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Abstract

I study the long run determinacy tradeoff – recurrent episodes of passive monetary policy are (in)*determinate* if their expected duration is long (*brief*) – when passive policy is at the zero bound. On-going regime change implies qualitatively different shock transmission from the standard New Keynesian model. For U.S. baseline parameter values, I find temporary fiscal stimulus is effective, while adverse supply shocks can be expansionary if the central bank's active policy stance is weak and/or if the liquidity trap's average duration exceeds 3 quarters.

Keywords: Zero bound; Monetary policy; Regime-switching; Determinacy.
JEL classification codes: E31, E52, E58, E61.

*This paper is dedicated to Anton Bruckner, master of musical regime-switching. I thank the editor and an anonymous referee for their valuable comments. The usual disclaimer applies.

1 Introduction

How do expectations of recurring zero interest rates affect the transmission of fundamental shocks? In an important contribution, Davig and Leeper (2007) generalized the Taylor (1993) principle – the proposition that central banks stabilize the economy by raising their policy rate more than one-for-one in response to higher inflation – from a fixed-regime to a regime-switching New Keynesian model. Monetary policy's reaction to fluctuations can then switch from an active regime, characterized by an aggressive inflation response, to a passive regime with less aggressive response following a Markov chain with exogenous transition probabilities.

A key insight of these authors is that determinate (i.e., unique and bounded) solution paths depend on *all* model parameters. Of these, in this paper I focus on the passive regime's expected duration when conventional monetary policy is *completely* passive and short-term nominal interest rates are at their zero bound (ZB). This matters for three reasons. *First*, the impact elasticities of output to fundamental shocks tend to be very different when conventional policy is switched off. For example, the New Keynesian model predicts that temporary adverse supply shocks are expansionary at the ZB, while the multiplier effect of government spending shocks is sharply higher.[1] *Second*, equilibrium indeterminacy leads to non-unique propagation of fundamental shocks and excess volatility caused by non-fundamentals (sunspots), and should thus be avoided; see Benhabib et al. (2002) and Lubik and Schorfheide (2004). And, *third*, the profession is gradually realizing that ZB episodes are not one-off but may recur. The U.S. 3-month Treasury Bill data shown in Figure 1 indicates the ZB constraint has been binding about 6.6 percent over the period 1934:01-2015:05:[2]

FIGURE 1 HERE

To be sure, one may question inference based on 80 years of data considering the profound structural changes in the economy, as well as the conduct of monetary policy in the last

[1] On the New Keynesian predictions of expansionary adverse supply shocks and amplified government spending multiplier at the ZB see Christiano et al. (2011), Wieland (2014) and Woodford (2011).

[2] A ZB episode is a period when 3-month T-Bill rates do not exceed 25 basis points. The resulting empirical frequency is 0.066, or 64/972 months, of which 28 were in the 1930s and 36 have occurred since December 2008. Note that Eggertsson and Woodford (2003), Christiano et al. (2011) and Mertens and Ravn (2014) all assume truncated Markov chains, so liquidity traps are non-recurring (*one-off*) by construction. Models with a recurring passive regime include Nakata (2014), Nakata and Schmidt (2014), Richter and Throckmorton (2015) and Tambakis (2014).

century. That said, Chung et al. (2012) conclude that, in the face of growing uncertainty about large negative demand shocks, 25 years is too small a sample for evaluating tail risks.[3]

I follow Davig and Leeper (2007) in specifying that regime change is an exogenous (*iid*) Markov process, based on what these authors call the *emergency response-driven trigger* of the ZB regime. In principle, the likelihood that future regimes differ from the present one may also be triggered by sunspot or confidence shocks (Mertens and Ravn (2014)). Following the long-run determinacy principle, large deviations from the original Taylor (1993) principle – as when conventional monetary policy is "switched off" – must be short-lived. It follows that requiring equilbrium paths to be determinate implies a maximum average duration for ZB episodes. Specifically, for U.S. baseline parameters values I establish that ZB episodes expected to last more than 3 quarters are inconsistent with determinacy, all else equal. Effectively, imposing a minimum exit rate from liquidity traps amounts to restricting attention to fundamental-based ZB episodes.

Assuming fundamental-shock fluctuations around a unique steady state, the New Keynesian DSGE consensus is that temporary adverse supply shocks are recessionary in normal times but expansionary at the ZB, while the government spending multiplier is amplified. Against that background, I show the "puzzling" implications only arise when equilibrium is indeterminate. Specifically, short-term fiscal stimulus is always expansionary and temporary adverse supply shocks are generally contractionary. During liquidity traps, however, the output impact of such shocks can be positive if the central bank active policy stance is weak. Intuitively, agents expect higher inflation because they anticipate a less reactive monetary policy in the active regime. In turn, that expectation lowers expected real interest rates and stimulates current output. Adopting a progressively less responsive active policy stance, the output elasticity of a 1 standard deviation inflation shock can be as high as 4.5. By contrast, the government spending multiplier at the ZB is around 4 regardless of the specific policy rule, provided the ZB episode is expected to be brief. Thus, temporary fiscal consolidation is never expansionary inside the determinacy region. Comparing across regimes, I find the spending multiplier is about 0.6 in the active regime but always exceeds one during liquidity traps, starting around 1.7 for one-off ZB events.

The results suggest the possibility that the on-going, long ZB episode is indeterminate. A similar uncomfortable implication was raised by Davig and Leeper (2007) commenting on Lubik and Schorfheide's (2004) conclusion about U.S. monetary policy prior to 1982. The reason is that, as in the regime-switching framework liquidity traps – determinate or

[3] Partly also in response to such uncertainty, influential commentators such as Krugman (2013) have argued that near-zero interest rates may constitute the *new normal* for monetary policy.

indeterminate – are recurrent, they cannot be avoided by policy measures including forward guidance (Eggertsson and Woodford (2003)), aggressive fiscal expansion (Benhabib et al. (2002)), or coordinating equilibrium selection (Mertens and Ravn (2014)), to name but a few. All these contributions assume ZB events are temporary. Rather, the empirical implication here is that, *given* their exogenous incidence, policymakers should strive to limit their duration, so as to prevent the welfare deterioration commonly associated with non-unique fundamental transmission and sunspot-driven business cycles.

