

Social Welfare Issues of Financial Literacy

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August 2010

CWPE 1036

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Abstract

In the matter of financial literacy it is often supposed that *more* is automatically preferable to *less*. This paper considers to what extent this may be true generally, and specifically focuses on the case of investment forecasting skill (a significant component of an individual's financial literacy). We show that while improved forecasting skill can increase an individual's own utility, the resulting increase in trading volume leads to higher asset price volatility. Under the plausible assumption that this volatility imposes disutility on non-investors, an interesting trade-off is exposed between the benefits of skill improvement which accrue to investors, and the costs suffered more broadly by society. The paper constructs a formal analytic framework in which to discuss these issues, examines under what conditions the marginal utility of skill is in fact monotonic for the individual and considers implications for policy-makers.

JEL Classification: D14, D53, D60, D82, D84, G01, G17, G18, G28

Keywords: financial literacy, welfare, skill, active management, financial crises

*The authors thank Robert Kosowski, Hamish Low and Susan Thorp for many useful comments. O.J. Williams acknowledges support from the Economic and Social Research Council (award number PTA-031-2004-00054).

1 Introduction

“Where ignorance is bliss, ’Tis folly to be wise” *Thomas Gray (1742)*

“All wisdom is foolish that does not adapt itself to the common folly” *Michel Eyquem de Montaigne (1533-1592)*¹

1.1 Financial Literacy

The topic of financial literacy has attracted a growing literature in recent years, with its relevance very much underlined by the prominent role played by consumer finance in global credit crises from 2007 onwards. The concept cuts across many diverse fields of economics, encompassing issues at the level of the individual, such as consumers’ ability to manage a household budget or make informed decisions about credit, as well as at the aggregate level, such as the impact of financial literacy on stock-market participation (with attendant consequences for asset price dynamics).²

A key theme of the literature is that the rapid pace of financial innovation has led to a wide array of savings, credit and investment products, provided not only by traditional institutions, but also by novel forms of intermediaries and agents (a process which has been accelerated by technological progress, particularly the internet). Comprehending this landscape, and evaluating competing product offerings, has placed non-trivial demands on consumers’ analytic capabilities, with consequently a high likelihood of sub-optimal financial decision-making. At the extremes, a plausible hypothesis is that the financial *litterati* benefit from access to superior investment opportunities while less well-informed members of society fall prey to predatory lending or fraudulent investment scams.

Simultaneously, financial literacy has become a prominent item on the public agenda world-wide. (Cox (2006), HM Treasury (2009) and EBF (2009) give insights into the treatment of this matter in the United States, United Kingdom and European Union respectively). Indeed, even when not stated explicitly, elements of investor education are often implicit in moves towards greater consumer protection in financial services, such as the Treating Customers Fairly principle devised by the UK Financial Services Authority (FSA (2005)).

Whilst at first glance it might appear that *more* financial literacy would be unequivocally preferable to *less*, it is essential to take account of how this is distributed across individuals in the economy. For instance it is clear that a substantial proportion of the world’s economic activity is concerned with the trading and management of financial products, and that this is executed by a highly-financially-literate technocracy. However the recent dramatic turbulence across financial markets (with severe knock-on effects to the real sector) vocally begs the question of whether such a concentration of expertise is socially desirable. This discussion continues to enjoy a particularly high profile in the popular media, e.g. Trillin (2009) (*‘...having smart*

¹quoted from ‘The Complete Essays of Montaigne’ translated by D. Frame (1957, book 3 chapter 3)

²A broad overview of the field is provided by Braunstein and Welch (2002), while representative empirical literature includes van Rooij *et al* (2007) who consider the relationship between financial literacy and stock market participation (finding that low levels of literacy are associated with low likelihoods of stock investing), Lusardi (2008) who presents evidence of financial illiteracy in the United States and Hung *et al* (2009) who discuss approaches to the definition and measurement of financial literacy itself.

guys there almost caused Wall Street to collapse.’) as well as in governments across the world, exemplified by the proposal of the ‘Volcker Rule’ by Obama (2010). On a related note, within the academic and practitioner communities there is much debate about future performance prospects of quantitative funds given an increasing degree of herding by investors into a relatively small number of correlated strategies (see Fabozzi *et al* (2008)).

This state of affairs presents us with a host of intriguing theoretical problems concerning the relationship between financial literacy, its distribution across the population and overall social welfare. A common thread running throughout these problems is that in each case the central objects of interest are the distributions of consumers’ future wealth which we can model as depending on some vector of consumer-specific financial literacy characteristics or attributes; according to the problem at hand these might be observable quantities, e.g. age or educational level, or alternatively qualitative survey data (a significant body of the empirical literature deals with such surveys, as reviewed by Hung *et al* (2009)). For the sake of argument we shall use the term *literacy parameter* to refer to any characteristic upon which future random wealth is dependent.

In some cases these problems can be approached by applying existing well-established analytic methods. For instance investigation of the impacts of sub-optimal saving and borrowing decisions can be performed within conventional frameworks of inter-temporal consumption and portfolio choice (e.g. Campbell and Viceira (2002)). Similarly, when considering the welfare effects suffered by consumers who do not make full use of certain financial products (either by choice or due to lack of information), we can find a starting point in the literature which deals with the effects of portfolio constraints, e.g. Vila and Zariphopoulou (1997) consider the impact of borrowing constraints while He and Pearson (1991) address short-selling constraints.

However many important problems remain which are somewhat specific to this domain. To that end, in this paper we present general results concerning the relationship between financial literacy and individuals’ expected utility as well as complementary results at the aggregate social level. Furthermore, as a topical example, we focus attention on one particular attribute of literacy which is *investment forecasting skill*.

There are many aspects of forecasting skill embedded in the concept of financial literacy. As we have already declared, financial literacy problems are often characterised by a random distribution of future wealth and forecasting skill concerns how well an individual can *anticipate* these random future outcomes. In its broadest sense the notion is very flexible, for instance a family’s decision over whether to rent or buy a house will (to some extent) depend on their expectation of future house price inflation, while their choice of a fixed or variable rate mortgage depends on expectations of the path of future interest rates. Similarly, decisions about investment in human capital (e.g. higher education) depend on expectations of future earnings, which in turn relate - to some degree - to broader macroeconomic factors. The apposite issue here is that the vast majority of individuals are engaged in a forecasting process at some point in their lifetimes and that their effectiveness in this process is influenced by their level of financial literacy.

Notwithstanding the variety of themes which come under the forecasting umbrella, we devote our attentions here to the example of forecasting future asset prices from an investment perspective, since this is one of the most prominent and explicit forecasting processes in which

an individual might participate. Nevertheless, a substantial proportion of households in advanced economies typically ‘outsource’ this forecasting problem to professional fund managers. Within that industry active, rather than passive, management of assets is often equated with forecasting skill and presumably active managers are highly financially literate. Their skill attracts fees and underpins a large part of the financial industry however their claims to be able to predict financial outcomes are controversial and their activities are from time to time deemed to be prime examples of the negative effects of ‘too much’ financial literacy in the ‘wrong hands’. This raises the specific questions of what level of forecasting skill is desirable and how should this be distributed in the population. These issues are clearly of both academic and political interest and in this paper we set out a framework in which to address them.

1.2 Investment Forecasting Skill

Discussion of asset price predictability has raged in the economics and finance literature for centuries. Contributions range from the technical analysis methods devised in 18th century Japan³ to the efficient market hypotheses debates throughout the 20th century (e.g. Bachelier (1900), Fama (1965), Samuelson (1965), Malkiel (1973)) and more recent work on behavioural finance (Kahneman and Tversky (1979), Simon (1987), Shleifer (1999)). Regardless of where one stands on this topic it is evident that investors’ appetite for actively managed funds has continued unabated and their trading activities constitute a vast proportion of everyday financial market turnover.

Against this background an orthodoxy has developed in certain political-economic quarters as regards the preferred structure of financial markets. For instance arguments are frequently advanced in favour of wider household ownership of equities (often connected with privatization schemes, e.g Perotti (1995)), greater control by individuals of their pension portfolios (Emmerson and Wakefield (2003)) and investment by the general public in hedge funds (e.g. FSA (2005)). In many jurisdictions this has led to the implementation of specific policy measures with profound consequences for the asset management industry as well as the welfare of individuals themselves. From the point of view of financial economics, a major substantive impact of these measures is to alter the distribution and levels of forecasting skill among investors. As the *dramatis personae* of markets change, so too does the character of their price dynamics.

From a theoretical perspective, an extensive body of literature demonstrates how active trading can lead to increased volatility; this trading may itself be the result of superior information (Grossman and Stiglitz (1980)), opportunistic ‘noisy’ behaviour (Campbell and Kyle (1988), De Long *et al* (1990)) or fundamental analysis (Graham and Dodd (1934)), but all of these channels involve some component of skilled forecasting.⁴ However when investors are relatively unskilled they may have no conscious forecast whatsoever or, if they do, update it erroneously and infrequently. Under relatively undemanding assumptions about rationality on the part of investors, low skill translates into negligible trading and, in turn, less volatile markets. With this in mind, the supposition that such policies are a ‘good thing’ sits uneasily with the intuition

³the so-called Sakata Rules reputedly devised by rice trader Munehisa Homma

⁴Value investors used to claim that their skills involved no notion of forecasting outside pure arbitrage, but the notion of value involves some projection of future earnings in most cases.

that they may bring increased volatility in their wake.

In the sections which follow we specifically set out to address two broad groups of policy questions which we categorise as follows:

Type I questions concern the *level* of skill in the market: *What is the optimal level of skill which an investor should attempt to attain? Is more skill always preferable to less skill? Is it socially desirable to increase the level of all investors' skill in general?* These are relevant to discussion of financial literacy and investor education.

Type II questions relate instead to the *distribution* of skill: *Is it in the interests of unskilled investors to delegate portfolio decisions to professional managers with superior skill? Is there a socially-optimal allocation of capital across differentially-skilled investor types?* These are relevant to financial product innovation (e.g. introduction of 130/30 funds) and broadening retail access to more sophisticated products such as hedge funds.

The importance of Type II questions is sharpened by the observation that many famous financial catastrophes (e.g. Barings, Long Term Capital Management and Madoff) have involved one group of investors accepting *uncritically* a real (or imaginary) complex, mathematical derivatives strategy purveyed by another group, *supposed* to have superior skill.⁵

Our results are illuminating: we find that increasing the skill level of one group of investors can indeed increase their own utility but the induced incremental volatility can simultaneously reduce the utility of others with a net impact on social welfare which is negative. We also find that even if competing groups of independent investors increase their level of skill in tandem it is perfectly plausible for overall welfare to decline; once again the culprit is increased volatility.

Existing theory provides us with helpful foundations from which to select appropriate *literacy parameters* θ to represent skill. While it is common practice to measure managers' *ex post* performance with metrics such as Sharpe ratios and Information Ratios these do not differentiate between returns achieved due to luck or forecasting skill. Fortunately, however, a range of alternatives is available which measure managers' forecasting ability directly; as well as being revealing statistics as regards skill we find that these parameters also have valuable economic content and indeed define skill as a quasi-commodity over which preferences can be established much like conventional goods.

Our approach is to first construct a general equilibrium model of agents with heterogeneous skill levels and solve for equilibrium prices and portfolios conditional on agents' respective proprietary forecasts. We then examine the resulting distributions of equilibrium price, wealth and utility over repeated forecasting instances. This gives us an *ex ante* picture of the relative merits of different market structures (in terms of agents' skill levels).

Existing literature treats the topic of performance measurement extensively; for instance Bain (1996) reviews methods from a practitioner viewpoint, Aragon and Ferson (2006) review theory and recent empirical evidence and Knight and Satchell (2002) covers a range of theoretical and practical themes. A separate branch of literature considers the appropriateness of performance measures and their relationship to the investor's optimal portfolio construction problem (e.g. Leland (1999)). Notwithstanding these contributions we believe that this paper

⁵See Fay(1997), Lowenstein (2001) and Markopolos (2010) for explanatory accounts of Barings, LTCM and Madoff respectively.

is among the first to address the topic of skill from a marginal utility or welfare perspective.

Much of the literature on dynamic models with heterogeneous investors deals with the case where agents are all price-takers. Examples include Detemple and Murthy (1994) and Zapatero (1998). These models feature exogenous primitives such as stochastic production processes and derive equilibrium interest rates which typically have a weighted average form dependent on investors' asymmetric beliefs and evolution of wealth. However the effect of interaction on risky asset prices is limited or nonexistent. For example, agents decide what quantity of capital to commit to alternative available investments based on an imperfect returns forecast however the actual return does not respond to the quantity demanded, rather the demand effect shows up in the equilibrium rate of return of the risk-free asset which soaks up residual wealth.

