

# Fiscal Federalism and Electoral Accountability

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## Abstract

We study the efficient allocation of spending and taxation authority in a federation in which federal politicians are exposed to electoral uncertainty. We show that centralization may, but need not, result in a loss of electoral accountability. We identify an important asymmetry between positive and negative externalities and show that centralization may not be efficient in economies with positive externalities even when regions are identical and centralization does not entail a loss of accountability. We also show that decentralization can only Pareto dominate centralization in economies with negative externalities.

*Keywords:* Fiscal federalism, local public goods, externalities, performance voting, turnout uncertainty, electoral accountability.

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# 1 Introduction

Should a society opt for a centralized fiscal system under which spending decisions are made by a central authority and financed from general tax revenues or should it opt for a decentralized system in which fiscal choices are made by local authorities and financed by local taxes? In his seminal work on economic federalism, Oates (1972) answered this question by highlighting a trade off between internalization of externalities and the capacity of the state to cater for regional differences in taste. His famous Decentralization Theorem states that decentralization is desirable if externalities are weak and regional differences in taste are large.<sup>1</sup> Clearly such economic trade offs are important, yet the design of the fiscal state has equally important *political economy* implications. This is because political inefficiencies are affected by the degree of centralization of the fiscal state. This has been explored in a growing literature on the political economy of fiscal federalism.<sup>2</sup> This literature, which we discuss in more detail below, has identified various political trade offs as well as reasons why the economic trade offs are affected by politics. This paper makes a contribution to this literature by pointing to a new and important political cost of centralization: governance uncertainty. It explores how governance uncertainty affects the trade off between internalization of externalities under centralization and the perceived benefits of electoral accountability under decentralization.

The general framework of our analysis is the common agency model with governance uncertainty studied by Aidt and Dutta (2004). This model portrays a society populated by heterogeneous groups of voters (e.g., living in different regions) with conflicting policy preferences. The groups of voters (the principals) use elections to hold an opportunistic or rent seeking politician (the agent) accountable for his policy choices while in office. They do so by voting retrospectively in an infinite sequence of elections, as in Ferejohn

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<sup>1</sup>This result is, as pointed out by Besley and Coate (2004), driven by the somewhat artificial assumption that the federal government cannot tailor spending to regional differences in taste. See Harstad (2007) for a rationale for why it might be politically optimal to select uniform federal policies.

<sup>2</sup>See, e.g., Seabright (1996); Crémer and Palfrey (1996, 2000); Edwards and Keen (1996); Dixit and Londregan (1998); Lockwood (2002, 2008); Luelfesman (2002); Besley and Coate (2003); Dur and Roelfsema (2005); Tommasi and Weinschelbaum (2007); Bordignon et al. (2008); Fredriksson et al. (2010).

(1986), Persson et al. (1997) or Coate and Morris (1999). The critical new feature of the analysis is that *ex ante* – before each election – the politician is uncertain about which group will be pivotal in deciding the outcome of the election. We call this governance uncertainty. Governance uncertainty has many different sources, as we discuss in more detail below. To be concrete, however, we relate it to random events that affect the electoral turnout rate of voters in different groups.<sup>3</sup> These random turnout shocks, which we assume to be correlated within groups but not between groups, introduce uncertainty from the point of view of the politician as to which of group holds the majority amongst those voters who actually turn out to vote in any given election. An example of what we have in mind is random fluctuations in weather conditions in different locations. As in Roemer (1998), such fluctuations are to a first approximation uncorrelated across regions and induce random turnouts in elections.

We adopt this general setting to revisit one of the classical questions of fiscal federalism: when should provision of local public goods be centralized? Our analysis highlights a new political cost of centralization. This cost arises because turnout uncertainty is more pronounced at the federal level than at the regional level. As a consequence, centralization may be associated with a loss of electoral accountability. The nature of this loss, however, depends on the direction of the externality associated with provision of local public goods. We identify an important asymmetry between situations with positive and negative externalities. With negative externalities, voters are forced to accept more rent seeking in a centralized federation than when fiscal decisions are decentralised to the regions. Consequently, centralization entails a loss of accountability that must be traded off against the benefits of internalizing externalities. Centralization is only Pareto efficient if the (negative) externality is sufficiently strong. With positive externalities, on the other hand, centralization does not entail a loss of accountability per se. Yet, even when the regions of the federation are identical in all respects, centralization is not necessarily Pareto efficient despite the presence of (positive) externalities.

The organization of the fiscal state is not just a question of theoretical interest. It is an issue of great practical importance as well. The ongoing debate about the

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<sup>3</sup>See Dhillon and Peralta (2002) for a good survey of the literature on voter turnout.

appropriate role of the European Union is just one example this. Another is the view that a reorganization of the fiscal state towards more a decentralized structure is one very promising way to increase efficiency and fairness in provision of public goods in less developed countries (Santos (1998); Bardhan (2000)). Finally, the analysis can provide insights into the forces that stabilize and destabilize federal fiscal structures.

The rest of the paper is organized as follows. In section 2, we discuss the related literature and put our contribution in context. In section 3, we present a general political (common) agency model with governance uncertainty and introduce and discuss the main assumptions. In section 4, we present the characterization results from Aidt and Dutta (2004). We provide complete proofs in Appendix I. In section 5, we tailor the general model to the case of local public goods and fiscal federalism and present the main results of the paper. In section 6, we discuss the implications of our analysis for fiscal integration and disintegration. In section 7, we summarize and discuss a number of extensions.

## 2 Related literature

The literature on the political economy of fiscal federalism has been surveyed by, for example, Inman and Rubinfeld (1997) and Lockwood (2006), and we shall only attempt to cover the most direct links to our analysis here.

Our paper is most directly related to the work by Seabright (1996), Tommasi and Weinschelbaum (2007), Bordignon et al. (2008) and Hindriks and Lockwood (2009). Seabright (1996) argues that political accountability is weakened when public spending decisions are centralized. He measures this effect as the reduction in the probability that a given region can determine the re-election of the government. In our model, this notion is made precise. The political clout of a region is determined by the probability that voters of that region holds the majority among those who turn out to vote in the federation. Importantly, whether the reduction, implied by centralization, in the probability that a given region can determine the re-election of the government leads to a loss of political accountability depends on the nature of the externalities associated

with provision of local public goods, as discussed above. This is a new insight. Tommasi and Weinschelbaum (2007) study the question of centralization versus decentralization within the framework of a common agency model. They allow the principals (citizens of the regions within the country) to offer monetary rewards to either the federal politician (under centralization) or to the regional politicians (under decentralization). They identify a trade off between internalization of externalities and the coordination failure that arises among the principles when fiscal decisions are centralized. One can interpret the trade off that we highlight in a similar way, but with two important differences. One difference is that we focus on the implicit incentives that the threat of termination of office can provide rather than the explicit incentives provided by monetary payments. Another difference is that we allow for positive as well as negative externalities and show that this distinction matters in important ways for the nature of the coordination failure. Bordignon et al. (2008) also find that the distinction between positive and negative externalities matters within a lobbying model similar to that of Tommasi and Weinschelbaum (2007). The reason is, however, very different from the one highlighted by our analysis. It has to do with the fact that lobbying under decentralization may partly compensate for the fact that local public goods are under-provided, but only if the externality is positive. Hindriks and Lockwood (2009) stress that voters are often poorly informed about policy outcomes, not only in other districts but also in their own, and that elections, in addition to their disciplining role, also serve as a selection device. As in our context, fiscal centralization reduces electoral accountability, but this effect is counteracted by a selection effect that encourages “bad” incumbents to pretend to be “good”. Our analysis abstracts from the selection effects in order to stress the effect of governance uncertainty on electoral accountability.

Our paper is also related to the works by Besley and Coate (2003), Dur and Roelfsema (2005) and Lockwood (2002; 2008). Besley and Coate (2003) identify two important political effects of centralization. These are related to different legislative procedures at the federal level. First, centralization induces uncertainty as to whether or not the representative from a particular region will be include in the minimum winning coalition that determines policy. Second, when policy making at the federal level is determined

by bargaining between representatives from different regions, regional voters may have an incentive to delegate strategically and elect a politician that cares a lot about public spending. In both cases, a trade off between the political distortion (uncertainty or strategic delegation) and the benefits of internalizing (positive) externalities determines whether or not centralization is beneficial. Besley and Coate find that centralization is, typically, beneficial if the externality is strong enough. Dur and Roelfsema (2005), however, extend this analysis to show that centralization may fail to internalize externalities if the cost of public policy cannot be shared among the regions. Luelfesmann et al. (2008) furthermore argue that Besley and Coate (2003) underplay the scope for bargaining amongst regions and show that decentralization tends to dominate centralization when this is taken into account.<sup>4</sup> Like Besley and Coate (2003), we also focus on the uncertainty that arises when fiscal decisions are centralized, but we stress governance uncertainty rather than uncertainty about being included in the minimum winning coalition. It is interesting to notice that decentralization, in our model, can only Pareto dominate centralization in the presence of a negative externality – a case that Besley and Coate (2003) do not consider.<sup>5</sup> Moreover, while we do not allow bargaining among regions, we do not allow this at the federal level either. This simplification allows us to isolate the key political economy trade off in a simple and transparent way.

Lockwood (2002) argues that centralization leads to inefficient outcomes when regional representatives vote over agendas that contain sets of region-specific projects. The problem is that the political choice is not tailored sufficiently to within-region benefits. Thus, centralization entails a classical trade off between catering for regional differences and internalizing externalities. Importantly, however, the political distortions imply that weaker externalities and heterogeneity between regions need not increase the efficiency gain from decentralizations. In our model, there is no regional differences with regard to the benefits of public goods. Nonetheless, we find an interesting asymmetry

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<sup>4</sup>A similar conclusion is reached by Cheikbossian (2000). He shows that with a decentralized fiscal structure voters strategically elect representatives to eliminate any element of cooperation between representatives at the decision-making stage and that this tend to work against centralization. See also Luelfesman (2002) who shows that with linear cost sharing rules decentralization typically is socially optimal.

<sup>5</sup>Besley and Coate (2003) make welfare comparisons based on aggregate public goods surplus.

between positive and negative externalities which provides a complementary example of how politics can change the classical trade offs in surprising ways. Lockwood (2008) further explores ways in which the Decentralization Theorem may break down under majority voting or lobbying even when federal policy is, by assumption, prevented from reflecting regional preferences.

### 3 A General Model of Governance Uncertainty

The starting point of our analysis is an infinite horizon model of repeated elections and performance voting, familiar from Ferejohn (1986), Persson et al. (1997), Coate and Morris (1999) and Aidt and Magris (2006) among others. We extend the standard formulation of the model by introducing voter heterogeneity and governance uncertainty.

Society consists of two groups of voters,  $i = 1, 2$ ; politicians are indexed by 0. A group is defined as a subset of voters who are affected in the same way by public policy. Group affiliation may be determined by observable characteristics such as age or gender, or by shared preferences for public policy. In the context of fiscal federalism, group affiliation is naturally defined along geographical lines and so we can think of the two groups as representing two regions within a federation. Per-period utility,  $u_{it}$ , is discounted with the common discount factor  $\beta \in (0, 1)$  and lifetime welfare is given by

$$V_{0i} = \sum_{t=0}^{\infty} \beta^t u_{it}; \quad i \in \{0, 1, 2\}. \quad (1)$$

There are  $n_1$  voters in group 1 and  $n_2$  voters in group 2. We assume that  $n_1 \geq n_2$ . The size of the total (voter) population is  $n = n_1 + n_2$ .

Each period, the politician collects taxes up to a maximum of  $T$ , spends some of this on providing amenities to his electorate, and keeps the rest for himself.<sup>6</sup> Denoting the cost of providing utilities to the two groups of voters by  $c_t$ , we can write the politician's per-period payoff as

$$u_{0t} = T - c_t \quad (2)$$

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<sup>6</sup>This formulation of the conflict of interest between voters and politicians is due to Persson et al. (1997) and used extensively in Persson and Tabellini (2000). It should be understood as a metaphor for the more general phenomenon that politicians can divert their efforts towards activities that are not in the interests of their electorate.

if in office, and  $u_{0t} = 0$  otherwise.

The cost of providing utility to voters is determined by the *political cost function*. We define  $C(x_{1t}, x_{2t})$  as the minimum cost to the politician of providing utility levels  $u_{1t} \geq x_{1t}$  and  $u_{2t} \geq x_{2t}$  simultaneously to voters in the two groups at time  $t$ . Likewise, we define  $C_i(x_{it})$  as the minimum cost of providing the utility level  $u_{it} \geq x_{it}$  to group  $i$ ,  $i = 1, 2$ , in isolation. We begin by specifying the political cost function directly, but shall derive it from more fundamental considerations in the application to fiscal federalism that follows. We make the following assumptions.