2 The regime-switching model

A closed economy with nominal rigidities is described by the forward-looking aggregate supply (AS/NKPC) and demand (AD) relationships

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t^S \quad (1)$$

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) + u_t^D \quad (2)$$

where the period-t inflation rate π_t, output gap x_t and nominal interest rate i_t – henceforth, the policy rate – are expressed in log deviations from a unique steady-state, and the parameters have the familiar interpretation.[4] The mean-zero AS and AD shocks, u_t^S and u_t^D respectively, are exogenous demand (government spending) and supply (inflation, or productivity) disturbances following independent stationary AR(1) processes:

$$u_t^D = \rho_D \; u_{t-1}^D + \varepsilon_t^D \;,\; u_t^S = \rho_S \; u_{t-1}^S + \varepsilon_t^S \quad (3)$$

where ρ_D, $\rho_S \in [0, 1)$ and ε_t^D, ε_t^S are iid random variables with standard deviations σ^D, σ^S.

In any period, the economy is either in the *active* regime, denoted a, where the policy rate is strictly positive and determined by a Taylor-type rule, or the passive (*ZB*) regime, z, where it is zero and the feedback portion of the rule is inactive:[5]

$$\begin{aligned} i_t^a &= \phi_\pi \pi_{at} + \phi_x x_{at} \;\;,\; \Pr\{s_t = a \mid \Omega_t\} = P_a \\ i_t^z &= 0 \;\;,\; \Pr\{s_t = z \mid \Omega_t\} = P_z \end{aligned} \quad (4)$$

where $\phi_\pi > 1$, $\phi_x \geq 0$, $s_t \in \{a, z\}$, $\Omega_t = \{s_{t-1}, s_{t-2}, ...\}$ and the regimes' ergodic probabilities P_a and P_z are defined below.

[4] $\beta \in (0, 1)$ is the discount factor; NKPC slope $\kappa > 0$ is proportional to the degree of nominal rigidity; and $\sigma^{-1} > 0$ is the interest-elasticity of output.

[5] I do not label the regimes conventional and unconventional because my model is highly stylized, abstracting from non-traditional policy instruments including large-scale asset purchases by the central bank. I also refrain from calling the active regime *normal* because of Japan's ZB experience since the 1990s.

Regime change in equations (4) is not triggered by large fundamental or taste shocks as in Eggertsson and Woodford (2003), where the policy rule is $i_t = \max\{r_t^n + \phi_\pi \pi_t, 0\}$ with a time-varying natural real interest rate. Rather, it captures monetary policy's periodic shift in focus from price stability to concerns such as systemic risk and financial stability. The short-run dynamics are driven by the homogeneous Markov transition matrix:

$$M = \begin{bmatrix} p_{aa} & p_{az} \\ p_{za} & p_{zz} \end{bmatrix}$$

The off-diagonal elements, $p_{ij} > 0$ $(i, j \in \{a, z\})$ satisfying $p_{ii} + p_{ij} = 1$, are defined as

$$\Pr\{s_t = z \mid s_{t-1} = a\} = p_{az} \ , \ \Pr\{s_t = a \mid s_{t-1} = z\} = p_{za} \tag{5}$$

where s_t is independent of u_t^D and u_t^S. Regime i *persistence* and *expected duration* are then given by $p_{ii} > 0$ and $(1 - p_{ii})^{-1} = T_i > 1$, respectively.[6] Each recurrent policy regime has positive ergodic probability given by:

$$P_z = \frac{p_{az}}{p_{az} + p_{za}} \ , \ T_z = \frac{P_z}{p_{az}(1 - P_z)} \tag{6}$$

$$P_a = \frac{p_{za}}{p_{az} + p_{za}} \ , \ T_a = \frac{P_a}{p_{za}(1 - P_a)} \tag{7}$$

where $P_a + P_z = 1$; see Hamilton (1994). Equations (6)-(7) imply a long-run tradeoff: if regime i's incidence increases then its expected duration must become shorter, and vice versa. As shown by Davig and Leeper (2007) and Richter and Throckmorton (2015), equilibrium determinacy requires the ZB regime's expected duration to be sufficiently short. Put differently, the long-run Taylor principle need not hold every period provided large deviations from it are short-lived; see also Nakata and Schmidt (2014).

Shock propagation under a fixed active regime is well understood: positive AD shocks are expansionary and inflationary; adverse AS shocks are stagflationary. Otherwise, the New Keynesian Markov-switching model is fully described by the linear system

$$\begin{aligned} B y_t &= A E_t y_{t+1} + C u_t \Leftrightarrow \\ \Gamma y_t &= E_t y_{t+1} + v_t \end{aligned} \tag{8}$$

where $\Gamma \equiv A^{-1}B$ provided A is invertible; $v_t \equiv A^{-1} C u_t$; y_t is the $4x1$ regime-dependent target variable vector; u_t is the $2x1$ fundamental shock vector:

[6] Nakata (2014) and Nakata and Schmidt (2014) also refer to p_{az} and p_{zz} as the ZB regime's frequency and persistence.

$$y_t = \begin{pmatrix} \pi_{at} \\ \pi_{zt} \\ x_{at} \\ x_{zt} \end{pmatrix}, \; E_t y_{t+1} = \begin{pmatrix} E_t \pi_{at+1} \\ E_t \pi_{zt+1} \\ E_t x_{at+1} \\ E_t x_{zt+1} \end{pmatrix}, \; u_t = \begin{pmatrix} u_t^S \\ u_t^D \end{pmatrix}$$

and

$$A = \begin{bmatrix} \beta p_{aa} & \beta p_{az} & 0 & 0 \\ \beta p_{za} & \beta p_{zz} & 0 & 0 \\ \sigma^{-1} p_{aa} & \sigma^{-1} p_{az} & p_{aa} & p_{az} \\ \sigma^{-1} p_{za} & \sigma^{-1} p_{zz} & p_{za} & p_{zz} \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 & -\kappa & 0 \\ 0 & 1 & 0 & -\kappa \\ \sigma^{-1}\phi_\pi & 0 & 1+\sigma^{-1}\phi_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \tag{9}$$

Matrix A includes the exogenous transition probabilities; matrix B includes the active response coefficients ϕ_π and ϕ_x; and C stacks the fundamental shocks. Davig and Leeper (2007) show that the solutions of (8) are determinate iff all four eigenvalues of Γ, denoted λ_i $(i = 1, ..., 4)$ are outside the unit circle. That is indeed the case for most parameter values when the passive regime is the ZB.[7] In general, the solution paths of (8) are given by:

$$\begin{aligned} y_t &= G u_t + V w_t && (10) \\ w_t &= \Lambda w_{t-1} + M u_t && (11) \end{aligned}$$

where w_t is a non-fundamental variable. Let k is the number of eigenvalues of Γ inside the unit circle. Then V is $2xk$, Λ is kxk, and M is any $kx2$ real matrix. If $k = 1$, matrix Λ becomes a scalar equal to the eigenvalue inside the unit circle, measuring sunspot persistence.