For our purposes it is essential to capture the notion of competition between agents for returns. Introducing non-price-taking behaviour into conventional continuous-time dynamic models is a complex undertaking in its own right, and Basak (1997) remains one of the few treatments of this problem. In view of the rich heterogeneity which is central to our analysis we focus here on static models which nevertheless expose our key results. Heterogeneity in static models has remained relatively unexplored for some time; Lintner (1969) covered much of the relevant ground in a non-price-taking model. We find that introducing forecasting skill into this context brings useful new insight and we look forward to extending this to a dynamic setting in future work.

The organisation of the paper is as follows: in section 2 we introduce our central financial literacy propositions cast in a general risk-sharing framework, in section 3 we present a model of two discrete wealth states in which financial literacy is characterised using *hit-rate* as a forecasting skill measurement and in section 4 we apply the model to the policy questions outlined above. Section 5 concludes, considering the relevance of our findings for empirical work and areas for further study. Proofs and detailed derivations are relegated to the Appendix.

2 Financial Literacy: Individual Utility and Social Welfare

2.1 Individual Expected Utility

Suppose we have K agents: each has a private signal $S^{(i)}$ which is jointly distributed (according to their subjective belief) with the actual state of the world which we denote by z . This joint density is denoted $q(\mathbf{S}, z)$ where \mathbf{S} is the vector of all agents' signals. For the sake of clarity of notation we have that all securities are in zero net supply and all cashflows are settled in the same period, i.e. the payoffs of securities occur at the same point in time as their settlement. Finally, each agent's joint distribution q depends on a literacy parameter $\theta^{(i)}$ and we denote by θ (no superscript) the vector of all agents' skill parameters together.

The agents trade with each other and construct optimal portfolios of wealth in all states which we represent by the function $w^{(i)}(\mathbf{S}, z; \theta)$. We make no assumptions on the form which $w^{(i)}$ might take. We do not explicitly solve here the optimization problem to determine $w^{(i)}(\mathbf{S}, z)$ (which is well-established elsewhere in the literature, dating back as far as Wilson (1968)),

however in order to understand what follows it is essential to note that w represents the state-dependent wealth of the agent in *equilibrium*. Also, due to the single period nature of our model, $w^{(i)}$ is the agent's final wealth net of the initial cost of securities and this will clearly depend on the entire vector of literacy parameters in the market (θ) since these influence the individual demands of other agents.

We begin by considering the perspective of an individual agent. For clarity in the algebra which follows we tend to suppress the (i) superscripts.

Proposition 2.1. *Marginal expected utility of the general literacy parameter θ^i is given by:*

$$\mathbb{E} [u_w(w(\mathbf{S}, z; \theta))w_\theta(\mathbf{S}, z; \theta)] + \text{COV} \left[u(w(\mathbf{S}, z; \theta)), \frac{q_\theta(\mathbf{S}, z; \theta)}{q(\mathbf{S}, z; \theta)} \right] \quad (1)$$

Proof. See Appendix. □

Clearly a sufficient condition for positive marginal expected utility of literacy will be having both terms positive and we consider these individually below.

The first term in (1) is the marginal expected utility of a literacy improvement while keeping the subjective density unchanged, i.e. we focus only on the influence which the literacy parameter has on position size (and hence future state-dependent wealth). For instance: a literacy improvement might take the form of an increase in forecasting skill which reduces the dispersion of an agent's future state distribution; *ceteris paribus*, depending on the precise form of the agent's utility function, that increase in precision may lead to the agent altering his portfolio, increasing positions in states believed to be more likely (and reducing positions in less likely states). However whether or not this also leads to an increase in the agent's expected wealth depends on the demand function(s) of other agent(s) and the resulting equilibrium price. The overall effect of this entire interaction is concentrated in this term which we call the *sizing* term.

In Figure 1 we provide a simple example. Here we have two agents A and B and a single risky asset. Each agent has a CARA utility function with equal risk aversion (λ) and they each believe that the future asset price is normal although they differ as regards its mean and variance. It is straightforward to show that their resulting demand for the risky asset is linear and given by $Q_i = \frac{m_i - p}{\lambda \sigma_i^2}$ where m_i and σ_i^2 denote type-specific forecasts of mean and variance (with i either A or B) and p is the equilibrium price.⁶ In the figure the individual agents' asset demands are represented by the lines AA' and BB' . Evidently a skill-increasing improvement in literacy (reflected in a decrease in σ_i^2) causes a flattening in these lines although they will always intersect the price axis (where $Q = 0$) at the respective mean forecast level m_i .

Hence we see that improving type A's literacy (flattening the line from $A_0A'_0$ to $A_1A'_1$) increases the quantity of securities sold by agent A in equilibrium. However note that agent A's expected wealth depends not only on this quantity but also on the equilibrium price. This expected net wealth (before and after the flattening) is illustrated on the graph by the appropriate shaded box (bordered vertically by the lines $Q = Q_i$ and the p -axis, and horizontally by the lines $p = m_A$ and $p = p_i$). Clearly increasing the equilibrium traded quantity Q_i (in absolute terms) does not necessarily increase expected wealth as this will depend on the precise

⁶A detailed derivation is provided by Williams (2009).

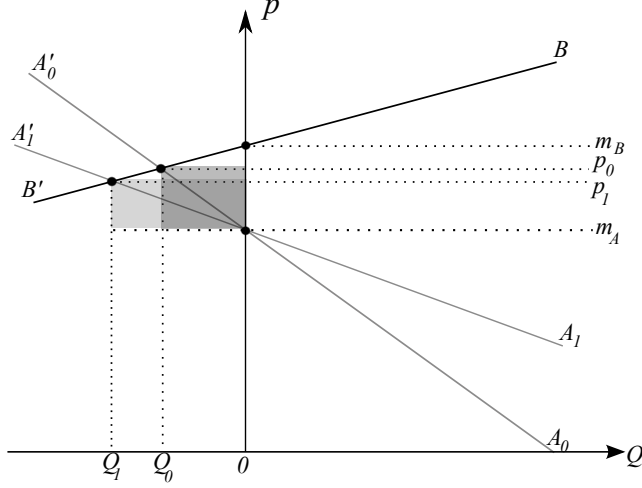


Figure 1: A simple example of interacting asset demands between two agents with heterogeneous forecasts of the future price; $A_0A'_0$ and $A_1A'_1$ are demand schedules for agent A at two different skill levels and BB' is the supply schedule for agent B (obtained by reflecting agent B's demand schedule in the price-axis). Equilibrium prices are p_i and the mean forecasts of the agents are m_A and m_B . At both equilibria agent A is a seller of assets while agent B is a buyer (represented graphically as a supplier of a negative quantity).

shape of the other agent's demand schedule (e.g. BB'). This is why the sign of the *sizing* term is of particular interest.

The second term in (1) is the covariance between utility of state-dependent wealth and the ratio $\frac{q_\theta(\mathbf{S}, z; \theta)}{q(\mathbf{S}, z; \theta)}$ (this ratio could approximately be thought of as the proportionate 'distortion' applied to the joint density function q at point (\mathbf{S}, z) when the literacy parameter is increased by one unit). For instance, suppose we consider the conditional $q(z|S)$ to be a standard univariate normal and treat the literacy parameter as a skill-induced precision adjustment to the (unity) variance, i.e. variance is defined as $(\frac{1}{\theta})^2$, then we have:

$$q(z; \theta) = \frac{\theta}{\sqrt{2\pi}} \exp\left[-\frac{\theta^2 z^2}{2}\right]$$

$$q_\theta(z; \theta) = \frac{q(z; \theta)}{\theta} - \frac{2\theta z^2 q(z; \theta)}{2}$$

$$\frac{q_\theta(z; \theta)}{q(z; \theta)} = \frac{1}{\theta} - \theta z^2$$

which we illustrate in Figure 2 for the case $\theta = 4$. Positive values of the ratio occur where the density is 'pulled upwards' by the improvement in literacy (indicating relatively more likely states) and negative values where the density is 'pushed downwards' (indicating the opposite). We are interested in the value of the covariance between this function and the agent's state dependent utility. Ideally, given an improvement in literacy $d\theta$ and knowing their private signal S , the agent would adjust their portfolio in perfect sympathy with the 'distortion' function (e.g. overweighting positions where 'distortion' is positive and underweighting where negative), leading to a positive covariance. This term therefore represents how closely state-dependent utility is 'tuned' to the 'improved' subjective density belief which the agent obtains. We call this therefore the *tuning* term.

We next present an example of a special case which helps to provide some intuition.

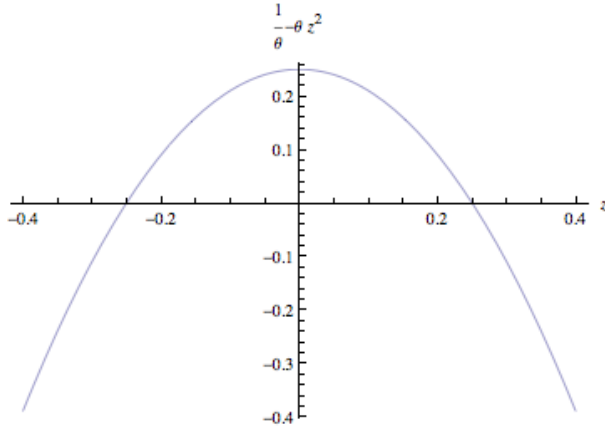


Figure 2: A simple example of a density ‘distortion’ function based on a standard normal where $\theta = 4$ plotted over the domain $(-0.4, +0.4)$

Corollary 2.2. *Suppose that the agent’s wealth is unconditionally normally-distributed, then the condition for positive marginal expected utility of the general literacy parameter θ^i is:*

$$\frac{\text{COV}\left[w, \frac{q\theta}{q}\right] + \mathbb{E}[w\theta]}{\text{COV}[w, w\theta]} > R_A$$

where R_A is Rubinstein’s (1973) coefficient of global risk-aversion.

Proof. See Appendix. □

Various remarks follow from this corollary:

(a) As regards $\mathbb{E}[w\theta]$: positivity of this term by itself is neither necessary nor sufficient for (2.2) to hold. This is the very essence of our proposition: an increase in literacy which increases future wealth on average is not guaranteed to deliver net positive marginal utility once other distributional characteristics are taken into account.

(b) The ‘tuning’ effect represented by $\text{COV}\left[w, \frac{q\theta}{q}\right]$ can take any sign depending on the precise relationship between the nature of the investor’s literacy parameter (as it influences the subjective distribution q) and the flexibility which is available to construct the state-dependent portfolio w . For instance (returning to the example where θ represents forecasting skill) if the agent is in the unfortunate position of skillfully forecasting parts of the distribution which cannot be traded using available securities (e.g. extreme events in the tails of the distribution) then this term could be negligible. In the opposite case of a complete market with no portfolio constraints we would expect this term to be positive.

2.2 Social Welfare

We now turn our attention to the overall profile of financial literacy in the market and its relationship to an equally-weighted social welfare function. We begin by assuming a number of heterogeneous investor types, each with potentially different initial wealth levels (denoted w_0) and utility functions. We define a cross-sectional distribution of agent types according to headcount, which we denote by f , and for each agent type we denote the j ’th moment of their

uncertain wealth gains by $M^{(j)}(\theta)$. Note that these moments are a function of the entire profile of literacy in the market represented by the vector of all agents' literacy parameters θ . For each type we also assume the existence of all derivatives of their utility function, where we denote the j 'th derivative by $u^{(j)}$.

Hence, from a cross-sectional point of view, we can think of both $u^{(j)}(w_0)$ and $M^{(j)}(\theta)$ as random variables with values which depend on the type of a randomly-selected agent, distributed according to f .

Proposition 2.3. *For an economy with an equally-weighted Social Welfare Function we define the Social Benefit of Trading (SBT) as the incremental social welfare achieved in equilibrium as a result of securities trading among agents with a profile of literacy θ . This is given by the following expression:*

$$SBT(\theta) = \sum_{j=1}^{\infty} \mathbb{E}_f \left[\frac{u^{(j)}(w_0)}{j!} M^{(j)}(\theta) \right]$$

where \mathbb{E}_f indicates expectation taken with respect to the cross-sectional distribution of agent types (f) and $u^{(j)}$ and $M^{(j)}$ are cross-sectional random variables as described above.

