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with Unemployment and Sticky Prices: The Case of a
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15 April 2011

CWPE 1146

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April 15, 2011

Abstract

Ravenna and Walsh (2010) develop a linear quadratic framework for optimal monetary policy analysis in a New Keynesian model featuring search and matching frictions and show that maximization of expected utility of the representative household is equivalent to minimizing a quadratic loss function that consists of inflation, and two appropriately defined gaps involving unemployment and labor market tightness. This paper generalizes their analysis, most importantly by relaxing the Hosios (1990) condition which eliminates the distortions resulting from labor market inefficiencies, such that the equilibrium level of unemployment under flexible prices would not necessarily be optimal. I take account of steady-state distortions using the methodology of Benigno and Woodford (2005) and derive a quadratic loss function that involves the same three terms, albeit with different relative weights and definitions for unemployment- and labor market tightness gaps. I evaluate the resulting loss function subject to a simple set of log-linearized equilibrium relationships and perform policy analysis. The key result of the paper is that search externalities give rise to an endogenous cost push term in the new Keynesian Phillips curve, suggesting a case against complete price stability as the only goal of monetary policy, because there is now a trade-off between stabilizing inflation and reducing inefficient unemployment fluctuations. Transitory movements of inflation in this environment helps job creation and hence prevents excessive volatility of unemployment.

JEL Classifications: E52, E61, J64.

Keywords: Optimal monetary policy, unemployment, search externalities.

*I am grateful to Tiago Cavalcanti, Chryssi Giannitsarou, Sean Holly, Kamiar Mohaddes, and Sergejs Saksonovs as well as the Macroeconomics Workshop participants at the University of Cambridge for constructive comments and suggestions.

1 Introduction

Given the wide-spread attention to rising unemployment figures associated with the recent financial crisis as well as the world-wide use of expansionary monetary policy in response to the global recession, a couple of questions arise in policy debates. What are the consequences of labour market inefficiencies for the conduct of optimal monetary policy? Is there a trade-off between stabilizing CPI inflation and reducing inefficient unemployment fluctuations? In this paper, I address these questions in a New Keynesian model featuring search and matching distortions. The main objective is to focus on the implications for optimal monetary policy of search externalities. By optimal policy I mean the one that minimizes an intertemporal loss function subject to the model's equilibrium relationships under commitment. I make a contribution to the literature by deriving an explicit expression for the welfare objective in the presence of search externalities; those that distort the steady state of the model and render the standard Linear Quadratic (LQ) methods of [Benigno and Woodford \(2003\)](#) and [Woodford \(2003\)](#) inapplicable. I also reduce the equilibrium dynamics of the model into a simple log-linear representation in inflation, unemployment gap, and the labor market tightness gap to make the framework more tractable. The resulting generalized LQ model enables me to evaluate the model-driven quadratic loss function and perform policy analysis. The key result of the paper is that labor market inefficiencies matter for optimal monetary policy. In contrast to recent findings in the New Keynesian Search literature, I show that the optimal inflation rate is typically non-zero because it is used to indirectly attenuate the externality that arises from search and matching frictions, one that is usually eliminated in earlier papers by assuming an efficient labor market allocation, see [Ravenna and Walsh \(2010\)](#) and [Thomas \(2008\)](#) for example. Since search externalities generate an endogenous cost push term in the new Keynesian Phillips curve, the policy maker faces a trade-off between stabilizing inflation and reducing inefficient unemployment fluctuations.

Two main features underlie the search and matching models of equilibrium unemployment. First, existing matches command a surplus in equilibrium as hiring firms and searching workers have to spend resources before matches can take place. Second, matching models exhibit congestion or search externalities due to the tightness of the labor market, the relative number of hiring firms to searching workers. These externalities are due to the fact that one additional searching worker in the market increases the probability that a hiring firm will match with a job-seeker but decreases the probability that a searching worker already in the market will match with a firm. [Hosios \(1990\)](#) shows that search externalities are balanced, and thereby labor market allocations (market tightness and unemployment) are Pareto efficient, when the bargaining power of workers equals the elasticity of the matching function with respect to vacancies. Although Hosios condition need not hold empirically most studies in the literature are constrained to this simplifying parameter configuration. I depart from this unappealing assumption to explicitly study the implications for optimal monetary policy of search externalities when the NK model is augmented with unemployment and policy is based on an intertemporal model-consistent loss function.

Given the attractiveness of the non-Walrasian search and matching model of equilibrium unemployment, a growing number of papers have incorporated it into the standard New Keynesian (NK) framework to explore its implications for macro dynamics and/or optimal

monetary policy.¹ The NK model featuring search frictions consists mainly of three distortions: 1) monopolistic competition, 2) staggered price setting, and 3) congestion externalities which create inefficient labor market allocations. The first two are present in a canonical NK approach to monetary policy analysis but the third one is absent due to the assumption of Walrasian labor markets. In a simple NK model without unemployment it is possible to show that under certain assumptions,² an optimizing policy maker can implement the efficient (i.e. flexible price) allocation through a zero inflation (optimal) policy and does not face a trade-off between stabilization of inflation and reducing the gap between actual output and the flexible price level of output. [Blanchard and Gali \(2008\)](#), [Ravenna and Walsh \(2010\)](#), and [Thomas \(2008\)](#) extend the optimal monetary policy analysis to a NK framework featuring search and matching frictions. They derive linear quadratic (LQ) models which consist of linear structural equations and quadratic loss functions and show that monetary policy prescriptions of standard new Keynesian models are preserved in this new integrated setting, albeit in the absence of wage rigidities. However, for the LQ approach to provide correct welfare rankings, they assume an efficient (non distorted) steady state by imposing the Hosios parameter configuration which eliminates the congestion externalities.³

[Benigno and Woodford \(2005\)](#) show that the LQ approach to the optimal policy problem can preserve correct welfare rankings even when the steady state is distorted to an arbitrary extent if second order approximations are taken to the model structural relations (specifically, to the New Keynesian Phillips Curve). Making use of this general approach I obtain a welfare-theoretic loss function that consists of inflation, and two appropriately defined gaps involving unemployment and labor market tightness as well as a simple and intuitive log-linear representation of the model's equilibrium dynamics in these three variables. The resulting model-driven welfare criterion differs significantly from those obtained in [Ravenna and Walsh \(2010\)](#), and [Thomas \(2008\)](#) because the degree of distortion of the steady state, owing to search externalities, affects the weights on the stabilization objectives of the policy maker in the loss function. The coefficients in the quadratic approximation depend on the underlying structural parameters of the model that govern preferences, the degree of nominal price rigidity, and the search and bargaining processes in the labor market.

The results obtained in this work add to the rich debate on optimal monetary policy. More specifically, congestion externalities suggest a case against complete price stability as the only goal of monetary policy and generate a trade-off between stabilizing inflation and reducing inefficient unemployment fluctuations.⁴ When the bargaining power of workers is higher than the vacancy elasticity of the matching function (search inefficiencies exist), productivity shocks create a gap between the flexible price equilibrium and the social planner's allocation (first best), and generate a cost push term in the new Keynesian Phillips curve. In this case,

¹Examples include [Blanchard and Gali \(2007\)](#), [Gertler, Sala, and Trigari \(2008\)](#), [Gertler and Trigari \(2009\)](#), [Ravenna and Walsh \(2008a\)](#), [Ravenna and Walsh \(2010\)](#), [Sala, Söderström, and Trigari \(2008\)](#), [Thomas \(2008\)](#), [Trigari \(2009\)](#), [Walsh \(2003\)](#), and [Walsh \(2005\)](#).

²The existence of an output subsidy that offsets the distortion due to the market power of monopolistically competitive price-setters is essential. With this assumption, the steady state under a zero-inflation policy involves an efficient level of output.

³Labor market efficiency ensures that the flexible price equilibrium is exactly the same as the social planner's allocation.

⁴[Blanchard and Gali \(2008\)](#) and [Thomas \(2008\)](#) find that real-wage rigidity and staggered wage adjustment create a case against price stability, respectively.

the policy maker can employ a zero inflation policy to achieve the flexible price equilibrium but not the first best. Consequently, complete price stability cannot ensure an efficient labor market allocation. The policy maker faces an unemployment/inflation trade-off because it can only steer firms' incentives to post vacancies towards the efficient level and reduce inefficient unemployment when prices are sticky but not when they are flexible. Deviation from complete price stability in this environment helps job creation and thereby reduces excessive unemployment fluctuations.

These findings are in line with what is being argued in [Faia \(2009\)](#). She uses a Ramsey framework with quadratic price adjustment costs and matching frictions in the labor market to study the implications of steady-state distortions (monopolistic competition and search externalities) for the conduct of optimal monetary policy. However, the methodology and approach that I adopt differ from hers in many respects, highlighted by the fact that I derive a micro-founded linear quadratic model under an inefficient steady state as opposed to the Ramsey approach that she takes. The LQ framework enables me to provide analytical insights on the Central Bank's objectives in presence of sticky prices and labor search frictions as well as a simple linear representation of the model's equilibrium equations. It is along the above dimensions that these two papers complement each other.

The remainder of the paper is organized as follows: [Section 2](#) presents the basic model. [Section 3](#) describes the first best allocation as well as the flexible price equilibrium. The linear quadratic model is derived in [section 4](#). The main findings of the paper are presented in [section 5](#), where policy analysis is conducted under alternative parameterization. Finally, [section 6](#) summarizes the results, concludes, and proposes some possible extensions.

2 The Model Economy

The model economy consists of four sectors: 1) households whose utility depends on consumption of final goods, and their members are either in a match (employed) or searching for a new match (unemployed).⁵ 2) wholesale firms who employ labor and produce intermediate goods in a perfectly competitive market. They face search frictions and bargain with workers over wages. 3) monopolistically competitive retailers who purchase intermediate goods from the wholesale sector, set the price of transformed goods in a staggered fashion and sell them to households. 4) a monetary authority who seeks to minimize a quadratic loss function. In order to provide a convenient separation of the two distortions in the model, I incorporate labor market frictions in the wholesale sector where prices are flexible and introduce sticky prices in the retail sector among firms who do not employ labor.⁶

The presence of search frictions in the labor market prevents some unemployed workers from finding jobs and some hiring firms from filling their vacancies in each period. The flow of matches between job-seekers and hiring firms is given by the so-called matching function,

$$M_t = \psi V_t^\varepsilon U_t^{1-\varepsilon},$$

⁵I abstract from labor force participation decisions.