It is important for policymakers to avoid actions that would induce indeterminacy, as the excess output volatility caused by sunspots reduces welfare if agents are risk-averse. In what follows I focus on linear minimum state variable (MSV) solutions within the determinate region ($k = 0$). The MSV solutions of system (8) involve four target variables:

$$\begin{aligned} \pi_{at} &= b_a^D \; u_t^D + b_a^S \; u_t^S \;, \; x_{at} = c_a^D \; u_t^D + c_a^S \; u_t^S && (12) \\ \pi_{zt} &= b_z^D \; u_t^D + b_z^S \; u_t^S \;, \; x_{zt} = c_z^D \; u_t^D + c_z^S \; u_t^S && (13) \end{aligned}$$

Coefficient vectors $[b_a^S, b_z^S, c_a^S, c_z^S]'$ and $[b_a^D, b_z^D, c_a^D, c_z^D]'$ are regime-specific impact elasticities. Assuming a determinate solution to the regime-switching model (8) exists and guessing the

[7] The determinacy region is very similar to Davig and Leeper (2007). It, and the proof that the Markov-switching model with a passive ZB regime has indeterminacy of degree one are available upon request.

inflation and output paths in (12)-(13), the equilibrium elasticities solve:

$$\begin{bmatrix} 1-\beta p_{aa}\rho_S & -\beta p_{az}\rho_S & -\kappa & 0 \\ -\beta p_{za}\rho_S & 1-\beta p_{zz}\rho_S & 0 & -\kappa \\ \sigma^{-1}(\phi_\pi - p_{aa}\rho_S) & -\sigma^{-1}p_{az}\rho_S & 1+\sigma^{-1}\phi_x - p_{aa}\rho_S & -p_{az}\rho_S \\ -\sigma^{-1}p_{za}\rho_S & -\sigma^{-1}p_{zz}\rho_S & -p_{za}\rho_S & 1-p_{zz}\rho_S \end{bmatrix} \begin{bmatrix} b_a^S \\ b_z^S \\ c_a^S \\ c_z^S \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

for AS shocks and

$$\begin{bmatrix} 1-\beta p_{aa}\rho_D & -\beta p_{az}\rho_D & -\kappa & 0 \\ -\beta p_{za}\rho_D & 1-\beta p_{zz}\rho_D & 0 & -\kappa \\ \sigma^{-1}(\phi_\pi - p_{aa}\rho_D) & -\sigma^{-1}p_{az}\rho_D & 1+\sigma^{-1}\phi_x - p_{aa}\rho_D & -p_{az}\rho_D \\ -\sigma^{-1}p_{za}\rho_D & -\sigma^{-1}p_{zz}\rho_D & -p_{za}\rho_D & 1-p_{zz}\rho_D \end{bmatrix} \begin{bmatrix} b_a^D \\ b_z^D \\ c_a^D \\ c_z^D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \tag{15}$$

for AD shocks. Analytical solutions to equation systems (14)-(15) are readily computed but difficult to interpret; note that one-off ZB events amount to the special case $p_{za} = 1$.

3 Expectation formation effects

Leading equations (12)-(13) by one period, applying definitions (3) and imposing $\rho_S = \rho_D = \rho$, wlog, the regime-specific inflation and output gap expectations are:

$$\begin{aligned} E_{s_t t}\pi_{t+1} &= E_t[\pi_{t+1} \mid s_t = i] = p_{ii}(\rho_D b_i^D u_t^D + \rho_S b_i^S u_t^S) && (16) \\ &\quad + p_{ij}(\rho_D b_j^D u_t^D + \rho_S b_j^S u_t^S) \\ E_{s_t t}x_{t+1} &= E_t[x_{t+1} \mid s_t = i] = p_{ii}(\rho_D c_i^D u_t^D + \rho_S c_i^S u_t^S) && (17) \\ &\quad + p_{ij}(\rho_D c_j^D u_t^D + \rho_S c_j^S u_t^S) \end{aligned}$$

where $i, j \in \{a, z\}$. All else equal, equations (16)-(17) indicate that expectation formation effects from regime i to j increase in $p_{ji} > 0$ and $p_{ii} > 0$. The two transition probabilities are independently determined by Markov matrix M.[8] To gauge expectation spillovers across regimes, I apply expectation definitions (16)-(17) to AD relation (2) and write:

$$\begin{aligned} x_{zt} &= \{1 + \rho p_{zz}[c_z^D - c_a^D + \sigma^{-1}(b_z^D - b_a^D)]\}u_t^D && (18) \\ &\quad + \rho p_{zz}[c_z^S - c_a^S + \sigma^{-1}(b_z^S - b_a^S)]u_t^S + (c_a^D + \sigma^{-1}b_a^D)\rho u_t^D + \rho(c_a^S + \sigma^{-1}b_a^S)u_t^S \\ x_{at} &= \{1 + \rho p_{aa}[c_z^D - c_a^D + \sigma^{-1}(b_z^D - b_a^D)]\}u_t^D && (19) \\ &\quad + \rho p_{aa}[c_z^S - c_a^S + \sigma^{-1}(b_z^S - b_a^S)]u_t^S + (c_a^D + \sigma^{-1}b_a^D)\rho u_t^D + \rho(c_a^S + \sigma^{-1}b_a^S)u_t^S \\ &\quad - \sigma^{-1}[(\phi_\pi b_a^D + \phi_x c_a^D)u_t^D + (\phi_\pi b_a^S + \phi_x c_a^S)u_t^S] \end{aligned}$$

[8] If the ZB constraint was approached gradually then setting a higher inflation target would lower ZB incidence, all else equal, and p_{az} could not be specified exogenously; see Chung et al. (2012).

The four regime-specific output elasticities are defined as $c_i^f = \frac{\partial x_{it}}{\partial u_t^f}$, $f \in \{D, S\}$, $i \in \{a, z\}$. A useful benchmark of on-going regime change on c_i^f is equal expected duration, $p_{aa} = p_{zz} = p$. Equations (18)-(19) then imply:

$$c_z^D = \frac{1 + \rho[(1-p)c_a^D + \sigma^{-1}b_a^D + p\sigma^{-1}(b_z^D - b_a^D)]}{1 - \rho p} \tag{20}$$

$$c_a^D = \frac{1 + \rho[c_z^D + \sigma^{-1}b_a^D + p\sigma^{-1}(b_z^D - b_a^D)] - \sigma^{-1}(\phi_\pi b_a^D + \phi_x c_a^D)}{1 - \rho p + \sigma^{-1}\phi_x} \tag{21}$$

for AD and

$$c_z^S = \frac{\rho[(1-p)c_a^S + p\sigma^{-1}(b_z^S - b_a^S) + \sigma^{-1}b_a^S]}{1 - \rho p} \tag{22}$$

$$c_a^S = \frac{\rho p[c_z^S + \sigma^{-1}(b_z^S - b_a^S)] - \sigma^{-1}b_a^S(\phi_\pi - \rho)}{1 - \rho(1-p) + \sigma^{-1}\phi_x} \tag{23}$$

for AS shocks. Equations (20)-(23) indicate fundamental disturbances are propagated as follows:

