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29 September 2016

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Labor Market Effects of Inconsistent Policy Interventions: Evidence from India's Employment Guarantees

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Abstract

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JEL: I38, J31, J68, O12

Keywords: Employment guarantee, wage rigidity, uncertainty, welfare, NREGA, India

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1 Introduction

Employment guarantee schemes have long been a standard response of the government in its role as an employer of last resort. Starting with the Poor Employment Act of 1817 in Britain (Blaug, 1963, 1964) and the New Deal Programs during the 1930s in the US (Kesselman, 1978; Bernstein, 1970), large public works have played an increasingly dominant role in providing income support for the poor in many developing economies.¹ The recent implementation of the largest public workfare program in the world, the National Rural Employment Guarantee Act (NREGA) has reignited the debate on the general equilibrium effects of such large public works programs.²

The recent empirical literature on the labor market impact of NREGA provides evidence that the program did increase low-skilled private sector wages (Azam, 2012; Zimmermann, 2012; Berg et al., 2013; Imbert and Papp, 2015; Bahal, 2016).³ Imbert and Papp further report substantial welfare gains to the poor from this wage increase. Juxtaposed with this body of work is the literature that highlights partial implementation of public workfare programs in India. Although NREGA guarantees 100 days of low-skilled employment to every rural household per year, studies have found high levels of rationing or unmet demand for NREGA work (Dutta et al., 2012; Imbert and Papp, 2014). Moreover, the level of employment provision has been shown to fluctuate greatly across districts and over time (Drèze and Khera, 2009; Drèze and Oldiges, 2009; Bahal, 2016).

Although many useful lessons have emerged from this recent work, there is a significant disconnect between these two strands of literature. Little or no attempt has been made to understand the consequences of variability in program provision on private employment and wages. Intuitively, partial program implementation by itself may already incur substantial losses in terms of unrealized potential gains for (i) the intended program beneficiaries who are rationed out of the workfare, and (ii) the non-beneficiaries who stand to gain from the consequent increase in low-skilled private wages. The key question, however, is to understand the impact of variability in program provision on market outcomes, especially in labor markets that are often fraught with distortions like wage rigidities and market power.

Put differently, we ask whether fluctuations in program provision over time dampen

¹See Drèze and Sen (1989), Lipton (1996), Subbarao (1997), Keddeman (1998) for discussion of public works in various countries of Latin America, Asia, and Africa over the last few decades. We use the terms public works, workfare programs, and employment guarantee schemes interchangeably.

²The program which started in 2006 was implemented in phases and covered all the districts of the country by 2008. The employment scheme was later renamed to Mahatma Gandhi NREGA.

³Although the impact estimates reported by these studies are fairly heterogeneous (owing primarily to differences in data and econometric methodologies).

or compress the wage increase that corresponds to the level of program provision today. The issue is even more relevant for developing economies where variability in program provision emanates mostly from limitations in administrative capabilities and constrained budgets. In the case of India, employment provision (or lack thereof) has indeed been associated with supply factors like administrative capacity or political will instead of demand factors like poverty or rate of unemployment (Imbert and Papp, 2015; Bahal, 2016).

To answer this question, we use a model of EGS that accommodates three key features of (i) imperfect competition and market power, (ii) nominal wage rigidities, and (iii) uncertainty regarding the level of program provision in the future. First, given the evidence of market outcomes that support imperfect competition and market power, we drop the assumption of a perfectly competitive rural labor market (see for e.g., Bardhan and Rudra, 1981; Bardhan, 1979, 1984; Binswanger et al., 1984; Datt, 1997).

Second, following the evidence of downward nominal wage rigidity (Kahn, 1997; Lebow et al., 1999; Knoppik and Beissinger, 2003, 2009; Dickens et al., 2006; Behr and Pötter, 2010) we restrict nominal wage cuts in our model. Specifically for the casual daily labor market in India, Drèze and Mukherjee (1989) note “The standard wage (in money terms) . . . appears to be, more often than not, rigid downwards during the slack season”. More recently, Kaur (2014) provides evidence of downward rigidity in nominal wages using wage and employment responses to rainfall shocks. Lastly, following the empirical literature that provides evidence on partial implementation and variability in program provision, we study market outcomes in the event rationing in future is different from its level today.

Using a static model of EGS we first show that public workfare leads to a contraction of labor supply for private employment which increases private wages.⁴ The level of program provision determines the extent of contraction in private labor supply and the consequent increase in private wages. Next, to accommodate temporal variation in program provision we recalculate labor market outcomes in a two-period model of EGS. Keeping the present level of program provision the same, we compare wage increases between the static and two-period cases. We find that in comparison to the static case, wage increase in the two-period framework is lower if the program provision deteriorates in future. Moreover, we find that the greater is the contraction in

⁴The fall in private employment associated with EGS implementation is consistent with the empirical evidence (see for e.g. Imbert and Papp, 2015). Since we incorporate rationing explicitly in our model and therefore assume EGS wage to be higher than the private wage, market power doesn't result in cases where both private wages and employment can increase due to the EGS. See Basu, Chau, and Kanbur (2009) for a discussion when EGS can raise private employment as well. In that spirit, a workfare is different from a minimum wage policy where both private employment and wages may increase due to efficiency gains in a monopsonistic or oligopsonistic labor market.

program provision in the future (vis-à-vis its level today), the greater is the compression of wage increases.

The compression of wage increases in the two-period framework follows from (i) a decline in the program provision in the future which ideally warrants a wage cut and (ii) the downward nominal wage rigidity which restricts such a reduction in wages. This apparent trade-off between current and future profit implies that present wages cannot be solely determined by the level of the program provision today. Instead, optimal private wage in the current period is based on present and future levels of program provision. This explains why a decline in program provision in the future dampens wage increases today. In reality, however, there is uncertainty regarding the level of program provision in the future. To incorporate this uncertainty, we assume the rate of rationing in future to be drawn from a given distribution of rationing rates. The optimal wage in the first period is then based on optimizing current and expected future profit. We find that greater variability in the level of program provision in the future is associated with higher compression of wage increases today. Importantly, greater compression of wage increases is shown to reduce both employer profit and worker utility.⁵

Finally, we empirically validate two key predictions of our model: (i) that the increase in private wages due to the workfare is lower when variation in program provision is high; and (ii) that the compression of wage increases is more severe in districts where inflation is low relative to where inflation is high. Using district-level expenditure data on employment schemes from Bahal (2016), we find that the increase in (real) private wages due to the workfare is lower as the variability in program provision increases.⁶ In support of the second prediction, we find that the compression of wage increases is significantly higher for low inflation districts relative to high inflation districts. These results have important policy implications as they suggest that by simply reducing the variability in program provision, without necessarily increasing the average program expenditure, can be welfare enhancing.

Our results contribute to the literature in three important ways. First, we supplement the general literature that studies compression of wage increases due to downward nominal wage rigidity (Elsby, 2009; Stüber and Beissinger, 2010). These studies model wage rigidity through a loss in worker productivity following a downward revision in nominal wages. In their model, firms compress wage increases today if they anticipate future wage cuts that may prove costly to implement. We share the same

⁵We assume risk neutral preferences for both employers and workers. Incorporating risk averse preferences will only exacerbate the loss in utility associated with the variability in program provision.

⁶We address the usual concerns regarding endogeneity. We use standard deviation in program expenditure as a measure of variability in program provision.

insight as these studies except that our objective is to associate the compression of wage increases with the *need* to cut wages in the future due to a decline in program provision; downward nominal wage rigidity is assumed in our model. A similar framework can be used to assess other policy interventions that affect labor market outcomes under downward nominal wage rigidity.

Second, we contribute to the recent theoretical literature on employment guarantee schemes by explicitly incorporating (i) partial implementation of the workfare and (ii) temporal variability in program provision. While [Basu et al. \(2009\)](#) and [Imbert and Papp \(2015\)](#) acknowledge rationing, their comparative static framework does not allow for a discussion on market outcomes due to the temporal variability in rationing and the ensuing uncertainty.⁷ On the other hand, studies like [Basu \(2013\)](#) and [Bahal \(2016\)](#) discuss labor market outcomes under a productive EGS in a framework with more than one period. However, since both the studies assume complete program implementation, there is no role for variability in program provision.

Finally, our study highlights the need to better understand the role of expectations in determining the labor market effects of large employment guarantees. If the continued existence of a workfare alleviates uncertainty regarding the level of program provision, then part of the increase in wages should simply be due to the accompanying decompression of wage increases. This may be a relevant factor that explains why recent empirical studies have found negligible contemporaneous impact of NREGA on wages in contrast to a sizeable impact over time ([Berg et al., 2013](#); [Shrivastava, 2015](#); [Bahal, 2016](#)).

The next section discusses some empirical regularities regarding recent employment guarantee programs and rural labor markets in India. Section 3.3 and 3.4 respectively discuss the model and the welfare implications of a variable and inconsistent employment guarantee. Section 3.5 tests the key empirical predictions of the model. Section 3.6 concludes.

2 Descriptive Evidence

This section discusses the key empirical regularities regarding employment guarantee schemes and nominal agricultural wage changes in India. We first present evidence that work under NREGA is rationed. Next, we discuss temporal variation in the level of program provision to highlight that employment provision is susceptible to deteriorate in the future. This evidence is important as a decline in program provision in the future

⁷[Basu, Chau, and Kanbur \(2009\)](#) for example take the choice of location and the ease off access as an implicit rationing device by the government.

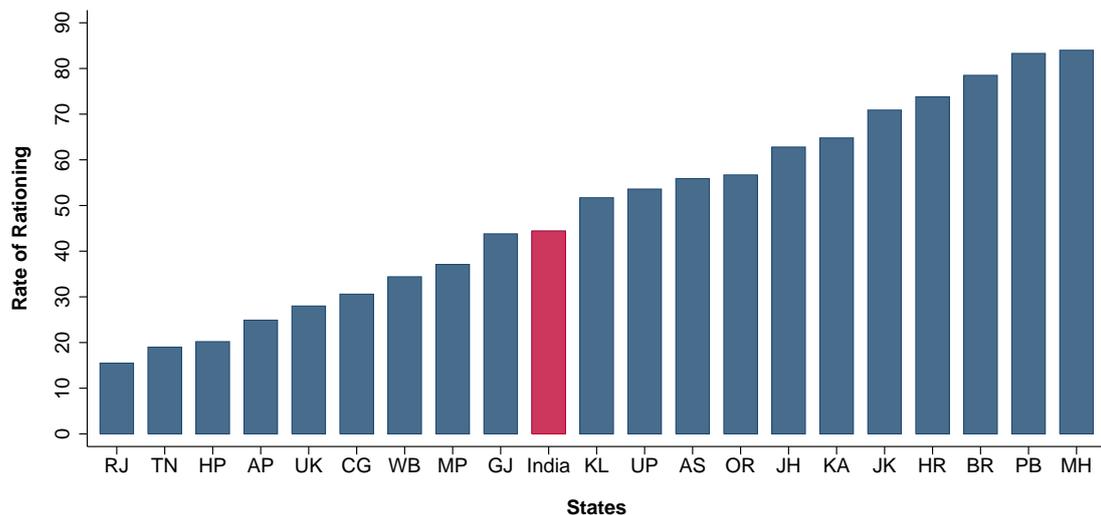
represents the *need* for future wage cuts in order to partly reverse the increase in wages today. In the presence of downward nominal wage rigidity, this ultimately leads to the compression of wage increases.

We further show that the level of program provision today is not very informative of the level of program provision in the future. This underscores the uncertainty associated with a variable and inconsistent program. Finally, we present evidence of downward nominal wage rigidity in rural labor markets in India. Together with these empirical regularities, we develop our theory in the next section which is later subject to empirical validation that completes the paper.

2.1 Rationing in Employment Guarantee Schemes

Although NREGA guarantees 100 days of low-skilled manual labor work to every rural household, work under NREGA is substantially rationed. Using evidence from the nationally representative National Sample Survey (NSS), [Dutta et al. \(2012\)](#) estimate rationing rates – defined as the proportion of workers who demand but do not get work under NREGA – for all the states of the country. [Figure 1](#) shows the rationing rates across Indian states calculated using estimates from [Dutta et al. \(2012\)](#). As the figure shows, while there is significant variation across states, rationing for work under NREGA is substantial with the rationing rate of $\approx 44\%$ at the national level.