Proof. See Appendix. □

Corollary 2.4. *For an economy composed of multiple types of investor where future wealth is normally-distributed we have*

$$SBT(\theta) = \text{COV}_f [u'(w_0), \mu_w(\theta)] + \frac{1}{2} \mathbb{E}_f [u''(w_0) M^{(2)}(\theta)]$$

where $\mu_w(\theta) \equiv M^{(1)}(\theta)$ and $M^{(2)}(\theta)$ respectively denote the type-specific mean and second moment of wealth gains, both of which are cross-sectional random variables distributed according to f . COV_f and \mathbb{E}_f denote covariance and expected value calculated across types in the population with respect to this f distribution.

Proof. This follows from Proposition 2.3 by noting that $\mathbb{E}_f [\mu_w(\theta)] = 0$ due to market clearing⁷ and hence $\mathbb{E}_f [u'(w_0)\mu_w(\theta)] = \text{COV}_f [u'(w_0), \mu_w(\theta)]$. □

This corollary shows us that for risk averse agents ($u''(w_0) < 0$) a necessary condition for positive SBT is that initial wealth covaries negatively with expected wealth gains across investor types. In other words we require that wealth enhancements due to literacy will accrue on average to the poorer members of society. Taking the particular example of investing skill: although we lack empirical evidence on this point we suspect that this covariance is more likely to be positive in reality, i.e. it is the wealthier investors in society who have the means to either improve their own skill or to delegate management of their funds to others with superior skill. Were our suspicion to be true, it would turn the sign of the first term negative in (2) and thereby militate against any social benefit of skill.

⁷Securities are in zero net supply hence aggregate demand equals aggregate supply *ex ante* and once uncertainty is resolved the value of aggregate profit will equal the absolute value of aggregate loss. Informally-speaking, trading is a 'zero sum game' in this model.

The corollary also gives us conditions under which changes in the profile of financial literacy will be socially beneficial. For instance if society has a choice between two alternative profiles θ_1 or θ_2 , both of which result in an equal cross-sectional second moment of wealth, then θ_1 will be preferred if

$$\mathbb{C}\mathbb{O}\mathbb{V}_f [u'(w_0), \mu_w(\theta_1)] > \mathbb{C}\mathbb{O}\mathbb{V}_f [u'(w_0), \mu_w(\theta_2)] \quad (2)$$

Corollary 2.5. *For an economy composed of multiple types of investor with identical utility functions \bar{u} where future wealth is normally-distributed and initial wealth is equal across all the types we have*

$$SBT(\theta) = \frac{1}{2} \mathbb{E}_f [\bar{u}''(w_0) M^{(2)}(\theta)] \quad (3)$$

Proof. This follows immediately from Corollary 2.4 above where the first term becomes zero when $u'(w_0)$ is equal across all types. \square

For risk averse investors the sign of SBT in (3) will be unambiguously negative since $M^{(2)} > 0$. However in a no-trade equilibrium we have zero mean and variance of wealth gains since future wealth equals initial wealth. The immediate consequence of this is that society would prefer an equilibrium with absolutely *no* trading ($M^{(2)} = 0$) to any equilibrium *with* trading ($M^{(2)} > 0$). Returning to our example where θ represents forecasting skill: if we assume that all unskilled investors have identical diffuse beliefs about future prices (resulting in zero trading) then this would be equivalent to a preference for no skill over any skill.

In summarising the results above it is clear that general arguments will be hard to find, therefore we consider further particular special cases in the section which follows. These will demonstrate the applicability of our propositions. Prior to that exposition however we briefly discuss parametric measurements of forecasting skill from a more practical viewpoint.

2.3 Forecasting Skill Metrics in Practice

Before we proceed to model forecasting skill as a particular attribute of financial literacy, it is instructive to briefly review candidates for the parameter which we have thus far labeled as θ .⁸ The fundamental requirement is that θ should be some measurement of dependence or association between actual future states of the world and a forecast which is made at the time when asset demands are determined. We outline two possibilities below.

2.3.1 Hit-Rate

Perhaps the simplest measure of association in this context is the *hit-rate* which we can define informally as the proportion of forecasts which are *directionally* correct. Here we ignore magnitudes of return entirely and focus simply on the signs of the forecasts and actual outcomes. In a basic incarnation we might consider a model of two discrete states which we represent by the outcomes 0 and 1 respectively and denote by the random variable Q . We also restrict forecasts

⁸A detailed discussion of skill measurement is beyond the scope of this paper. More complete treatments of this topic are provided by Aragon and Ferson (2006), Granger and Machina (2006) and Williams (2009) among others.

to be correspondingly either 0 or 1 and denote these by the random variable S . For the sake of parsimony we assume that the hit-rate is not conditional on the direction of the forecast (i.e. 0 forecasts are equally-likely to be correct as 1 forecasts) and therefore we define the hit-rate as:

$$\mathbb{P}[Q = 1|S = 1] = P[Q = 0|S = 0] = h$$

2.3.2 Information Coefficient

Grinold and Kahn (1999) define the Information Coefficient (IC) as the correlation between an investor's *forecast* of a future asset return (in terms of a standardised score) and the *actual* return where return is defined as

$$r_{t+1} \equiv \frac{P_{t+1}}{P_t} - 1$$

Formally, if we were to represent the forecast as a standardised normal random variable, $S \sim N(0, 1)$, unconditional expected future return as μ , and assume joint-normality between forecast and actual return r_{t+1} , then the conditional forecast return would be:

$$E[r_{t+1} | S] = \mu + IC\sigma S$$

where IC is the investor's Information Coefficient and σ is the unconditional standard deviation of the return. Similarly we obtain the investor's conditional forecast variance:

$$Var[r_{t+1} | S] = \sigma^2(1 - IC^2)$$

The Information Coefficient features extensively in the portfolio management literature. By way of examples: Coggin and Hunter (1983) discuss empirical IC measurement issues, Dimson and Marsh (1984) refer to the concept in their empirical study of analysts' forecasting ability, Khan *et al* (1996) apply IC in global asset allocation, Khan (1998) discusses its applicability in bond investing, Lee (2000) uses it as a key parameter in a range of analytics pertaining to tactical asset allocation, Herold (2003) discusses its relevance in qualitative forecasting, Satchell and Williams (2007) consider the question of optimal forecasting horizon and Fishwick (2007) analyses the case of multiple models.

For purposes of reconciliation we provide the proposition below as a rule-of-thumb to aid in comparison of hit-rates and Information Coefficients.

Proposition 2.6. *For the case of forecasting a future state which takes only one of the two discrete values 0 or 1: if forecasts are also restricted to the discrete values 0 and 1 where the probability of a forecast of 1 is denoted by g , then the relationship between hit-rate h and Information Coefficient (IC) is given by:*

$$IC = \frac{h - g}{1 - g}$$

For example when $g = \frac{1}{2}$ then $IC = 2h - 1$.

Proof. See Appendix. □

In the analysis which follows we have chosen to focus on the use of *hit-rate* as a skill measurement within the context of a two-state model. Equivalent analysis for the case of the Information Coefficient (with a continuum of states) is more involved but raises several further subtle insights. We consider the *IC* case in a separate paper⁹ where, despite the incremental complexity, we find several results which are qualitatively similar to those which follow in this paper.

3 A Model with Two Discrete States

3.1 The Model and Equilibrium Solution

Suppose we have K agents, divided into two types (A and B), two states of the world and a single risky asset which pays off one unit in state 1 and zero units in state 0. We denote the actual probability of state 1 by g and the proportion of agents who are type A by ϕ (with $0 < \phi < 1$). Each agent receives a type-dependent private signal S^A or S^B and has a hit-rate h^A or h^B which represents the probability of their signal being correct. Therefore for each agent we need to solve the expected utility maximisation problem to obtain the quantity of risky asset which they buy *given* their signal. We denote this quantity x_0 (when $S = 0$) or x_1 (when $S = 1$). We assume the agents have exponential utility functions with differing risk aversions λ_A and λ_B and to ease the algebra we make the additional assumption that each agent can borrow and lend unlimited amounts at a risk-free rate of zero.¹⁰

Furthermore all settlement and payoffs take place in the same time period so the assets have the character of futures-type contracts. We denote equilibrium price by p and hence agents' expected utilities (conditional on their signal) are given by:

$$\begin{aligned}\mathbb{E}[U|S = 0] &= -\frac{1}{\lambda}(1-h)\exp[-\lambda x_0(1-p)] - \frac{1}{\lambda}h\exp[\lambda x_0 p] \\ \mathbb{E}[U|S = 1] &= -\frac{1}{\lambda}h\exp[-\lambda x_1(1-p)] - \frac{1}{\lambda}(1-h)\exp[\lambda x_1 p]\end{aligned}$$

Optimal position amounts x are easily found by differentiation:

$$\begin{aligned}\frac{dU_0}{dx_0} &= (1-p)(1-h)\exp[-\lambda x_0(1-p)] - ph\exp[\lambda x_0 p] = 0 \\ x_{S=0} &= \frac{1}{\lambda} \log \left[\frac{1}{HP} \right] \\ \frac{dU_1}{dx_1} &= (1-p)h\exp[-\lambda x_1(1-p)] - p(1-h)\exp[\lambda x_1 p] = 0 \\ x_{S=1} &= \frac{1}{\lambda} \log \left[\frac{H}{P} \right]\end{aligned}$$

⁹Satchell and Williams (2010)

¹⁰While clearly unrealistic this allows us to highlight key details in the model which concern forecast formulation, price determination and utility realisation. Including more complex heterogeneity in wealth and preferences is straightforward and does not significantly alter the basic results we present here.

where for convenience we refer to the probabilities in terms of *odds*:

$$H = \frac{h}{1-h}$$

$$P = \frac{p}{1-p}$$

and for notational clarity these quantities will often feature in our expressions instead of the underlying probabilities. If the signal $S = 1$, for instance, then the odds ratio $\frac{H}{P}$ can be interpreted as indicating the agent's level of optimism or pessimism relative to the market.

The equilibrium price p is now found by imposing market clearing. As an example we consider below the case where $S^A = 0$ and $S^B = 1$:

$$\frac{\phi}{\lambda_A} \log \left[\frac{1}{H^A P} \right] + \frac{1-\phi}{\lambda_B} \log \left[\frac{H^B}{P} \right] = 0$$

$$\left[\frac{1}{H^A P} \right]^{\frac{\phi}{\lambda_A}} = \left[\frac{P}{H^B} \right]^{\frac{1-\phi}{\lambda_B}}$$

$$P = [H^B]^{\frac{1-\phi}{\lambda_A + \frac{1-\phi}{\lambda_B}}} \left[\frac{1}{H^A} \right]^{\frac{\frac{\phi}{\lambda_A}}{\lambda_A + \frac{1-\phi}{\lambda_B}}}$$

$$P = [H^B]^{1-f} \left[\frac{1}{H^A} \right]^f$$

where superscripts and subscripts index H and λ according to agent type and we use f to denote the risk-tolerance weighted proportion of type A 's in the market. We note also that $0 < f < 1$ and that when $\lambda_A = \lambda_B$ (i.e. we have common constant risk aversion between the two agent types) then f and ϕ are equal. Given our assumption of constant absolute risk aversion it is clear that an increase in risk-aversion of type i is equivalent to a reduction in the proportion of type i agents in the market. Therefore much of the following analysis proceeds in terms of the f parameter without discussion of any differential risk aversion between agents.¹¹

In all we have four cases to consider depending on values of S^A and S^B and the corresponding solutions appear in Table 1 where in the interests of tidy notation we define $M = (H^A)^f (H^B)^{1-f}$ and $Q = (H^A)^f (H^B)^{f-1}$.

Table 1: Equilibrium in two-agent two-state model, conditional on each agent having formed their forecasts but before state of the world is revealed

S^A, S^B	P	p
0, 0	$\frac{1}{M}$	$\frac{1}{1+M}$
0, 1	$\frac{1}{Q}$	$\frac{1}{1+Q}$
1, 0	Q	$\frac{1}{1+Q}$
1, 1	M	$\frac{1}{1+M}$

¹¹Nevertheless when we eventually address the question of social welfare we are careful to aggregate utilities using weightings given by ϕ although equilibrium prices and allocations are determined by f .

3.2 Equilibrium Price and Wealth Dynamics

Although our model is inherently static it can be thought of as a transmission mechanism where agents' signals (either 0 or 1) are converted into equilibrium price. In this sense it is apparent that equilibrium price dynamics are driven by the changing nature of agents' beliefs over time. Given knowledge of the agents' skill levels (as embodied in h) along with further details of the probability distribution of forecasts and outcomes we can gain insight into plausible characteristics which the unconditional price distribution may have over time. (Here we interpret time to mean a series of repeated forecasting instances.¹²)

In order to fully specify the joint distribution of (S^A, S^B, z) it remains for us to specify the marginal univariate probabilities of S^A and S^B and also the nature of dependency between them. We consider simple extreme examples below but note that more complexity could potentially be incorporated here, for instance via copula-type representations (e.g. Tajar *et al* (2001)).