AD shocks. Temporary fiscal expansions are more potent at the ZB by the standard (fixed) active transmission channel: $c_z^D > c_a^D$.[9] From that consensus ranking, consider the effect on c_z^D of a persistence increase, $\Delta p > 0$ wlog. Noting the elasticity functional forms are hyperbolic in p, the denominator of c_z^D in equation (20) increases unambigously. Provided positive AD shocks are more inflationary at the ZB ($b_z^D > b_a^D$), the c_z^D numerator rises by $\rho\sigma^{-1}(b_z^D - b_a^D)$ as agents anticipate the ZB to last longer. It also declines by ρc_a^D , however, because a return to active policy is less likely. The sign of Δc_z^D depends on the level of p.

Now consider shortening average ZB duration, $p_{zz} < p_{aa}$ wlog. This has two complementary effects. It directly lowers c_z^D and, by symmetry, relatively higher p_{aa} raises c_a^D. Note that lower c_z^D enters as $+\rho c_z^D$ in c_a^D, while higher c_a^D enters as $-\rho p_{zz} c_a^D$ in c_z^D. It follows that the indirect effect of lower p_{zz} is higher c_a^D. Conversely, a more persistent active regime lowers c_z^D so the two effects are reinforcing. It is also easy to check that white noise fundamentals ($\rho = 0$) imply $c_a^D = \frac{1-\sigma^{-1}(\phi_\pi b_a^D + \phi_x c_a^D)}{1+\sigma^{-1}\phi_x} < 1$ and $c_z^D = 1$.

AS shocks. The impact of lower p_{zz} on c_z^S in equation (23) is ambiguous *a priori*. First, note that $\rho = 0$ implies $c_z^S = 0$: the output gap is insulated from supply shocks at the ZB unless they are persistent. Then, given $b_z^S > b_a^S$, the negative shock to p_{zz} lowers c_z^S by $\rho\sigma^{-1}(b_z^D - b_a^D)$. It also means a return to active policy is more likely, $\Delta p_{za} > 0$. In turn, given $c_a^S < 0$, that implies c_z^S declines more than before, by ρc_a^S. At the same time the

[9] To see this, note that the numerator (denominator) of (20) is less (greater) than of (21) for all $\sigma^{-1} > 0$. The regime-specific inflation rates are similarly obtained by substituting (16)-(17) to AS relation (1).

denominator of c_z^S rises for all $\rho > 0$. The net output impact of adverse supply (inflationary) shocks is thus more negative. Conversely, however, such shocks are less contractionary, or even expansionary for longer-lasting liquidity traps.

Further, the sign of c_z^S in (23) is independent of the magnitude of c_z^D. Hence, the model can generate contractionary inflation shocks at the ZB without violating the consensus view on large (above one) government spending multipliers.[10]

One-off ZB events. If the active regime is absorbing ($p_{za} = 1$), the regime-specific spending multipliers and ZB-specific output elasticity to AS shocks become:

$$c_a^D = \frac{1 + \rho[c_z^D + \sigma^{-1} b_a^D + (1 - p_{az})\sigma^{-1}(b_z^D - b_a^D)] - \sigma^{-1}(\phi_\pi b_a^D + \phi_x c_a^D)}{1 - \rho(1 - p_{az}) + \sigma^{-1}\phi_x} \tag{24}$$

$$c_z^D = \frac{1 + \rho(c_a^D + \sigma^{-1} b_a^D)}{1 - \rho} \; , \; c_z^S = \rho(c_a^S + \sigma^{-1} b_a^S) \tag{25}$$

More severe ZB episodes (higher p_{az}) lower c_a^D, indicating that ignoring recurrent regime change is likely to underestimate the average government spending multiplier.

4 Fundamental shock transmission

4.1 Baseline calibration

In calibrating the regime-switching model, I adopt as baseline the U.S. parameter estimates of Lubik and Schorfheide (2004) and Davig and Leeper (2007), referred to as *LS* and *DL*, with the exception of the choice of p_{zz} and the active policy stance where I use the optimal policy response coefficients of Schmitt-Grohé and Uribe (2007), referred to as *SU*. They are summarized in Table 1:

Table 1. Baseline parameterization	
Quarterly discount factor	$\beta = 0.99$
Price rigidity (NKPC slope)	$\kappa = 0.17$
Interest-elasticity of output	$\sigma^{-1} = 1$
Shock persistence	$\rho_S = \rho_D = 0.75$
Shock variability	$\sigma^S = 0.80$, $\sigma^D = 0.23$
Active regime responses	$\phi_\pi = 3, \phi_x = 0$ (*SU*)
Regime persistence	$p_{aa} = 0.975$, $p_{zz} = 0.647$

[10] It is worth contrasting this property with Wieland (2014), where the spending multiplier at the ZB exceeds one ($c_z^D > 1$) if and only if adverse AS shocks are expansionary ($c_z^S > 0$).

I combine $p_{aa} = 0.975$ – one ZB episode per decade, on average – with $p_{zz} = 0.647$ ($T_z = 2.83$ quarters), the passive regime persistence matching $P_z = 0.066$, the U.S. historical ZB frequency from Figure 1.[11] That is because the LS choice of $p_{zz} = 0.93$ – liquidity traps expected to last 3.5 years – is inconsistent with determinacy. To see why, I establish a *maximum* persistence threshold, $p_{zz}^{\max} < 1$ – i.e., a minimum ZB exit rate $p_{za}^{\min} > 0$ – beyond which equilibrium solution paths become indeterminate. I illustrate this threshold in Figure 2, fixing $p_{aa} = 0.975$ and plotting $\lambda_{\min}$, the minimum eigenvalue of system (8) against p_{zz}:

FIGURE 2 HERE

In all four cases, $\lambda_{\min}$ crosses the indeterminacy point ($\lambda = 1$) only once because the regime-switching New Keynesian model displays degree-one indeterminacy. Under the baseline (*Panel A*), the maximum persistence is $p_{zz}^{\max} = 0.668$, implying $p_{za}^{\min} = 0.332$ or $T_z^* \simeq 3$ quarters. The tentative implication is that historical ZB persistence is narrowly consistent with equilibrium determinacy, as defined in the recurrent regime-switching model.[12] In *Panel B*, a weaker active transmission mechanism extends the threshold to $p_{zz}^{\max} = 0.753$**,** that is $T_z^{\max} \simeq 4$ quarters. In *Panel C*, more price flexibility limits the threshold to $p_{zz}^{\max} = 0.501$. Liquidity traps expected to last beyond six months then yield indeterminate equilibria. In *Panel D* the less reactive policy rules of LS-DL and Taylor (1993) reduce the threshold only marginally, to $p_{zz}^{\max} = 0.656$ and 0.620, respectively.

