Nonlinearities in Cross-Country Growth Regressions: A Bayesian Averaging of Thresholds (BAT) Approach

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Abstract

We propose a Bayesian Averaging of Thresholds (BAT) approach for assessing the existence and quantifying the effect of threshold effects in cross-country growth regressions in the presence of model uncertainty. The BAT method extends the Bayesian Averaging of Classical Estimates (BACE) approach proposed by Sala-i-Martin, Doppelhofer, and Miller (2004) by allowing for uncertainty over nonlinear threshold effects. We apply our method to a set of determinants of long-term economic growth in a cross section of 88 countries. Our results suggest that when model uncertainty is taken into account there is no evidence for robust threshold effects caused by the *Initial Income*, measured by GDP per capita in 1960, but that the *Number of Years an Economy Has Been Open* is an important source of nonlinear effects on growth.

Keywords: Model Uncertainty, Model Averaging, Threshold Estimation, Non-Linearities, Determinants of Economic Growth

JEL Classifications: C11, C15, O20, O50

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1 Introduction

Following the influential contributions of Kormendi and Meguire (1985) and Barro (1991), the empirical growth literature has used cross-country regressions to identify variables that are robustly (partially) correlated to growth of per capita GDP. Many different economic, social and political variables have been proposed as important determinants of economic growth. Durlauf and Quah (1999), for instance, name more than eighty variables that have been included at least once in a cross-country growth regression. Brock and Durlauf (2001) refer to this problem also as "open-endedness" of theories of economic growth.

Levine and Renelt (1992) gave a first assessment of the robustness of growth determinants by applying a version of Leamer (1983)'s extreme bounds analysis. Levine and Renelt checked for robustness when changing the set of conditioning variables and concluded that almost no variable used by Kormendi and Meguire (1985) and Barro (1991) is robustly correlated with average GDP per capita growth. Sala-i-Martin (1997) criticizes the extreme bounds test as too strong for any variable to pass it in the framework of empirical growth research. Instead Sala-i-Martin proposes to analyze the entire distribution of coefficients of interest when changing the set of explanatory variables. For computational reasons, both Levine and Renelt (1992) and Sala-i-Martin (1997) restrict models to a particular size and fix a number of regressors.

Bayesian model averaging (BMA) addresses model uncertainty over several dimensions of the model space, such as the choice of explanatory variables and estimation of associated parameters of interest or size of the model. The basic idea of model averaging is essentially an application of Bayes rule, and can at least be traced back to Jeffreys (1961). Leamer (1978, chapter 4) discusses model averaging in detail in an important contribution and shows how to use either standard (conjugate) Bayesian methods or limiting Bayesian cases, such as diffuse priors and sample-dominated (Classical) priors. Following Leamer (1978), Sala-i-Martin, Doppelhofer and Miller (2004) - henceforth SDM (2004) - introduce an approach which they call Bayesian Averaging of Classical Estimates (BACE) which introduces minimal prior information by using sample-dominated Bayesian priors and least-squares (Classical) estimates. The resulting posterior model weights are proportional to the Bayesian Information Criterion (BIC).[1] Raftery (1995) also proposes to combine BIC model

[1]Wasserman (2000) contains a nice dicussion of BMA with Bayesian, limiting Bayesian (BIC)

weights and maximum likelihood (OLS) estimates for model selection. SDM (2004) differ from Raftery (1995) in the specification of prior probability over the model space and sampling method. Our proposed BAT approach uses BACE to estimate the posterior distribution of the parameter of interest. In particular, the prior over the model space introduced in section 2.1 of this paper builds on the idea of specifying relatively parsimonious models *a priori* (see also Mitchell and Beauchamp, 1988, and SDM ,2004).

Several studies have applied model averaging techniques to growth empirics. Using cross-country data, Fernandez, Ley and Steel (2001) and SDM (2004) find a sizeable group of variables that are important determinants of long-term economic growth, supporting Sala-i-Martin's (1997) findings. However, all the methods mentioned above approach the issue of model uncertainty in growth regressions under the assumption that the relationship between the explanatory variables and the growth rate is *linear*. This essentially implies that the effect associated with a particular variable is constant across subsamples of the data used. Various deviations from the linear paradigm have been tested in the empirical literature and there is ample evidence of parameter heterogeneity, multiple regimes and threshold nonlinearities in cross country growth regressions (see e.g. Durlauf and Johnson, 1995, Durlauf, Kourtellos and Minkin, 2001, Masanjala and Papageorgiou, 2004, or Papageorgiou, 2002).[2]

Many theoretical growth models deliver multiple steady states (e.g. Azariadis and Drazen, 1990). Masanjala and Papageorgiou (2004) explicitly model nonlinearities in the aggregate production function. Easterly (2006) critically investigates the importance of "poverty traps" that have been very influential in economic policy-making, motivating for example some of the "Millenium Goals" proposed by the United Nations. The existence and economic importance of nonlinearities and threshold effects among determinants of economic growth plays thus a major role in the present policy discussion on global development strategies.