⁶This modeling device is common in the literature. See [Ravenna and Walsh \(2008a\)](#), [Ravenna and Walsh \(2010\)](#), [Thomas \(2008\)](#), [Trigari \(2009\)](#), [Walsh \(2003\)](#), [Walsh \(2005\)](#).

in which M_t is the number of matches created in each period; U_t denotes the stock of unemployed workers; V_t measures the number of vacancies; ψ is a scaling parameter; and ε is the elasticity of the matching function with respect to vacancies. It is also convenient to introduce $\theta_t = \frac{V_t}{U_t}$ as a measure of labor market tightness. At each point in time, a vacant job is matched to an unemployed worker with probability $q(\theta_t) = \frac{M_t}{V_t}$. Similarly, the probability that any worker looking for a job is matched with an open vacancy at time t is denoted with $p(\theta_t) = \theta_t q(\theta_t) = \frac{M_t}{U_t}$.

2.1 Households

The model contains a continuum of large identical households on the unit interval with a measure one of individuals, indexed by $m \in [0, 1]$, that live within each household. A fraction, $N_{t-1} = \int_0^1 N_{mt-1} dj$, of the representative household's members are employed by competitive firms in production activities at the start of period t , receiving real wage w_t . The remaining members, $U_t = 1 - N_{t-1}$, are unemployed and search for jobs. Those who are employed might separate from their jobs during period t at an exogenous rate λ , while unemployed members have a probability $p(\theta_t)$ of finding a new job within the period. Therefore, the household's employment rate evolves according to the following law of motion

$$N_t = (1 - \lambda)N_{t-1} + p(\theta_t)(1 - N_{t-1}), \quad (1)$$

which together with $U_t = 1 - N_{t-1}$ and $\theta_t = \frac{V_t}{U_t}$ describe the so-called Beveridge curve, a downward sloping relationship between unemployment and vacancies.

The representative household chooses asset holdings, A_t , and consumption levels, C_t , to maximize the intertemporal welfare function,⁷

$$H_t(C_t, A_t) = \max \{u(C_t) + \beta E_t H_{t+1}(C_{t+1}, A_{t+1})\},$$

subject to equation (1) and a budget constraint given by

$$C_t + \frac{A_t}{P_t} = w_t N_t + (1 + i_{t-1}) \frac{A_{t-1}}{P_t} + \pi_t^r,$$

where $u(\cdot)$ is the instantaneous utility function, $C_t = \left(\int_0^1 C_{jt}^{(\gamma-1)/\gamma} dj \right)^{\gamma/(\gamma-1)}$ is the Dixit-Stiglitz basket of final goods purchased from the continuum of monopolistic retailers in which γ is the elasticity of substitution between different varieties, A_{t-1} are holdings of one-period riskless nominal bonds with nominal interest rate between periods t and $t-1$ equal to i_{t-1} , and π_t^r are real profits from the retail sector. $P_t \equiv \left(\int_0^1 P_{jt}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}$ measures the price of a unit of the consumption basket. Accordingly, the optimal allocation of expenditure on each variety is given by $C_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\gamma} C_t$.

⁷I follow the literature in assuming that consumption risks are fully pooled, see Merz (1995) among others. Since consumption is equalized across members, I can use the same notation for consumption of the representative household and that of each member.

The intertemporal first order condition for the household's decision problem with respect to A_t yields the standard Euler equation

$$u'(C_t) = \beta(1 + i_t) E_t \left[\frac{P_t}{P_{t+1}} u'(C_{t+1}) \right]. \quad (2)$$

It is also possible to obtain the welfare enjoyed by the household from supplying an additional worker as

$$\frac{\partial H_t}{\partial N_{mt}} = u'(C_t)w_{mt} - \beta E_t p(\theta_{t+1}) \frac{\partial H_{t+1}}{\partial N_{mt+1}} + \beta(1 - \lambda) E_t \frac{\partial H_{t+1}}{\partial N_{mt+1}}, \quad (3)$$

where the contribution of an additional worker to the households' welfare is given by the real wage times the marginal utility of consumption at period t , minus the cost this worker would incur on the household should the job search continue for another period, plus the future value of the job conditional on non-separation. Letting $V_{mt}^W = \left(\frac{\partial H_t}{\partial N_{mt}} \right) \left(\frac{1}{u'(C_t)} \right)$ denote the value of an employed worker to the household in consumption units at period t , equation (3) can be expressed as

$$V_{mt}^W = w_{mt} + E_t [1 - \lambda - p(\theta_{t+1})] \beta_{t,t+1} V_{mt+1}^W,$$

where $\beta_{t,t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}$ is the stochastic discount factor between periods t and $t + 1$.

2.2 Competitive Producers (Search Frictions)

A measure-one continuum of perfectly competitive wholesale firms, indexed by $i \in [0, 1]$, produce a homogenous intermediate good, Y_{it}^w , during period t which they sell to retailers at real price $\frac{P_t^w}{P_t}$. The production function is given by

$$Y_{it}^w = Z_t N_{it},$$

and is identical across firms. Each wholesale firm employs N_{it} workers at real wage w_{it} and faces a common state of technology Z_t . Hiring in this sector is subject to search and matching frictions because firstly, firms have to post vacancies to obtain new employees which is assumed to come at a utility cost κ for each job posting, and secondly, they lose their existing workers at rate λ during period t . The stock of employment at firm i evolves according to the following law of motion

$$N_{it} = (1 - \lambda)N_{it-1} + V_{it}q(\theta_t), \quad (4)$$

where V_{it} is the number of vacancies the firm posts at time t , and $q(\theta_t)$ is the probability of filling a vacancy during that period. The timing assumption in this paper is such that searching workers who find a match start their jobs immediately within the period, while those separating from their jobs at time t are not allowed to search until next period. Accordingly, fluctuations in unemployment arise from cyclical variation in hirings as opposed to separations. See [Hall \(2005\)](#) and [Shimer \(2005\)](#) for evidence in support of this phenomenon.

The firm chooses V_{it} and N_{it} to maximize the expected present discounted sum of real profits (written in recursive form) subject to equation (4).

$$J_{it}(N_{it}, V_{it}) = \frac{P_t^w}{P_t} Z_t N_{it} - w_{it} N_{it} - \frac{\kappa V_{it}}{u'(C_t)} + E_t \beta_{t,t+1} J_{it+1}(N_{it+1}, V_{it+1}),$$

where $\frac{\kappa}{u'(C_t)}$ measures the cost of posting a vacancy in consumption terms. The first order condition with respect to V_{it} yields the so-called job posting condition which implies that the value of a filled job must be equal to the search costs associated with hiring, or

$$\frac{\partial J_{it}}{\partial N_{it}} = \frac{\kappa}{u'(C_t) q(\theta_t)}. \quad (5)$$

The resulting first order condition with respect to N_{it} describes the value to the firm of an additional worker and is given by

$$\begin{aligned} \frac{\partial J_{it}}{\partial N_{it}} &= \frac{P_t^w}{P_t} Z_t - w_{it} + (1 - \lambda) E_t \beta_{t,t+1} \frac{J_{it+1}}{\partial N_{it+1}}, \\ V_{it}^J &= MRP N_t - w_{it} + (1 - \lambda) E_t \beta_{t,t+1} V_{it+1}^J, \end{aligned} \quad (6)$$

in which the contribution of the worker to the firm's lifetime profits is given by the marginal revenue product of labor, $MRP N_t = \frac{P_t^w}{P_t} Z_t$, minus the real wage, plus the discounted value of having a match in the following period. In other words, the value of a filled job is equal to the firm's current period profit plus the continuation value of the job. Combining (5) and (6) yields the firm's hiring decision

$$\frac{\kappa}{q(\theta_t)} = u'(C_t) \left(\frac{P_t^w}{P_t} Z_t - w_{it} \right) + (1 - \lambda) \beta E_t \frac{\kappa}{q(\theta_{t+1})}. \quad (7)$$

According to (7), the search costs associated with hiring (in utils) is equal to the firm's current profit (marginal revenue product minus the real wage) plus the discounted recruitment cost savings if an existing match survives into the following period. In the absence of search and matching frictions (when $\kappa = 0$), equation (7) simplifies to $\frac{P_t^w}{P_t} Z_t = w_{it}$. This condition corresponds to the standard new Keynesian model, where the marginal revenue product of an employee is equal to the marginal cost of a worker to the firm (real wage); or equivalently, the marginal cost $\frac{P_t w_{it}}{Z_t}$ must be equated to the nominal price P_t^w .

2.3 Monopolistic Firms (Sticky Prices)

There exists a continuum of monopolistically competitive retailers indexed by $j \in [0, 1]$, who purchase the wholesale good in a competitive market at real price $\frac{P_t^w}{P_t}$, differentiate it with a technology that converts one unit of intermediate good into one unit of final good, $C_{jt} = Y_{jt} = Y_{it}^w$, and then re-sell it to households. Since the only input in the production function of the retail firm is the intermediate good, each retailer's real marginal cost is $\frac{P_t^w}{P_t}$. This is just the inverse of the markup of retail over wholesale goods which in turn depends on matching frictions that characterize the wholesale sector.

The retail firm seeks to maximize its lifetime profits by setting the price of its product subject to constraints implied by the demand for its good, the production technology it has

access to and a restriction on the frequency of price adjustment. I make use of the Calvo (1983) model of price setting and assume that each period only a randomly chosen fraction, $1 - \delta$, of firms can adjust their prices. A retailer that can re-set its price in period t chooses P_{jt} to maximize

$$\max_{P_{jt}} E_t \sum_{i=0}^{\infty} \delta^i \beta_{t,t+i} \left((1+s) \frac{P_{jt}}{P_{t+i}} - \frac{P_{t+i}^w}{P_{t+i}} \right) C_{jt+i}$$

subject to

$$C_{jt+i} = \left(\frac{P_{jt}}{P_{t+i}} \right)^{-\gamma} C_{t+i}, \quad (8)$$

where s is the subsidy rate on sales revenues. This output subsidy is introduced in the model to offset the distortions due to the market power of monopolistically competitive price-setters.