It is essential to note that the dependence structure assumed between the agents' signals places strict constraints on values which h^A and h^B can take, e.g. it is nonsense for both h^A and h^B to equal unity and still be independent. This topic is considered for general multivariate Bernoulli distributions by Chaganty and Joe (2006); for the trivariate case they find necessary and sufficient conditions on the bivariate marginals such that the overall joint distribution is valid. Here we make the plausible assumption that unconditional signal probabilities exactly equal the actual probability of the state of the world which, in turn, we fix at $\frac{1}{2}$, i.e.

$$\mathbb{P}[S^A = 1] = \mathbb{P}[S^B = 1] = g = \frac{1}{2} \quad (4)$$

We further note that a hit-rate of $h = \frac{1}{2}$ ($H = 1$) represents the *least* level of possible skill (greatest uncertainty) since for $h < \frac{1}{2}$ an agent with signal S will invest identically as he would if receiving the signal $(1 - S)$ and having hit-rate $(1 - h) > \frac{1}{2}$. Therefore we restrict our exploration to cases where $h > \frac{1}{2}$.

(1) In the case of perfect correlation (which we might see in, for instance, a herding scenario) we first note that $H^A = H^B$ and there is in fact zero trade. Hence we trivially find that:

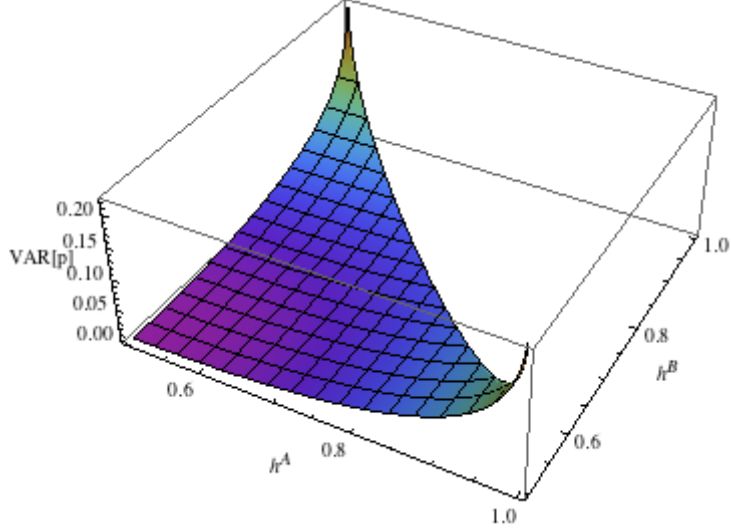
$$\begin{aligned} \mathbb{E}[p] &= (1 - g)\frac{1}{1 + M} + g\frac{M}{1 + M} \\ \mathbb{E}[p^2] &= (1 - g)\frac{1}{1 + 2M + M^2} + g\frac{1}{1 + 2\frac{1}{M} + \frac{1}{M^2}} \end{aligned}$$

and when hit-rates are both equal to $\frac{1}{2}$ ($H = M = 1$) we have expectation of $\frac{1}{2}$ and variance of 0. As the hit-rate increases towards unity ($H \rightarrow \infty$) the mean price tends to g (the frequency of forecasts equal to 1) and variance $g(1 - g)$ as we would expect from a Bernoulli distribution.

From this we note that (quite intuitively) the equilibrium price is a convex combination of a 'bullish' price and a 'bearish' price and the variance depends on both the degree of switching between bullish/bearish states (represented by g) and the profile of agents' skill (upon which

¹²This approach is similar in spirit to techniques employed in the market microstructure literature where the process by which trading orders arrive asynchronously into the market is modeled and the pace of this flow is a more appropriate scale on which to measure time than wall-clock time.

Figure 3: Variance of price with two equiprequent independent forecasters



the size of position-taking depends).

(2) We now consider independence. Again for comparison we consider the case of equal skill. Hence we have:

$$\mathbb{E}[p] = (1-g)^2 \frac{1}{1+M} + g(1-g) \frac{1}{1+Q} + g(1-g) \frac{1}{1+\frac{1}{Q}} + g^2 \frac{1}{1+\frac{1}{M}}$$

$$\mathbb{E}[p^2] = (1-g)^2 \left[\frac{1}{1+M} \right]^2 + g(1-g) \left[\frac{1}{1+Q} \right]^2 + g(1-g) \left[\frac{1}{1+\frac{1}{Q}} \right]^2 + g^2 \left[\frac{1}{1+\frac{1}{M}} \right]^2$$

In this case we must apply the conditions of Chaganty and Joe to determine the admissible range of hit-rates. Given the assumptions in (4) these reduce to

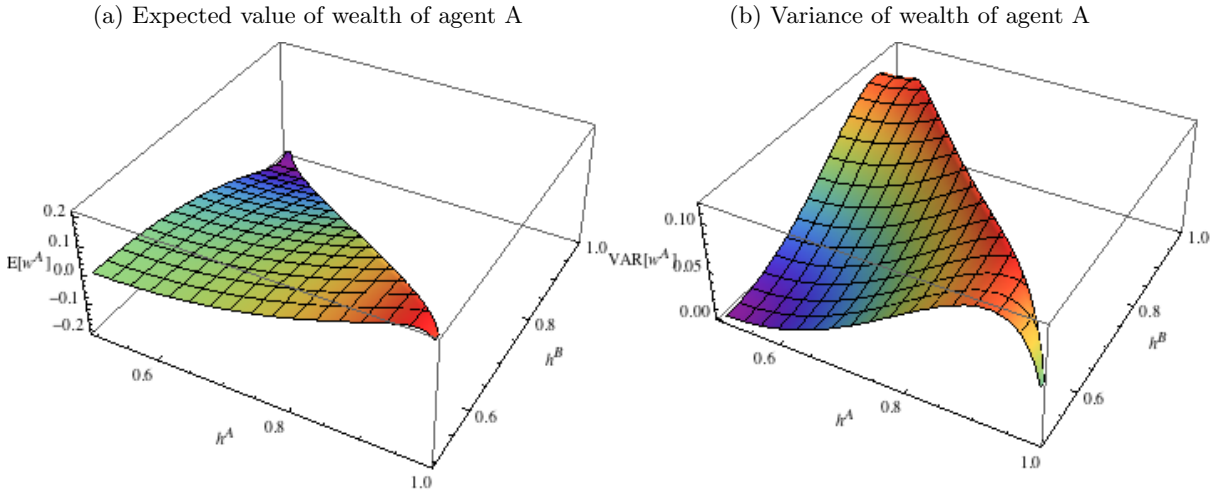
$$h^A + h^B < \frac{3}{2}; h^A < 1, h^B < 1$$

demonstrating that if $h^A = h^B$ then the maximum acceptable common hit rate will be $h = \frac{3}{4}$ where $H = M = 3$.

As illustration here we consider the case of equiprequent types (i.e. $f = 0.5$). This simplification conveniently means that when both agents are equally uninformed we have $H^A = H^B = 1 = M = Q$. Once again in this case the mean price is $\frac{1}{2}$ and variance is 0 and as both $h \rightarrow 1$ we find $\mathbb{E}[p] \rightarrow g$ (mean price simply represents the average of the agents' forecasts) and variance increases. The latter is illustrated in Figure 3 where we carefully restrict the plot to the region of (h^A, h^B) points consistent with independence. It is apparent, however, that variance in price is at its highest with a single skilled forecaster (e.g. $h^A \rightarrow 1$ and $h^B = \frac{1}{2}$).

Now we consider an agent's unconditional expected *wealth* over repeated forecasting instances. We compute this by the weighted sum of wealth levels which we list in Table 2 from the perspective of type *A* (the equivalent values for type *B* agents are straightforward to deter-

Figure 4: Distribution of future wealth over repeated forecasting instances (two equiprequent independent agents)



mine but we omit the details in the interests of brevity).

$$\mathbb{E}[w^A] = \sum_{(S^A, S^B, z)} w^A(S^A, S^B, z) q(S, z; h^A) \quad (5)$$

We plot the overall expected level of wealth in Figure 4. This plot at once highlights the challenge we face in determining the marginal benefit of skill improvement. Although expected wealth gradually increases for the agent if we hold the other agent's skill level constant, if we increase both together it is apparent that there is an offsetting effect. Although the agent gets better at forecasting future market states, the competition which he faces from the other agent diminishes the returns which can be captured.

We also note that increased skill levels in general lead to increased variance in equilibrium price over repeated forecasting instances. This raises the question of whether the disutility of the increased variance in returns may actually outweigh the positive utility of the improved mean. We investigate this next.

3.3 Unconditional Expected Utility

In Table 2 we have presented the possible levels of utility which an agent can achieve together with the probability of each state and from this it is straightforward to compute expected utility over repeated realisations of forecasts and states. We therefore plot these levels in Figure 5 against agents' skill levels

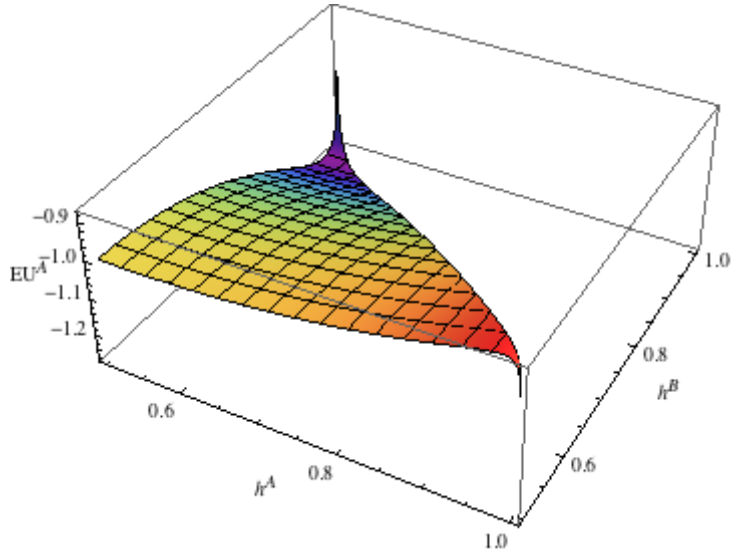
It is apparent from the plot that the marginal expected utility of skill is positive for most skill levels. In principle the optimal level of skill can be computed analytically however we do not present that analysis here as the expression is rather unwieldy and we do not find it particularly insightful.

Nevertheless inspection of Figure 5 shows how the increasing variance of wealth tends to detract from the wealth benefits which type A expects to gain from increasing his own skill (keeping the other type's skill unchanged). Similarly if agent A keeps his skill unchanged while

Table 2: *Ex-post* characteristics of equilibrium in equiprequent two-agent two-state model

S^A, S^B, z	$w^A(S^A, S^B, z)$	$U(w^A)$	$q(S, z; h^A)$	$\frac{q_{h^A}(S, z; h^A)}{q(S, z; h^A)}$
0, 0, 0	$-\frac{1}{\lambda} \frac{1}{1+M} \log \frac{1}{Q}$	$-\frac{1}{\lambda} Q^{\frac{-1}{1+M}}$	$(1-g)h^A h^B$	$\frac{1}{h^A}$
0, 0, 1	$\frac{1}{\lambda} \frac{M}{1+M} \log \frac{1}{Q}$	$-\frac{1}{\lambda} Q^{\frac{M}{1+M}}$	$g(1-h^A)(1-h^B)$	$-\frac{1}{1-h^A}$
0, 1, 0	$-\frac{1}{\lambda} \frac{1}{1+Q} \log \frac{1}{M}$	$-\frac{1}{\lambda} M^{\frac{-1}{1+Q}}$	$(1-g)h^A(1-h^B)$	$\frac{1}{h^A}$
0, 1, 1	$\frac{1}{\lambda} \frac{Q}{1+Q} \log \frac{1}{M}$	$-\frac{1}{\lambda} M^{\frac{Q}{1+Q}}$	$g(1-h^A)h^B$	$-\frac{1}{1-h^A}$
1, 0, 0	$-\frac{1}{\lambda} \frac{1}{1+\frac{1}{Q}} \log M$	$-\frac{1}{\lambda} M^{\frac{Q}{1+Q}}$	$(1-g)(1-h^A)h^B$	$-\frac{1}{1-h^A}$
1, 0, 1	$\frac{1}{\lambda} \frac{\frac{1}{Q}}{1+\frac{1}{Q}} \log M$	$-\frac{1}{\lambda} M^{\frac{-1}{1+Q}}$	$gh^A(1-h^B)$	$\frac{1}{h^A}$
1, 1, 0	$-\frac{1}{\lambda} \frac{1}{1+\frac{1}{M}} \log Q$	$-\frac{1}{\lambda} Q^{\frac{M}{1+M}}$	$(1-g)(1-h^A)(1-h^B)$	$-\frac{1}{1-h^A}$
1, 1, 1	$\frac{1}{\lambda} \frac{\frac{1}{M}}{1+\frac{1}{M}} \log Q$	$-\frac{1}{\lambda} Q^{\frac{-1}{1+M}}$	$gh^A h^B$	$\frac{1}{h^A}$

Figure 5: Expected utility of type A for the case of two equiprequent independent agent types



his competitor (agent B) improves, then A suffers the dual blow of declining expected wealth along with increasing variance, reflected in a rapidly declining expected utility.