In this paper we propose a *Bayesian Averaging of Thresholds* (BAT) approach that explicitly allows for non-linearities in the form of level-dependent parameter hetero-

and alternative weights based on information criteria. For a further introduction to model averaging see Hoeting, Madigan, Raftery and Volinsky (1999) or Doppelhofer and Durlauf (2007).

[2]Crespo Cuaresma (2002) presents a robustness exercise where threshold nonlinearity is explicitly accounted for but model size uncertainty is not dealt with.

geneity as usually specified by threshold models (see Hansen, 1996, 2000). We allow for uncertainty over possible threshold effects and associated threshold observations by extending the BACE method of SDM (2004) and estimating the posterior distribution of these quantities of interest. Our proposed method estimates threshold values under model uncertainty based on the inspection of the posterior inclusion probability of the threshold parameter. Note that the distribution of threshold effects and interactions are calculated by averaging over many possible specifications and are therefore not conditional on a particular model. The resulting inference and policy analysis is therefore taking into account uncertainty over models, including nonlinear effects.

The paper contributes to the literature on the empirics of economic growth nonlinearities as follows: First, our proposed BAT method estimates the entire posterior distribution of thresholds and associated nonlinear effects. We only need to specify a set of candidate threshold variables (motivated by the literature on growth nonlinearities discussed above) and prior parameters for the expected number of explanatory and threshold variables being present in the model. Second, we show that there is a relatively small set of robust nonlinear effects once we allow for uncertainty over the number of threshold variables and threshold observations. In particular, conditioning on the *Number of Years an Economy Has Been Open* affects the size and significance of the effect of some other determinants of growth, whereas *Initial Income* appears to play a much less prominent role as a threshold variable when allowing for uncertainty over the number of variables causing nonlinearities. This result can be contrasted with the vast evidence of (model specific) nonlinearities found in earlier studies. Third, a technical contribution of the paper is to extend the BACE sampling method to the estimation of the distribution of nonlinear effects and associated thresholds. A key role is played by the specification of priors of inclusion of threshold variables and thresholds. We choose a relatively modest prior model size, implying a preference for more parsimonious models a priori.

Our method is closely related to Bayesian methods of model-based clustering using Bayes factors (see for example Fraley and Raftery, 2002, Handcock, Raftery and Tantrum, 2007) and can be reinterpreted as a parametric cluster building procedure such as that put forward in Raftery and Dean (2006). Although our proposal differs from the recent literature in its implementation, it also makes use of model averaging through Bayes factors and retains the modelling spirit of the Bayesian model-based clustering literature. The BAT method is also closely related to the integrated BMA (or IBMA) approach used by Eicher, Papageorgiou and Roehn (this issue). The main difference between BAT and this approach is the prior over the model space and the sampling method discussed in section 2 below.

The paper is organized as follows. Section 2 presents the methodology proposed to account for threshold nonlinearity in cross-country growth regressions in the presence of model uncertainty, which we call Bayesian Averaging of Thresholds (BAT). Section 3 reports the results of the robustness analysis for a dataset formed by the 21 variables that SDM (2004) find robust in their analysis and two potential threshold variables: *Initial Income* measured by the level of GDP per capita in 1960 and the *Number of Years an Economy Has Been Open*. Section 4 concludes and presents further paths of research.

2 Bayesian Averaging of Thresholds (BAT)

2.1 Thresholds and model uncertainty: BAT

The BAT approach is aimed at evaluating the existence and robustness of nonlinearities in regressions with model uncertainty. It is a generalization of the BACE approach in SDM (2004) which allows for threshold effects of certain variables on the regression parameters.

Consider a set of variables that are potentially related to growth, $\mathbf{X}$, and a set of variables that are potentially causing threshold-nonlinearity in the growth regression, $\mathbf{Z}$. $\mathbf{Z}$ may or may not be a subset of $\mathbf{X}$. The stylized nonlinear model we are considering is

$$\gamma = \alpha + \sum_{k=1}^{n} \beta_k x_k + \sum_{j=1}^{m} \left[(\alpha_j^* + \sum_{k=1}^{n} \beta_{jk}^* x_k) \mathsf{I}(z_j \leq \tau_j) \right] + \varepsilon, \tag{1}$$

where γ is a vector of T observations of growth rates of GDP per capita, $x_1, \ldots, x_n \in \mathbf{X}$, $z_1, \ldots, z_p \in \mathbf{Z}$, $\mathsf{I}(\cdot)$ is the indicator function, taking value one if the argument is true and zero otherwise and ε is an error term assumed uncorrelated across cross-sectional units and with constant variance σ^2. There are therefore m variables inducing nonlinearity in equation (1). For simplicity we assume that the nonlinearity which is induced by variable z_i is independent from the regime in which an observation is according to another threshold variable z_j, for $i \neq j$. Although the BAT method can be generalized in a straightforward manner to the setting with dependent nonlinearities, this assumption avoids having to use cross-products of indicator functions in (1), which would increase the computational time of the procedure significantly.