The optimal pricing equation is then given by

$$E_t \sum_{i=0}^{\infty} \delta^i \beta_{t,t+i} P_{t+i}^\gamma C_{t+i} \left((1+s) \frac{P_t^*}{P_{t+i}} - \frac{\gamma}{\gamma-1} \frac{P_{t+i}^w}{P_{t+i}} \right) = 0, \quad (9)$$

where P_t^* is the common price chosen by all price-setters. Therefore, retailers set prices as a constant mark-up over real marginal costs for the expected duration of the price contract. Using the definition of $\beta_{t,t+i}$, I can re-write equation (9) as

$$\frac{P_t^*}{P_t} = \frac{E_t \sum_{i=0}^{\infty} (\delta\beta)^i u'(C_{t+i}) C_{t+i} \frac{\gamma}{\gamma-1} \frac{P_{t+i}^w}{P_{t+i}} \left(\frac{P_{t+i}}{P_t} \right)^\gamma}{E_t \sum_{i=0}^{\infty} (\delta\beta)^i u'(C_{t+i}) C_{t+i} (1+s) \left(\frac{P_{t+i}}{P_t} \right)^{\gamma-1}} = \frac{K_t}{F_t}. \quad (10)$$

From $P_t \equiv \left(\int_0^1 P_{jt}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}$, the average price in period t satisfies the following law of motion

$$P_t^{1-\gamma} = (1-\delta) P_{t-1}^{1-\gamma} + \delta P_t^{1-\gamma}. \quad (11)$$

3 Equilibrium

This section characterizes the social planner's allocation as well as equilibrium in the decentralized economy under a flexible wage setting mechanism.

3.1 Decentralized Equilibrium with Flexible Wages

To find the equilibrium in the decentralized economy, it is required to determine the real wage which appears in both equations (3) and (6). I follow the search and matching literature in assuming that an intermediate-good producer and a worker determine the real wage according to the Nash solution to a bargaining problem. Each participant in the bargain

will receive a fixed share of the joint match surplus which is the sum of the surpluses of the firm, $\frac{\partial J_{it}}{\partial N_{it}}$, and the worker in consumption units, $\left(\frac{\partial H_t}{\partial N_t}\right) \left(\frac{1}{u'(C_t)}\right)$, or

$$V_t^J = \xi(V_t^J + V_t^W),$$

where $\xi \in (0, 1)$ is the firm's share of the job surplus in period t . The equilibrium real wage is then given by

$$w_t^{Nash} = (1 - \xi) \left(\frac{P_t^w}{P_t} Z_t + \frac{\kappa \beta E_t \theta_{t+1}}{u'(C_t)} \right). \quad (12)$$

Substituting (12) into equation (7) for the real wage yields the following job creation condition or the equilibrium in a decentralized economy.

$$\frac{\kappa}{q(\theta_t)} = \xi u'(C_t) \frac{P_t^w}{P_t} Z_t + \beta E_t [1 - \lambda - (1 - \xi)p(\theta_{t+1})] \frac{\kappa}{q(\theta_{t+1})}. \quad (13)$$

It will also prove useful to find equilibrium in intermediate goods market which requires that total supply be equal total demand by retailers, $Y_t^w = \int_0^1 Y_{jt} dj$. Using (8), this condition can be written as

$$C_t \Delta_t = Y_t^w = Z_t N_t,$$

where $\Delta_t \equiv \int_0^1 \left(\frac{P_{jt}}{P_{t+i}}\right)^{-\gamma} dj$ is a measure of price dispersion.

3.2 Efficient Equilibrium

The constrained-efficient allocation which serves as a benchmark for monetary policy evaluation is derived by solving the optimization problem of a benevolent social planner who is faced with the aggregate technological and resource constraints as well as the labor market frictions that are present in a decentralized economy. However, the planner internalizes the effects of search and matching distortions and avoids any inefficient dispersion in relative prices. It seeks to maximize the joint welfare of households and managers given by

$$\sum_{t=0}^{\infty} \beta^t \{u(C_t) - \kappa V_t\}$$

subject to the following set of constraints

$$\begin{aligned} C_t &= Y_t = Y_t^w = Z_t N_t, \\ N_t &= (1 - \lambda)N_{t-1} + M_t, \\ M_t &= \psi V_t^\varepsilon U_t^{1-\varepsilon}, \\ U_t &= 1 - N_{t-1}. \end{aligned}$$

Combining the first order conditions of the social planner's problem with respect to V_t and N_t yields the following optimality condition or the efficient equilibrium

$$\frac{\kappa}{q(\theta_t)} = \varepsilon u'(C_t) Z_t + \beta E_t [1 - \lambda - (1 - \varepsilon)p(\theta_{t+1})] \frac{\kappa}{q(\theta_{t+1})}. \quad (14)$$

Comparing the decentralized outcome (13) with the social planner's equilibrium (14) shows that the efficient allocation (first best) can only be enforced in the disaggregated economy if there is no dispersion in relative prices, $\Delta_t = 1$, the retail markup is equal to one, $\frac{P_t^w}{P_t} = 1$, as in the standard new Keynesian model, and the firm's bargaining power, ξ , is equal to the elasticity of the matching function with respect to vacancies, ε . In contrast to the standard NK model, employing a subsidy that offsets the allocative effects of the steady-state markup is not sufficient to ensure efficiency of the resulting outcome. A too high bargaining power of workers (firms) can lead to an inefficient level of unemployment which is above (below) the Pareto optimum. Hence, the so-called Hosios (1990) condition, in which $\xi = \varepsilon$, is required for efficient vacancy creation. Provided the above conditions are satisfied, the economy's steady state is efficient meaning that the flexible price equilibrium is exactly the same as the first best allocation.

4 Linear-Quadratic Analysis

This section derives the appropriate stabilization objectives for monetary policy analysis in the model economy developed above and log-linearizes its equilibrium conditions. The resulting outcome is a Linear Quadratic (LQ) model which consists of a set of linear structural equations and a welfare-theoretic quadratic loss function. Rotemberg and Woodford (1997) and Woodford (2003) show that under certain conditions (efficiency of the steady state being one), a second order Taylor approximation to the expected present discounted value of utility of the representative household is related inversely to a conventional quadratic loss function. In a more general case of an inefficient steady state (owing to monopolistic competition or search externalities), the LQ method can preserve correct welfare rankings only if second order approximations are also taken to the model structural relations so that all linear terms in the welfare criterion are eliminated, see Benigno and Woodford (2005) for details.

In what follows, I assume the following functional form for preferences

$$u(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma},$$

where $\sigma > 0$ is the coefficient of relative risk aversion.

4.1 The Quadratic Approximation to Welfare

The first step in deriving a LQ model is to take a second-order Taylor expansion to the welfare of the representative household. Letting $\hat{X}_t = \log\left(\frac{X_t}{X}\right)$ denote the log deviation of any variable X_t around its flexible price steady-state value X and letting $\tilde{X}_t = \hat{X}_t - \hat{X}_t^e$ denote the gap between \hat{X}_t and its stochastic efficient (flexible price) equilibrium counterpart \hat{X}_t^e , the household's welfare criterion admits the following approximation

$$\mathbf{W} \simeq -E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\kappa V C^{\sigma-1}}{\xi} (\xi - \varepsilon) \hat{\theta}_t + q_\pi \pi_t^2 + q_u \hat{U}_{t+1}^2 + q_\theta \hat{\theta}_t^2 + q_{uz} \hat{U}_{t+1} \hat{Z}_t \right\}. \quad (15)$$

Derivation details as well as expressions for q_π , q_u , q_θ , and q_{uz} are given in Appendix A. When Hosios condition is not satisfied, $\xi \neq \varepsilon$, there is a non zero linear term in (15)

involving labor market tightness. Consequently, evaluating this function to second order using an approximate solution for the path of $\hat{\theta}_t$ that is accurate only to first order leads to incorrect welfare rankings. In other words, one cannot determine the optimal policy, even up to first order, using this welfare criterion together with log-linear approximations to the model's structural equations. [Ravenna and Walsh \(2010\)](#) and [Thomas \(2008\)](#) avoid this problem by assuming $\xi = \varepsilon$ which leads to an efficient steady state. An alternative way of dealing with this issue is to take second order approximations to the firm's optimal price setting relation (10) and job creation condition (13) and substitute the outcomes in (15) to eliminate the linear term involving $\hat{\theta}_t$. Pursuing the latter option yields

$$\mathbf{W} \simeq -E_t \sum_{t=1}^{\infty} \beta^t \left\{ \lambda_{\pi} \pi_t^2 + \lambda_u \left(\hat{U}_{t+1} - \hat{U}_{t+1}^* \right)^2 + \lambda_{\theta} \left(\hat{\theta}_t - \hat{\theta}_t^* \right)^2 \right\}, \quad (16)$$

where the coefficients in (16) depend on the underlying structural parameters of the model that govern preferences, the degree of nominal price rigidity, and the search and bargaining processes in the labor market, see Appendix A for details. \hat{U}_{t+1}^* and $\hat{\theta}_t^*$ are target levels of unemployment and labor market tightness respectively. They both depend on the evolution of exogenous productivity disturbances (are functions of \hat{U}_{t+1}^e and $\hat{\theta}_t^e$) but need not necessarily correspond to the flexible price equilibrium. Note that equation (16) can nest the welfare criterion obtained in [Ravenna and Walsh \(2010\)](#) if an efficient labor market allocation is assumed. In this case $\hat{U}_{t+1}^* = \hat{U}_{t+1}^e$ and $\hat{\theta}_t^* = \hat{\theta}_t^e$ and the second order approximation reads

$$\mathbf{W} \simeq -E_t \sum_{t=1}^{\infty} \beta^t \left(\lambda_{\pi} \pi_t^2 + \lambda_u \tilde{U}_{t+1}^2 + \lambda_{\theta} \tilde{\theta}_t^2 \right).$$

Since the obtained welfare criterion, (16), is purely quadratic in inflation, unemployment, and labor market tightness (i.e., lacking linear terms), it is possible to evaluate it to second order using only a set of first-order approximations to the model's structural relationships.