4 Application to Policy Issues

Equipped with knowledge of the distribution of future wealth we can now turn attention to the questions we presented in the introduction. In the course of doing this we will show the applicability of the theory we presented in Section 2.

4.1 Type I: Effects of the Level of Skill

First of all we consider matters regarding the overall level of skill in the market, e.g. effects of changing the level of skill and whether we can say anything about an ‘optimal’ level of skill. While there are clearly many approaches to tackling this question, the approach which we follow here demonstrates the usefulness of our simple model along with the intuition provided by Proposition 2.1. In order to keep our analysis separate from skill distributional factors (which we consider Type II issues) we begin by assuming an equal number of investors of each type in the market, i.e. the equifrequent case to which we have already made some reference in the previous section. We now address three related questions.

(1) *Suppose both agent types always have equal skill and we increase the shared skill level in tandem. What are the consequences of skill adjustment for expected utility?*

In this case $h^A = h^B = h$, thus $H^A = H^B = H = M$ and $Q = 1$. The equilibrium properties of this structure are shown in Table 3. We find that expected utility in this case has a convenient analytical form which we reach with the aid of hyperbolic functions as follows:

$$\begin{aligned}\mathbb{E}[U] &= -\frac{1}{\lambda} \left[h^2 + (1-h)^2 + h(1-h)M^{-\frac{1}{2}} + (1-h)hM^{\frac{1}{2}} \right] \\ \mathbb{E}[U] &= -\frac{1}{\lambda} [1 - 2h(1-h) + h(1-h) [\exp(-0.5 \log M) + \exp(0.5 \log M)]] \\ \mathbb{E}[U] &= -\frac{1}{\lambda} [1 + 2h(1-h) [\cosh 0.5 \log M - 1]]\end{aligned}$$

Using the fact that

$$\frac{dM}{dh} = \frac{1}{(1-h)^2}$$

it is straightforward to now derive

$$\frac{d\mathbb{E}[U]}{dh} = -\frac{1}{\lambda} [\sinh 0.5 \log H - (2 - 4h) + (2 - 4h) \cosh 0.5 \log H]$$

We can compare this expression with the results of applying (1) from section 2, which enables us to break the marginal utility down into intuitive components. First we have the ‘sizing’ component which we find to be *strictly negative* for $h > 0.5$:

$$\mathbb{E}[u_w(w(S, z; h))w_h(S, z; h)] = \frac{1}{2\lambda} H^{-\frac{1}{2}} - \frac{1}{2\lambda} H^{\frac{1}{2}} = -\frac{1}{\lambda} \sinh 0.5 \log H < 0$$

Table 3: Characteristics of equilibrium in two-agent two-state model where $h^A = h^B = h$

S^A, S^B, z	$w^A(S^A, S^B, z)$	$U(w^A)$	$q(S, z; h)$	$\frac{q_h(S, z; h)}{q(S, z; h)}$	w_h
0, 0, 0	0	$-\frac{1}{\lambda}$	$(1-g)h^2$	$\frac{2}{h_2}$	0
0, 0, 1	0	$-\frac{1}{\lambda}$	$g(1-h)^2$	$-\frac{1}{1-h}$	0
0, 1, 0	$\frac{1}{2\lambda} \log H$	$-\frac{M^{-\frac{1}{2}}}{\lambda}$	$(1-g)h(1-h)$	$\frac{1-2h}{h(1-h)}$	$\frac{1}{2\lambda h(1-h)}$
0, 1, 1	$-\frac{1}{2\lambda} \log H$	$-\frac{M^{\frac{1}{2}}}{\lambda}$	$g(1-h)h$	$\frac{1-2h}{h(1-h)}$	$-\frac{1}{2\lambda h(1-h)}$
1, 0, 0	$-\frac{1}{2\lambda} \log H$	$-\frac{M^{\frac{1}{2}}}{\lambda}$	$(1-g)(1-h)h$	$\frac{1-2h}{h(1-h)}$	$-\frac{1}{2\lambda h(1-h)}$
1, 0, 1	$\frac{1}{2\lambda} \log H$	$-\frac{M^{-\frac{1}{2}}}{\lambda}$	$gh(1-h)$	$\frac{1-2h}{h(1-h)}$	$\frac{1}{2\lambda h(1-h)}$
1, 1, 0	0	$-\frac{1}{\lambda}$	$(1-g)(1-h)^2$	$-\frac{2}{1-h}$	0
1, 1, 1	0	$-\frac{1}{\lambda}$	gh^2	$\frac{2}{h}$	0

This result is intuitively appealing: if both agents have the same skill level then increases in skill will lead to larger position taking on average *but* equilibrium prices will become more accurate representations of future prices. These two factors work against each other with a net wealth impact which is negative.

Separately we have the ‘tuning’ component which is *strictly positive* for $h > 0.5$:

$$\begin{aligned} \mathbb{E} \left[u(w(S, z; h)) \frac{q_h(S, z; h)}{q(S, z; h)} \right] &= -\frac{1}{\lambda} \left[2h - 2(1-h) + (1-2h)H^{-\frac{1}{2}} + (1-2h)H^{\frac{1}{2}} \right] \\ &= -\frac{1}{\lambda} \left[-(2-4h) + (2-4h) \cosh 0.5 \log H \right] > 0 \end{aligned}$$

This is consistent with our expectations: our model is a complete market since we have two states and two assets (one risky asset and an unlimited risk-free asset) hence agents have maximum flexibility to construct state-dependent portfolios which closely match their subjective distributions.

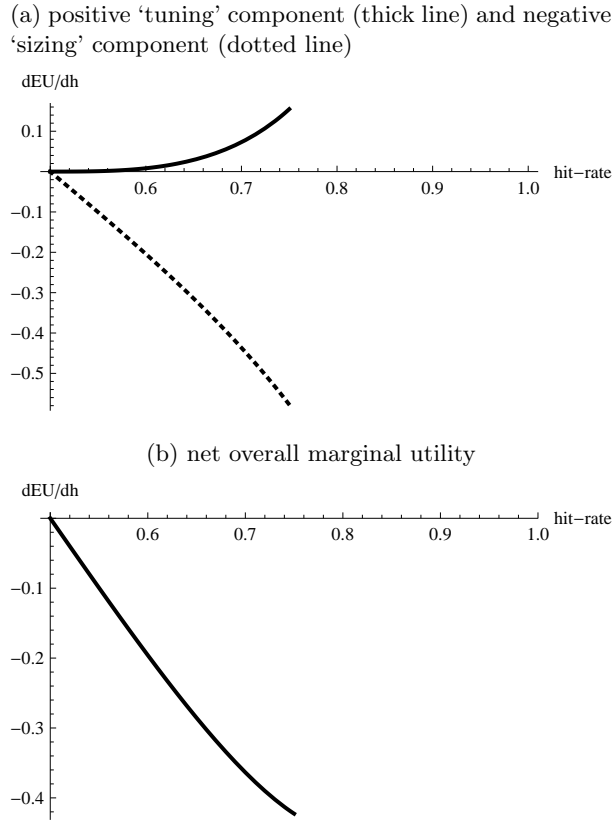
By way of summary we plot overall marginal expected utility for this case in Figure 6, showing the breakdown between the ‘sizing’ and ‘tuning’ components. Evidently increasing skill has a negative effect on each agent’s utility. This immediately addresses one of our Type I questions: increasing the skill of competing investor types in tandem may not lead to utility benefits overall.

(2) *Suppose only one type is skilled. What is the effect on their ex ante utility as skill is increased, assuming the other type remains unskilled?*

In this case the analytic expression is more complex and so we resort to inspection of the expected utility plot in Figure 5 and the cross-section of this surface in Figure 7. In contrast to the previous example we find here that marginal expected utility is positive at all but the very highest levels of skill from the perspective of the skilled investor (shown in panel (a)). Note however that the expected utility of the unskilled investor declines (panel (b)). One policy issue here is that targeting particular investor groups for skill improvements can easily have negative spillover effects on others.

We do not go to the extent of decomposing marginal utility in this case but use this as a demonstration of how sensitive the impact of skill improvement is to the exact structure of skill

Figure 6: Marginal expected utility for two identically-skilled independent types



in the market. This raises the important question of how altering individual investors’ skill levels may affect overall social welfare. This constitutes our third question.

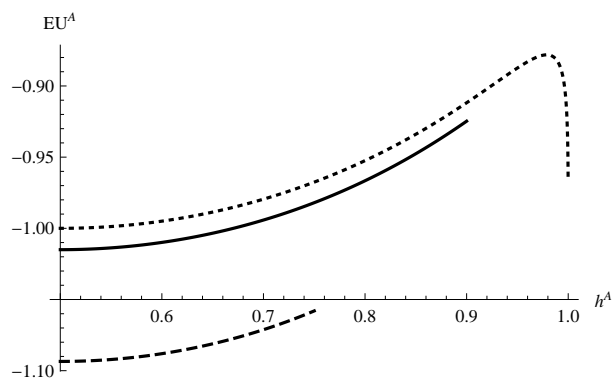
(3) *Is it socially desirable to increase skill levels?*

To address this question we adopt the device of a Social Welfare Function (SWF). We refer to Figure 8 which plots both Utilitarian (equally-weighted) and Rawlsian (maximin) social welfare functions for our model market (see Mas-Colell *et al* 1995). An immediate consequence of the assumptions of our model is that the surface is downward-sloping. For all plausible skill levels (i.e. with the exception of extreme levels bordering on perfect foresight) we find that improving skill of one type results in decreasing social welfare overall (if we keep the other type’s skill constant). Indeed the optimal social allocation of skill in this economy occurs when both types are perfectly unskilled, hence our original quotation by Thomas Gray.

Naturally this surprising result is heavily driven by the simple structure of our model market, in particular the assumption of equal initial wealth across types. Nevertheless this is very much consistent with the more general intuition suggested by our theoretical analysis in Proposition 2.3. Indeed in Corollary 2.4 we demonstrated (under the assumption of normality) the necessity of *negative* covariance between initial wealth and wealth gains in order to achieve a social benefit from trading. Our example here has *zero* covariance, however we reiterate our suspicion that in reality this covariance may well be *positive*, which would reinforce the notion that ignorance may indeed be bliss from a social point of view.