Since we are explicitly dealing with model uncertainty, n and m are not assumed to be known. In the spirit of SDM (2004), we assume prior inclusion probabilities for the elements of $\mathbf{X}$ and $\mathbf{Z}$. In particular, we assume a prior inclusion probability of $\bar{n}/N$ for the variables in $\mathbf{X}$ and a prior inclusion probability of $\bar{m}/M$ for the variables in $\mathbf{Z}$, where $N = card(\mathbf{X})$ and $M = card(\mathbf{Z})$. This choice of prior inclusion probability implies a prior preference for relatively parsimonious models.[3] This implies that the prior expected number of included $\mathbf{X}$-variables in the regression (excluding the constant) is $\bar{n}$ and the prior expected number of variables inducing nonlinearities is $\bar{m}$, leading to an expected model size of $(\bar{n}+1)(\bar{m}+1)$. Figure 1 shows the implied prior distributions of our benchmark case with prior model sizes $\bar{n} = 5$, $\bar{m} = 1$, as well as the uniform prior case with $\bar{n} = 10.5$, $\bar{m} = 1$. Notice that we use a uniform prior of including threshold variables in $\mathbf{Z}$, implying prior inclusion probability equal to 0.25, 0.5 and 0.25 for including no threshold, one threshold and two threshold variables, respectively.

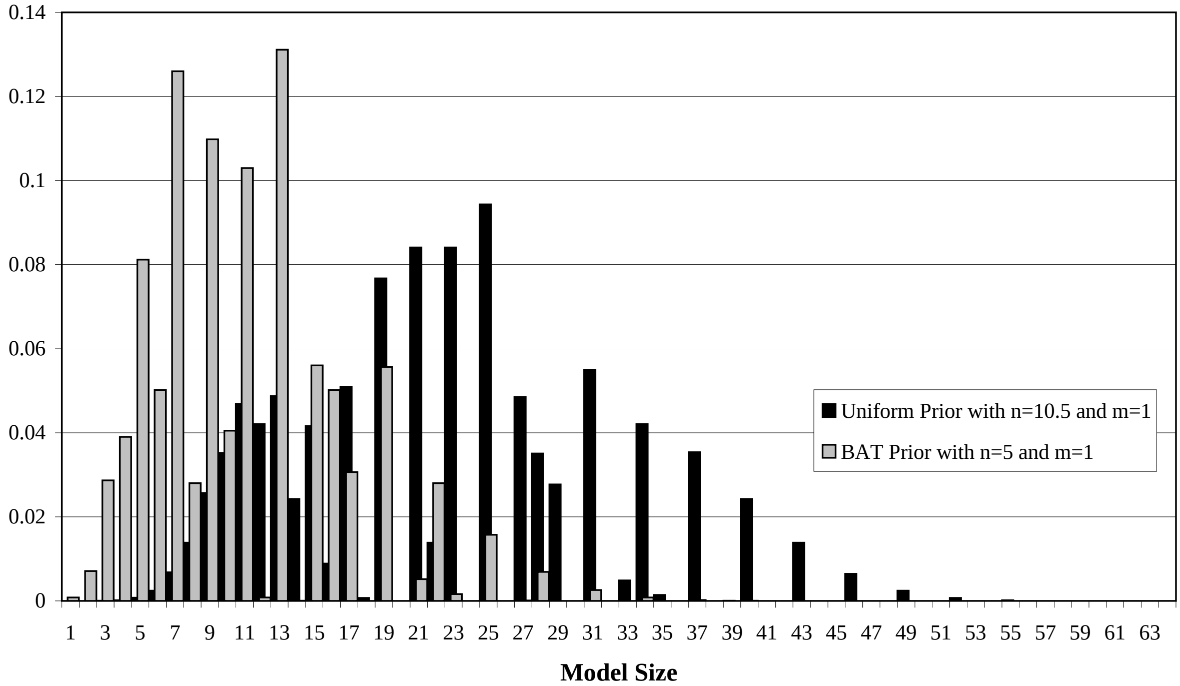
Given the choice of regressors from $\mathbf{X}$ and threshold variable from $\mathbf{Z}$ we proceed as follows to choose a threshold value z_j. We assign a diffuse prior to values of z_j actually observed in the sample after trimming $100\times\theta\%$ of the observations from each extreme of the empirical distribution. We impose this trimming of the distribution to avoid that one of the resulting regimes contains too few observations which could lead to unreliable estimation results. Therefore, the prior inclusion probability of $z_{j,i}$ (observation i in threshold variable z_j) as a threshold in (1) conditional on the inclusion of z_j as a threshold variable is uniform and given by $1/[T(1-2\theta)]$.[4]

It should be noted that this prior specification for the threshold values uses sample information and could thus be controversial if the Bayesian approach is to be taken

[3]For a discussion of a prior with a mass at zero corresponding to the prior probability of *exclusion* see Mitchell and Beauchamp (1988). The choice of prior model size can be contrasted with a *uniform* prior probability over the space of models which implies a prior inclusion probability equal to 1/2 for variables in $\mathbf{X}$ and $\mathbf{Z}$.