Equation (16) illustrates the central bank's policy objectives. Staggered price adjustment means that inflation volatility reduces welfare as it generates relative price dispersion and leads to an inefficient composition of retail goods for a given level of wholesale output. Welfare is also reduced by inefficient variation of (un)employment as in the standard new Keynesian model. Therefore, the second term in (16) measures the success of monetary policy in stabilizing the welfare relevant (un)employment gap. Labor market tightness gap arises because of the existence of search externalities and reduces utility of the representative household. These three gaps can be closed simultaneously only when the steady state is efficient, otherwise the policymaker has to trade off dealing with two separate goals: inefficient price dispersion, and socially suboptimal matches that result in a misallocation of workers between employment and unemployment.

4.2 Linear Structural Equations

The second step in deriving a LQ model is to take log-linear approximations to the model's structural equilibrium conditions. Appendix B shows how these equilibrium relationships can

be reduced to a system of three equations in labor market tightness gap, $\tilde{\theta}_t$, unemployment gap, \tilde{U}_t , and inflation, π_t .

$$\tilde{\theta}_t = -\frac{1}{\varepsilon p} \left(\tilde{U}_{t+1} - \rho_u \tilde{U}_t \right), \quad (17)$$

$$\tilde{U}_{t+1} = E_t \tilde{U}_{t+2} + \frac{C}{U} \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r_t^e), \quad (18)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_p \left\{ \frac{\kappa C^\sigma}{\xi q} (1 - \varepsilon) \left[\tilde{\theta}_t - \beta (1 - \lambda) \tilde{\theta}_{t+1} \right] - \sigma \frac{U}{C} \tilde{U}_{t+1} \right\} + \zeta_t, \quad (19)$$

in which $\kappa_p = \frac{(1-\delta\beta)(1-\delta)}{\delta}$ and r_t^e is the real interest rate in the stochastic efficient equilibrium.

Equation (18) is analogous to the conventional IS curve but expressed in terms of unemployment rather than output-gap; while equation (19) is the new Keynesian Phillips curve in the presence of search frictions. The term in the bracket is the real marginal cost which is a decreasing (increasing) function of current unemployment⁸ and expected future (current) labor market tightness. ζ_t is a composite *cost-push* term, indicating the degree to which the exogenous productivity shocks preclude simultaneous stabilization of inflation and welfare relevant unemployment gap. The cost push shock arises endogenously in my model as a result of congestion externalities and is given by

$$\zeta_t = \beta \kappa_p \frac{\kappa V C^{\sigma-1}}{\xi} \left(\frac{C}{U} \right) \left[(1 - \xi) \hat{\theta}_{t+1} - (1 - \varepsilon) \hat{\theta}_{t+1}^e \right].$$

When steady state of the model is efficient, $\xi = \varepsilon$, this cost push term is zero and there is no inflation/unemployment trade-off.

5 Optimal Policy from a "Timeless Perspective"

Equation (16) serves as an objective function for the central bank's policy problem. Optimal monetary policy is obtained by minimizing this equation subject to a sequence of log-linearized equilibrium constraints given by (17)-(19). The form of the optimization problem just stated (i.e. distorted steady state case) is the same as in a model with an efficient steady state; the only differences made by allowing $\xi \neq \varepsilon$ have to do with the expressions that I have derived for λ_π , λ_u and λ_θ as functions of underlying model parameters, the expression for ζ_t as a function of underlying technological disturbances, and the definition of the welfare relevant unemployment gap $\hat{U}_{t+1} - \hat{U}_{t+1}^*$ as well as labor market tightness gap $\hat{\theta}_t - \hat{\theta}_t^*$. See Appendix A for details.

5.1 Welfare Gaps

The model economy developed in this paper involves three distortions: 1) staggered price setting, 2) search externalities, and 3) monopolistic competition. We can offset the distortion

⁸Unemployment is a predetermined variable.

due to the market power of monopolistically competitive price-setters by assuming an appropriate output (sales revenue) subsidy and deal with the remaining two frictions. Let W^s denote the welfare of the representative household when prices are sticky and labor market is subject to search externalities, and let W^f denote welfare under flexible prices and search inefficiencies. In the absence of all three frictions, the first best level of welfare would be W . Therefore,

$$W - W^s = (W - W^f) + (W^f - W^s).$$

Using the same terminology as in [Ravenna and Walsh \(2008b\)](#), I define $W - W^f$ as the "*search gap*" and $W^f - W^s$ as the "*nominal rigidity gap*". The former gap, which arises exclusively as a result of search externalities, is the welfare distance between the first best equilibrium and the flexible price allocation. The latter gap, which is generated as a result of price dispersion, is the welfare difference between the flexible price equilibrium and the allocation under which prices are sticky. The Hosios parameter configuration eliminates the search gap ($W - W^f = 0$), while price stability ensures ($W^f - W^s = 0$). When $\xi = \varepsilon$, the policy maker is able to eliminate the only existing gap (nominal rigidity gap) through a zero inflation policy. When we depart from the Hosios condition by allowing $\xi \neq \varepsilon$, the search gap is not zero anymore. Accordingly, the policy maker should aim to minimize the sum of the two gaps. In this case a policy that eliminates the effects of imperfect competition and nominal rigidity does not necessarily implement the first best allocation. Reducing inefficient unemployment fluctuations (closing the search gap) therefore requires a policy that allows for transitory movements in inflation (deviations from price stability). In what follows I will examine the optimal monetary policy and the role of alternative assumptions about the efficiency of the steady state in more details.

5.2 The Case for Price Stability (Efficient Steady State)

The welfare of the representative household defined in [\(16\)](#) is clearly maximized by a policy under which inflation is zero at all times if two conditions are met: (i) the retail markup is one, $\frac{P_t^w}{P_t} = 1$, and ii) the firm's share of surplus, ξ , is equal to the vacancy elasticity of the matching function, ε . The former condition is imposed by an output subsidy, $s = 1/(1 - \gamma)$, that offsets the distortion resulting from the market power of monopolistically competitive retailers. The latter condition, [Hosios \(1990\)](#), implies an efficient level of vacancies and unemployment and renders a zero search gap. These conditions jointly ensure that steady state of the model is efficient and if prices were perfectly flexible, the equilibrium allocation of resources would be optimal. Even with the staggered price setting, a zero inflation policy leads to an equilibrium allocation of resources that is the same as if prices were flexible; hence the policy is optimal.

5.3 Inflation-Unemployment Trade-Off (Distorted Steady State)

In [5.2](#), I have described a special case in which complete price stability was optimal. In a more realistic case, when I allow for a distorted steady state (due to search externalities), the cost-push term, ζ_t , in the new Keynesian Phillips relationship will be non-zero. In general, congestion externalities affect the flexible-price equilibrium level of unemployment/labor market tightness while they are irrelevant for the efficient allocation of resources

and therefore they give rise to fluctuations in the cost-push term. When $\zeta_t \neq 0$, it is not possible simultaneously to fully stabilize both inflation and the welfare-relevant unemployment gap; the optimal trade-off between the two stabilization objectives generally involves some degree of variation in both variables in response to technological shocks. To better understand the nature of this trade-off, I perform a simulation exercise in the remaining parts of this section.

5.3.1 Baseline Calibration (Efficient Allocation)

The baseline monthly calibration of the model parameters to US data is summarized in Table 1. The discount factor, β , is set to 0.997, corresponding to an annual real interest rate of 4 percent in the steady state. Using thirty-three sets of estimates of wage and income elasticities, Chetty (2006) argues that the mean implied value of relative risk aversion in the utility function is 0.71, with a range of 0.15 to 1.78 in the additive utility case. Therefore I choose a coefficient of relative risk aversion, σ , of 0.71. The elasticity of substitution among differentiated goods, γ , is set to 7.67. This value translates into a 15% mark-up of prices over retailer’s marginal cost. Price adjustment probability, $1 - \delta$, is assumed to be 0.25, implying an average duration of price contracts of 4 months. I set the vacancy elasticity of matches, ε , to 0.6 following the US evidence in Blanchard and Diamond (1989). To focus on the implications of search externalities for the conduct of optimal monetary policy, the firm’s share of surplus, ξ , is allowed to vary in simulations. Following Gertler and Trigari (2009), I choose a monthly separation rate, λ , of 0.035 to match the evidence that jobs last for two and a half years. Based on the evidence presented in Shimer (2005), the value of job finding rate is set to $p = 0.3$, which implies a steady state unemployment rate of 0.1. The vacancy filling rate, q , is equal to 0.7, following the evidence presented by Haan, Ramey, and Watson (2000). The value for the utility cost of posting vacancies, κ , is obtained from the steady state relationships. Finally, the aggregate productivity shock, Z_t , follows an AR(1) process with a persistence of 0.8.

5.3.2 Steady State Analysis

Search and matching models exhibit congestion or search externalities due to the tightness of the labor market, the relative number of hiring firms to searching workers. One additional searching worker in the market increases the probability that a hiring firm will match with a job-seeker but decreases the probability that a searching worker already in the market will match with a firm. Hosios (1990) shows that congestion externalities are balanced, and labor market allocations are optimal when the bargaining power of workers equals the elasticity of the matching function with respect to vacancies. The Hosios parameter configuration, $\xi = \varepsilon$, ensures that the decentralized allocation is the same as the planner’s solution which internalizes the search externalities. However, the Hosios condition need not hold empirically. Table 2 presents the implied steady state values of the model’s key variables for two cases: (i) $\xi = \varepsilon$ and (ii) $\xi \neq \varepsilon$. To find the steady state values of unemployment and labor market tightness, I jointly solve the following two equations for U and θ using the parameter values

Table 1: Parameter Values

Discount factor	β	0.997
Coefficient of relative risk aversion	σ	0.71
Elasticity of substitution across goods	γ	7.67
Price adjustment probability	$1 - \delta$	0.25
Vacancy elasticity of matches	ε	0.6
Separation rate	λ	0.035
Job finding rate	p	0.3
Vacancy filling rate	q	0.7
Persistence of the productivity shock	ρ	0.8

given in Table 1.

$$1 - \beta(1 - \lambda - (1 - \xi)\psi\theta^\varepsilon) = \frac{\xi\psi\theta^{\varepsilon-1}}{\kappa(1 - U)^\sigma}, \quad (20)$$

$$U = \frac{\lambda}{\psi\theta^\varepsilon + \lambda}, \quad (21)$$

where (20) is the steady state evaluation of equation (13) augmented with $p = \psi\theta^\varepsilon$ and $q = \psi\theta^{\varepsilon-1}$, while (21) is derived from $\frac{p}{\lambda} = \frac{N}{U}$ and $N = 1 - U$, see Appendix C for details. Having found the steady state value of θ , I can solve for the job finding rate, p , and the vacancy filling probability, q . Steady state value of vacancies, V , is then calculated from the definition of labor market tightness.⁹ Second column of Table 2 shows that if workers bargaining power increases to $(1 - \xi) = 0.7 > \varepsilon$, the steady state unemployment rate will be inefficiently high and firms' incentives to post vacancies will be low. Consequently, the steady state level of the labor market tightness will be inefficiently low.