To put these numbers in perspective, by 2015q2 the current ZB episode has lasted 22 quarters, amounting to $p_{zz} = 0.945$**.** When passive means less active, DL find that the minimum exit rate for determinacy is $p_{za}^{\min} = 0.10$ ($p_{zz}^{\max} = 0.90$). The fact that my minimum exit rates are higher also conforms with LS's key point that loose Fed monetary policy in the 1970s is consistent with multiple equilibria. Hence, insisting on ZB persistence below $p_{zz}^{\max}$ focuses attention on *determinate* liquidity traps.

Table 2 below reports the unique and bounded regime-specific solutions implied by three different active policy stances:

[11] Substituting this ergodic probability and the exogenous ZB incidence of *LS-DL*, $p_{az} = 0.025$, into equation (6) yields $p_{zz} = 0.647$.

[12] Validating this conclusion would require further long-run identifying restrictions, especially concerning the lag lengths specified for endogenous variables; see the debate between Lubik and Schorfheide (2004) and Beyer and Farmer (2007) and the discussion in Tambakis (2014).

Table 2. Regime-specific impact elasticities[13]

	Active	Zero Bound
$p_{aa} = 0.975$, $p_{zz} = 0.647$	$P_a = 0.934$	$P_z = 0.066$
Inflation		
SU baseline: $p_{zz}^{\max} = 0.674$	$\pi_{at} = 0.42u_t^D + 0.62u_t^S$	$\pi_{zt} = 1.46u_t^D + 2.14u_t^S$
LS-DL: $p_{zz}^{\max} = 0.656$	$\pi_{at} = 0.50u_t^D + 1.53u_t^S$	$\pi_{zt} = 1.57u_t^D + 3.36u_t^S$
T: $p_{zz}^{\max} = 0.620$	$\pi_{at} = 0.62u_t^D + 2.52u_t^S$	$\pi_{zt} = 1.73u_t^D + 4.68u_t^S$
Output gap		
SU baseline	$x_{at} = 0.53u_t^D - 5.11u_t^S$	$x_{zt} = 3.80u_t^D - \mathbf{0.29}u_t^S$
LS-DL	$x_{at} = 0.65u_t^D - 3.76u_t^S$	$x_{zt} = 4.01u_t^D + \mathbf{2.02}u_t^S$
T	$x_{at} = 0.82u_t^D - 2.30u_t^S$	$x_{zt} = 4.32u_t^D + \mathbf{4.52}u_t^S$

Comparing the SU baseline to *LS-DL* and T suggests several transmission properties: (i) Both shocks are more inflationary at the ZB, and the less aggressive the central bank's active policy stance. (ii) The output impact of temporary positive AD shocks is much higher at the ZB. By comparison, Christiano et al. (2011) report the interval $[0.8, 1.2]$. (iii) Adverse AS shocks are contractionary in normal times, relatively less if output is shielded by more accommodating active policy.[14] (iv) Adverse AS shocks are mildly contractionary at the ZB only under SU. However, if the active regime's anti-inflation stance weakens (*LS-DL*, T) they become expansionary.

4.2 Sensitivity analysis

In Figure 3 I present the evolution of the four output elasticities with passive persistence:

FIGURE 3 HERE

The results suggest expectation formation effects *gain* (lose) significance with ZB expected duration in the *passive* (active) regime. In *Panel A*, the active government spending multiplier ranges from $c_a^D = 0.57$, when ZB events are one-off, to 0.53 approaching $p_{zz}^{\max}$. Fiscal stimulus is much more effective at the ZB (*Panel B*): the multiplier grows from $c_z^D = 1.73$ to

[13]Row 2 reports the ergodic probabilities of each policy regime. The parameter values are as in Table 1 with the exception of *LS-DL* and T. These denote $\phi_\pi = 2.19, \phi_x = 0.3$, the response coefficients of Lubik and Schorfheide (2004) and Davig and Leeper (2007), and $\phi_\pi = 1.5, \phi_x = 0.5$ from Taylor (1993), respectively.

[14]This policy tradeoff is familiar from the fixed regime model. The sensitivities of the inflation elasticities to p_{zz} and ρ are not reported to save space.

3.96 around $p_{zz}^{\max}$. It becomes even more effective if active monetary policy is less reactive: c_z^D ranges from $[0.65, 4.17]$, under *LS-DL*, to $[0.82, 4.49]$ under *T*.[15]

In Fig. 3, *Panel C* temporary inflation shocks are contractionary in the active regime, in line with the consensus view. The impact elasticities are very stable around -5. Against that, in liquidity traps output is very sensitive to their expected duration (*Panel D*). Starting at -3.33 as $p_{zz} \to 0$, c_z^S declines in absolute to -0.06 as $p_{zz}^{\max} = 0.67$ and then turns positive. The ensuing positive range, however, is incompatible with determinacy. Consistent with equations (20)-(23), longer liquidity traps *boost* (dampen) the impact of *AD* (AS) shocks within the determinacy region.

One can interpret the above fundamental asymmetry as follows. If the ZB regime becomes more likely, expected inflation in equation (16) rises. As argued by Wieland (2014), this lowers expected real interest rates, which stimulates consumption and output. As at the ZB the negative AS shock would be recessionary, the greater expected duration amplifies the inflation expectation effect, mitigating the slowdown. And if the active stance weakens, expected real interest rates decline further so the net impact may be expansionary.

Turning to fundamental persistence, Figure 4 presents the behavior of the output elasticities maintaining regime persistence at baseline and varying ρ on the unit interval:

FIGURE 4 HERE

In *Panel A*, the active spending multiplier declines with AD shock persistence, turning negative ("expansionary austerity") around $\rho \simeq 0.95$, while the ZB-specific multiplier grows smoothly with ρ (*Panel B*). Similarly, in the active regime the contractionary impact of negative AS shocks strengthens with ρ (*Panel C*). These elasticities are not very sensitive to the interest-rate rule the central bank implements in normal times. By contrast, in *Panel D* the sign of c_z^S from $\rho = 0$ is determined by the active policy stance. Temporary inflation shocks are recessionary for the aggressive baseline *SU* policy response (bold line), but *expansionary* under the less reactive *LS-DL* and *T* response coefficients (dotted line). Expectation formation effects from a to z are stronger if the active stance weakens.