Figure 1: Rationing rates in NREGA across Indian states



Using data from a RICE survey conducted for 70 villages in Gujarat, Rajasthan, and Madhya Pradesh, [Imbert and Papp \(2014\)](#) also note that “households with at least one member employed under the act . . . report a mean of only 38 days of work . . . well

below the guaranteed 100 days”. Moreover, not all the demand for work is even documented. As a World Bank study notes, many job card holders do not actively demand for work and instead passively wait for work to be provided (The World Bank, 2011). Overall, there is conclusive evidence that demand for work under such employment schemes is rationed. In fact, rationing excess demand for work may be inevitable under such large programs if the government does not have a lot of flexibility in changing the wage rates.

If wages are fixed at socially acceptable level and the budget for employment guarantees is limited, then the amount of employment provided under the EGS has to be adjusted accordingly. However, since both supply and demand of EGS work determine the level of program provision, it is important to know whether differences in program provision are due to the differences in the demand for EGS work. The evidence from recent studies, however, suggests that variation in employment provision is mostly due to supply factors like administrative capabilities and not from demand factors like local economic conditions or the level of poverty.

Imbert and Papp (2014) for example show that a host of district and worker level characteristics are unable to account for the stark differences in the level of employment provision across districts.⁸ Similarly, Dutta et al. (2012) find the number of NREGA days provided to be only weakly correlated with the level of poverty. Hence, there is evidence to support that fluctuations in program provision are mostly supply driven like bureaucratic idiosyncrasies, political will, or experience in managing similar programs in the past.

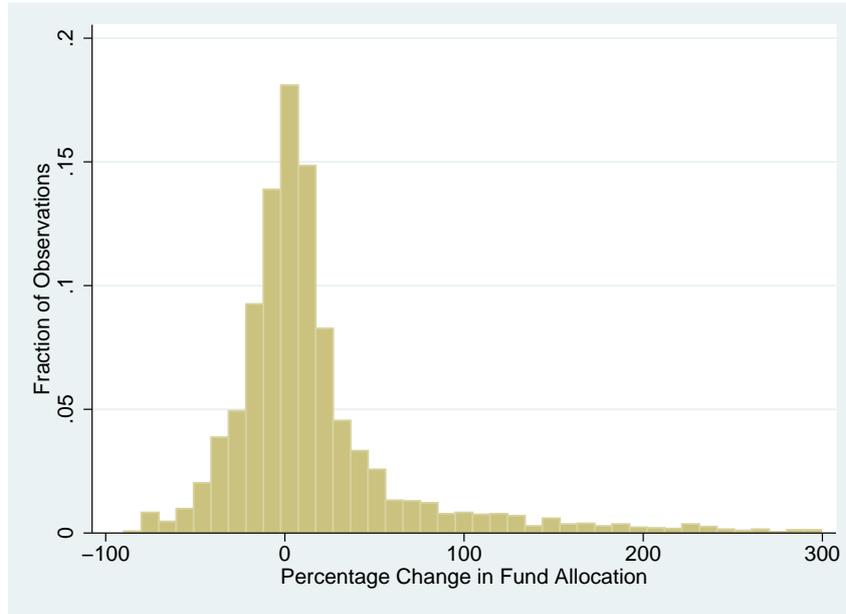
2.2 Inconsistent and Variable Employment Guarantee

In this section, we show that the program provision in the future can deteriorate vis-à-vis its level today. The existence of such temporal variability in program provision is important since the compression of wage increases in our model is associated with the need to cut (downwardly rigid) wages in the future because of a decline in program provision. Using employment expenditure data from Bahal (2016), we use the funds made available at the start of each fiscal year as a measure of the level of program provision.⁹ Figure 2 shows the distribution of year-on-year growth of funds made available for two recent employment guarantee schemes in India at the district level from 2001-2010.

⁸Their district controls include for e.g., the rate of literacy, poverty rate, agricultural productivity, the level of wages, local elections, etc. Worker controls include age, sex, education, marital status, etc.

⁹Given that program provision is largely noted to be supply driven, funds made available at the start of a fiscal year are a good indicator of the intended level of program provision at the district level.

Figure 2: Distribution of Fund Allocation Changes



The graph shows the distribution of year-on-year percentage change in real per capita fund allocation made under SGRY and NREGA. The unit of observation is a district-year. Data is for 442 districts from 2001-2010 (4420 observations). Fund allocation changes are top coded at 300%.

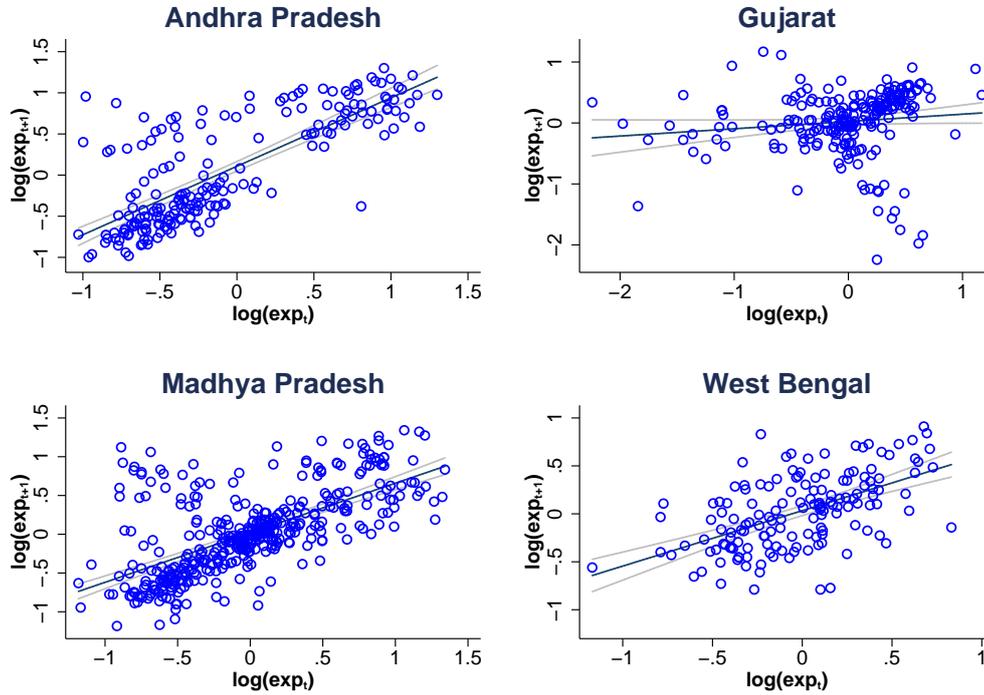
As can be seen, there are substantial fluctuations in fund allocation growth. While the distribution is skewed towards the right with an average of 16% growth, a standard deviation of 50% indicates a sizeable probability with which program provision may decline in the future. This is reaffirmed from the substantial mass of the distribution that lies to the left of zero.¹⁰ Hence, the program provision in all likelihood may deteriorate in the future vis-à-vis its level today.

Further, there is a high level of uncertainty associated with forecasting the level of program provision in the future. We expand on this by employing actual program expenditure data at the district level for 10 years from Bahal (2016). Figure 3 highlights the uncertainty involved in predicting the future level of program expenditure based on the current expenditure levels. The figure plots the lead versus the current program expenditure (in logs) for 109 districts of four major states of India. As can be seen, actual expenditure in the future can be substantially higher or lower than the predicted 95% confidence interval obtained from a linear regression of future expenditure $\ln(e_{i,t+1})$ on current expenditure $\ln(e_{i,t})$.¹¹ We use this evidence to introduce uncertainty in our

¹⁰More than one-third of the observations correspond to a negative growth in fund allocation. There is considerable heterogeneity in the probability of a decline in fund allocation across states, from just 15% for Andhra Pradesh to nearly 50% for Haryana.

¹¹The regression also controls for district and year fixed effects along with a binary variable I_{NREGA} which indicates whether or not NREGA was implemented in a district by year t .

Figure 3: Predicted versus actual expenditure using past expenditure



model later in the next section.

2.3 Downward Nominal Wage Rigidity

Finally, our assumption of downward nominal wage rigidity follows from the evidence presented in Kaur (2014) regarding year-on-year changes in nominal agricultural wages in India. Figure 4 (from Kaur, 2014) shows nominal wage changes for 256 districts over a period of 30 years.

The distribution of wage changes exhibits large clustering at zero and a discontinuous drop to the left of zero. This may be indicative of potential wage cuts that did not materialize due to downward rigidities.¹² Kaur further concludes that while positive rainfall shocks increase nominal wages, negative shocks do not result in wage cuts. Such asymmetric wage adjustment to labor demand shocks is consistent with downward rigidities.¹³

¹²A large cluster at nominal zero changes is not expected under a continuous distribution of shocks (McLaughlin 1994, Kahn 1997). In contrast, Kaur (2014) shows the distribution of real agricultural wage changes to be symmetric around zero.

¹³Survey evidence from Kaur further suggests that nominal wage cuts are regarded as unfair by agricultural workers and employers.

Figure 4: Changes in nominal wages



Source: [Kaur \(2014\)](#). The figure shows the histogram of year-on-year percentage changes in nominal agricultural wages for 256 districts from 1956-1987 (7,680 observations) taken from World Bank Climate and Agriculture dataset. Wage changes are top coded at 50% and bottom coded at -50%.

3 Model

We construct a model of the labor market at the village level. We assume labor to be of measure 1. Wage from private employment is w . The cost of working c is distributed uniformly over the interval $[0, C]$. This cost includes the cost of effort and other costs such as the opportunity cost of not migrating. The utility of private employment is given by $u = w - c$. We normalize reservation utility as 0. For those who are employed, $u \geq 0$, which implies $c \leq w$. Labor supply in the village is given by the fraction of people who are willing to work in private employment at wage w : $l = w/C$. Hence, the inverse labor supply is given by $w(l) = Cl$. Below we make our first assumption.

Assumption 1. Employers have market power.

Even with perfect competition, it is possible that the wage increases due to an EGS are compressed. This is because wages may already be higher than the current competitive level due to business cycle fluctuations and downward wage rigidity. As [Elsby \(2009\)](#) notes “...even in the absence of forward-looking behavior, downward wage rigidity raises the level of wages that firms inherit from the past. As a result, firms do not have to raise wages as often or as much to obtain their desired wage level.” However, following the literature on agricultural labor markets in India, perfect

competition may be a strong assumption.¹⁴ Further, under perfect competition, the introduction of a more intense public employment program like NREGA should result in an immediate increase in wages.¹⁵

However, the recent empirical literature on NREGA (see for e.g., [Berg et al., 2013](#); [Shrivastava, 2015](#); [Bahal, 2016](#)) does not find any immediate adjustment of wages due to the introduction of NREGA. This too is evidence against perfectly competitive agricultural labor markets in India. Therefore, we assume employers to have some degree of market power. For the ease of algebra, we assume a monopsonistic labor market structure. However, our results do not depend on the degree of imperfect competition. We show in the Appendix that our results are also valid under oligopsony.

We model the agricultural production function as increasing and concave in l . For ease of calculation, we use a quadratic function. The profit of the employer is given by

$$\pi = a(l - l^2/2) - wl = a(l - l^2/2) - Cl^2$$

where a signifies labor productivity. The employer chooses the optimum amount of labor to maximize its profit. We can use the first order condition to get this optimum level, which will be the equilibrium level of employment and which will also give us the equilibrium wage.

$$l_0^* = \frac{a}{a + 2C}$$

$$w_0^* = \frac{aC}{a + 2C}$$

As expected, both employment and wage increase with productivity a . On the other hand, wage increases while employment decreases with cost C .

3.1 EGS

Now we introduce an EGS into the model. We assume that the government, under the EGS, offers a wage w_g . The wage is set at a socially acceptable level that is higher than the prevailing market wage w_0^* and the government cannot change it.¹⁶ This implies

¹⁴For example, [Bardhan \(1984\)](#) notes: "... in about 45 percent of our sample villages (in West Bengal) 7 or fewer employers account for most of the casual labor employment in the village ... In many villages there is open or tacit collusion of big employers in the labor market" (p.60).

¹⁵In a competitive setting, large employment schemes can increase private wages in order to clear the labor supply and demand ([Imbert and Papp, 2015](#)). Even in a multi-period environment, there will still be a complete upward adjustment of wages since the benefit of raising wages now outweighs the expected decline in future profits ([Kaur, 2014](#)).