**Assumption 1** *The political cost functions are monotonically increasing in each argument, i.e.,*

$$\begin{aligned} \text{(M)} \quad & x_t > x'_t \Rightarrow C(x_t) \geq C(x'_t) \\ & x_{it} > x'_{it} \Rightarrow C_i(x_{it}) \geq C_i(x'_{it}) \end{aligned}$$

where  $x_t = (x_{1t}, x_{2t})$ . Further,  $\lim_{x_i \rightarrow \infty} C(x_1, x_2) = \lim_{x_i \rightarrow \infty} C_i(x_{it}) = \infty$ .

**Assumption 2** *The political cost functions are continuous, i.e.,*

$$\text{(K)} \quad \begin{aligned} & C(x_{1t}, x_{2t}) \in \mathcal{C}^1 \\ & C_i(x_{it}) \in \mathcal{C}^1 \end{aligned} .$$

The first assumption says that it is costly for the politician to generate utility for each group of voters. This is clearly the case whenever tax resources that could otherwise have been extracted as rents have to be devoted to the task. However, when the politician can generate utilities by providing public goods, the cost functions may not be strictly increasing. The second assumption rules out discontinuities in the cost of generating utilities. Both of these assumptions can be relaxed.

The property of the political cost function that really matters for outcomes is whether it is sub- or super-additive. The political cost function is sub-additive if

$$\text{(C}^+\text{)} \quad C(x_{1t}, x_{2t}) \leq C_1(x_{1t}) + C_2(x_{2t}) \tag{3}$$

and super-additive if

$$\text{(C}^-\text{)} \quad C(x_{1t}, x_{2t}) > C_1(x_{1t}) + C_2(x_{2t}). \tag{4}$$

A sub-additive political cost function makes it cheaper to provide utility to all voters jointly than to provide the same utility levels to the two groups separately. In public finance, sub-additivity is, typically, associated with pure public goods or positive externalities. A super-additive cost function makes it more expensive to please all groups of voters jointly than to please them separately. Super-additivity is caused by negative externalities associated with, for example, provision of local public goods, pollution, or with envy effects.

The politician, elected at  $t$ , cannot make binding promises on the level and pattern of public spending before he enters office. Since his own payoff decreases with  $c_t$ , he would, in the absence of further incentives, choose  $c_t = 0$  and provide no amenities to the electorate. Voters know this, and threaten to vote retrospectively against a politician who does not provide them with a minimum level of utility. At the beginning of each period, voters in each group announce simultaneously a performance standard, denoted  $x_{1t}$  and  $x_{2t}$ . They then vote in favor of reelection of the incumbent politician if, and only if the policy implementation observed at the end of the period generates at least that level of utility, i.e., if, and only if  $u_{it} \geq x_{it}$ .

The key feature of the model is that politicians are exposed to governance uncertainty. At the most general level this means that the incumbent cannot be sure ex ante which of the two groups is decisive in determining his reappointment. Governance uncertainty can arise for many different reasons. A leading example is electoral turnout uncertainty, and this is the interpretation we shall follow here for concreteness. In particular, we generate governance uncertainty by assuming that neither group can guarantee to turn out in full force at elections. Consequently, a politician may deliver on the performance standard set by group 1, who, say, holds the majority ex ante, by incurring the cost  $C_1(x_{1t})$ , but fail to deliver on the standard set by group 2 ( $u_{2t} < x_{2t}$ ). On the day of the election,  $\tilde{n}_{it}$  voters from group  $i$  actually show up to vote, and the politician can lose his bid for reelection if  $\tilde{n}_{2t} > \tilde{n}_{1t}$ . The central assumption of our analysis is that electoral turnout is uncertain, and that individual voters vote according to the announced performance standards *if they show up to vote*, but that they cannot, as a group, guarantee a particular turnout rate. This is captured by the next assumption.

**Assumption 3** *Electoral turnout,  $\tilde{n}_t = (\tilde{n}_{1t}, \tilde{n}_{2t})$ , is random. The ex ante probability that the turnout of group 1 is greater than that of group 2,  $P(\tilde{n}_{1t} \geq \tilde{n}_{2t})$ , is equal to  $p_1$  and constant over time. The complementary probability is  $p_2 = 1 - p_1$ . We assume that  $p_1 \in (0, 1)$ .*

Here, we specify the parameters  $p_1$  and  $p_2$  directly. They can be derived from more basic considerations, however, and different distributions of turnout shocks map into alternative specifications of  $p_1$  and  $p_2$ . It is important that  $0 < p_1 < 1$ , so that neither group can guarantee reelection. This is more likely to be the case when turnout shocks are correlated within groups and when differences in group sizes are not too large. An example of this is weather shocks. These are typically uncorrelated across space and can affect the turnout rate in particular geographical locations or keep certain types of voters, such as the poor, at home (Roemer, 1998).

It is important to stress that governance uncertainty can arise for many other reasons than turnout uncertainty in elections. It may, for example, reflect fluctuations in inter-group power relations with one group becoming more powerful and therefore more pivotal than another due to unpredictable events. The lobbying power of social groups may well fluctuate in this way. Under this interpretation, the probability of being pivotal,  $p_i$ , can be seen as a manifestation of randomness in the cost of political mobilization. Combined with the insights from Olson (1965), a minority could be as likely as a majority group to be pivotal, not because it may in fact hold the majority among those who turn out to vote, but because it is better at organizing an effective lobby group. Another example is random preferences. Suppose that some people like education spending while others want spending on care for the elderly and that the proportions of individuals of these two types fluctuate in unpredictable ways. In this case,  $p_i$  represents the probability that one of the “preference types” is pivotal.

The game between the incumbent politician and the two groups of voters unfolds over time as follows. At the beginning of each period, voters in each group announce the (utility) standard that the politician needs to satisfy to get their votes in the next election. The standards are chosen by the two groups non-cooperatively and at the same time. The politician observes the standards and determines whether to comply,

and if so, how many standards to meet. We denote the set of actions available to the politician by  $A = \{(00), (10), (01), (11)\}$  with elements  $a_t = (00)$  (meet neither standard);  $a_t = (10)$  (meet group 1’s standard only);  $a_t = (01)$  (meet group 2’s standard only); and  $a_t = (11)$  (meet both standards). At the end of the period, a new election is held and voters randomly turn up to vote. Those who turn up vote according to the announced performance standard. The politician either wins or loses. In the latter case, he is replaced by an identical challenger; in the former case, he gets (at least) another term in office. After the election, the game continues to the next period where a similar sequence of events takes place. We restrict attention to history-independent subgame perfect Nash equilibria of this game.<sup>7</sup> In addition, we assume that the politician, if indifferent between two or more actions (which are then preferred to the remaining ones), chooses the action that maximizes reelection chances. Below, when we refer to equilibrium this is what we have in mind.

## 4 Equilibrium Paths

We can apply Theorem 1 from Aidt and Dutta (2004) to characterize the set of equilibria. The theorem, which we formally state and prove in Appendix I, says that all equilibrium paths of the political game described above have a property called *strategic consensus*: the politician prefers to meet all performance standards at all times, all those voters who turn out to vote in the election vote for the incumbent, and the incumbent is reelected with certainty, irrespective of turnout shocks. While this outcome, perhaps, is to be expected when the political cost function is sub-additive and it is cheaper for the politician to satisfy the standards jointly than separately, it is surprising that the same result obtains with super-additive costs. In this case, the fact that it is *more* expensive to satisfy the standards jointly than separately suggests that “partisan” outcomes would be more likely. This intuition is, however, wrong. To see why, consider the special case

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<sup>7</sup>Formally, the model describes a dynamic common agency game with absorbing states and perfect information. The two groups of voters are principals, and the elected politician their common agent. Uncertainty in rewards arises from uncertainty about which of the two principals will have the “casting vote”, or final say, in the only reward available: re-election. There is no aggregate uncertainty, as one of the principals will have the casting vote for sure.

where the only policy instrument is a group-specific transfer. This makes the political cost function additive. To please voters, the politician must either be partisan and give transfers to one group only or seek consensus and give to both. The two groups of voters set their standards simultaneously. Suppose that group 1 announces a standard that is so high that the politician prefers to take his chances and offer transfers only to group 2. This cannot be an equilibrium. This is because group 1 gets nothing and it would do better by reducing its standard to a level such that it is in the best interest of the politician to offer it a transfer. In other words, whenever the politician is willing to implement a “partisan” outcome, the disfavored group has an incentive to lower its standard to induce the politician to make a “partisan” choice in its favor. This logic continues until the standards are such that the politician is just willing to implement a policy that satisfies both groups. The result is strategic consensus. Importantly, it does not follow from this logic that the two groups will “under-bid” each other until the politician captures the entire rent. This would only happen if the two groups were “perfect substitutes” in the sense that either of them can guarantee reelection for sure (see Ferejohn, 1986). In our model, however, the two groups of voters avoid Bertrand-style competition precisely because they are not “perfect substitutes” from the point of view of the politician: the consent of both is needed to secure the reelection reward with certainty. As a consequence, voters retain some control power, even when political costs are additive. A similar logic applies when the political cost function is either sub- or super-additive.

Although all equilibrium paths display strategic consensus, the distribution of payoffs depends critically on the properties of the political cost function. Let  $X = \{x_{1t}, x_{2t}\}_{t=0}^{\infty}$  be a sequence of *equilibrium* performance standards. In an economy with sub-additive political costs, the following characterization result holds. We provide a formal proof in Appendix I.

**Proposition 1 (Sub-additive Costs)** *If the political cost functions satisfy assumptions [M] and [K] and are sub-additive, then  $X$  must satisfy*

$$(\mathbf{SC}_1^+) \quad C(x_{1t}, x_{2t}) = \beta T;$$

$$(\mathbf{SC}_2^+) \quad C_1(x_{1t}) \geq \beta p_1 T;$$

$$(\mathbf{SC}_3^+) \quad C_2(x_{2t}) \geq \beta p_2 T.$$

Moreover,  $(\mathbf{SC}_2^+)$  and  $(\mathbf{SC}_3^+)$  hold with equality for additive political cost functions. Along all equilibrium paths, the politician receives payoffs  $(1 - \beta)T$  per period.

The proposition explores the fact that the politician must, at equilibrium, be indifferent between satisfying both and satisfying none of the standards. As a consequence, the politician *always* gets per period payoff  $(1 - \beta)T$ , while the remaining share of tax revenues,  $\beta T$ , is devoted to the task of generating utilities to voters. Importantly, this distribution of resources is unaffected by turnout uncertainty. Thus, strategic consensus provides the politician with “full insurance” against random voter turnout and voters with insurance against “partisan” choices by the politician. When the political cost function is additive, the allocation of utility between the two groups of voters is uniquely determined by  $p_1$  and  $p_2$ . In contrast, economies with *strictly* sub-additive costs exhibit multiple equilibria in performance standards at each  $t$ , and any equilibrium allocation what arises with sub-additive costs (weakly) Pareto dominates the utility allocation with additive costs.

In an economy with super-additive political costs, the utility allocation is very different, as shown by the second characterization result (see Appendix I for a proof).

**Proposition 2 (Super-additive Costs)** *If the political cost functions satisfy assumptions [M] and [K] and are super-additive, then  $X$  must satisfy*

$$(\mathbf{SC}_1^-) \quad C(x_1, x_2)(1 + \eta_1) - C_1(x_1) = \eta_1 T$$

$$(\mathbf{SC}_2^-) \quad C(x_1, x_2)(1 + \eta_2) - C_2(x_2) = \eta_2 T$$

where  $\eta_i = \frac{(1-p_i)\beta}{1-\beta}$  for  $i = 1, 2$ . The politician receives payoffs  $T - C(x_1, x_2) > (1 - \beta)T$  every period. Moreover, if the cost functions are differentiable and  $\frac{\partial C}{\partial x_1 \partial x_2} > 0$ , then the solution to  $(\mathbf{SC}_1^-)$  and  $(\mathbf{SC}_2^-)$  is unique.

In this case, the politician must, at equilibrium, be indifferent between satisfying both standards and satisfying just one of them. The politician receives  $T - C(x_1, x_2)$

each period. This is more that he receives along *any* equilibrium path with sub-additive costs, but the payoff is no longer independent of turnout shocks. Intuitively, super-additive costs make it costly for the politician to implement consensus outcomes. This enables him to extract more rents: the two groups of voters have, *ceteris paribus*, to lower their standards to prevent “partisan” outcomes.