Figure 7: Expected utility for skilled agents of each type

(a) $\mathbb{E}(U^A)$ with h^B fixed at 0.5 (dotted), 0.6 (solid) or 0.75 (dashed)



(b) $\mathbb{E}(U^B)$ with h^B fixed at 0.5 (dotted), 0.6 (solid) or 0.75 (dashed)

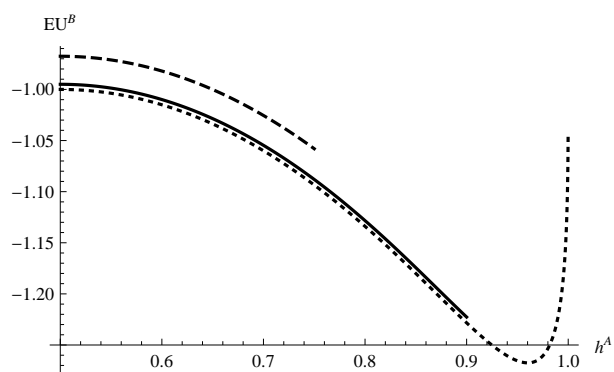
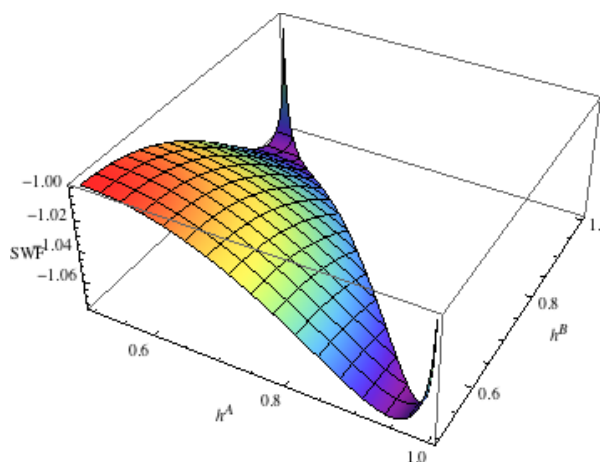
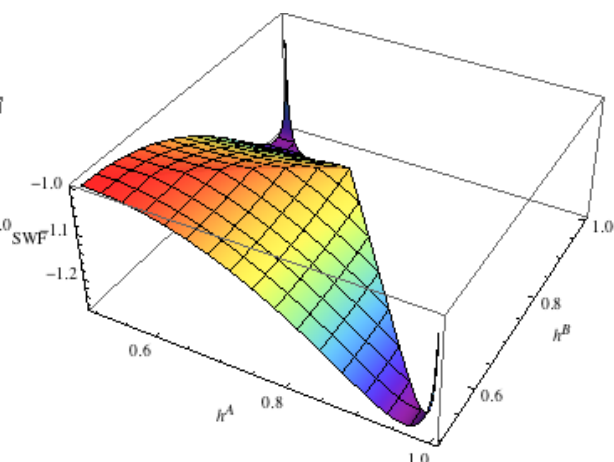


Figure 8: Social Welfare Functions for two skill agents

(a) Utilitarian (equally-weighted)



(b) Rawlsian (maximin)



4.2 Type II: Effects of Distribution of Skill

In the preceding section we have demonstrated the ambiguous effects of raising investor skill while keeping the distribution of investor types constant (with an equal weighting between the two types after adjusting for potentially differing risk tolerance). We now consider the effects of adjusting the proportion of investors of each type while keeping skill levels constant. As a reminder, we are concerned here with questions of the form:

Is it in the interests of unskilled investors to delegate portfolio decisions to professional managers with superior skill? Is there a socially-optimal allocation of capital across differentially-skilled investor types?

A central aspect of our analysis so far was typically whether or not incremental skill-induced price volatility was justified by increased investor utility. However from a policy perspective a drawback of this approach is its assumption that all members of the population are investors, with heterogeneity characterised only by their differential levels of forecasting skill. This might be justified (at least on a *ceteris paribus* basis) if we could demonstrate that the non-investors were an entirely independent group whose welfare was unaffected by events taking place in financial markets. Naturally this is an impossible case to make; in practice we see ubiquitous reminders of the (largely negative) welfare effects which asset price volatility have on non-investors via the real economy.¹³

We now recognise that increased volatility is a ‘bad’ from the point of view of *both* investors *and* non-investors, however the former group enjoy the offsetting benefit of (potentially) increased wealth due to the trading opportunities which volatility presents. From a quantitative point of view we can apply the analysis of the preceding sections to summarise the average welfare of the *investor* group while the disutility of the *non-investor* group can be represented by the standard deviation of the equilibrium price over repeated forecasting instances.

We illustrate this approach in Figures 9 to 12 where we measure utility and social welfare in terms of certainty equivalent wealth. In Figure 9 we consider the perspective of a skilled type (A) while we keep type B’s skill level fixed with $h^B = 0.5$. The proportion of skilled types in the population is denoted by f .¹⁴ The skilled types enjoy maximum welfare when they are as small as possible a group in the overall population. In this case they can derive positive benefit from their superior forecasting ability. However the magnitude of this benefit declines monotonically as they become more frequent (f increases) and we find that when they constitute the entire population ($f = 1.0$) they obtain zero incremental utility. Reminiscent of the case in Section 4.1 we find that the increased welfare of the skilled types comes at the cost of declining welfare for the unskilled types and monotonically increasing volatility.

For comparison in Figure 10 we illustrate the case where type B actually does have some forecasting skill (h^B is fixed at 0.6) but this is nevertheless lower than type A. Whilst at first glance the overall picture is similar we find that in this case the switching of investors from type

¹³This theme is prevalent in many strands of the macroeconomics literature: by way of example: Agenor and Aizenman (1999) and Caballero (2000) consider volatility in the context of emerging financial markets, Aghion and Banerjee (2005) model relationships between volatility and production and Ul Haq *et al* (1996) collect various perspectives on the celebrated Tobin Tax, specifically intended to combat the damaging real effects of spurious volatility.

¹⁴again we consider f to have been adjusted for differing risk-tolerances if relevant

B to type A (i.e. f increasing from zero) actually reduces volatility at first. This effect occurs because the two types have independent forecasts (by the deliberate construction of our model) and equilibrium price is, loosely speaking, a weighted average of the two types' forecasts.

In Figures 11 and 12 we show society's trade-off between investors' welfare and standard deviation of price (which we use as a measure of the volatility which affects investors and non-investors alike). The plots consist of two separate overlaid components:

(1) In the background of each figure we have sketched convex social indifference curves which are a stylized representation of society's preferences over investor-welfare/volatility pairs; these curves are for indicative purposes only and have not been chosen with any particular preferences in mind, although we feel that convexity is a reasonable assumption on the usual basis that average combinations are (plausibly) preferable to extremes. These curves radiate outwards from the origin, with the optimal point being the origin itself. Hence curves further from the origin represent less desirable outcomes.

(2) Overlaid on the indifference curves are - in each case - three separate curves, each of which is the locus of the feasible investor-welfare/volatility pairs which can be achieved *for given fixed levels of h^A and h^B* as the proportion of types (f) is varied.¹⁵ For the sake of argument we call these curves *isoskill* curves since each one is drawn by keeping available skill levels fixed and varying only f . All the *isoskills* have a point in common which is where all investors are type B (the relatively unskilled type whose skill level is always kept the same across *isoskills* in each figure). Note that in Figure 11 (where $h^B = 0.5$ representing no skill) both investors' welfare and volatility are zero at this common point, as would be expected.¹⁶

In view of our previous results it is unsurprising to find that investors' *average* welfare is negative (in certainty equivalent wealth terms) irrespective of the particular levels of skill h^A and h^B , i.e. all points on all *isoskills* lie below the x-axis. As the proportion of (relatively higher-skilled) type A's is increased (f increasing) the market moves along the *isoskill* away from the common point *initially* in the direction of declining average investor welfare. From the perspective of investors, this trajectory is akin to the 'tragedy of the commons': although they may continue to enjoy positive welfare individually, this declines in magnitude *per capita*.

While the average investor welfare falls, the effect on volatility is interesting. In Figure 11 (where type B's are unskilled with $h^B = 0.5$) we find that volatility increases throughout this process, however in Figure 12 (where type B's are somewhat skilled but less so than type A's) we can see conditions where volatility initially declines along with investor welfare (specifically on the *isoskill* where $h^A = 0.65$ and $h^B = 0.6$). Indeed one might imagine circumstances where society views this relationship as an acceptable trade-off, i.e. tangential to a social indifference curve.

In due course, as f continues to rise, a turning point is reached beyond which average investor welfare increases back towards zero; this comes about since the proportion of less skilled Type B's is a gradually reducing part of the average welfare calculation. Nevertheless, even as average investor welfare improves beyond the turning point, we find that volatility continues to climb. Given the convex shape of the *isoskill* there is therefore a segment representing investor-

¹⁵For this calculation we use f to compute prices and allocations and assume equal risk aversion between types so we also use f to calculate weightings in the welfare calculation.

¹⁶We might informally describe this as the 'ignorance is bliss' point.

Figure 9: Investor certainty-equivalent wealth and volatility as the mixture of types is varied from fewer skilled to more skilled types; h_B fixed at 0.5 (i.e. unskilled) and h_A set at 0.7 (dashed line), 0.6 (solid line) and 0.55 (dotted line); f represents the proportion of Type A's in the investor population

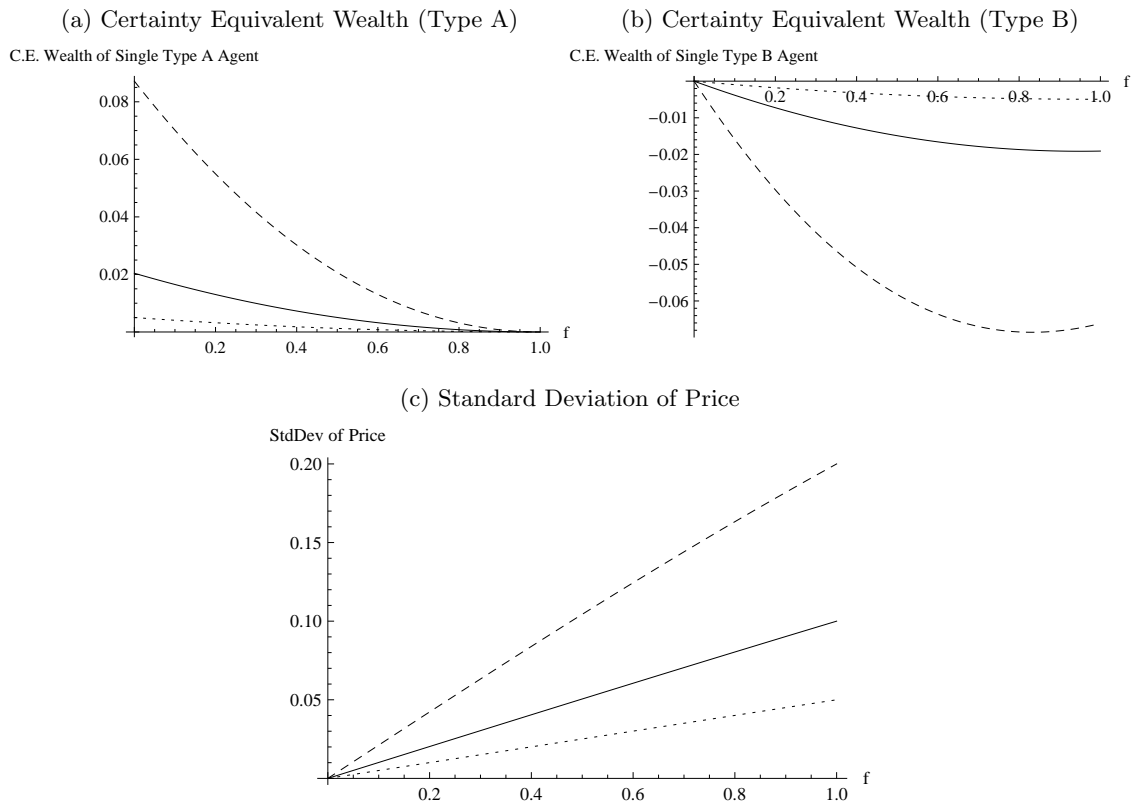


Figure 10: Investor certainty-equivalent wealth and volatility as the mixture of types is varied from fewer skilled to more skilled; h_B fixed at 0.6 and h_A set at 0.8 (dashed line), 0.7 (solid line) and 0.65 (dotted line)