[4]For two consecutive ordered observations $z_{j,i}$ and $z_{j,i+1}$, any threshold value τ_j in the interval $[z_{j,i}, z_{j,i+1})$ leads to the same variable $\mathsf{I}(z_j \leq \tau_j)$. This implies that we only need to define a discrete prior probability for each realized value of z_j, instead of a continuous prior density on the support of z_j.

Figure 1: Prior Probabilities by Model Size: Benchmark BAT Case with Prior Model Sizes $\bar{n} = 5$, $\bar{m} = 1$ and Uniform Prior with $\bar{n} = 10.5$, $\bar{m} = 1$.



literally. A related issue is the ordering of variables in the cross-sectional context, which is straightforward in the time-series context.[5] We proceed in the estimation by assuming a "natural ordering" of threshold variables **Z** from smallest to largest realized value and applying the trimming and selection of threshold to the ordered observations. Given the obvious difficulties involved in setting bounds to the prior distribution of the threshold values without observing the realized sample of the threshold variable and since using sample information for the prior specification is standard in the Bayesian literature of threshold estimation (see, for example, Koop and Potter, 1999), we decided to use this mixed approach to threshold estimation (see Phillips (1991) for a reference to this controversy).

Given the setting put forward above, the prior probability attached to a model

[5]We thank Hashem Pesaran for raising this point in discussions with us.

containing n **X**-variables and m threshold variables with thresholds $\{\tau_1, \ldots, \tau_m\}$ is[6]

$$P(M_{n,m,\tau_1,\ldots,\tau_m}) = (\bar{n}/N)^n(1-\bar{n}/N)^{N-n}(\bar{m}/M)^m(1-\bar{m}/M)^{M-m}[1/[T(1-2\theta)]]^m. \tag{2}$$

With this diffuse prior specification and further assuming a diffuse prior with respect to σ, the odds ratio for two models can be approximated (see Leamer, 1978, and Schwarz, 1978) as

$$\frac{P(M_0|Y)}{P(M_1|Y)} = \frac{P(M_0)}{P(M_1)}T^{(k_0-k_1)/2}\left(\frac{SSE_0}{SSE_1}\right)^{-T/2}, \tag{3}$$

where k_i is the size of model i, $P(\cdot|Y)$ refers to posterior probabilities and SSE_i is the sum of squared residuals from the estimation of model i. Therefore, given our model space $\mathcal{M}$ the posterior probability of model i can be computed as

$$P(M_i|Y) = \frac{P(M_i)T^{-k_i/2}SSE_i^{-T/2}}{\sum_{j=1}^{card(\mathcal{M})} P(M_j)T^{-k_j/2}SSE_j^{-T/2}}. \tag{4}$$

The posterior model probabilities allow us to easily compute the first and second moment of the posterior densities of the α, β and τ parameters in (1), given by

$$E(\xi|Y) = \sum_{l=1}^{card(\mathcal{M})} P(M_l|Y)E(\xi|Y, M_l) \tag{5}$$

and

$$\begin{aligned}\mathrm{var}(\xi|Y) &= \sum_{l=1}^{card(\mathcal{M})} P(M_l|Y)\mathrm{var}(\xi|Y, M_l) + \\ &+ \sum_{l=1}^{card(\mathcal{M})} P(M_l|Y)(E(\xi|Y, M_l) - E(\xi|Y))^2\end{aligned} \tag{6}$$

where ξ is the parameter of interest and $E(\xi|Y, M_l)$ is the OLS estimator of ξ for the constellation of **X**- variables, **Z**-variables and threshold values implied by model l. The posterior probability that a given **X**-variable, **Z**-variable or threshold value is part of the regression can be computed as the sum of posterior model probabilities of those models containing the variable or threshold value of interest.

[6]This is the prior model probability assuming that there are no repeated observations in the central 100(1-2θ)% of the empirical distribution of the variables in the **Z** group. If an observation for variable z_j repeated r times, its prior inclusion probability as a threshold value conditional on the inclusion of z_j as a threshold variable would be $r/[T(1-2\theta)]$, and $P(M_{n,m,\tau_1,\ldots,\tau_m})$ could be adjusted conveniently.

2.2 Random sampling in the BAT framework

For reasonable sizes of **X** and **Z** the number of possible regressions is enormous.[7] We are therefore estimating the posterior model probabilities (4) and posterior moments (5), (6) of the parameter of interest ξ by drawing directly from its posterior distribution. The results presented below are calculated using a random sampling procedure proposed by SDM (2004).[8] The random sampler (RS) uses prior inclusion probabilities of variables in **X** and **Z** and the uniform prior over threshold values z_j to obtain (5), (6) and the posterior inclusion probabilities for **X**- variables, **Z**-variables and threshold values. The sampling design is as follows.

1. We sample n_j variables from **X** and m_j variables from **Z**. Each variable in these sets has an inclusion probability of $\bar{n}/N$ and $\bar{m}/M$ for the set **X** and **Z** respectively.

2. For each one of the m_j **Z**-variables sampled, we independently sample a threshold value from the empirical distribution of realized values after trimming $100 \times \theta\%$ of the observations from the extremes.