¹⁶Having an EGS wage lower than the prevailing market wage in our model would render the EGS completely ineffective as it will be unable to attract any workers. In practice, the EGS wage is linked

that everyone employed at wage w_0^* , as well as some of those not employed would want to work in the EGS at the higher wage w_g .

Let e_0 be the expenditure required to employ everyone who wants a job under the EGS. We assume that the actual budget e is lower than this and hence everyone cannot be employed. The people who do not get employed in the EGS despite wanting to work in it are said to be rationed out and the rate of rationing is given by

$$r = 1 - \frac{e}{e_0}$$

As mentioned earlier, the rate of rationing is around 40% for India on average (Dutta et al., 2012), justifying the assumption that $e < e^0$.

We assume that the government randomly selects the people who do get employed in the EGS. This means that for the workers wanting to work in the EGS, the cost of working c is not correlated with the probability of getting to work in the EGS (we show in the Appendix that relaxing this assumption does not change the results).

The new labor supply for private employment consists of two parts. For the private wage w less than the EGS wage w_g , all the people working in private employment at this wage when there was no EGS will now want to work in the EGS. However, a fraction r of them will be rationed out and they will form the labor supply for private employment. For $w > w_g$, everyone will want to work in the higher paying private employment and the situation will be the same as when there was no EGS.

$$l = \begin{cases} \frac{wr}{C} & \text{if } w \leq w_g \\ \frac{w}{C} & \text{if } w > w_g \end{cases}$$

This is shown graphically in Figure 5, which depicts the discontinuity in the labor supply at $w = w_g$.

Hence, the new inverse labor supply is given by

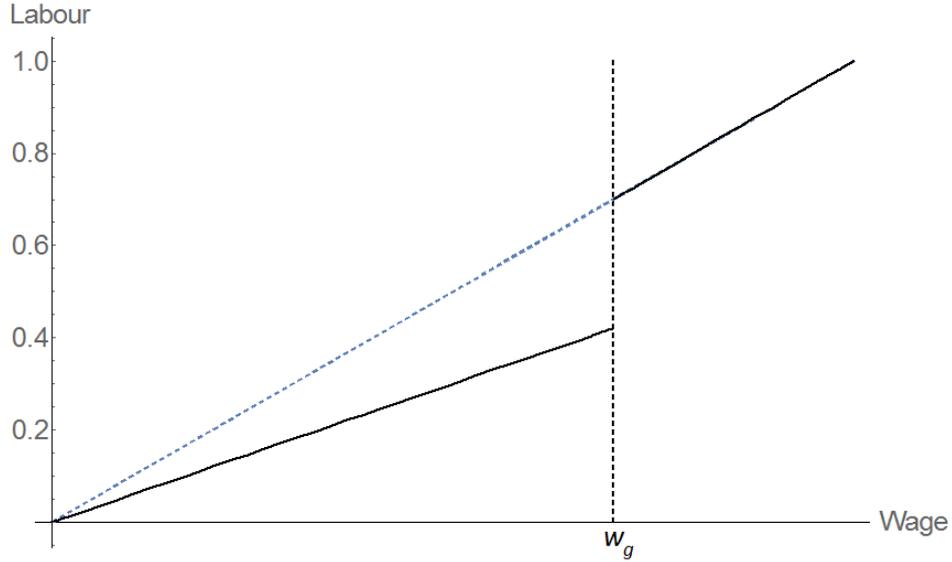
$$w(l) = \begin{cases} \frac{Cl}{r} & \text{if } w \leq w_g \\ Cl & \text{if } w > w_g \end{cases}$$

Using the F.O.C. of the employer's optimization problem, we get the equilibrium private sector employment and wage.

$$l_1^* = \min \left\{ \frac{ar}{ar + 2C}, \frac{w_g r}{C} \right\} < l_0^*$$

to the statutory minimum wage, which is generally much higher than the actual wages being paid, and which the government does not have too much flexibility in changing.

Figure 5: Labor supply for private employment when there is an EGS with wage w_g



Note: The upward sloping dashed line represents the labor supply when there is no EGS. The solid line is the labor supply for private employment when there is an EGS with the EGS wage being 70 and rationing rate 0.6. The labor supply only includes the workers being rationed out of the EGS as long as the private wage is below the EGS wage. When the private wage exceeds the EGS wage, the labor supply reverts to the original case as workers no longer prefer working in the EGS.

$$w_1^* = \min \left\{ \frac{aC}{ar + 2C}, w_g \right\} > w_0^*$$

This shows that wage increases and private employment decreases as the EGS is implemented. Overall employment will increase as the EGS employs all the people who left private employment and also some people who were not previously employed in the private sector. We can also see that in the case where $w_1^* < w_g$, $\frac{dw_1^*}{dr} < 0$, which implies $\frac{dw_1^*}{de} > 0$. Hence, ceteris paribus, as the government expenditure increases, rationing decreases and the wage in the private sector increases. In the case where $w_1^* = w_g$, any further expenditure on the EGS will be ineffective as it will only shift employment from the private sector to the EGS without causing any change to total employment.

For simplifying the rest of the analysis, we will assume that the EGS wage w_g is higher than the maximum possible value of the optimal private wage w_1^* , i.e. $w_g \geq \frac{a}{2}$. Since the EGS wage is fixed and the analysis does not seek to find the impact of changes in the EGS wage on the private wage, relaxing this assumption will not affect the results obtained. Hence, we can now write the optimal private wage and private employment in the simplified form.

$$\begin{aligned}
l_1^* &= \frac{ar}{ar+2C} < l_0^* \\
w_1^* &= \frac{aC}{ar+2C} > w_0^*
\end{aligned}
\tag{1}$$

3.2 Two-period Model

Now we move from a static model to a two-period model of the labor market with an EGS. As we have shown earlier, rationing rates vary a lot within a district. Hence the rationing rates in the two periods are likely to be different. Let the employer observe the rationing in the first period, t . Let us assume for now that the employer is naive and sets the wage in the first period at the statically optimum level described in the previous section. In the second period, $t + 1$, if the rationing rate is lower than that in the first period, then the labor supply would decrease and the optimum wage would be higher than that in the first period. Conversely, if the rate of rationing increases then the optimum wage would be lower. Here we introduce the downward rigidity constraint on wages.

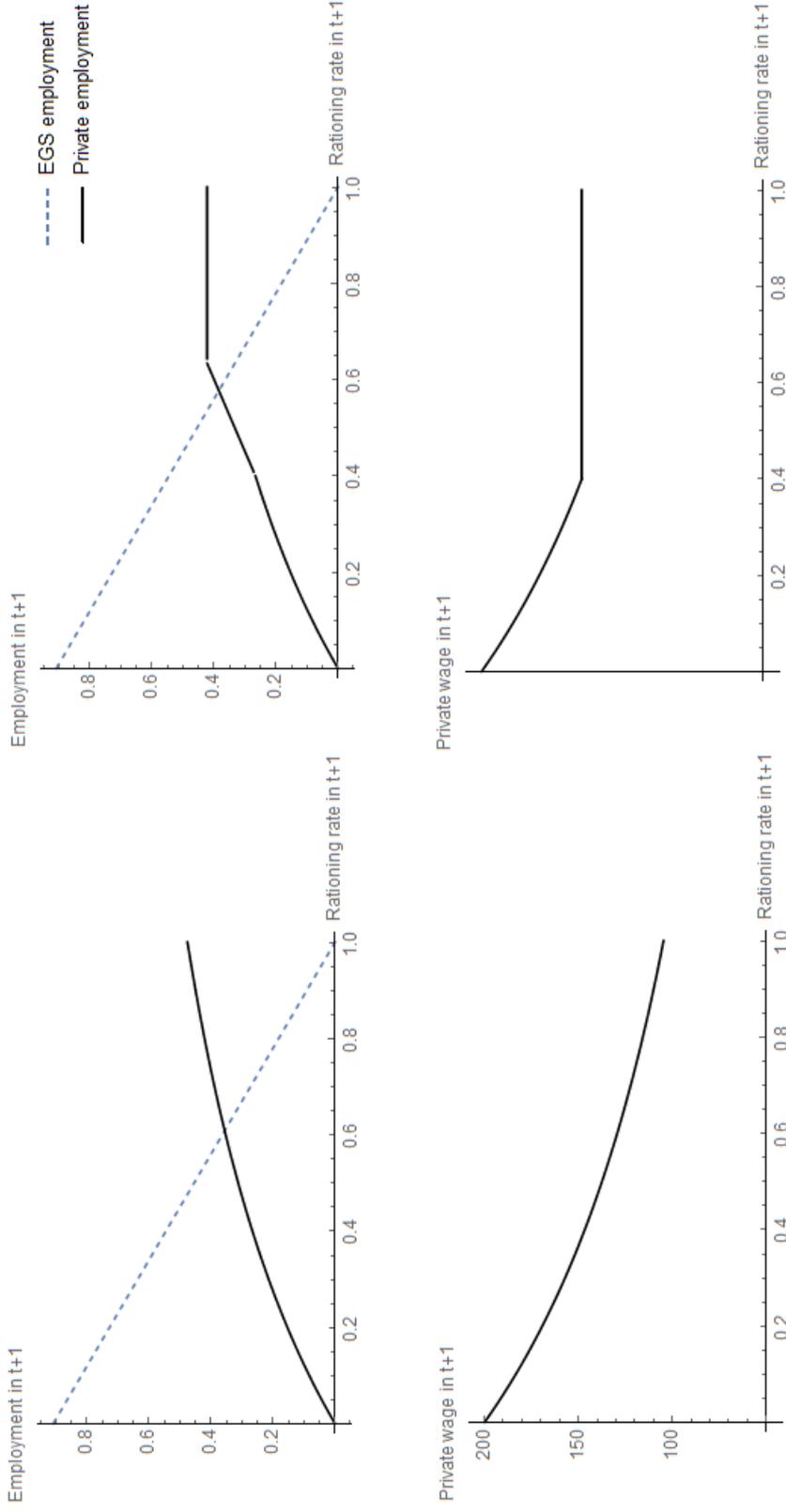
Assumption 2. Private wages are downwardly rigid i.e. wages cannot decrease from period 1 to 2.

We have discussed the supporting evidence for this earlier with [Kaur \(2014\)](#). We are simplifying the treatment of downward rigidity in two ways. First, we are considering perfect rigidity, i.e. we are, by construction, preventing wages from decreasing. In a more complex model, one could introduce a cost of wage reduction, for example, reduced productivity because of perceived unfairness. Here we abstract from this by assuming that such costs are large enough to prevent any downward revision of wages. Secondly, we are abstracting away from price changes and differences between real and nominal wage changes. We will show later that our results will continue to hold, albeit more weakly, in the case of price inflation.

Now the naive employer, having set the first-period wage at the statically optimum level, observes the rationing rate in the second period and then tries to optimize given the rigidity constraint. [Figure 6](#) shows how the employment and wage in period $t + 1$ change with the rationing rate r_{t+1} , when there is no rigidity and when there is rigidity.

The graphs on the left show the “no rigidity” case. As r_{t+1} increases, the EGS employment decreases linearly. The private employment increases and private wage decreases as given by the expressions for l_1^* and w_1^* , the optimal employment and wage in the static case. The graphs on the right show the case with downward wage rigidity, with rationing in the first period $r_t = 0.4$. Here, the private employment increases and

Figure 6: Employment and wages in the second period, with (right) and without (left) downward wage rigidity



Note: The dashed sloping line in both employment graphs is the employment in the EGS which mechanically decreases from 1 to 0 as rationing increases from 0 to 1. The solid line is the employment in the private sector. The graphs on the left show the case without downward nominal wage rigidity where private employment, as well as private wage, are smooth curves as the employer adjusts both optimally. The graphs on the right show the case with nominal wage rigidity with the rationing rate in the first period assumed to be 0.4, and the wage in the first period being set as optimal for that period only. Both private wage and private employment for rationing rate less than 0.4 are exactly the same as the case without rigidity. After that, the rigidity constraint binds and the employer is unable to lower the wage further. Hence, the private employment increases linearly absorbing a fixed fraction of the workers rationed out of the EGS. When the private employment reaches around 0.55, the employer no longer finds it profitable to hire more labor at that wage. Private employment stagnates at this level and any workers who are rationed out because of a further increase in rationing become unemployed.

private wage decreases as before till $r_{t+1} = 0.4$. At this point, the wage w_{t+1} equals the wage in period t and any further increase in r_{t+1} will mean that the wage cannot be further reduced to an optimal level and will be stuck at the first-period wage w_t . As the rationing rate increases and $r_{t+1} > r_t$, more workers are rationed out of the EGS. Some of these workers whose costs are low enough are employed by the private employer. Hence, private employment increases linearly. As r_{t+1} keeps increasing it is possible that the number of workers rationed out of the EGS is higher than the optimal labor requirement of the private employer with the wage fixed at w_t . Let this value of r_{t+1} where the labor supply is equal to the optimal labor requirement for the employer, i.e. where marginal productivity equals wage, be \bar{r}_{t+1} . After this point, the private employment no longer increases although the EGS employment keeps decreasing.