5.3.3 Impulse Response Analysis

This subsection analyzes the optimal responses of unemployment, labor market tightness and inflation to productivity disturbances under different assumptions about the efficiency of the labor market allocation. I express the inflation rate in annual percent deviation from the flexible price steady-state, while unemployment and labor market tightness are denoted

⁹Note that in case (i), I used the parameter values in Table 1 for unemployment, job finding probability and vacancy filling rate and calculated κ ; while in case (ii), I found the values of labor market tightness, unemployment, p and q directly from the steady state relationships.

Table 2: Steady State Properties, Efficient vs Inefficient Bargaining

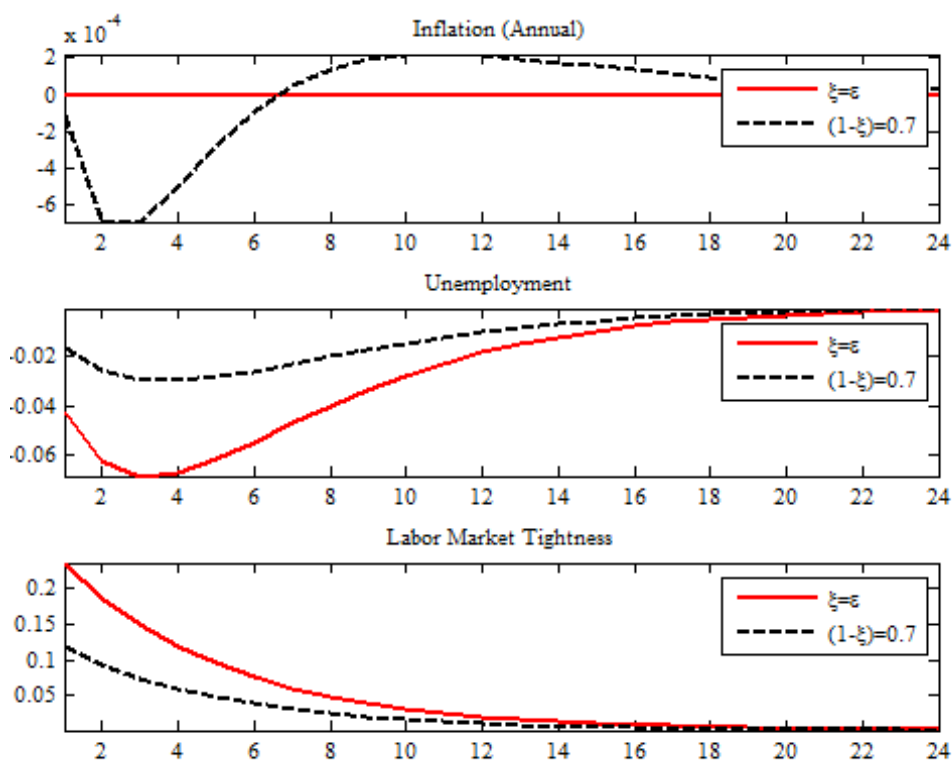
Variable	Efficient Allocation, $\xi = \varepsilon$	Inefficient Allocation, $(1 - \xi) > \varepsilon$
Unemployment U	0.10	0.12
Vacancies V	0.043	0.036
Tightness θ	0.43	0.29
Job finding rate p	0.30	0.24
Vacancy filling rate q	0.70	0.81

in monthly percentage deviations. Figure 1 plots the impulse responses of these variables to an unexpected rise of the productivity by one percent. To explore the explicit implications of search externalities (efficiency of the steady state) for the determination of optimal monetary policy, simulation results are produced for different values of the bargaining power. When the firm's share of surplus, ξ , is equal to the vacancy elasticity of the matching function, ε , the flexible price equilibrium is exactly the same as the social planner's allocation. Consequently, in response to productivity shocks, actual unemployment moves with the efficient (flexible price) unemployment and as such optimal policy calls for complete price stability. In this case, the search gap and cost push shocks are both zero and monetary policy is able to insulate inflation and unemployment gap from disturbances (solid lines in Figure 1).

Recall that the vacancy elasticity of the matching function, ε , is set to 0.6. For values of ξ different from ε , the flexible price steady state is different from social planner's equilibrium (the labor market allocation is inefficient), and the search gap is non-zero. Therefore, I expect the policy maker to be faced with a trade-off between moving the policy instrument to stabilize inflation or to correct for inefficient unemployment fluctuations in response to productivity shocks. This scenario is represented by dashed lines in Figure 1. A positive productivity shock raises the surplus of a match between a firm and a worker, leads firms to post more vacancies, pushes down the unemployment rate, and increases the labor market tightness. When the worker's bargaining power is inefficiently high $(1 - \xi) = 0.7$, the impact of the productivity shock on labor market tightness is smaller as firms take a lower share of the surplus and have less incentives to post vacancies. This renders the unemployment rate larger than the efficient one. The same shock also creates a gap between the flexible price equilibrium and the social planner's allocation, generates a cost push term, ζ_t , in the new Keynesian Phillips curve (owing to search externalities), and produces policy trade-offs.

The decision to post vacancies and to hire workers depends on expected labor market tightness for which future monetary policy position matters. Under commitment, the policy maker can credibly anchor expectations about future variables and as such it might deviate from complete price stability for some periods in order to steer firms' incentives to post

Figure 1: Impulse responses of selected variables under optimal policy to a one percent productivity shock for two cases: (i) efficient allocation and (ii) inefficient equilibrium.



vacancies towards the efficient level and speed up the process of convergence to the unemployment target, \hat{U}_{t+1}^* . In this case the monetary authority tries to correct for the inefficiently high unemployment through time-varying inflation, hence we observe larger deviations from price stability and lower unemployment fluctuations. Another reason for this observation is the higher weight the policy maker puts on unemployment gap stabilization, λ_u . As shown in Appendix A, λ_u depends positively on the steady state value of unemployment, U , and negatively on $\xi - \varepsilon$. With an inefficient equilibrium, ξ is smaller than ε and U is larger than its efficient counterpart; both contributing to a higher λ_u in the loss function.

6 Concluding remarks

I have derived an explicit second order approximation to the welfare of the representative agent when the flexible price equilibrium is different from the first best allocation, due to search externalities. I have shown that the resulting welfare-theoretic loss function depends on inflation, unemployment gap, and an additional quadratic term involving labor market tightness. These gaps could be interpreted as the percentage deviation of unemployment (and market tightness) from a target variable that depends on the evolution of exogenous shocks. In general, there is thus no reason for the target level of unemployment (and market tightness) to correspond to the flexible price allocation.

It is shown that productivity shocks may preclude simultaneous stabilization of inflation and the welfare-relevant unemployment gap; the extent to which this is true depends on the degree of variability of the cost push term in the new Keynesian Phillips curve, the relative weight on unemployment stabilization in the quadratic loss function, and the degree of the violation of the Hosios condition. The analysis of optimal monetary policy above assumes (i) perfect unemployment risk sharing among household members, (ii) full labor force participation, and (iii) only one source of disturbance (technological shock). How the policy implications may vary once we allow for imperfect risk sharing, variable labor market participation, and shocks other than technology are topics worthy of investigation.

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Appendix A: Second Order Expansions

Approximation to the Representative Household's Welfare

To drive a second order approximation to the representative household's welfare, it is necessary to introduce some additional notation. Let $\hat{X} = \log\left(\frac{X_t}{X}\right)$ be the log deviation of any variable X_t around its steady-state value X . Notice that we can write any function X_t^y as $e^{y \ln(X_t)}$ which can then be expanded in the logarithm of its arguments around the logarithm of their steady state levels such that the outcome is in log deviation terms. Employing this notation and assuming a Constant Relative Risk Aversion (CRRA) utility, the household's welfare can be approximated by

$$U(C_t, N_t, V_t) \simeq C^{1-\sigma} \left(\hat{C}_t + \frac{1-\sigma}{2} \hat{C}_t^2 \right) - \kappa V \left(\hat{V}_t + \frac{1}{2} \hat{V}_t^2 \right) + t.i.p. + O^3, \quad (22)$$

where O^k indicates terms of order k -th and higher in the size of the shocks and *t.i.p.* represents terms independent of policy. In order to substitute for \hat{C}_t and \hat{V}_t in (22), I perform the following second order expansions for *market clearing condition*, $C_t \Delta_t = Z_t N_t$, and *vacancies*, $V_t = \theta_t U_t$, or

$$\hat{C}_t = -\hat{\Delta}_t + \hat{N}_t + \frac{1}{2} \hat{N}_t^2 + \hat{Z}_t \hat{N}_t + \hat{Z}_t + \frac{1}{2} \hat{Z}_t^2 - \frac{1}{2} \hat{C}_t^2, \quad (23)$$

$$\hat{V}_t + \frac{1}{2} \hat{V}_t^2 = \hat{\theta}_t + \hat{U}_t + \frac{1}{2} \left(\hat{\theta}_t + \hat{U}_t \right)^2. \quad (24)$$