3. Equation (1) is estimated for the constellation of variables and threshold values which has been sampled. The information necessary in order to obtain equations (4), (5) and (6) are saved for the model sampled.

4. Steps 1.-3. are replicated R (a large number of) times and (4), (5) and (6) are computed using the replicated models, replacing $card(\mathcal{M})$ by R. Changes in parameters of interest are monitored to ensure convergence of averages of sampling distributions to the posterior distribuition[9].

The procedure allows us to obtain the posterior inclusion probability of all possible interactions of variables in **X** with indicator functions for a given variable of **Z** and a threshold value z_j. This posterior inclusion probability is computed as the sum of posterior model probabilities for models including that threshold variable and

[7]Notice that for a given **Z**-variable, $T(1-2\theta)$ threshold values are possible, and each threshold value defines a different model in our setting. This implies that, for a given group of **X** variables and two threshold variables, $[T(1-2\theta)]^2$ models are possible. For example, if T=90, θ=0.15, N=20 and M=2, $\mathcal{M}$ contains more than 4200 million models.

[8]For details see the Technical Appendix to SDM (2004), which is available at: `www.econ.cam.ac.uk/doppelhofer`.

[9]See Doppelhofer and Durlauf (2007) for a discussion of model averaging techniques.

threshold value and allows us to obtain an estimate for the threshold value corresponding, for instance, to the mode of the posterior inclusion probability. Comparisons with the prior inclusion probabilities enable us to identify the threshold values whose inclusion probability increases or decreases after observing the data. In a similar fashion, the nonlinear effect can be evaluated by computing the posterior expected value and posterior variance of the parameter of the interaction for the corresponding threshold value.

3 Nonlinearities and growth: Empirical application

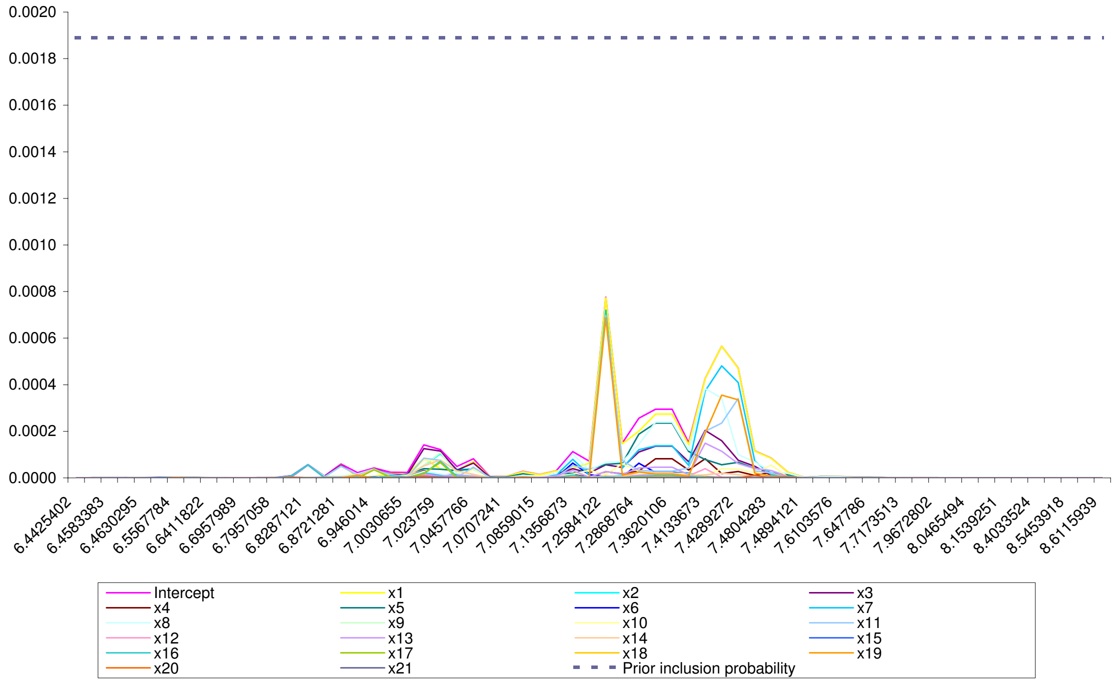
In this section we apply the BAT procedure to a reduced set of growth determinants in order to evaluate the existence and nature of nonlinearities in growth regressions. We choose the 21 variables that SDM (2004) find to be robustly related to growth using the (linear) BACE approach as the set $\mathbf{X}$. The variables are presented in a table in the Data Appendix,[10] For this application, we will use a relatively small group of variables as $\mathbf{Z}$, formed by two variables that have often been reported to cause threshold-nonlinearity in growth regressions: the *initial level of GDP per capita* and the *proportion of years an economy has been open between 1950-1994* according to the criteria in Sachs and Warner (1995). Durlauf and Johnson (1995), Hansen (2000), Masanjala and Papageorgiou (2004) and Crespo Cuaresma (2002) report evidence on nonlinearity induced by initial GDP per capita levels. Papageorgiou (2002) finds evidence that sets of countries with different openness levels tend to differ in the statistical model relating economic growth to other economic variables.[11]