Now we consider the case of the sophisticated employer who takes into account the wage rigidity and the variability of rationing rates and hence optimizes over two periods rather than just one. Let us first assume that the employer knows in advance what the rationing rate is going to be in the second period. In the next subsection, we will consider the more realistic case where the employer is uncertain about future rates of rationing. In the present case, the employer maximizes the sum of the profits in the two periods.

$$\Pi = a(l_t - \frac{l_t^2}{2}) - w_t l_t + a(l_{t+1} - \frac{l_{t+1}^2}{2}) - w_{t+1} l_{t+1}$$

The profit maximization is done under the rigidity constraint: $w_t \leq w_{t+1}$. The optimal value of w_t will depend on r_{t+1} . If $r_{t+1} \leq r_t$, then the constraint will not bind, and w_t will be given by the expression for static wage w_1^* given in Equation 1. If $r_{t+1} > r_t$, then the constraint will bind and the second-period wage will no longer be able to be set optimally. There will be a trade-off between profits in the two periods and the optimal w_t in the two-period case will be lower than the corresponding value of w_1^* for the single period case.

Proposition 1. The optimal wage in the first period w_t^ will depend on the value of r_{t+1} in comparison with r_t . For a given value of r_t , r_{t+1} can lie in one of three ranges ¹⁷:*

1. For $r_{t+1} \leq r_t$,

$$w_t^* = \frac{aC}{ar_t + 2C} = w_1^*$$

¹⁷We show in the appendix that when both r_t and r_{t+1} are known, then $\bar{r}_{t+1} = \frac{r_t(ar_t + C)}{(ar_t - C)}$. Note that the third range will only arise for values of $r_t \in [0, 1]$ that are such that $0 \leq \frac{r_t(ar_t + C)}{(ar_t - C)} \leq 1$.

2. For $r_t < r_{t+1} \leq \bar{r}_{t+1}$,

$$w_t^* = \frac{aC}{a\phi(r_t, r_{t+1}) + 2C} < w_1^*$$

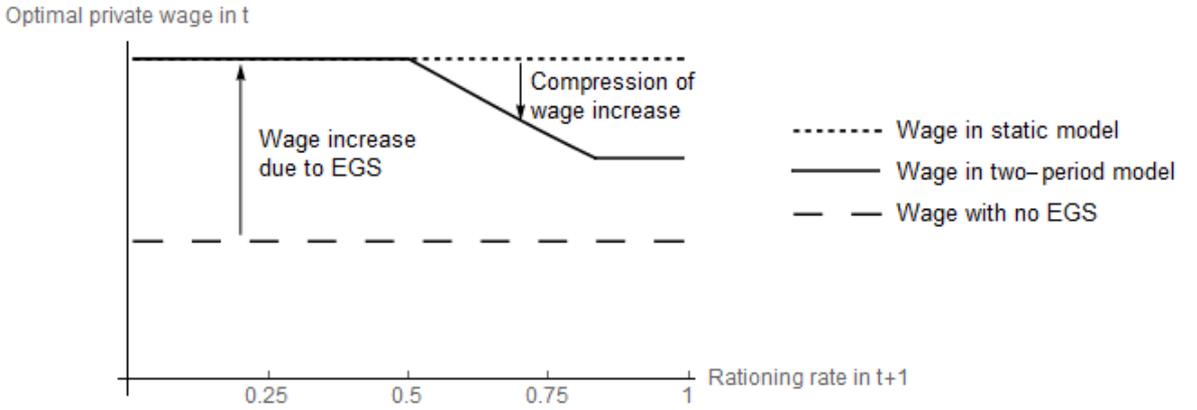
Where,¹⁸ $\phi(r_t, r_{t+1}) = \frac{r_{t+1}^2 + r_t^2}{r_{t+1} + r_t} > r_t$ for $r_t < r_{t+1}$

3. $r_{t+1} > \bar{r}_{t+1}$,

$$w_t^* = \frac{aC}{a\phi(r_t, \bar{r}_{t+1}) + 2C} < w_1^*$$

The proof is given in the Appendix. Note that the optimal private employment l_t^* will change proportionally to w_t^* and will hence be lower than the statically optimum value in the second and third ranges.

Figure 7: Wage in the three ranges showing compression of wage increases



Note: The graph shows the increase in private wage due to the EGS in the static case as the gap between the optimal wages in the 'No EGS' and the static models. The solid line representing the optimal wage in the two-period model shows how this wage increase gets compressed once the rationing rate in the second period exceeds that in the first period, which is assumed to be 0.5 here. Further, it shows that the optimal wage in the two-period model stops declining with the rationing rate after a threshold where it enters the third range.

Figure 7 illustrates the three cases. The horizontal axis is r_{t+1} and the vertical axis is w_t^* . The optimal wage in the absence of an EGS, w_0^* , is also shown for comparison. The graph is drawn for $r_t = 0.5$. For $r_{t+1} \leq r_t$, we are in range 1 and the wage increase is highest. For $r_{t+1} > r_t$ we enter range 2 where the increase in wage gets compressed and the compression increases as r_{t+1} increases. For even higher values of r_{t+1} we enter range 3. The wage increase does not get compressed any further and w_t^* is constant at its minimum value.

¹⁸We also see that $\phi(r_t, r_{t+1}) < 1$. Hence, $w_t^* > w_0^* = \frac{aC}{a + 2C}$

In the analysis above we have abstracted away from price changes. The empirical observations are regarding downward rigidity in *nominal* wages. Hence if there is price inflation, the rigidity condition will change. Let all the variables above signify real values. Let the price level in the first period be 1 and that in the second period be $1 < p < (a + 2C)/(ar_t + 2C)$. The rigidity condition would now be $w_t \leq pw_{t+1}$. The only difference this would make in the analysis above is to change the bounds of the three ranges. Range 1 would be larger, $r_{t+1} \leq pr_t + 2C(p - 1)/a$ while ranges 2 and 3 would be smaller at $r_{t+1} > pr_t + 2C(p - 1)/a$. Hence, the results that we obtain assuming zero inflation would still be qualitatively valid with price inflation.

3.3 Uncertainty

Up to now we have assumed that the employer has perfect information about the rate of rationing in the next period. In reality, however, there is uncertainty associated with forecasting the level of program provision in the future (see Figure 3). Therefore, we make the following assumption.

Assumption 3. There is uncertainty about future rates of rationing.

Introducing uncertainty in the two-period model implies that there is always a probability that the rate of rationing may be higher in the future. We will show that this uncertainty leads to a reduction in the optimal wage compared to the case when the rationing rate is fixed.

Lemma 1. There is a decrease in the optimal wage in the first period, w_t^ , when we move from the case when it is known with certainty that rationing is going to be the same in both periods i.e. $r_t = r_{t+1}$, to a case where there is uncertainty such that there is a chance that rationing may go up, i.e. $P[r_{t+1} > r_t] > 0$.*

We show the proof in the Appendix but the intuition is evident. As soon as there is a small probability that the optimal wage in the second period is going to be lower, the nominal rigidity constraint binds and the statically optimal wage in the first period is no longer optimal for both periods. The new optimal wage would be lower, trading off some reduction in profit in the first period with increased profit in the second period.

Lemma 1 deals with the extensive margin of uncertainty, i.e. what happens when we move from certainty to uncertainty. Now we look at the intensive margin, i.e. what happens when uncertainty increases. To do this we need to define what we mean by more uncertainty.

When there is uncertainty about future rationing, the optimization by the employer will be done based on expectations about future rationing. Let the employer believe

that the rationing rate is drawn from a distribution. We can think of uncertainty as the variance of this distribution. A distribution with zero variance would imply a fixed rationing rate and would correspond to certainty as discussed earlier. As the variance increases the uncertainty about future rationing increases. Let us assume for simplicity that r is uniformly distribution with the support $2x$, given by $U[m - x, m + x]$, where m is the mean of the distribution. Increasing x increases variance while preserving the mean, hence we can use x as a measure of uncertainty. We want to see how an increase in x changes the optimal wage.

The employer maximizes the current profit and the expected future profit given by

$$\Pi = a(l_t - \frac{l_t^2}{2}) - w_t l_t + E[a(l_{t+1} - \frac{l_{t+1}^2}{2}) - w_{t+1} l_{t+1}]$$

To calculate the expectation, we will need to divide the distribution of r_{t+1} into three parts corresponding to the three ranges listed in the previous subsection. The first section is where $w_{t+1} \geq w_t$ and it extends from $m - x$ to the point where $w_t = w_{t+1}$. Let this value of r_{t+1} be $r'(w_t)$.¹⁹ The second section extends from $r'(w_t)$ to a point where wage equals marginal productivity. Let this point be $r_{t+1} = r''(w_t)$.²⁰ The third section extends from $r''(w_t)$ to $m + x$. Hence the expression for the sum of current and expected profit is

$$\begin{aligned} \Pi(w_t, r_t, m, x) = & \pi(w_t, l(w_t, r_t)) + \frac{1}{2x} \left(\int_{m-x}^{r'(w_t)} \pi(w_{t+1}^*, l(w_{t+1}^*, r_1)) dr_1 \right. \\ & + \int_{r'(w_t)}^{r''(w_t)} \pi(w_t, l(w_t, r_2)) dr_2 + \left. \int_{r''(w_t)}^{m+x} \pi(w_t, l_{max}(w_t)) dr_3 \right) \end{aligned}$$

Where,

$$\begin{aligned} \pi\{w, l\} &= a(l - \frac{l^2}{2}) - wl \\ l(w, r) &= \frac{wr}{C} \\ w_{t+1}^* &= \frac{aC}{ar_1 + 2C} \\ l_{max}(w) &= \frac{a-w}{a} \end{aligned}$$

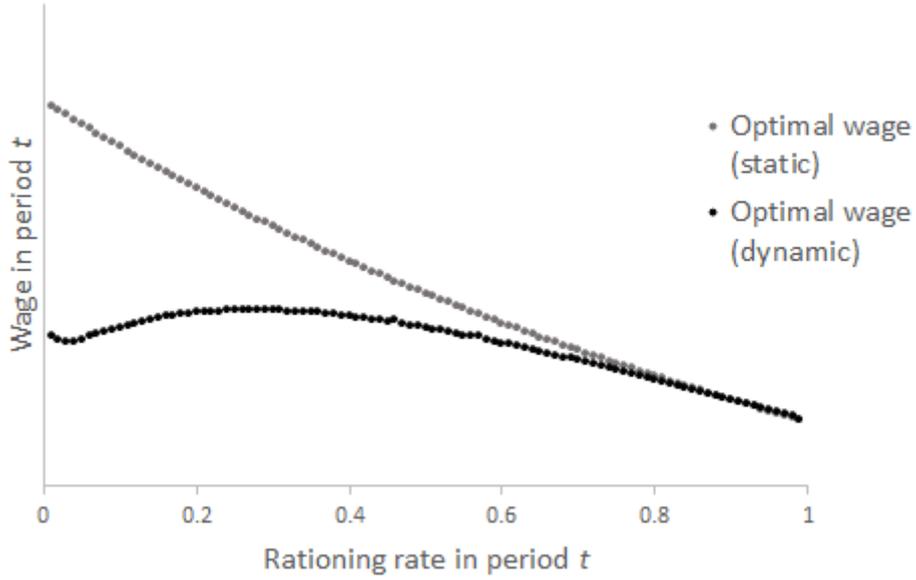
The profit maximizing value of w_t will be a function of r_t , m and x . [Figure 8](#) shows

¹⁹The expression for $r'(w_t)$ is obtained using $w_t = w_{t+1} = \frac{ac}{ar' + 2c}$.