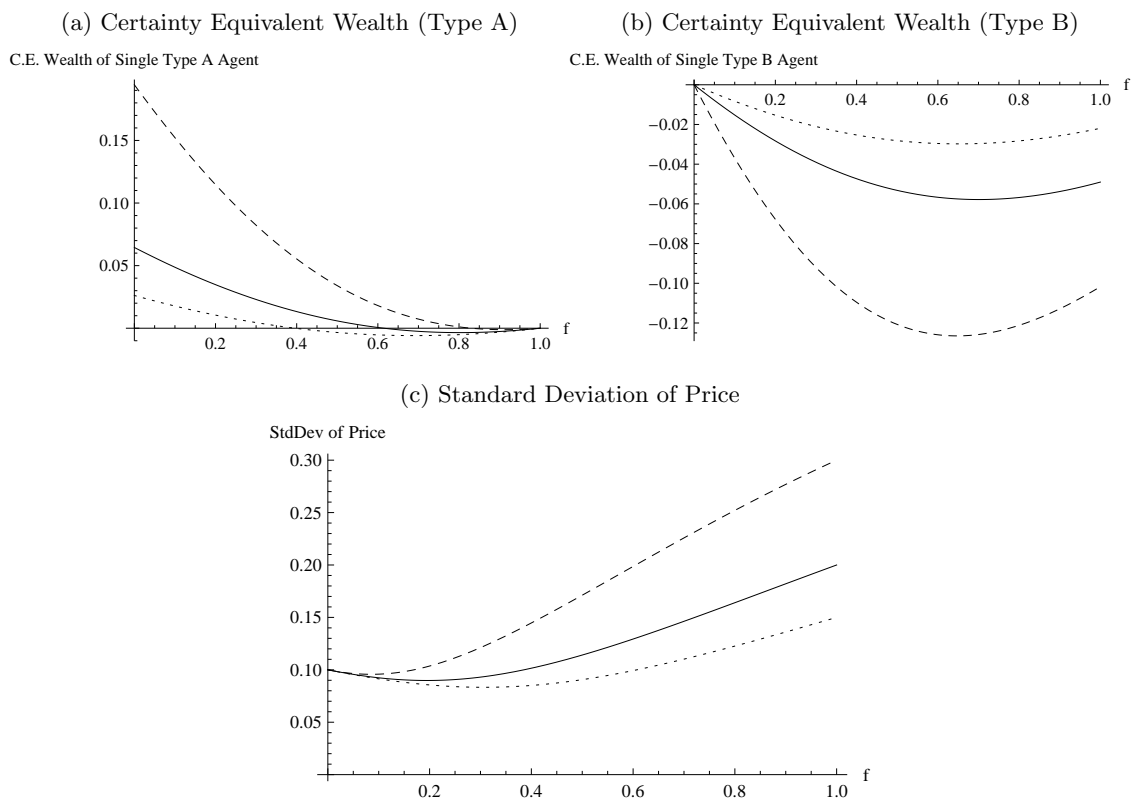


Figure 11: Social trade-off between investors' certainty-equivalent wealth and asset price volatility as proportion of skilled investors increases; h_B is fixed at 0.5 (i.e. unskilled) for all the curves and h_A is fixed at either 0.55 (thinnest curve), 0.6 (middle curve) or 0.7 (thickest curve); arrows indicate direction of increasing proportion of skilled types

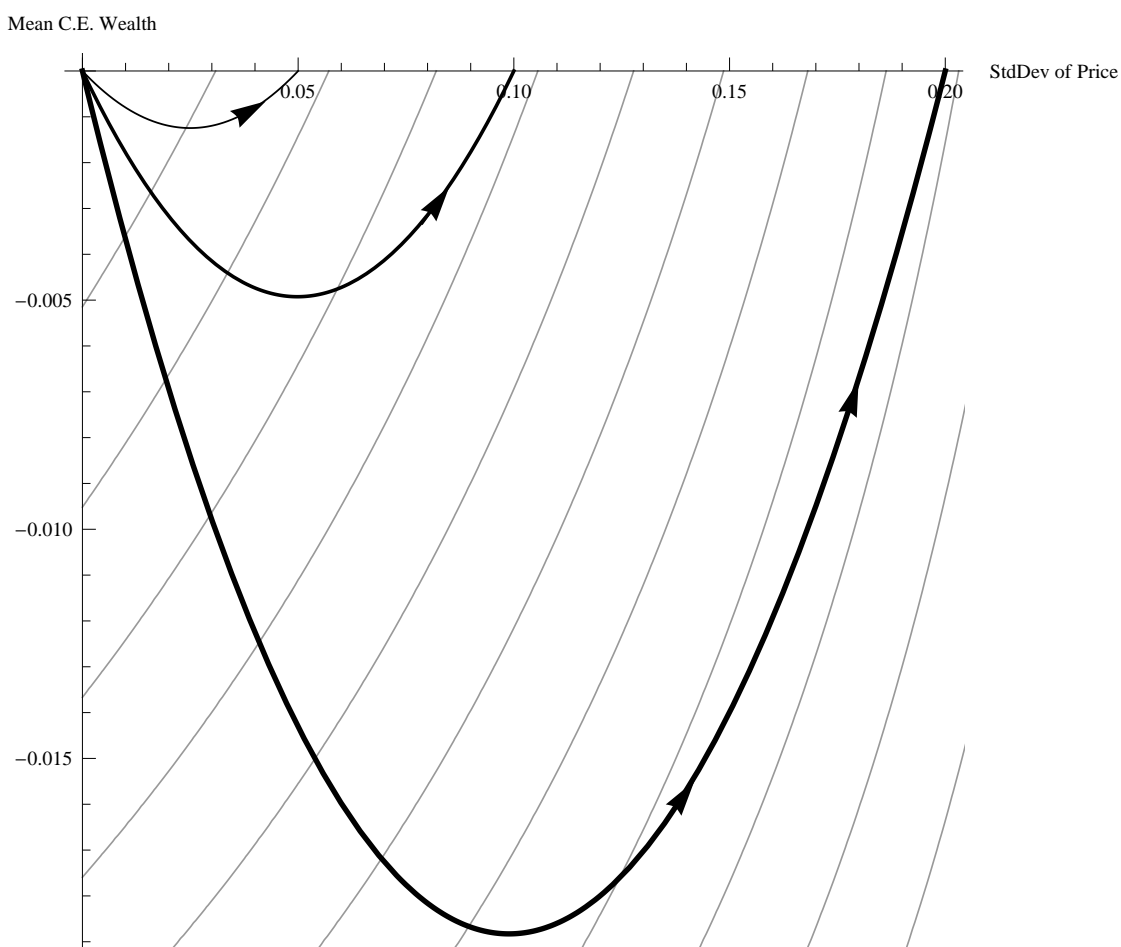


Figure 12: Social trade-off between investors' wealth and asset price volatility as proportion of skilled investors increases; h_B is fixed at 0.6 for all the curves and h_A is fixed at either 0.65 (thinnest curve), 0.7 (middle curve) or 0.8 (thickest curve); arrows indicate direction of increasing proportion of skilled types; point a represents entire population at the lower hit-rate (which is always 0.6), while b , c and d represent entire population at the higher hit-rates (0.65, 0.7 and 0.8 respectively)

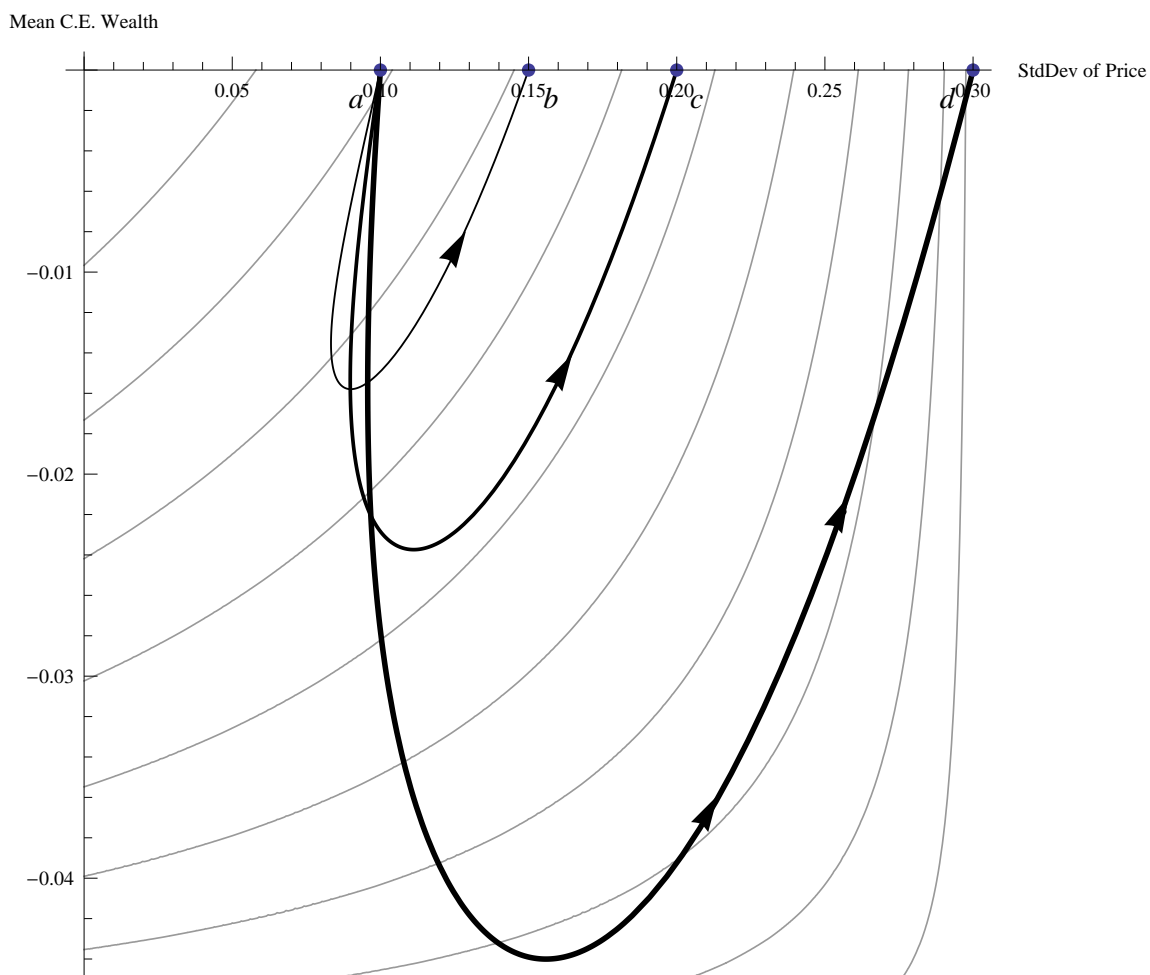
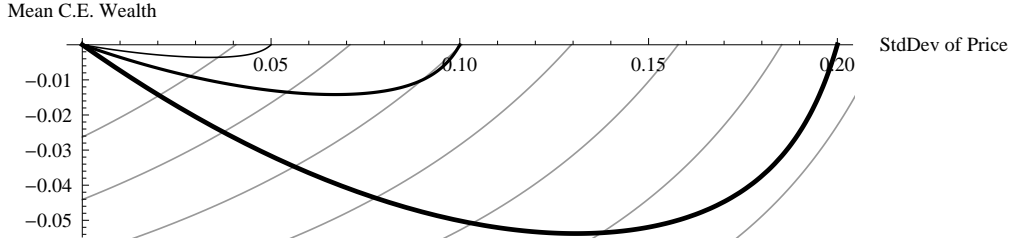
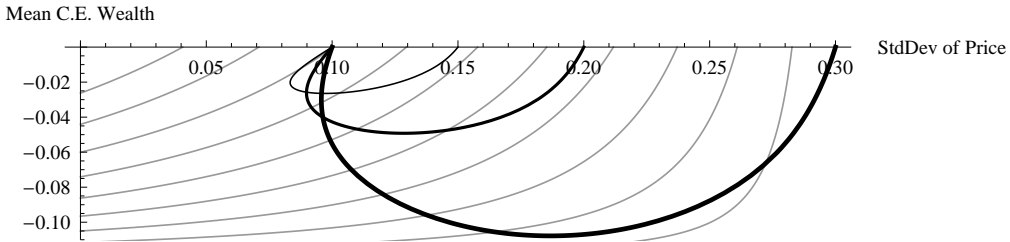


Figure 13: Social trade-off between investors' certainty-equivalent wealth and asset price volatility as proportion of skilled investors increases; $\frac{\lambda_A}{\lambda_B} = 0.1$.

(a) h_A fixed at either 0.55 (thinnest curve), 0.6 (middle curve) or 0.7 (thickest curve); h_B fixed at 0.5 in all cases



(b) h_A fixed at either 0.65 (thinnest curve), 0.7 (middle curve) or 0.8 (thickest curve); h_B fixed at 0.6 in all cases



welfare/volatility outcomes where equal investor-welfare could be achieved but with a lower volatility. This is loosely reminiscent of the concept of efficient versus inefficient portfolios as classically described by Markowitz (1952) and leads to a notion of a skill-efficient set which will correspond to the solution of a Mean-Variance Social Welfare Function.

In Figure 13 (a) and (b) we redraw the charts of Figures 11 and 12 but we impose different levels of risk aversion between the two agent types: specifically $\frac{\lambda_A}{\lambda_B} = 0.1$, i.e. the higher skilled type is the less risk-averse of the two. In the notation of Section 3.1 we recall that:

$$f = \frac{\phi}{\phi + (1 - \phi) \frac{\lambda_A}{\lambda_B}}$$

and while the 'weighted' frequency f determines equilibrium prices and allocations we plot *isoskills* with weightings given by ϕ . Clearly this effects both the shape and level of the *isoskill* curves. The effect of the differential risk-aversion is that $f > \phi$ so the skilled types have a disproportionately high impact on pricing (and volatility) compared to their headcount in the population. Hence the mean certainty-equivalent wealth now reaches much lower levels than under equal risk aversion since - for a given level of volatility - a greater proportion of the population are in the low-skilled category.