The results presented below were obtained with ten million replications of the BAT procedure with random sampling setting $\bar{n}$=5, $\bar{m}$=1 and $\theta = 0.15$. We also ran the BAT procedure with other parameter constellations and the results concerning the existence and nature of nonlinearities appear robust to sensible changes in the expected number of included variables in the $\mathbf{X}$ group, $\bar{n}$, the expected number of

[10] The first 18 variables are robustly related to growth meaning that, in the linear BACE setting, the posterior inclusion probability is higher than the prior inclusion probability. The other three variables used as part of $\mathbf{X}$ (DENS60, RERD and OTHFRAC) are marginally related to growth: the posterior inclusion probability is slightly smaller than the prior inclusion probabilities, but their corresponding effect is estimated with high precision when they are included in the growth regression. The full set of countries included in the analysis can be found in SDM (2004).

[11] See also Huang and Chang (2006) and Papageorgiou (2006).

Figure 2: Posterior and prior inclusion probability, threshold value in *Initial GDP per capita*

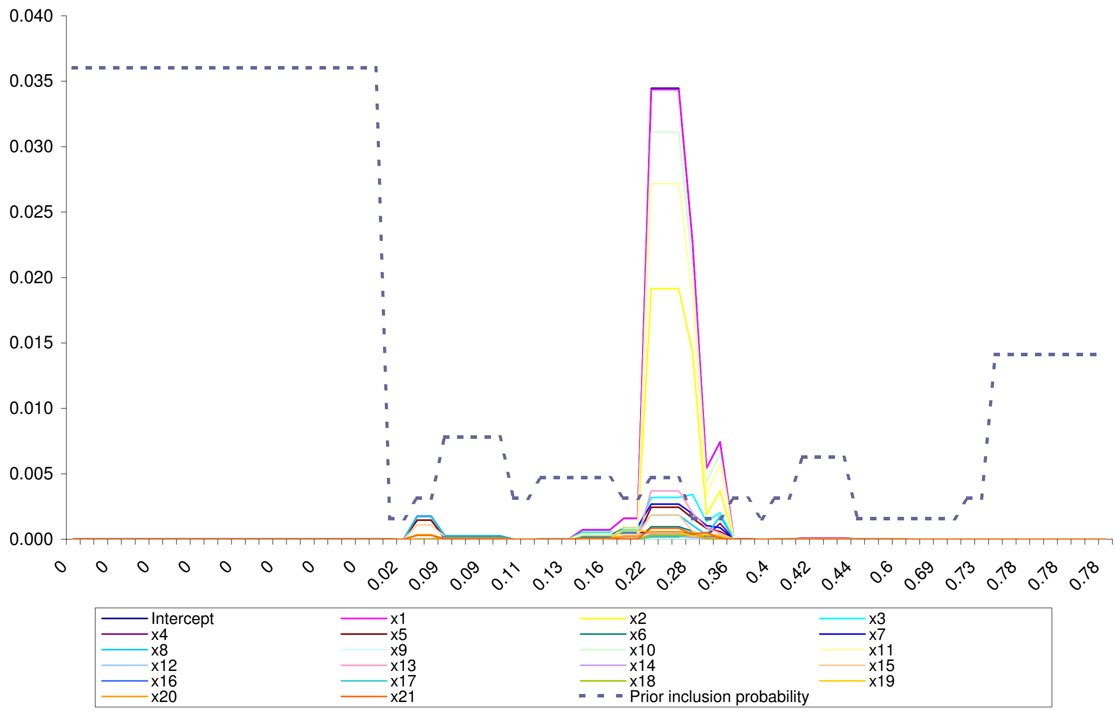


included variables from the **Z** group, $\bar{m}$ and the trimming parameter.

Figure 2 and Figure 3 present the posterior inclusion probabilities for the threshold value in all possible interactions of the **X** group variables with each one of the threshold variable (initial GDP per capita in 1960 in Figure 2 and number of years an economy has been open in Figure 3). The prior inclusion probability for each realized value is also plotted in the figures.[12] While in the case of initial income the prior inclusion probability is the same for all threshold values, in the case of the openness variable the repetition of identical values in the sample leads to different prior inclusion probabilities for each potential threshold value. The most remarkable feature of the posterior inclusion probabilities of the threshold values

[12]For a given interaction and a threshold value, the prior inclusion probability is given by the product of the prior inclusion variable of the corresponding **X** variable ($\bar{n}/N$), the corresponding **Z** variable ($\bar{m}/M$) and the corresponding threshold value ($r/[T(1-2\theta)]$, where r is the number of times the threshold value is repeated in the range of potential threshold values of the **Z** variable).