In the next section, we tailor the general model to the case of fiscal federalism. We identify the two groups of voters with voters living in different regions of a country and argue that governance uncertainty generated by turnout shocks is more pronounced at the federal than at the regional level.

## 5 Fiscal Federalism

We consider a country with two regions,  $i = 1, 2$ . This corresponds to the two groups in the general model. The two regions can be of different sizes, with  $n_i$  voters living in region  $i$ . We suppose that  $n_1 \geq n_2$ . The regions may also differ with regard to tax potential and electoral turnout patterns. Individuals in each region derive utility from local public goods  $g_{it}$  and private goods  $y_{it}$ . Consumption of local public goods in one region generates externalities for individuals in the other region. To capture this, we write the utility function of a typical individual living in region  $i$  as

$$u_{it} = y_{it} + g_{it} - \gamma g_{-it} \tag{5}$$

where  $\gamma \in (-1, \frac{n_2}{n_1})$  captures the strength of the externality.  $\gamma > 0$  corresponds to a negative and  $\gamma < 0$  to a positive externality. Public goods are produced by the following technology

$$g_{it} = \frac{1}{\alpha} k_{it}^\alpha \tag{6}$$

where  $k_{it}$  is an input required to produce the public good, bought at a constant price of one. For simplicity, we assume that  $\alpha = \frac{1}{2}$ .<sup>8</sup> The maximum revenue that can be raised each period in region  $i$  is  $T_i$ , and so the maximum revenue that can be raised in

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<sup>8</sup>This assumption can be relaxed, but doing so yields no additional insights and complicates the mathematical exposition.

the country is  $T = T_1 + T_2$ . We use the convention that politicians raise the maximum revenue each period, spent some of it on providing local public goods, some on transfers  $s_{it} > 0$  to individuals, and keep the rest as rents.

We compare two institutional arrangements: Regionalism [ $R$ ] and federalism [ $F$ ]. Regionalism means that each region elects its own politician who can finance local public goods (and transfers) out of local tax revenues. Federalism means that a single elected politician is in charge of the whole country and can use general tax revenues to provide public goods and transfers to the two regions.<sup>9</sup>

The key assumption of the application is that turnout uncertainty is more pronounced at the federal than at the regional level. This assumption can be justified in many ways. Most importantly, the federal politician must, by definition, cater to more principals than each of the regional politicians. In particular, the federal politician needs the support of the majority of the whole country while a regional politician only needs the majority support of his own region. Turnout shocks at the regional level renders regional turnout unpredictable. Consequently, the federal politician faces an additional layer of uncertainty that is not present in regional elections. To make the contrast as sharp as possible, we assume that regional politicians can guarantee reelection if they satisfy the performance standard set by voters in their region: there is no turnout uncertainty within a region. In contrast to the two regional politicians, the politician in charge of the federation is exposed to turnout uncertainty and needs the support of voters in both regions to get reelected for sure. We denote the ex ante probability that voters in region  $i$  holds the majority among those who turn out to vote by  $p_i$  with  $p_1 = 1 - p_2$ . Again, it is important to stress that turnout uncertainty is not the only valid interpretation of  $p_i$ . For example,  $p_i$  can also be interpreted as a power index that captures the influence of region  $i$  in federal decisions. All regions may be pivotal occasionally because of random shifts in power relations, but some regimes are more likely to be pivotal than others.

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<sup>9</sup>This formulation rules out cooperation among regions in regime [ $R$ ]. In some cases, this could be important, although high transaction costs typically rule such cooperation out in practice.

## 5.1 The Benevolent Planner's Solution

As a benchmark, suppose that all public finance decisions were made by benevolent planners. When fiscal decisions are decentralized to the regional level, two regional planners decide independently and simultaneously how much local public good to provide to their region. They do so by maximizing regional aggregate public goods surplus taken the spending decision in the other region as given:

$$s_{it}^D(k_{it}; k_{-it}) = 2(k_{it}^{\frac{1}{2}} - \gamma k_{-it}^{\frac{1}{2}}) - k_{it}, \quad i = 1, 2. \quad (7)$$

In a federation, on the other hand, decisions are made by one benevolent planner who maximizes aggregate public goods surplus for the whole country, i.e.,  $s_t^F(k_{1t}, k_{2t}) = \sum s_{it}^D(\cdot)$ . It is easy to verify that federalism under these ideal circumstances Pareto denominates regionalism for all  $\gamma \neq 0$ .<sup>10</sup> The intuition is straight forward. Rent seeking is not an issue with benevolent planners, so the level of centralization does not create or eliminate political distortions. Moreover, federal and regional planners are equality good at catering to local tastes. Hence, the only concern is to internalize externalities. This provides a clear-cut benchmark against which we can measure political distortions.

## 5.2 The Political Cost Functions

To characterize equilibrium allocations, we need to derive the political cost functions. This is done in Appendix II. In the following, we focus on the situation in which both federal and regional politicians provide local public goods *and* transfers at equilibrium. This basically requires that tax resources are sufficiently large in each region and in the federal as a whole.<sup>11</sup>

<sup>10</sup>For  $\gamma = 0$ , the institutional arrangement makes no difference.

<sup>11</sup>Necessary conditions that guarantee that politicians, at equilibrium, provide local public goods and transfers in all regimes are:  $T_i > \frac{(n_i)^2}{\beta}$  for  $i = 1, 2$ ; for  $\gamma < 0$

$$T > \max_i \frac{n_i}{\beta p_i} (4\gamma n_{-i} + n_i (\gamma^2 + 1));$$

and for  $\gamma \geq 0$

$$T > \max_i \left[ \frac{n_i^2 (1 + \gamma^2 (1 + \beta p_i)) - 4\beta \gamma n_1 n_2 p_i + n_{-i}^2 \gamma^2 (1 - \beta p_i)}{\beta p_i} \right].$$

See Appendix II for details.

Under regionalism, the two regional politicians face a separate performance standard. They make decisions about public spending without (direct) regard for the welfare of voters in the other region, i.e., each politician takes the spending decisions by the other politician as given. Consider the politician in region  $i$  who in period  $t$  faces the performance standard  $x_{it}$ . The minimum cost of satisfying this standard for a given input to the production of local public goods in the other region is

$$C(x_{it}; k_{-i}) = \min_{k_{it} \geq 0, s_{it} \geq 0} k_{it} + n_i s_{it} \quad (8)$$

subject to  $x_{it} \leq s_{it} + 2k_{it}^{\frac{1}{2}} - 2\gamma k_{-it}^{\frac{1}{2}}$  and the regional budget constraint. It follows that  $k_{it} = (n_i)^2$  and  $s_{it} = x_{it} - 2(n_i - \gamma n_{-i})$ . The political cost functions are

$$C_i^R(x_{it}) = (n_i)^2 + n_i(x_{it} - 2(n_i - \gamma n_{-i})) \text{ for } i = 1, 2. \quad (9)$$

We notice that the externality is not internalized: both regions spend on local public goods up to the point where the regional marginal benefit is equal to the marginal cost. The transfer must, therefore, “compensate” regional voters for the impact of spending on local public goods in the other region. In each region, voters set the performance standard in period  $t$  taking the standard of the other region as given. At equilibrium, the standards are set to make each regional politician indifferent between satisfying the standard and getting reelected (for sure) and not satisfying it, in case of which he is replaced but keeps all local tax revenues  $T_i$  for himself. This yields the following stationary equilibrium allocation:

$$x_{it}^R = \frac{\beta T_i}{n_i} - n_i + 2(n_i - \gamma n_{-i}) \text{ for } i = 1, 2. \quad (10)$$

The politician of region  $i$  keeps a share  $(1 - \beta)T_i$  of regional tax revenues each period, and uses the rest to provide local public goods and transfers to voters of his region. A negative externality reduces voters’ welfare ( $\gamma > 0$ ), while a positive externality ( $\gamma < 0$ ) enhances their well-being, as one would expect.

Under federalism, decision making power rests with a single elected politician who faces the performance standards  $\{x_{1t}, x_{2t}\}$  set by voters in the two regions each period. The politician minimizes the cost of satisfying the two standards jointly by

spending  $k_{it} = (n_i - n_{-i}\gamma)^2$  on local public goods and by providing transfers  $s_{it} = x_{it} - 2n_i(1 + \gamma^2) + 4\gamma n_{-i}$  to voters in each of the two regions. The political cost function is therefore given by

$$\begin{aligned} C^F(x_{1t}, x_{2t}) &= (n_1 - n_2\gamma)^2 + (n_2 - n_1\gamma)^2 \\ &\quad + n_1(x_{1t} - 2n_1(1 + \gamma^2) + 4\gamma n_2) \\ &\quad + n_2(x_{2t} - 2n_2(1 + \gamma^2) + 4\gamma n_1) \end{aligned} \quad (11)$$

If the politician decides to satisfy the standard of one of the regions, say, region  $i$ , only, then it is clear that  $s_{-it} = 0$ . However, if local public goods generate a positive externality, it is cost effective to provide some local public goods to region  $-i$ : not because the politician cares about the welfare of voters in that region as such, but because it is, up to a point, cheaper to provide utility to voters in region  $i$  this way than to give them transfers. Hence, for  $\gamma < 0$ , the cost minimizing choice of spending on local public goods is  $k_{it} = (n_i)^2$  and  $k_{-it} = (n_i\gamma)^2$  and the transfer to each voter of group  $i$  is  $x_{it} - 2n_i(1 + \gamma^2)$ . If, on the other hand, local public goods generate negative externalities, then  $k_{-it} = 0$  minimizes political costs and the politician spends  $k_{it} = (n_i)^2$  on local public goods to region  $i$  and provides the voters of that region with the transfer  $s_{it} = x_{it} - 2n_i$ . With this in mind, we can for  $i = 1, 2$  write the political cost functions as follows

$$C_i^F(x_{it}) = (n_i)^2 + n_i(x_{it} - 2n_i) \text{ for } \gamma \geq 0 \quad (12)$$

$$C_i^F(x_{it}) = (1 + \gamma^2)(n_i)^2 + n_i(x_{it} - 2n_i(1 + \gamma^2)) \text{ for } \gamma < 0. \quad (13)$$

We notice that for  $\gamma < 0$

$$C^F(x_{1t}, x_{2t}) - \sum_i C_i^F(x_{it}) = 4n_2n_1\gamma < 0, \quad (14)$$

and for  $\gamma \geq 0$ ,

$$C^F(x_{1t}, x_{2t}) - \sum_i C_i^F(x_{it}) = \gamma(4n_1n_2 - \gamma(n_1^2 + n_2^2)) > 0. \quad (15)$$

The political cost function is sub-additive for  $\gamma < 0$  and additive for  $\gamma = 0$ . For  $\gamma > 0$ , the cost function is super-additive for all  $\gamma \in \left(0, \frac{n_2}{n_1}\right)$  as  $(3n_1^2 - n_2^2)\frac{n_2}{n_1} > 0$  for  $n_1 \geq n_2$ .

Below we apply propositions 1 and 2 to characterize stationary equilibrium allocations. Our main goal is to compare regime [F] and [R] under different assumptions about the magnitude of the externality. We use Pareto efficiency as our welfare criterion. In doing so, we adopt a citizens-centric approach and exclude the rents captured by the politicians. That is, we say that regime [F] Pareto dominates [R] if all voters prefer [F] to [R]. This approach has several advantages. Firstly, Besley and Coate (2003) propose to use aggregate public goods surplus to evaluate the costs and benefits of centralization. We prefer the Pareto criterion because it, in contrast to a criterion based on aggregate public goods surplus, has a clear-cut positive implication: if one institutional arrangement Pareto dominates another, *all* voters would support a change in the institutional arrangement if the decision to change was put to a vote in, e.g., a referendum (as in Crémer and Palfrey (1996)). Secondly, the citizens-centric approach has the advantage that the comparisons are not distorted by whether politicians can extract more or less rents. Since the rents are pure social waste in our model, this seems a reasonable choice from a normative point of view. However, from a positive point of view, it is interesting also to study how regional politicians, who may have a disproportionate say in whether centralization takes place or not, rank the different regimes, and we do so in section 6.

### 5.3 No Externality

To set the stage, we begin by considering the case in which there is no externality. In this case, political costs are additive and the total rent  $((1 - \beta)T)$  captured by the federal politician corresponds precisely to the sum of those captured by the two regional politicians  $((1 - \beta)T_1 + (1 - \beta)T_2)$ . An implication, then, is that the only effect of centralization is to allow redistribution between the two regions: with additive costs, centralization is a zero-sum game and if one region gains it must be at the expense of the other. Consequently, the two regimes cannot be Pareto ranked.