I can express (23) in terms of \hat{U}_t but it requires performing the following approximation to $U_t = 1 - N_{t-1}$, or

$$\hat{N}_{t-1} + \frac{1}{2} \hat{N}_{t-1}^2 = -\frac{U}{C} \left(\hat{U}_t + \frac{1}{2} \hat{U}_t^2 \right) = -\frac{\lambda}{p} \left(\hat{U}_t + \frac{1}{2} \hat{U}_t^2 \right), \quad (25)$$

in which I have used the steady state relationship $\frac{p}{\lambda} = \frac{N}{U}$. Inserting (25) into (23) and using $N = 1 - U = C$, I obtain the following approximation for the market clearing condition

$$\hat{C}_t = -\hat{\Delta}_t - \frac{U}{C} \left(\hat{U}_{t+1} + \frac{1}{2} \hat{U}_{t+1}^2 \right) + \hat{Z}_t + \frac{1}{2} \hat{Z}_t^2 - \frac{1}{2} \left(\frac{U}{C} \right)^2 \hat{U}_{t+1}^2, \quad (26)$$

from which I can find an expression for \hat{C}_t^2 :

$$\hat{C}_t^2 = \left(\frac{U}{C} \right)^2 \hat{U}_{t+1}^2 - 2 \frac{U}{C} \hat{U}_{t+1} \hat{Z}_t.$$

The next step is to approximate *the Beveridge curve*. The second order expansion for the law of motion of employment, $N_t = (1 - \lambda)N_{t-1} + \psi \theta_t^\varepsilon (1 - N_{t-1})$, reads

$$\hat{N}_t + \frac{1}{2} \hat{N}_t^2 = (1 - \lambda - p) \left(\hat{N}_{t-1} + \frac{1}{2} \hat{N}_{t-1}^2 \right) + \lambda \left(\varepsilon \hat{\theta}_t + \frac{1}{2} \varepsilon^2 \hat{\theta}_t^2 \right) - p \varepsilon \hat{N}_{t-1} \hat{\theta}_t. \quad (27)$$

Combining the above equation with (25) yields

$$\hat{U}_{t+1} + \frac{1}{2}\hat{U}_{t+1}^2 = \rho_u \left(\hat{U}_t + \frac{1}{2}\hat{U}_t^2 \right) - \varepsilon p \left(\hat{\theta}_t + \frac{1}{2}\varepsilon\hat{\theta}_t^2 + \hat{U}_t\hat{\theta}_t \right), \quad (28)$$

where $\rho_u = 1 - \lambda - p$. Multiplying both sides of (28) by β^t and integrating across t , admits the following equation for the present discounted value of *unemployment*

$$\sum_{t=0}^{\infty} \beta^t \hat{U}_{t+1} = \sum_{t=0}^{\infty} \beta^t \left\{ -\frac{\varepsilon\beta\kappa VC^\sigma}{\xi(\beta\kappa VC^\sigma + U)} \left(\hat{\theta}_t + \frac{1}{2}\varepsilon\hat{\theta}_t^2 + \frac{\varepsilon p}{\rho_u}\hat{\theta}_t^2 + \frac{1}{\rho_u}\hat{U}_{t+1}\hat{\theta}_t \right) - \frac{1}{2}\hat{U}_{t+1}^2 \right\}, \quad (29)$$

in which I have used the steady state relationship $\frac{\varepsilon p}{1-\beta(1-\lambda-p)} = \frac{\varepsilon\beta\kappa VC^\sigma}{\xi(\beta\kappa VC^\sigma + U)}$ and $\hat{\theta}_t\hat{U}_t = \frac{\varepsilon p}{\rho_u}\hat{\theta}_t^2 + \frac{1}{\rho_u}\hat{U}_{t+1}\hat{\theta}_t$. Inserting (26) and (24) into (22) and integrating forward, I can write the expected present discounted value of *Household's Utility* as

$$\begin{aligned} \mathbf{W} \simeq & E_t \sum_{t=0}^{\infty} \beta^t \left\{ -\kappa VC^{\sigma-1}\hat{\theta}_t - \frac{\beta\kappa VC^\sigma + U}{C}\hat{U}_{t+1} \right\} + \\ & + E_t \sum_{t=0}^{\infty} \beta^t \left\{ \begin{aligned} & -\hat{\Delta}_t - \frac{1}{2} \left\{ \sigma \left(\frac{U}{C} \right)^2 + \frac{\beta\kappa VC^\sigma + U}{C} \right\} \hat{U}_{t+1}^2 - \kappa VC^{\sigma-1} \left(\frac{1}{2} + \frac{\varepsilon p}{1-\lambda-p} \right) \hat{\theta}_t^2 \\ & -\kappa VC^{\sigma-1} \frac{1}{\rho_u} \hat{U}_{t+1} \hat{\theta}_t - (1-\sigma) \frac{U}{C} \hat{U}_{t+1} \hat{Z}_t \end{aligned} \right\}. \end{aligned} \quad (30)$$

Substituting (29) into (30) eliminates the linear term involving \hat{U}_t . Therefore

$$\begin{aligned} \mathbf{W} \simeq & -\frac{\kappa VC^{\sigma-1}}{\xi} (\xi - \varepsilon) E_t \sum_{t=0}^{\infty} \beta^t \hat{\theta}_t + \\ & + E_t \sum_{t=0}^{\infty} \beta^t \left\{ \begin{aligned} & -\frac{\gamma}{2\kappa_p} \pi_t^2 - \frac{1}{2} \sigma \left(\frac{U}{C} \right)^2 \hat{U}_{t+1}^2 - \frac{1}{2} \frac{\kappa VC^{\sigma-1}}{\xi} \left(\xi - \varepsilon^2 + 2(\xi - \varepsilon) \frac{\varepsilon p}{\rho_u} \right) \hat{\theta}_t^2 \\ & -\frac{\kappa VC^{\sigma-1}}{\xi} \frac{\xi - \varepsilon}{\rho_u} \hat{U}_{t+1} \hat{\theta}_t - (1-\sigma) \frac{U}{C} \hat{U}_{t+1} \hat{Z}_t \end{aligned} \right\}, \end{aligned} \quad (31)$$

in which I have used the approximation of *price dispersion term*, $\sum_{t=0}^{\infty} \beta^t \hat{\Delta}_t = \frac{\gamma}{2\kappa_p} \sum_{t=0}^{\infty} \beta^t \pi_t^2$.

The unemployment transition equation,

$$\hat{U}_{t+1} = \rho_u \hat{U}_t - \varepsilon p \hat{\theta}_t, \quad (32)$$

implies the recursive expression

$$\hat{U}_{t+1}^2 = \rho_u^2 \hat{U}_t^2 + (\varepsilon p)^2 \hat{\theta}_t^2 - 2\varepsilon p \rho_u \hat{\theta}_t \hat{U}_t,$$

which can then be integrated forward to obtain

$$\sum_{t=1}^{\infty} \beta^t \hat{U}_{t+1} \hat{\theta}_t = -\frac{1}{2} \sum_{t=1}^{\infty} \beta^t \left\{ \varepsilon p \hat{\theta}_t^2 + \frac{1 - \beta \rho_u^2}{\varepsilon p} \hat{U}_{t+1}^2 \right\}. \quad (33)$$

Using (33), I can write equation (31) as

$$\mathbf{W} \simeq -E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\kappa V C^{\sigma-1}}{\xi} (\xi - \varepsilon) \hat{\theta}_t + q_\pi \pi_t^2 + q_u \hat{U}_{t+1}^2 + q_\theta \hat{\theta}_t^2 + q_{uz} \hat{U}_{t+1} \hat{Z}_t \right\}, \quad (34)$$

where the coefficients in equation (34) are defined as

$$\begin{aligned} q_\pi &= \frac{\gamma}{2\kappa_p}, \\ q_u &= \frac{1}{2} \left\{ \sigma \left(\frac{U}{C} \right)^2 - \frac{\kappa V C^{\sigma-1}}{\xi} (\xi - \varepsilon) \frac{1 - \beta \rho_u^2}{\varepsilon p \rho_u} \right\}, \\ \lambda_\theta &= \frac{1}{2} \frac{\kappa V C^{\sigma-1}}{\xi} \left(\xi - \varepsilon^2 + (\xi - \varepsilon) \frac{\varepsilon p}{\rho_u} \right), \\ \lambda_{uz} &= (1 - \sigma) \frac{U}{C}. \end{aligned}$$

Approximations to the Decentralized Equilibrium Conditions

The linear term involving $\hat{\theta}_t$ in equation (34) can be eliminated using second order approximations to the firm's optimal price setting relation (10) and job creation condition (13).

Optimal Price Setting Condition

Dividing both sides of (11) by $P_t^{1-\gamma}$ and substituting an expression for $\frac{P_t^*}{P_t}$ using (10) yields

$$\frac{1 - \delta \Pi_t^{\gamma-1}}{1 - \delta} = \left(\frac{F_t}{K_t} \right)^{\gamma-1},$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$. This equation can be written exactly as

$$\log \left(1 - \frac{\delta \Pi_t^{\gamma-1}}{1 - \delta} \right) = (\gamma - 1) (\log K_t - \log F_t). \quad (35)$$

A second order approximation to the left hand side of (35) takes the form

$$\log \left(1 - \frac{\delta \Pi_t^{\gamma-1}}{1 - \delta} \right) = \frac{\delta}{1 - \delta} (\gamma - 1) \left\{ \pi_t + \frac{1}{2} \frac{\gamma - 1}{1 - \delta} \pi_t^2 + O^3 \right\}. \quad (36)$$

Substituting (36) into (35) yields

$$\pi_t - \frac{1}{2} \frac{\gamma - 1}{1 - \delta} \pi_t^2 + O^3 = \frac{1 - \delta}{\delta} (\hat{K}_t - \hat{F}_t). \quad (37)$$

K_t and F_t are defined as

$$\begin{aligned} K_t &= E_t \sum_{i=0}^{\infty} (\delta \beta)^i k_{t+i}, \quad k_{t+i} = C_{t+i}^{1-\sigma} m c_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^\gamma, \\ F_t &= E_t \sum_{i=0}^{\infty} (\delta \beta)^i f_{t+i}, \quad f_{t+i} = C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}}{P_t} \right)^{\gamma-1}. \end{aligned}$$