²⁰ $r''(w_t)$ is given by wage equals marginal productivity $w_t = a(1 - l_{t+1}) = a(1 - \frac{w_t r''}{c})$.

how the dynamically optimal wage obtained by maximizing the expression above, as well as the statically optimal value given by the expression for w_1^* in Equation 1 varies with r_t . Hence, the compression of the wage increase, represented by the difference between the statically and dynamically optimal wages is different for different values of r_t . The compression is high for low values of r_t as the probability of $r_{t+1} > r_t$ is higher, and vice versa.

Figure 8: Variation of statically and dynamically optimum wages in period t with rationing rate in period t



Note: The optimal values have been numerically computed for a uniform distribution over $[0,1]$, i.e. for $m = 0.5$ and $x = 0.5$. The statically optimal wage is given by the formula in Equation 1, and the dynamically optimal wage is obtained by numerically locating the value of w_t that maximizes $\Pi(w_t, r_t, m, x)$

We want to show that the dynamically optimal wage, $w_t^*(r_t, m, x)$, decreases with an increase in variance represented by an increase in x . Since the algebra is quite complicated, we provide the analytical solution for a special case, when rationing rate in the first period is the mean of the distribution, i.e. $r_t = m$.

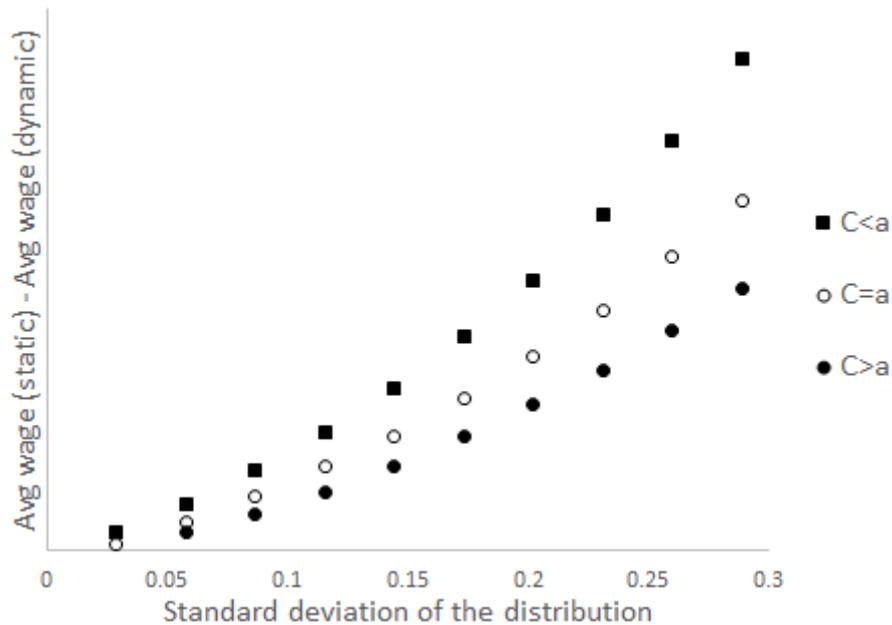
Proposition 2. The profit maximizing value of the wage at the mean of the distribution of r , expressed as $w_t^|_{r_t=m}$ decreases with an increase in variance represented by an increase in x , i.e. $\frac{dw_t^*|_{r_t=m}}{dx} < 0$.*

We show the proof in the Appendix using the implicit function theorem.

When we think about the welfare implications of a change in uncertainty, we would be concerned about the average change in wage across the entire distribution of r rather than just at the mean. To show a result similar to Proposition 2 for the average wage

across the distribution, we conducted simulations using different values of the parameters. The average wage is obtained by integrating the optimal wage $w_t^*(r_t, m, x)$ over the uniform distribution $[m - x, m + x]$. This average wage is now only a function of m and x . The simulations were used to compute this average optimal wage for different distributions with m fixed at 0.5 but changing values of x . Figure 9 shows the results of the simulations for three cases, $C < a$, $C = a$, and $C > a$. We find that the average optimal wage (dynamic) obtained from maximizing Π , the sum of current and expected future profits, is always lower than the average optimal wage (static) obtained by maximizing just the current profit. More importantly, we also find that this difference increases with an increase in variance signified by an increase in x .

Figure 9: Graph showing increase in wage compression with increase in variance



Note: The horizontal axis shows the standard deviation, which is equal to $x/\sqrt{3}$, of the uniform distribution with mean $m = 0.5$. The vertical axis is the difference between the statically optimal and dynamically optimal wages averaged over the entire distribution of r_t under an EGS.

This result further strengthens our intuition that increasing uncertainty will lead to increased compression of the wage increases due to the EGS. This is an important result as the government's average expenditure is directly correlated with mean rationing rate - higher average expenditure will imply lower mean rationing. But the variance of rationing is related to the variance of expenditure (keeping the EGS wage constant). Our results hence suggest that by simply lowering the variability in the provision of public employment while keeping the average expenditure constant, the government can achieve higher private wages and employment.

4 Welfare

We now look at how the compression in the wage increase impacts welfare. The social welfare function consists of three main components representing the utility of the employer, the utility of the workers, and the cost of the program. Additionally, we would also like to incorporate the social planner's distributional preference over the relative importance of the utilities of the employer and the workers. This can be done in two ways. One is to attach weights directly to the utilities of the employer and the workers. The other way, as used by [Basu, Chau, and Kanbur \(2009\)](#), is to not weight the utilities but have an additional weighted component representing employment, to account for the social planner's concern with increasing employment using an EGS. The results shown in this section would be valid for either formulation of social welfare but we follow [Basu, Chau, and Kanbur \(2009\)](#) as the implementation of an EGS by a government indicates its interest in increasing employment. We define the social welfare function as consisting of the following parts

1. The utility (profit) of the employer (π)
2. The utility of the workers (W)
3. The benefit of increasing employment (ηE)
4. The cost of implementing the program (μB)

Note that as we are using risk neutral utility functions, all four parts can be directly interpreted in money terms.

$$SW = \pi + W - \mu B + \eta E$$

where, $E = (l_p + l_g)$; l_p and l_g are fractions of the labor pool employed in the private sector and the EGS respectively,

$\eta > 0$ signifies the marginal social benefit of employment in addition to the direct benefit through the workers' utility,

$B = w_g l_g$ is the money spent on wages paid in the EGS,

$\mu > 1$ is a multiplier that incorporates as a proportion of the wage bill, the administrative cost of running the EGS, as well as the marginal cost of public funds to account for distortionary taxation.

4.1 No EGS Case

In this case, we first compute the expressions for the employer's profit and for the workers' utility using the expressions for equilibrium wage and employment derived earlier.

Profit is

$$\pi_0 = a(l_p - l_p^2/2) - wl_p = \frac{a^2}{2(a+2C)}$$

Workers' utility is

$$W_0 = \int_0^{l_p} (w - c) dx$$

We integrate the employed worker's utility over the range 0 to l_p . The assumption is that the utility of the workers not working in either the private sector or the EGS is 0. Since the cost c is uniformly distributed over $[0, C]$, we can write $c = xC$. Thus integrating we get

$$W_0 = wl_p - \frac{Cl_p^2}{2} = \frac{a^2C}{2(a+2C)^2}$$

The benefit from employment is

$$\eta E_0 = \eta l_p = \frac{\eta a}{a+2C}$$

Social welfare is the sum of π_0 , W_0 and ηE_0 . As there is no EGS the cost μB is zero.

4.2 EGS with Fixed Rationing Rate

With the rationing rate being fixed, there is no uncertainty and employers optimize statically. We assume that the rationing rate is r and that the EGS wage is set at $w_g > w$. The profit π_1 expectedly reduces as the labor supply gets lowered by the EGS leading to an increase in wage and a decrease in private employment.

$$\begin{aligned} \pi_1 &= a(l_p - l_p^2/2) - wl_p \\ &= \frac{a^2r}{2(ar+2C)} < \pi_0 \end{aligned}$$

For the workers, since the current private wage as well as the EGS wage are higher than the private wage in the no-EGS case, and the total employment is higher, it is evident that their utility will increase with the introduction of EGS. Formally, the workers'

utility will have two components, one for workers working in the private sector at wage w and the other for workers working in the EGS at wage w_g .

$$W_1 = \int_0^{l_p} (w - c_x) dx + \int_0^{l_g} (w_g - c_y) dy$$

We know that all the workers working in the private sector would want to work in the EGS but are rationed out. Hence out of all the workers with cost $c < w$, a fraction r would be working in the private sector. Hence, $c_x = \frac{x\mathcal{C}}{r}$. Similarly, out of all workers with cost $c < w_g$, a fraction $(1 - r)$ would be working in the EGS. Hence, $c_y = \frac{y\mathcal{C}}{1 - r}$. Integrating, we get

$$W_1 = wl_p - \frac{Cl_p^2}{2r} + w_g l_g - \frac{Cl_g^2}{2(1 - r)}$$

The employment in private sector and in EGS are given by

$$l_p = \frac{wr}{\mathcal{C}}; l_g = \frac{w_g(1 - r)}{\mathcal{C}}$$

Using the expression for optimal wage under EGS we get,

$$W_1 = \frac{a^2 Cr}{2(ar + 2\mathcal{C})^2} + \frac{w_g^2(1 - r)}{2\mathcal{C}}$$

We can show that when $w_g = w$,

$$W_1 = \frac{a^2 \mathcal{C}}{2(ar + 2\mathcal{C})^2} > W_0$$

Therefore, $\forall w_g \geq w, W_1 > W_0$.

The benefit from employment is

$$\eta E_1 = \eta(l_p + l_g) = \eta \left(\frac{ar}{ar + 2\mathcal{C}} + \frac{w_g(1 - r)}{\mathcal{C}} \right)$$

We can again show that for $w_g \geq w, E_1 > E_0$.

The cost of providing the EGS is

$$\mu B_1 = \mu w_g l_g = \frac{\mu w_g^2(1 - r)}{\mathcal{C}}$$

Hence, the introduction of EGS reduces welfare through lower employer profit and increased cost to the government, and increases welfare through increased worker util-

ity and increased employment. Whether this increases or decreases the total welfare $SW1$ depends on the values of the two parameters μ and η , which depict the inefficiency of the government and the social weight attached to achieving higher employment.

4.3 EGS with Uncertainty

With uncertainty, the employer no longer knows the future level of rationing. Instead, the current optimal wage is based on a distribution of probable future rationing rates. For comparison with the previous case, we assume that the distribution has the same mean rationing rate and it represents an increase of variance from zero. We have shown that uncertainty leads to the compression of the wage increase possible under certainty,²¹ and also that the compression is higher with increased uncertainty. To see how this compression affects welfare, we check how the components of the welfare function as derived for an EGS with a fixed rationing rate respond to a reduction in the wage. The expressions for an EGS with and without uncertainty are comparable as the labor supply function remains the same ($l = \frac{wr}{C}$) and we can compare the two for the same level of rationing.

First, let us consider profit. Compared to the case with a fixed rationing rate, both wage and private employment are reduced. Hence, it is not obvious if the employer's profit will increase or decrease.

$$\begin{aligned}\pi_2 &= a(l_p - l_p^2/2) - wl_p \\ &= a\left(\frac{wr}{C} - \frac{w^2r^2}{2C^2}\right) - \frac{w^2r}{C}\end{aligned}$$

We can show that $\frac{d\pi_2}{dw} > 0$, when w is less than the optimal static wage w_1^* , which is always the case according to Lemma 1. Hence, the employer's profit with EGS under uncertainty π_2 is always lower than that under certainty π_1 . This is because the reduction in production due to a reduction in employment is not offset by the reduction in wages. This is expected since wage is always less than the marginal product of labor when employers have market power.

For the workers, it is obvious that the utility will reduce as both wage and employment are declining.

$$W_2 = \frac{w^2r}{2C} + \frac{w_g^2(1-r)}{2C}$$

²¹By 'certainty' here we imply fixed rationing rates since if rationing rates are not fixed then uncertainty about the future is inevitable.

We can again show that $\frac{dW_2}{dw} > 0$ when $w < w_1^*$. Hence, $W_2 < W_1$.

The cost μB will remain the same for the same EGS wage w_g and the same actual level of rationing r . The benefit from employment will decrease because of lower private employment l_p . Hence, regardless of the values of the parameters μ and η , uncertainty will lead to loss of welfare i.e $SW_2 < SW_1$.