**Proposition 3 (No externality  $\gamma = 0$ )** *Regime [F] and [R] cannot be Pareto ranked. Region  $i$  prefers regime [F] to [R] if, and only if*

$$p_i > \frac{T_i}{T} \quad \text{for } i = 1, 2.$$

**Proof.** Using proposition 1, we can derive the equilibrium utility allocation in regime [F] as follows

$$x_{it}^F = \frac{\beta p_i T}{n_i} + n_i \text{ for } i = 1, 2.$$

The utility differences between regime [F] and [R] is

$$x_{it}^F - x_{it}^R = n_i^{-1} \beta (T p_i - T_i) = n_i^{-1} \beta (p_i T_{-i} - p_{-i} T_i) \equiv \widehat{\Delta}_i \text{ for } i = 1, 2. \quad (16)$$

where  $x_{it}^R$  is defined by equation (10). The proposition follows immediately from the fact that  $\widehat{\Delta}_1 > 0 \Leftrightarrow \widehat{\Delta}_2 < 0$  ■

Individuals in region  $i$  receives  $\frac{p_i \beta T}{n_i} + n_i$  from the federal government and  $\frac{\beta T_i}{n_i} + n_i$  from the regional government. Intuitively, therefore, whether a region gains or loses from centralization depends on  $p_i$  – the probability that each region holds the majority among those who turn out to vote in the federal election – relative to the region’s contribution to federal tax revenues. An implication, then, is that poor regions are, *ceteris paribus*, more likely to favor centralization than rich regions. For given tax resources, the size of the region as such does not matter for the costs and benefits of centralization. However, if the tax resources are, say, proportional to the population size of a region, i.e.,  $T_i = n_i I_i$  where  $I_i$  is per capita income of region  $i$ , then size becomes a consideration. Supposing, for example, that  $I_i = I_{-i}$  and  $p_i = p_{-i}$ , then region  $i$  benefits from federal redistribution if and only if it is smaller than region  $-i$ . In other words, in the absence of externalities, centralization tends to be favored by small and poor regions and/or by regions that are likely to be pivotal in federal decision making.

## 5.4 Negative Externalities

The situation is more complex and interesting when local public goods generate a negative externality ( $\gamma > 0$ ) and political costs become super-additive. In this case, centralization is associated with three effects. The first effect is the *redistribution effect* described above: centralization pools revenues from the two regions and thus allows redistribution to take place. The second effect is the *internalization effect*: centralization induces the federal politician to internalize the externality in order to minimize the cost

of getting reelected. This benefits all voters. The third effect is the *rent effect*. The rent effect arises because political costs are super-additive. Recall from proposition 2 that the federal politician's share of total revenues, at equilibrium, is *larger* than  $(1 - \beta)T$ . This implies that less is available in total to generate amenities to voters in the federation than in the two regions separately. This harms all voters. In the next proposition, we isolate the externality and rent effect from the redistribution effect by assuming that  $p_1 = \frac{1}{2}$  and that  $T_1 = T_2$ .

**Proposition 4 (Negative Externalities  $\gamma > 0$ )** *Let  $\theta = \frac{n_1}{n_2} \geq 1$ . Assume that  $p_1 = \frac{1}{2}$  and  $T_1 = T_2$ . Then for  $\beta > \frac{(1+\theta)(\theta-1)}{2(3\theta^2-1)}$*

1. *[R] is Pareto superior to [F] for  $\gamma \in (0, \frac{4(1-\beta)\theta}{(2-\beta)(1+\theta^2)})$ .*
2. *[F] is Pareto superior to [R] for  $\gamma \in (\frac{4(1-\beta)\theta}{(2-\beta)(1+\theta^2)}, \theta^{-1})$ .*

**Proof.** Using proposition 2 and equations (11) and (12), we can derive the (unique) stationary utility allocation as follows:

$$x_{it}^F = \frac{\beta p_i T + n_i^2 + \gamma(1 - \beta p_i)(\gamma n_{-i}^2 - 4n_i n_{-i} + \gamma n_i^2)}{n_i} \quad \text{for } i = 1, 2.$$

The utility differences between regime [F] and [R] are

$$\Delta_{it} = x_{it}^F - x_{it}^R = \widehat{\Delta}_i + \frac{\gamma(\gamma(1 - \beta p_i)(n_i^2 + n_{-i}^2) - 2n_i n_{-i}(1 - 2\beta p_i))}{n_i} \quad \text{for } i = 1, 2,$$

where  $\widehat{\Delta}_i$  is defined in equation (16). For  $p_1 = \frac{1}{2}$  and  $T_1 = T_2$ , we have that

$$\Delta_{it} = \frac{\gamma(\gamma(1 - \frac{1}{2}\beta)(n_i^2 + n_{-i}^2) - 2n_i n_{-i}(1 - \beta))}{n_i}.$$

We note that  $\Delta_{it} \geq 0 \Leftrightarrow \Delta_{-it} \geq 0$ . In particular,  $\Delta_{it} < 0$  for  $i = 1, 2$  for  $\gamma \in (0, \frac{4(1-\beta)\theta}{(2-\beta)(1+\theta^2)})$  and (weakly) positive for  $\gamma \in [\frac{4(1-\beta)\theta}{(2-\beta)(1+\theta^2)}, \theta^{-1})$  where  $\theta = \frac{n_1}{n_2}$ . Notice that  $\frac{4(1-\beta)\theta}{(2-\beta)(1+\theta^2)} < \theta^{-1} \Leftrightarrow \beta > \frac{(1+\theta)(\theta-1)}{2(3\theta^2-1)}$  ■

The proposition shows that centralization is efficient only with strong negative externalities. This is in contrast to the social planner's solution which showed that centralization is a Pareto improvement for all  $\gamma > 0$ . The result, however, echoes the classical

finding by Oates (1972), although the logic is entirely different. While Oates focused on the trade off between internalizing externalities and catering for differences in the preference for public goods in different regions, the trade off behind proposition 4 has nothing to do with heterogenous taste: it is driven by the rent effect. Centralization implies a transfer of resources from voters in the two regions to the federal politician. For weak externalities, both regions are, for that reason, worse off in a federation. However, for  $\gamma > \frac{4(1-\beta)\theta}{(2-\beta)(1+\theta^2)}$ , the externality effect is sufficiently strong to dominate the rent effect, and federalism Pareto dominates regionalism. It is interesting to notice that this threshold is decreasing in  $\theta$ . This means that two unequally sized regions are *more* likely to benefit from joining a federation than two equal-sized regions. The reason is that it is relatively expensive for a regional politician to compensate his voters through transfers for any un-internalized externalities when the two regions are of unequal size.

Proposition 4 ignores the redistribution effect which, as we noted above, is driven by turnout uncertainty as captured by  $p_i$  and differences in tax resources in the two regions. Taking this effect into account, we can define the values of  $p_1$  for which the two regions are indifferent between the two regimes as:

$$p_1^1(\gamma, \lambda) = \frac{\beta\lambda T_2 + \gamma(2n_1n_2 - \gamma(n_1^2 + n_2^2))}{\beta(T_2(1 + \lambda) + (4n_1n_2 - \gamma(n_1^2 + n_2^2))\gamma)}; \quad (17)$$

$$p_1^2(\gamma, \lambda) = \frac{\beta\lambda T_2 + \gamma^2(1 - \beta)(n_1^2 + n_2^2) - 2\gamma(1 - 2\beta)n_1n_2}{\beta(T_2(1 + \lambda) + (4n_1n_2 - \gamma(n_1^2 + n_2^2))\gamma)}; \quad (18)$$

where  $\lambda = \frac{T_1}{T_2}$ . Region 1 prefers regime [F] to [R] if, and only if  $p_1 > p_1^1(\gamma, \lambda)$  and region 2 prefers regime [F] to [R] if, and only if  $p_1 < p_1^2(\gamma, \lambda)$ . The two functions,  $p_1^1(\gamma, \lambda)$  and  $p_1^2(\gamma, \lambda)$ , are drawn in Figure 1 in  $(\gamma, p_1)$  space for a given value of  $\lambda$ . We can identify two main areas: in area 1 regime [R] is Pareto superior to [F], while in area 2, regime [F] Pareto dominates [R]. Outside these areas, the distribution effect is sufficiently strong to make one of the regions better off at the expense of the other. An increase in  $\lambda$  (which makes region 1 relatively richer) shifts  $p_1^1(\gamma, \lambda)$  and  $p_1^2(\gamma, \lambda)$  up making it less likely that region 1 and more likely that region 2 benefits from federalism.

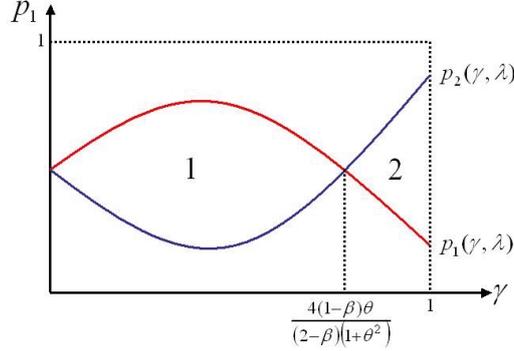


Figure 1: Welfare analysis with super-additive political costs.

## 5.5 Positive Externalities

The situation in which local public goods generate positive externalities is very different. In this case, political costs are sub-additive and proposition 1 shows that there exists multiple equilibria under federalism. Along all equilibrium paths, the aggregate utility of the two regions is, however, uniquely determined by

$$n_1 x_{1t} + n_2 x_{2t} = \beta T + ((n_1^2 + n_2^2)(1 + \gamma^2) - 4\gamma n_1 n_2). \quad (19)$$

Moreover, the lower bounds on the utility provided to each region is given by  $x_i \geq \frac{1}{n_i} (\beta p_i T + n_i^2 (1 + \gamma^2))$  for  $i = 1, 2$ . The federal politician collects the rent  $(1 - \beta)T$  each period. This is the same as the total rent collected by the two regional politicians: there is no rent effect with sub-additive costs. In the absence, then, of significant redistribution effects (i.e., for  $p_1 = \frac{1}{2}$ ,  $T_1 = T_2$ ), one might expect that centralization is always a Pareto improvement. The next proposition shows that this is not the case. To state the result, we denote the share of total utility that goes to region 1 by  $\varphi$ . This allows us to index equilibrium allocations by  $\varphi$ . We also, for simplicity and without loss of any important insights, assume that  $n_1 = n_2 = 1$ . This this case  $\varphi = \frac{x_{1t}}{\beta T + 2(1 - \gamma)^2}$ .