These definitions imply second order expansions

$$\hat{K}_t + \frac{1}{2}\hat{K}_t^2 + O^3 = (1 - \delta\beta) E_t \sum_{i=0}^{\infty} (\delta\beta)^i \left(\hat{k}_{t+i} + \frac{1}{2}\hat{k}_{t+i}^2 \right) + O^3, \quad (38)$$

$$\hat{F}_t + \frac{1}{2}\hat{F}_t^2 + O^3 = (1 - \delta\beta) E_t \sum_{i=0}^{\infty} (\delta\beta)^i \left(\hat{f}_{t+i} + \frac{1}{2}\hat{f}_{t+i}^2 \right) + O^3, \quad (39)$$

where \hat{k}_{t+i} and \hat{f}_{t+i} are given by

$$\hat{k}_{t+i} = (1 - \sigma) \hat{C}_{t+i} + \widehat{m}c_{t+i} + \gamma \left(\hat{P}_{t+i} - \hat{P}_t \right) = (1 - \sigma) \hat{C}_{t+i} + \widehat{m}c_{t+i} + \gamma \sum_{j=t+1}^{t+i} \pi_j,$$

$$\hat{f}_{t+i} = (1 - \sigma) \hat{C}_{t+i} + (\gamma - 1) \left(\hat{P}_{t+i} - \hat{P}_t \right) = (1 - \sigma) \hat{C}_{t+i} + (\gamma - 1) \sum_{j=t+1}^{t+i} \pi_j.$$

Equations (38) and (39) can be used to obtain a second order expansion for the right hand side of (37) as

$$\begin{aligned} \hat{K}_t - \hat{F}_t &= (1 - \delta\beta) E_t \sum_{i=0}^{\infty} (\delta\beta)^i \left[\hat{k}_{t+i} - \hat{f}_{t+i} + \frac{1}{2} \left(\hat{k}_{t+i}^2 - \hat{f}_{t+i}^2 \right) \right] - \frac{1}{2} \left(\hat{K}_t - \hat{F}_t \right) \left(\hat{K}_t + \hat{F}_t \right) \\ &= (1 - \delta\beta) \left\{ E_t \sum_{i=0}^{\infty} (\delta\beta)^i \left[\hat{k}_{t+i} - \hat{f}_{t+i} + \frac{1}{2} \left(\hat{k}_{t+i}^2 - \hat{f}_{t+i}^2 \right) \right] - \frac{1}{2} \frac{\delta}{1 - \delta} \pi_t Z_t \right\}, \quad (40) \end{aligned}$$

where I have used (37) to substitute for $\left(\hat{K}_t - \hat{F}_t \right)$ and

$$\begin{aligned} \hat{K}_t + \hat{F}_t &= (1 - \delta\beta) A_t \\ A_t &= E_t \sum_{i=0}^{\infty} (\delta\beta)^i \left(\hat{k}_{t+i} + \hat{f}_{t+i} \right). \end{aligned}$$

I can use the definitions of \hat{k}_{t+i} and \hat{f}_{t+i} as well as (37) to further simplify (40). The resulting equation can be written recursively as

$$\begin{aligned} &\pi_t + \frac{1}{2} \frac{\gamma - 1}{1 - \delta} \pi_t^2 + \frac{1}{2} (1 - \delta\beta) \pi_t A_t \\ &= \frac{(1 - \delta\beta)(1 - \delta)}{\delta} \left[\hat{k}_t - \hat{f}_t + \frac{1}{2} \left(\hat{k}_t^2 - \hat{f}_t^2 \right) \right] + \\ &+ \frac{1}{2} \gamma \beta E_t \pi_{t+1}^2 + \beta E_t \pi_{t+1} + \frac{1}{2} \beta \frac{\gamma - 1}{1 - \delta} E_t \pi_{t+1}^2 + \frac{1}{2} \beta (1 - \delta\beta) E_t \pi_{t+1} A_{t+1}. \end{aligned}$$

The above equation when integrated forward yields

$$V_{t0} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \kappa_p \left(\widehat{m}c_t + \frac{1}{2} \widehat{m}c_t^2 + (1 - \sigma) \hat{C}_t \widehat{m}c_t \right) + \frac{\gamma}{2} \pi_t^2 \right\},$$

where $\kappa_p = \frac{(1 - \delta\beta)(1 - \delta)}{\delta}$.

Job Creation Condition

Assuming a CRRA utility specification and using the relationships $mc_t = \frac{P_t^W}{P_t}$, $q(\theta_t) = \psi\theta_t^{\varepsilon-1}$, $p(\theta_t) = \psi\theta_t^\varepsilon$, $C_t = \Delta_t^{-1}Z_tN_t$, and $U_t = 1 - N_{t-1}$, I can write (13) as

$$\frac{\kappa}{\psi}\theta_t^{1-\varepsilon} = \xi mc_t \Delta_t^\sigma N_t^{-\sigma} Z_t^{1-\sigma} + \beta E_t [1 - \lambda - (1 - \xi)\psi\theta_{t+1}^\varepsilon] \frac{\kappa}{\psi}\theta_{t+1}^{1-\varepsilon}.$$

The second order expansion of the above equation reads

$$\begin{aligned} & \frac{\kappa}{q} \left[(1 - \varepsilon)\hat{\theta}_t + \frac{1}{2}(1 - \varepsilon)^2\hat{\theta}_t^2 \right] \\ &= \xi C^{-\sigma} \left\{ \begin{aligned} & \widehat{mc}_t + \frac{1}{2}\widehat{mc}_t^2 + \sigma\hat{\Delta}_t + \sigma\frac{U}{C}\hat{U}_{t+1} + \frac{1}{2}\sigma\left(\frac{C}{U} + 1 + \sigma\right)\left(\frac{U}{C}\right)^2\hat{U}_{t+1}^2 \\ & + \sigma\frac{U}{C}\hat{U}_{t+1}\widehat{mc}_t + (1 - \sigma)\sigma\frac{U}{C}\hat{U}_{t+1}\hat{Z}_t + (1 - \sigma)\hat{Z}_t\widehat{mc}_t + (1 - \sigma)\hat{Z}_t \end{aligned} \right\} \\ & + \beta\frac{\kappa}{q} \left\{ (1 - \lambda) \left[(1 - \varepsilon)\hat{\theta}_{t+1} + \frac{1}{2}(1 - \varepsilon)^2\hat{\theta}_{t+1}^2 \right] - p(1 - \xi) \left(\hat{\theta}_{t+1} + \frac{1}{2}\hat{\theta}_{t+1}^2 \right) \right\}. \end{aligned} \quad (41)$$

Multiplications of the first order approximation to the real marginal cost, \widehat{mc}_t , and (32) imply

$$\hat{U}_{t+1}\widehat{mc}_t = \frac{\kappa VC^\sigma}{\xi pU} \left\{ (1 - \varepsilon)\hat{U}_{t+1}\hat{\theta}_t - \beta\rho_{mc}\hat{U}_{t+1}\hat{\theta}_{t+1} \right\} - \sigma\frac{U}{C}\hat{U}_{t+1}^2 - (1 - \sigma)\hat{U}_{t+1}\hat{Z}_t,$$

where $\rho_{mc} = (1 - \lambda)(1 - \varepsilon) - p(1 - \xi)$. I can substitute this expression into (41) for the cross-product term involving $\hat{U}_{t+1}\widehat{mc}_t$ and integrate the result forward to obtain

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t \left[\widehat{mc}_t + \frac{1}{2}\widehat{mc}_t^2 + \sigma\hat{\Delta}_t + \sigma\frac{U}{C}\hat{U}_{t+1} + \frac{1}{2}\sigma\left(\frac{1}{U} - \sigma\right)\left(\frac{U}{C}\right)^2\hat{U}_{t+1}^2 \right] \\ &= \sum_{t=0}^{\infty} \beta^t \left\{ \begin{aligned} & \frac{\kappa VC^{\sigma-1}}{\xi} \left(\xi - \varepsilon + \frac{1-\xi}{U} \right) \hat{\theta}_t \\ & + \left\{ \frac{1}{2}\frac{\kappa VC^{\sigma-1}}{\xi} \left(\varepsilon^2 - 2\varepsilon + \xi + \frac{1-\xi}{U} \right) + \varepsilon\sigma\frac{\kappa VC^{\sigma-1}}{\xi}\frac{\rho_{mc}}{\rho_u} \right\} \hat{\theta}_t^2 \\ & + \sigma\frac{\kappa VC^{\sigma-1}}{\xi}\frac{\xi-\varepsilon}{\rho_u}\hat{U}_{t+1}\hat{\theta}_t - (1 - \sigma)\hat{Z}_t\widehat{mc}_t \end{aligned} \right\}, \end{aligned} \quad (42)$$

where I have used the steady state relations $\frac{\kappa}{\psi}\theta^{1-\varepsilon} = \frac{\kappa}{q}$ and $\psi\theta^\varepsilon = p$ and the fact that $\hat{\theta}_{t-1}$ and $\hat{\theta}_{t-1}\hat{U}_{t-1}$ are independent of policy as of date zero. The second order approximation to the firm's optimal price setting condition can be substituted into (42) for $\widehat{mc}_t + \frac{1}{2}\widehat{mc}_t^2$ to get

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t \left\{ -\sigma\frac{U}{C}\hat{U}_{t+1} + \frac{\kappa VC^{\sigma-1}}{\xi} \left(\xi - \varepsilon + \frac{1 - \xi}{U} \right) \hat{\theta}_t \right\} \\ &= \sum_{t=0}^{\infty} \beta^t \left[\begin{aligned} & -\frac{\gamma}{2\kappa p}\pi_t^2 + \sigma\hat{\Delta}_t + \frac{1}{2}\sigma\left(\sigma - 2 + \frac{1}{U}\right)\left(\frac{U}{C}\right)^2\hat{U}_{t+1}^2 \\ & \left\{ -\frac{1}{2}\frac{\kappa VC^{\sigma-1}}{\xi} \left(\varepsilon^2 - 2\varepsilon + \xi + \frac{1-\xi}{U} \right) - \varepsilon\frac{\kappa VC^{\sigma-1}}{\xi}\frac{\rho_{mc}}{\rho_u} \right\} \hat{\theta}_t^2 \\ & -\frac{\kappa VC^{\sigma-1}}{\xi}\frac{\xi-\varepsilon}{\rho_u}\hat{U}_{t+1}\hat{\theta}_t - (1 - \sigma)^2\frac{U}{C}\hat{U}_{t+1}\hat{Z}_t \end{aligned} \right], \end{aligned}$$

in which I have used

$$\hat{C}_t \widehat{m}c_t = \sigma \left(\frac{U}{C} \right)^2 \hat{U}_{t+1} + \frac{\kappa V C^{\sigma-1} \varepsilon \rho_{mc}}{\xi \rho_u} \hat{\theta}_t^2 + \frac{\kappa V C^{\sigma-1} (\xi - \varepsilon)}{\xi \rho_u} \hat{U}_{t+1} \hat{\theta}_t + \frac{U}{C} (1 - \sigma) \hat{U}_{t+1} \hat{Z}_t + \hat{Z}_t \widehat{m}c_t.$$

