbf{C} &= 0, \; \mathbf{D}_1=(\mathbf{I}-\mathbf{BC})^{-1}\mathbf{G}_1, \\
(\mathbf{I}-\mathbf{BC})\mathbf{D}_0-\mathbf{BD}_0\mathbf{R} &= \mathbf{G}_0+\mathbf{BD}_1,
\end{aligned}$$

that is,

$$(\mathbf{I}-\mathbf{BC})\mathbf{D_0}-\mathbf{BD}_0\mathbf{R}=\mathbf{G}_0+\mathbf{B}(\mathbf{I}-\mathbf{BC})^{-1}\mathbf{G}_1,$$

which is the solution given by (A19). ■

A2.6 Impulse responses

We compute the impulse responses to a credit shock following Koop, Pesaran, and Potter (1996) and Pesaran and Shin (1998), since the generalized impulse response functions offer more modelling flexibility.

Definition 1 *The "generalized impulse response function"(GIRF) of a vector process $\mathbf{x}_t$ of dimension $p \times 1$ is defined by*

$$GI_{\mathbf{x}}(h,\delta,\Omega_{t-1})=E(\mathbf{x}_{t+h}|\boldsymbol{\psi}_t=\delta,\Omega_{t-1})-E(\mathbf{x}_{t+h}|\Omega_{t-1}),$$

where Ω_{t-1} is the common information set at time $t-1$, and δ is a vector of shocks.

Recall that model solutions are given by equation (A19). Using definition 1 of GIRFs and denoting $GI_{\mathbf{x}}(h,\delta,\Omega_{t-1})$ by $GI_{\mathbf{x}}(h)$ for simplicity, we have:

$$\begin{aligned}
GI_{\mathbf{x}}(h) &= \mathbf{C}GI_{\mathbf{x}}(h-1)+\mathbf{D}_0GI_{\xi}(h)+\mathbf{D}_1GI_{\xi}(h-1), \text{ for } h=0,1,2,3..., \\
GI_{\xi}(h) &= \mathbf{R}GI_{\xi}(h-1), \quad \text{for } h=1,2,3..., \\
GI_{\mathbf{x}}(h) &= 0, \quad \text{for } h<0, \quad \text{and} \quad GI_{\xi}(h)=0, \quad \text{for } h<0.
\end{aligned}$$

For the technology shock on impact, we have

$$GI_{\xi}(0)=GI_{\psi}(0)=\frac{1}{\sqrt{\mathbf{e}_1'\mathbf{Cov}(\boldsymbol{\psi}_t)\mathbf{e}_1}}\mathbf{Cov}(\boldsymbol{\psi}_t)\mathbf{e}_1 \; , \tag{A25}$$

where $\mathbf{e}_1 = (1,0)'$ and

$$\mathbf{Cov}(\boldsymbol{\psi}_t) = \begin{pmatrix} \sigma_\varepsilon^2 & \rho_{\varepsilon\eta}\sigma_\varepsilon\sigma_\eta \\ \rho_{\varepsilon\eta}\sigma_\varepsilon\sigma_\eta & \sigma_\eta^2 \end{pmatrix}.$$

To obtain the GIRFs for the credit shock, we need to replace $\mathbf{e}_1$ in (A25) by $\mathbf{e}_2 = (0,1)'$. In the standard case where technology shocks and credit shocks are assumed to be uncorrelated, we have

$$\mathbf{Cov}(\boldsymbol{\psi}_t) = \begin{pmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix},$$

and (A25) can be simplified such that $GI_\xi(0) = (\sigma_\varepsilon, 0)'$ for a one-standard-deviation positive shock to technology, and $GI_\xi(0) = (0, \sigma_\eta)'$ for a one-standard-deviation positive shock to credit.

B Data Appendix

B1 Data sources

The main sources of the time-series data are Datastream and the Federal Reserve.

B1.1 Deposit rate

The Datastream series "U.S. CD Secondary Market 1 Month - Middle Rate" (FRCDS1M), "U.S. CD Secondary Market 3 Month - Middle Rate" (FRCDS3M), "U.S. CD Secondary Market 6 Month - Middle Rate" (FRCDS6M) are used to construct the deposit rate series used in the paper. The deposit rate is given by the arithmetic average of the one-month, three-month and six-month certificate of deposit (CD) series. The source for the Datastream series is the Federal Reserve and the series are measured in per cent per annum. The middle rate refers to the midpoint between the bid and offered rates.