Furthermore, we can use the simulation results presented in Section 3.3 to extend the analysis done above and state that an increase in uncertainty, represented by an increase in the variance of the distribution of r , reduces welfare as it reduces both private wage and private employment. Hence, the government can improve welfare by reducing the variation of EGS expenditure, without changing the average expenditure.

5 Testing the Theory

The theory generates two testable predictions: (i) that average positive marginal effect of the workfare on private wages should be lower when variation in program provision is high; and (ii) that the compression of wage increases should be more severe when inflation is low relative to when inflation is high. The first prediction follows from section 3.3 where we show that greater variability in program provision results in a larger compression of wage increases (Figure 9). Next, even though we abstract from price changes in our model, it is straightforward to see that price inflation makes the constraint of downward nominal wage rigidity less restrictive. This is so since relative to the constant price assumption, price inflation alleviates the need for future wage cuts (due to a decline in program provision) resulting in a smaller compression of wage increases today.

To empirically validate the predictions of the model, we use district-level expenditure data from 2001-2010 for two large public workfare programs in India: SGRY and NREGA as used in Bahal (2016).²² The annual frequency data is reported in financial year format (1 April to 31 March) and includes information on the opening balance, funds made available at the start of the fiscal year, and expenditure incurred over the year. While data on SGRY is from the year of its implementation in 2001 to its last operational financial year 2007-08, data on NREGA starts from 2006-07 to 2010-11.²³ Next, we use district-level wage data on agricultural activities as reported by the Agricultural Wages of India (AWI). The monthly frequency AWI data is from July 2001 - June 2011. We use the daily wage rate data for *field labor* which includes agricultural

²²The data is collected from district-level financial statements of SGRY and NREGA from (i) Ministry of Rural Development (MoRD); (ii) Datanet (India); and (iii) nrega.nic.in.

²³The two schemes never co-existed together in a district.

activities like ploughing, sowing, reaping, and weeding. Since wage rates are reported separately for men and women, we take the average of male and female wages to construct our measure of field wages.²⁴

A shortcoming of the monthly AWI wage series is that it often contains missing data for some of the months. Furthermore, the annual publications of AWI sporadically exclude data for some districts and states.²⁵ We first improve the signal to noise ratio by converting the monthly wage series to annual frequency by taking 12-month averages in the financial year format to match the frequency and the period of the employment expenditure data. Second, we restrict our attention to a complete balanced panel of 134 districts from 12 major states of India. Hence we have 1340 annual observations. We deflate both the wage and employment expenditure data to 2001 prices using state-wise Consumer Price Index for Rural Laborers (CPI-RL) collected by the Labor Bureau, Government of India. Unless otherwise mentioned, all the variables in the empirical analysis are in real, per-capita terms.

5.1 Effect of Variability in Expenditure on Wage Increase

To test the first prediction of our theory, we estimate Equation 2 where the subscripts i and t denote district and year respectively. The dependent variable is agricultural wages $w_{i,t}$ (in rupees), while expenditure under the program $e_{i,t}$ is a measure of program provision (also in rupees). Ex-ante, we expect wages to increase as expenditure under the program increases. For a district, we capture the variability in program provision by the standard deviation σ_i of the expenditure incurred over 10 years $\{e_{i,2001}, \dots, e_{i,2010}\}$.

$$w_{i,t} = \beta_1 e_{i,t} + \beta_2 (e_{i,t} \times \sigma_i) + I_{NREGA} + \alpha_i + \gamma_t + \xi_{it} + \beta \mathbf{X}_{i,t} \quad (2)$$

Since greater variability in program provision is expected to compress wage increases, we expect the marginal effect of expenditure on wages to decrease as σ_i increases. That is, we expect the coefficient of interaction β_2 to be negative. Among other controls, α_i and γ_t represent district and year fixed effects respectively; ξ_{it} controls for district specific trends; I_{NREGA} signifies a change in program regime for a district from SGRY to NREGA (see footnote 11); and $\mathbf{X}_{i,t}$ represents a vector of other controls discussed below. The inclusion of district fixed effects is to address potential endogeneity concerns that may result in biased estimates of β_1 and β_2 if variations in program expenditure are correlated with time-invariant district characteristics. By way

²⁴Although AWI reports wages for children as well, we exclude wages reported under children as most of the observations are missing under this category.

²⁵For example, data for states like Chhattisgarh, Jharkhand, and Uttarakhand is not available before 2005.

of example, if program expenditure in drought prone districts is high on average, then β_1 and β_2 may be spuriously low.

Year fixed effect addresses two main endogeneity concerns. First, it controls for fluctuations in the size of the EGS at the national level that affect all districts. As variations in program provision at national level may be arguably endogenous to cyclical developments, it may lead to spurious estimates due to reverse causation. Secondly, year fixed effects control for extreme weather events that may increase both $e_{i,t}$ and σ_i for all the districts in a year. Failure to control for such aggregate fluctuations may downward bias the estimates. Similarly, we filter out any changes in $w_{i,t}$ that are associated with the phase-wise implementation of NREGA using the indicator variable I_{NREGA} which controls for the regime change from SGRY to NREGA.

Next, district specific linear trends address any potential concerns that may arise if growth in program expenditure is related to how development indicators in a district evolve over time. Finally, $\mathbf{X}_{i,t}$ is a vector of additional controls that may simultaneously affect wages and program provision in a district. These include an indicator for state election-year, the proportion of Scheduled Castes and Scheduled Tribes population in a district, and the average rainfall (in millimeters) that a district receives in a year during the rainy season.²⁶ Since the district sizes vary in our panel, we weight all the regressions by district population. Further, we relax the assumption that within district observations are independent by clustering standard errors at the district level which are robust to heteroskedasticity as well.

Column 1 of [Table 1](#) reports the OLS estimates of [Equation 2](#). Although both $\hat{\beta}_1$ and $\hat{\beta}_2$ have expected signs, both the coefficients are statistically insignificant.²⁷ While the specification in [Equation 2](#) controls for a host of endogeneity concerns, it is still susceptible to district-year fluctuations in program expenditure that can be arguably endogenous. For example, any local shock like natural calamity or conflict may adversely affect the local agricultural labor market and result in higher than expected EGS expenditure due to an increase in demand for public work.²⁸

Following [Bahal \(2016\)](#), we exploit the fund allocation process of the programs to check for potentially endogenous expenditure fluctuations by using fund availability as an instrument for actual expenditure. Due to the fiscal federalism in India, virtually all of the expenditure under such programs is financed by the central government. This

²⁶We use remote sensed rainfall data from the Tropical Rainfall Measuring Mission (TRMM) satellite. See Fetzer (2014) for a detailed discussion on the consistency and the quality of TRMM data over any other remote sensed or ground-based data. We thank Thiemo Fetzer for sharing the rainfall data.

²⁷Note that we do not explicitly control for σ_i in [Equation 2](#) since it is just a linear combination of district fixed effects that are already controlled for.

²⁸Drèze (1990) for example discusses a high take up of public employment by laborers under Maharashtra's EGS during the famine of 1970-73.

entails a certain amount of funds that are made available at the start of the fiscal year for all the districts. However, expenditure during the whole year is not necessarily restricted by the fund availability since the financial accounts roll over to the next year. This allows actual expenditure to exceed fund availability (at district level) in case the demand for work under the program is unusually high. In the event of such over-expenditure, the central government clears the negative opening balance next year by making the appropriate releases to (i) meet previous obligations and (ii) make funds available for the following year.

To measure the degree of under or over utilization of funds, we define utilization ratio as $100 * e_{i,t} / e_{i,t}^a$ where $e_{i,t}^a$ is the availability of funds for district i in year t . [Figure 10](#) plots the year-wise utilization ratio for 134 districts. The observations marked in red show over-utilization while the observations in blue represent under-utilization. If the under/over utilization of funds is related to the demand for EGS work due to private labor market conditions, then this may result in biased OLS estimates of [Equation 2](#). Similarly, error in measuring actual expenditure may attenuate both $\hat{\beta}_1$ and $\hat{\beta}_2$ towards zero. To check for such potentially endogenous district-year fluctuations, we instrument actual expenditure $e_{i,t}$ with fund availability $e_{i,t}^a$. In support of instrument validity, Bahal (2016) shows that while factors that determine fund availability are mostly controlled for in [Equation 2](#), any residual variation in fund availability is largely pre-determined and supply driven. Bahal further shows that fund availability is not sensitive to future rainfall shocks. The flexibility to overspend if the need arises ([Figure 10](#)) may explain the absence of any systematic adjustment in fund availability to anticipated future shocks.

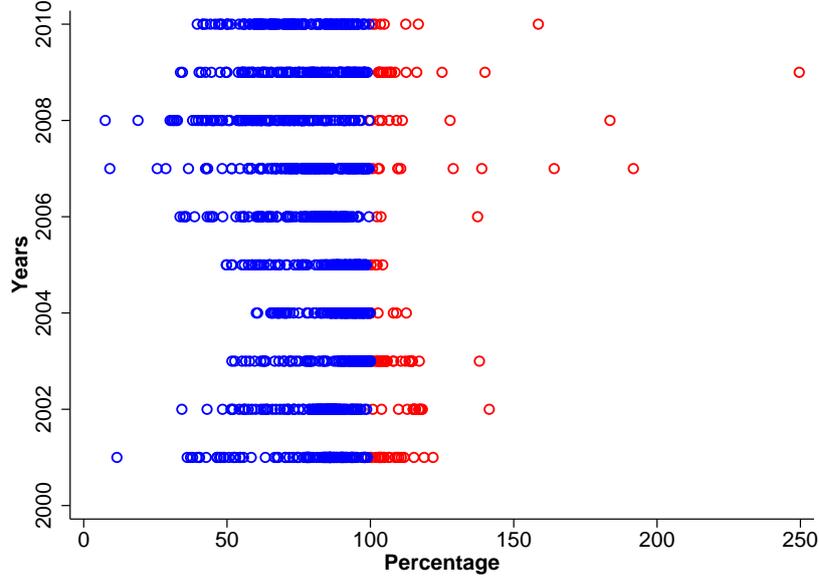
We, therefore, estimate [Equation 2](#) using two-stage least squares where $e_{i,t}$ and $e_{i,t} \times \sigma_i$ are respectively instrumented with $e_{i,t}^a$ and $e_{i,t}^a \times \sigma_i^a$ in the first stage regression along with the rest of the controls. Here σ_i^a is the standard deviation of fund availability during 2001-2010. Column 2 of [Table 1](#) reports the second stage estimates of the 2SLS regression.²⁹ As can be seen, both $\hat{\beta}_1$ and $\hat{\beta}_2$ have expected signs and are statistically different from zero. Comparing $\hat{\beta}_1$ and $\hat{\beta}_2$ in columns 1 and 2 suggests OLS estimates to be biased towards zero.

The coefficient of 1.8×10^{-2} for $e_{i,t}$ suggests a wage increase of 4.8% (of the average wage) computed at mean expenditure \bar{e} and $\sigma_i = 0$.³⁰ This estimated increase in wages closely matches the wage increase reported by ([Berg et al., 2013](#); [Imbert and Papp, 2015](#)). However, σ_i is not equal to zero in reality and instead follows a

²⁹The first stage regression equations (not reported) yield expected estimates where the F-test on instruments exclusion comfortably exceeds the threshold of 20.

³⁰The average per-capita expenditure is 152 rupees while the average real wage is around 57 rupees (in 2001 prices).

Figure 10: Actual Expenditure as a Percent of Fund Availability



Note: The figure shows the year-wise utilization of funds as a percent of funds made available. The observations marked in red show over-utilization while the observations in blue represent under-utilization.

distribution. Hence, we compute the marginal effect of expenditure on wages at the average ($\bar{\sigma}_i \approx 114$ rupees). We find that relative to when $\sigma_i = 0$, the marginal effect of expenditure on wages contracts by approximately a 31% (to 1.24×10^{-2}) at $\bar{\sigma}_i$. This corresponds to a wage increase of 3.3% (of the average wage) at mean expenditure \bar{e} , which is 1.5 percentage points lower when $\sigma_i = 0$.