**Proposition 5 (Positive externalities  $\gamma < 0$ )** *Assume that  $p_1 = \frac{1}{2}$ ,  $T_1 = T_2$  and  $n_1 = n_2 = 1$ . Then there exists a  $\bar{\varphi} \in (0, \frac{1}{2})$  such that for  $\varphi \in [\bar{\varphi}, 1 - \bar{\varphi}]$  regime [F] Pareto dominates regime [R].*

**Proof.** Using proposition 1, we can calculate the “best” and the “worst” equilibrium allocation for each region under regime [F]:

$$\begin{aligned}x_{it}^{\max} &= p_i\beta T + 1 + \gamma^2 - 4\gamma \\x_{it}^{\min} &= \beta p_i T + 1 + \gamma^2\end{aligned}$$

for  $i = 1, 2$ . Region  $i$  is better off under [R] than under [F] in the “worst” equilibrium if

$$x_{it}^{\min} - x_{it}^D = \widehat{\Delta}_i + \gamma^2 + 2\gamma < 0,$$

and is better off under [F] than under [R] in the “best” equilibrium if

$$x_{it}^{\max} - x_{it}^D = \widehat{\Delta}_i + \gamma^2 - 2\gamma > 0,$$

where  $\widehat{\Delta}_i$  is defined in equation (16). For  $p_1 = \frac{1}{2}$  and  $T_1 = T_2$ , we see that  $x_{it}^{\min} - x_{it}^D < 0$  and  $x_{it}^{\max} - x_{it}^D > 0$  for  $i = 1, 2$ . Thus, at least one region prefers [F] to [R]. Along any equilibrium path

$$x_{1t} + x_{2t} = \beta T + 2(1 - \gamma)^2.$$

Define the share of total utility obtained by region  $i$  by  $\varphi_i$ . Region  $i$  is then indifferent between the two regimes for

$$\varphi_i = \frac{\beta T_i + 1 - 2\gamma}{\beta T + 2(1 - \gamma)^2} \equiv \bar{\varphi}_i.$$

Note that for  $T_1 = T_2$ ,  $0 < \bar{\varphi}_1 < 1 - \bar{\varphi}_2 < 1$  and that  $\bar{\varphi}_1 = \bar{\varphi}_2 < \frac{1}{2}$ . Since  $\sum_i \varphi_i = 1$ , we conclude that for  $\varphi_1 \in (\bar{\varphi}_1, 1 - \bar{\varphi}_2)$  both regions prefer [F] to [R]. Substitution of  $\varphi_1 = \varphi$  and  $\bar{\varphi}_1 = \bar{\varphi}$  yields the proposition ■

**Corollary 1** *For  $p_1 = \frac{1}{2}$ ,  $T_1 = T_2$ ,  $n_1 = n_2 = 1$  and  $\gamma < 0$ , there exist equilibrium allocations for which centralization is not a Pareto improvement.*

The proposition shows that federalism Pareto dominates regionalism in some, but not all, equilibria. In the absence of the rent and redistribution effect, it is surprising that centralization is not always efficient. Why is it not better for all voters to allow internalization of the external benefits? The reason is that the selection of equilibria,

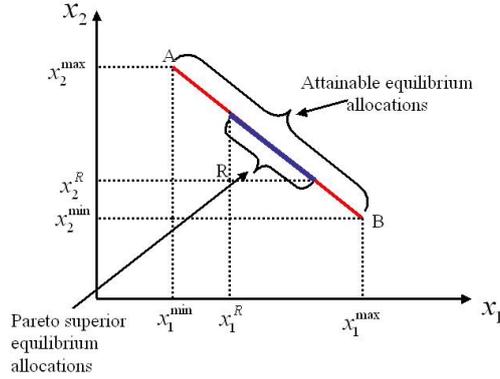


Figure 2: Utility allocations with sub-additive political costs.

in fact, re-opens the door to redistribution, but now redistribution is driven by the selection of equilibria, rather than by differences in  $p_i$  and  $T_i$  as such. For example, in the “worst” equilibrium under regime [F], the external benefit captured by region 1 is  $\gamma^2$  which is less than what it “receives” under [R], namely  $-2\gamma$ . The point is that in this equilibrium most of the benefits from having the positive externality internalized are captured by region 2 and region 1 is better off with the “external” benefits unintentionally bestowed on it by region 2 under regionalism. This – and the proposition more generally – is illustrated in Figure 2. The Figure shows the utility allocations attainable in the federation under the assumptions of the proposition. The segment  $A - B$  indicated with bold on the utility frontier contains the equilibrium allocations that Pareto dominate regionalism (represented by point  $R$ ). The remaining allocation on the frontier cannot be Pareto ranked. In these cases, contrary to the Decentralization Theorem, it is not efficient to centralize despite the fact that there are no regional differences in neither taste nor income, but there are (positive) externalities to be internalized. An implication of this, then, is that regionalism cannot ever Pareto dominate federalism with positive externalities. This stands in sharp contrast to the case with negative externalities discussed above.

## 6 Fiscal Integration and Disintegration

Logically, fiscal integration among otherwise independent regions or countries must either be fully voluntary or forced upon reluctant regions by more powerful neighbors. Voluntary integration leads to a stable fiscal structure, while the end result of forced integration must be considered unstable with a tendency to break down over time. Leading examples of the former include Switzerland, where the independent Cantons in 1848 agreed to form a federation, and the United States in the formative years. As an example of the latter one may point to the United Kingdom. England has traditionally played the leading role within the Union, but over the years her power has gradually been curtailed, first, by Ireland seceding in 1921, and more recently by the push to devolve power to Wales and Scotland. Our analysis can speak directly to the forces that create and destroy federations.<sup>12</sup>

In the absence of strong externalities (and economies of scale), federations are simply vehicles for redistribution and must be forced in one way or the other. Federal structures are, typically, supported by small, relatively poor regions that stand gain from integration and opposed by rich and populous regions that stand to lose. Of course, if the rich and populous regions are sufficiently powerful (in the sense of being more likely to be pivotal in federal decision making), this preference ordering may be reversed, but it remains that, in the absence of externalities, federalism cannot be based on consensus. As a consequence, federations born in this context are likely to be unstable with regions continuously trying to secede.

Voluntary formation of a federation, then, as in Oates (1972), requires strong externalities. Our analysis suggests that the logic leading to the formation of a stable federation differs significantly depending on whether externalities are predominately negative or positive. With negative externalities, the strength of the externality *is* the key driver of integration: a strong negative externality makes all regions favor a federation and accept the loss of accountability that comes with it. But heterogenous population sizes also play a role. In fact, federations are more likely to form among

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<sup>12</sup>See Alesina and Spolaore (1997) for analysis of the state formation.

regions of different sizes than among equal-sized regions. Surprisingly, a strong positive externality is not sufficient to make federalism the preferred organization structure of the fiscal state. The reason is that turnout uncertainty opens up the door for redistribution through equilibrium selection even among otherwise symmetric regions. Depending on the distributional outcome some regions may lose out and veto integration even when externalities are strong or, if they are already in the federation, attempt to secede.

We have so far taken a citizens-centric approach and ignored the interests of the regional politicians when making regime comparisons. In practice, however, regional politicians may have disproportionate influence on integration decisions and be able to supersede the interests of the voters they represent. To consider this possibility, suppose that the fiscal architecture of the country is decided by consent of the two regional politicians irrespective of what voters want and that each perceives that there is a probability  $q_i$ , with  $\sum_i q_i = 1$ , that he will become the “federal politician”.<sup>13</sup> Given that, centralization cannot be voluntary if the externality is (weakly) positive. The reason is that the total rent that can be extracted by the federal politician is equal to the sum of the rents extracted by the two regional politicians. As a consequence, one of them will lose, in expectation, by agreeing to a federation. With negative externalities, the situation is very different. Recall that the aggregate rent that can be extracted by the federal politician is greater than the sum of the rents extracted by the two regional politicians. This implies that federalism may be preferred to regionalism by all regional politicians. To see this, suppose that the two regions are symmetric with  $p_i = \frac{1}{2}$ ,  $n_1 = n_2 = 1$  and  $T_1 = T_2 = \frac{1}{2}T$ . In this case, the rent collected by the federal politician is:<sup>14</sup>

$$R^F = (1 - \beta) T + 2\gamma (1 - \beta) (2 - \gamma). \quad (20)$$

The politician of region  $i$  prefers federalism to regionalism if  $q_i R^F > \frac{1}{2} (1 - \beta) T$ . Then, for  $q_1$  such that

$$\bar{q}_1 > q_1 > 1 - \bar{q}_1 \quad (21)$$

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<sup>13</sup>If they do not have any chance of becoming the federal politician, they will veto any attempt at centralization since they will lose their rents.

<sup>14</sup>See Appendix III for details.

both politicians prefer [F] to [R] where

$$\bar{q}_1 = \frac{\frac{1}{2}(1-\beta)T + 2\gamma(1-\beta)(2-\gamma)}{(1-\beta)T + 2\gamma(1-\beta)(2-\gamma)} > \frac{1}{2}. \quad (22)$$

The threshold  $\bar{q}_1$  is increasing in the strength of the externality. Hence, the two regional politicians are most likely to consent to a federation if externalities are strong, not because they have any interest in internalizing these externalities, but because they can extract extra rents from voters in this case. Combined with proposition 4, this provides a very strong positive prediction: in the presence of strong negative externalities, all voters and all politicians support a federation.

## 7 Conclusion and Discussion

This paper revisits the classical question about whether fiscal decisions should be centralized or decentralized. We show how governance uncertainty – exemplified by turnout uncertainty – affects the trade off between internalization of externalities and political accountability. We highlight a novel asymmetry between positive and negative externalities and show that centralization only weakens political accountability in the presence of negative externalities. We also show that in the presence of positive externalities centralization may not be Pareto efficient despite the fact that policy can be tailored to regional tastes and centralization internalizes regional spillover effects. These results, however, ignore a potentially important benefit of decentralization, namely yardstick competition. As shown by Besley and Case (1995), voters can make comparisons between jurisdictions and use information about what is happening in other jurisdictions to overcome political agency problems. This forces incumbents into (yardstick) competition in which they care about what other incumbents are doing. This benefit is, of course, lost if fiscal decisions are centralized. It would be interesting in future research to extend the analysis to include the possibility of yardstick competition.

More generally, the paper explores the consequences of turnout uncertainty in a political agency model with repeated elections, retrospective voting, and heterogeneous voters. The general framework and the characterization results in Aidt and Dutta

(2004) can be adopted to many other applications than the one studied here. This includes other public finance problems, e.g., the choice between targeted transfers and universal public goods (see Aidt and Dutta (2010)), but applications in many other fields, including corporate governance and labor economics, also come to mind.

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## 8 Appendix I

To prove the main characterization results, we must first prove that all equilibria exhibits strategic consensus. We begin by introducing some extra notation. Let  $\underline{x}_{-i}(x_i)$  be the level of utility group  $-i$  obtains when the politician provides utility level  $x_i$  to group  $i$  at minimum cost without regard to the welfare of group  $-i$ . Then, the following is true:

$$\begin{aligned} [B_1] \quad C(x_{1t}, \underline{x}_{2t}(x_{1t})) &= C_1(x_{1t}) \\ [B_2] \quad C(\underline{x}_{1t}(x_{2t}), x_{2t}) &= C_2(x_{2t}). \end{aligned}$$

A special case of this is when  $C(x_{1t}, 0) = C_1(x_{1t})$  and  $C(0, x_{2t}) = C_2(x_{2t})$  as assumed in Aidt and Dutta (2004). We also assume that  $C(0, 0) = C_i(0) = 0$ ,  $i = 1, 2$ . We can now state the main Theorem.

**Theorem 1 (Strategic Consensus)** *Assume that  $\beta \in (0, 1)$ . Let  $x_t = (x_{1t}, x_{2t})$  be a pair of performance standards set by the two groups of voters for period  $t$  and define  $X = \{x_t\}_{t=0}^{\infty}$  as a sequence of such standards. Let  $a_t^*$  be the action implemented by the politician in period  $t$ ; define  $V_{0t}(a_t)$  as the politician's payoff.*

1. *A stationary subgame perfect Nash equilibrium exists.*
2. *Suppose (M), (K) and (C<sup>+</sup>) hold. Along any stationary equilibrium path, X satisfies*

$$(\mathbf{SC}^+) \quad V_{0t}(11) = V_{0t}(00) \geq \max\{V_{0t}(10), V_{0t}(01)\}.$$

*Any sequence X satisfying (SC<sup>+</sup>) is a stationary subgame perfect Nash equilibrium in performance standards. Along any stationary equilibrium path, the politician chooses  $a_t^* = (11)$  at every  $t$  and he is reelected for sure.*

3. *Suppose (M), (K) and (C<sup>-</sup>) hold. Along any stationary equilibrium path, X satisfies*

$$(\mathbf{SC}^-) \quad V_{0t}(11) = V_{0t}(10) = V_{0t}(01) > V_{0t}(00).$$

*Any sequence X satisfying (SC<sup>-</sup>) is a stationary subgame perfect Nash equilibrium in performance standards. Along any stationary equilibrium path, the politician chooses  $a_t^* = (11)$  at every  $t$  and he is reelected for sure.*

**Corollary 2** *Every stationary subgame perfect Nash equilibrium path displays strategic consensus at each  $t$ .*

We prove the Theorem with a series of Lemmas. We begin by introducing some notation. Denote for each action  $a_t \in A$ , the politician's payoff by  $V_{0t}(a_t)$  and write

$$V_{0t}(00) = T; \quad (23)$$

$$V_{0t}(10) = T - C_1(x_{1t}) + p_1\beta V_{t+1}; \quad (24)$$

$$V_{0t}(01) = T - C_2(x_{2t}) + p_2\beta V_{t+1}; \quad (25)$$

$$V_{0t}(11) = T - C(x_{1t}, x_{2t}) + \beta V_{t+1}. \quad (26)$$

where  $V_{t+1} > 0$  is the value of being reelected at time  $t + 1$ . Note that the politician is only reelected with some probability ( $p_1$  or  $p_2$ ) if he chooses to be "partisan" and satisfy one of the standards only.