We decided to use the Datastream (Federal Reserve) series instead of the International Finance Statistics series "U.S. Certificate of Deposit rate 3 months (secondary market)" (60LC.ZF), since the Federal Reserve has a broader coverage of CD rates, to include the one-month and six-month CD rates, which we use to construct the final deposit-rate series.

B1.2 Loan rate

We take the Datastream series "U.S. Bank Prime Loan - Middle Rate" (FRBKPRM) as our preferred measure of the loan rate. The source of this series is the Federal Reserve. The U.S. Bank Prime Loan Rate is the rate posted by a majority of top 25 (by assets in domestic offices) insured U.S. chartered commercial banks. Prime is one of the several base rates used by banks to price short-term business loans. Weekly figures are averages of the seven calendar days ending on Wednesday of the current week; monthly figures include each calendar day in the month. The interest rate is annualized using bank interest, and the middle rate refers to the midpoint between the bid and offered rates. We decided to use the Datastream (Federal Reserve) series instead of the IFS series "Bank prime loan rate" (60P..ZF) to be consistent with the source of our deposit-rate series.

B1.3 CPI series

Note that, both the series for the loan rate and deposit rate are in nominal terms. In order to estimate our model, we need to convert the nominal loan and deposit rates to real series, using a measure of the inflation rate. We take the CPI series "Consumer Price Index for All Urban Consumers: All Items" from the Federal Reserve Bank of St Louis (CPIAUCSL). The series is seasonally adjusted and indexed at the years 1982-84 (=100).

B1.4 Consumption

We use the data series "U.S. Real Personal Consumption Expenditures" (PCECC96) from Federal Reserve Bank of St Louis as our measure for consumption. The data source is U.S. Department of Commerce: Bureau of Economic Analysis, and the quarterly series is seasonally adjusted in billions of chained 2005 dollars.

B1.5 Output

The gross domestic product series is taken from the Federal Reserve Bank of St Louis (GDPC96). The series is seasonally adjusted in billions of chained 2005 dollars.

B1.6 Bank deposits

We use the Federal Reserve series "U.S. commercial bank liabilities–deposits and borrowing" (Federal Reserve H.8 Table) as a measure of bank deposits. This series is measured in billions of U.S. dollars and current prices, and is seasonally adjusted. According to the Federal Reserve, www.federalreserve.gov/releases/h8/current/default.htm (page 3), deposits are composed of large time deposits and other deposits.

B1.7 Bank credit

The data series on bank credit ("U.S. Commercial bank assets–bank credit") is taken from the Federal Reserve H.8 Table, http://www.federalreserve.gov/releases/h8/current/default.htm (page 2). According to the Federal Reserve, bank credit is composed of securities in bank credit, and loans and leases in bank credit. Loans and leases in bank credit include commercial and industrial loans, real estate loans and consumer loans. We use the credit series from the Federal Reserve, rather than the IFS measure "Bank credit to the private sector" used in the empirical paper, because there is no matching deposit series from the IFS, while such series exists in the Federal Reserve. The data series on bank credit is seasonally adjusted and expressed in billions of dollars in current prices.

B1.8 Wages

The Federal Reserve series on "Average Hourly Earnings of Production and Nonsupervisory Employees: Total Private" is used as a proxy for wages. The data source is the Federal Reserve Bank of St Louis (AHETPI), taken from the U.S. Department of Labor: Bureau of Labor Statistics. The data series is seasonally adjusted and expressed in dollars per hour.

B1.9 Hours worked

The data series on "Average Weekly Hours Private Non-Farm United States" is taken from Datastream (USHKIP..O). The primary source of the data is the U.S. Department of Labor: Bureau of Labor Statistics (USDOL). The series is measured in hours and seasonally adjusted. It captures the expected or actual period of employment for the week, usually expressed in the number of hours.

B1.10 Employment

The employment data ("All Employees: Total Private Industries") are from the Federal Reserve Bank of St Louis (USPRIV). The series is measured in thousands of persons and seasonally adjusted. The source of the data is the U.S. Department of Labor: Bureau of Labor Statistics.

B1.11 U.S. population

The U.S. population data ("U.S. Population: Mid-Month") are from the Federal Reserve Bank of St Louis (POPTHM). The series is measured in thousands. The source is the U.S. Department of Commerce: Bureau of Economic Analysis.

B1.12 Liabilities of non-financial non-farm corporate business

We take the Federal Reserve Flow of Funds series (levels data, Table L.102) on the liabilities of non-financial non-farm corporate business. In particular, we are interested in the series on corporate bonds (Z1/FL103163003.Q), corporate equities (Z1/FL103164103), and loans and short-term paper (Z1/FL104140005.Q). The series are measured in millions of U.S. dollars.

The data series are available upon request.