Up to now we have measured variation in program provision for a district by calculating the standard deviation of its entire expenditure series. In reality, however, the information set of employers in the private labor market is limited to current and past levels of program provision (expenditure) while this information set is continuously updated every period. Consequently, we construct an alternative measure of variability in program provision where $\sigma_{i,t}$ is the standard deviation of program expenditure in district i and year t using expenditure till year t : $\{e_{i,2001}, \dots, e_{i,t}\}$ which is calculated recursively for $t = 2002, \dots, 2010$. To incorporate this alternative measure of program variability, we replace σ_i with $\sigma_{i,t}$ in Equation 2 to estimate Equation 3.³¹

$$w_{i,t} = \beta_1 e_{i,t} + \beta_2 (e_{i,t} \times \sigma_{i,t}) + \beta_3 \sigma_{i,t} + I_{NREGA} + \alpha_i + \gamma_t + \xi_{it} + \beta \mathbf{X}_{i,t} \quad (3)$$

³¹Note that Equation 3 explicitly controls for $\sigma_{i,t}$ since unlike σ_i , it is not a linear combination of any categorical variables.

Columns 3 and 4 of [Table 1](#) report the OLS and 2SLS estimates of [Equation 3](#). While the instrument for actual expenditure in column 4 is same as in column 2, the instrument for $\sigma_{i,t}$: $\sigma_{i,t}^a$ is constructed analogously to $\sigma_{i,t}$ but for fund availability. Similar to the previous set of regressions, we find the OLS estimates to be attenuated towards zero. The coefficient of $e_{i,t} \times \sigma_{i,t}$ in column 4 is significantly less than zero at 90% confidence level and is very comparable in size to the coefficient of interaction in column 2. Consistent with the hypothesis that variability in program provision should have no impact on wages at zero expenditure, the coefficient of $\sigma_{i,t}$ is insignificantly different from zero.³² In column 5 we show that the 2SLS estimation of [Equation 3](#) excluding $\sigma_{i,t}$ makes the interaction term significant at 99% confidence level while keeping the size of the estimate largely the same. The alleviation of multicollinearity may explain the reduction in standard error of the interaction term in column 5. Nevertheless, we continue with column 4 as our preferred specification for [Equation 3](#) which controls for all the base and interacted effects.

The estimates of column 4 suggest that in comparison to the case with $\sigma_{i,t} = 0$, the marginal effect of expenditure on wages is approximately 13 percent lower at average $\sigma_{i,t}$. The lower compression of wage increases in the 2SLS estimation of [Equation 3](#) relative to [Equation 2](#) can be mostly explained by a smaller mean of $\sigma_{i,t} = 55$. Given the scale up of program expenditure due to the implementation of NREGA during the later half of the sample, σ_i may over-estimate wage compression since it computes standard deviation based on the entire expenditure series of a district. In that respect, $\sigma_{i,t}$ may better reflect the extent of compression since it recursively computes the standard deviation of each district for every period. Overall, our results give strong support to the hypothesis that greater variability in program provision results in greater compression of wage increases.

5.2 Compression under Low and High Inflation Districts

Next, we test whether the rate of inflation has any influence on the degree of compression. We calculate average Consumer Price Index-Rural Laborer (CPI-RL) for each district over the 10 years. We then divide the sample of 134 districts between low inflation and high inflation districts based on whether a district is respectively below or above the median value of the average CPI. [Table 2](#) re-estimates regressions of columns 1 – 4 of [Table 1](#) with the interaction terms estimated separately for low and high inflation districts.

³²Since $\frac{\partial w}{\partial \sigma} = \beta_2 e + \beta_3 \Big|_{e=0} \implies \beta_3 = 0$.

Both OLS (column 1) and 2SLS (column 2) estimates of Equation 2 show that the coefficient of interaction is a few times larger in low inflation districts relative to high inflation districts. The 2SLS estimates in column 2 imply a compression of wage increases at approximately 42% and 70% for the high and low inflation sub-samples respectively.³³ However, as explained earlier, the compression of wage increases calculated using σ_i may be an overestimate.

Columns 3 and 4 respectively report the OLS and 2SLS estimates of Equation 3 with the interaction term estimated separately for low and high inflation sub-samples. Similar to the previous set of regressions, the coefficient of interaction is around 3 times larger in low inflation districts relative to high inflation districts. Like before, we calculate the degree of compression by comparing the increase in wages at average expenditure when $\sigma_{i,t} = \bar{\sigma}_{i,t}$ relative to when $\sigma_{i,t} = 0$. The estimates in column 4, our preferred specification, suggest compression of wage increases at approximately 14% and 25% for high and low inflation districts respectively.

Hence, in agreement with studies like [Elsby \(2009\)](#) and [Stüber and Beissinger \(2010\)](#), we find compression of wage increases to be larger when inflation is low. These results further corroborate the central message of the paper, inconsistent and variable EGS can result in compression of wage increases in the presence of downward nominal wage rigidity. Further, the degree of compression is more severe when inflation is low than when inflation is high.

6 Conclusion

In this article, we develop a theoretical model to show that variability in the level of program provision and the ensuing uncertainty compresses wage increases that occur due to employment guarantees. Consistent with our theory, we present empirical evidence to show that greater variability in program provision results in a larger compression of wage increases. Further, we find that compression of wage increases is more severe in districts where inflation is low relative to where inflation is high.

The study has important policy implications. First, we show that incomplete and variable program implementation can diminish the increase in wages which results in an unambiguous welfare loss. This is especially a concern for developing economies which usually lack proper institutions or political will to implement large-scale programs. Second, our findings suggest that simply reducing the variability in program provision, without necessarily increasing the average expenditure, can be welfare en-

³³Calculated by comparing wage increase at average expenditure when $\sigma_i = \bar{\sigma}_i$ relative to when $\sigma_i = 0$. We calculate $\bar{\sigma}_i$ separately for each sub-sample.

hancing. These findings are hence very relevant for program design as well as pre-implementation cost-benefit analysis.

While not a part of this study, a promising area for future research is to understand how employers and workers expectations about employment guarantees affect labor market outcomes. This is important if the continued existence of the workfare alleviates uncertainty regarding the level of program provision in the future. Such a revision in expectations will increase wages simply due to the accompanying decompression of wage increases. This may provide an alternative explanation to the delayed effect of employment guarantees on wages that has been noted in the recent empirical literature on employment guarantees in India.³⁴

³⁴See for example, [Berg et al. \(2013\)](#), [Bahal \(2016\)](#), and [Shrivastava \(2015\)](#) who find wages to increase due to the overall exposure to the workfare as opposed to an immediate increase in wages. [Bahal \(2016\)](#) suggests the build up of productive capital under public workfare as a possible explanation for the observed “stock effect”.

Tables

Table 1: Effect of Variability in Expenditure on Wages

	(1)	(2)	(3)	(4)	(5)
	OLS	2SLS	OLS	2SLS	2SLS
$e_{i,t}$	0.553 [0.688]	1.811** [0.860]	1.273* [0.683]	1.981*** [0.638]	1.843*** [0.680]
$e_{i,t} \times \sigma_i$	-0.001 [0.002]	-0.005** [0.003]			
$\sigma_{i,t}$			-0.004 [0.023]	-0.008 [0.031]	
$e_{i,t} \times \sigma_{i,t}$			-0.004 [0.003]	-0.005* [0.003]	-0.006*** [0.002]
District Effects	Yes	Yes	Yes	Yes	Yes
Year Effects	Yes	Yes	Yes	Yes	Yes
Trend Effects	Yes	Yes	Yes	Yes	Yes
Other Controls	Yes	Yes	Yes	Yes	Yes
Observations	1340	1340	1206	1206	1206
Ftest > 20		Yes		Yes	Yes

All estimates in the table are of the order of magnitude -2. The unit of observation is a district-year. Data is for 134 districts from 2001-2010. The dependent variable in all the regressions is real agricultural wage. $e_{i,t}$ is real per-capita expenditure. For a district, σ_i is the standard deviation of expenditure incurred over 10 years $\{e_{i,2001}, \dots, e_{i,2010}\}$. While $\sigma_{i,t}$ is the standard deviation calculated till year t $\{e_{i,2001}, \dots, e_{i,t}\}$ for $t = 2002, \dots, 2010$. In columns 2 and 4 we use real per-capita fund availability $e_{i,t}^a$ as instrument for $e_{i,t}$. σ_i in column 2 and $\sigma_{i,t}$ in column 4 is respectively instrumented with σ_i^a and $\sigma_{i,t}^a$. Here σ_i^a and $\sigma_{i,t}^a$ are standard deviation of funds made available calculated analogous to σ_i and $\sigma_{i,t}$ respectively. Standard errors clustered at district level are reported in parentheses.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Compression under High and Low Inflation

	(1)	(2)	(3)	(4)
	OLS	2SLS	OLS	2SLS
$e_{i,t}$	1.554 [1.017]	3.291** [1.285]	1.830** [0.763]	2.576*** [0.688]
<i>Low Inflation</i> $\overbrace{e_{i,t} \times \sigma_i}$	-0.017* [0.009]	-0.029** [0.012]		
<i>High Inflation</i> $\overbrace{e_{i,t} \times \sigma_i}$	-0.004 [0.003]	-0.010*** [0.004]		
$\sigma_{i,t}$			-0.003 [0.022]	-0.012 [0.030]
<i>Low Inflation</i> $\overbrace{e_{i,t} \times \sigma_{i,t}}$			-0.015** [0.006]	-0.017*** [0.006]
<i>High Inflation</i> $\overbrace{e_{i,t} \times \sigma_{i,t}}$			-0.005* [0.003]	-0.006* [0.003]
District Effects	Yes	Yes	Yes	Yes
Year Effects	Yes	Yes	Yes	Yes
Trend Effects	Yes	Yes	Yes	Yes
Other Controls	Yes	Yes	Yes	Yes
Observations	1340	1340	1340	1340
Ftest > 20		Yes		Yes

All estimates in the table are of the order of magnitude -2. The unit of observation is a district-year. Data is for 134 districts from 2001-2010. The dependent variable in all the regressions is real agricultural wage. The table re-estimates Table 1 regressions where the interactions of σ_i and $\sigma_{i,t}$ with $e_{i,t}$ are now estimated separately for high and low inflation districts. See Table 1 for variable definitions. 2SLS regressions involve the use of same instruments as in Table 1. Standard errors clustered at district level are reported in parentheses.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

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Appendix

Proof of Proposition 1

The employer's problem is

$$\begin{aligned} \max_{w_t, l_t, w_{t+1}, l_{t+1}} \quad & a(l_t - \frac{l_t^2}{2}) - w_t l_t + a(l_{t+1} - \frac{l_{t+1}^2}{2}) - w_{t+1} l_{t+1} \\ \text{s.t.} \quad & w_t \leq w_{t+1} \end{aligned}$$

r_{t+1} can be in one of three ranges.

$$1. r_{t+1} \leq r_t$$

The wages in the two periods will be given by the respective statically optimal values as the constraint does not bind.

$$w_z^* = \frac{aC}{ar_z + 2C} \quad z \in \{t, t+1\}$$

$$2. r_t < r_{t+1} \leq \bar{r}_{t+1}$$

Here the constraint will bind. So, using $w_t = \frac{Cl_t}{r_t}$ and $w_{t+1} = \frac{Cl_{t+1}}{r_{t+1}}$, we can write the employer's problem as

$$\max_{l_t, l_{t+1}} \quad a(l_t - \frac{l_t^2}{2}) - \frac{Cl_t^2}{r_t} + a(l_{t+1} - \frac{l_{t+1}^2}{2}) - \frac{Cl_{t+1}^2}{r_{t+1}} \quad \text{s.t.} \quad \frac{Cl_t}{r_t} \leq \frac{Cl_{t+1}}{r_{t+1}}$$

The Lagrangian is given by

$$\mathcal{L} = a(l_t - \frac{l_t^2}{2}) - \frac{Cl_t^2}{r_t} + a(l_{t+1} - \frac{l_{t+1}^2}{2}) - \frac{Cl_{t+1}^2}{r_{t+1}} + \lambda (\frac{Cl_{t+1}}{r_{t+1}} - \frac{Cl_t}{r_t})$$

F.O.C.

$$\frac{\partial \mathcal{L}}{\partial l_t} = a(1 - l_t) - \frac{2Cl_t}{r_t} - \lambda \frac{C}{r_t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial l_{t+1}} = a(1 - l_{t+1}) - 2\frac{Cl_{t+1}}{r_{t+1}} + \lambda \frac{C}{r_{t+1}} = 0$$

Complimentary slackness

$$\lambda \frac{\partial \mathcal{L}}{\partial \lambda} = \lambda (\frac{Cl_{t+1}}{r_{t+1}} - \frac{Cl_t}{r_t}) = 0$$

i.e. either $\lambda = 0$ and $\frac{Cl_{t+1}}{r_{t+1}} - \frac{Cl_t}{r_t} > 0$, or $\lambda > 0$ and $\frac{Cl_{t+1}}{r_{t+1}} - \frac{Cl_t}{r_t} = 0$

When the constraint binds, i.e. $r_{t+1} > r_t$, then the second condition will hold.