[15] Christiano al. (2011) also find a positive monotonic link between c_z^D and T_z. By contrast, Mertens and Ravn (2014) show fiscal multipliers are much larger for brief, fundamental-based liquidity traps. See also Nakata (2014).

5 Concluding remarks

In contrast to Tambakis (2014), in the present paper I studied *short-term* fundamental shock transmission in a New Keynesian recurrent regime-switching model when passive policy involves the ZB. I showed that relatively *short* (long) ZB episodes are (in)*determinate*, and explored the impact elasticities' sensitivity to ZB expected duration and fundamental shock persistence. For determinate liquidity traps, I found that temporary inflation shocks are contractionary provided the central bank's active policy stance is aggressive enough; such a stance dampens somewhat the increased efficacy of temporary fiscal stimulus; and "expansionary austerity" does not arise.

References

[1] Benhabib, J., Schmitt-Grohé, S., Uribe, M., 2002. Avoiding liquidity traps, Journal of Political Economy 110, 535-563.

[2] Beyer, A., Farmer, R., 2007. Testing for indeterminacy: An application to U.S. monetary policy: Comment, American Economic Review 97, 524-529.

[3] Chung, H., Laforte, J.-P., Reifschneider, D., Williams, J., 2012. Have we underestimated the likelihood and severity of zero lower bound events? Journal of Money, Credit and Banking 44, 47-87.

[4] Christiano, L., Eichenbaum, M., Rebelo, S., 2011. When is the government spending multiplier large? Journal of Political Economy 119, 78-121.

[5] Davig, T., Leeper, E., 2007. Generalizing the Taylor principle, American Economic Review 97, 607-635.

[6] Eggertsson, G., Woodford, M., 2003. The zero bound on interest rates and optimal monetary policy, Brookings Papers on Economic Activity 34, 139-235.

[7] Hamilton, J., 1994. *Time Series Analysis.* Princeton University Press, Princeton, NJ.

[8] Krugman, P., 2013. Secular stagnation, coalmines, bubbles, and Larry Summers, The New York Times, November 16.

[9] Lubik, T., Schorfheide, F., 2004. Testing for indeterminacy: An application to U.S. monetary policy, American Economic Review 94, 190-217.

[10] Nakata, T., 2014. Reputation and liquidity traps, Finance and Economics Discussion Series 2014-50, Board of Governors of the Federal Reserve System, Washington, D.C.

[11] Nakata, T., Schmidt, S., 2014. Conservatism and liquidity traps, Finance and Economics Discussion Series 2014-105, Board of Governors of the Federal Reserve System, Washington, D.C.

[12] Richter, A., Throckmorton, N., 2015. The zero lower bound: Frequency, duration, and numerical convergence, B.E. Journal of Macroeconomics: Contributions 15, 157-182.

[13] Mertens, K., Ravn, M., 2014. Fiscal policy in an expectations driven liquidity trap, Review of Economic Studies 81, 1637-1667.

[14] Schmitt-Grohé, S., Uribe, M., 2007. Optimal simple and implementable monetary and fiscal rules, Journal of Monetary Economics 54, 1702-1725.

[15] Tambakis, D., 2014. On the risk of long-run deflation, Economics Letters 122, 176-181.

[16] Taylor, J., 1993. Discretion versus policy rules in practice, Carnegie-Rochester Conference Series on Public Policy 39, 195-214.

[17] Wieland, J., 2014. Are negative supply shocks expansionary at the zero lower bound? Inflation expectations and financial frictions in sticky price models, University of California, Berkeley, working paper.

[18] Woodford, M., 2011. Simple analytics of the government expenditure multiplier, American Economic Journal: Macroeconomics 3, 1-35.