Now, suppose, in some period  $t$ , that the two groups of voters announce the standards  $x_t = \{x_{1t}, x_{2t}\}$ . Given these standards, the politician chooses an action from the set  $\{a_t \in A : \arg \max_{a_t \in A} V_{0t}(a_t)\}$ . If the politician is indifferent between two or more actions in this set, he chooses the action that maximizes reelection chances. This is anticipated by the two groups of voters when they, simultaneously, set their standards at the beginning of the period. With these preliminary remarks we can state the first Lemma.

**Lemma 1** *Suppose that [M] and [K] hold. If the performance standards  $x_t = \{x_{1t}, x_{2t}\}$  constitute a subgame perfect Nash equilibrium at time  $t$ , then  $x_t$  must satisfy*

$$(E0) \quad V_{0t}(11) \geq \max\{V_{0t}(10), V_{0t}(01), V_{0t}(00)\}.$$

**Proof:** We argue by contradiction. Suppose that  $\tilde{x}_t = \{\tilde{x}_{1t}, \tilde{x}_{2t}\}$  constitutes a stationary subgame perfect Nash equilibrium in performance standards *and* that

$$V_{0t}(11) < \max\{V_{0t}(10), V_{0t}(01), V_{0t}(00)\}$$

at time  $t$ . There are four separate cases to consider. We show in each case that at least one of the two groups of voters has an incentive to deviate from  $\tilde{x}_t$ , leading to the required contradiction.

1. Suppose that

$$V_{0t}(10) = \max\{V_{0t}(10), V_{0t}(01), V_{0t}(00)\} > V_{0t}(11)$$

or that

$$V_{0t}(10) = V_{0t}(00) = \max\{V_{0t}(10), V_{0t}(01), V_{0t}(00)\} > V_{0t}(11).$$

Rewrite (24) and (26) to get

$$V_{0t}(10) - V_{0t}(11) = C(x_{1t}, x_{2t}) - C_1(x_{1t}) - p_2\beta V_{t+1}.$$

By [M] and [K], property  $[B_1]$  implies that there must exist a  $x'_{2t} > \underline{x}_{2t}$  such that

$$C(\tilde{x}_{1t}, x'_{2t}) - C_1(\tilde{x}_{1t}) - p_2\beta V_{t+1} < 0.$$

This implies that group 2 can gain by announcing the standard  $x'_{2t}$  instead of  $\tilde{x}_{2t}$ .

2. Suppose instead that

$$V_{0t}(01) = \max\{V_{0t}(10), V_{0t}(01), V_{0t}(00)\} > V_{0t}(11)$$

or that

$$V_{0t}(01) = V_{0t}(00) = \max\{V_{0t}(10), V_{0t}(01), V_{0t}(00)\} > V_{0t}(11).$$

By an argument similar to the previous case, there must exist a  $x'_{1t} > \underline{x}_{1t}$  such that

$$C(x'_{1t}, \tilde{x}_{2t}) - C_2(\tilde{x}_{2t}) - p_1\beta V_{t+1} < 0.$$

This implies that group 1 can gain by announcing the standard  $x'_{1t}$  instead of  $\tilde{x}_{1t}$ .

3. Suppose that

$$V_{0t}(00) = \max\{V_{0t}(10), V_{0t}(01), V_{0t}(00)\} > V_{0t}(11).$$

Rewrite equations (23) and (24) to get

$$V_{0t}(00) - V_{0t}(10) = C(x_{1t}, \underline{x}_{2t}) - p_1\beta V_{t+1}.$$

By [M] and [K] there must exist a  $x''_{1t} > 0$  such that

$$C(x''_{1t}, \underline{x}_{2t}) - p_1\beta V_{t+1} < 0.$$

This implies that group 1 can at least gain  $x''_{1t} > 0$  by announcing the standard  $x''_{1t}$  instead of  $\tilde{x}_{1t}$ . A similar argument can be made for group 2.

4. Suppose that

$$V_{0t}(10) = V_{0t}(01) = \max\{V_{0t}(10), V_{0t}(01), V_{0t}(00)\} > V_{0t}(11)$$

or

$$V_{0t}(10) = V_{0t}(01) = V_{0t}(00) > V_{0t}(11).$$

We need to consider two sub-cases. First, suppose the politician chooses  $a_t = (10)$ . We can then repeat the argument from case 1 to show that there exists a deviation for group 2. Second, suppose the politician chooses  $a_t = (01)$ . We can then repeat the argument from case 2 to show that there exists a deviation for group 1.

**Lemma 2** *A pair of performance standards  $x_t = (x_{1t}, x_{2t})$  is a stationary subgame perfect Nash equilibrium at time  $t$  if, and only if*

$$\text{(E1)} \quad V_{0t}(11) = \max\{V_{0t}(01), V_{0t}(00)\};$$

$$\text{(E2)} \quad V_{0t}(11) = \max\{V_{0t}(10), V_{0t}(00)\}.$$

**Proof:** Suppose that  $p_1 \geq \frac{1}{2}$ . The per-period payoff of group 1 is

$$\begin{aligned} u_{1t} &= x_{1t} && \text{if } \max\{V_{0t}(11), V_{0t}(10)\} \geq \max\{V_{0t}(01), V_{0t}(00)\}; \\ u_{1t} &= \underline{x}_{1t} && \text{otherwise.} \end{aligned}$$

The per-period payoff of group 2 is

$$u_{2t} = \begin{cases} x_{2t} & \text{if } \begin{cases} V_{0t}(11) \geq \max\{V_{0t}(10), V_{0t}(00), V_{0t}(01)\} \\ V_{0t}(01) > \max\{V_{0t}(10), V_{0t}(00), V_{0t}(11)\} \\ V_{0t}(01) = V_{0t}(00) > \max\{V_{0t}(10), V_{0t}(11)\} \end{cases} ; \\ \underline{x}_{2t} & \text{otherwise.} \end{cases}$$

Recall that  $C(x_{1t}, x_{2t})$  and  $C_i(x_{it})$  are monotonically increasing in their arguments by [M]. Suppose that  $\tilde{x}_t$  is a (stationary subgame perfect Nash) equilibrium. Then, by Lemma 1, (E0) is satisfied by  $\tilde{x}_t$ . It follows that the payoff of group 1 is maximized by the standard,  $x_{1t}$ , that satisfies (E1), and that the payoff of group 2 is maximized by the standard,  $x_{2t}$ , that satisfies (E2). Finally, notice that if (E1) and (E2) are satisfied by a set of performance standards at time  $t$ , then these standards constitute a stationary subgame perfect Nash Equilibrium. This completes the proof for the case with  $p_1 \geq \frac{1}{2}$ . The proof for the case where  $p_1 < \frac{1}{2}$  is similar and is omitted ■

The following two Lemmas explore the implications of assumptions (C<sup>+</sup>) and (C<sup>-</sup>), respectively.

**Lemma 3** *Conditions (E1), (E2), and (C<sup>+</sup>) hold at  $t$  if, and only if*

$$V_{0t}(11) = V_{0t}(00) \geq \max\{V_{0t}(10), V_{0t}(01)\}.$$

**Proof:** Note that (C<sup>+</sup>) implies that

$$(C0^+) \quad V_{0t}(11) + V_{0t}(00) \geq V_{0t}(10) + V_{0t}(01)$$

at any  $t$ . We prove the Lemma by contradiction. Suppose  $V_{0t}(11) > V_{0t}(00)$ . Condition (E2) implies that

$$V_{0t}(11) = V_{0t}(10).$$

Substitute into (C0<sup>+</sup>) to get that

$$V_{0t}(00) \geq V_{0t}(01).$$

Combing this with (E1) yields

$$V_{0t}(11) \leq V_{0t}(00).$$

This is a contradiction, so  $V_{0t}(11)$  cannot be greater than  $V_{0t}(00)$ . It follows directly from (E1) that  $V_{0t}(11)$  cannot be smaller than  $V_{0t}(00)$ . Finally,  $V_{0t}(11) = V_{0t}(00)$  is compatible with (C0<sup>+</sup>), (E1), and (E2) only if  $V_{0t}(10) \leq V_{0t}(00)$  and  $V_{0t}(01) \leq V_{0t}(00)$  ■

The next Lemma considers the case of super-additive costs.

**Lemma 4** *Conditions (E1), (E2), and (C<sup>-</sup>) hold at  $t$  if, and only if*

$$V_{0t}(11) = V_{0t}(10) = V_{0t}(01) > V_{0t}(00).$$

**Proof:** Note that  $(\mathbf{C}^-)$  implies that

$$(\mathbf{C0}^-) \quad V_{0t}(11) + V_{0t}(00) < V_{0t}(10) + V_{0t}(01)$$

at any  $t$ . We begin by proving that  $V_{0t}(11) = V_{0t}(10)$ . This is done by contradiction. First, suppose that  $V_{0t}(11) > V_{0t}(10)$ .  $(\mathbf{E2})$  implies that

$$V_{0t}(00) > V_{0t}(10).$$

Combining this with  $(\mathbf{C0}^-)$  implies that

$$V_{0t}(11) < V_{0t}(01).$$

However,  $(\mathbf{E1})$  implies that  $V_{0t}(11) \geq V_{0t}(01)$ . This is a contradiction, so  $V_{0t}(11)$  cannot be greater than  $V_{0t}(10)$ . Second, suppose that  $V_{0t}(10) > V_{0t}(11)$ .  $(\mathbf{E2})$  implies that

$$V_{0t}(11) \geq V_{0t}(10);$$

This is a contradiction, and so  $V_{0t}(10)$  cannot be greater than  $V_{0t}(11)$ . We conclude that  $V_{0t}(10) = V_{0t}(11)$ . The proof that  $V_{0t}(01) = V_{0t}(11)$  is similar and omitted. Finally,  $V_{0t}(11) = V_{0t}(10) = V_{0t}(01)$  is compatible with  $(\mathbf{C0}^-)$  only if  $V_{0t}(11) = V_{0t}(10) = V_{0t}(01) > V_{0t}(00)$  ■

The last Lemma establishes that a stationary subgame perfect equilibrium exists.

**Lemma 5** *A stationary subgame perfect equilibrium exists for  $\beta \in (0, 1)$ .*

**Proof:** Suppose first that  $(\mathbf{C}^+)$  holds. In this case, a stationary equilibrium  $\hat{x} = \{\hat{x}_1, \hat{x}_2\}$  satisfies  $(\mathbf{SC}^+)$  at every  $t$ . This implies

$$\frac{T - C(\hat{x}_1, \hat{x}_2)}{1 - \beta} = T; \tag{27}$$

and that

$$T \geq \max\left[\frac{T - C_1(\hat{x}_1)}{(1 - p_1\beta)}, \frac{T - C_2(\hat{x}_2)}{(1 - p_2\beta)}\right].$$

Equation (27) rewrites as

$$C(\hat{x}_1, \hat{x}_2) = \beta T.$$

Equilibrium levels of  $\hat{x}$  satisfy

$$C(\hat{x}_1, \hat{x}_2) = \beta T \tag{28}$$

and

$$T \leq \min\left[\frac{C_1(\hat{x}_1)}{p_1\beta}, \frac{C_2(\hat{x}_2)}{p_2\beta}\right]. \tag{29}$$

It follows from conditions  $(\mathbf{C}^+)$ ,  $(\mathbf{M})$  and  $(\mathbf{K})$  that there exists a solution to equations (28) and (29).

Suppose instead that  $(\mathbf{C}^-)$  holds. In this case, a stationary equilibrium  $\bar{x} = \{\bar{x}_1, \bar{x}_2\}$  must satisfy

$$\frac{T - C(\bar{x}_1, \bar{x}_2)}{1 - \beta} = \frac{T - C_1(\bar{x}_1)}{1 - p_1\beta} \tag{30}$$

and

$$\frac{T - C(\bar{x}_1, \bar{x}_2)}{1 - \beta} = \frac{T - C_2(\bar{x}_2)}{1 - p_2\beta} \quad (31)$$

along with

$$\frac{T - C(\bar{x}_1, \bar{x}_2)}{1 - \beta} > T. \quad (32)$$

Define the quantities  $x_{11}, x_{12}, x_{21}, x_{22}$  as solutions to equations (30) and (31) when  $x_1 = \underline{x}_1$  and  $x_2 = \underline{x}_2$  respectively. Then,

$$\begin{aligned} \frac{T - C(x_{11}, \underline{x}_2)}{1 - \beta} &= \frac{T - C(x_{11}, \underline{x}_2)}{1 - p_1\beta}, \\ \frac{T - C(\underline{x}_1, x_{21})}{1 - \beta} &= \frac{T}{1 - p_1\beta} \\ \frac{T - C(x_{12}, \underline{x}_2)}{1 - \beta} &= \frac{T}{1 - p_2\beta} \\ \frac{T - C(\underline{x}_1, x_{22})}{1 - \beta} &= \frac{T - C(\underline{x}_1, x_{22})}{1 - p_2\beta}. \end{aligned}$$

Solving these equations yields

$$T = C(x_{11}, \underline{x}_2) = C(\underline{x}_1, x_{22});$$

in addition,

$$x_{12} \leq x_{11};$$

and

$$x_{21} \leq x_{22}$$

whenever  $\beta \in (0, 1)$ . It follows that a solution to equations (30) and (31) exists.