From the two FOCs

$$ar_t(1 - l_t) - 2Cl_t + ar_{t+1}(1 - l_{t+1}) - 2Cl_{t+1} = 0 \quad \dots (1)$$

Using complimentary slackness,

$$\frac{l_{t+1}}{r_{t+1}} = \frac{l_t}{r_t}$$

Replacing l_{t+1} in (1) with the above expression, we get

$$l_t^* = \frac{ar_t}{a \frac{r_{t+1}^2 + r_t^2}{r_{t+1} + r_t} + 2C}$$

$$w_t^* = \frac{aC}{a \frac{r_{t+1}^2 + r_t^2}{r_{t+1} + r_t} + 2C} = w_{t+1}^*$$

3. $\bar{r}_{t+1} \leq r_{t+1}$

Here the constraint will bind and with the wage in period $t + 1$ fixed at its minimum value, an increase in the rationing rate in period $t + 1$ will not increase private employment any further as it has already reached its optimum value. We can obtain this by maximizing the profit in period $t + 1$ for a fixed wage.

$$\max_{l_{t+1}} a(l_{t+1} - \frac{l_{t+1}^2}{2}) - \bar{w}_t l_{t+1}$$

F.O.C.

$$\bar{l}_{t+1} = \frac{a - \bar{w}_t}{a}$$

At the lower bound of the range $[\bar{r}_{t+1}, 1]$, all the labor supply available for private employment is absorbed i.e. $\bar{l}_{t+1} = \frac{\bar{w}_t \bar{r}_{t+1}}{C}$.

Also, the wage is still the optimal wage as derived in the Range 2.

$$\bar{w}_t = \frac{aC}{a \frac{\bar{r}_{t+1}^2 + r_t^2}{\bar{r}_{t+1} + r_t} + 2C}$$

Using the expressions for \bar{l}_{t+1} and \bar{w}_t in the F.O.C., we obtain the expression for \bar{r}_{t+1}

$$\bar{r}_{t+1} = \frac{r_t(ar_t + C)}{ar_t - C}$$

Relaxing the Assumption that Selection into EGS is Independent of Cost of Working

Let workers with cost of working c lower than a threshold value of \bar{c} be selected into EGS for sure. For workers with $c > \bar{c}$, let a fraction $(1 - r)$ be selected into EGS and the rest be rationed out. Note that the overall rationing rate is not r but $\frac{r(w_g - \bar{c})}{w_g}$.

The labor supply is given by

$$l = \begin{cases} 0 & w < \bar{c} \\ \frac{(w - \bar{c})r}{C} & \bar{c} \leq w \leq w_g \\ \frac{w}{C} & w > w_g \end{cases}$$

In the static case, profit maximization by the employer gives us the optimal private wage w_1^* and private employment l_1^* .

$$w_1^* = \frac{(a - \bar{c})C}{ar + 2C} + \bar{c}$$

$$l_1^* = \frac{(a - \bar{c})r}{ar + 2C}$$

In the two-period case with the corresponding values of r being r_t and r_{t+1} , we get

$$w_t^* = \frac{(a - \bar{c})C}{a\phi(r_t, r_{t+1}) + 2C} + \bar{c}$$

$$l_t^* = \frac{(a - \bar{c})r_t}{a\phi(r_t, r_{t+1}) + 2C}$$

We can see that as $\phi(r_t, r_{t+1}) > r_t$, $w_t^* < w_1^*$ and we get the same result as before.

Oligopsony

Now we show the same results for oligopsony.

Employers

There are N identical employers. The profit for employer i is given by

$$\pi_i = a(l_i - l_i^2/2) - wl_i = a(l_i - l_i^2/2) - C(l_i + l_{-i})l_i$$

F.O.C.

$$\frac{\partial \pi_i}{\partial l_i} = 0$$

$$\implies a(1 - l_i) - 2Cl_i - Cl_{-i} = 0$$

For symmetric solution, $l_{-i} = (N - 1)l_i$

$$\implies a(1 - l_i) - 2Cl_i - C(N - 1)l_i = 0$$

$$\implies l_i = \frac{a}{a + C + CN}$$

Therefore,

$$l_{N0}^* = Nl_i = \frac{aN}{a + C + CN}$$

$$w_{N0}^* = \frac{aCN}{a + C + CN}$$

EGS

The inverse labor supply with rationing rate r is

$$w(l) = \frac{Cl}{r}$$

We just replace C with C/r in the no EGS case to get the results for EGS

$$l_{N1}^* = \frac{arN}{ar + C + CN}$$

$$w_{N1}^* = \frac{aCN}{ar + C + CN}$$

Dynamic Model

We will show the analysis only for Range 2 here as the rest is analogous to the monopsony case. The employer i 's problem is

$$\max_{l_{it}, l_{it+1}} a(l_{it} - \frac{l_{it}^2}{2}) - \frac{Cl_{it}(l_{it} + l_{-it})}{r_t} + a(l_{it+1} - \frac{l_{it+1}^2}{2}) - \frac{Cl_{it+1}(l_{it+1} + l_{-it+1})}{r_{t+1}}$$

s.t. $\frac{Cl_t}{r_t} \leq \frac{Cl_{t+1}}{r_{t+1}}$

The Lagrangian is given by

$$\mathcal{L} = a(l_{it} - \frac{l_{it}^2}{2}) - \frac{Cl_{it}(l_{it} + l_{-it})}{r_t} + a(l_{it+1} - \frac{l_{it+1}^2}{2}) - \frac{Cl_{it+1}(l_{it+1} + l_{-it+1})}{r_{t+1}}$$

$$+ \lambda(\frac{Cl_{t+1}}{r_{t+1}} - \frac{Cl_t}{r_t})$$

F.O.C.

$$\frac{\partial \mathcal{L}}{\partial l_{it}} = a(1 - l_{it}) - \frac{2Cl_{it} + Cl_{-it}}{r_t} - \lambda \frac{C}{r_t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial l_{it+1}} = a(1 - l_{it+1}) - \frac{2Cl_{it+1} + Cl_{-it+1}}{r_{t+1}} + \lambda \frac{C}{r_{t+1}} = 0$$

For symmetric solution, $l_{-i} = (N - 1)l_i$

$$a(1 - l_{it}) - \frac{(C + CN)l_{it}}{r_t} - \lambda \frac{C}{r_t} = 0$$

$$a(1 - l_{it+1}) - \frac{(C + CN)l_{it+1}}{r_{t+1}} + \lambda \frac{C}{r_{t+1}} = 0$$

From the two FOCs

$$ar_t(1 - l_{it}) - (C + CN)l_{it} + ar_{t+1}(1 - l_{it+1}) - (C + CN)l_{it+1} = 0 \quad \dots (1)$$

Using the complimentary slackness,

$$\frac{l_{t+1}}{r_{t+1}} = \frac{l_t}{r_t}$$

Replacing l_{t+1} in (1) with the above expression, we get

$$l_t^* = \frac{arN}{a \frac{r_t^2 + r_{t+1}^2}{r_t + r_{t+1}} + C + CN}$$

$$w_t^* = \frac{aCN}{a \frac{r_t^2 + r_{t+1}^2}{r_t + r_{t+1}} + C + CN}$$

$$\therefore w_{N1}^* > w_t^* > w_{N0}^*$$

Proof of Lemma 1

When the rationing rate is fixed, $r_t = r_{t+1} = r$, then optimal wages are given by $w_t^* = w_{t+1}^* = \frac{aC}{ar+2C} = w$. Now, let there be some probability of the rationing rate in the second period being higher or lower than r , while still keeping the expected rationing rate at r .

$$P[r_{t+1} = r+x] = P[r_{t+1} = r-x] = p$$

Let w_{t+1} in the cases where r_{t+1} is $r-x$, r and $r+x$ be denoted by w'_{t+1} , w''_{t+1} and w'''_{t+1} respectively.

The profit function in terms of wage and rationing rate is given by

$$\pi(w, r) = a\left(\frac{wr}{C} - \frac{w^2r^2}{2C^2}\right) - \frac{w^2r}{C}$$

The sum of current and future expected profit is given by

$$\Pi = \pi(w_t, r) + p\pi(w'_{t+1}, r-x) + (1-2p)\pi(w''_{t+1}, r) + p\pi(w'''_{t+1}, r+x)$$

Now let the wage in the first period be fixed at the statically optimum level w_1^* . Now, w'_{t+1} is independent of w_t since the rationing rate is lower. $w''_{t+1} = w_t$, since the rationing rate is the same. Also, $w'''_{t+1} = w_t$, since the rigidity will bind and the wage will not be lowered beyond w . Hence,

$$\Pi|_{w_t=w_1^*} = \pi(w_t, r) + p\pi(w'_{t+1}, r-x) + (1-2p)\pi(w_t, r) + p\pi(w_t, r+x)$$

Now we differentiate Π with respect to w_t . Since w_1^* is the statically optimal wage at rationing rate r , therefore, $\frac{\partial \pi(w_t, r)}{\partial w_t} \Big|_{w_t=w_1^*} = 0$.

$$\begin{aligned} \frac{\partial \Pi}{\partial w_t} \Big|_{w_t=w_1^*} &= p \frac{\partial \pi(w_t, r+x)}{\partial w_t} \Big|_{w_t=w_1^*} \\ &= \frac{p(r+x)}{C^2} (a(C - w_1^*(r+x)) - 2w_1^*C) \end{aligned}$$

Using $w_1^* = \frac{aC}{ar+2C}$, we get

$$\frac{\partial \Pi}{\partial w_t} \Big|_{w_t=w_1^*} = -\frac{pa^2x(r+x)}{C(ar+2C)} < 0 \quad \forall p, x > 0$$

Hence, the dynamically optimal wage will be lower than the statically optimal wage. QED.

Proof of Proposition 2

The sum of current profit and future expected profit when $r_t = m$ is given by

$$\begin{aligned} \Pi(w_t, r_t, x) = & \pi(w_t, l(w_t, r_t)) + \frac{1}{2x} \left(\int_{r_t-x}^{r'(w_t)} \pi(w_{t+1}^*, l(w_{t+1}^*, r_1)) dr_1 \right. \\ & \left. + \int_{r'(w_t)}^{r''(w_t)} \pi(w_t, l(w_t, r_2)) dr_2 + \int_{r''(w_t)}^{r_t+x} \pi(w_t, l_{\max}(w_t)) dr_3 \right) \end{aligned}$$

At optimal wage w_t^* , the derivative of Π with respect to w_t is zero. Ignoring r_t for now, we write the optimal wage as a function of x , which is proportional to the standard deviation.

$$\Pi^{w_t}(w_t^*(x), x) = 0$$

Using the implicit function theorem, we obtain the derivative of the optimal wage with respect to x in terms of the double differential and the cross differential of Π

$$\frac{dw_t^*}{dx} = - \frac{\Pi^{w_t x}}{\Pi^{w_t w_t}}$$

We know that $\Pi^{w_t w_t}$ will be negative at the profit maximizing value w_t^* . Hence to show that $\frac{dw_t^*}{dx} < 0$, we need to show that $\Pi^{w_t x} < 0$.

We obtain the expression for $\Pi^{w_t x}$ as

$$\Pi^{w_t x} = \frac{15aw_t C - 10w_t^2 C - 6a^2 C + 6aw_t r_t (a - w_t)}{12a^2 x^2 w_t}$$

As the denominator is always positive, we only consider the numerator. Using *Lemma 1*, we know that for any level of uncertainty, $w_t^* < w_1^* = \frac{aC}{ar_t + 2C}$. This implies, $r_t < \frac{aC - 2w_t C}{aw_t}$. Since $w_t < a$, the multiplier of r_t is positive in the expression and we can replace r_t with $\frac{aC - 2w_t C}{aw_t}$ to get an expression which is always larger than the numerator of $\Pi^{w_t x}$. Again using $w_t < a$, we show that this expression is negative, implying that the numerator is also negative.

$$\text{Numerator}(\Pi^{w_t x}) < w_t C (2w_t - 3a) < 0$$

QED.