I can now substitute in for the linear term involving unemployment (29) and simplify the result by using (33) to obtain

$$\begin{aligned} & -\frac{\kappa V C^{\sigma-1}}{\xi} \left(\vartheta + \xi - \varepsilon + \frac{1 - \xi}{U} \right) \sum_{t=1}^{\infty} \beta^t \hat{\theta}_t \\ & = \sum_{t=1}^{\infty} \beta^t \left\{ \begin{aligned} & + (1 - \sigma) \frac{\gamma}{2\kappa_p} \pi_t^2 + \left(\frac{1}{2} \sigma (1 - \sigma) \left(\frac{U}{C} \right)^2 - \frac{1}{2} \frac{\kappa V C^{\sigma-1}}{\xi} (\xi - \varepsilon + \vartheta) \frac{1 - \beta \rho_u^2}{\varepsilon p \rho_u} \right) \hat{U}_{t+1}^2 \\ & + \left(\frac{1}{2} \frac{\kappa V C^{\sigma-1}}{\xi} (\vartheta \varepsilon + \varepsilon^2 - 2\varepsilon + \xi + \frac{1 - \xi}{U}) + \frac{\varepsilon \kappa V C^{\sigma-1}}{\xi \rho_u} \left(\frac{1}{2} \vartheta p + \rho_{mc} - \frac{1}{2} p (\xi - \varepsilon) \right) \right) \hat{\theta}_t^2 + (1 - \sigma)^2 \frac{U}{C} \hat{U}_{t+1} \hat{Z}_t \end{aligned} \right\}, \end{aligned} \quad (43)$$

where $\vartheta = \frac{\sigma \varepsilon U}{\beta \kappa V C^{\sigma+U}}$.

Evaluating the CB's Loss Function

I can now multiply both sides of (43) by $\varpi = \frac{\xi - \varepsilon}{\vartheta + \xi - \varepsilon + \frac{1 - \xi}{U}}$ and insert the resulting expression in (31) to eliminate all remaining linear terms in the second order approximations.

$$\mathbf{W} \simeq -E_t \sum_{t=1}^{\infty} \beta^t \left(\lambda_{\pi} \pi_t^2 + \lambda_u \hat{U}_{t+1}^2 + \lambda_{\theta} \hat{\theta}_t^2 + \lambda_{uz} \hat{U}_{t+1} \hat{Z}_t \right). \quad (44)$$

The coefficients in equation (44) are defined below

$$\lambda_{\pi} = \frac{\gamma}{2\kappa_p} (1 - (1 - \sigma) \varpi),$$

$$\lambda_u = \frac{1}{2} \left\{ \sigma \left(\frac{U}{C} \right)^2 (1 - (1 - \sigma) \varpi) - \frac{\kappa V C^{\sigma-1}}{\xi} \frac{(\xi - \varepsilon)(1 - \xi)}{U (\xi - \varepsilon + \vartheta + \frac{1 - \xi}{U})} \frac{1 - \beta \rho_u^2}{\varepsilon p \rho_u} \right\},$$

$$\lambda_{\theta} = \frac{1}{2} \frac{\varepsilon \kappa V C^{\sigma-1}}{\xi} \left\{ (1 - \varepsilon) \left(\frac{2(\xi - \varepsilon) + \vartheta + \frac{1 - \xi}{U}}{\xi - \varepsilon + \vartheta + \frac{1 - \xi}{U}} \right) + \frac{\xi - \varepsilon}{\rho_u} \left(\frac{p \frac{1 - \xi}{U} - 2\rho_u (1 - \varepsilon)}{\xi - \varepsilon + \vartheta + \frac{1 - \xi}{U}} \right) \right\},$$

$$\lambda_{uz} = (1 - \sigma) \frac{U}{C} (1 - (1 - \sigma) \varpi).$$

Letting $\tilde{X}_t = \hat{X}_t - \hat{X}_t^e$ denote the gap between \hat{X}_t and its stochastic efficient (flexible price) equilibrium counterpart \hat{X}_t^e , I can proceed to obtain a version of the loss function that

consists of a set of appropriately defined gaps involving inflation, unemployment, and labor market tightness. To deal with the only cross product term in (44), I use

$$\hat{Z}_t = \frac{\kappa C^\sigma}{\xi q} \frac{1-\varepsilon}{1-\sigma} \hat{\theta}_t^e - \beta \frac{\kappa C^\sigma}{\xi q} \rho_u \frac{1-\varepsilon}{1-\sigma} \hat{\theta}_{t+1}^e - \frac{\sigma}{1-\sigma} \frac{U}{C} \hat{U}_{t+1}^e, \quad (45)$$

and multiply it by \hat{U}_{t+1} and integrate it forward to obtain

$$\sum_{t=1}^{\infty} \beta^t \hat{U}_{t+1} \hat{Z}_t = - \sum_{t=1}^{\infty} \beta^t \left(\frac{\varepsilon \kappa V C^\sigma}{\xi U} \frac{1-\varepsilon}{1-\sigma} \hat{\theta}_t \hat{\theta}_t^e + \frac{\sigma}{1-\sigma} \frac{U}{C} \hat{U}_{t+1} \hat{U}_{t+1}^e \right).$$

Thus, we have that

$$\mathbf{W} \simeq -E_t \sum_{t=1}^{\infty} \beta^t \left\{ \lambda_\pi \pi_t^2 + \lambda_u \left(\hat{U}_{t+1} - \hat{U}_{t+1}^* \right)^2 + \lambda_\theta \left(\hat{\theta}_t - \hat{\theta}_t^* \right)^2 \right\}, \quad (46)$$

where $\hat{U}_{t+1}^* = \frac{1}{2\lambda_u} \sigma \left(\frac{U}{C} \right)^2 (1 - (1 - \sigma) \varpi) \hat{U}_{t+1}^e$ and $\hat{\theta}_t^* = \frac{1}{2\lambda_\theta} \frac{\varepsilon \kappa V C^{\sigma-1}}{\xi} (1 - \varepsilon) (1 - (1 - \sigma) \varpi) \hat{\theta}_t^e$.

Appendix B: The Linearized Equilibrium Conditions

The model economy involves the following core log-linearized equations.

Consumption Euler equation

$$\hat{C}_t = E_t \hat{C}_{t+1} - \left(\frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1}).$$

Aggregate resource constraint and production function

$$\hat{C}_t = \hat{Y}_t = \hat{Z}_t + \hat{N}_t.$$

Unemployment

$$\hat{N}_{t-1} = -\frac{U}{C} \hat{U}_t = -\frac{\lambda}{p} \hat{U}_t.$$

Evolution of employment

$$\hat{N}_t = \rho_u \hat{N}_{t-1} + \lambda \varepsilon \hat{\theta}_t.$$

Job market clearing condition or decentralized equilibrium

$$\widehat{m}c_t = \frac{\kappa C^\sigma}{\xi q} \left\{ (1 - \varepsilon) \hat{\theta}_t - \beta \rho_{mc} \hat{\theta}_{t+1} \right\} - \sigma \frac{U}{C} \hat{U}_{t+1} - (1 - \sigma) \hat{Z}_t.$$

Price adjustment equation

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_p \widehat{m}c_t.$$

I can proceed to obtain a version of the model that consists of three structural equations in $\hat{\theta}_t$, \hat{U}_t , and π_t as follows.

$$\begin{aligned}\hat{\theta}_t &= -\frac{1}{\varepsilon p} \left(\hat{U}_{t+1} - \rho_u \hat{U}_t \right), \\ \hat{U}_{t+1} &= E_t \hat{U}_{t+2} + \frac{C}{U} \frac{1}{\sigma} (i_t - E_t \pi_{t+1}) + \frac{C}{U} (1 - \rho) \hat{Z}_t, \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa_p \left\{ \frac{\kappa C^\sigma}{\xi q} \left[(1 - \varepsilon) \hat{\theta}_t - \beta \rho_{mc} \hat{\theta}_{t+1} \right] - \sigma \frac{U}{C} \hat{U}_{t+1} - (1 - \sigma) \hat{Z}_t \right\}\end{aligned}$$

Using (45), I can re-write these equations in gap terms - variables expressed relative to their stochastic efficient equilibrium counterparts.

$$\begin{aligned}\tilde{\theta}_t &= -\frac{1}{\varepsilon p} \left(\tilde{U}_{t+1} - \rho_u \tilde{U}_t \right), \\ \tilde{U}_{t+1} &= E_t \tilde{U}_{t+2} + \frac{C}{U} \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r_t^e), \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa_p \left\{ \frac{\kappa C^\sigma}{\xi q} (1 - \varepsilon) \left[\tilde{\theta}_t - \beta (1 - \lambda) \tilde{\theta}_{t+1} \right] - \sigma \frac{U}{C} \tilde{U}_{t+1} \right\} + \zeta_t,\end{aligned}$$

where $\zeta_t = \beta \kappa_p \frac{\kappa V C^{\sigma-1}}{\xi} \left(\frac{C}{U} \right) \left[(1 - \xi) \hat{\theta}_{t+1} - (1 - \varepsilon) \hat{\theta}_{t+1}^e \right]$.

Appendix C: Useful Steady State Relationships

$$\begin{aligned}\frac{1}{\beta} - (1 - \lambda - p) &= \xi \left(p + \frac{q}{\beta \kappa C^\sigma} \right) = \frac{\xi p (\beta \kappa V C^\sigma + U)}{\beta \kappa V C^\sigma} \\ \frac{p}{\lambda} &= \frac{N}{U} \\ \theta &= \frac{p}{q} = \frac{V}{U} \\ C &= N = 1 - U \\ U &= \frac{\lambda}{p + \lambda}\end{aligned}$$