Figure 3: Posterior and prior inclusion probability, threshold value in proportion of *Years Open*



for initial GDP per capita is that they systematically fall below the prior inclusion probability, therefore lending little evidence to the existence of threshold nonlinearities caused by initial development levels once that model uncertainty is explicitly taken into account. The bigger bulk of posterior inclusion probability appears for many interactions in the interval between 7.26 (corresponding to the initial GDP per capita of Malaysia) and 7.45 (corresponding to the initial GDP per capita of Algeria). It should be noted that for simulations run setting $\bar{m}$=2 (that is, considering only nonlinear threshold models with both threshold variables as the relevant class), posterior inclusion probabilities in this range appeared greater than the prior inclusion probabilities, but as long as model uncertainty with respect the existence of nonlinearities is taken into account (that is, for parameter constellations with $\bar{m} < 2$ such as the one reported here), the evidence of threshold effects caused by initial income disappears.

The importance of model uncertainty in this setting appears even more evident if the results are compared with those in Crespo Cuaresma (2002), who performs a similar analysis without taking into account uncertainty in model size and in the nature of the threshold variable. Crespo Cuaresma (2002) obtains model averaged estimates from specifications with a given number of regressors and a fixed threshold variable, but uncertainty about the threshold value. With these assumptions, Crespo Cuaresma finds strong evidence of nonlinearities induced by the initial level of GDP per capita. The results in our BAT analysis, however, imply that initial income is no longer a robust threshold variable once uncertainty about the number of threshold variables and implied model size is taken into account. Our findings therefore shed new light on the results in Crespo Cuaresma (2002) and other contributions referred to above.

In Figure 3 the posterior inclusion probabilities for the threshold value corresponding to the openness variable are presented. In the case of this threshold variable posterior inclusion probabilities are higher than prior inclusion probabilities in the range delimited by 0.22 (corresponding to the openness experience of Gambia and Ghana in our sample) and 0.33 (the proportion of years open for Nicaragua and Syria in our data) for the interactions with the following variables: the regression intercept, East Asian dummy, primary schooling 1960, investment price, fraction of tropical area, malaria prevalence, life expectancy in 1960, African dummy, Latin American dummy and Spanish colony. For these variables, Table 1 presents the posterior mean and standard deviation of β and β^* in (1) conditional on inclusion

Variable	β: Posterior mean	β: Posterior s.d.	β^*: Posterior mean	β^*: Posterior s.d.
Intercept	0.060352	0.022257	-0.009038	0.014481
East Asian dummy	0.019399	0.006475	-0.038349	0.010120
Primary schooling 1960	0.025717	0.010226	0.017526	0.015585
Investment price	-0.000083	0.000027	0.000011	0.000085
Fraction tropical area	-0.013797	0.004492	0.008528	0.008710
Malaria prevalence	-0.011639	0.008979	-0.018294	0.019087
Life expectancy 1960	0.000708	0.000351	-0.000719	0.000653
African dummy	-0.008482	0.011845	-0.031641	0.009293
Latin American dummy	-0.012638	0.005627	0.008361	0.007484
Spanish colony	-0.009723	0.005534	0.010712	0.007640

Values obtained with ten million replications of the BAT procedure for the group of robust variables in SDM (2004) (first 21 variables in the Data Appendix), for $\bar{n}$=5, $\bar{m}$=1 and $\theta = 0.15$. Posterior mean and standard deviation of β^* evaluated at the threshold value of openness corresponding to the mode of the posterior inclusion probability of each interaction reported.

Table 1: Posterior mean and standard deviations of β and β^* conditional on inclusion for openness as a threshold variable

of the respective variables, evaluated at the threshold value of the openness variable corresponding to the mode of the posterior inclusion probability for each interaction. In this case, similar results concerning the importance of trade openness as a threshold variable are obtained if model size and threshold variable uncertainty is not taken into account. Crespo Cuaresma (2002) reports strong evidence of nonlinear effects induced by this openness variable, and the analysis in this study reinforces this result and proves that the threshold effects reported in the literature tend to be robust to broader definitions of model uncertainty such as the one employed here.

The interaction effect is very well estimated for the case of the East Asian and African dummies, and the results shed an interesting light on the effects which are picked up by these variables in cross country growth regressions. The posterior mean of the East Asian dummy parameter (conditional on inclusion) corresponding to the regime of "open countries" (defined by a threshold parameter of 0.22 in the variable "Years open", which corresponds to the mode of the posterior inclusion parameter) is very similar to the result obtained in SDM (2004)[13] for the linear setting and is

[13]In SDM (2004)'s results, the East Asian dummy is found to be the most robust variable of a set of 67 growth determinants. Conditional on inclusion of this variable in the linear setting, the posterior mean of the parameter attached to the dummy in SDM (2004) is 0.022, with posterior

estimated very precisely. The posterior mean of the additive effect for observations in the regime of "closed countries" is -0.038, with a posterior standard deviation of 0.010, which deems the positive effect of the East Asian dummy inexistent for this subsample. This result implies that the positive and robust coefficient found for the East Asian dummy in other studies is driven exclusively by a group of East Asian countries which were relatively open (as defined by the threshold estimate) in the period under analysis. If the East Asian dummy was to be interpreted in a geographical sense, our results indicate that it is trade institutions and economic policy that make a difference in the growth experience of East Asia. A similar conclusion is reached for the case of the African dummy: when the interaction effects with openness are taken into account, this variable appears only robust and estimated with a high degree of precision in the regime corresponding to the subsample of relatively closed countries. Furthermore, the quantitative effect in this regime is estimated to be higher in absolute value than the linear elasticity obtained in SDM (2004).[14]

These results suggest that these regional dummies are basically picking up the effect of subsamples of countries with a differential openness experience in the period under consideration. Our results, thus, call for care in the interpretation of the parameters associated to such regional dummies, since they appear to be mainly picking the effect of trade policy on economic growth. The result is of particular importance since the East Asian and African dummy tend to appear systematically robust in linear growth regression settings and are routinely included as control variables in the empirical implementation of models of economic growth.