Figure 1. U.S. historical zero bound frequency

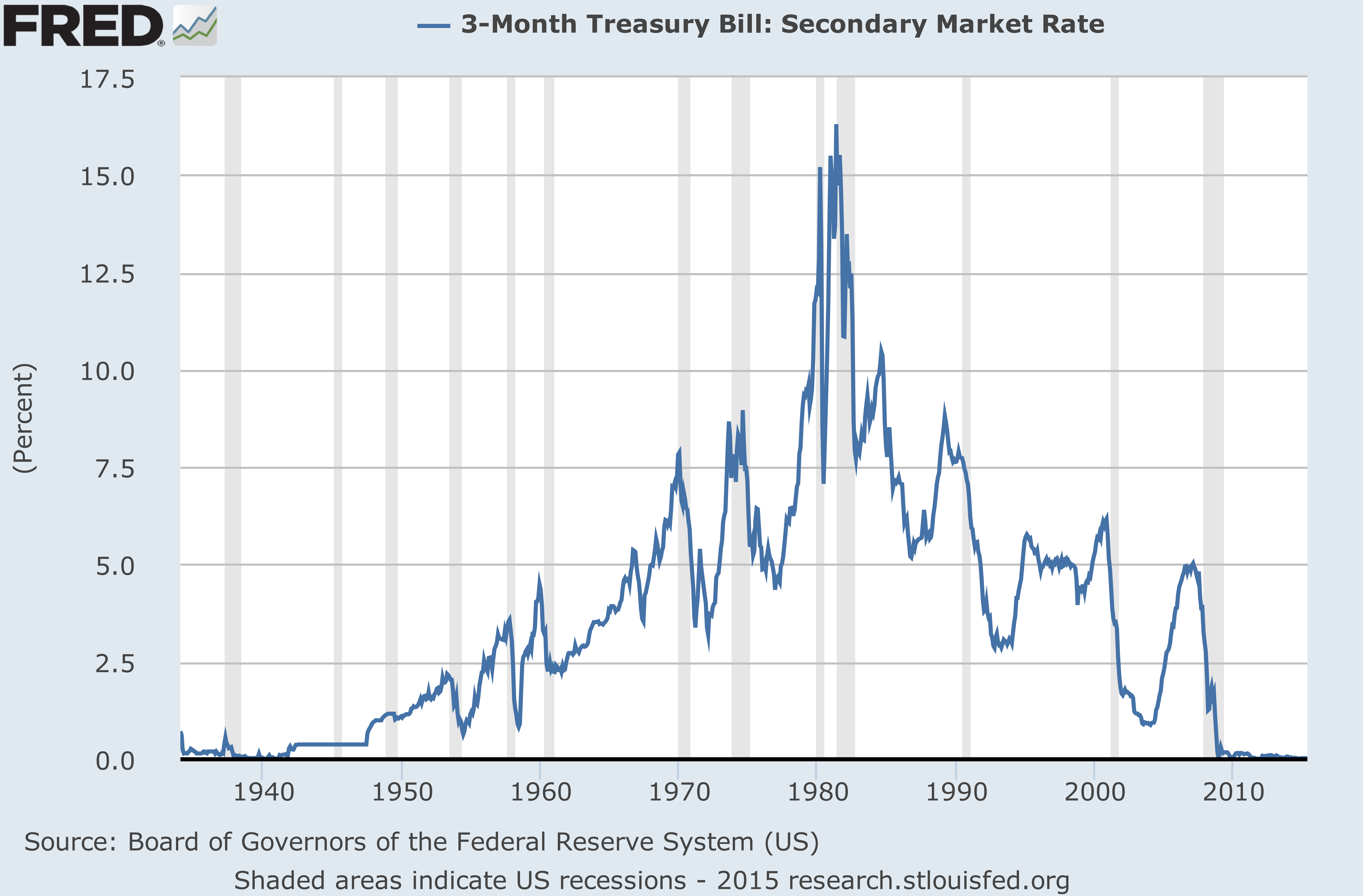
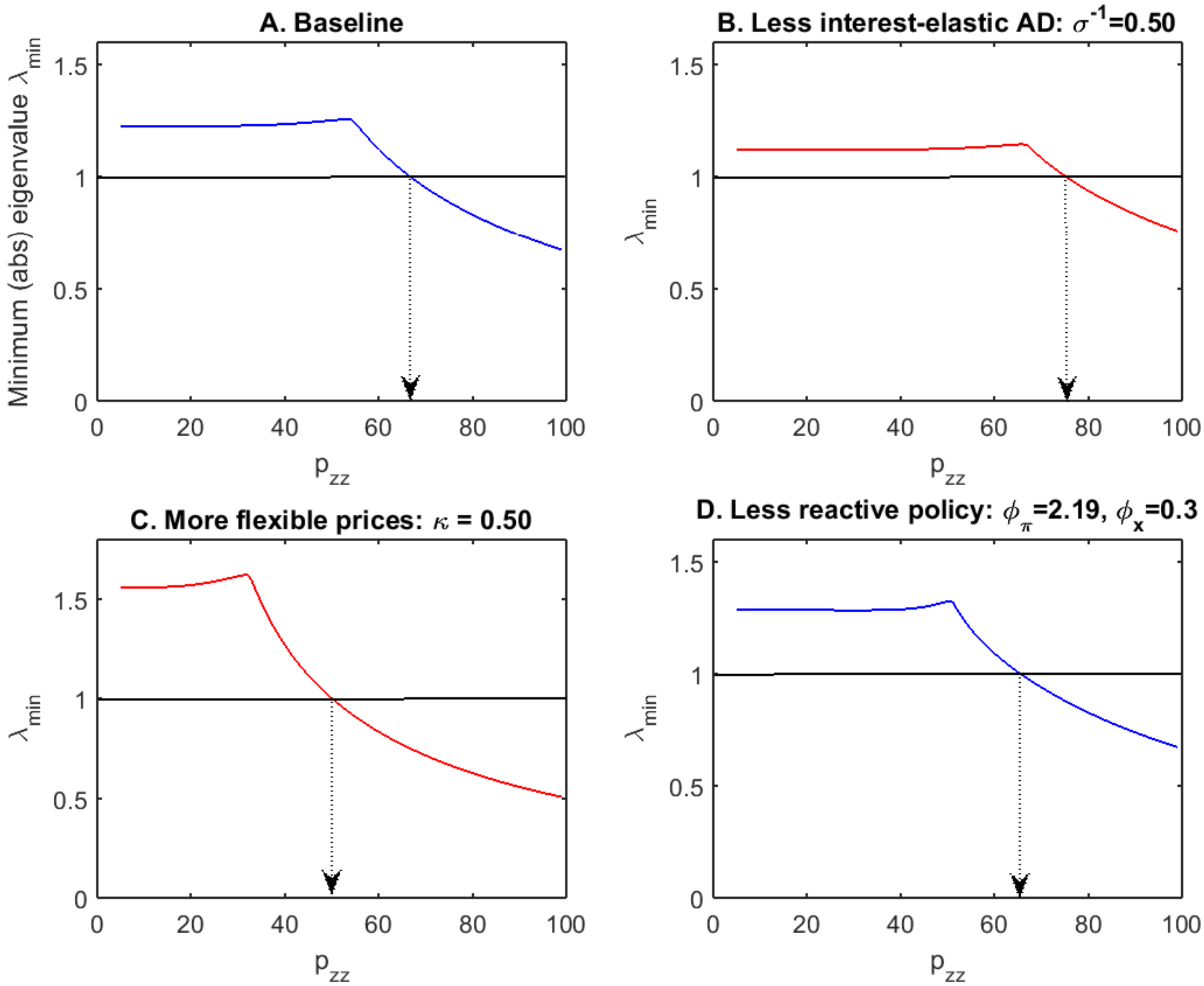
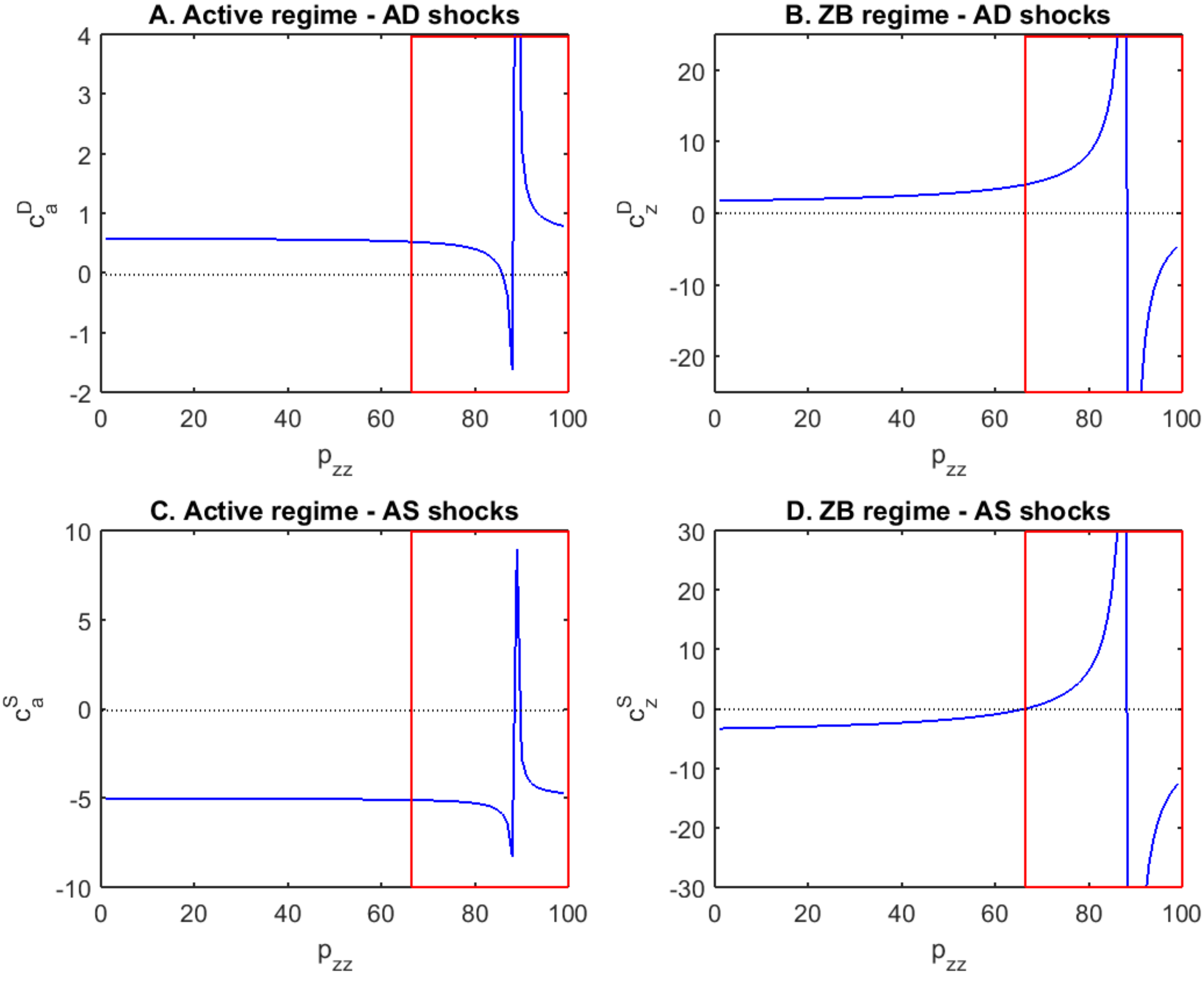