Additionally, if  $\bar{x}$  satisfies equations (30) and (31) then restriction (32) holds for all  $\beta \in (0, 1)$ . To show that an equilibrium exists for all  $\beta \in (0, 1)$ , rewrite (30) and (31) as

$$\begin{aligned} T\theta &= (1 + \theta)C(\bar{x}_1, \bar{x}_2) - C(\bar{x}_1, \underline{x}_2); \\ T\eta &= (1 + \eta)C(\bar{x}_1, \bar{x}_2) - C(\underline{x}_1, \bar{x}_2); \end{aligned}$$

where  $\theta = \frac{p_2\beta}{1-\beta}$  and  $\eta = \frac{p_1\beta}{1-\beta}$ . Adding the two equations, we obtain

$$(\theta + \eta)(T - C(\bar{x}_1, \bar{x}_2)) - C(\bar{x}_1, \bar{x}_2) = C(\bar{x}_1, \bar{x}_2) - C(\bar{x}_1, \underline{x}_2) - C(\underline{x}_1, \bar{x}_2) > 0 \quad (33)$$

by [C-]. Note also that  $\theta + \eta = \frac{\beta}{1-\beta}$  and that (33) implies

$$C(\bar{x}_1, \bar{x}_2) < \beta T$$

as assumed  $\blacksquare$

Based on this fundamental result, it is relatively straight forward to prove propositions 1 and 2.

**Proof of proposition 1.** The value of reelection starting from any period  $t$  is  $V_{0t} = \max[V_{0t}(01), V_{0t}(10), V_{0t}(11), V_{0t}(00)]$ . We obtain from Lemma 3 and equation (23) that  $V_{0t} = V_{0t}(00) = T$ . This implies that

$$V_{0t+1} = T.$$

We obtain, from Theorem 1 and equations (23) to (26), that

$$V_{0t}(11) = V_{0t}(00) \Rightarrow C(x_{1t}, x_{2t}) = \beta T;$$

and that

$$V_{0t}(00) \geq V_{0t}(10) \Rightarrow C_1(x_{1t}) \geq \beta p_1 T;$$

$$V_{0t}(00) \geq V_{0t}(01) \Rightarrow C_2(x_{2t}) \geq \beta p_2 T.$$

The politician's per period payoff is  $T - C(x_{1t}, x_{2t}) = (1 - \beta)T$ . Moreover, suppose  $C(x_{1t}, x_{2t}) = C_1(x_{1t}) + C_2(x_{2t})$ . Then, there exist a unique stationary equilibrium,  $x_{1t} = x_1^*$  and  $x_{2t} = x_2^*$ , with

$$C_1(x_1^*) = \beta p_1 T;$$

$$C_2(x_2^*) = \beta p_2 T.$$

**Proof of proposition 2.** The value of reelection starting from any period  $t$  is  $V_{0t} = \max[V_{0t}(01), V_{0t}(10), V_{0t}(11), V_{0t}(00)]$ . We obtain from Lemma 4 that  $V_{0t} = V_{0t}(11)$  for all  $t$ . Iterative, forward substitution, using equation (26), yields

$$V_{0t} = \sum_{k=0}^{\infty} \beta^k (T - C(x_{1t+k}, x_{2t+k})).$$

For sequences of stationary standards, we get

$$V_{0t} = V_{0t+1} = \frac{T - C(x_1, x_2)}{1 - \beta}.$$

Substituting for  $V_{0t+1} = \frac{T - C(x_1, x_2)}{1 - \beta}$ , we get that

$$V_{0t}(11) = V_{0t}(10) \Rightarrow (\mathbf{SC}_1^-)$$

and

$$V_{0t}(11) = V_{0t}(01) \Rightarrow (\mathbf{SC}_2^-).$$

Finally,  $V_{0t} = \frac{T - C(x_1, x_2)}{1 - \beta}$  for all  $t$  implies that the politician gets  $T - C(x_1, x_2)$  per period. This is strictly greater than  $(1 - \beta)T$  because  $V_{0t}(11) > V_{0t}(00)$  by Lemma 4. For uniqueness, see proposition 3 in Aidt and Dutta (2004) ■

## 9 Appendix II

In this appendix, we derive the political cost function under federalism. Suppose the politician wants to satisfy both regions. He, then, solves the following problem each period (where we have omitted subscript  $t$  for simplicity):

$$\min_{k_1, k_2, s_1, s_2} k_1 + k_2 + n_1 s_1 + n_2 s_2$$

subject to

$$x_1 \leq 2k_1^{\frac{1}{2}} - 2\gamma k_2^{\frac{1}{2}} + s_1$$

$$x_2 \leq 2k_2^{\frac{1}{2}} - 2\gamma k_1^{\frac{1}{2}} + s_2$$

Under the assumption that  $\gamma < \frac{n_2}{n_1}$ ,  $k_1$  and  $k_2$  are (weakly) positive at the optimum. It is useful to distinguish between four cases:

1.  $s_1 > 0, s_2 > 0$
2.  $s_1 = s_2 = 0$
3.  $s_1 = 0, s_2 > 0$
4.  $s_1 > 0, s_2 = 0$

Case 1: Substituting the two constraints, which must be binding at the optimum, into the objective function and taking the first derivatives with respect to  $k_1$  and  $k_2$  yields:

$$\begin{aligned} 1 - n_1 k_1^{-\frac{1}{2}} + n_2 \gamma k_1^{-\frac{1}{2}} &= 0 \\ 1 - n_2 k_2^{-\frac{1}{2}} + n_1 \gamma k_2^{-\frac{1}{2}} &= 0. \end{aligned}$$

Solving this, we get  $k_1 = (n_1 - n_2 \gamma)^2$  and  $k_2 = (n_2 - n_1 \gamma)^2$ . The per capita transfers are

$$\begin{aligned} s_1 &= x_1 - 2(n_1 - n_2 \gamma) + 2\gamma(n_2 - n_1 \gamma) = x_1 - 2n_1(1 + \gamma^2) + 4\gamma n_2 \\ s_2 &= x_2 - 2(n_2 - n_1 \gamma) + 2\gamma(n_1 - n_2 \gamma) = x_2 - 2n_2(1 + \gamma^2) + 4\gamma n_1 \end{aligned}$$

Notice that  $s_i > 0$  requires that  $x_i > 2n_i(1 + \gamma^2) - 4\gamma n_{-i}$ . The political cost function is

$$\begin{aligned} C(x_1, x_2) &= (n_1 - n_2 \gamma)^2 + (n_2 - n_1 \gamma)^2 \\ &\quad + n_1(x_1 - 2n_1(1 + \gamma^2) + 4\gamma n_2) \\ &\quad + n_2(x_2 - 2n_2(1 + \gamma^2) + 4\gamma n_1) \end{aligned}$$

Case 2: This case applies for  $x_1 \leq n_1(1 + \gamma^2) - 2\gamma n_2$  and  $x_2 \leq n_2(1 + \gamma^2) - 2\gamma n_1$ . We need to make a distinction between three sub-cases. Firstly, let  $\gamma \geq 0$  and  $\min\left\{\frac{x_1}{x_2}, \frac{x_2}{x_1}\right\} > -\gamma$  or  $\gamma < 0$  and  $\min\{x_1, x_2\} > 0$ . Then, both constraints are binding and we can solve them to get the lowest spending level on the two local public goods that will generate the required utility levels:

$$\begin{aligned} k_1 &= \left(\frac{x_1 + \gamma x_2}{2(1 - \gamma^2)}\right)^2 \\ k_2 &= \left(\frac{x_2 + \gamma x_1}{2(1 - \gamma^2)}\right)^2 \end{aligned}$$

and the cost function is

$$C(x_1, x_2) = \left(\frac{x_1 + \gamma x_2}{2(1 - \gamma^2)}\right)^2 + \left(\frac{x_2 + \gamma x_1}{2(1 - \gamma^2)}\right)^2.$$

Notice that  $C(0, 0) = 0$ . Secondly, suppose that the constraint for group 1 is not binding. First, if  $\gamma \geq 0$ , then  $k_1 = 0$  and  $k_2 = \left(\frac{x_2}{2}\right)^2$  and

$$C(x_1, x_2) = \left(\frac{x_2}{2}\right)^2 \quad \text{for } x_1 \leq -\gamma x_2.$$

Second, if  $\gamma < 0$ , the politician solves

$$\min k_1 + k_2$$

subject to

$$x_2 \leq 2k_2^{\frac{1}{2}} - 2\gamma k_1^{\frac{1}{2}}.$$

Letting  $v$  be the multiplier on the constraint, we can write the first order conditions as

$$\begin{aligned} 1 - k_2^{-\frac{1}{2}}v &= 0 \\ 1 + \gamma k_1^{-\frac{1}{2}}v &= 0 \end{aligned}$$

Solving for  $k_1$  and  $k_2$  and substituting into the constraint yields

$$v = \frac{x_2}{2(1 + \gamma^2)}$$

and we find that  $k_2 = \left(\frac{x_2}{2(1+\gamma^2)}\right)^2$  and  $k_1 = \left(\frac{\gamma x_2}{2(1+\gamma^2)}\right)^2$  for  $x_2 \geq 0$ . The political cost function is

$$\begin{aligned} C(x_1, x_2) &= \left(\frac{x_2}{2(1 + \gamma^2)}\right)^2 + \left(\frac{\gamma x_2}{2(1 + \gamma^2)}\right)^2 \\ &= \frac{x_2^2}{4(1 + \gamma^2)} \text{ for } x_1 < 0. \end{aligned}$$

Third, suppose that the constraint for group 2 is not binding. By analogy we get for  $\gamma \geq 0$  that  $k_2 = 0$  and  $k_1 = \left(\frac{x_1}{2}\right)^2$  and

$$C(x_1, x_2) = \left(\frac{x_1}{2}\right)^2 \text{ for } x_2 \leq -\gamma x_1.$$

For  $\gamma < 0$ , we get

$$C(x_1, x_2) = \frac{x_1^2}{4(1 + \gamma^2)} \text{ for } x_2 < 0.$$

Case 3: Substituting  $s_2$  out from the beginning, we can write the Lagrange function as

$$L = k_1 + k_2 + n_2 \left(x_2 - 2k_2^{\frac{1}{2}} + 2\gamma k_1^{\frac{1}{2}}\right) + \psi \left(x_1 - 2k_1^{\frac{1}{2}} + 2\gamma k_2^{\frac{1}{2}}\right)$$

where  $\psi$  is a Lagrange multiplier. We can calculate the first order conditions:

$$1 + n_2 \gamma k_1^{-\frac{1}{2}} - \psi k_1^{-\frac{1}{2}} = 0 \tag{34}$$

$$1 - n_2 k_2^{-\frac{1}{2}} + \psi \gamma k_2^{-\frac{1}{2}} = 0 \tag{35}$$

$$x_1 - 2k_1^{\frac{1}{2}} + 2\psi k_2^{\frac{1}{2}} = 0 \tag{36}$$

Solve equations (34) and (35) to get

$$\begin{aligned} k_1 &= (\psi - n_2 \gamma)^2 \\ k_2 &= (n_2 - \gamma \psi)^2. \end{aligned}$$