4 Conclusions and further research

In this paper we have proposed a Bayesian Averaging of Thresholds (BAT) approach that jointly investigates uncertainty over explanatory variables and threshold effects. Our methodology makes use of Bayesian model averaging in the spirit of SDM (2004) and puts forward a method for estimating thresholds in the presence of model uncertainty based on the evaluation of the posterior inclusion probability of potential threshold values. We apply our BAT method to a set of explanatory variables that were found to be robustly related to long-term growth in the linear setting by SDM

standard deviation of 0.006.

[14] The posterior mean conditional on inclusion for the African dummy in SDM (2004) is -0.015, with a posterior standard deviation of 0.007.

(2004) and two threshold variables that have been suggested by the literature on non linearities and growth, initial GDP per capita and the number of years an economy has been open.

The results suggest that the nonlinear growth effect that the initial level of income has been shown to have in other empirical studies (see for instance Durlauf and Johnson, 1995, and Hansen, 2000) is not robust when model uncertainty (in the sense of uncertainty about the size of the model, the threshold values and the nature of the interactions) is explicitly taken into account. On the other hand, we find evidence for robust interaction effects of several variables with the number of years an economy has been open since 1950. In particular, our results imply that the widely used East Asian dummy and African dummy are basically picking up the effect of subsamples of countries with a high and low degree of openness, respectively.

We are working on extending our approach in several ways. The combination of uncertainty on the existence of nonlinearities and the structure of the covariance matrix of the error term in our BAT framework can be a fruitful avenue of future research in this topic. This method can be embedded in methods of Bayesian model-based clustering. Finally, our BAT approach can be readily extended to more complex models with a larger number of nonlinearities. Stratified sampling (see SDM, 2004) and Markov-Chain Monte Carlo model composite methods may prove to be useful in the implementation of our model averaging methodology for larger models.

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A Data Appendix

Rank	Short Name	Variable Description	PIP	Mean	S.D.
Dep. Var.	**GROWTH**	**Growth of GDP per capita at PPP between 1960–1996.**	–	0.0182	0.0191
1	**EAST**	**East Asian Dummy**	0.82	0.11364	0.31919
2	**P60**	**Primary Schooling Enrollment**	0.80	0.72614	0.29321
3	**IPRICE1**	**Investment Price**	0.77	92.47	53.68
4	**GDPCH60L**	**Log GDP in 1960**	0.68	7.35494	0.90108
5	**TROPICAR**	**Fraction of Tropical Area**	0.56	0.57024	0.47160
6	**DENS65C**	**Population Coastal Density**	0.43	146.87	509.83
7	**MALFAL66**	**Malaria Prevalence**	0.25	0.33943	0.43089
8	**LIFE060**	**Life Expectancy**	0.21	53.72	12.06
9	**CONFUC**	**Fraction Confucian**	0.21	0.01557	0.07932
10	**SAFRICA**	**Sub-Saharan Africa Dummy**	0.15	0.30682	0.46382
11	**LAAM**	**Latin American Dummy**	0.15	0.22727	0.42147
12	**MINING**	**Fraction GDP in Mining**	0.12	0.05068	0.07694
13	**SPAIN**	**Spanish Colony Dummy**	0.12	0.17045	0.37819
14	**YRSOPEN**	**Years Open 1950-94**	0.12	0.35545	0.34445
15	**MUSLIM00**	**Fraction Muslim**	0.11	0.14935	0.29616
16	**BUDDHA**	**Fraction Buddhist**	0.11	0.04659	0.16760
17	**AVELF**	**Ethnolinguistic Fractionalization**	0.10	0.34761	0.30163
18	**GVR61**	**Gov't Consumption Share**	0.10	0.11610	0.07454
19	**DENS60**	**Population Density**	0.09	108.07	201.44
20	**RERD**	**Real Exchange Rate Distortions**	0.08	125.03	41.71
21	**OTHFRAC**	**Fraction Speaking Foreign Language**	0.08	0.32092	0.41363

Explanatory variables are ranked by Posterior Inclusion Probability $P(\xi_j \neq 0|Y)$ (PIP) using the BACE method (SDM, 2004). The set of regressors **X** is given by variables 1 to 21. The threshold variables **Z** are ranked 4 (Log GDP in 1960) and 14 (Years Open 1950-94), respectively, but this is not necessarily informative of their role as threshold variable. Variables ranked 22 to 67 by SDM (2004) were not included in the results presented but are available at http://www.econ.cam.ac.uk/faculty/doppelhofer/research/bace.htm.