Figure 2. The onset of indeterminacy*



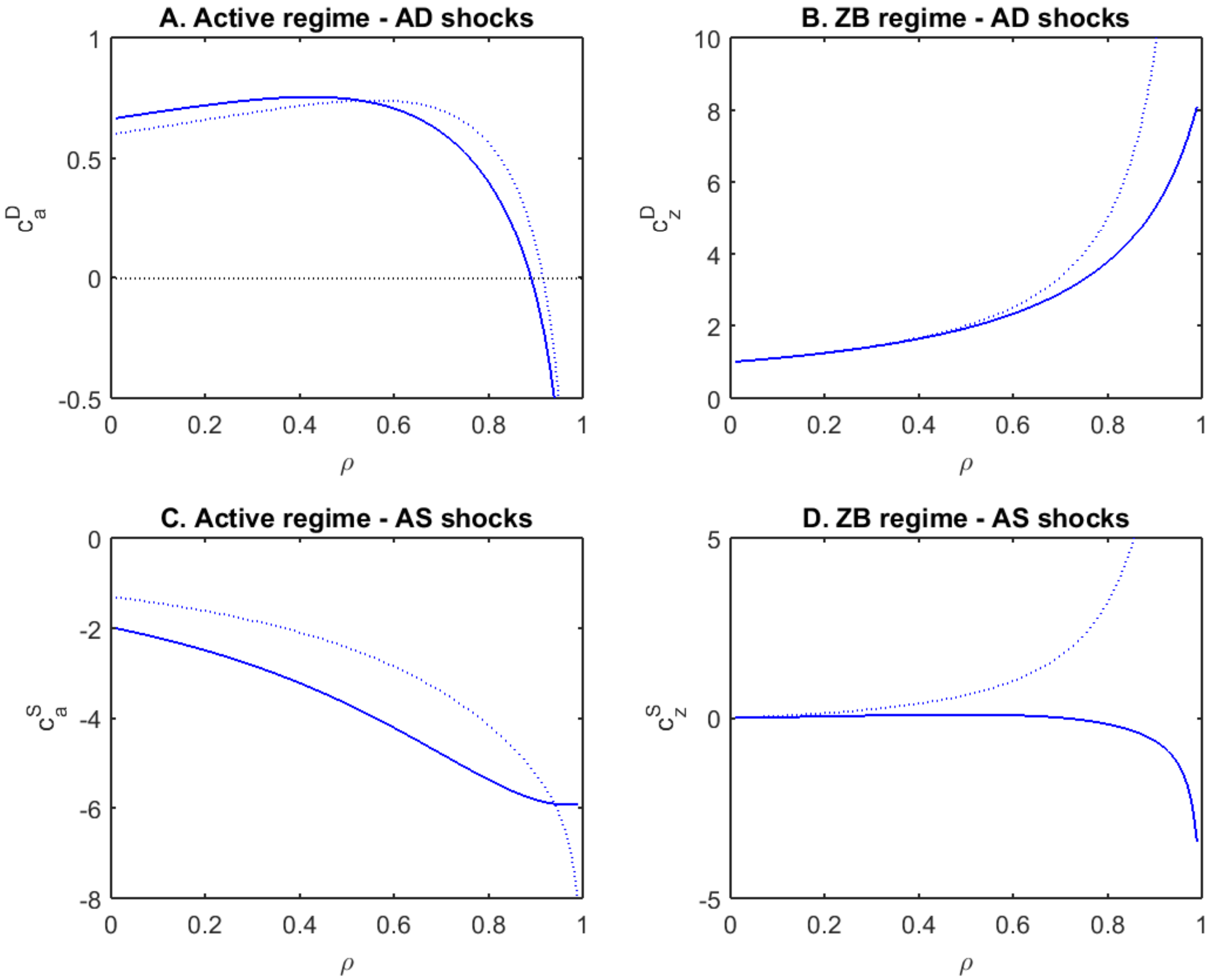
*On the x-axis ZB persistence varies on [0,1]. The y-axis displays the minimum eigenvalue of system (10). All parameter values other than the one considered are in Table 1.

Figure 3. Output elasticities and regime persistence*



*All parameter values are in Table 1. The red box in each panel indicates the ZB persistence range incompatible with equilibrium determinacy in the regime-switching model.

Figure 4. Output elasticities and fundamental persistence*



* All parameter values are in Table 1. The solid and dotted blue lines employ the baseline (SU) and LS-DL active policy response coefficients, respectively.