We now apply these tools to our Type II policy questions but find that direct answers are elusive without more specific knowledge of skill levels, market structure and distribution of wealth across the population.

Although these examples indicate that a 'first best' allocation of skill across investors would be our 'ignorance is bliss' point, unless society explicitly outlaws the accumulation of investor

skill it seems likely that wealth incentives for skill accumulation *on an individual basis* will persist (as depicted, for instance, in Figure 9). This means that to a large extent society may have to take the location of the prevailing *isoskill* curve as exogenously given, depending on the limitations of investors' own human capital rather than any structural arrangements in the economy itself.

We cautiously hypothesize therefore that policy interventions intended to shift the location of the *isoskill* are likely to be slow and costly. In contrast, governments may be able to use faster-acting policy tools to influence the exact location *on the isoskill*, e.g. supply side measures aimed at adjusting the structure of the asset management industry, targeted tax incentives to reward specific types of investment, etc..

In a 'second best' world, this highlights the need for policy-makers to attempt to establish where on the *isoskill* their economy is located: if on an 'inefficient' segment then there are clear welfare incentives to reducing the proportion of skilled types (ϕ), thereby reducing volatility while maintaining (or increasing) average investor welfare. However unless the economy can be immediately relocated to the new 'efficient' point then the challenge here is that a gradual reduction in ϕ may lead to reduced investor welfare during the transition period: potentially an unpopular consequence.

If the economy is on the 'efficient' segment there may be a perfectly socially-acceptable trade-off between investor-welfare and volatility; this depends on the precise levels of skill in the market as well as society's preferences, for instance we see possible examples in Figures 12 and 13(b) but not in Figures 11 or 13(a). It is also possible in this case that an increase in the skilled population (ϕ) may move society onto a higher indifference curve.

5 Concluding Remarks

To some degree governments have the policy tools to influence the distribution of financial literacy, albeit rather crudely. We have put forward general propositions which provide a theoretical framework to consider the welfare effects of such alterations to an economy's financial literacy profile, including required conditions for achieving welfare improvements. Specifically, under the assumption of normally-distributed wealth, we have shown that social benefits will depend partly on the relationship between each individual's initial wealth and subsequent (literacy-induced) gains, and partly on the impact which any measures have on volatility of wealth.¹⁷ Ideally, policy measures to increase financial literacy should strive for a negative covariance between initial wealth and wealth gains in order to make a positive contribution to social welfare. Nevertheless if these policies have the side-effect of increasing volatility in asset markets then their overall welfare impact may still be negative and we have provided a theoretical basis to analyse this trade-off.

To make our propositions more vivid we demonstrated their applicability to the case of investment forecasting skill, being a particular facet of financial literacy. Although our model was predicated on simple assumptions we tentatively found that circumstances favouring skill improvement may be less commonplace than conventionally believed and we were able to demon-

¹⁷In the case of non-normal wealth distributions we extend the calculation to include higher moments of wealth and higher derivatives of utility functions but similar intuition applies.

strate several scenarios where skill improvement had questionable value. In this context we addressed two broad policy categories:

Type I policies concern the *levels* of skill in the population, keeping the *distribution* constant. Here we would argue that the benefits of *universal* investor education must be weighed against the effects which higher levels of skilled trading can have on volatility. Clearly investor education which does not increase skill would be a suboptimal social investment, but even increased skill combined with greater volatility may have negative welfare implications for society as a whole as we demonstrated in Section 4. However, given limited resources to improve financial literacy *in specific social groups*, our general proposition argues that society should focus educational efforts very much on the poorest, and such programmes may indeed be welfare-enhancing if appropriately designed to be volatility-neutral (or volatility-reducing).

Type II policies consider the effects of the *distribution* of skill in the market, keeping *levels* constant. In this case we have shown that *in appropriate circumstances* it can indeed be socially desirable to switch investors between skill levels. This might be achieved by - for instance - encouraging broader consumer participation in actively-managed funds (including 130/30 and hedge funds), or by altering the regulatory environment to allow conventional funds greater active management flexibility (such as the European Union's UCITS III directive). Nevertheless we have been careful to demonstrate that the desirability of such measures depends on the initial levels and distribution of skill in the market. Once again it is perfectly possible for such policies to have adverse effects on volatility and to therefore be self-defeating. We have shown how analysis of this welfare/volatility trade-off can be carried-out by using methods analogous to those of portfolio theory and hence given a theoretical explanation for the quote '*...having smart guys there almost caused Wall Street to collapse.*'

Although these results apparently introduce new complexity into policy making, there is great scope for helpful empirical modeling of markets along the lines we have described. In particular a wealth of analysts' forecast data is available from which forecasting skill can be measured, along with reports of institutional Assets Under Management which give some sense of distribution of investors across levels of skill. The methods we have introduced in this paper enable the debate to benefit from proper quantitative treatment and raise it from the level of folk myths and value judgements.

This leads us to empirical applications. Assuming access to a suitably broad database of forecasts, trading positions and outcomes, our results suggest a variety of testable hypotheses:

(a) Do investors typically operate close to their optimal skill level? (In section 2 we specialised our proposition to the case of normally-distributed wealth to find simple conditions for the turning point in marginal utility).

(b) Do more complete markets lead to higher skill levels in general? (Our example in section 4 hints at this possibility due to more positive 'tuning' effects).

(c) Do markets segment into separating equilibria of skilled and unskilled types? (This is suggested by the negative marginal utility achieved when all agents increase skill together).

Finally, it is clear that analysis of skill parameters enables society to detect the extreme implausibility (even impossibility) of fraudulent investment offerings which are indeed 'too good to be true'. Our framework enhances this by providing a setting in which to model the wide

social damage which can be wrought by such schemes. Financial literacy can indeed be an equitable servant as well as a pernicious master.

A Derivations and Proofs

A.1 Proofs

A.1.1 Proposition 2.1

Proof. Agent i has unconditional expected utility given by:

$$\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} u(w(\mathbf{S}, z; \theta)) q(\mathbf{S}, z; \theta) dS^{(1)} dS^{(2)} \cdots dS^{(n)} dz$$

Differentiating with respect to the agent's literacy parameter θ^i gives:

$$\begin{aligned} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} u_w(w(\mathbf{S}, z; \theta)) w_\theta(\mathbf{S}, z; \theta) q(\mathbf{S}, z; \theta) dS^{(1)} \cdots dz + \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} u(w(\mathbf{S}, z; \theta)) q_\theta(\mathbf{S}, z; \theta) dS^{(1)} \cdots dz \\ = \mathbb{E} [u_w(w(\mathbf{S}, z; \theta)) w_\theta(\mathbf{S}, z; \theta)] + \mathbb{E} \left[u(w(\mathbf{S}, z; \theta)) \frac{q_\theta(\mathbf{S}, z; \theta)}{q(\mathbf{S}, z; \theta)} \right] \end{aligned}$$

Since q is a probability density we can think of $\frac{q_\theta(\mathbf{S}, z; \theta)}{q(\mathbf{S}, z; \theta)}$ as if it were a score function; from well-known properties of the score function it therefore follows that

$$\mathbb{E} \left[\frac{q_\theta(\mathbf{S}, z; \theta)}{q(\mathbf{S}, z; \theta)} \right] = 0 \tag{6}$$

□

A.1.2 Corollary 2.2

Proof. We deploy Stein's Lemma to rewrite (1) in more familiar terms. This results in a condition for positive marginal utility of literacy as follows:

$$\begin{aligned} \text{COV} [u_w(w), w_\theta] + \text{COV} \left[u(w), \frac{q_\theta}{q} \right] + \mathbb{E} [u_w(w)] \mathbb{E} [w_\theta] &> 0 \\ \mathbb{E} [u_{ww}(w)] \text{COV} [w, w_\theta] + \mathbb{E} [u_w(w)] \text{COV} \left[w, \frac{q_\theta}{q} \right] + \mathbb{E} [u_w(w)] \mathbb{E} [w_\theta] &> 0 \\ \frac{\text{COV} \left[w, \frac{q_\theta}{q} \right] + \mathbb{E} [w_\theta]}{\text{COV} [w, w_\theta]} &> R_A \end{aligned}$$

where

$$R_A \equiv - \frac{\mathbb{E} [u_{ww}(w)]}{\mathbb{E} [u_w(w)]}$$

is the coefficient of risk aversion defined by Rubinstein (1973).

□

A.1.3 Proposition 2.3

Proof. For clarity we consider here only the case of discrete types: suppose we have N agent types, each with a heterogeneous utility function $u^{[i]}$ and being a proportion $f^{(i)}$ of the population. Agent type i has initial (certain) wealth denoted by $w_0^{(i)}$ and future state-dependent wealth represented by the function $w^{(i)}(\mathbf{S}, z; \theta)$. We assume we have a utilitarian (i.e. equally-weighted) social welfare function. We denote the j 'th derivative of $u^{[i]}$ by $u^{[i](j)}$. If agents refrain entirely from investing (i.e. there is no trading with each other) then the value of this welfare function is given by

$$SWF_0 = \sum_{i=0}^{N-1} f^{(i)} u^{(i)}(w_0^{(i)})$$

However when agents *do* engage in trade (with a vector of skill parameters θ) then *ex ante* expected social welfare is given by

$$\begin{aligned} SWF(\theta) &= \sum_{i=0}^{N-1} f^{(i)} \mathbb{E}[u^{(i)}(w^{(i)}(\mathbf{S}, z; \theta))] \\ &= \sum_{i=0}^{N-1} f^{(i)} \mathbb{E} \left[u(w_0^{(i)}) + \sum_{j=1}^{\infty} \frac{1}{j!} (w^{(i)}(\mathbf{S}, z; \theta) - w_0^{(i)})^j u^{[i](j)}(w_0^{(i)}) \right] \\ &= SWF_0 + \sum_{i=0}^{N-1} f^{(i)} \mathbb{E} \left[\sum_{j=1}^{\infty} \frac{1}{j!} (w^{(i)}(\mathbf{S}, z; \theta) - w_0^{(i)})^j u^{[i](j)}(w_0^{(i)}) \right] \\ &= SWF_0 + \sum_{j=1}^{\infty} \sum_{i=0}^{N-1} f^{(i)} \frac{u^{[i](j)}(w_0^{(i)})}{j!} \mathbb{E} \left[(\Delta w^{(i)}(\mathbf{S}, z; \theta))^j \right] \end{aligned}$$

Hence the net welfare effect of trading is given by the second term. We call this the Social Benefit of Trading (*SBT*) which is therefore given by:

$$SBT(\theta) = \sum_{j=1}^{\infty} \mathbb{E}_f \left[\frac{u^{(j)}(w_0)}{j!} M^{(j)}(\theta) \right]$$

where \mathbb{E}_f indicates that expectation is taken with respect to the cross-sectional distribution of types f and $M^{(j)}(\theta)$ denotes the j 'th moment of the type-specific change in wealth which is equal to $\mathbb{E} \left[(\Delta w^{(i)}(\mathbf{S}, z; \theta))^j \right]$. \square

A.1.4 Proposition 2.6

Proof. Suppose we make a discrete forecast S of either 0 or 1 in respect of a discrete outcome Q which is also either 0 or 1. We define the following probabilities:

$$\begin{aligned} \mathbb{P}[S = 1] &= g \\ \mathbb{P}[Q = 1|S = 1] &= P[Q = 0|S = 0] = h \end{aligned}$$

We list the possible outcomes and joint probabilities of (S, Q) pairs in Table 4 and hence compute the following:

$$\begin{aligned}\mathbb{E}[SQ] &= gh \\ \mathbb{E}[S] &= \mathbb{E}[Q] = g \\ \mathbb{V}[S] &= \mathbb{V}[Q] = g(1-g) \\ \text{COV}[S, Q] &= gh - g^2 = g(h-g) \\ IC = \text{COR}[S, Q] &= \frac{\text{COV}[S, Q]}{\sqrt{\mathbb{V}[S]\mathbb{V}[Q]}} = \frac{h-g}{1-g}\end{aligned}$$

For example when $g = \frac{1}{2}$ we have $IC = 2h - 1$. □

Table 4: Joint distribution of forecast S and actual outcome Q where h denotes hit-rate and g denotes marginal probability of a forecast equal to 1

S	Q	SQ	$\mathbb{P}[S, Q]$
0	0	0	$(1-g)h$
0	1	0	$(1-g)(1-h)$
1	0	0	$g(1-h)$
1	1	1	gh

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