Substitute this in equation (36) and solve for  $\psi$ :

$$\psi = \max \left\{ \frac{x_1 + 4\gamma n_2}{2(1 + \gamma^2)}, 0 \right\}.$$

Using this, we get that for  $\psi > 0 \Leftrightarrow x_1 > 4\gamma n_2$

$$\begin{aligned} k_1 &= \left( \frac{x_1 + 4\gamma n_2}{2(1 + \gamma^2)} - n_2\gamma \right)^2 = \left( \frac{x_1 + 2\gamma n_2(1 - \gamma^2)}{2(1 + \gamma^2)} \right)^2 \\ k_2 &= \left( n_2 - \gamma \frac{x_1 + 4\gamma n_2}{2(1 + \gamma^2)} \right)^2 = \left( \frac{2n_2(1 - \gamma^2) - \gamma x_1}{2(1 + \gamma^2)} \right)^2 \\ s_2 &= x_2 - 2 \left( \frac{2n_2(1 - \gamma^2) - \gamma x_1}{2(1 + \gamma^2)} \right) + 2\gamma \left( \frac{x_1 + 2\gamma n_2(1 - \gamma^2)}{2(1 + \gamma^2)} \right) \end{aligned}$$

and the political cost function is

$$\begin{aligned} C(x_1, x_2) &= \left( \frac{x_1 + 2\gamma n_2(1 - \gamma^2)}{2(1 + \gamma^2)} \right)^2 + \left( \frac{2n_2(1 - \gamma^2) - \gamma x_1}{2(1 + \gamma^2)} \right)^2 \\ &\quad + n_2 \left( x_2 - 2 \left( \frac{2n_2(1 - \gamma^2) - \gamma x_1}{2(1 + \gamma^2)} \right) + 2\gamma \left( \frac{x_1 + 2\gamma n_2(1 - \gamma^2)}{2(1 + \gamma^2)} \right) \right). \end{aligned}$$

We notice that  $s_2 > 0$  requires that  $x_2 > \frac{2(n_2(1 - 2\gamma^2 + \gamma^4) - \gamma x_1)}{(1 + \gamma^2)}$ . For  $x_1 \leq 4\gamma n_2$ ,  $\psi = 0$ . The first order conditions are

$$\begin{aligned} 1 + n_2\gamma k_1^{-\frac{1}{2}} &\geq 0 \\ 1 - n_2k_2^{-\frac{1}{2}} &\geq 0. \end{aligned}$$

For  $\gamma \geq 0$ ,  $k_1 = 0$  and  $k_2 = (n_2)^2$  and the cost function is

$$C(x_1, x_2) = (n_2)^2 + n_2(x_2 - 2n_2).$$

For  $\gamma < 0$ ,  $k_2 = (n_2)^2$  and  $k_1 = (\gamma n_2)^2$  and the cost function is

$$C(x_1, x_2) = (n_2)^2(1 + \gamma^2) + n_2(x_2 - 2n_2(1 + \gamma^2)).$$

Case 4: This is similar to case 3. For  $\psi > 0 \Leftrightarrow x_2 > 4\gamma n_1$ , we get

$$\begin{aligned} k_1 &= \left( n_1 - \gamma \frac{x_2 + 4\gamma n_1}{2(1 + \gamma^2)} \right)^2 = \left( \frac{2n_1(1 - \gamma^2) - \gamma x_2}{2(1 + \gamma^2)} \right)^2 \\ k_2 &= \left( \frac{x_2 + 4\gamma n_1}{2(1 + \gamma^2)} - n_1\gamma \right)^2 = \left( \frac{x_2 + 2\gamma n_1(1 - \gamma^2)}{2(1 + \gamma^2)} \right)^2 \\ s_1 &= x_1 - 2 \left( \frac{2n_1(1 - \gamma^2) - \gamma x_2}{2(1 + \gamma^2)} \right) + 2\gamma \left( \frac{x_2 + 2\gamma n_1(1 - \gamma^2)}{2(1 + \gamma^2)} \right) \end{aligned}$$

and the political cost function is

$$C(x_1, x_2) = \left( \frac{x_1 + 2\gamma n_1(1 - \gamma^2)}{2(1 + \gamma^2)} \right)^2 + \left( \frac{2n_1(1 - \gamma^2) - \gamma x_2}{2(1 + \gamma^2)} \right)^2 + n_1 \left( x_1 - 2 \left( \frac{2n_1(1 - \gamma^2) - \gamma x_2}{2(1 + \gamma^2)} \right) + 2\gamma \left( \frac{x_2 + 2\gamma n_1(1 - \gamma^2)}{2(1 + \gamma^2)} \right) \right).$$

We notice that  $s_1 > 0$  requires that  $x_1 > \frac{2(n_1(1 - 2\gamma^2 + \gamma^4) - \gamma x_2)}{(1 + \gamma^2)}$ . For  $x_2 \leq 4\gamma n_1$ ,  $\psi = 0$ . For  $\gamma \geq 0$ ,  $k_2 = 0$  and  $k_1 = (n_1)^2$  and the cost function is

$$C(x_1, x_2) = (n_1)^2 + n_1(x_1 - 2n_1).$$

For  $\gamma < 0$ ,  $k_1 = (n_1)^2$  and  $k_2 = (\gamma n_1)^2$  and the cost function is

$$C(x_1, x_2) = (n_1)^2(1 + \gamma^2) + n_1(x_1 - 2n_1(1 + \gamma^2)).$$

Now, suppose that the politician will only try to satisfy the demands of group  $i$ . There are two cases to consider:

1.  $s_i > 0$ .
2.  $s_i = 0$ .

Case 1: The politician solves

$$\min_{k_i, k_{-i}, s_i} k_i + k_{-i} + n_i s_i$$

subject to  $x_i \leq s_i + 2k_i^{\frac{1}{2}} - 2\gamma k_{-i}^{\frac{1}{2}}$ . The optimal choice is

$$k_i = (n_i)^2 \text{ and } k_{-i} = 0 \text{ for } \gamma \geq 0$$

and

$$k_i = (n_i)^2 \text{ and } k_{-i} = (n_i \gamma)^2 \text{ for } \gamma < 0.$$

The transfer is

$$s_i = \begin{cases} x_i - 2n_i & \text{for } \gamma \geq 0 \\ x_i - 2n_i(1 + \gamma^2) & \text{for } \gamma < 0 \end{cases}$$

The political cost function is

$$C_i(x_i) = \begin{cases} (n_i)^2 + n_i(x_i - 2n_i) & \text{for } \gamma \geq 0 \\ (1 + \gamma^2)(n_i)^2 + n_i(x_i - 2n_i(1 + \gamma^2)) & \text{for } \gamma < 0 \end{cases}.$$

Notice that for  $\gamma \geq 0$ ,  $x_i > 2n_i$  for  $s_i > 0$  and for  $\gamma < 0$ ,  $x_i > n_i 2(1 + \gamma^2)$  for  $s_i > 0$ .

Case 2: First, if  $\gamma \geq 0$ , then  $k_i = \left(\frac{x_i}{2}\right)^2$  and  $k_{-i} = 0$  and

$$C(x_i) = \left(\frac{x_i}{2}\right)^2 \text{ for } x_{-i} \leq -\gamma x_i.$$

Second, if  $\gamma < 0$ , the politician solves

$$\min_{k_i, k_{-i}} k_i + k_{-i}$$

subject to

$$x_i \leq 2k_i^{\frac{1}{2}} - 2\gamma k_{-i}^{\frac{1}{2}}.$$

Letting  $\chi$  be the multiplier on the constraint, we can write the first order conditions as

$$\begin{aligned} 1 - k_i^{-\frac{1}{2}} \chi &= 0; \\ 1 + \gamma k_{-i}^{-\frac{1}{2}} \chi &= 0. \end{aligned}$$

Solving for  $k_1$  and  $k_2$  and substituting into the constraint yields

$$\chi = \frac{x_i}{2(1 + \gamma^2)},$$

and we find that  $k_i = \left(\frac{x_i}{2(1 + \gamma^2)}\right)^2$  and  $k_{-i} = \left(\frac{\gamma x_i}{2(1 + \gamma^2)}\right)^2$  for  $x_i \geq 0$ . The political cost function is

$$\begin{aligned} C(x_i) &= \left(\frac{x_i}{2(1 + \gamma^2)}\right)^2 + \left(\frac{\gamma x_i}{2(1 + \gamma^2)}\right)^2 \\ &= \frac{x_i^2}{4(1 + \gamma^2)} \text{ for } x_i < 0. \end{aligned}$$

In the text, we focus on the case where, at equilibrium, the politician offers local public goods *and* transfers. This requires that tax revenues are sufficiently large. More specifically, it requires the following.

1. Under [R], each regional politician spends  $k_i = (n_i)^2$  and  $x_{it} - 2n_i + 2\gamma n_{-i}$  on transfers. The equilibrium payoff is

$$x_{it}^R = \frac{\beta T_i}{n_i} - n_i + 2(n_i - \gamma n_{-i}).$$

Substitute this into the expression for the transfer and note that  $s_i > 0$  requires that  $T_i > \frac{(n_i)^2}{\beta}$  for  $i = 1, 2$ .

2. Under [F], two cases can arise. For  $\gamma \geq 0$ , we can, using proposition 2, write the payoff to group  $i$  at time  $t$  as

$$\begin{aligned} x_{1t}^F &= \frac{T\beta p_1 + n_1^2 + \gamma(1 - \beta p_1)(\gamma n_2^2 - 4n_1 n_2 + \gamma n_1^2)}{n_1} \\ x_{2t}^F &= \frac{T\beta p_2 + n_2^2 + \gamma(1 - \beta p_2)(\gamma n_1^2 - 4n_1 n_2 + \gamma n_2^2)}{n_2} \end{aligned}$$

The transfers are

$$\begin{aligned} s_1 &= x_1 - 2(n_1 - n_2\gamma) + 2\gamma(n_2 - n_1\gamma) = x_1 - 2n_1(1 + \gamma^2) + 4\gamma n_2 \\ s_2 &= x_2 - 2(n_2 - n_1\gamma) + 2\gamma(n_1 - n_2\gamma) = x_2 - 2n_2(1 + \gamma^2) + 4\gamma n_1 \end{aligned}$$

At equilibrium, they are positive for

$$T > \max_i \left\{ \frac{n_i^2(1 + \gamma^2(1 + \beta p_i)) - 4\beta\gamma n_1 n_2 p_i + n_{-i}^2 \gamma^2(1 - \beta p_i)}{\beta p_i} \right\}.$$

For  $\gamma < 0$ , we can, using proposition 1, define the minimum equilibrium payoffs as

$$(1 + \gamma^2)(n_i)^2 + n_i(x_i - 2n_i(1 + \gamma^2)) = \beta p_i T.$$

Solving this yields  $x_{it} = \frac{1}{n_i}(T\beta p_i + n_i^2(\gamma^2 + 1))$ .  $s_i > 0$ , then, requires that

$$x_i > 2n_i(1 + \gamma^2) + 4\gamma n_{-i}$$

or

$$T > \max_i \left\{ \frac{n_i}{\beta p_i} (4\gamma n_{-i} + n_i(\gamma^2 + 1)) \right\}.$$

So, overall we need

$$T > \max_i \left\{ \frac{n_i}{\beta p_i} (4\gamma n_{-i} + n_i(\gamma^2 + 1)); \frac{n_i^2(1 + \gamma^2(1 + \beta p_i)) - 4\beta\gamma n_1 n_2 p_i + n_{-i}^2 \gamma^2(1 - \beta p_i)}{\beta p_i} \right\}.$$

## 10 Appendix III

The rent,  $R^F$ , extracted by the federal politician is  $T - C(x_1^F, x_2^F)$  where  $C(x_1^F, x_2^F)$  is given in equation (11) and

$$x_i^F = \frac{1}{2}\beta T + 1 + 2\gamma(1 - \frac{1}{2}\beta)(\gamma - 2) \quad \text{for } i = 1, 2. \quad (37)$$

Substitution yields equation (20). A comparison yields that  $q_1 R^F > \frac{1}{2}(1 - \beta)T$  if and only if

$$q_1 > \frac{\frac{1}{2}(1 - \beta)T}{R^F} \quad (38)$$

and that  $(1 - q_1)R^F > \frac{1}{2}(1 - \beta)T$  if and only if

$$q_1 < 1 - \frac{\frac{1}{2}(1 - \beta)T}{R^F}. \quad (39)$$

Substitution of  $R^F$  into equation (39) yields  $\bar{q}_1$ . We notice that  $\bar{q}_1 > \frac{1}{2}$  because

$$\begin{aligned} & \frac{\frac{1}{2}(1 - \beta)T + 2\gamma(1 - \beta)(2 - \gamma)}{(1 - \beta)T + 2\gamma(1 - \beta)(2 - \gamma)} - \frac{1}{2} \\ &= \frac{(2 - \gamma)\gamma}{(T + 4\gamma - 2\gamma^2)} > 0. \end{aligned} \quad (40)$$

This implies that there exist values of  $q_1$  such that both regional politician benefit from centralization. Moreover,

$$\frac{\partial \bar{q}_1}{\partial \gamma} = \frac{2(1-\gamma)T}{(T+4\gamma-2\gamma^2)^2} > 0 \quad (41)$$

for  $\gamma < \frac{n_2}{n_1} < 1$ .